Test bodies and naked singularities: is the self-force the cosmic censor?

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Outline

- Naked singularities and the Cosmic Censorship Conjecture (CCC)
- Creating naked singularities by shooting test-bodies into a BH: is the CCC violated? (Jacobson & Sotiriou 2009)
- Is the JS process still valid beyond the test-body approximation?
  - Part 1: GW fluxes (radiation reaction, aka dissipative self-force)
  - Part 2: conservative self-force

What is a curvature singularity?

- Curvature invariants diverge (GR loses predictive power)

- Near singularities quantum effects must be important

  Same as in QED: if $E^2 - B^2$ is large, Schwinger pair production, but the curvature invariant $R_{\alpha\beta\mu\nu} R^{\alpha\beta\mu\nu}$ is the analog of $E^2 - B^2$

- Near singularities there may be closed timelike curves (time machines)

  but singularities are cloaked by an event horizon in BH spacetimes

What if the singularity is not cloaked by event horizon?

- ”Naked” singularity
- Unpleasant properties (breakdown of GR eqs, quantum effects, time machines) exposed to outside observers
- Kerr with a>1 contains naked singularity, but is classically unstable

  (Dotti, Gleiser, Ranea-Sandoval, Vucetich 2008; Cardoso, Pani, Cadoni, Cavaglia 2008, Pani, EB, Berti, Cardoso 2010)

Irrespective of stability, can naked sings even be formed under reasonable initial conditions?

Cosmic Censorship Conjecture (Penrose 1969)

- Postulates classical GR eqs in 4D contain mechanism preventing naked singularities from forming under regular initial conditions

- Counterexamples involve unphysical eqs of state (eg pressureless matter) or higher dimensional spacetimes (Lehner & Pretorius 2010)

Can we form naked singularities by shooting at BHs?

- Conceivably possible because bullets carry angular momentum which can spin BH up to a>1, but...
- .... naked sings do not form in relativistic collisions of comparable mass BHs (Sperhake et al 2009, Shibata et al 2008)
- … and if you shoot test particles into a BH with a=1, you end up with a=1 (Wald 1974)
- But if we shoot test particles into *almost* extremal BH, we can spin it up to a>1 (Jacobson & Sotiriou 2009)

On what orbit do we shoot?

- $a = 1 - 2\epsilon^2$
- Bullet cannot have too much ang mom otherwise it is just scattered: $L < L_{max}$
- Final spin of the BH needs to be >1:
  $a_{fin} = (a + L)/(1 + E)^2 > 1$ → $L > L_{min}$
- $L_{min} < L_{max}$ for orbit to exist
- Orbit needs to go from spatial infinity to horizon (if not, body created at finite radius → need to check if size << distance to horizon and if destroyed by tidal forces)

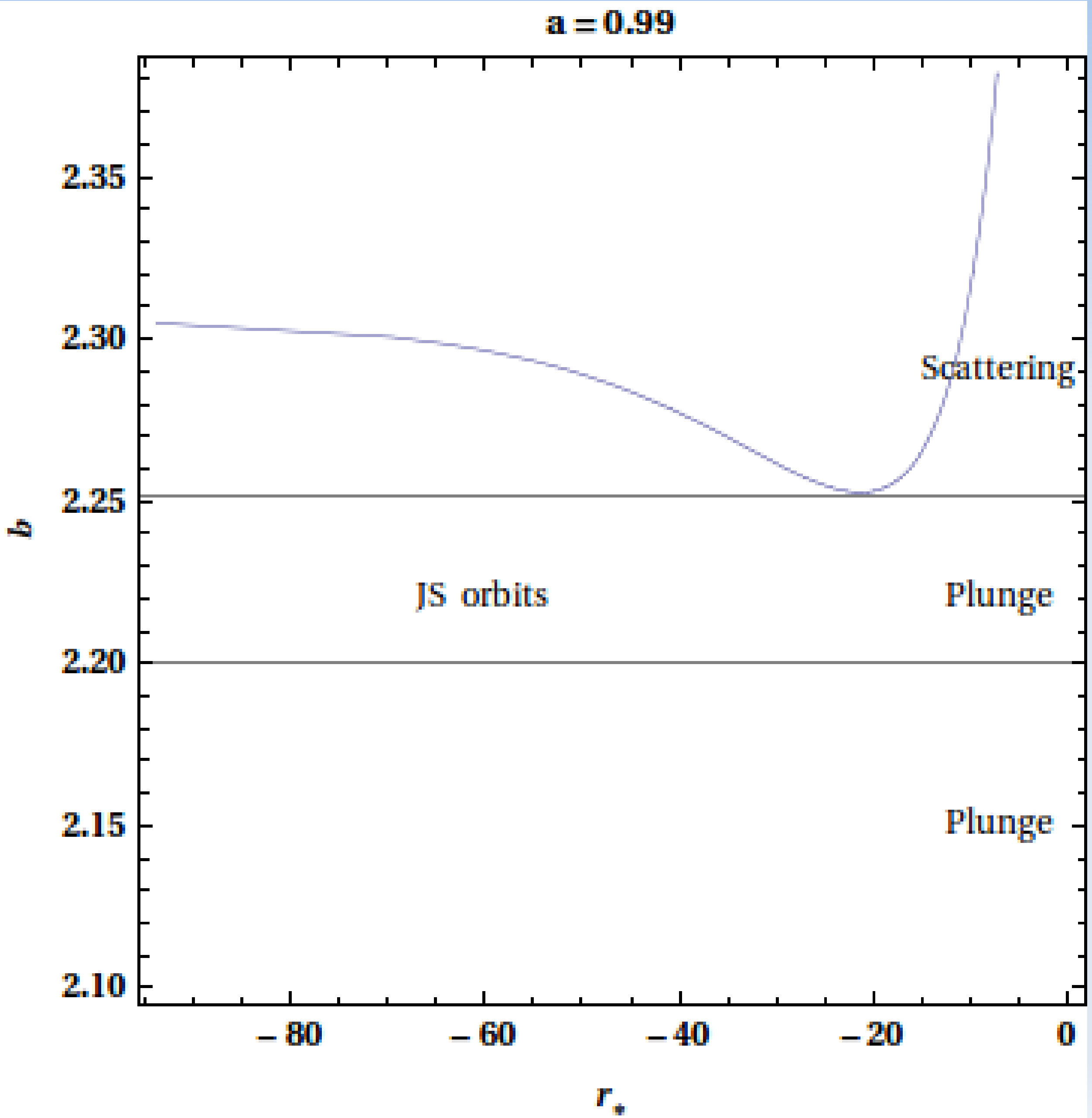
→ E/m, L/m >>1 (almost a photon)

On what orbit do we shoot the particle?

- Combining all constraints, allowed range is
  $b = L/E = 2 + \delta\epsilon, \;\; 2\sqrt{2} < \delta < 2\sqrt{3}$
  $(2 - 2\sqrt{2})\epsilon < E < (2 + 2\sqrt{2})\epsilon$

- $b_{ph} = 2 + 2\sqrt{3}\epsilon$ is the impact parameter of the circular photon orbit (light ring) →
  if $b \sim 2 + 2\sqrt{3}\epsilon$ particle orbits the LR many times, and emission of GWs (radiation reaction) must be important

The effective potential for a photon in Kerr



Effect of radiation reaction

$$a_{fin} = 1 + 8\epsilon^2 (1-x) x y + 2 E_{rad} - L_{rad} + O(\epsilon)^3$$

$$E = E_{min} + x(E_{max} - E_{min}), L = L_{min} + y(L_{max} - L_{min})$$

Radiation reaction might prevent overspinning!

How do the fluxes scale?

- E_{rad}, L_{rad} propotional to N_{cycles} at LR:

  $E_{rad} = N_{cycles}\ \Delta E, \quad L_{rad} = N_{cycles}\ \Delta L$

- Using geodesics eqs

$$N_{cycles} \approx [A + B \log(k\epsilon)]\left(\frac{8}{3} + \frac{\sqrt{3}}{2\epsilon}\right), \quad b = b_{ph}(1-k),\ k \ll \epsilon$$

- From FD analysis $\Delta E / \Delta L \approx \Omega_{ph} \approx 1/2 - (\sqrt{3}/2)\epsilon$

$$E_{rad} = N_{cycles}\,\Delta E = N_{cycles} E_1 (1 + e_2 \epsilon)$$

$$L_{rad} = N_{cycles}\,\Delta L = 2 N_{cycles} E_1 [1 + (e_2 + \sqrt{3})\epsilon]$$

  where E_1 is flux in one orbit at the LR at leading order in ε

How does E_1 scale?

- Normally scale with body's mass $E_1 \sim m E^2$
- But here we have a relativistic, so $m \to E$

  $E_1 \sim E^3 \sim \varepsilon^3$ because $E \sim \varepsilon$
- Using $N_{cycles} \approx [A + B \log(k\epsilon)]\left(\frac{8}{3} + \frac{\sqrt{3}}{2\epsilon}\right) \sim \frac{\log(k\epsilon)}{\epsilon}$

$$L_{rad} = N_{cycles} \Delta L = 2 N_{cycles} E_1 [1 + (e_2 + \sqrt{3})\epsilon] \sim \epsilon^2 \log\epsilon$$

$$E_{rad} = N_{cycles} \Delta E = N_{cycles} E_1 (1 + e_2 \epsilon) \sim \epsilon^2 \log\epsilon$$

$$a_{fin} = 1 + 8\epsilon^2 (1-x) x y + 2 E_{rad} - L_{rad}$$

$$= 1 + 8\epsilon^2 (1-x) x y - 2\sqrt{3}\,\epsilon N_{cycles} E_1$$

$$O(\epsilon^3 \log k\epsilon)$$

Do the fluxes spin the BH up or down?

$$a_{fin} = 1 + 8\epsilon^2 (1 - x) x y - 2\sqrt{3}\epsilon N_{cycles} E_1$$

- $E_1 > 0$ because $\Omega_{ph} > \Omega_{hor}$ (i.e. no superradiance)

  → spin-*down*

- Subtlety: analysis valid both both ingoing and outgoing fluxes. But fluxes down the horizon might spin BH *up* before body is captured

$$a_{before\,capture} = 1 - 2\epsilon^2 + 2\sqrt{3}\epsilon N_{cycles} E_1$$

Do GW fluxes affect JS's analysis?

$$a_{fin} = 1 + 8\epsilon^2 (1-x)\, x\, y - O(\epsilon^3 \log k\epsilon)$$

- If k < exp(-1/ε), no naked sings form by particle capture, but might be formed by ingoing fluxes
- If k > exp(-1/ε), and fluxes cannot prevent formation of naked singularities

For fixed k, fluxes unimportant for a ~ 1

How do we test this picture?

- Calculate GW fluxes for JS orbits numerically

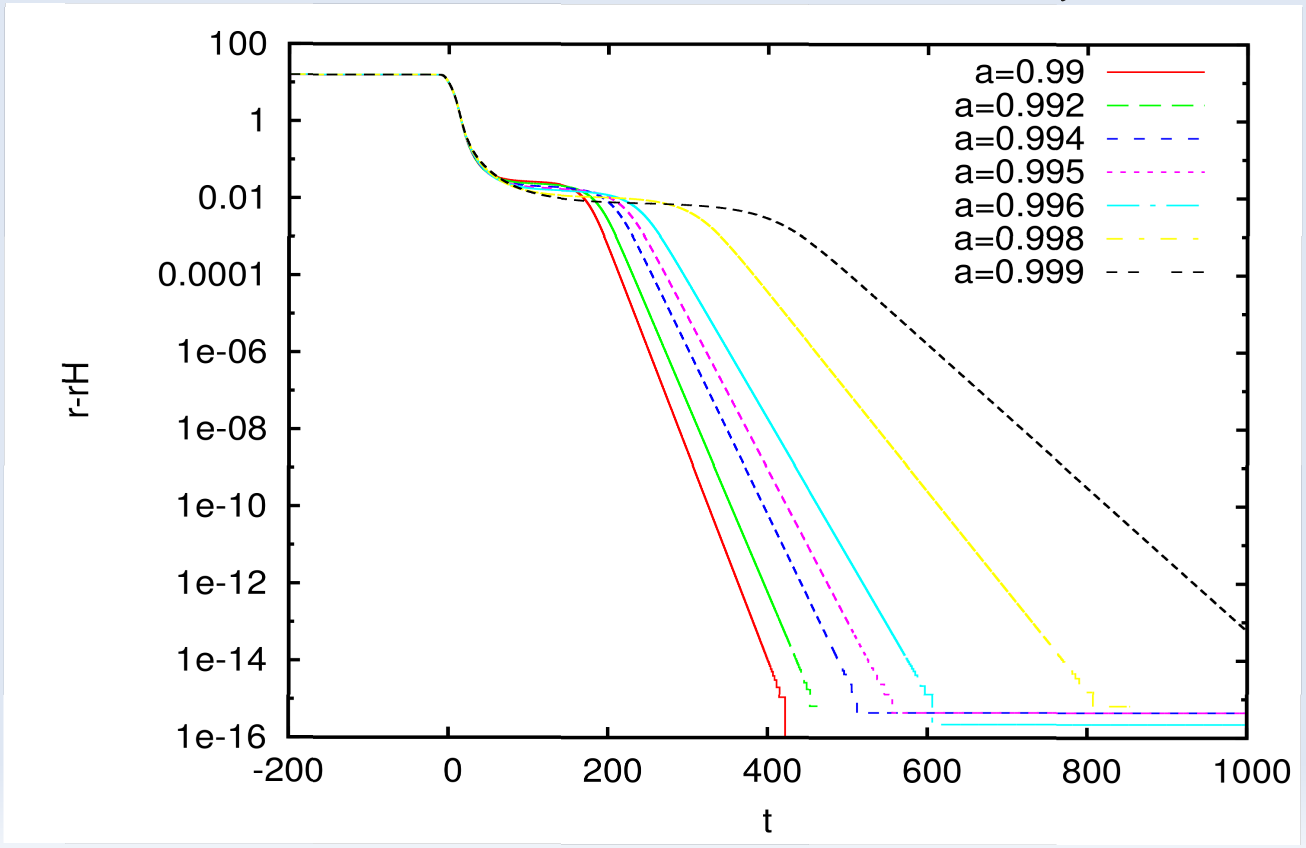
- Time domain code solving Teukolsky eqs describing GW perturbations for extreme mass-ratio binaries

$$\nabla_{\mu} \nabla^{\mu} h_{\alpha \beta} = 16 \pi T_{\alpha \beta}$$

Code tested in previous publications (Burko & Khanna 2007, Sundararajan, Khanna & Hughes 2007, 2008, 2010), but calculation of JS fluxes challenging

Source of Teuk eqs $\nabla_\mu \nabla^\mu h_{\alpha\beta} = 16\pi T_{\alpha\beta}$

- JS geodesics around BHs with a = 0.99, 0.992, 0.994, 0.996, 0.998, 0.999, 0.9998
- $E=(E_{max}+E_{min})/2=2\varepsilon$, $L=b_{ph}E(1-k)$ with k=1.e-5, and m=1.e-5<< E
- Extract A, B appearing in $N_{cycles} \approx [A + B\log(k\epsilon)]\left(\frac{8}{3}+\frac{\sqrt{3}}{2\epsilon}\right)$



Numerical challenges

- Relativistic plunging orbits: little time to dissipate junk radiation → need to create particle gradually and add artificial cycles
- Almost extremal BHs: junk is long-lived, LR and horizon freqs are very close (need accuracy to avoid spurious super-radiance effects) → use tortoise coords, check convergence (with particle's size, grid size and extraction radius)
- All multipole moments important. Higher moments damped by finite grid resolution, but can be recostructed because they are in geometric progression (Finn & Thorne 2000)

Numerical fluxes E_{rad} and L_{rad}

- Converge with extraction radius, grid resolution and particle's size
- Check high multipole moments are in geometric progression (Finn & Thorne 2000)
- $E_{rad}/L_{rad} \approx \Omega_{ph} \approx 1/2-(\sqrt{3}/2)\epsilon$ to within 1%
- Fit with $L_{rad}=E_1[1+(e_2+\sqrt{3})\epsilon]$,

  $$E_{rad}=N_{cycles}E_1(1+e_2\epsilon),\quad E_1=C\epsilon^n$$

  gives n=2.91
- Data fit with n=3 to within 2-5% (~ numerical errors due to extrapolation to high multipoles)

Numerical fluxes E_{rad} and L_{rad}

Fluxes alone cannot prevent formation of naked singularities when a~1

a	0.99	0.992	0.994	0.996	0.998	0.999	0.9998
a_{fin}^{JS}	1.0043	1.0035	1.0026	1.0018	1.0009	1.00045	1.00009
a_{fin}	0.882	0.928	0.961	0.984	0.997	0.9996	1.00006

The gravitational self-force

Motion of small BH with mass m in a curved spacetime with curvature radius L

- Near BH, $g=g_{BH}+O(r/L)+O(r/L)^2$
- Far away, $g=g_{bkgd}+O(R_g/L)+O(R_g/L)^2$, $R_g=2\ G\ m/c^2$
- Matching in a buffer region where both pictures are valid, one finds the BH's eqs of motion

$$u^\mu \nabla_\mu u^\nu = f^\nu_{cons} + f^\nu_{diss}$$

$f^\nu_{cons}, f^\nu_{diss} = O(R_g/L)$ are the SF

Derived for BH, but result valid also for classical "particle" (any body with size $R_g \ll L$)

Physical meaning of the SF

- Can be written in terms of derivatives of h^{reg} (perturbation produced by particle, but regularized to avoid divergence at particle's position)

  ⇨ SF = interaction of particle with itself

- $$u^\mu \nabla_\mu u^\nu = f^\nu_{cons} + f^\nu_{diss}, \qquad f^\nu_{cons}, f^\nu_{diss} = O(R_g/L)$$

  ⇕

  $$\tilde{u}^\mu \tilde{\nabla}_\mu \tilde{u}^\nu = 0, \quad \tilde{g} = g + h^{reg}, \qquad h^{reg} = O(R_g/L)$$

  Particle moves on geodesic of ”perturbed” metric

Effect of the SF

- Dissipative SF = radiation reaction

$$u^\mu \nabla_\mu u^\nu = f^\nu_{cons} + f^\nu_{diss}, \quad E = -m u_t \quad \Rightarrow$$

$$dE/d\tau = -m f_t^{diss} = O(R_g/L)^2$$

- From $\tilde{u}^\mu \tilde{\nabla}_\mu \tilde{u}^\nu = 0, \; h^{reg} = O(R_g/L)$ the conservative self force changes effective potential by $O(R_g/L)$

$$\Rightarrow \quad \Delta\Omega_{ISCO}, \Delta\Omega_{ph}, \Delta b_{ph} \sim O(R_g/L)$$

- For a non-relativistic particle $R_g \sim G m/c^2$

What if the particle is relativistic?

- Expect $R_g \sim E$ because in GR energy gravitates

  e.g. BH boosted to relativistic energy E (Aichelburg-Sexl metric) has "size" ~ 2 G E/c^2

- Energy flux for JS orbits:

  $$dE/dt = -m f_t^{diss} d\tau/dt \sim O(R_g/L)^2 \epsilon$$

  because $dt/d\tau \sim 1/(r - r_H) \sim 1/\epsilon$

- TD code gives $E_1 \sim \varepsilon^3$

- Numerical results confirm that $R_g \sim E \sim \varepsilon$ for a relativistic particle

Use R_g ~ E ~ ε to calculate conservative self-force

- For relativistic orbits we expect $\Delta\Omega_{ph}, \Delta b_{ph} \sim O(R_g/L) \sim O(\epsilon)$ but in what direction are the changes?

- Barack & Sago (2009): for non-relativistic orbits in Schwarzschild $\Delta\Omega_{ISCO} > 0$ →

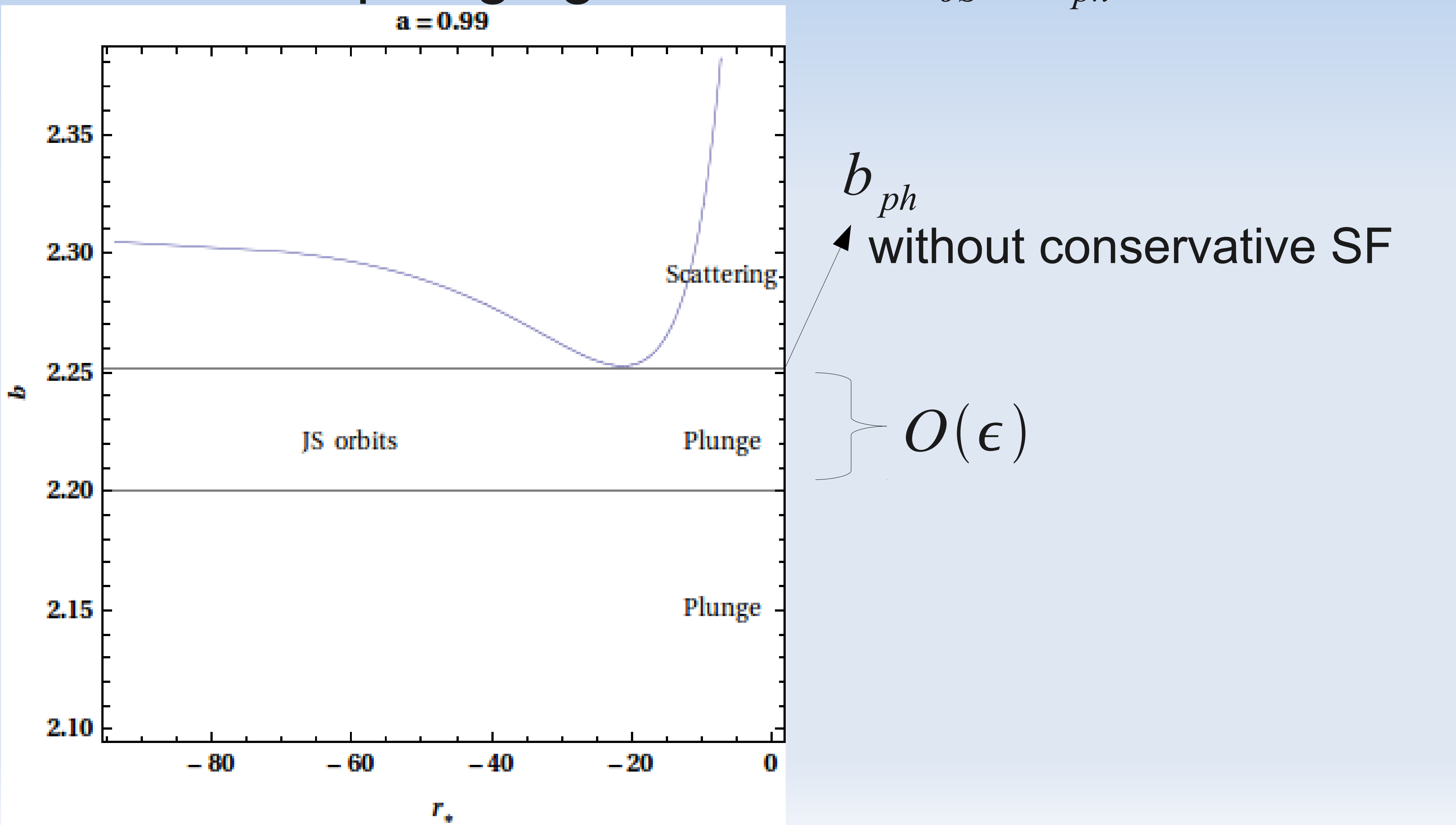
  We might expect $\Delta\Omega_{ph} > 0$ for relativistic orbits in Kerr

- For nearly extremal BHs, $b_{ph} \approx 1/\Omega_{ph}$ →

  $\Delta\Omega_{ph} > 0$ would imply $\Delta b_{ph} < 0$

$\Delta b_{ph} < 0$: BH shrinks and dodges bullet!

- $\Delta b_{ph} \sim O(\epsilon)$ may be enough to prevent JS orbits from plunging, because $b_{JS} = b_{ph} - O(\epsilon)$



Conservative self-force (1)

- Has right magnitude and sign to prevent JS orbits from falling into BH by raising potential barrier
- Effective radial potential for radial motion ~ effective potential for gravitational waves

  (eikonal approximation, but valid also for $l \sim 2$)

  ⇒ conservative SF has right magnitude and sign to prevent ingoing fluxes from forming naked sings

Conservative self-force (2)

JS also proposed creating naked singularity by dropping spinning particle with L=0 and

$$(2-2\sqrt{2})\epsilon < E < (2+2\sqrt{2})\epsilon$$

$$S/E = 2+\delta\epsilon\,,\quad 2\sqrt{2} < \delta < 2\sqrt{3}$$

from very close to BH horizon

Conservative SF changes background metric by O(ε): important also in spinning case but sign unclear

Conclusions

- Radiation reaction prevents formation of naked sings in some cases, but less and less effective when a~1

- BH cross section decreases due to conservative SF: BH shrinks and dodges the bullet!

  The self-force is the cosmic censor!

- Numerical tests of this picture:

  - Done for radiation reaction

  - Few yrs away for conservative SF