Holographic No-Boundary Measure

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(also: arXiv:0803.1663, 1009.2525)

Intro

Novel approach to measure in (eternal) inflation,

based on old idea – no-boundary state.

$\rightarrow$ amplitude for different cosmological backgrounds and fluctuations

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Novel approach to measure in (eternal) inflation,

based on old idea – no-boundary state.

$\rightarrow$ amplitude for different cosmological backgrounds and fluctuations

Main result:

No-boundary wave function can be viewed as a Euclidean ADS wave function

$\rightarrow$precise AdS/CFT dual formulation of no-boundary state

Intro

Alternative viewpoints on results:

- a step towards placing no-boundary state on firm footing

  $\rightarrow$ precise ‘holographic’ measure in eternal inflation?

- a novel application of AdS/CFT to cosmology

- a realization of dS/CFT

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  $\rightarrow$ precise 'holographic' measure in eternal inflation?

- a novel application of AdS/CFT to cosmology

- a realization of dS/CFT

In more general terms:

"The universe's quantum state provides a natural connection between Euclidean (asymptotic) AdS and Lorentzian inflationary cosmologies."

Outline

- No-Boundary measure: review
- its ADS form and holographic representation
- Application to eternal inflation

No-Boundary Wave Function

$$\Psi[b,h,\chi] = \int_C \delta g \delta\phi \exp(-I[g,\phi])$$

"The amplitude of configurations (b,h,χ) *on a three-surface* Σ *is given by the integral over all regular metrics* g *and matter fields* ϕ *that match* (b,h,χ) *on their only boundary."* [Hartle & Hawking '83]

Low energy toy models:

$$I[g,\phi] = -\frac{1}{2}\int \sqrt{g}(R-2\Lambda) + \int \sqrt{g}\left[(\nabla\phi)^2 + V(\phi)\right]$$

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Motivated by analogy with ground state wave function in QM and QFT, e.g. SHO:

Euclidean PI: $\psi(x_0) = \int \delta x \exp\{-I[x(\tau)]/\hbar\}$

with $I[x(\tau)] = \frac{1}{2}\int d\tau[\dot{x}^2 + \omega^2 x^2]$

Saddle pt appr: $\psi(x_0) \propto \exp\{-\omega x_0^2/2\}$

(no tunneling involved)

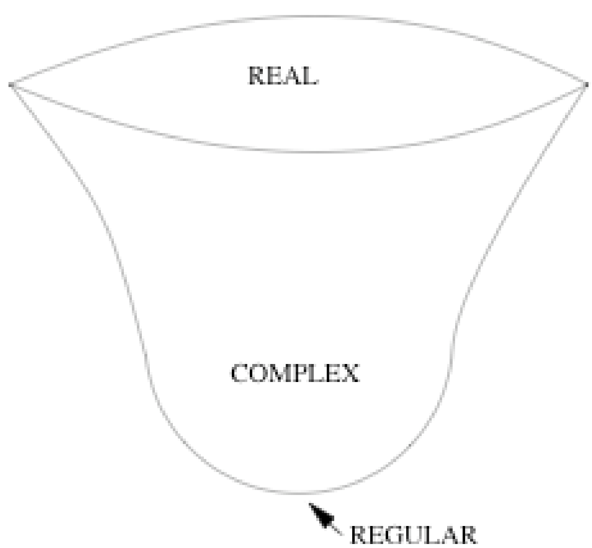
Semiclassical Approximation

In some regions of (mini)superspace the wave function can be evaluated in the steepest descents approximation.

To leading order in $\hbar$ the NBWF will then have the semiclassical form,

$$\Psi(b, h, \chi) \approx \exp\{[-I_R(b, h, \chi) + iS(b, h, \chi)]/\hbar\}$$

In general the extremal geometries will be complex:



Classical Universes in Quantum Cosmology

$$\Psi(b, h, \chi) \approx \exp\{[-I_R(b, h, \chi) + iS(b, h, \chi)]/\hbar\}$$

Classical predictions $\rightarrow$ via WKB interpretation

The semiclassical wave function predicts Lorentzian, classical evolution in regions of superspace where [Hawking '84, Grischuk & Rozhansky '90]

$$|\nabla_A I_R| \ll |\nabla_A S|$$

The predicted classical histories of the universe are the integral curves of S_L:

$$p_A = \nabla_A S$$

and have probabilities

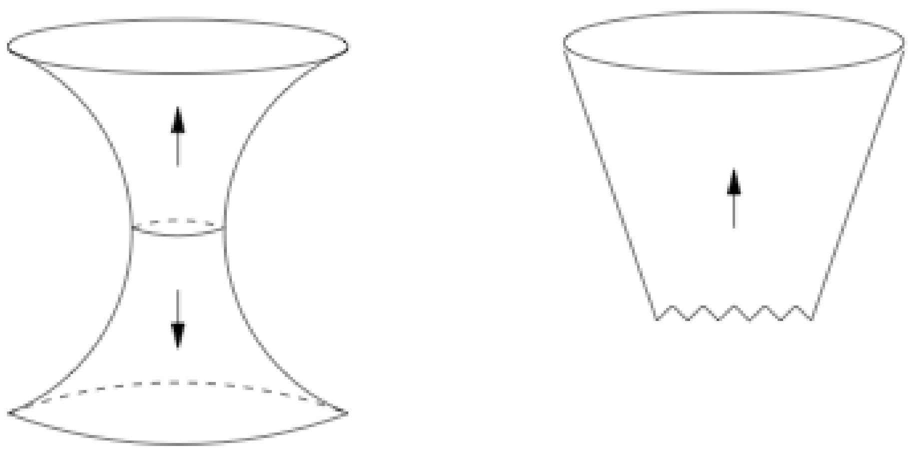
$$P_{history} \propto \exp[-2I_R/\hbar]$$

$\rightarrow$ no-boundary measure: probabilities for an ensemble of *cosmological backgrounds* and their fluctuations.

Classical histories

The Lorentzian histories of the universe that are predicted by the NBWF are distinct from the complex extrema that provide the semiclassical approximation to the wave function.

$$p_A = \nabla_A S$$



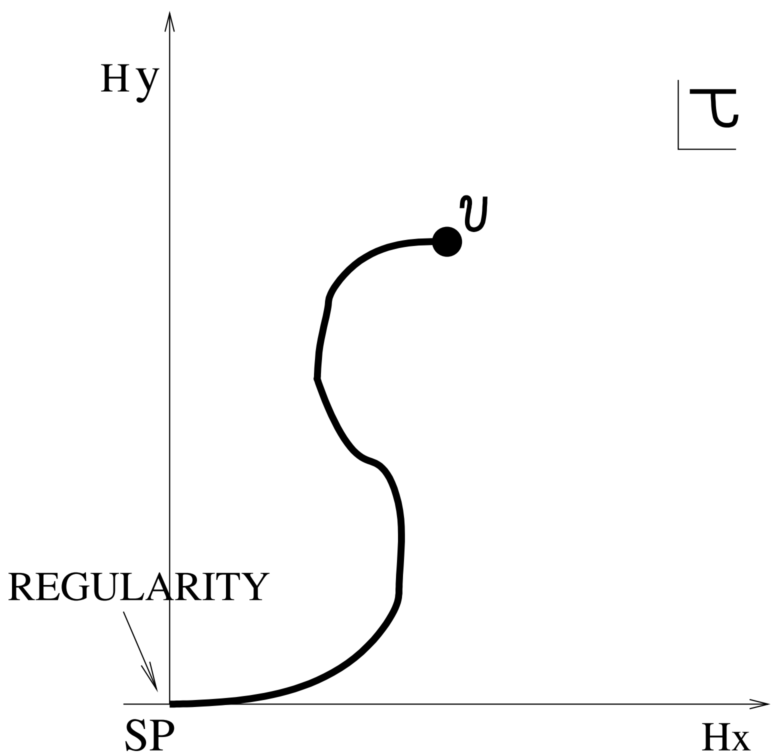
The complex extrema assign a relative probability to different coarse-grained four dimensional histories, including for local observations like the CMB.

Complex Saddle points

Saddle points:

$$ds^2 = N^2(\lambda)d\lambda^2 + g_{ij}(\lambda, x)dx^i dx^j, \quad \phi(\lambda, x)$$

In terms of complex $\tau(\lambda) = \int_0^\lambda d\lambda' N(\lambda')$,



SP: $\quad g_{ij}(0) \to 0, \quad \dot{\phi}(0) \to 0$

Boundary: $\quad g_{ij}(\upsilon) = b^2 h_{ij}, \quad \phi(\upsilon) = \chi$

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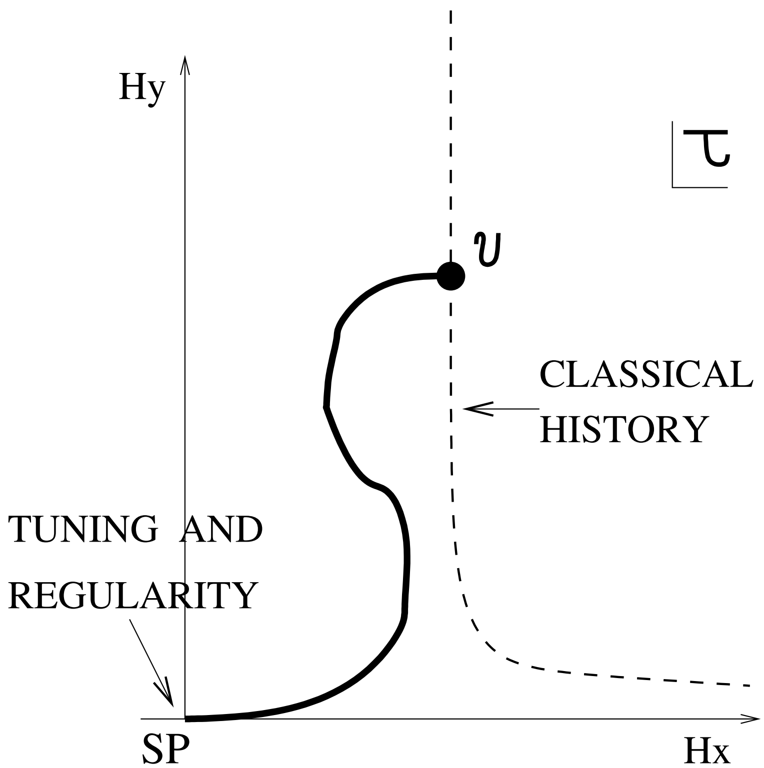

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Boundary: $\quad g_{ij}(\upsilon) = b^2 h_{ij}, \quad \phi(\upsilon) = \chi$

Tuning at SP: $\quad \phi(0) = \phi_0 e^{i\gamma}, \ldots$

$\to$ classical history!

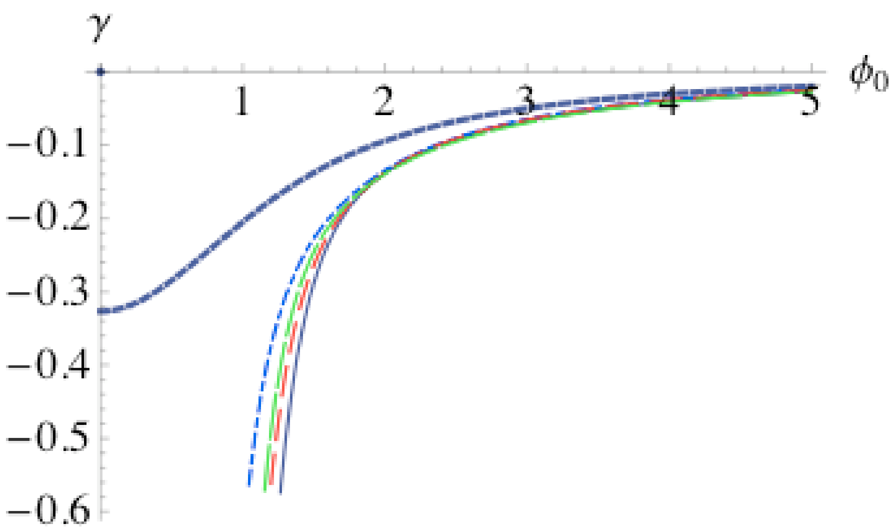
Example

Homogeneous/isotropic ensemble:

$$ds^2 = d\tau^2 + a^2(\tau)d\Omega_3, \quad \phi(\tau)$$

$$V(\phi) = \Lambda + \frac{1}{2}m^2\phi^2$$

Classical spacetime at late times requires:



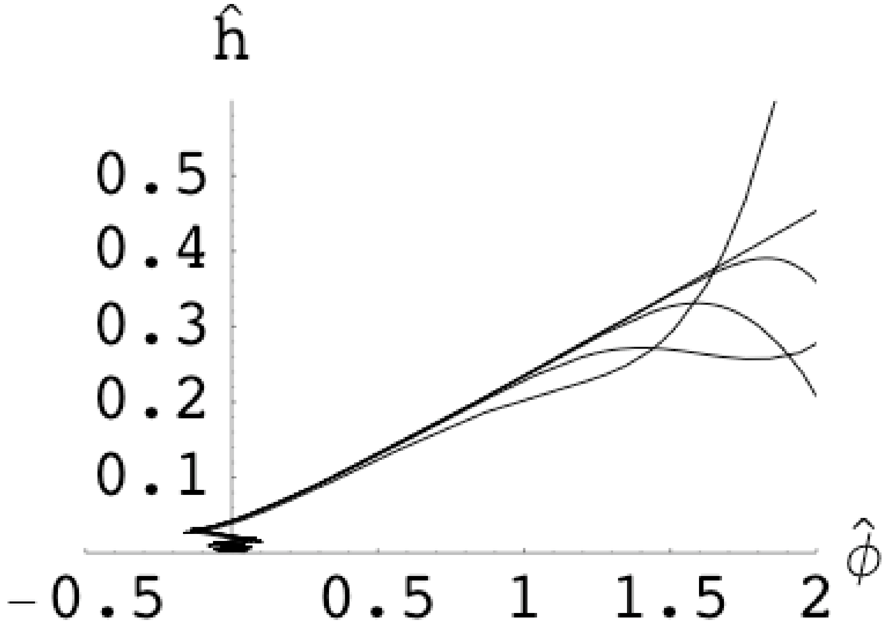
→ a 1-parameter set of FLRW universes

Inflation

Lorentzian histories:

$$p_A = \nabla_A S$$

Lorentzian evolution *backwards* in time:



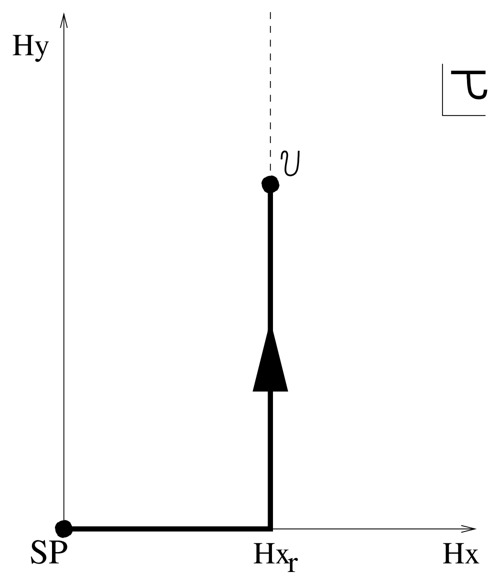
$\rightarrow$ NBWF predicts ensemble of inflating universes.

$$N_{efolds} \approx \phi_0^2$$

Part II: Its AdS representation and AdS/CFT dual

Complex Saddle points

Lorentzian histories lie on asymptotically vertical curves in complex τ-plane:



with $Hx_r \to \pi/2$ for $\phi_0 \to 0$

horizontal part: $ds^2 \approx d\tau^2 + \frac{1}{H^2}\sin^2(H\tau)d\Omega_3^2$

vertical part: $ds^2 \approx -dy^2 + \frac{1}{H^2}\cosh^2(Hy)d\Omega_3^2$

when no matter: complete solution $a(\tau) = \frac{1}{H}\sin(H\tau)$

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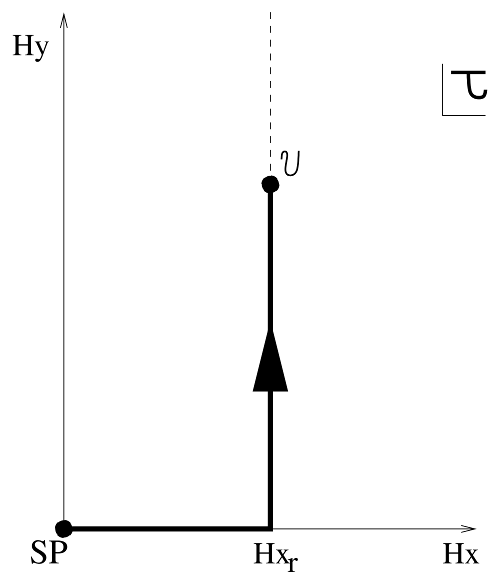
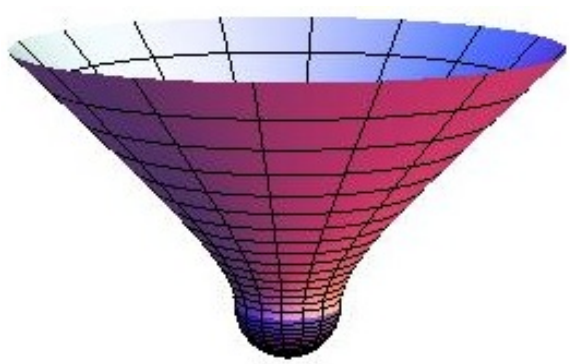
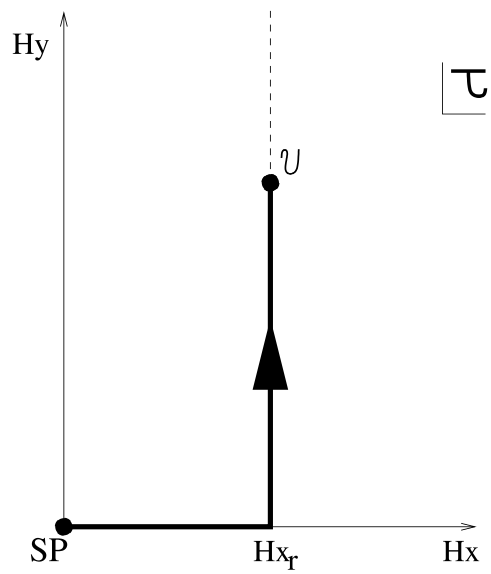


Embedding of saddle point:



Saddle point Action

$$\Psi(b,\chi) \approx \exp\{[-I_R(\chi) + iS(b,\chi)]/\hbar\}$$



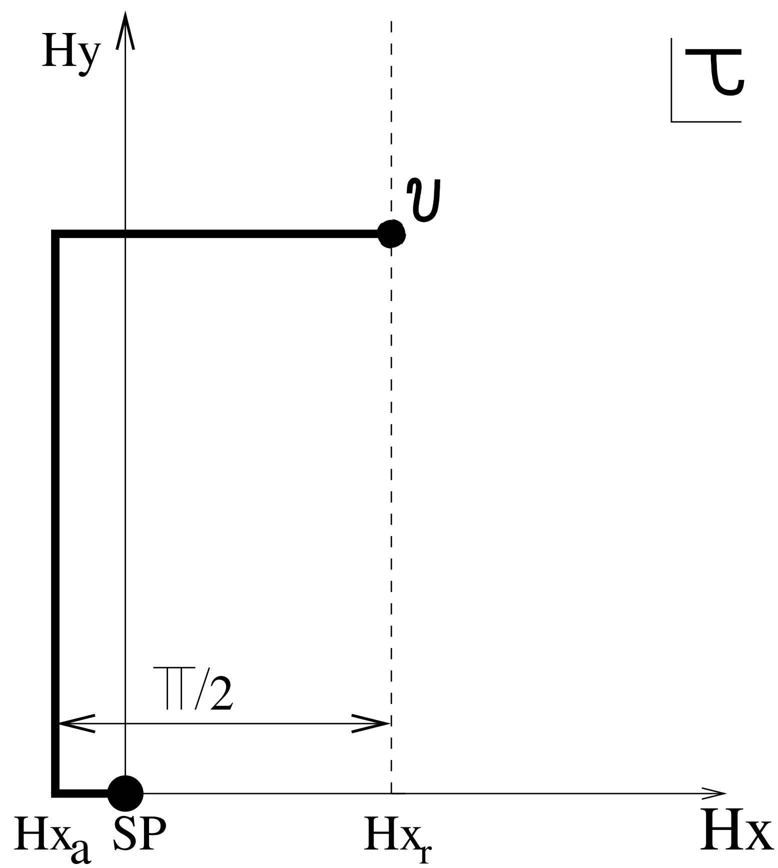
$$I(\upsilon) = \frac{3\pi}{2}\int_{C(0,\upsilon)} d\tau a[a^2(H^2 + 2V(\phi)) - 1]$$

$\rightarrow$ I_R tends to constant along vertical part

$\rightarrow$ probability measure on *classical histories.*

Representations of Saddle points

Different representation of the same saddle point:



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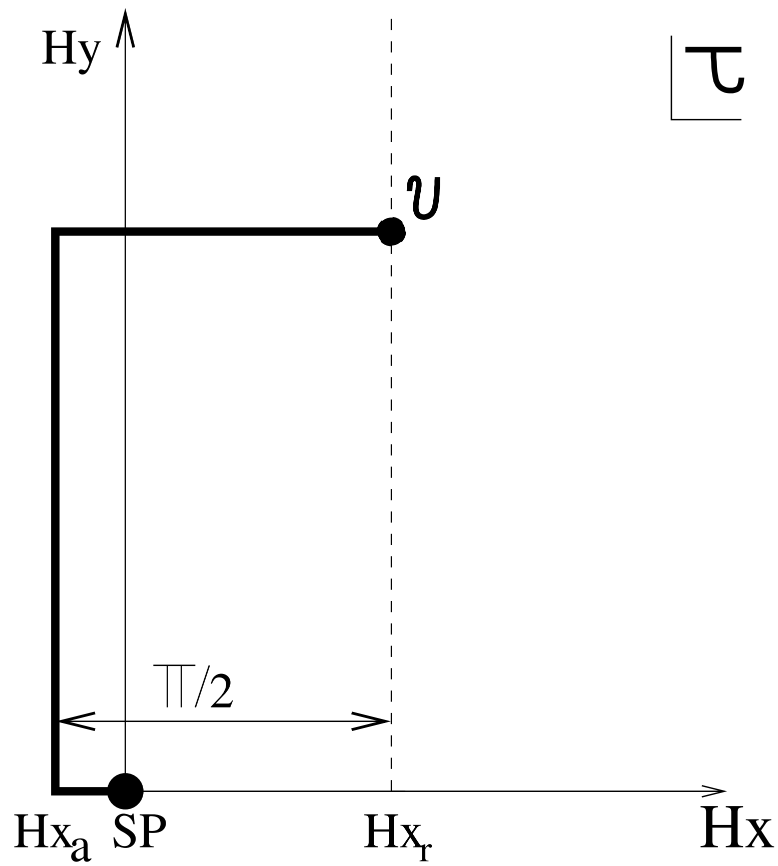


No matter: $x_a \to 0, \quad a(\tau) = \frac{1}{H} \sin(H\tau)$

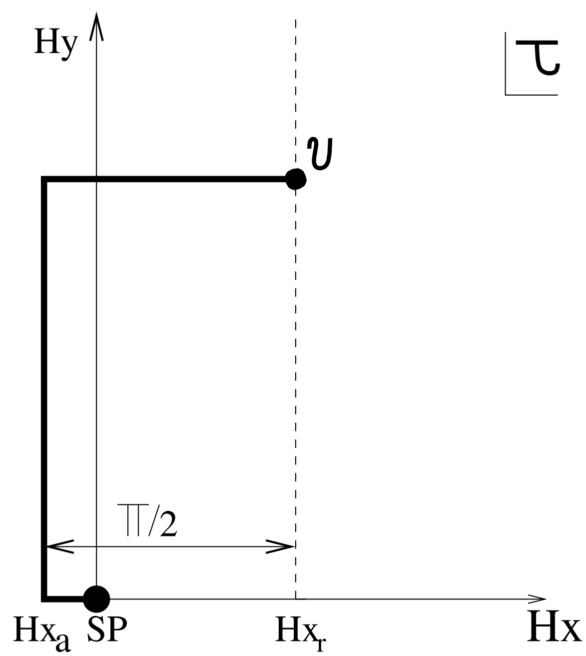
$\to$ Euclidean ADS along vertical part contour!

$$ds^2 = -dy^2 - \frac{1}{H^2} \sinh^2(Hy) d\Omega_3^2$$

Representations of Saddle points

With homogeneous matter:

$$ds^2 = d\tau^2 + a^2(\tau)d\Omega_3, \quad \phi(\tau)$$



Vertical part: $ds^2 \approx -dy^2 - \frac{1}{H^2}\sinh^2(Hy)d\Omega_3^2$

→ Euclidean ADS domain wall of $-(V+\Lambda)$ theory.

At $x = x_r$: $ds^2 \approx -dy^2 + \frac{1}{H^2}\cosh^2(Hy)d\Omega_3^2$

Along horizontal branch?

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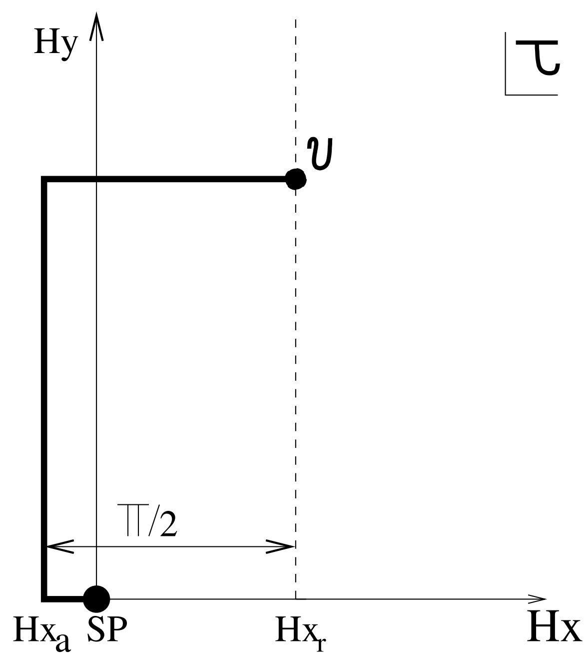

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$\rightarrow$ Euclidean ADS domain wall of $-(V+\Lambda)$ theory.

At $x = x_r$: $ds^2 \approx -dy^2 + \frac{1}{H^2}\cosh^2(Hy)d\Omega_3^2$

Along horizontal branch? $\rightarrow$ complex metric

Action integral along AdS contour

General Saddle Points:

$$ds^2 = d\tau^2 + g_{ij}(\tau, \Omega) d\Omega$$

Asymptotic expansion in small $u \equiv e^{i\tau} = e^{-y+ix}$

Asymptotic metric and field [Skenderis,...]:

$$g_{ij}(u, \Omega) = \frac{-1}{4u^2}[h_{ij}(\Omega) + h_{ij}^{(2)}(\Omega)u^2 + h_{ij}^{(-)}(\Omega)u^{\lambda_-} + h_{ij}^{(3)}(\Omega)u^3 + \cdots]$$

$$\phi(u, \Omega) = u^{\lambda_-}(\alpha(\Omega) + \alpha_1(\Omega)u + \cdots) + u^{\lambda_+}(\beta(\Omega) + \beta_1(\Omega)u + \cdots)$$

with $\lambda_\pm \equiv \frac{3}{2}[1 \pm \sqrt{1-(2m/3)^2}]$

and arbitrary ‘boundary values’ (h_{ij}, α)

Action integral along AdS contour

- Action integral along vertical part:

$$I_v = \int_v I[g,\phi] = -I^R_{AdS}(h,\chi) - S_{ct}(b,h,\chi)$$

where I^R_{AdS} is finite when $a \to \infty$.

- Surface terms:

$$S_{ct} = a_0 \int \sqrt{h} + a_1 \int \sqrt{h} R^{(3)} + a_2 \int \sqrt{h}\phi^2$$

- Action integral along horizontal part:

$$I_h = \int_h I[g,\phi] = +S_{ct}(b,h,\chi) - iS_{ct}(b,h,\chi)$$

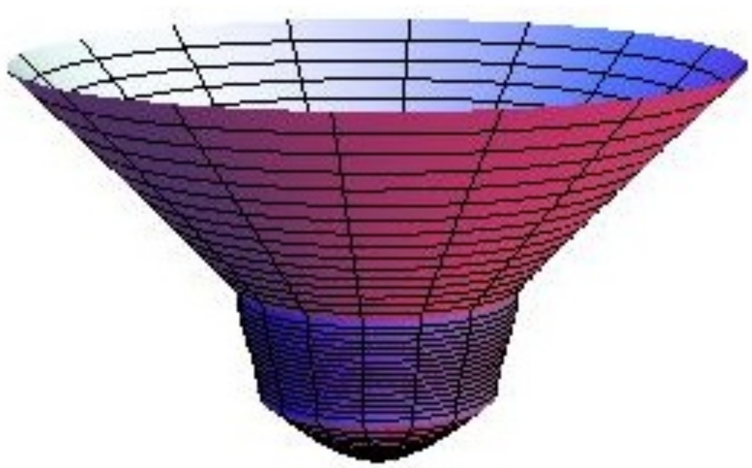
and no finite contribution.

Classicality automatically regularizes volume divergences of the AdS regime

$\rightarrow$ probabilities from I_v; surface terms etc from I_h.

A holographic dual?

$$\Psi(b,h,\chi) \approx \exp\{[+I^R_{AdS}(h,\chi) + iS_{ct}(b,h,\chi)]/\hbar\}$$



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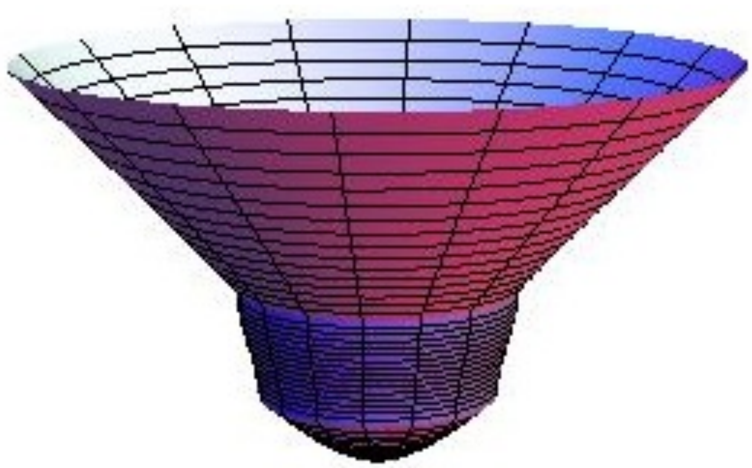


AdS/CFT [Maldacena,Witten,...]:

$$\exp(-I^R_{ADS}[h,\chi]/\hbar) = Z_{QFT}[h,\chi] = \langle \exp \int d^3x \sqrt{h} \alpha \mathcal{O} \rangle$$

→ 'dS/CFT dual' formulation of NBWF:

$$\Psi(b,h,\chi) \approx \frac{1}{Z_{QFT}[h,\chi,\epsilon]} \exp\{[iS_{ct}(b,h,\chi)]/\hbar\}$$

Remarks

$$\Psi(b,h,\chi) \approx \frac{1}{Z_{QFT}[h,\chi,\epsilon]} \exp\{[iS_{ct}(b,h,\chi)]/\hbar\}$$

- Dual partition function provides measure on configurations (h,χ).
- Physical interpretation of counterterms
- Duality involves coarse-graining over UV modes on both sides
- Result can also be viewed simply as application of Euclidean AdS/CFT to cosmology
- Eucl AdS/CFT is in line with notion of unique wave function of the universe
- Regularity at origin implemented in AdS/CFT

Conjecture: duality valid beyond leading order

Part III: Eternal Inflation

Probabilities of histories

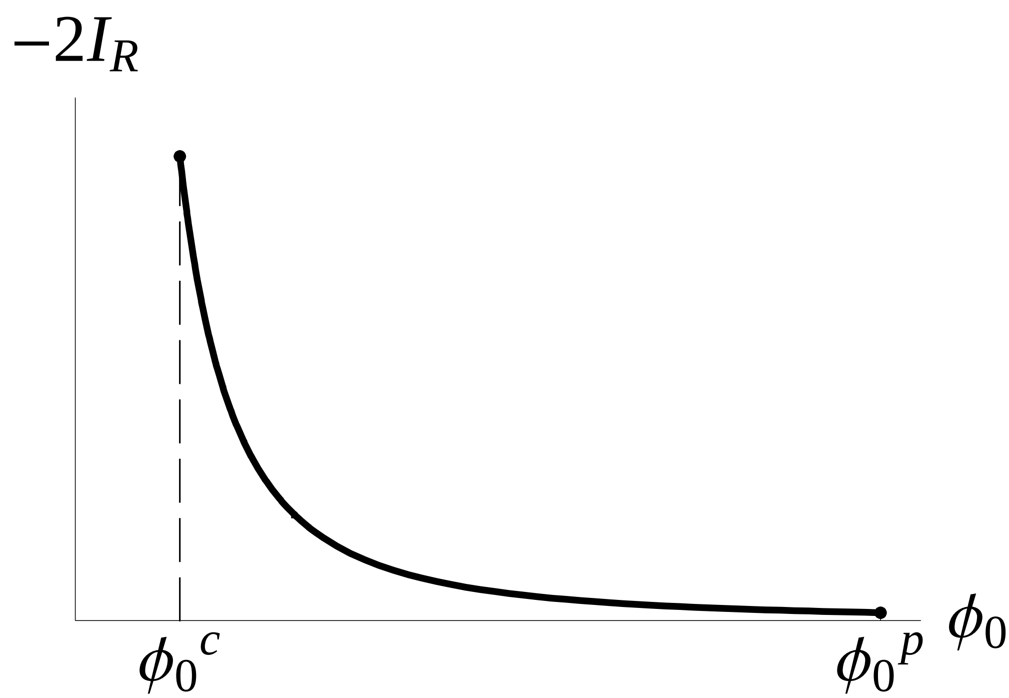
Why worry about eternal inflation?

Probabilities of histories

Why worry about eternal inflation?

The value of the real part of the Euclidean action of the saddle points is conserved along each Lorentzian history and determines its bottom-up probability.

$$p(\phi_0) \propto \exp[-2I_R/\hbar], \quad I_R \approx -\frac{\pi}{2}\frac{1}{V(\phi_0)}$$



$\rightarrow$ NBWF seemingly predicts few efolds of inflation.

Probabilities of Observations

State gives the probability of an entire universe.

But our observations are limited to a small patch...

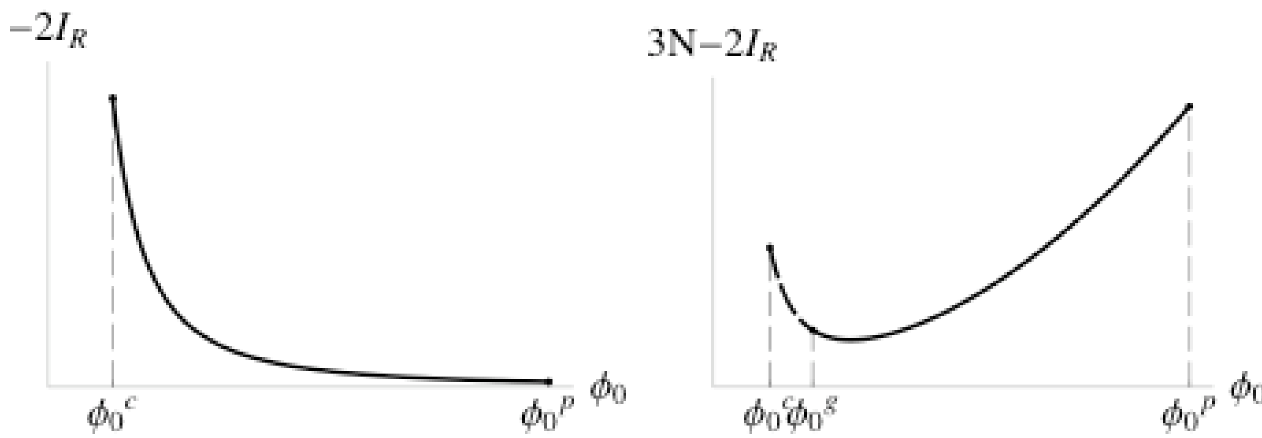
Probabilities for local observations therefore involve a sum – a coarse graining – over the unobserved three-metrics and fields on Σ, which is weighted by the volume of the surface to take into account our different possible locations. [Hartle & TH, 2009]

$$p(\mathcal{O}) \sim \frac{1}{V_m} \int_{\mathcal{O}}^{V_m} dh_{ij} d\chi |\Psi(h_{ij}, \chi)|^2 \mathrm{Vol}(h_{ij})$$

Volume weighting has a significant effect on the distribution in models of eternal inflation.

Eternal Inflation

$$|\Psi|^2 \mathrm{Vol}(h) \sim N_h(\phi_0)\; p(\phi_0) \;\; \propto \exp\left[\frac{3\phi_0^2}{4} + \frac{2\pi}{m^2\phi_0^2}\right]$$



→ Dominant contribution comes from histories with *many efolds*.

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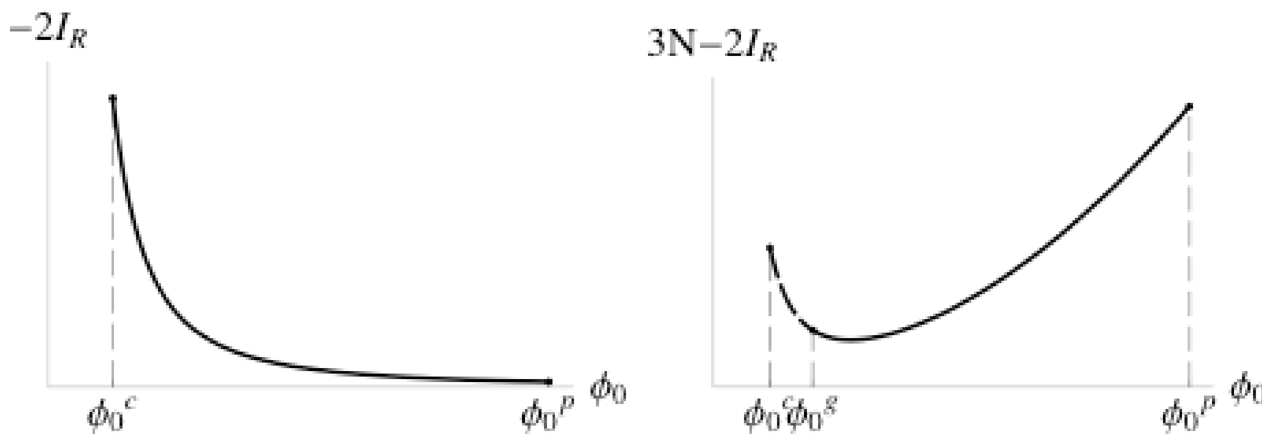

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Landscape models: $p(\mathcal{O})$ involves relative probability of different regions of eternal inflation,

$$p(\mathcal{O}) \sim p(\mathcal{O}|EI1)p(EI1) + p(\mathcal{O}|EI2)p(EI2) + \cdots$$

Eternal Inflation

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$\rightarrow$ no unique classical background...

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$\rightarrow$ no unique classical background...

Apply AdS/CFT dual formulation of no-boundary state to regime of eternal inflation.

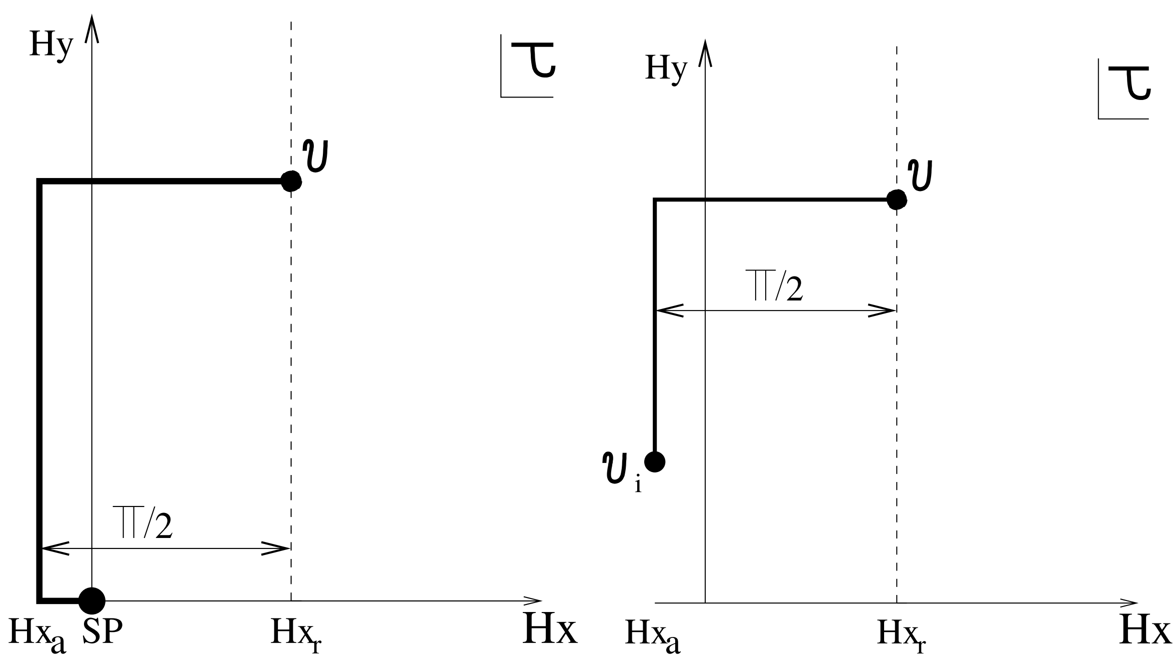
$\rightarrow$ dual automatically sums backreaction effects during eternal inflation, given a boundary configuration on a surface at the threshold.

Euclidean Eternal Inflation

$$\Psi(b, h, \chi) \approx \exp\{[+I^R_{AdS}(h, \chi) + iS_{ct}(b, h, \chi)]/\hbar\}$$

AdS with finite radius/dual CFT with cutoff

$\rightarrow$ replace only inner region of eternal inflation:



$\rightarrow$ inner boundary at threshold of eternal inflation

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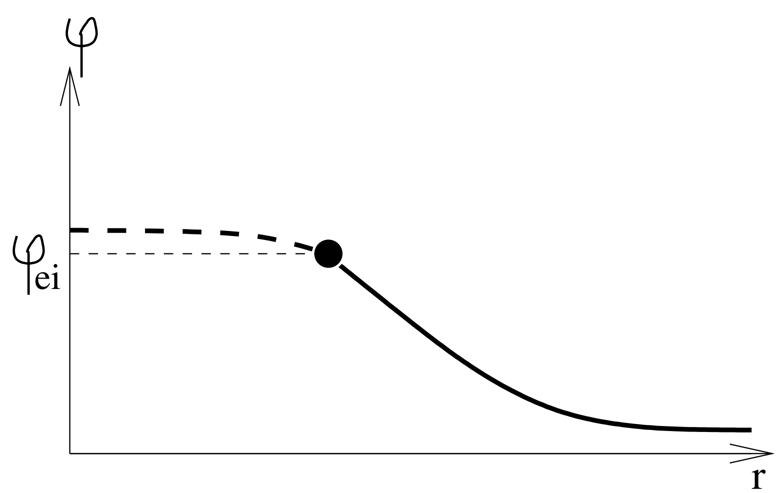

Dual description of eternal inflation:

- IR CFT with deformation given by $\phi = \phi_{EI}$.
- $< \mathcal{O} >$ on inner boundary replaces regularity at SP.

Euclidean Eternal Inflation

Improved no-boundary measure in eternal inflation:

$$|\Psi(b,\hat{h},\chi)|^2 \approx \frac{1}{|Z_{QFT}[\phi_{EI},h]|^2}\exp\{[+2\tilde{I}^R_{AdS}(h,\phi_{EI},\chi)/\hbar\}$$

with $\tilde{I}^R_{AdS}$ is the action of the "remaining" saddle point interpolating between the inner boundary at the threshold of eternal inflation and the final boundary.

Conclusion

- In the no-boundary quantum state, the action of Euclidean AdS domain walls gives the probabilities of different inflationary cosmologies.

- This naturally leads to a dual description of the no-boundary measure in terms of the partition function of relevant deformations of the CFTs that occur in AdS/CFT.

- The duality at finite scale factor involves a coarse-graining over UV modes on both sides.

- The duality can be used to reinterpret the regime of eternal inflation in terms of a dual field theory on an inner boundary at the threshold of eternal inflation.

- If the duality extends beyond leading order the dual at finite N would yield a more secure way to define and to calculate the no-boundary measure.