Infrared Issues in Inflation

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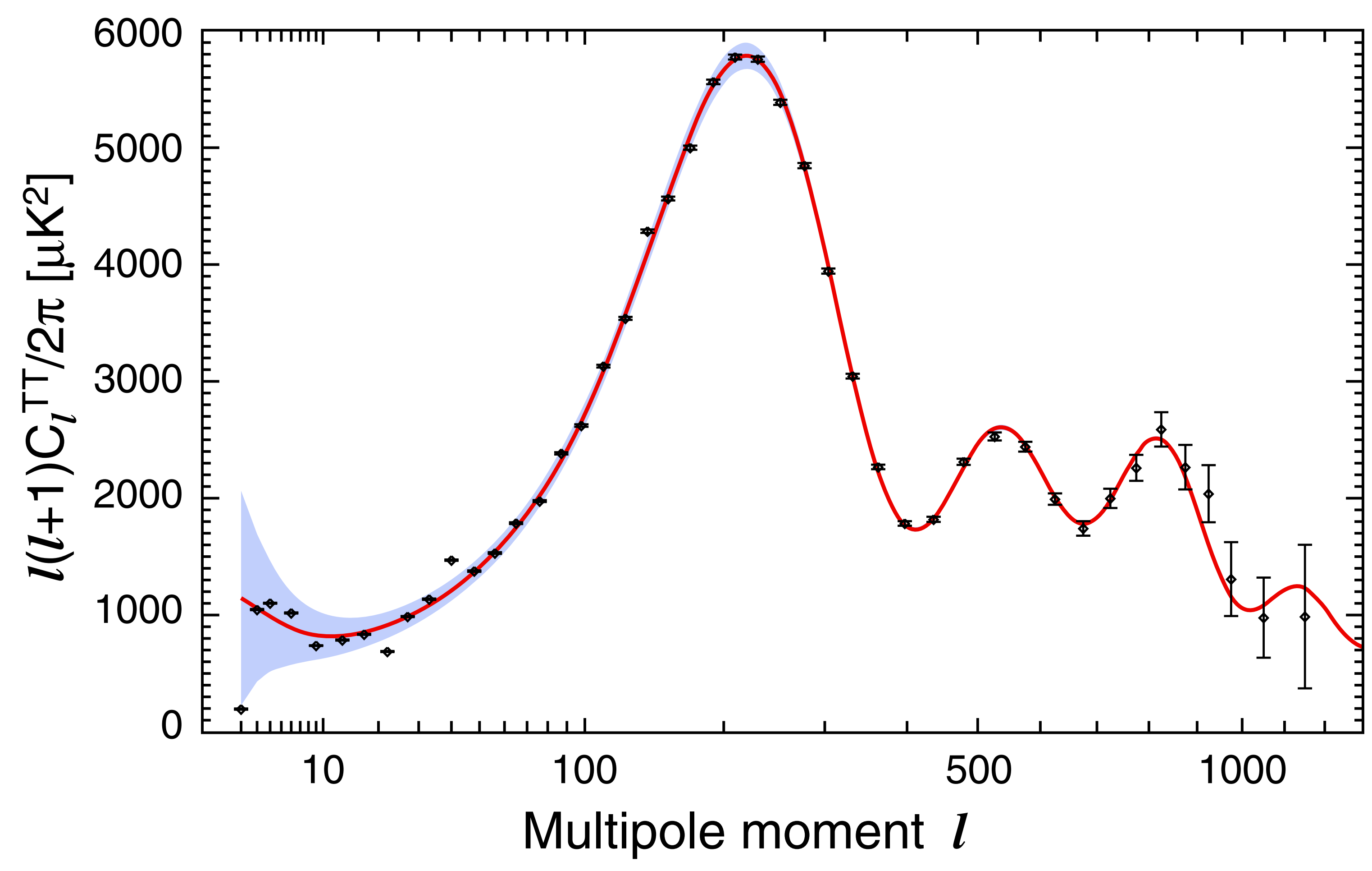
Outline

- What IR issues?
- Dynamical RG resummation of secular terms
- Conclusions

Want to extract fundamental physics from this:



But... how well do we trust our calculations?

What IR Issues?

- Light fields in DS have long been known to have IR problems.
  - Secular growth in time of the two point function (time dependent logs)
  - Box-size dependent logs
- These long-distance issues impair our ability to trust perturbative corrections to the power spectrum, bi-spectrum etc.

IR divergences are the signal that we're not describing the long distance physics sufficiently well:

ex: Scattering of charged particles: missing physics is due to radiating soft photons and choosing states of definite photon number

ex: Finite T divergences tell us about the breakdown of the loop expansion. Missing physics comes from the resummation of all higher loops

DRG Resummation of super-Hubble Fluctuations

DRG Resummation of Secular Growth

Two types of secular logs coming from quantum corrections

$$\ln \frac{\tau_0}{\tau} = \ln \frac{a(\tau)}{a(\tau_0)}$$

DeS inv. broken by a beginning of inflation

$$\ln(-k\tau) = \ln \frac{a(\tau_k)}{a(\tau)}, \; a(\tau_k) = \frac{k}{H}$$

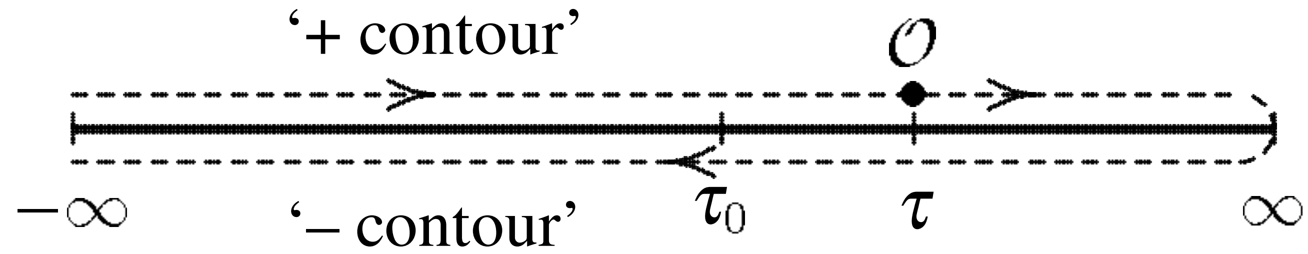
These show up in higher order corrections even in De S

In-In formalism

In cosmology we need to calculate time dependent expectation values

$$\langle \mathcal{O}(t) \rangle \equiv \mathrm{Tr}(\rho(t)\mathcal{O}(t)) = \mathrm{Tr}(\rho(t_0)U^\dagger(t,t_0)\mathcal{O}(t)U(t,t_0))$$

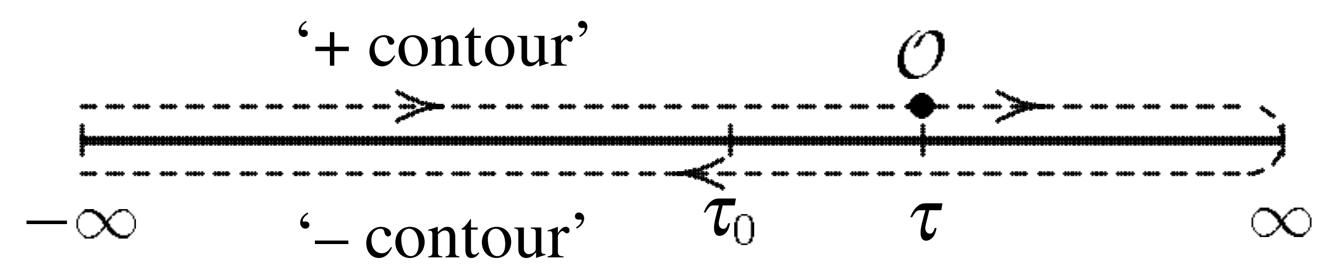
This corresponds to a path integral defined on a closed time contour



Field content doubled

$$\Phi \to \{\Phi^+, \Phi^-\} \qquad \Phi_C = \frac{1}{2}\left(\Phi^+ + \Phi^-\right)$$
$$\Phi_\Delta = \Phi^+ - \Phi^-$$

Times on - contour are later than those on + contour



3 Green's functions

$$\langle \Phi_C(x)\Phi_C(y)\rangle = -iG_C(x,y)$$
$$\langle \Phi_C(x)\Phi_\Delta(y)\rangle = G_R(x,y),$$
$$\langle \Phi_\Delta(x)\Phi_C(y)\rangle = G_A(x,y)$$

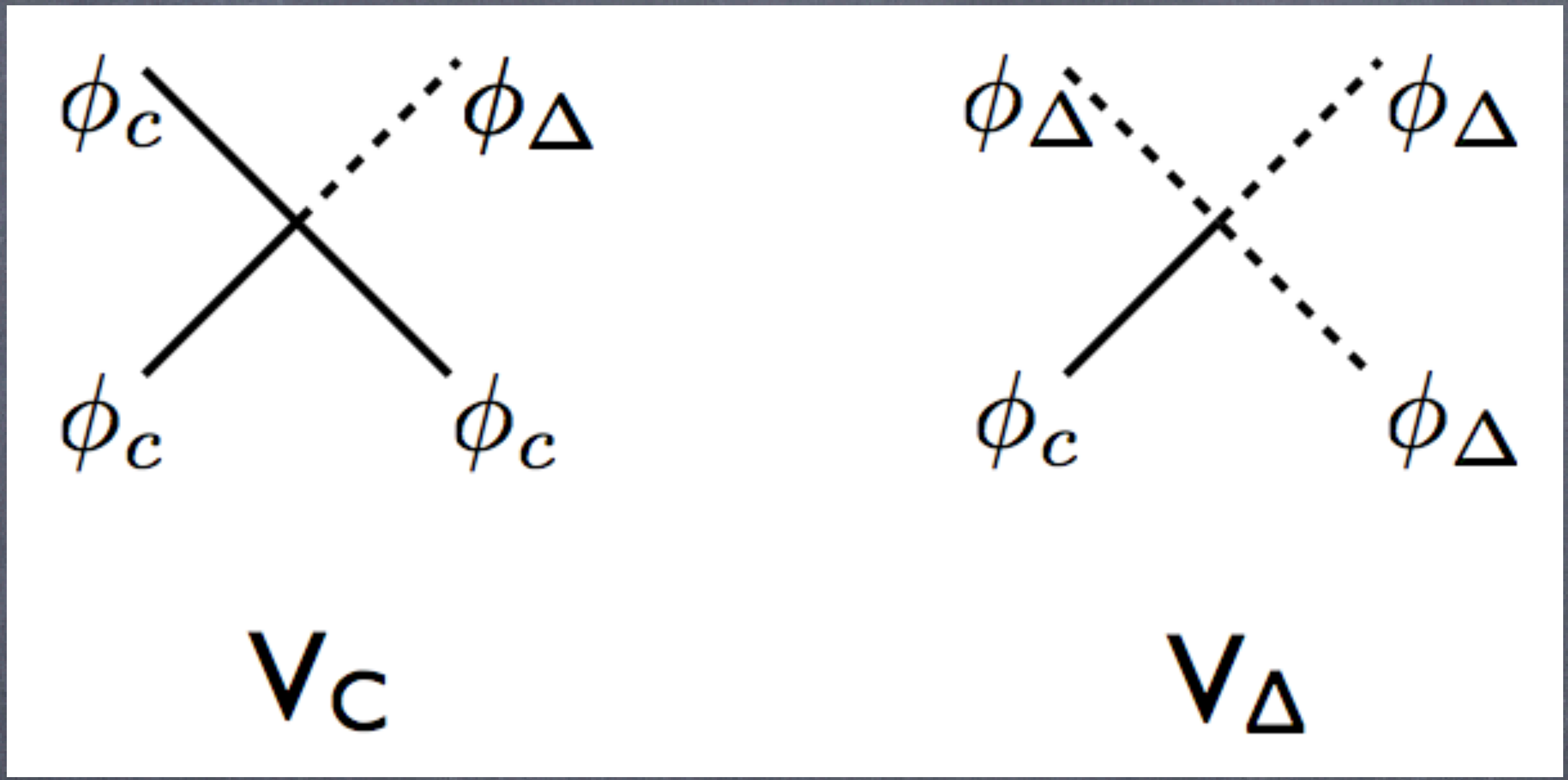
$$\mathcal{L} = \sqrt{-g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2}\left(m^2 + \xi R\Phi^2\right) - \frac{\lambda}{4!}\Phi^4\right)$$

$$\mathcal{L}(\Phi_c, \Phi_\Delta) = \sqrt{-g}\left(g^{\mu\nu}\partial_\mu\Phi_C\partial_\nu\Phi_\Delta - \frac{\lambda}{4!}\left(4\Phi_C^3\Phi_\Delta + \Phi_C\Phi_\Delta^3 + \text{c.t.}\right)\right)$$

$$G_C^0(k,\tau_1,\tau_2) \simeq \frac{H^2}{2k^3}\left\{1 + \mathcal{O}((k\tau)^2)\right\}$$

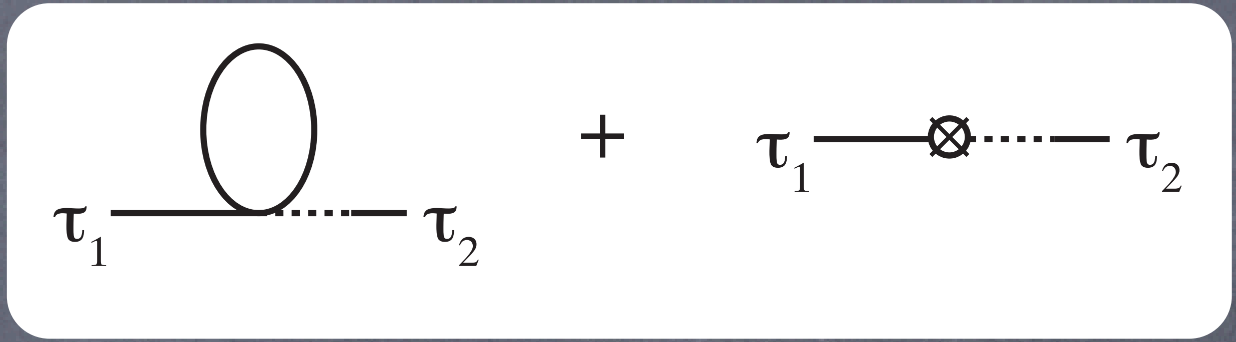
$$G_R^0(k,\tau_1,\tau_2) \simeq \Theta(\tau_1-\tau_2)\frac{H^2}{3}(\tau_1^3-\tau_2^3)\left\{1 + \mathcal{O}((k\tau)^2)\right\}$$

The vertices are



Loop Corrections and Secular behavior

Let's go to the tadpole graph



$$\Lambda(\tau) \equiv \langle \phi^2(x) \rangle = -iG^{-+}(x,x) = G^0_C(x,x) = \int \frac{d^3k}{(2\pi)^3}\, G^0_C(k,\tau,\tau) + \text{c.t.}$$
$$= \frac{1}{(2\pi)^2}\left[\int_{a\,\Lambda_{IR}}^{a\,\mu} \frac{dk}{k}\left\{H^2\left[1+\left(\frac{k}{aH}\right)^2\right]\right\}\right]$$
$$\simeq \frac{1}{(2\pi)^2}\left[H^2 \ln\left(\frac{\mu}{\Lambda_{IR}}\right) + \frac{1}{2}\left(\mu^2 - \Lambda^2_{IR}\right)\right]$$

IR cutoff is unphysical; should be replaced by physical scale L due to missing physics

$$
\begin{aligned}
G_C(k,L) &= G_C^{UV}(\mu/\Lambda_{IR}) + G_C^{IR}(\Lambda_{IR}L) \\
&= \left[A + B\ln\left(\frac{\mu}{\Lambda_{IR}}\right) + \cdots\right] + \left[C + B\ln(\Lambda_{IR}L) + \cdots\right] \\
&= (A+C) + B\ln(\mu L) + \cdots
\end{aligned}
$$

For logs, IR cutoff from UV calculation can give us full dependence on L

In our case, the choice is whether L depends on time or not.

mass term: L is time indep

Pre-inflationary physics: L time dep

Now use this to correct propagator

$$G_C(k,\tau) = \frac{H^2}{2k^3}\left[1 + \frac{\lambda}{3(2\pi)^2}\ln\left(\frac{\mu}{\Lambda_{IR}}\right)\ln\left(-k\tau\right) + \cdots\right]$$
$$\Rightarrow \frac{H^2}{2k^3}\left[1 + \frac{\lambda}{3(2\pi)^2}\ln\left(\mu L\right)\ln\left(-k\tau\right) + \cdots\right]$$

How to resum the secular terms?

How can we fix L?

Let's recall how the RG works:

1. Compute 1-loop corrected coupling

$$\alpha(\mu) = \alpha(\mu_0) + b\,\alpha^2(\mu_0)\,\ln\left(\frac{\mu}{\mu_0}\right)$$

valid for $\alpha(\mu_0) \ll 1,\ \alpha(\mu_0)\ln(\mu/\mu_0) \ll 1$

2. Differentiate then integrate wrt subtraction point

$$\frac{1}{\alpha(\mu)} = \frac{1}{\alpha(\mu_0)} - b\,\ln\left(\frac{\mu}{\mu_0}\right),\ \text{valid for } \alpha \ll 1$$

Domain of validity has been extended

What is the time dependent analog?

Another (Easier) Secular problem: Damped SHO in PT

$$\ddot{y} + y = -\epsilon\, \dot{y}, \;\; \epsilon \ll 1$$

Exact solution:

$$y(t) = y_0\; e^{-\frac{\epsilon}{2}t} \cos\left(t\sqrt{1 - \frac{\epsilon^2}{4}} + \delta \right)$$

Perturbative solution

$$y(t) = y_0\; e^{it} \left(1 - \frac{\epsilon}{2}t + \frac{\epsilon^2}{8}t^2 + i\frac{\epsilon^2}{8}t \right) + \text{c.c.} + \text{non} - \text{secular}$$

DRG Resummation

$$y_0 = A(\tau)Z(\tau)$$

$$Z(\tau) = 1 + \epsilon z_1(\tau) + \epsilon^2 z_2(\tau) + \cdots$$

$$z_1(\tau) = \frac{\tau}{2},\ \ z_2(\tau) = \frac{\tau^2}{8} - i\frac{\tau}{8}$$

Coefficients chosen to cancel secular behavior at a time tau

$$y(t,\tau) = A(\tau)\ e^{it}\left(1 - \frac{\epsilon}{2}(t-\tau) + \frac{\epsilon^2}{8}(t-\tau)^2 + i\frac{\epsilon^2}{8}(t-\tau)\right) + $$
$$+\text{c.c.} + \text{non} - \text{secular}$$

Now demand tau independence: DE for A(tau)

$$\frac{dy(t,\tau)}{d\tau} = 0 \Rightarrow$$

$$A(\tau) = A(0)\exp(-\frac{\epsilon}{2}\tau + i\frac{\epsilon^2}{8}\tau)$$

Finally use arbitrariness of tau to set tau=t

$$y(t) = A(0)e^{it}\exp(-\frac{\epsilon}{2}t + i\frac{\epsilon^2}{8}t) + \mathrm{c.c}$$

Suppose approximation scheme generates secular growth

$$y(t) = y_0(t) + \varepsilon\, y_1(t) + c_0 + \mathcal{O}\left(\varepsilon^2\right)$$
$$= y_0(c,t) + \varepsilon\, y_1(c,t) + c_0 + \mathcal{O}\left(\varepsilon^2\right),$$

with c the integration constant for $y_0(t)$.

1. Introduce an arbitrary time scale

$$y(t) = y_0(t) + \varepsilon\, [y_1(t) - y_1(\vartheta) + y_1(\vartheta)] + \mathcal{O}\left(\varepsilon^2\right) \Rightarrow$$
$$y(t) = y_0[c(\vartheta), t] + \varepsilon\, [y_1(t) - y_1(\vartheta)] + \mathcal{O}\left(\varepsilon^2\right)$$

with $y_0[c(\vartheta), t] \equiv y_0(c,t) + \varepsilon\, y_1(\vartheta)$

2. Use independence from new time scale to get DRG eqn

$$\left(\frac{\partial y_0}{\partial c}\right) \frac{dc}{d\vartheta} - \varepsilon\, \frac{\partial y_1(c,\vartheta)}{\partial \vartheta} \Rightarrow c = \tilde{c}(\vartheta).$$

3. Set new scale equal to t. Solution has greater domain of validity

$$y(t) = y_0[\tilde{c}(\vartheta), t] + \varepsilon\, [y_1(t) - y_1(\vartheta)] + \mathcal{O}\left(\varepsilon^2\right)$$
$$= y_0[\tilde{c}(t), t] + \mathcal{O}\left(\varepsilon^2\right)$$

Example: If
$$y(t) = c\,\left[1 + \varepsilon\, f(t) + \mathcal{O}\left(\varepsilon^2\right)\right]$$

the DRG improvement is
$$y(t) = c\, e^{\varepsilon f(t)}\left[1 + \mathcal{O}\left(\varepsilon^2\right)\right]$$

Now let's work this on the two point function:

$$G_C(k,\tau) = \frac{H^2}{2k^3}\left[1 + \frac{\lambda}{3(2\pi)^2}\ln\left(\mu L\right)\ln\left(-k\tau\right) + \cdots\right] \Rightarrow$$
$$G_C(k,\tau) = \frac{H^2}{2k^3}\exp\left[+\frac{\lambda}{3(2\pi)^2}\ln\left(\mu L\right)\ln\left(-k\tau\right)\right]\left(1 + \cdots\right)$$
$$= \frac{H^2}{2k^3}\left(\frac{k}{aH}\right)^{\delta}\left(1 + \mathcal{O}(\delta^2)\right)$$
$$\delta = \frac{\lambda}{3(2\pi)^2}\ln\left(\mu L\right)$$

We can do this for other situations

Massive (but light) field

$$G_C^0(k,\tau_1,\tau_2) \simeq \frac{H^2}{2k^3}(k^2\tau_1\tau_2)^\epsilon$$

$$G_R^0(k,\tau_1,\tau_2) \simeq \theta(\tau_1-\tau_2)\frac{H^2}{3}(\tau_1^{3-\epsilon}\tau_2^\epsilon - \tau_1^\epsilon\tau_2^{3-\epsilon})$$

$$\epsilon = \frac{m^2}{3H^2}$$

$$G_C(k,\tau) \simeq \frac{H^2}{2k^3}(-k\tau)^{2\epsilon}\left[1+\frac{\lambda}{6(2\pi)^2\epsilon}\left(\frac{\mu}{H}\right)^{2\epsilon}\ln(-k\tau)+\cdots\right]$$

Identify

$$\ln(\mu L) \to \frac{1}{2\epsilon}\left(\frac{\mu}{H}\right)^{2\epsilon} = \frac{3H^2}{2M^2}\left(\frac{\mu}{H}\right)^{2M^2/3H^2}$$

What does the DRG say?

$$G_C(k,\tau) \simeq \frac{H^2}{2k^3}(-k\tau)^{2\epsilon+\delta_m}$$
$$\delta_m = \frac{\lambda}{6(2\pi)^2\epsilon}\left(\frac{\mu}{H}\right)^{2\epsilon}$$

Coupling dominates mass if

$$\frac{\lambda}{(4\pi)^2} > 3\epsilon^2\left(\frac{H}{\mu}\right)^{2\epsilon} = \frac{M^4}{3H^4}\left(\frac{H}{\mu}\right)^{2M^2/3H^2}$$

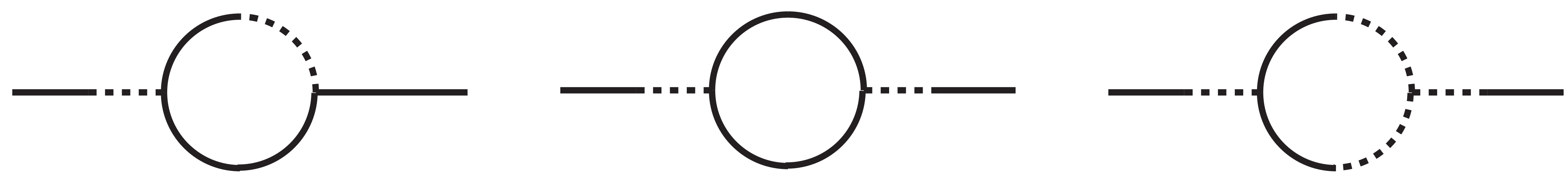
Equivalent mass

$$M_{\rm eff}^2 = \frac{3H^2}{2}\,\delta_m = \frac{\lambda H^2}{(4\pi)^2\epsilon}\left(\frac{\mu}{H}\right)^{2\epsilon} \simeq \frac{3\lambda H^4}{(4\pi)^2 M^2}$$

Same result as for mean field! Also

$$M_{\rm mf}^2 = \frac{1}{2}\lambda\langle\phi^2\rangle \Rightarrow$$
$$\langle\phi^2\rangle = \frac{3H^4}{8\pi^2 M^2}$$

Finally, try a cubic theory. Does the IR regulating physics look like a mass?



$$G_C(k,\tau,\tau) = G_C^0(k,\tau,\tau)\exp\left\{\frac{h^2}{9H^2}\left[\frac{1}{(2\pi)^2}\ln^3(-k\tau) + \frac{4\Lambda}{H^2}\ln^2(-k\tau) + \dots\right]\right\}$$

Not a mass; no surprise since potential is ill behaved.

Conclusions

- DRG resums secular terms in two point function
- DRG automatically resums leading logs; actual diagrams need not be singled out
- For quartic potential, missing IR physics is the generation of a dynamical mass.
- This is the same mass found in gap equations in stochastic program.
- DRG can distinguish different types of IR physics: quartic vs cubic potential.