Odd Parity in the CMB and Stringy Topologies

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IAP
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Outline

- Introduction and motivation
- Anomalous parity in the CMB
- Non-trivial topologies
  - Classical topologies
  - Stringy topologies
    - Orbifold point
    - Orbifold line
- Conclusions

Introduction – The Standard Model

Assumption:

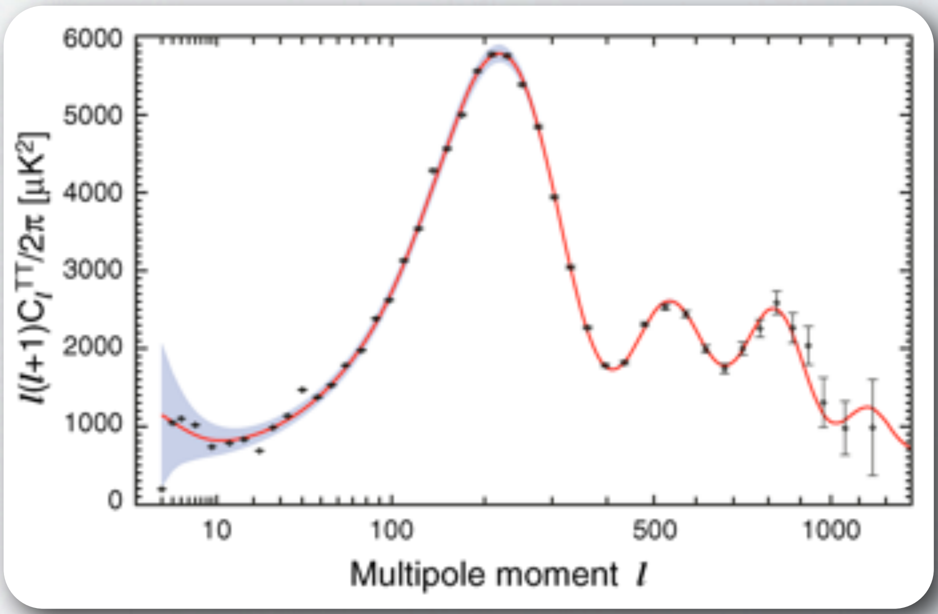
On large enough scales the universe is homogeneous and isotropic → ΛCDM

- Quantum fluctuations of the inflaton field seed Gaussian fluctuations in the metric perturbation field Φ

$$\langle \Phi_{\mathbf{k}} \rangle = 0 \qquad \langle \Phi_{\mathbf{k}} \Phi^*_{\mathbf{k}'} \rangle = (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}') P_\Phi(k)$$

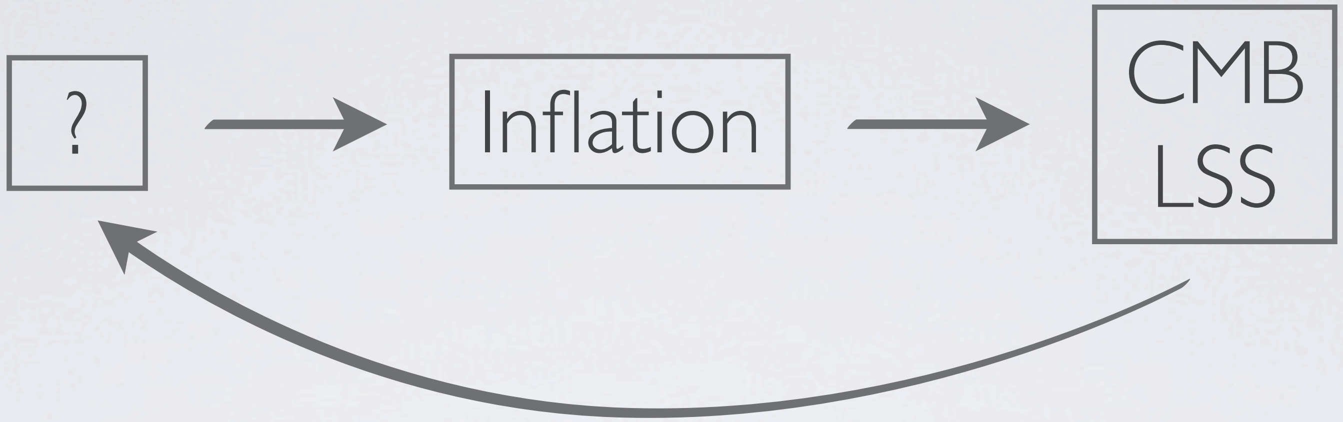
- CMB temperature anisotropy $T(\hat{\mathbf{n}}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{\mathbf{n}})$
- The coefficients are uncorrelated Gaussian random variables

$$\langle a_{\ell m} \rangle = 0 \qquad \langle a_{\ell m} a^*_{\ell' m'} \rangle = \delta_{\ell \ell'} \delta_{m m'} C_\ell$$

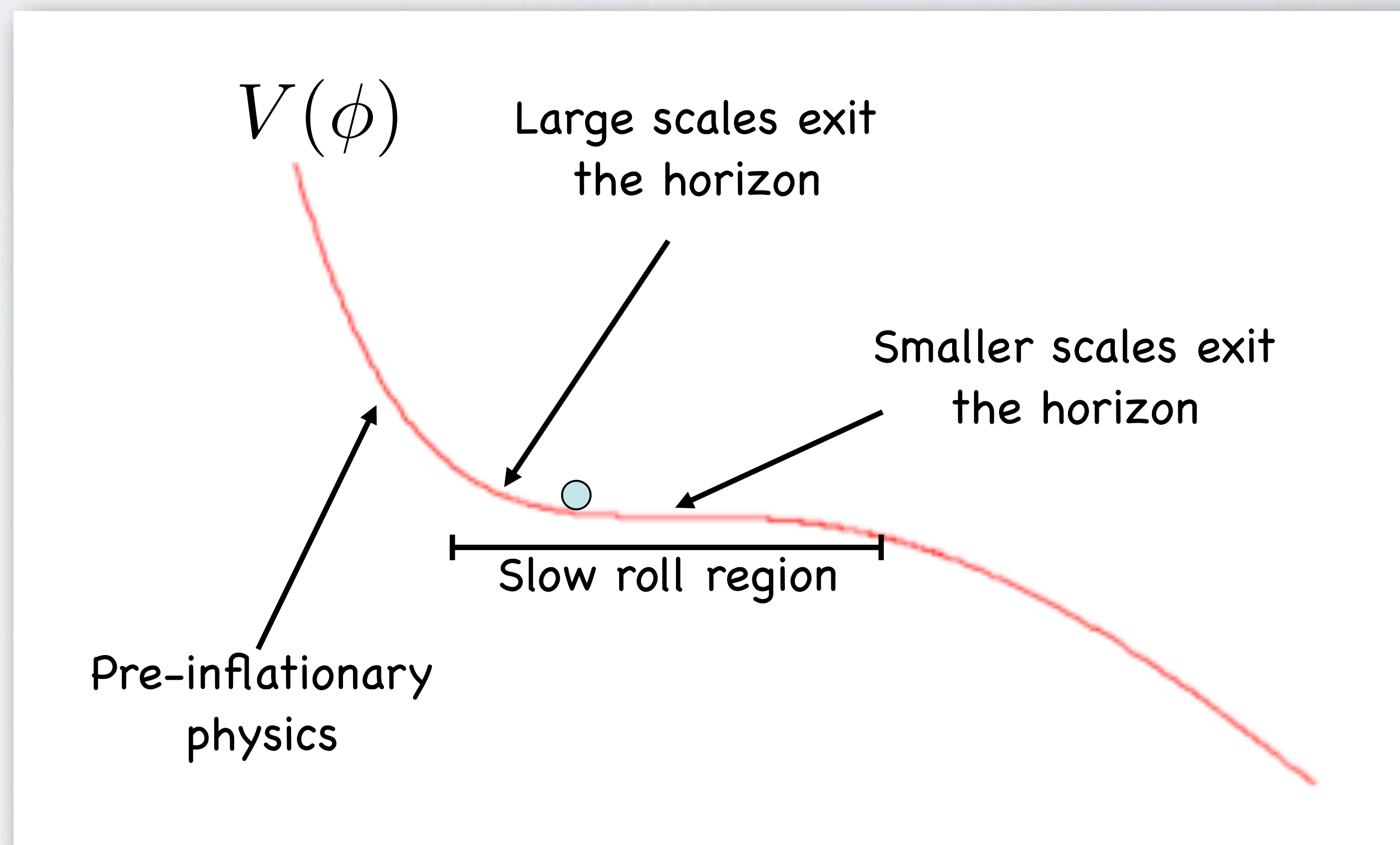


Larson et al., ApJS (2011)

Motivation



- Short inflation is theoretically preferred.
- Pre-inflationary physics affect the largest scales.
- Test the assumption of isotropy on largest scales.
- Indeed, there are several large scale anomalies ($\sim 3\sigma$).



Basic Deformations

$$\cancel{\langle \Phi_{\mathbf{k}} \rangle = 0}$$

Pre-inflationary relics

- Example: Pre-inflationary particle model (Fialkov, Itzhaki and Kovetz, JCAP 2010) and the search for giant concentric rings (Kovetz, ABD and Itzhaki, ApJ 2010).

Anomalous Parity in the CMB

ABD, Kovetz and Itzhaki, ApJ 2012

- Parity with respect to reflections through a plane:

$$\mathcal{P}_{\hat{\mathbf{n}}}: \quad \hat{\mathbf{r}} \to \hat{\mathbf{r}} - 2(\hat{\mathbf{r}} \cdot \hat{\mathbf{n}})\hat{\mathbf{n}}$$

de Oliveira-Costa, Smoot, Starobinsky, ApJ 1996
de Oliveira-Costa, Tegmark, Zaldarriaga, Hamilton, PRD 2004

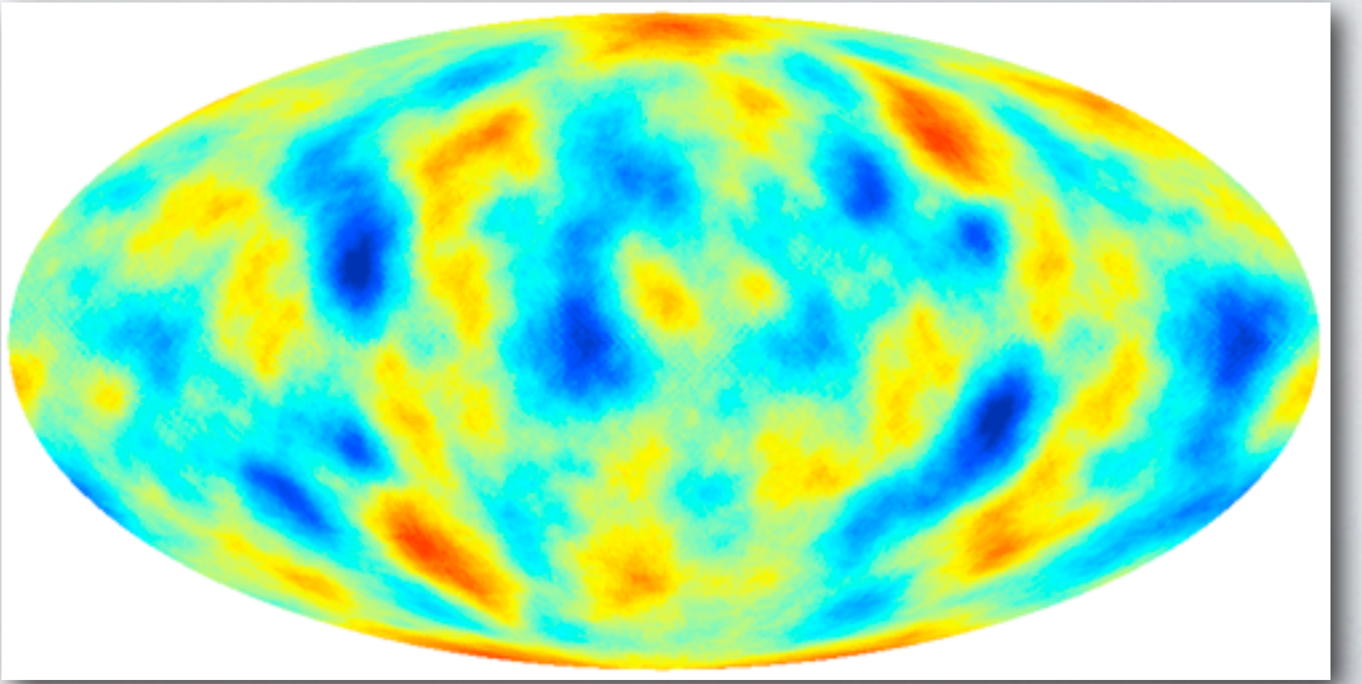
- "S" statistic:

$$S(\hat{\mathbf{n}}) = \int \mathrm{d}^2\hat{\mathbf{n}}' \left[T(\hat{\mathbf{n}}') - T(\mathcal{P}_{\hat{\mathbf{n}}}(\hat{\mathbf{n}}'))\right]^2$$

$$\downarrow$$

$$S(\hat{\mathbf{n}}) = \int \mathrm{d}^2\hat{\mathbf{n}}' \, T(\hat{\mathbf{n}}') T(\mathcal{P}_{\hat{\mathbf{n}}}(\hat{\mathbf{n}}'))$$

"S" Map



The Problems with Pixel-Space

- Difficult to study the scale dependence.
- There are two-point correlations $C(\theta) \equiv \langle T(\hat{\mathbf{n}})T(\hat{\mathbf{n}}')\rangle\Big|_{\hat{\mathbf{n}}\cdot\hat{\mathbf{n}}'=\cos\theta}$
- Masking the galactic plane results in strong bias.

Harmonic Space Score

$$T(\hat{\mathbf{n}}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{\mathbf{n}})$$

- Harmonic coefficients are uncorrelated $\langle a_{\ell m} a^*_{\ell' m'} \rangle = \delta_{\ell \ell'} \delta_{m m'} C_\ell$
- Under reflection through $\hat{\mathbf{z}}$ axis,

$$Y_{\ell m}(\mathcal{P}_{\hat{\mathbf{z}}}(\hat{\mathbf{n}})) = (-1)^{\ell+m} Y_{\ell m}(\hat{\mathbf{n}})$$

- For each direction $\hat{\mathbf{n}}$, compare for each ℓ the distribution of power between even and odd $\ell + m$ multipoles

$$S(\hat{\mathbf{n}}) = \sum_{\ell=2}^{\ell_{\max}} \left[\sum_{m=-\ell}^{\ell} (-1)^{\ell+m} \frac{|a_{\ell m}(\hat{\mathbf{n}})|^2}{\hat{C}_\ell} - 1 \right]$$

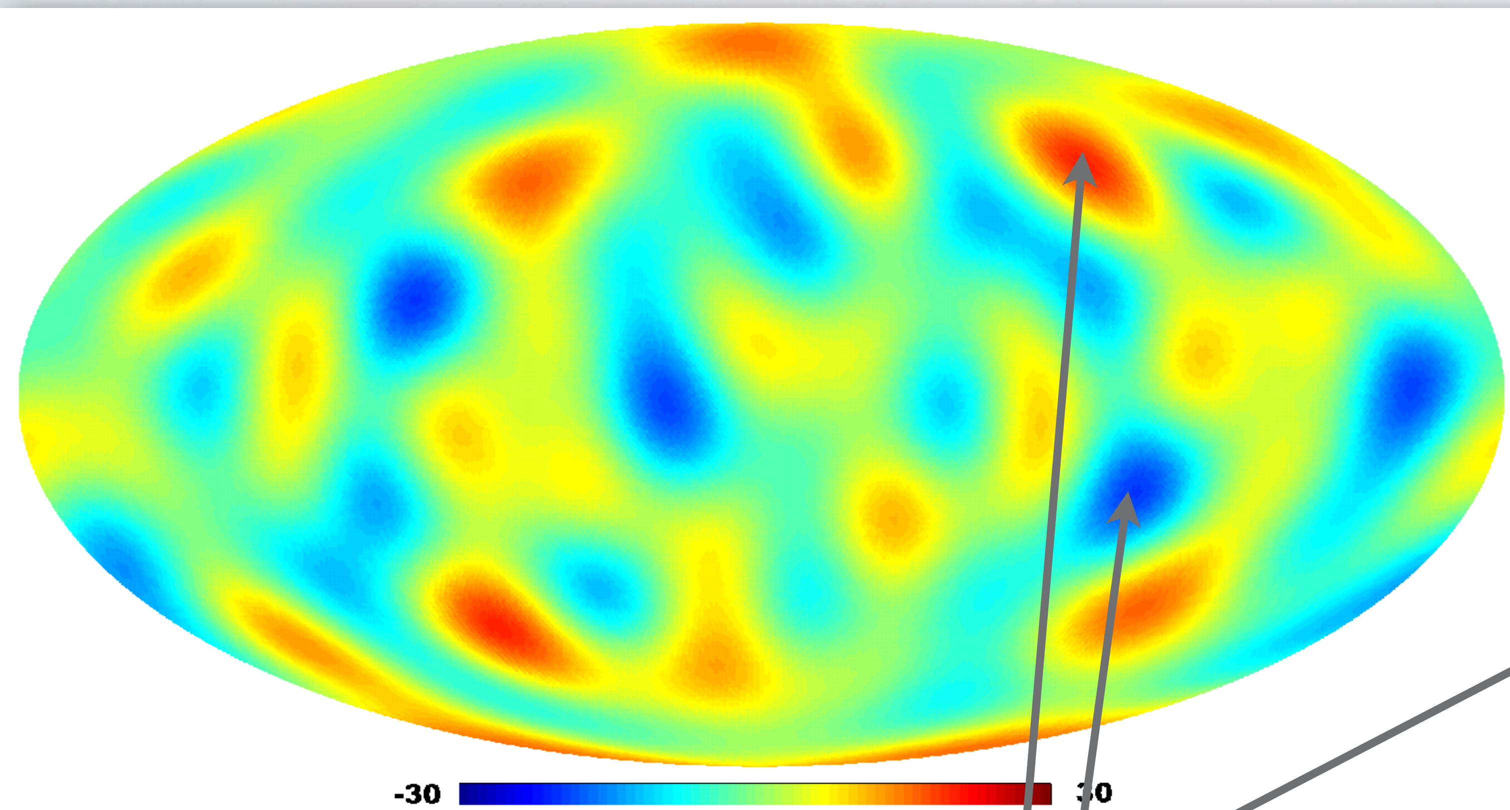
$$\hat{C}_\ell = \frac{1}{2\ell+1} \sum_m |a_{\ell m}|^2$$

- Standard ΛCDM signal should give $\langle S \rangle = 0$.

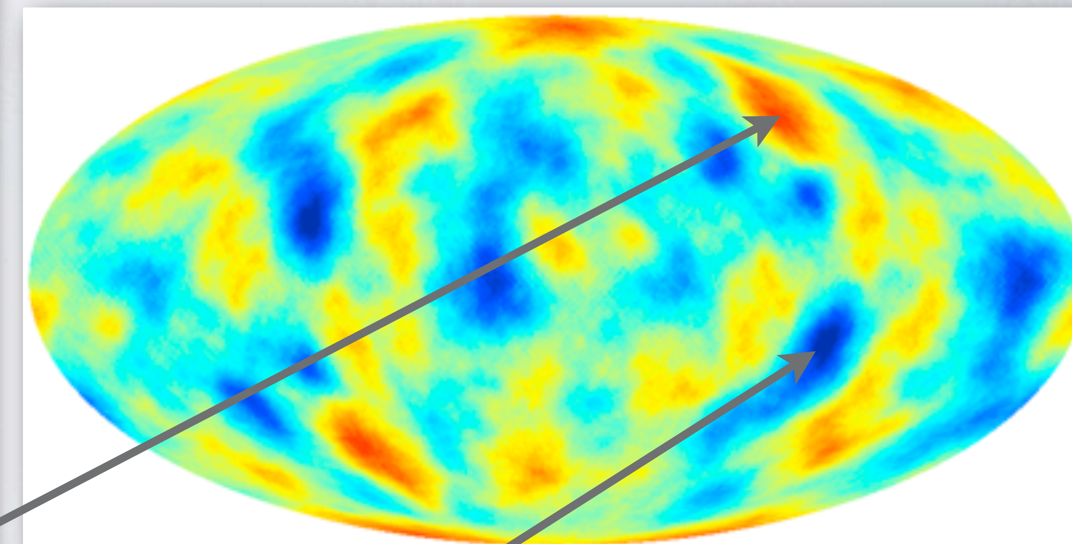
Parity Score – Full Sky Results

- Results for WMAP 7-year ILC map, taking $\ell_{\max} = 5$:

Parity Map



"S" Map



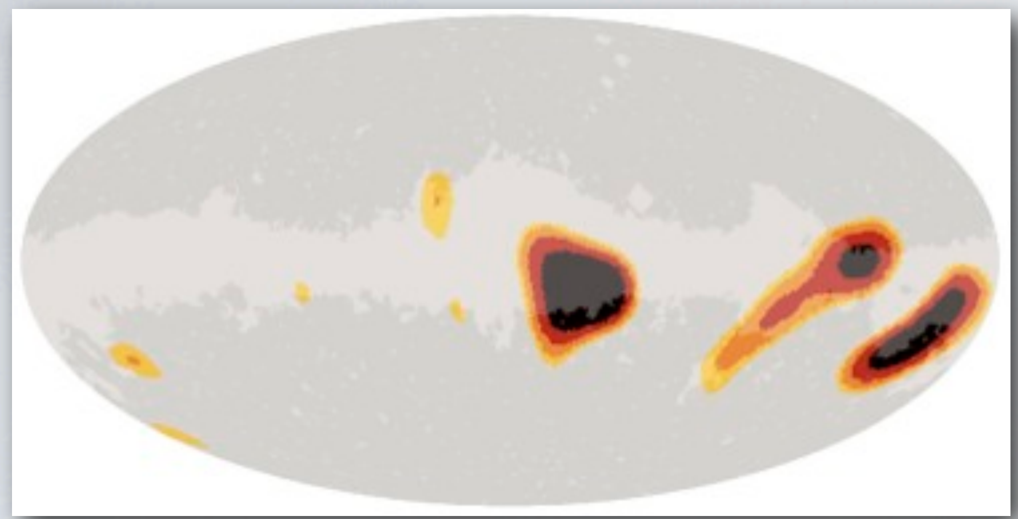
- A maximum at $(l, b) \simeq (260^\circ, 60^\circ)$, the direction of the "Axis of Evil".

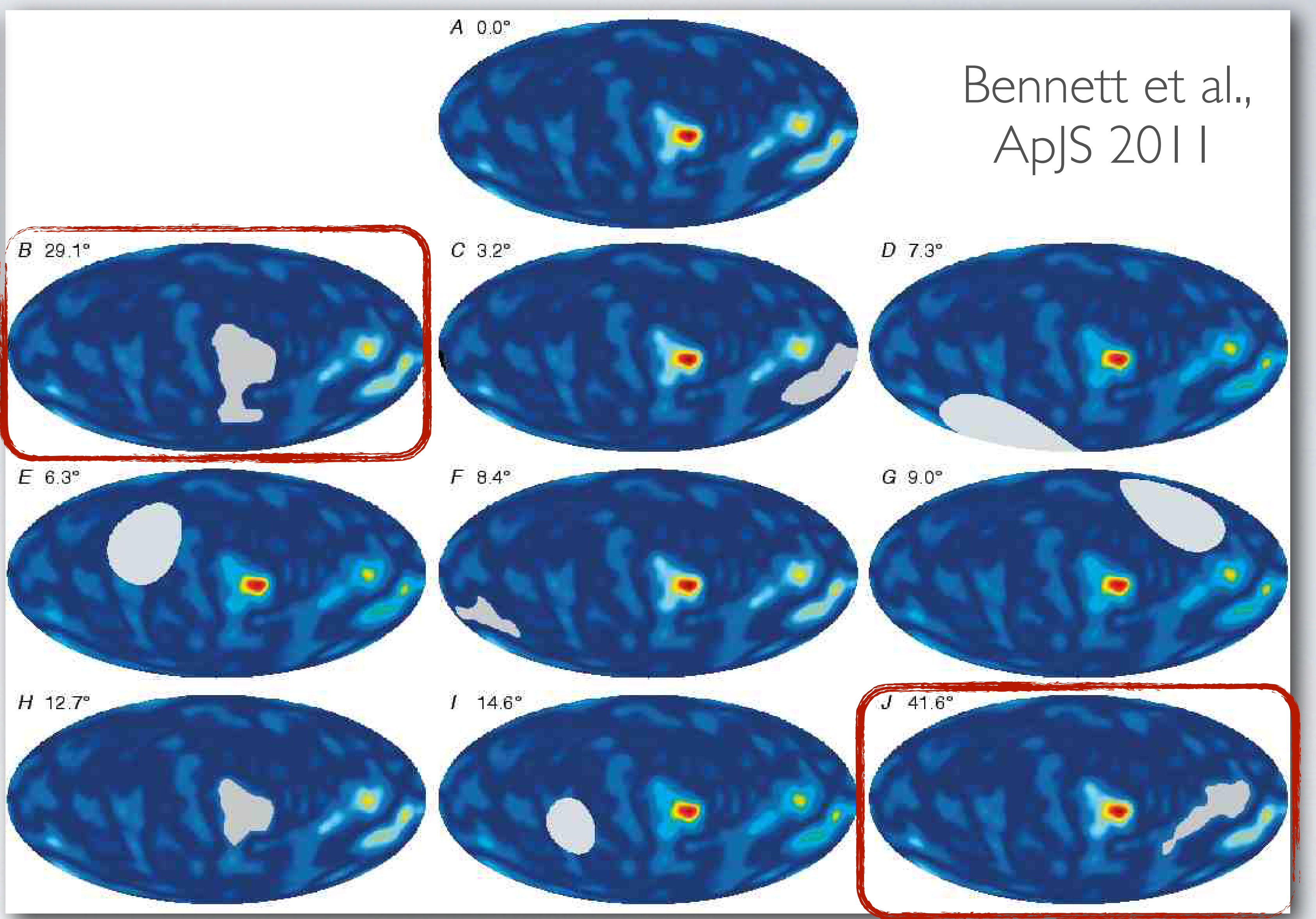
(e.g. de Oliveira-Costa et al., PRD 2004; Copi et al., PRD 2004)

- Planarity $\longleftrightarrow$ high $m = \pm\ell$ $\longleftrightarrow$ even.

- A minimum at $(l, b) \simeq (266^\circ, -19^\circ)$.

Masking Galactic Noise



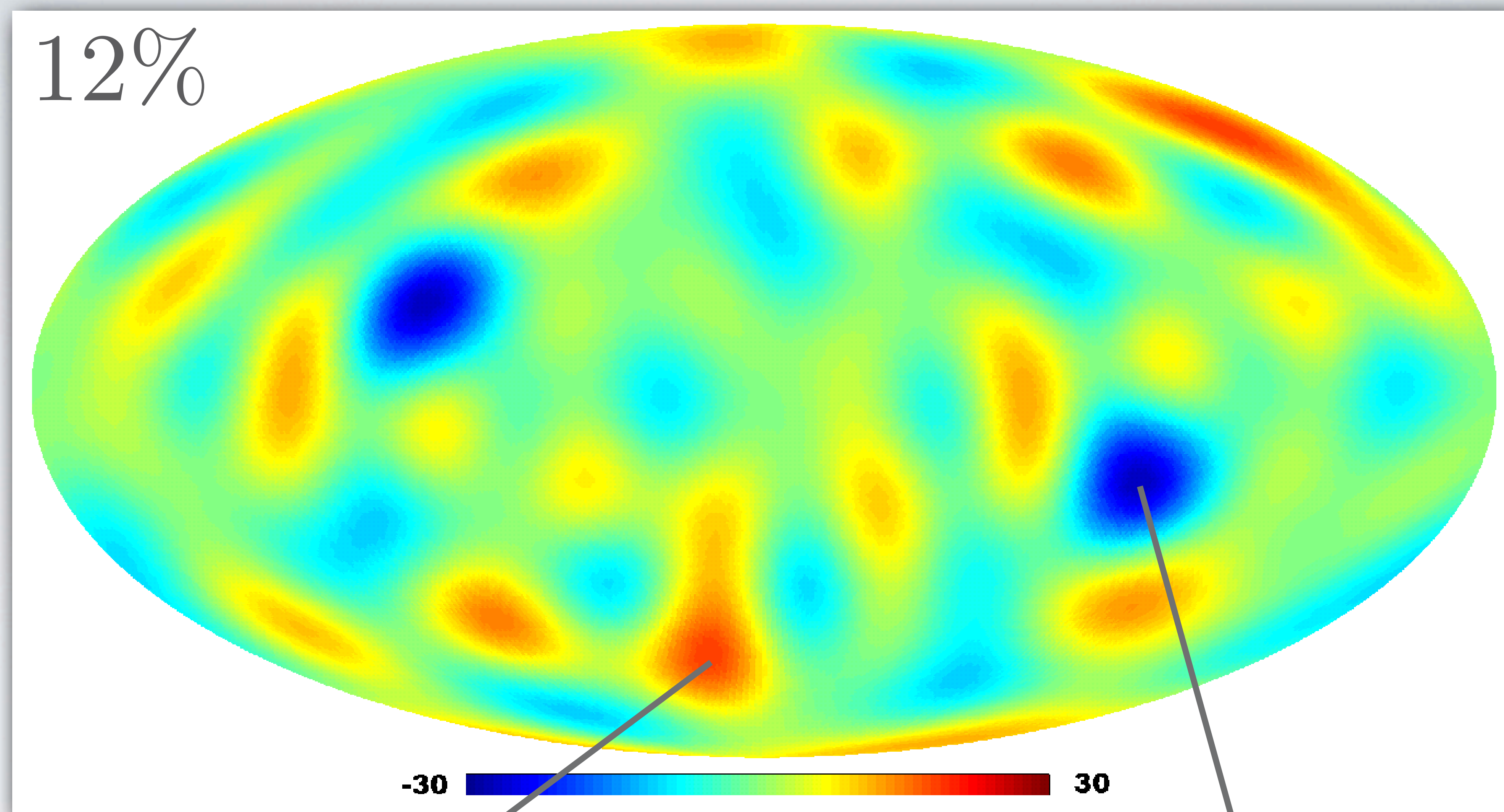


- Masking is crucial!

- Reconstruct $a_{\ell m}$ using covariance inversion method.
  (de Oliveira-Costa, Tegmark, PRD 2006; Efstathiou et al., MNRAS 2010; Aurich, Lustig, MNRAS 2011)

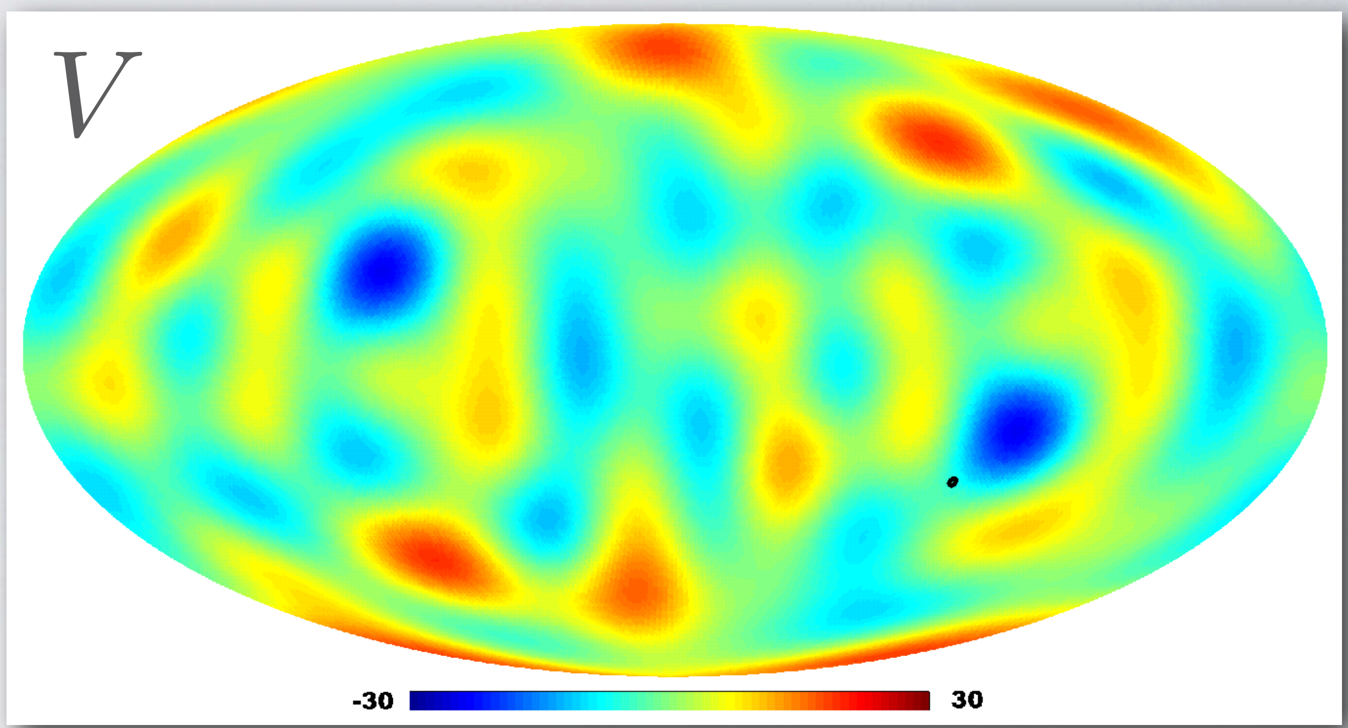
- Mask out a fixed total area A of most intensive pixels.

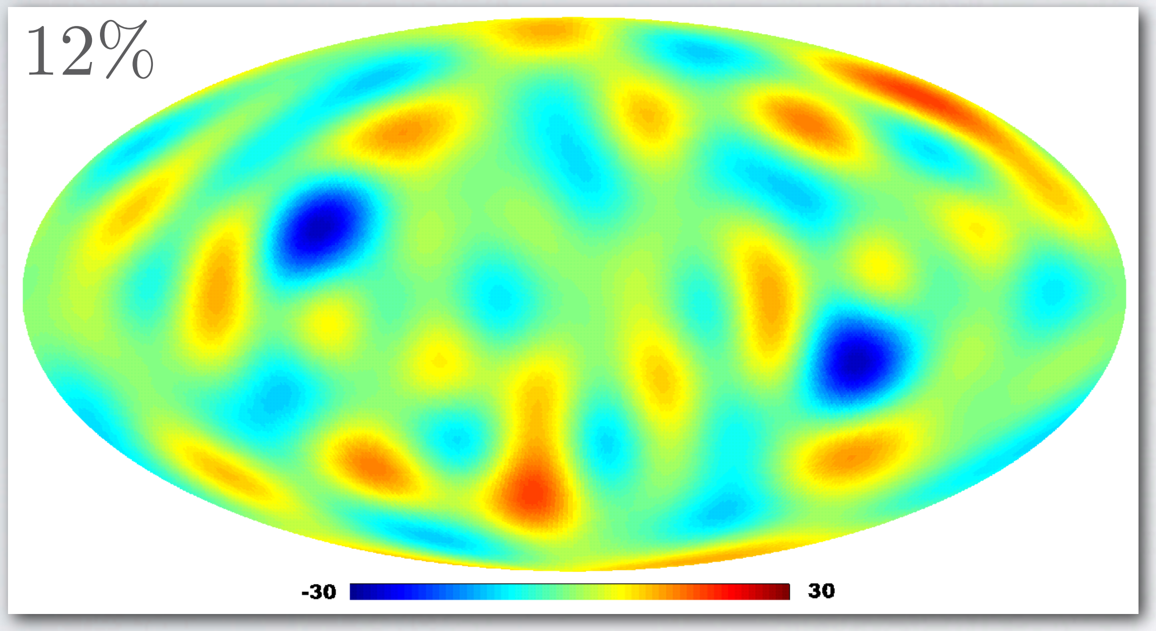
Parity Score – Masked Sky Results



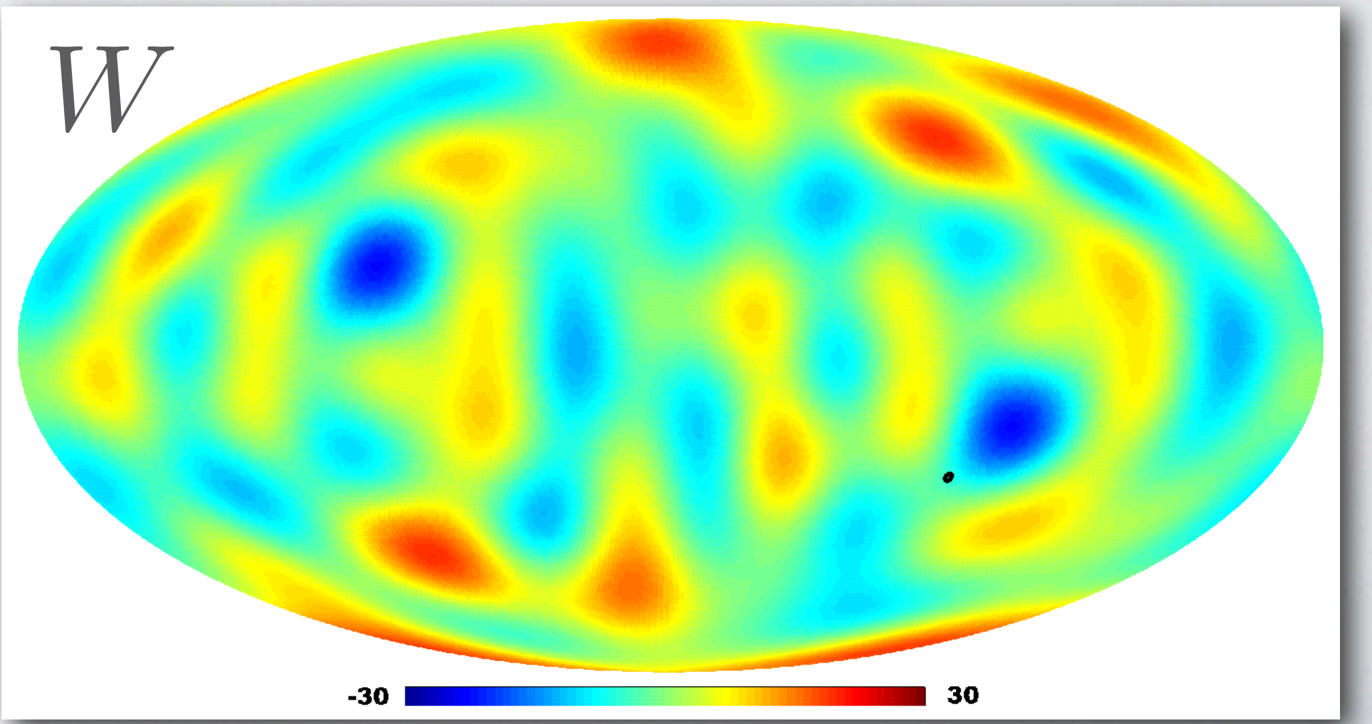
- At $A \sim 7\%$ jumps by almost 40°.
- Does not appear significant.
- Does not move, for all masks.
- Appears much more significant.

Separate Frequency Bands





Masked with KQ85



Significance of the Results

- Normalize score:

$$S_+(A) = \max_{\hat{\mathbf{n}}} S\left(\hat{\mathbf{n}}, A\right)$$

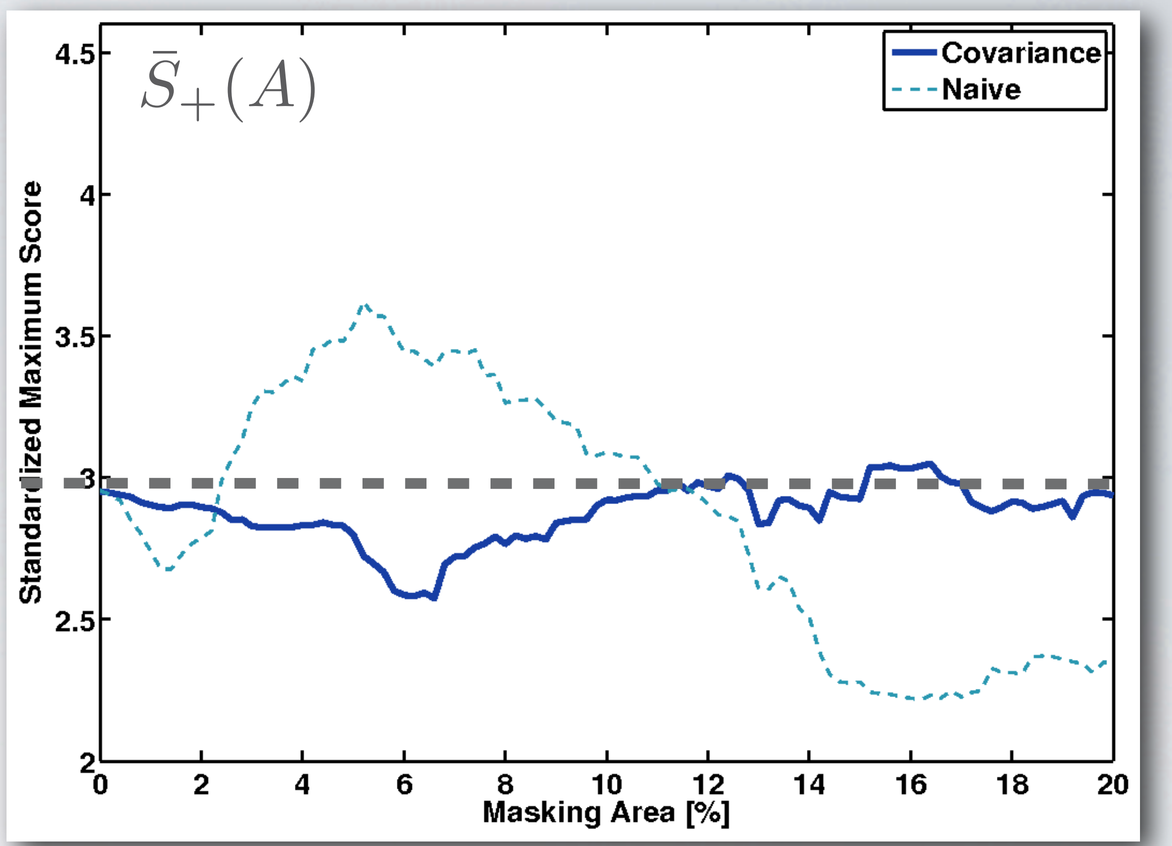
$$S_-(A) = \min_{\hat{\mathbf{n}}} S\left(\hat{\mathbf{n}}, A\right)$$

$$\bar{S}_\pm(A) = \left|\frac{S_\pm(A) - \mu(A)}{\sigma(A)}\right|$$

- Compare with random ΛCDM simulations.

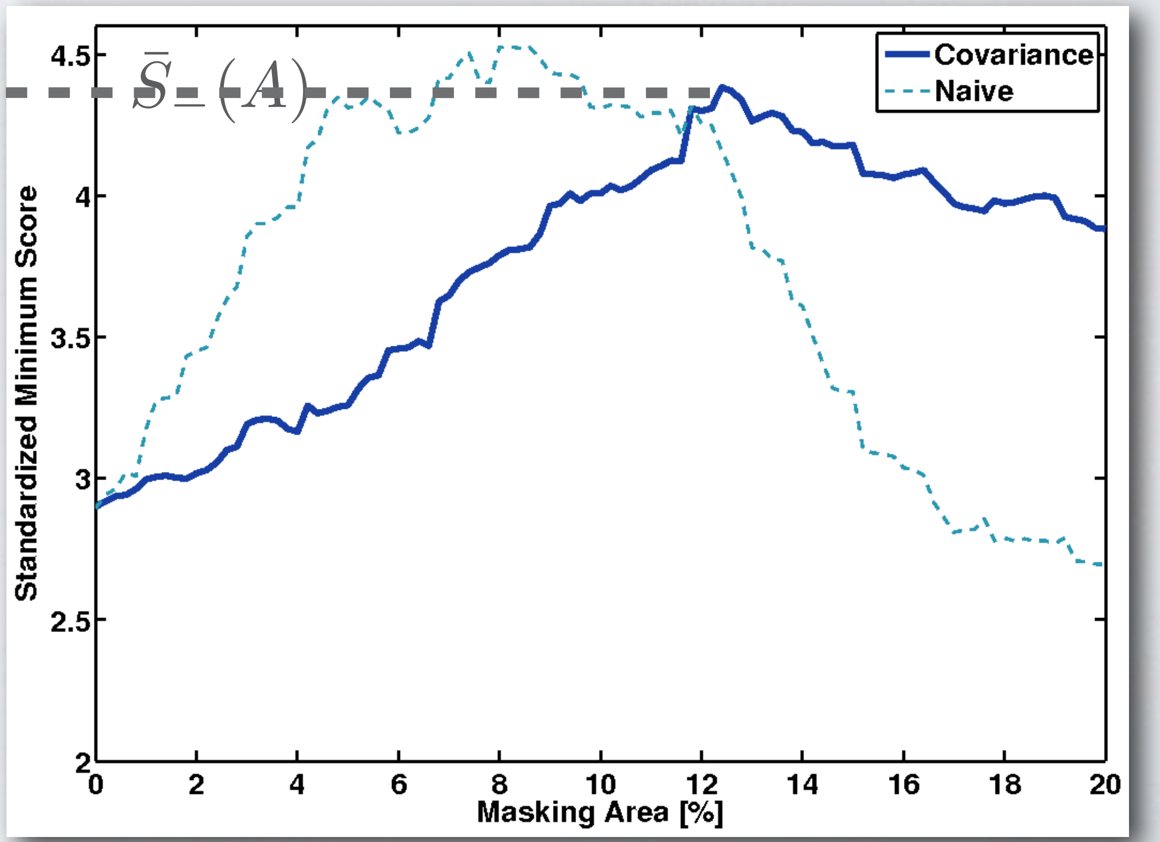
$\left(\ell_{\max} = 6 \Rightarrow 4.3\sigma \right)$

12%



0.03%

3.6σ



Basic Deformations

$$\langle \Phi_{\mathbf{k}} \rangle = 0$$

$$\langle \Phi_{\mathbf{k}} \Phi^*_{\mathbf{k}'} \rangle = (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}') P_\Phi(k)$$

Pre-inflationary relics

Non-trivial topologies

- Example: Pre-inflationary particle model (Fialkov, Itzhaki and Kovetz, JCAP 2010) and the search for giant concentric rings (Kovetz, ABD and Itzhaki, ApJ 2010).
- Perturbation drops with ℓ .

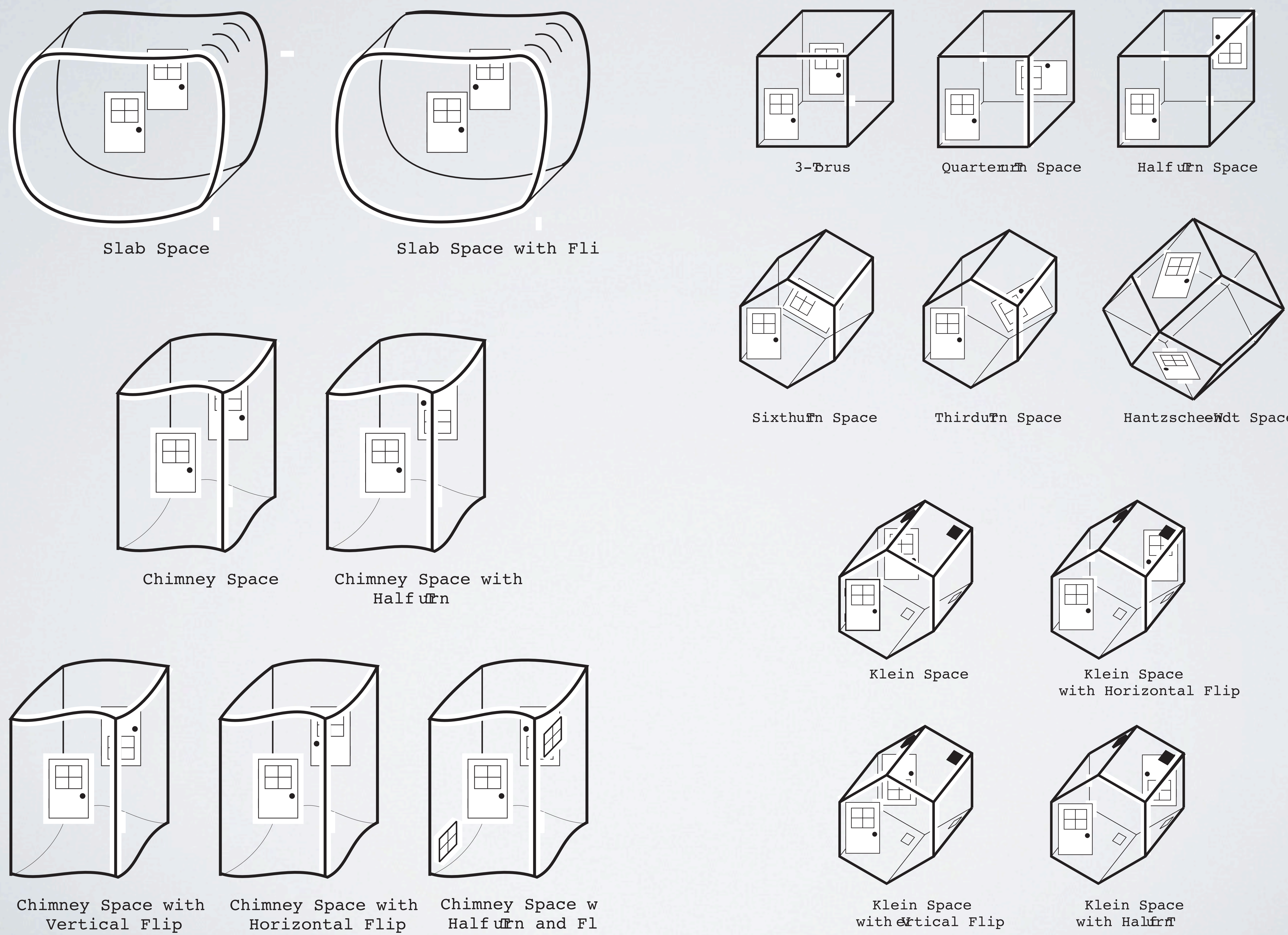
- Can also change the correlations in small scales.
- S/N can be significantly higher.
- Planck

Non-Trivial Topologies

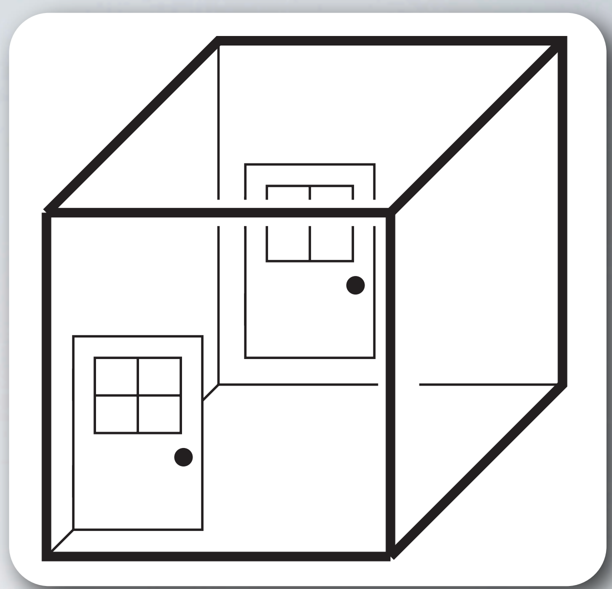
- GR only constrains geometry, not topology.
- Universe is flat, but not necessarily $\mathbb{R}^3$.
- Can identify points in space.
    - Classically, no fixed points allowed.
- Compact dimensions.

Classical 3D Flat Spaces

Riazuelo, Weeks, Uzan, Lehoucq, and Luminet, PRD 2004



The 3-Torus



- Fundamental domain with edges $L_x \times L_y \times L_z$ constrains

$$\mathbf{k} = 2\pi \left(\frac{n_x}{L_x}, \frac{n_y}{L_y}, \frac{n_z}{L_z} \right) \qquad\qquad n_i \in \mathbb{Z}$$

- Breaks isotropy but not homogeneity.

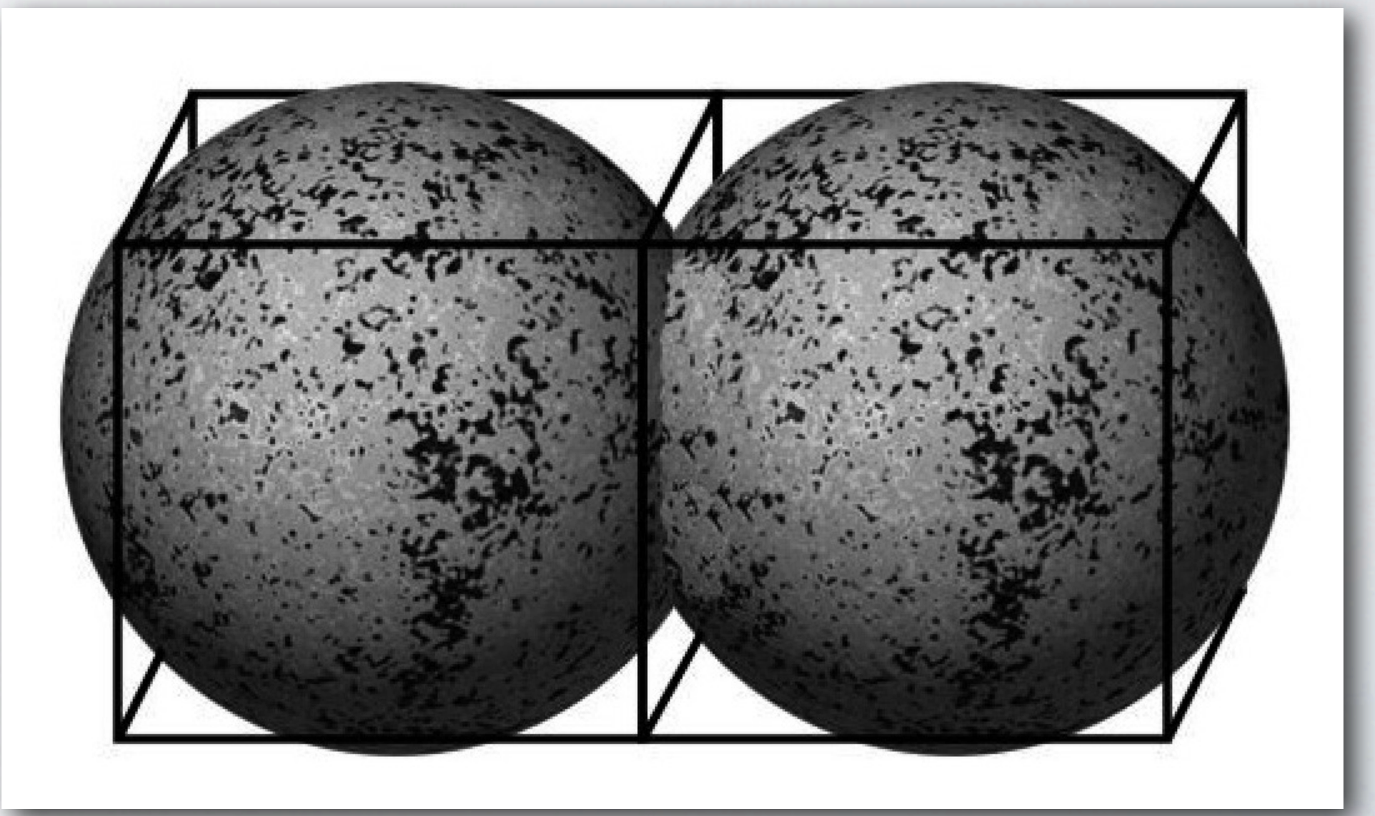
- Observational signatures:

  - Correlation matrix $C_{\ell m \ell' m'} = \langle a_{\ell m} a^*_{\ell' m'} \rangle$ is no longer diagonal.
    (Kunz, Aghanim, Cayón, Forni, Riazuelo, and Uzan, PRD 2006; Phillips and Kogut, ApJ 2006)

  - Matched circles in the sky.

Cornish et al., CQG 1998, PRL 2004
Key et al., PRD 2007
Bielewicz and Banday, MNRAS 2011
Vaudrevange et al., arXiv:1206.2939

$$S_{pq}(\alpha, \phi_*) = \frac{2 \sum_m m T_{p,m}(\alpha) T^*_{q,m}(\alpha) e^{-im\phi_*}}{\sum_n n \left(|T_{p,n}(\alpha)|^2 + |T_{q,n}(\alpha)|^2 \right)}$$



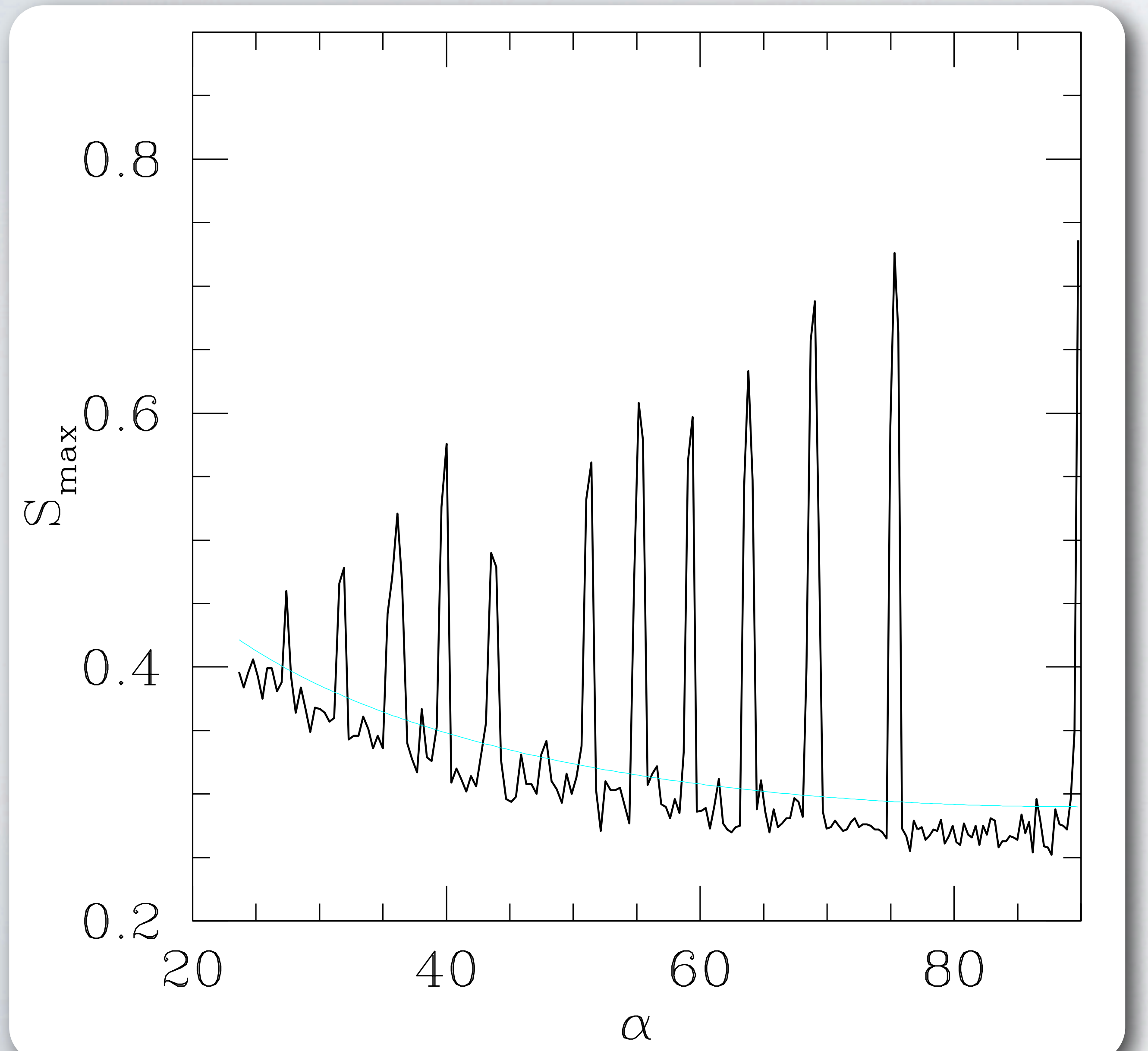
Matched Circles – Results

Cornish, Spergel, Starkman, Komatsu, PRL 2004

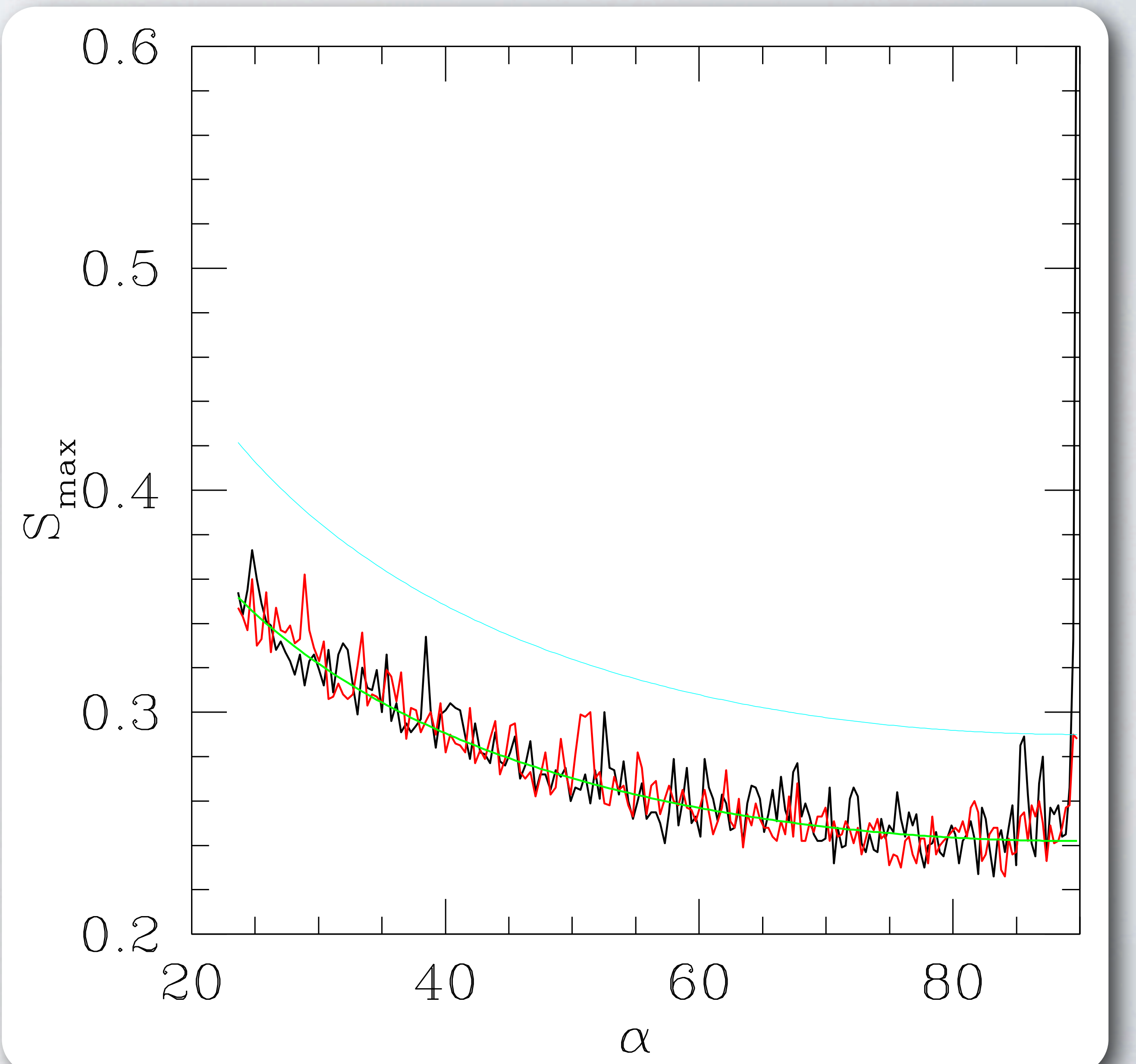
Size of fundamental domain $> 24\,\mathrm{Gpc}$.

Simulation with

$$L_x = L_y = L_z = 0.513\, r_{\mathrm{LSS}}$$



ILC Map

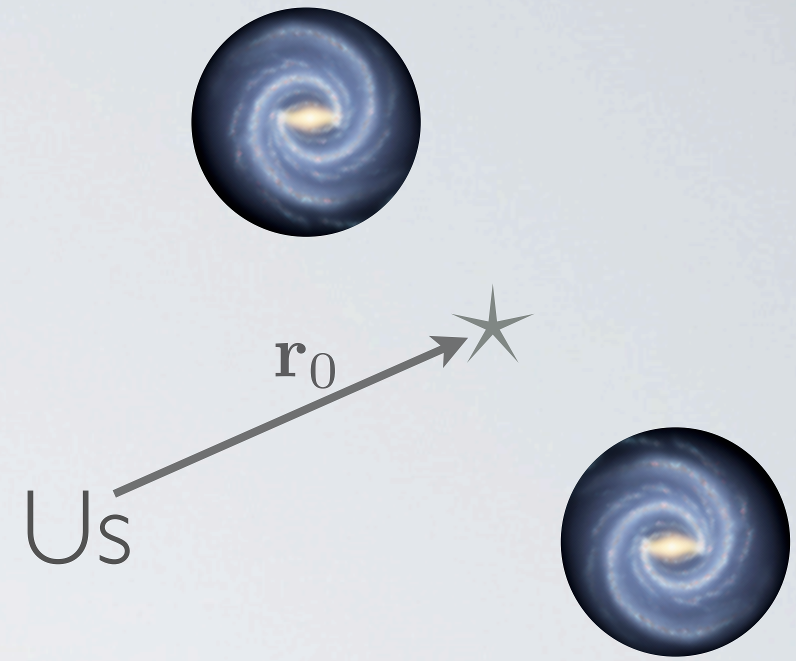


Stringy Topologies

ABD, Rathaus and Itzhaki, arXiv:1207.6218

- Identifications that include fixed points.
- DOF confined to the fixed points.
- As legitimate, in string theory, as the classical topologies.
- Doesn't have to include compact dimensions.
- Simplest examples:
    - Orbifold point
    - Orbifold line

Orbifold Point



- Universe is even with respect to some point.

- If the orbifold is located at $\mathbf{r}_0$, we identify

$$\mathbf{r} - \mathbf{r}_0 \sim \mathbf{r}_0 - \mathbf{r}$$

- The metric perturbation must then satisfy

$$\Phi(\mathbf{r}) = \Phi(2\mathbf{r}_0 - \mathbf{r}) \quad \Longrightarrow \quad \Phi_{\mathbf{k}} = e^{-2i\mathbf{k}\cdot\mathbf{r}_0}\Phi^*_{\mathbf{k}}$$

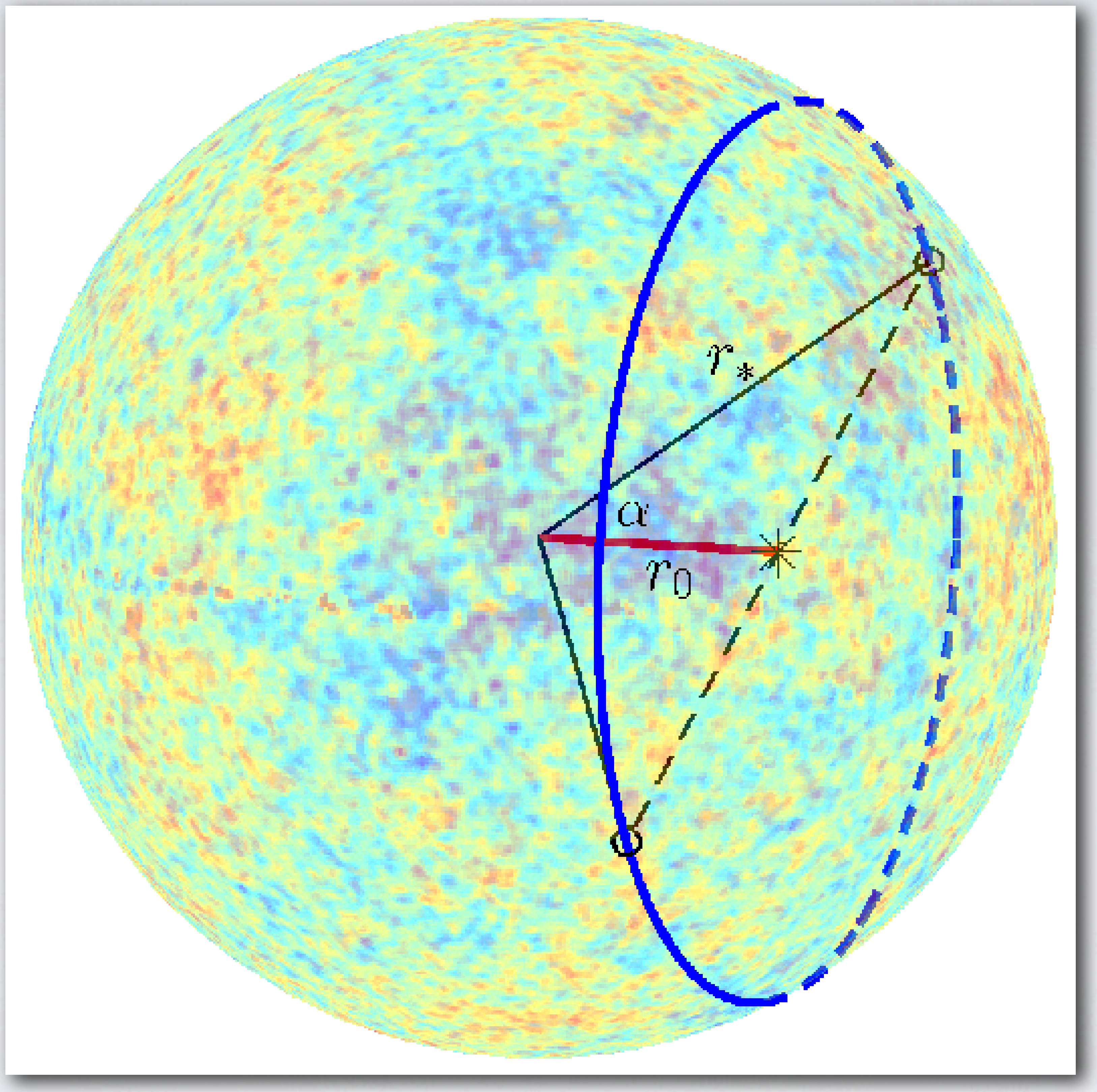
- The two-point function is now

$$\langle \Phi_{\mathbf{k}}\Phi^*_{\mathbf{k}'} \rangle = (2\pi)^3 \left(\delta(\mathbf{k}-\mathbf{k}') + e^{-2i\mathbf{k}\cdot\mathbf{r}_0}\delta(\mathbf{k}+\mathbf{k}')\right) P_\Phi(k)$$

- Observational signatures:
  - Off-diagonal angular correlations
  - A self matching circle

- Isotropy ✗
- Homogeneity ✗
- Gaussianity ✓

A Self Matching Circle (SMaC)



Angular Correlation Matrix

- Harmonic coefficients can be calculated as

$$a_{\ell m} = -\frac{i^\ell}{2\pi^2}\int \mathrm{d}^3k\,\Phi_{\mathbf{k}}\Delta_\ell(k)Y^*_{\ell m}(\hat{\mathbf{k}})$$

- Calculate angular correlation matrix $C_{\ell m \ell' m'} \equiv \langle a_{\ell m} a^*_{\ell' m'}\rangle$ using the two-point function $\langle \Phi_{\mathbf{k}}\Phi^*_{\mathbf{k}'}\rangle$.

- We get

$$C_{\ell m \ell' m'}(\mathbf{r}_0) = \delta_{\ell\ell'}\delta_{mm'}C_\ell^{(0)} + \Delta C_{\ell m \ell' m'}(\mathbf{r}_0)$$

where

$$C_\ell^{(0)} = \frac{2}{\pi}\int k^2\mathrm{d}k\,P_\Phi(k)|\Delta_\ell(k)|^2$$

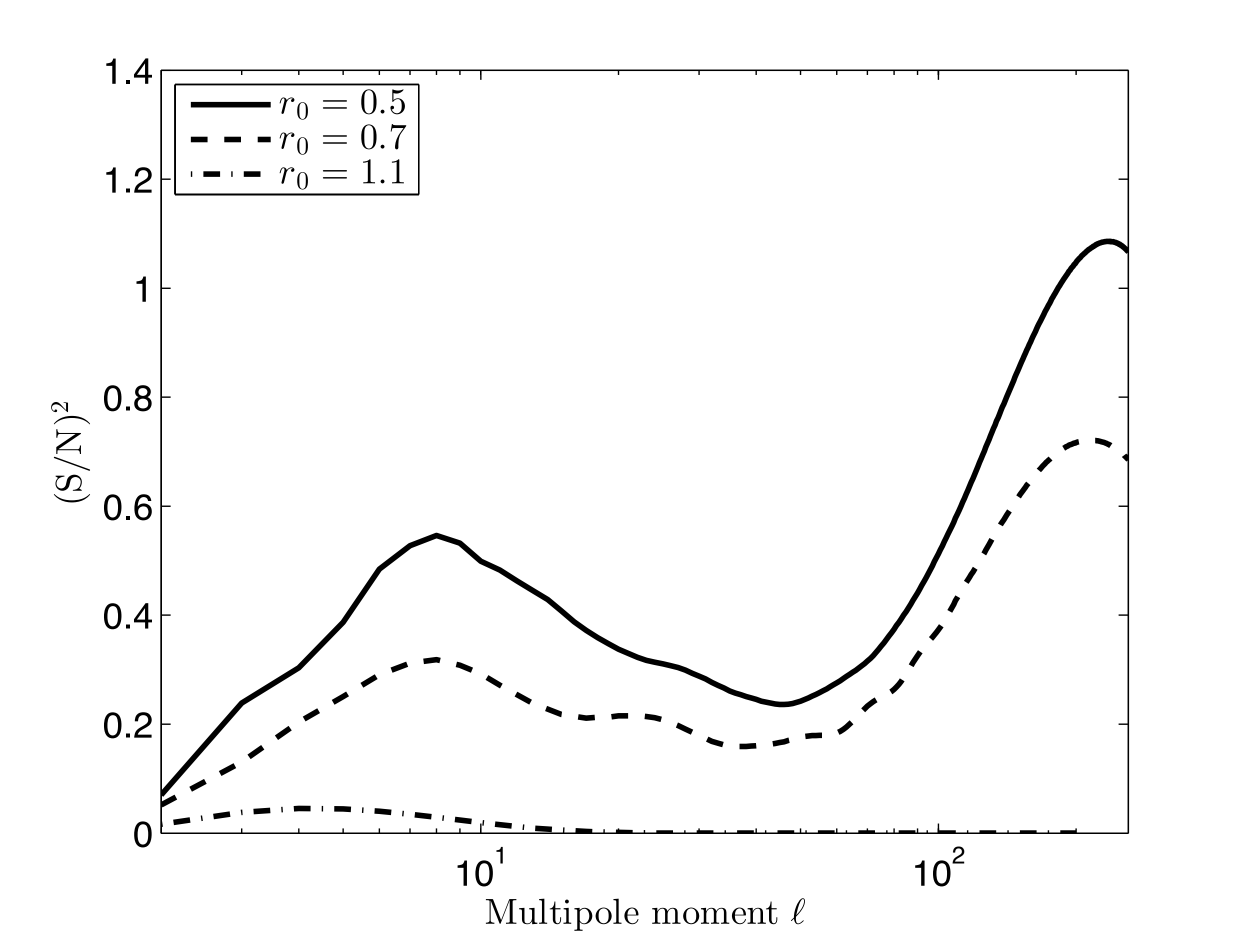
is the standard ΛCDM power spectrum.

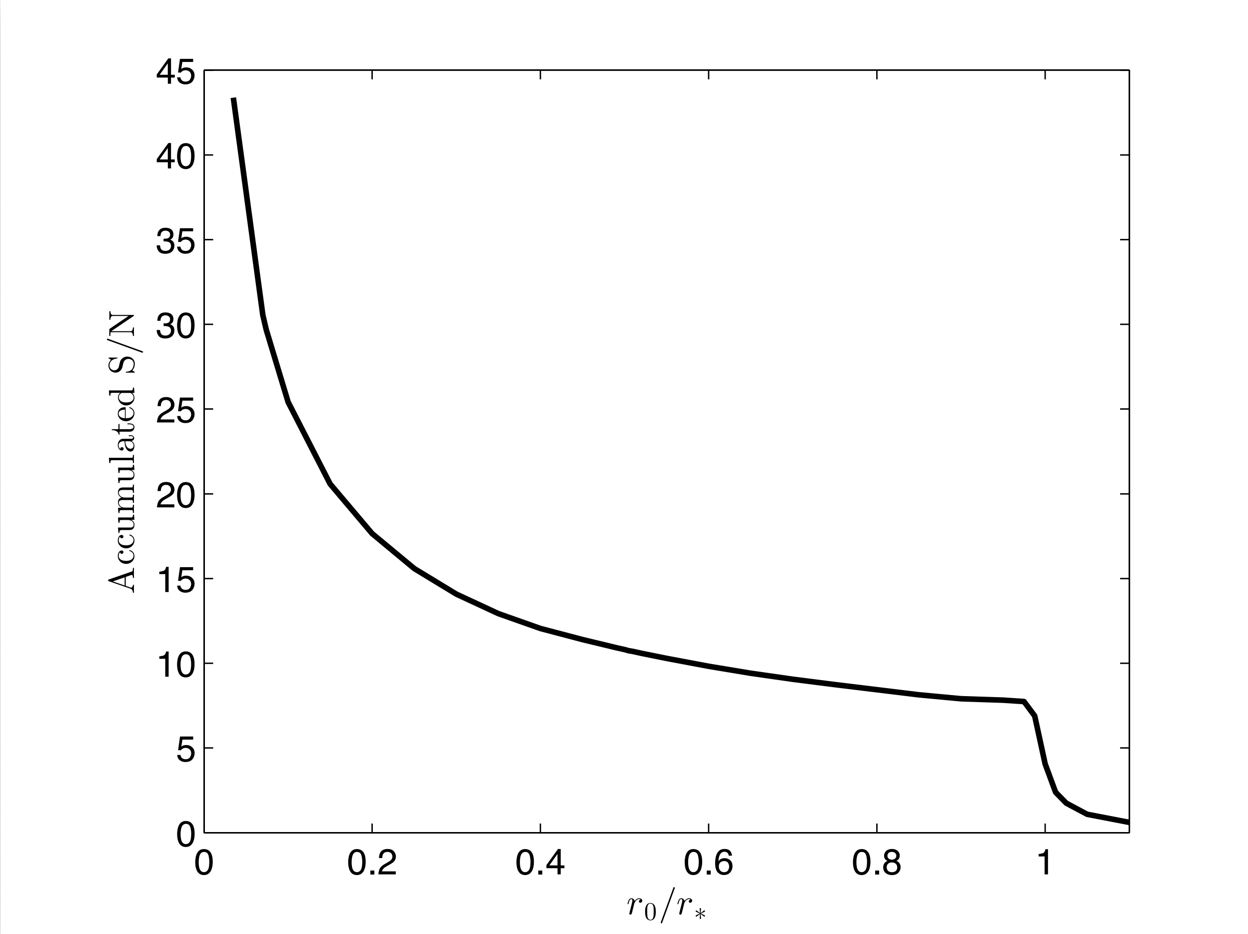
Signal to Noise for Detection

- Since the theory is Gaussian, all statistical information is encoded in the correlation matrix.

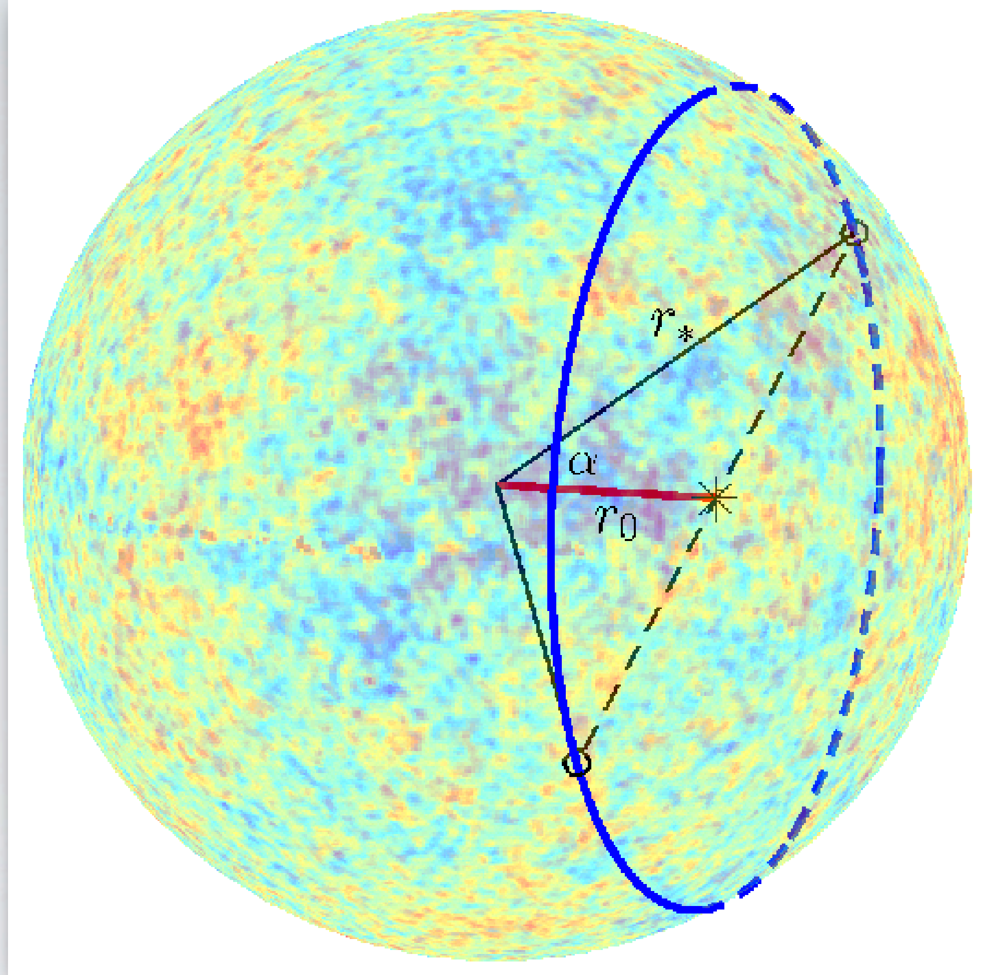
- Ideal S/N for detection, to leading order

$$\left(\frac{\mathrm{S}}{\mathrm{N}}\right)^2 \simeq \frac{1}{2}\sum_{\ell m \ell' m'} \frac{|\Delta C_{\ell m \ell' m'}|^2}{C_\ell^{(0)} C_{\ell'}^{(0)}}$$





Searching for a SMaC



- Use same score as for the matching circles of the torus $S_{pq}(\alpha, \phi_*)$, with $p = q$ and $\phi_* = \pi$

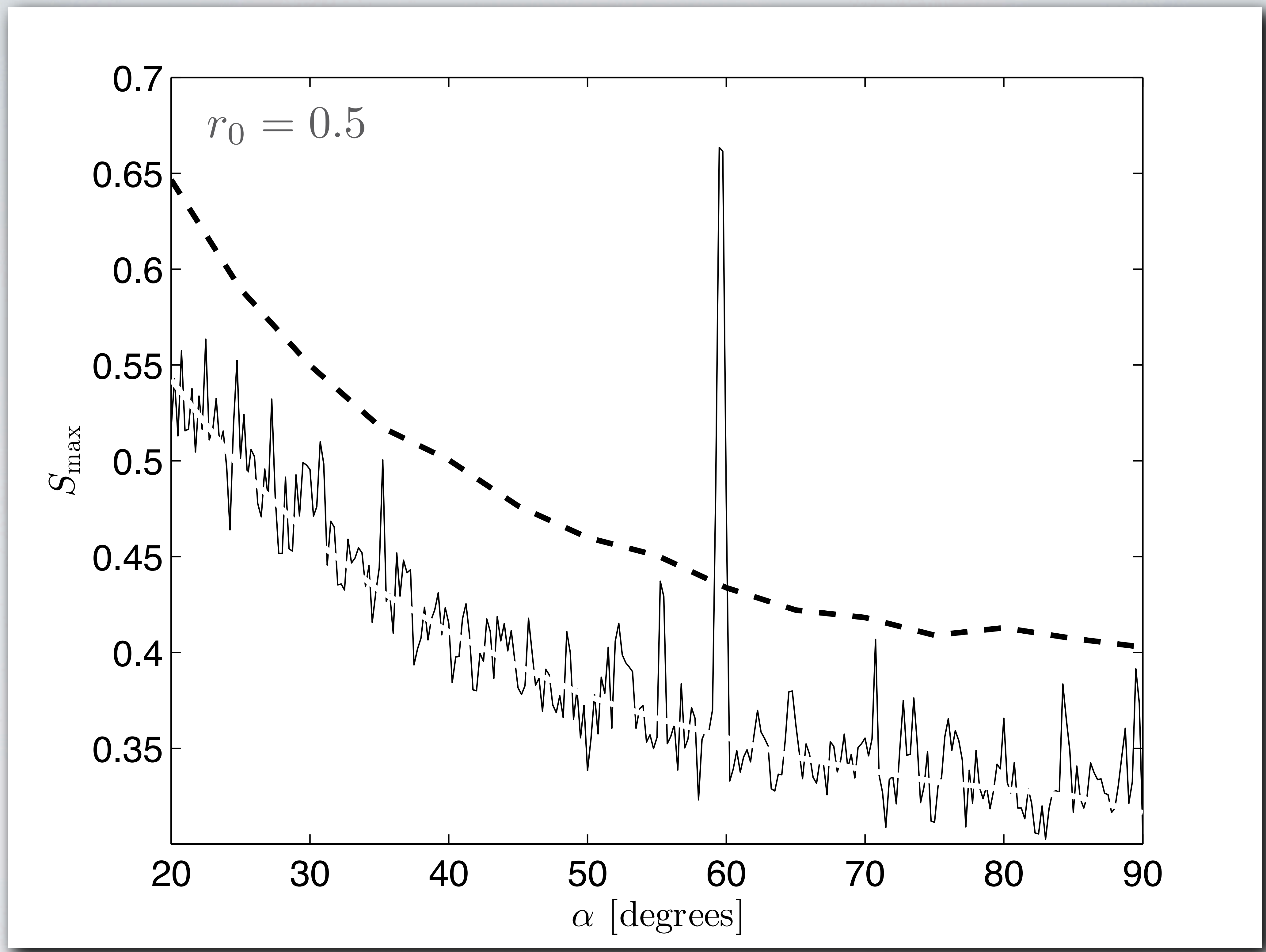
$$S_p(\alpha) = \frac{\langle T_p(\alpha, \phi) T_p(\alpha, \phi + \pi) \rangle}{\langle T_p^2(\alpha, \phi) \rangle}$$

- Fourier transform as $T_p(\alpha, \phi) = \sum_m T_{p,m}(\alpha) e^{im\phi}$ and add weights

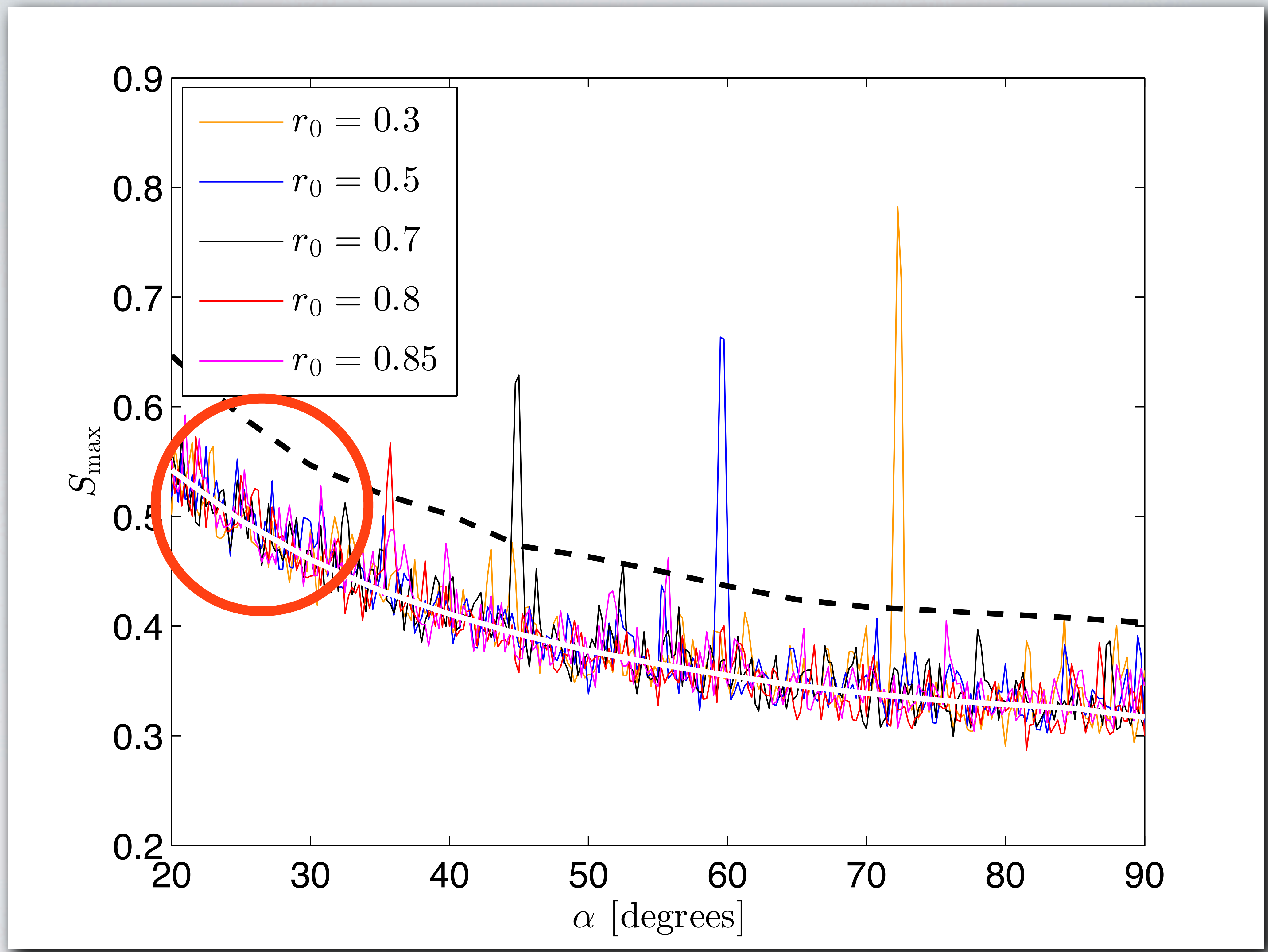
$$S_p(\alpha) = \frac{\sum_m (-1)^m m \left| T_{p,m}(\alpha) \right|^2}{\sum_n n \left| T_{p,n}(\alpha) \right|^2}$$

- Calculate $S_{\max}(\alpha)$, maximized over all pixels p.

SMaC Search – Simulations

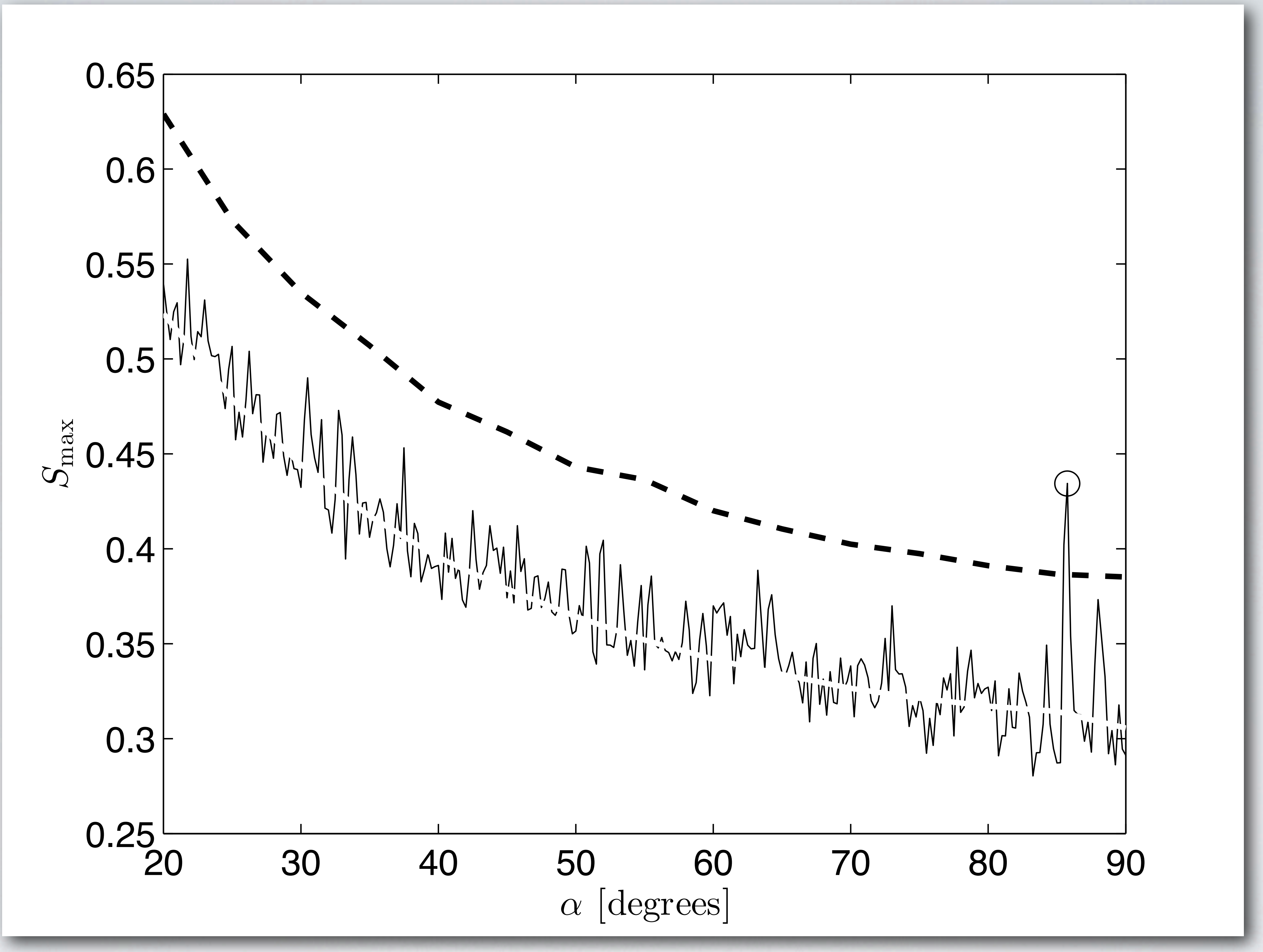


SMaC Search – Simulations



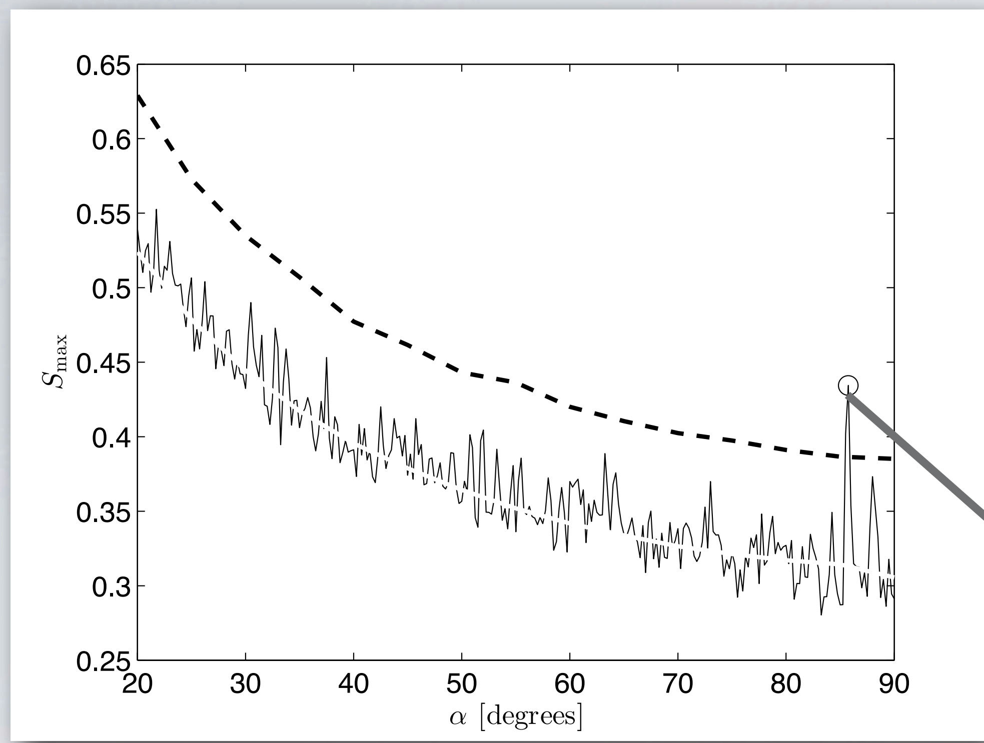
Detection threshold $\sim 0.85\, r_{\rm LSS}$

SMaC Search – Results



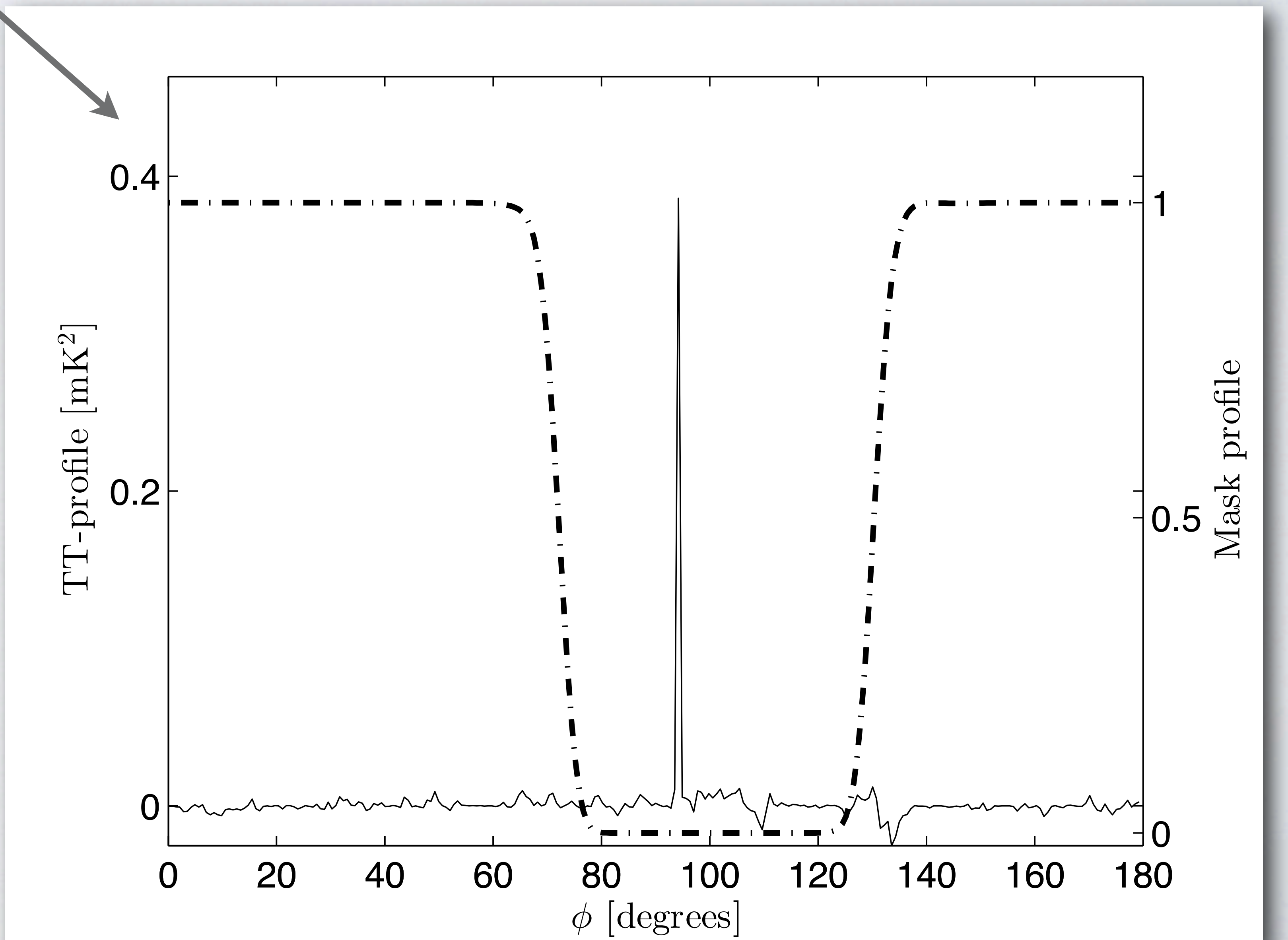
WMAP 7yr ILC Map

SMaC Search – Results



WMAP 7yr ILC Map

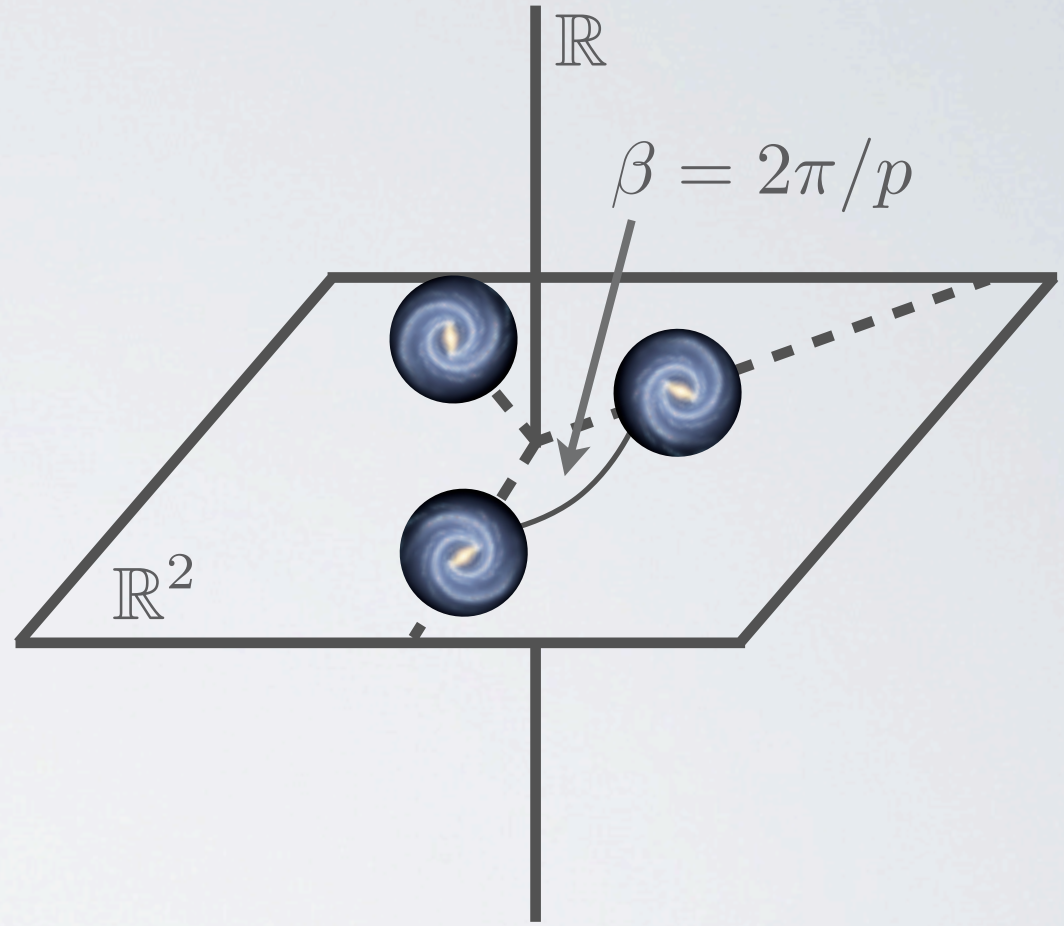
$T(\phi)T(\phi+\pi)$ profile



No detection

Orbifold Line

Work in progress, with B. Rathaus and N. Itzhaki



- Fixed line, instead of a point.
- $\mathbb{R} \times \mathbb{R}^2/\mathbb{Z}_p$ – quotient space under the action of the cyclic group $\mathbb{Z}_p$, where p is prime.
- Space is divided to p replicated sectors.
- If the line is located at $\mathbf{r}_0$, pointing towards $\hat{\mathbf{n}}$, we identify

$$\mathbf{r} - \mathbf{r}_0 \sim R_{\hat{\mathbf{n}}}(j\beta)[\mathbf{r} - \mathbf{r}_0]$$
$$(j = 1, 2, \ldots, p-1)$$

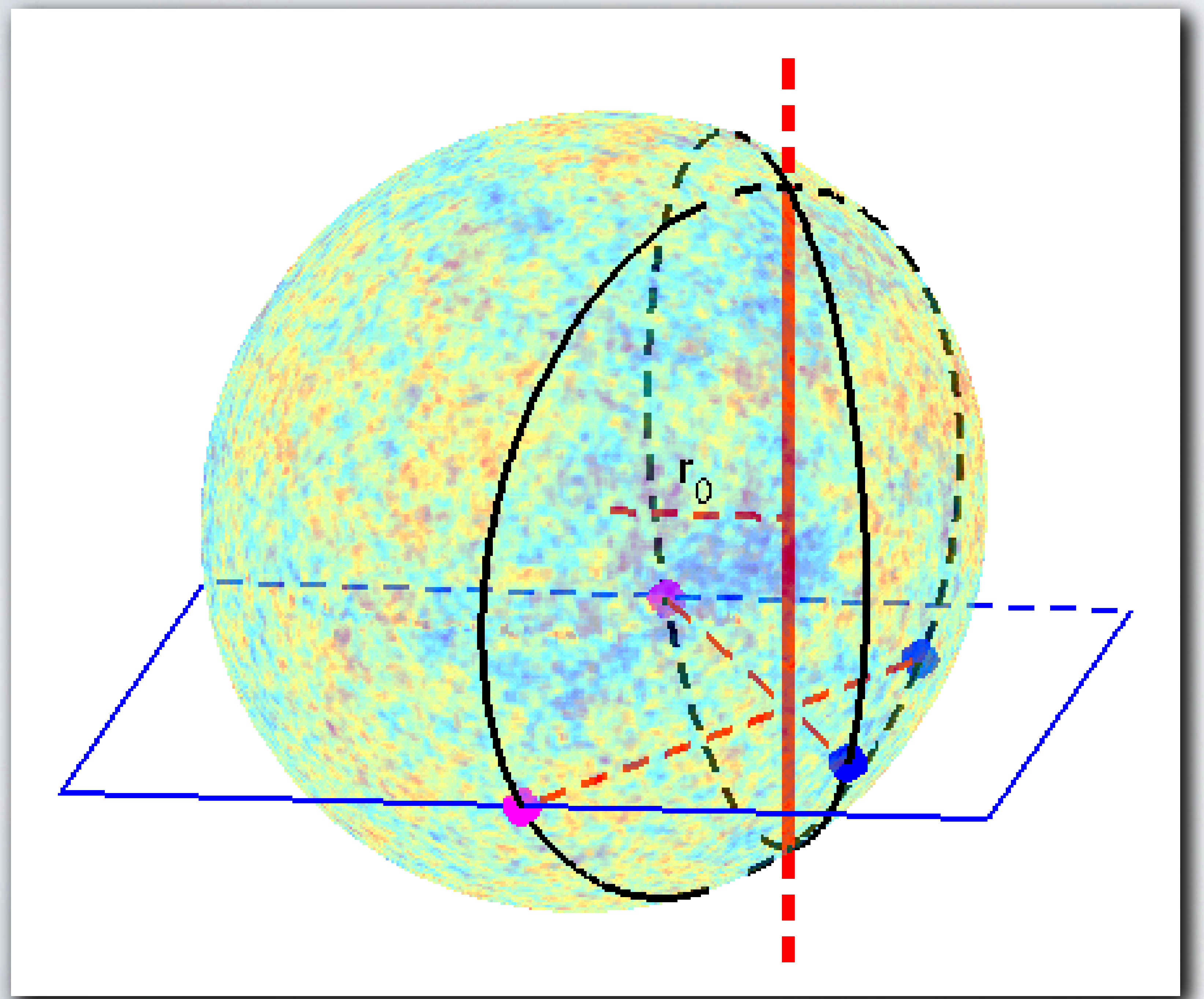
- Isotropy ✗
- Homogeneity ✗
- Gaussianity ✓

Orbifold Line – Observational Signatures

- Off-diagonal angular correlations

Work in progress

- Matched circles



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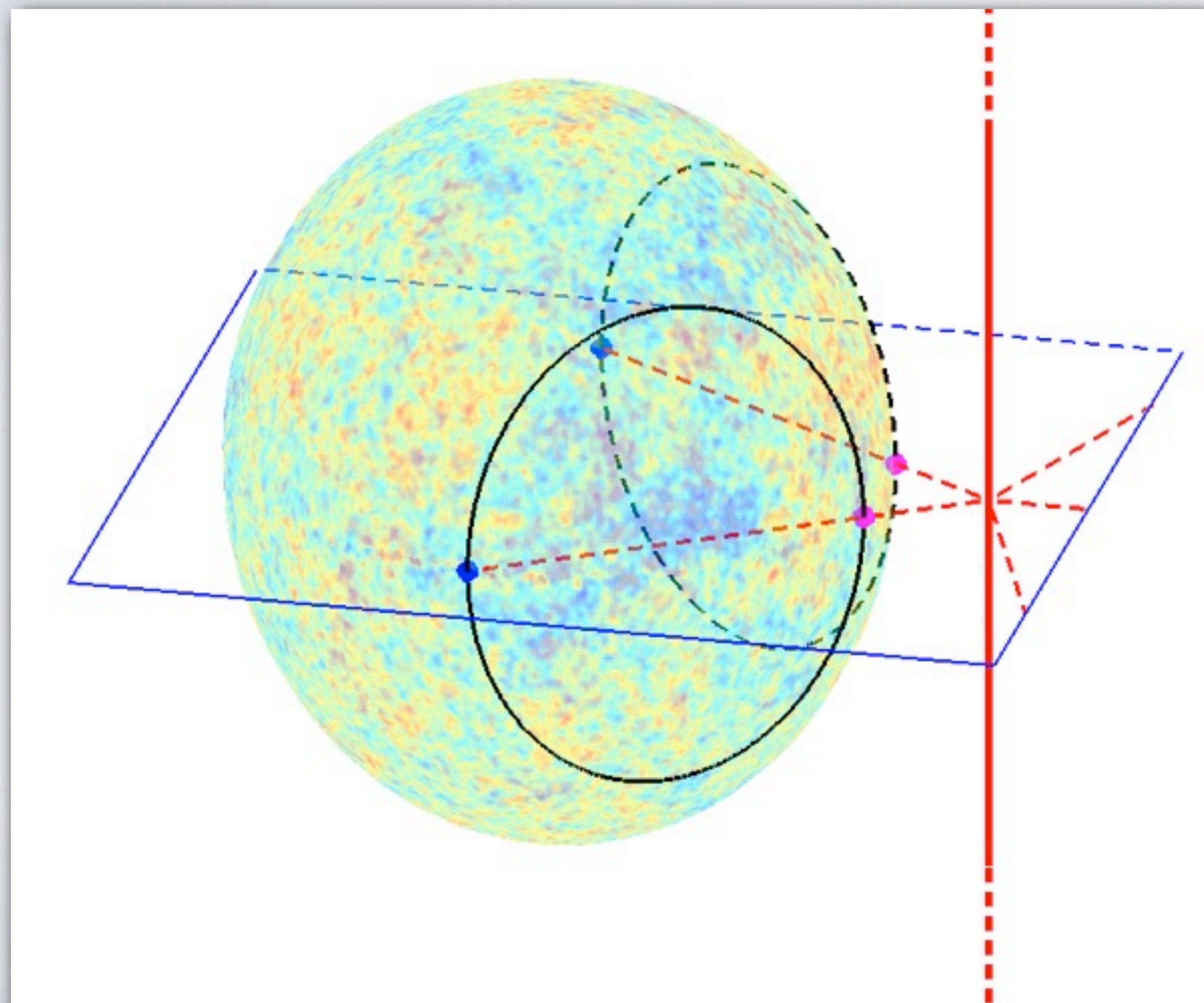
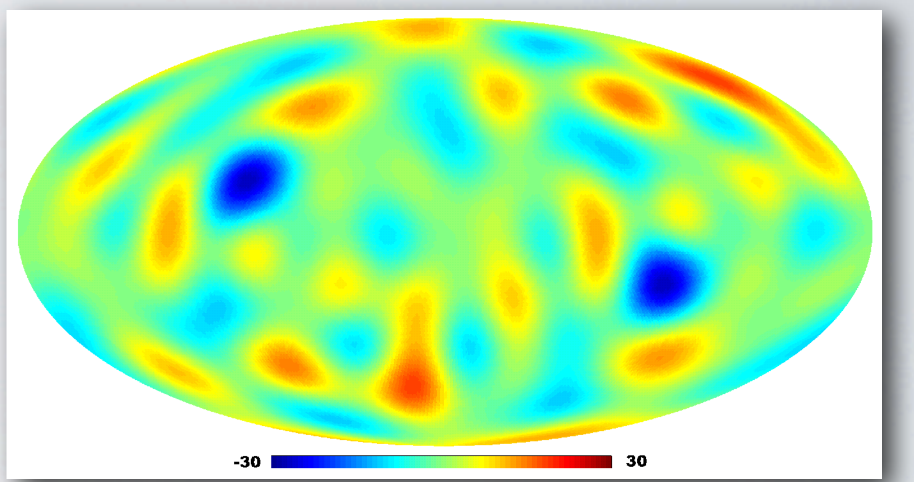
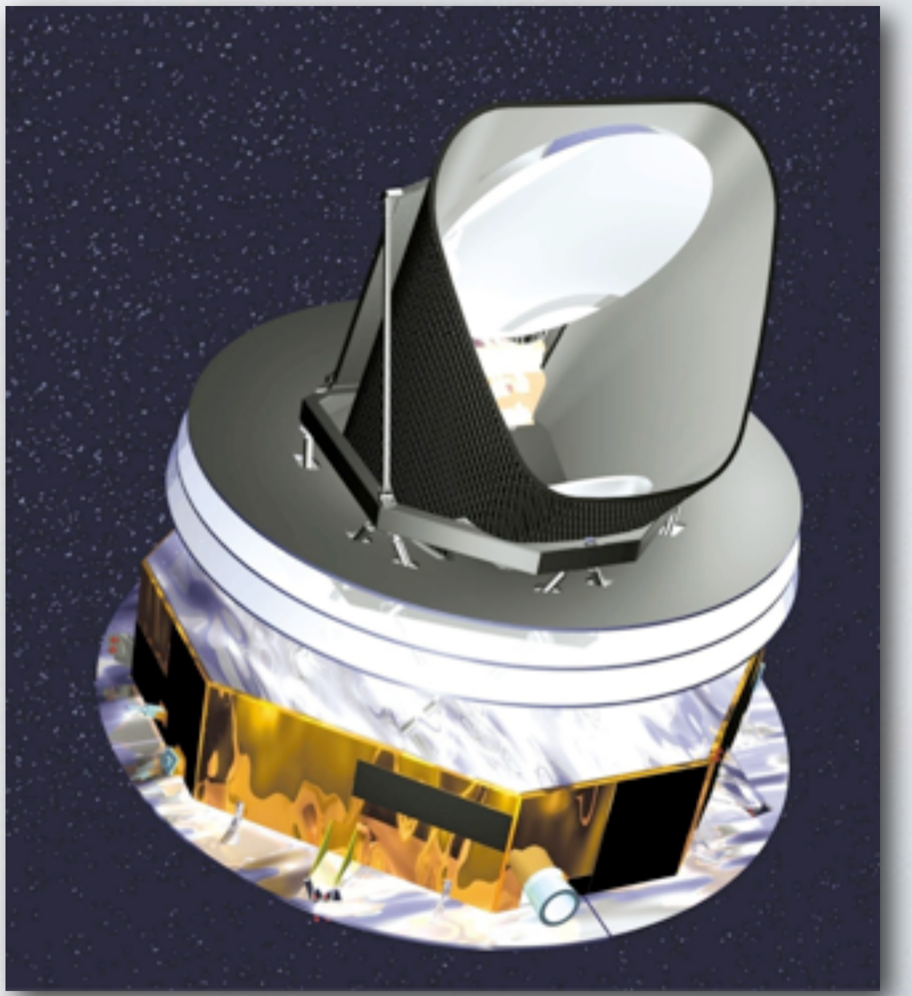


- Same detection score as before.
- Much more computationally intensive.
- So far – no detection.

Conclusions

- Large scale anomalies can be the result of pre-inflationary physics or a non-trivial topology.
- Search for parity: "Axis of Evil" ⇒ odd parity.
- Stringy topologies can also be considered.
  - Orbifold point: No detection.
  - Orbifold line: No detection, so far.
- Can also consider combinations of stringy and classical topologies.
- Data from Planck:
  - Small scales: Better resolution for searches of matching patterns
  - Weak lensing of small scales (Rathaus and Itzhaki, JCAP 2012)
  - Large scale anomalies





Thank You!