Higher order Post-Newtonian Corrections via Effective Field Theory

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Outline

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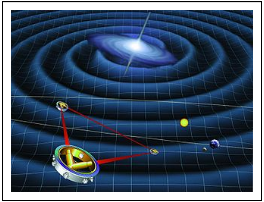
Gravitational Radiation

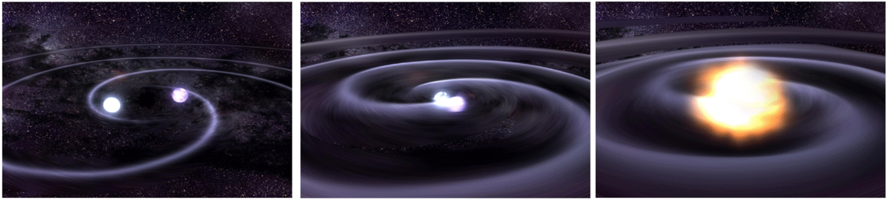
Prediction	1916 Einstein
Indirect Evidence	1974 Hulse & Taylor
Direct Observation	20?? Not yet!

Gravitational wave detectors:

- Ground-based
  - LIGO
  - GEO 600
  - Virgo
- Future ground-based
  - Advanced LIGO 2014
  - LCGT 2018
- Future space-based
  - eLISA 2025?



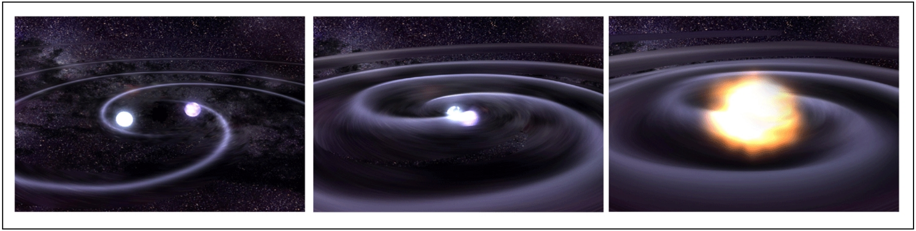
Sources of Gravitational Waves



Three phases in the life of a compact binary:

1. Inspiral
2. Merger
3. Ringdown

Sources of Gravitational Waves



Three phases in the life of a compact binary:

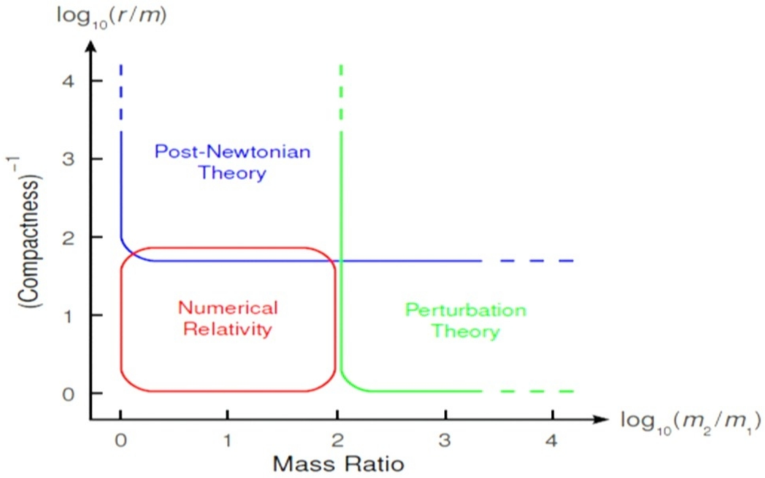
1. Inspiral
2. Merger
3. Ringdown

Detection by matched filtering $\Rightarrow$ Theoretical waveform templates

$\Rightarrow$Slight deviations in gravitational wave phase
would prevent successful detection

$\Rightarrow$ PN corrections required at least up to 4th order

Methods to compute gravitational-wave templates



⇒ Effective-One-Body approach, Damour and Buonanno 1999

*Plot courtesy of Luc Blanchet (Alexandre Le Tiec), talk presented at the EFT workshop, Nov. 2011, Perimeter Institute

What is the *post-Newtonian (PN) approximation of GR*?

Non-relativistic gravitationally bound systems,
i.e. such that satisfy

1. $v \ll 1$ $(c \equiv 1)$ with v the typical velocity
2. $Gm/r \sim v^2$ with m, r the typical mass, length

$nPN \equiv v^{2n}, n \in \mathbb{N}$, correction in GR to Newtonian gravity

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What is an *effective field theory (EFT)*?

An approximate theory that includes appropriate degrees of freedom to describe physical phenomena occurring at a chosen length scale, while ignoring substructure and degrees of freedom at shorter distances (or equivalently at higher energies).

Setup of the binary inspiral problem: Binary systems of compact objects emitting gravitational waves at the inspiral phase

$$\text{Length scales} \begin{cases} \text{size of compact object} & \equiv & r_s \\ \text{orbital separation of binary} & \equiv & r \\ \text{wavelength of radiation} & \equiv & \lambda \end{cases}$$

Setup of the binary inspiral problem:
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The binary inspiral is a multiple scale problem
$\Rightarrow$ EFT approach, Goldberger & Rothstein 2006

$$\left. \begin{array}{rcl} r_s/r & = & 2Gm/r \sim v^2 \\ r/\lambda & = & \underbrace{\frac{2\pi}{\lambda}}_{2\omega} \underbrace{\frac{r}{v}}_{1/\omega} v/2\pi \sim v \end{array} \right\} \Rightarrow r_s \lll r \ll \lambda$$

STAGE 1

- Isolated black hole described by $S_{EH}[g_{\mu\nu}]$.
- Interested in the dynamics at length scales $L \gg r_s$.
- Would have liked to define an effective action $S_{eff}[x^\mu, \bar{g}_{\mu\nu}]$ by integrating out the strong field modes $g^s_{\mu\nu}$, corresponding to the length scale r_s

$$e^{[iS_{eff}[x^\mu,\bar{g}_{\mu\nu}]]} \equiv \int \mathcal{D}g^s_{\mu\nu} e^{\left[iS_{EH}[g_{\mu\nu}\equiv g^s_{\mu\nu}+\bar{g}_{\mu\nu}]\right]},$$

equivalent to defining

$$S_{eff}[x, \bar{g}] \equiv S_{EH}[\bar{g}, g^s(x, \bar{g})]$$

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$$\Downarrow$$

Full theory may be unknown or known but strongly coupled

$\Downarrow$

$S_{eff}[x^\mu, \bar{g}_{\mu\nu}]$ can be expressed by introducing an infinite tower of worldline operators $O_i(\tau)$:

$$S_{eff}[x^\mu, \bar{g}_{\mu\nu}] = -\frac{1}{16\pi G}\int d^4x\sqrt{\bar{g}}R(x) + \underbrace{\sum_i c_i \int d\tau O_i(\tau)}_{\text{point particle action}}$$

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- All UV dependence shows up only in the Wilson coefficients $c_i(r_s)$.
- If the symmetries of the full theory that survive at low energies are known, the operators $O_i(\tau)$ must respect those symmetries.

Decoupling Theorem, Appelquist & Carazonne 1975

By writing down an effective action containing the most general set of worldline operators consistent with the symmetries of the full theory, we are accounting for the UV physics.

$$\Downarrow$$

The short distance field degrees of freedom around the black hole are integrated out by replacing them with an effective action

$$S_{eff}[x^\mu, \bar{g}_{\mu\nu}] = -\frac{1}{16\pi G}\int d^4x\sqrt{\bar{g}}R(x) - \int m d\tau + \underbrace{c_{10}\int d\tau\left(R_{\mu\alpha\nu\beta}\dot{x}^\alpha\dot{x}^\beta\right)^2 + \cdots}_{\text{finite size effects - not relevant here}}$$

$\Rightarrow$ Effective action of a binary, Goldberger & Rothstein 2006

$$S_{eff}[x_a^\mu, \bar{g}_{\mu\nu}] = S_{EH}[\bar{g}_{\mu\nu}] + \sum_{a=1}^{2} S_{pp}^{(a)}[x_a^\mu, \bar{g}_{\mu\nu}]$$

STAGE 2

- Now interested in the dynamics at length scales $L \gg r$.
- Metric is decomposed into $\bar{g}_{\mu\nu} \equiv \eta_{\mu\nu} + \underbrace{H_{\mu\nu}}_{\text{potential}} + \underbrace{\bar{h}_{\mu\nu}}_{\text{radiation}}$

	$H_{\mu\nu}$	$\bar{h}_{\mu\nu}$
k_0	v/r	v/r
$\vec{k}$	$1/r$	v/r

Effective action of composite object, Goldberger & Rothstein 2006

$S_{2bd}[x_a^\mu, \bar{h}_{\mu\nu}]$ is defined by integrating out the potential modes $H_{\mu\nu}$, corresponding to the length scale r

$$e^{\left[iS_{2bd}[x_a^\mu, \bar{h}_{\mu\nu}]\right]} \equiv \int \mathcal{D}H_{\mu\nu}\; e^{\left[iS_{eff}[x_a^\mu, \bar{g}_{\mu\nu}\equiv\eta+H+\bar{h}]\right]},$$

considering the classical limit.

- To obtain the conservative dynamics, set the radiation modes $\bar{h}_{\mu\nu}$ to 0.
- To obtain the radiative effects, consider the part of S_{2bd} with powers of $\bar{h}_{\mu\nu}$.

Reduction over the time dimension, Kol & Smolkin 2008

- Before integrating out the potential modes $H_{\mu\nu}$,
  note that they are instantaneous at LO!
  $\Rightarrow$ Apply nonrelativistic parametrization for the metric

$$d\tau^2 = e^{2\phi}(dt - A_i dx^i)^2 - e^{-2\phi}\gamma_{ij}dx^i dx^j$$

- Reduces over the time dimension à la Kaluza-Klein
  $\Rightarrow$ Change of field variables $\bar{g}_{\mu\nu} \to (\phi, A_i, \gamma_{ij} \equiv \delta_{ij} + \sigma_{ij})$
  — the non relativistic gravitational (NRG) fields

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NRG fields – Advantageous for PN applications, ML et al. 2010

Physical interpretation of field components:

- ϕ – Newtonian potential
- A_i – Gravito-magnetic vector, mediating LO interaction between 2 spins

Preferable over the common Lorentz covariant parametrization,
or Arnowitt-Deser-Misner decomposition.

Adding spin degrees of freedom

- Addition of a *spinning* part to the PP effective action at LO is equivalent to the pole-dipole approximation
- Two tetrads involved:

1. Background tetrad satisfying $g_{\mu\nu} = \eta_{ab} e^a_\mu e^b_\nu$ is fixed
2. Body fixed tetrad related by $e^\mu_A = \Lambda^a_A e^\mu_a$ to the background tetrad

- Generalized angular velocity in flat spacetime $\Omega^{ab} \equiv \Lambda^a_A \frac{d\Lambda^{Ab}}{d\tau} \Rightarrow \Omega^{\mu\nu} \equiv e^\mu_A \frac{De^{A\nu}}{D\tau}$ for curved spacetime

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$S_{pp} = \int d\tau L_{pp}(u^\mu, \Omega^{\mu\nu})$ containing 4 scalars,
that can be formed from u^μ and $\Omega^{\mu\nu}$:

1. $s_1 \equiv u^\mu u_\mu$
2. $s_2 \equiv \Omega^{\mu\nu}\Omega_{\mu\nu}$
3. $s_3 \equiv u^\mu \Omega_{\mu\nu}\Omega^{\nu\rho} u_\rho$
4. $s_4 \equiv \Omega^{\mu\nu}\Omega_{\nu\rho}\Omega^{\rho\kappa}\Omega_{\kappa\mu}$ *or* $det[\Omega^{\mu\nu}]$

- $\frac{\partial L_{pp}}{\partial u^\mu} \equiv -p_\mu$, $2\frac{\partial L_{pp}}{\partial \Omega^{\mu\nu}} \equiv -S_{\mu\nu}$
  + Reparametrization invariance
  $$\Rightarrow L_{pp} = -p_\mu u^\mu \underbrace{-\frac{1}{2} S_{\mu\nu}\Omega^{\mu\nu}}_{L_{pp(\mathbf{S})}} \leftarrow \text{minimal coupling}$$
- $L_{pp(\mathbf{S})}$ can be rewritten in terms of the spin connection $\omega_\mu^{ab} \equiv e^{b\nu} D_\mu e^a_\nu$ (Ricci rotation coefficients):
  $$L_{pp(\mathbf{S})} = \underbrace{-\frac{1}{2} S_{ab}\Omega^{ab}}_{\text{kinetic term}} - \frac{1}{2} S_{ab}\omega_\mu^{ab} u^\mu$$

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- Redundant unphysical degrees of freedom associated with the spin require to choose a spin supplementary condition (SSC)
- Take SSC of the form $C^{\mu} \equiv S^{\mu\nu} t_{\nu} = 0$;
  Covariant SSC $\Rightarrow S^{i0} = S^{ij} v^{j}$

- To derive the Feynman rules solve for the background tetrad in terms of $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- Expanding in $h_{\mu\nu}$ we obtain the following spin couplings

$$L_{pp(\mathbf{S})} = \frac{1}{2} S^{ab} h_{a\mu,b} u^{\mu} + \frac{1}{4} S^{ab} h^{\nu}_{b} \left(\frac{1}{2} h_{a\nu,\mu} + h_{\nu\mu,a} - h_{a\mu,\nu} \right) u^{\mu} + \cdots$$

- Note: the spin is derivative-coupled $\Rightarrow$ raises complexity of spin computations
- Power counting spin of compact object: $S \sim m v_{rot} r_s < m r_s \sim m v^2 r \sim L v$

The pure gravitational action: $S_g \equiv S_{EH} + S_{GF}$

$$S_g = \underbrace{-\frac{1}{16\pi G}\int dt\, d^3x \sqrt{\gamma}\left[-R[\gamma_{ij}] + 2\gamma^{ij}\partial_i\phi\partial_j\phi - \frac{1}{4}e^{4\phi}F_{ij}F_{kl}\gamma^{ik}\gamma^{jl}\right]}_{\text{Kaluza-Klein part of action}}$$

$$+ \underbrace{\frac{1}{8\pi G}\int d^4x\, (\partial_t\phi)^2}_{\text{treated as perturbation}} + \cdots + \frac{1}{32\pi G}\int d^4x \sqrt{g}\, g_{\mu\nu}\Gamma^{\mu}\Gamma^{\nu}$$

$$\gamma \equiv det(\gamma_{ij}), \quad F_{ij} \equiv \partial_i A_j - \partial_j A_i, \quad \Gamma^{\mu} \equiv \Gamma^{\mu}_{\rho\sigma} g^{\rho\sigma}$$

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The propagators

$$\phi \text{ ——— } \phi = 4\pi G \int \frac{d^3k}{(2\pi)^3}\frac{e^{i\vec{k}\cdot(\vec{x}_1-\vec{x}_2)}}{\vec{k}^2}\, \delta(t_1 - t_2)$$

$$A_i \text{ - - - - } A_j = -16\pi G\, \delta_{ij} \int \frac{d^3k}{(2\pi)^3}\frac{e^{i\vec{k}\cdot(\vec{x}_1-\vec{x}_2)}}{\vec{k}^2}\, \delta(t_1 - t_2)$$

$$\sigma_{ij} \text{ ═══ } \sigma_{kl} = 32\pi G\; P_{ij;kl} \int \frac{d^3k}{(2\pi)^3}\frac{e^{i\vec{k}\cdot(\vec{x}_1-\vec{x}_2)}}{\vec{k}^2}\, \delta(t_1 - t_2)$$

$$P_{ij;kl} \equiv \frac{1}{2}\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - 2\delta_{ij}\delta_{kl}\right)$$

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$$\gamma \equiv det(\gamma_{ij}), \quad F_{ij} \equiv \partial_i A_j - \partial_j A_i, \quad \Gamma^\mu \equiv \Gamma^\mu_{\rho\sigma}g^{\rho\sigma}$$

The propagator correction vertices

 $= \frac{1}{8\pi G} \int d^4x\, [\partial_t\phi]^2$

 $= -\frac{1}{32\pi G} \int d^4x\, [\partial_t A_i]^2$

The pure gravitational action: $S_g \equiv S_{EH} + S_{GF}$

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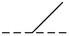
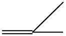
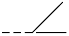
The 3-graviton vertices



$$= \frac{1}{8\pi G}\int d^4x\ \phi\left[\partial_i A_j\left(\partial_i A_j - \partial_j A_i\right) + \left(\partial_i A_i\right)^2\right]$$



$$= \frac{1}{16\pi G}\int d^4x\ \left[2\sigma_{ij}\partial_i\phi\partial_j\phi - \sigma_{jj}\partial_i\phi\partial_i\phi\right]$$



$$= -\frac{1}{4\pi G}\int d^4x\ \left[A_i\partial_i\phi\partial_t\phi\right]$$

The point particle action

$$
\begin{aligned}
S_{pp} &= -m \int dt \left[e^{\phi} \sqrt{(1 - A_i v^i)^2 - e^{-4\phi} \gamma_{ij} v^i v^j} \right] \\
&= -m \int dt \left[1 - \frac{1}{2} v^2 + \phi - A_i v^i + \frac{3}{2} \phi v^2 \right. \\
&\quad \left. - \frac{1}{2} \sigma_{ij} v^i v^j - \frac{1}{2} A_i v^i v^2 + \cdots + \frac{1}{2} \phi^2 - \phi A_i v^i + \cdots \right]
\end{aligned}
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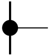
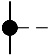
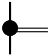
The 1-graviton mass couplings

 $= -m \int dt \; \phi \;\; \left[1 + \frac{3}{2} v^2 + \cdots\right]$

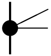
 $= \;\; m \int dt \; A_i v^i \left[1 + \frac{1}{2} v^2 + \cdots\right]$

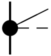
 $= \frac{m}{2} \int dt \; \sigma_{ij} v^i v^j \left[1 + \cdots\right]$

The point particle action

$$S_{pp} = -m\int dt\,\left[e^{\phi}\sqrt{\left(1-A_iv^i\right)^2-e^{-4\phi}\gamma_{ij}v^iv^j}\,\right]$$
$$= -m\int dt\,\left[1-\frac{1}{2}v^2+\phi-A_iv^i+\frac{3}{2}\phi v^2\right.$$
$$\left.-\frac{1}{2}\sigma_{ij}v^iv^j-\frac{1}{2}A_iv^iv^2+\cdots+\frac{1}{2}\phi^2-\phi A_iv^i+\cdots\right]$$

The 2-graviton mass couplings

 $= -\frac{m}{2}\int dt\ \phi^2\quad[1+\cdots]$

 $= \ m\int dt\ \phi A_iv^i\,[1+\cdots]$

The *spin* point particle action

$$S_{pp(\mathbf{S})} = \int dt \left[\frac{1}{4} S^{ij} F_{ij} + S^{ij} \partial_j \phi v^i + S^{0i} \partial_i \phi + \frac{1}{2} S^{ij} \partial_i \sigma_{jk} v^k \right.$$
$$- \frac{1}{2} S^{0i} \partial_i A_j v^j + \frac{1}{2} S^{0i} \partial_t A_i - S^{0i} \partial_t \phi v^i + \cdots$$
$$\left. + S^{ij} F_{ij} \phi - \frac{1}{2} S^{ij} A_i \partial_j \phi + 2 S^{0i} \phi \partial_i \phi + \cdots \right]$$

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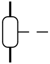
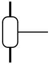
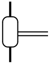
$$\left. + S^{ij} F_{ij} \phi - \frac{1}{2} S^{ij} A_i \partial_j \phi + 2 S^{0i} \phi \partial_i \phi + \cdots \right]$$

The 1-graviton *spin* couplings


$$= \int dt \, \frac{1}{2} \left[S^{ij} \partial_i A_j - S^{0i} \partial_i A_j v^j + S^{0i} \partial_t A_i \right]$$

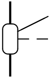

$$= \int dt \left[S^{ij} \partial_j \phi v^i + S^{0i} \partial_i \phi - S^{0i} \partial_t \phi v^i \right]$$


$$= \int dt \left[\frac{1}{2} S^{ij} \partial_i \sigma_{jk} v^k + \cdots \right]$$

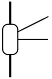
The *spin* point particle action

$$S_{pp(\mathbf{S})} = \int dt \Big[\frac{1}{4} S^{ij} F_{ij} + S^{ij} \partial_j \phi v^i + S^{0i} \partial_i \phi + \frac{1}{2} S^{ij} \partial_i \sigma_{jk} v^k$$
$$- \frac{1}{2} S^{0i} \partial_i A_j v^j + \frac{1}{2} S^{0i} \partial_t A_i - S^{0i} \partial_t \phi v^i + \cdots$$
$$+ S^{ij} F_{ij} \phi - \frac{1}{2} S^{ij} A_i \partial_j \phi + 2 S^{0i} \phi \partial_i \phi + \cdots \Big]$$

The 2-graviton *spin* couplings



$$= \int dt \left[2 S^{ij} \partial_i A_j \phi - \tfrac{1}{2} S^{ij} A_i \partial_j \phi + \cdots \right]$$



$$= \int dt \left[2 S^{0i} \phi \partial_i \phi + \cdots \right]$$

Construction of Feynman diagrams

- No graviton loops!
- Only connected diagrams when worldlines are stripped off
- Certain diagram topologies contribute at each order of G
- Each Feynman diagram contributes a definite power of v

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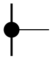

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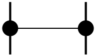

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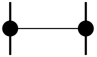
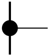


$$= \int dt \ \frac{G m_1 m_2}{r}, r \equiv \left|\vec{x}_1 - \vec{x}_2\right|, \vec{n} \equiv \frac{\vec{r}}{r}$$

From now on we suppress $\int dt$ in diagram values.

1PN correction (Einstein, Infeld & Hoffmann, 1938)

1PN potential – one-graviton exchange



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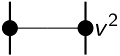

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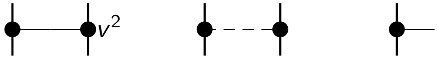

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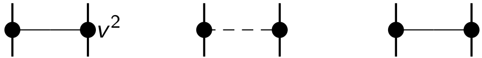

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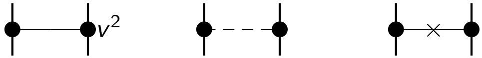

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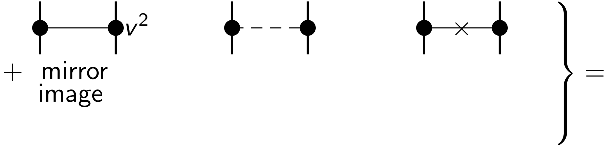
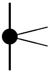


$$= \frac{Gm_1 m_2}{2r}\left(3v_1^2 + 3v_2^2 - 7\vec{v}_1\cdot\vec{v}_2 - \vec{v}_1\cdot\vec{n}\vec{v}_2\cdot\vec{n}\right)$$

1PN potential – nonlinear part



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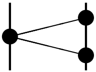

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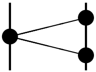
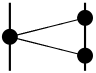


$+$ mirror image

$$= -\frac{G^2 m_1 m_2 (m_1 + m_2)}{2r^2}$$

1PN potential – nonlinear part



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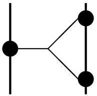

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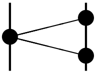

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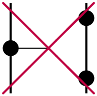
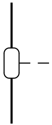
$$= -\frac{G^2 m_1 m_2 (m_1+m_2)}{2r^2}$$



NRG fields – no one-loop diagram on 1PN calculation!

LO (2PN) spin1-spin2 potential (Barker &O'Connell 1975)



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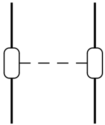

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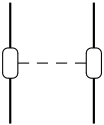
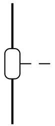


$$= \frac{G}{r^3}\left(\vec{S}_1\cdot\vec{S}_2 - 3\vec{S}_1\cdot\vec{n}\vec{S}_2\cdot\vec{n}\right), S^{ij}\equiv\epsilon^{ijk}S^k$$

- From the contraction of the LO 1-graviton spin coupling – the coupling of the A_i field – the gravitomagnetic vector
- Analogous to the magnetostatic interaction between two magnetic dipole moments

LO (1.5PN) spin-orbit correction (Tulczyjew 1959) – LO PN spin effect!



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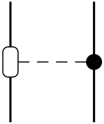

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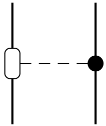

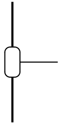

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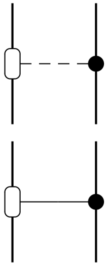

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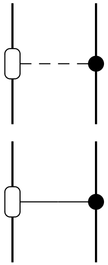


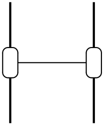
\+ mirror images

$$\Bigg\} = 2\frac{Gm_2}{r^2}\vec{S}_1\cdot\vec{v}\times\vec{n} + [1\leftrightarrow 2],\;\; \vec{v}\equiv\vec{v}_1-\vec{v}_2$$

- SSC is required at LO – unlike the LO spin1-spin2 case. Here the covariant SSC is used.

NLO spin1-spin2 interaction, ML 2008

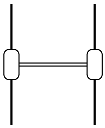
NLO (3PN) spin1-spin2 potential – one-graviton exchange



$$
=
$$

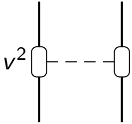
$$
\begin{aligned}
&= \frac{G}{r^3}\left(\vec{S}_1\cdot\vec{S}_2\vec{v}_1\cdot\vec{v}_2 - \vec{S}_1\cdot\vec{v}_2\vec{S}_2\cdot\vec{v}_1 - 3\vec{S}_1\times\vec{v}_1\cdot\vec{n}\vec{S}_2\times\vec{v}_2\cdot\vec{n}\right)\\
&+\frac{G}{r^3}\left(S_1^{0i}S_2^{0i} - 3S_1^{0i}n^iS_2^{0j}n^j\right.\\
&\qquad + S_1^{0i}\left[(\vec{S}_2\times\vec{v}_2)^i - 3n^i\vec{S}_2\times\vec{v}_2\cdot\vec{n}\right]\\
&\qquad \left. + S_2^{0i}\left[(\vec{S}_1\times\vec{v}_1)^i - 3n^i\vec{S}_1\times\vec{v}_1\cdot\vec{n}\right]\right)
\end{aligned}
$$

NLO (3PN) spin1-spin2 potential – one-graviton exchange

 =

$$= \frac{G}{r^3}\Big(-2\vec{S}_1\cdot\vec{S}_2\vec{v}_1\cdot\vec{v}_2 - \vec{S}_1\cdot\vec{v}_1\vec{S}_2\cdot\vec{v}_2 + 2\vec{S}_1\cdot\vec{v}_2\vec{S}_2\cdot\vec{v}_1 + 3\vec{S}_1\cdot\vec{n}\vec{S}_2\cdot\vec{n}\vec{v}_1\cdot\vec{v}_2 + 6\vec{S}_1\times\vec{v}_1\cdot\vec{n}\vec{S}_2\times\vec{v}_2\cdot\vec{n} - 3\vec{S}_1\times\vec{v}_2\cdot\vec{n}\vec{S}_2\times\vec{v}_1\cdot\vec{n}\Big)$$

NLO (3PN) spin1-spin2 potential – one-graviton exchange

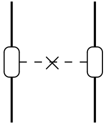


$+$ mirror image $=$

$$
\begin{aligned}
= \; & \frac{G}{r^3}\left(3S_1^{0i}\left[n^i \vec{S}_2 \times \vec{v}_1 \cdot \vec{n} - (\vec{S}_2 \times \vec{n})^i \vec{v}_1 \cdot \vec{n}\right]\right.\\
& \left. +3S_2^{0i}\left[n^i \vec{S}_1 \times \vec{v}_2 \cdot \vec{n} - (\vec{S}_1 \times \vec{n})^i \vec{v}_2 \cdot \vec{n}\right]\right)\\
& +\frac{G}{r^2}\left(\partial_t S_1^{0i}(\vec{S}_2 \times \vec{n})^i - \partial_t S_2^{0i}(\vec{S}_1 \times \vec{n})^i\right)
\end{aligned}
$$

- May be evaluated in two ways due to time derivative
- Acceleration and precession, i.e. $\dot{S}$ terms arise in the evaluation

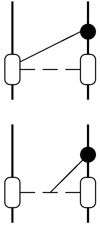
NLO (3PN) spin1-spin2 potential – one-graviton exchange

 =

$$
\begin{aligned}
= \quad \frac{G}{2r^3}\Big(&-\vec{S}_1\cdot\vec{S}_2\vec{v}_1\cdot\vec{v}_2+\vec{S}_1\cdot\vec{v}_1\vec{S}_2\cdot\vec{v}_2+\vec{S}_1\cdot\vec{v}_2\vec{S}_2\cdot\vec{v}_1\\
&+3\vec{S}_1\cdot\vec{S}_2\vec{v}_1\cdot\vec{n}\vec{v}_2\cdot\vec{n}-3\vec{S}_1\cdot\vec{n}\vec{S}_2\cdot\vec{n}\vec{v}_1\cdot\vec{v}_2\\
&-3\vec{S}_1\cdot\vec{v}_1\vec{S}_2\cdot\vec{n}\vec{v}_2\cdot\vec{n}-3\vec{S}_2\cdot\vec{v}_2\vec{S}_1\cdot\vec{n}\vec{v}_1\cdot\vec{n}\\
&-3\vec{S}_1\cdot\vec{v}_2\vec{S}_2\cdot\vec{n}\vec{v}_1\cdot\vec{n}-3\vec{S}_2\cdot\vec{v}_1\vec{S}_1\cdot\vec{n}\vec{v}_2\cdot\vec{n}\\
&+15\vec{S}_1\cdot\vec{n}\vec{S}_2\cdot\vec{n}\vec{v}_1\cdot\vec{n}\vec{v}_2\cdot\vec{n}\Big)
\end{aligned}
$$

- Precession terms arise

NLO (3PN) spin1-spin2 potential – nonlinear part: two-graviton exchange and cubic self-gravitational interaction



$+$ mirror images

$$= -\frac{G^2(m_1+m_2)}{2r^4}\left(5\vec{S}_1\cdot\vec{S}_2 - 17\vec{S}_1\cdot\vec{n}\vec{S}_2\cdot\vec{n}\right)$$

NNLO spin1-spin2 interaction , ML 2011

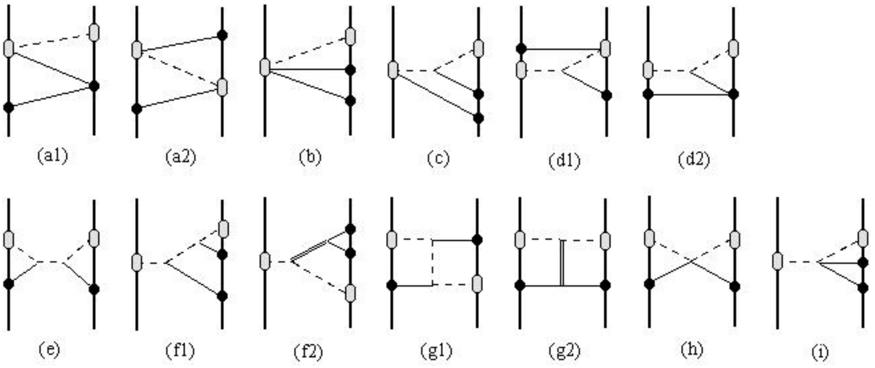


Figure: NNLO spin1-spin2 potential: order G^3 part

- Of these diagrams the 2-loop topologies (bottom) are the more complex
- The 2-loops divide into 3 kinds: the irreducible kind (g1, g2) is the nasty one!

Spin effects: Results

Spin effects: Results

- NLO (3PN) spin1-spin2 interaction (Porto & Rothstein; ML 2008)

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Future tasks

- Complete radiative effects of spinning binaries to NLO

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Future tasks

- Complete radiative effects of spinning binaries to NLO
- Complete spin 4PN corrections: conservative spin-squared, cubic.. and also radiative effects.

Further implementations: Results

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- Caged black holes (Chu, Goldberger & Rothstein 2006)

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- Tests of GR and alternative theories of gravity (Sturani et al. 2009)

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  - Rotating caged black holes (Kol & Smolkin 2008)
  - Charged caged black holes (Gilmore, Ross & Smolkin 2009)
- First steps: nPN corrections for the N-body problem (Chu 2008)
- First steps: Extreme mass ratio inspirals (Galley 2008)
- Radiation reaction in classical electrodynamics (Galley et al. 2010)
- Tests of GR and alternative theories of gravity (Sturani et al. 2009)

Prospective works

- 4PN conservative dynamics for binaries (Foffa & Sturani, underway)

Further implementations: Results

- Caged black holes (Chu, Goldberger & Rothstein 2006)
  - Rotating caged black holes (Kol & Smolkin 2008)
  - Charged caged black holes (Gilmore, Ross & Smolkin 2009)
- First steps: nPN corrections for the N-body problem (Chu 2008)
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Prospective works

- 4PN conservative dynamics for binaries (Foffa & Sturani, underway)
- Higher order PN radiative effects for binaries

Thank yoU!