Cosmology and GR limit of Horava-Lifshitz gravity

Shinji Mukohyama
(Kavli IPMU, U of Tokyo)

ref. Horava-Lifshitz Cosmology: A Review, arXiv: 1007.5199
also arXiv: 1105.0246 with K.Izumi
arXiv: 1109.2609 with E.Gumrukcuoglu & A.Wang

Power counting

$$I \supset \int dt dx^3 \dot{\phi}^2$$

$$\int dt dx^3 \phi^n$$

$$\propto E^{-(1+3+ns)}$$

- **Scaling dim of ϕ**
  t → b t (E → b^{-1}E)
  x → b x
  ϕ → b^s ϕ
  1+3-2+2s = 0
  s = -1

- Renormalizability

$$n \leq 4$$

- Gravity is highly non-linear and thus non-renormalizable

Abandon Lorentz symmetry?

$$I \supset \int dt dx^3 \dot{\phi}^2$$

- Anisotropic scaling

  t → b^z t (E→b^{-z}E)

  x → b x

  ϕ → b^s ϕ

  z+3-2z+2s = 0

  s = -(3-z)/2

- s = 0 if z = 3

$$\int dt dx^3 \phi^n$$

$$\propto E^{-(z+3+ns)/z}$$

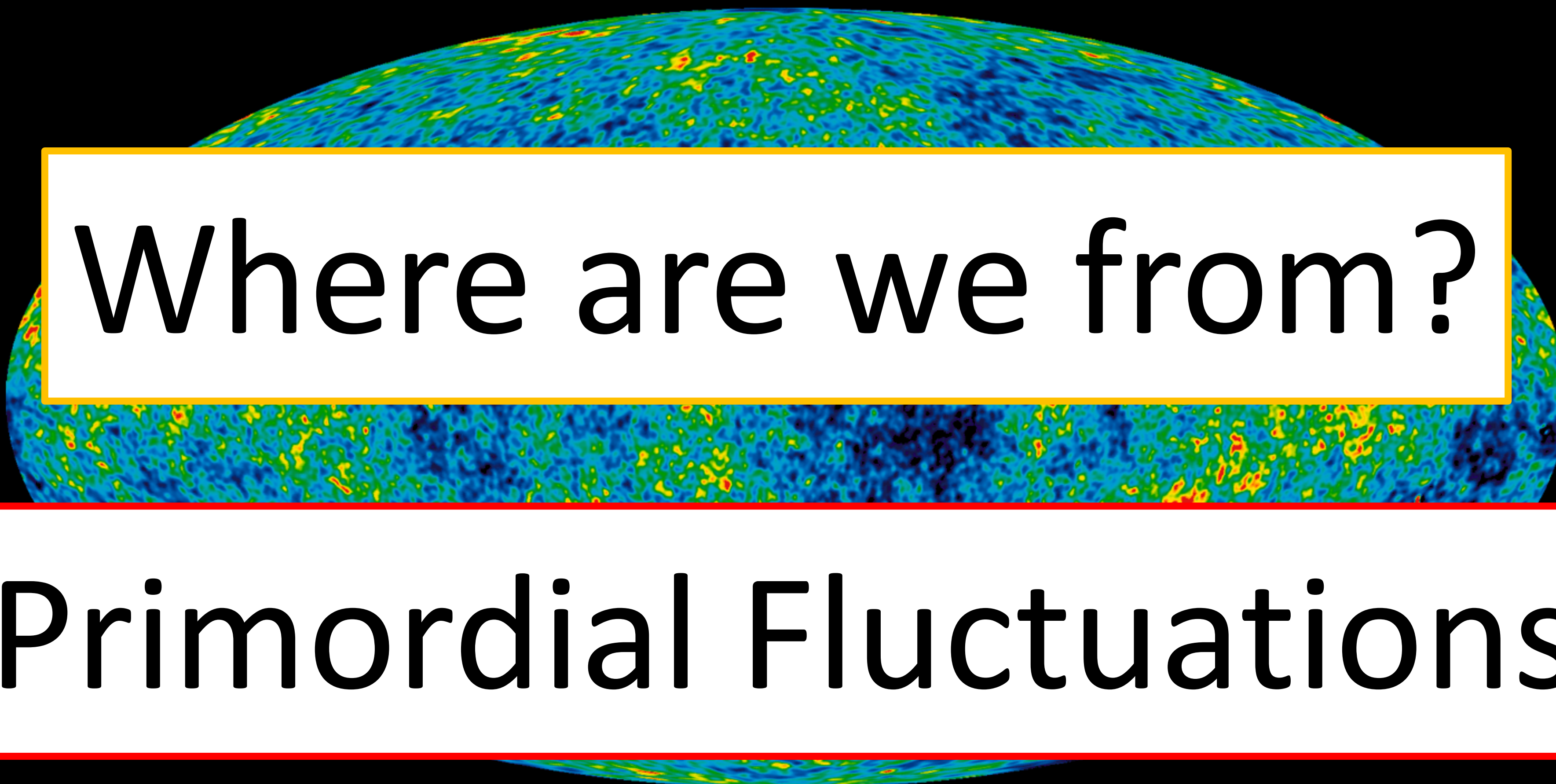
- For z = 3, any nonlinear interactions are renormalizable!
- **Gravity becomes renormalizable!?**

Cosmological implications

Horava-Lifshitz Cosmology: A Review, arXiv: 1007.5199

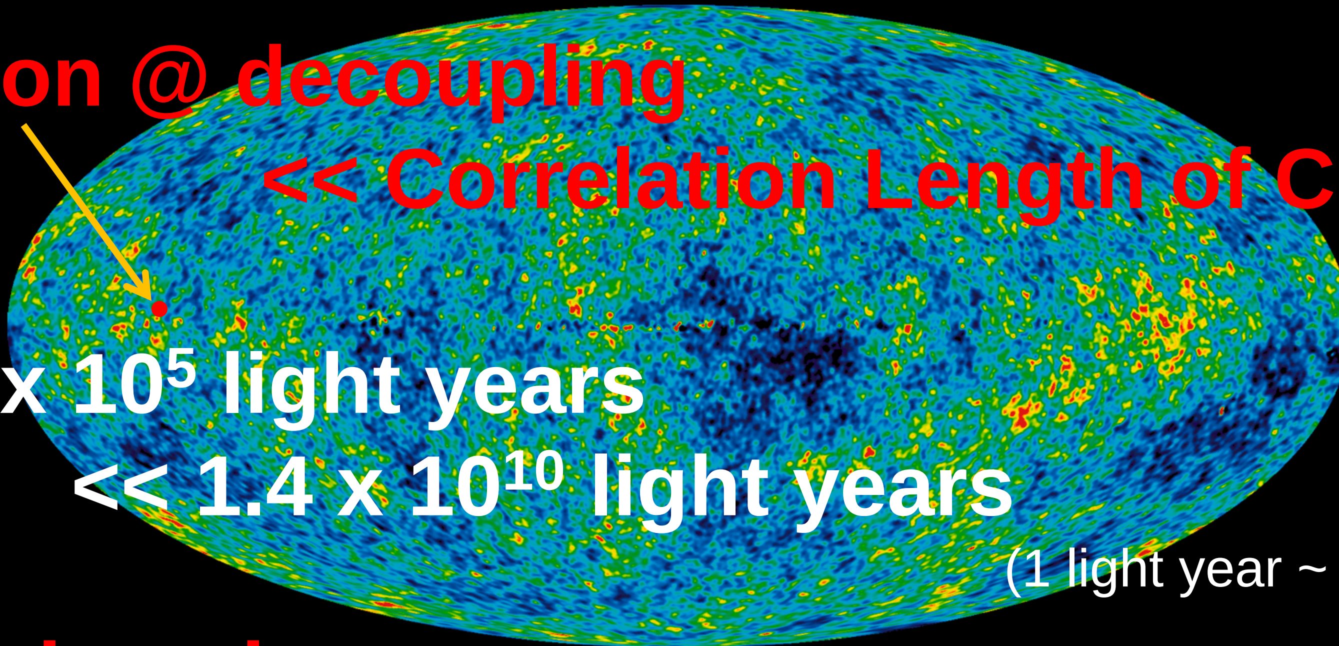
- The z=3 scaling solves the horizon problem and leads to scale-invariant cosmological perturbations without inflation (Mukohyama 2009).
- New mechanism for generation of primordial magnetic seed field (S.Maeda, Mukohyama, Shiromizu 2009).
- Higher curvature terms lead to regular bounce (Calcagni 2009, Brandenberger 2009).
- Higher curvature terms ($1/a^6$, $1/a^4$) might make the flatness problem milder (Kiritsis&Kofinas 2009).
- Absence of local Hamiltonian constraint leads to DM as integration "constant" (Mukohyama 2009).

Where are we from?



Primordial Fluctuations

Horizon Problem & Scale-Invariance



Horizon @ decoupling

<< Correlation Length of CMB

3.8 x 10^5 light years

<< 1.4 x 10^{10} light years

(1 light year ~ 10^{18} cm)

Scale-invariant spectrum

Δ ~ constant

$$\langle \zeta_{\vec{k}} \zeta_{\vec{k}'} \rangle = (2\pi)^3 \delta^3(\vec{k} + \vec{k}') \frac{\Delta}{|\vec{k}|^3}$$

Usual story

- $\omega^2 >> H^2$: oscillate $H = (da/dt) / a$

  $\omega^2 << H^2$: freeze   a : scale factor

  oscillation → freeze-out iff $d(H^2/\omega^2)/dt > 0$

  $\omega^2 = k^2/a^2$ leads to $d^2a/dt^2 > 0$

  Generation of super-horizon fluctuations requires accelerated expansion, i.e. inflation.

Usual story

- $\omega^2 \gg H^2$: oscillate   H = (da/dt) / a
  $\omega^2 \ll H^2$: freeze   a : scale factor
  oscillation → freeze-out iff $d(H^2/\omega^2)/dt > 0$
  $\omega^2 = k^2/a^2$ leads to $d^2a/dt^2 > 0$
  Generation of super-horizon fluctuations requires accelerated expansion, i.e. inflation.
- Scaling law
  $t \to b\,t$ ($E \to b^{-1}E$)
  $x \to b\,x$
  $\phi \to b^{-1}\,\phi$ ⇒ $\delta\phi \propto E \sim H$
  Scale-invariance requires almost const. H, i.e. inflation.

New story with z=3

Mukohyama 2009

- oscillation → freeze-out iff $d(H^2/\omega^2)/dt > 0$

  $\omega^2 = M^{-4}k^6/a^6$ leads to $d^2(a^3)/dt^2 > 0$

  OK for $a \sim t^p$ with $p > 1/3$

New story with z=3

Mukohyama 2009

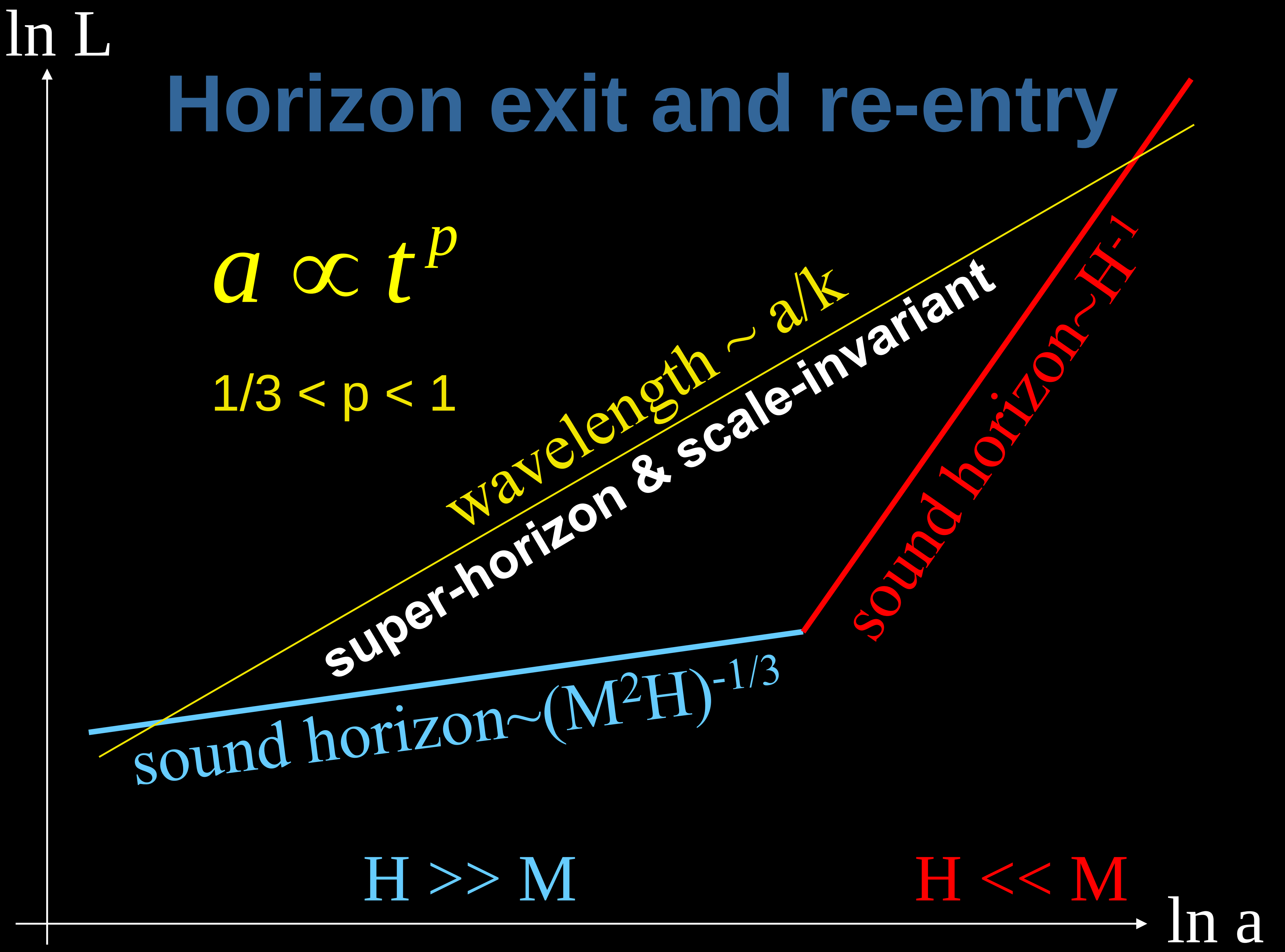
- oscillation → freeze-out iff $d(H^2/\omega^2)/dt > 0$
  $\omega^2 = M^{-4}k^6/a^6$ leads to $d^2(a^3)/dt^2 > 0$
  OK for $a\sim t^p$ with $p > 1/3$
- Scaling law
  $t \to b^3 t$ ($E \to b^{-3}E$)
  $x \to b\, x$
  $\phi \to b^0 \phi$ ⇒ $\delta\phi \propto E^0 \sim H^0$
  Scale-invariant fluctuations!

New story with z=3

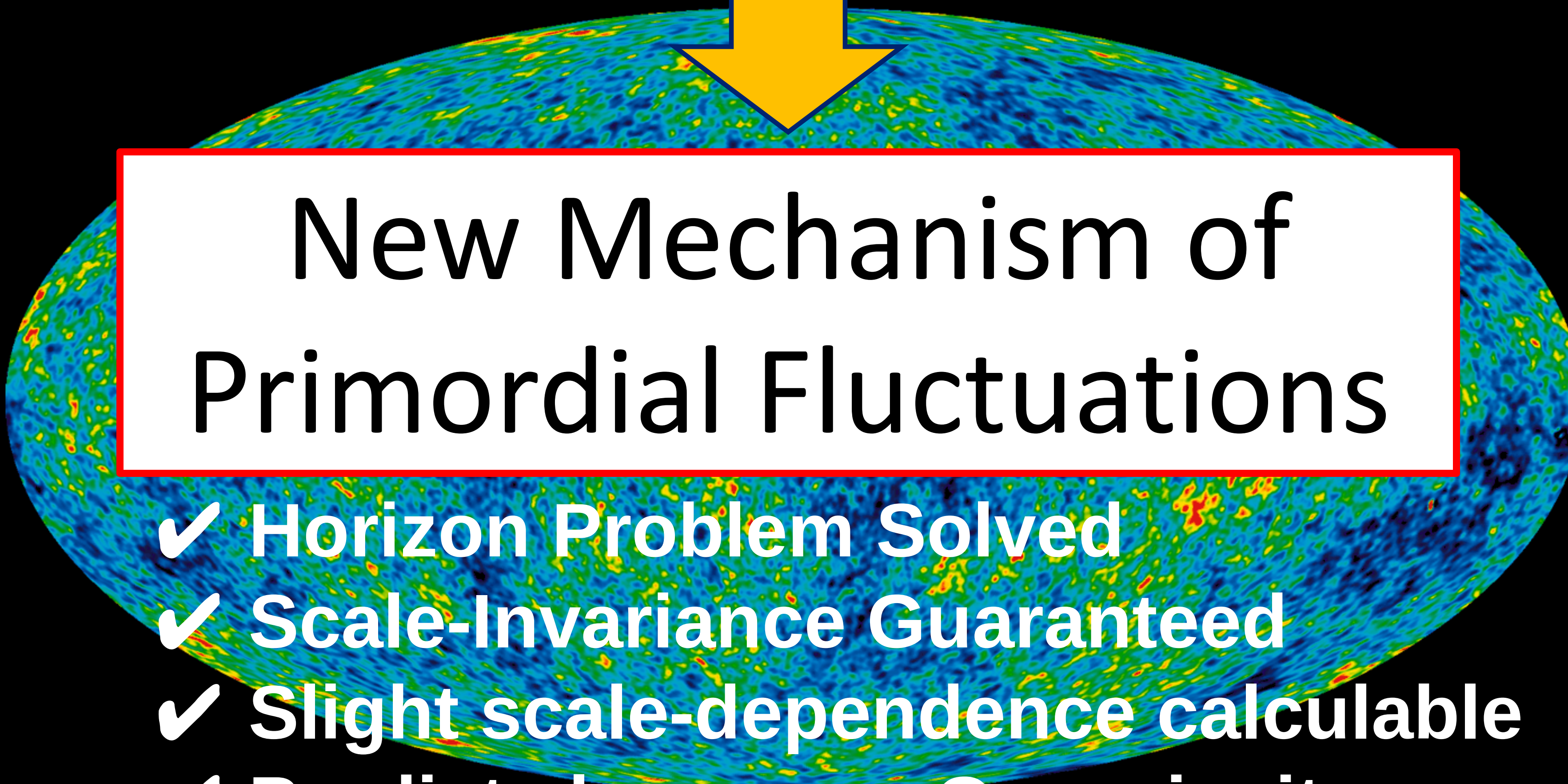
Mukohyama 2009

- oscillation → freeze-out iff $d(H^2/\omega^2)/dt > 0$
  $\omega^2 = M^{-4}k^6/a^6$ leads to $d^2(a^3)/dt^2 > 0$
  OK for $a \sim t^p$ with $p > 1/3$
- Scaling law
  $t \to b^3 t$ ($E \to b^{-3}E$)
  $x \to b\, x$
  $\phi \to b^0\, \phi$ ⇒ $\delta\phi \propto E^0 \sim H^0$
  Scale-invariant fluctuations!
- Tensor perturbation $P_h \sim M^2/M_{Pl}^2$

Horizon exit and re-entry



New Quantum Gravity



New Mechanism of Primordial Fluctuations

- ✔ **Horizon Problem Solved**
- ✔ **Scale-Invariance Guaranteed**
- ✔ **Slight scale-dependence calculable**
- ✔ **Predicts large non-Gaussianity**

Horava-Lifshitz gravity

Horava (2009)

- Basic quantities:
  lapse $N(t)$, shift $N^i(t,x)$, 3d spatial metric $g_{ij}(t,x)$
- ADM metric (emergent in the IR)
  $ds^2 = -N^2 dt^2 + g_{ij} (dx^i + N^i dt)(dx^j + N^j dt)$
- Foliation-preserving deffeomorphism
  $t \to t'(t), \quad x^i \to x'^i(t, x^j)$
- Anisotropic scaling with z=3 in UV
  $t \to b^z t, \quad x^i \to b\, x^i$
- Ingredients in the action

$$N dt \qquad \sqrt{g}\, d^3x \qquad g_{ij} \qquad D_i \qquad R_{ij}$$

$$K_{ij} = \frac{1}{2N}\left(\partial_t g_{ij} - D_i N_j - D_j N_i\right)$$

($C_{ijkl} = 0$ in 3d)

UV action with z=3

- Kinetic terms (2nd time derivative)

$$\int N dt \sqrt{g} d^3x \left(K_{ij} K^{ij} - \lambda K^2 \right)$$

c.f. $\lambda = 1$ for GR

- z=3 potential terms (6th spatial derivative)

$$\int N dt \sqrt{g} d^3x [\quad D_i R_{jk} D^i R^{jk} \quad D_i R D^i R$$

$$R_i^j R_j^k R_k^i \quad R R_i^j R_j^i \quad R^3 \quad]$$

c.f. $D_i R_{jk} D^j R^{ki}$ is written in terms of other terms

Relevant deformations (with parity)

- z=2 potential terms (4th spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad R_i^j R_j^i \quad R^2 \quad \right]$$

- z=1 potential term (2nd spatial derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad R \quad \right]$$

- z=0 potential term (no derivative)

$$\int N dt \sqrt{g} d^3x \left[\quad 1 \quad \right]$$

IR action

- **UV: z=3** , power-counting renormalizability

  ⬇ RG flow

- **IR: z=1** , seems to recover GR iff λ → 1

kinetic term

$$\frac{1}{16\pi G_N}\int N dt \sqrt{g}\, d^3x \left(K_{ij}K^{ij} - \lambda K^2 + c_g^2 R - 2\Lambda \right)$$

IR potential

note:

Renormalizability has not been proved.

RG flow has not yet been investigated.

Projectability condition N=N(t)

- Infinitesimal tr. $\delta t = f(t)$, $\delta x^i = \zeta^i(t,x^j)$

$$\delta g_{ij} = \partial_i \zeta^k g_{jk} + \partial_j \zeta^k g_{ik} + \zeta^k \partial_k g_{ij} + f \dot{g}_{ij}$$

$$\delta N_i = \partial_i \zeta^j N_j + \zeta^j \partial_j N_i + \dot{\zeta}^j g_{ij} + \dot{f} N_i + f \dot{N}_i$$

$$\delta N = \zeta^i \partial_i N + \dot{f} N + f \dot{N}$$

- Space-independent N cannot be transformed to space-dependent N.
- N is gauge d.o.f. associated with the space-independent time reparametrization.
- It is natural to restrict N to be space-independent.
- Consequently, Hamiltonian constraint is an equation integrated over a whole space.

Different versions of HL gravity

- There are versions w/wo the projectability condition.
- Horava's original proposal was **with the projectability condition, N=N(t).**
- **Naïve non-projectable extension is inconsistent** [c.f. Henneaux, et.al. 2009].
- Inclusion of $a_i = (\ln N)_{,i}$ (and thus more terms) in the action can cure the non-projectable extension [Blas, Pujolas and Sibiryakov 2009].
- U(1) extension [Horava-Melby-Thompson 2010]
- In the rest of this talk I will consider the projectable version, i.e. the theory with N=N(t), without U(1).

“Black holes” with N=N(t)?

- Schwarzschild BH in PG coordinate

$$ds^2 = -dt_P^2 + \left(dr \pm \sqrt{\frac{2m}{r}} dt_P \right)^2 + r^2 d\Omega^2$$

exact sol
for λ = 1

- Gaussian normal coordinate

$$ds^2 = -dt_G^2 + \cdots$$

approx sol
for λ = 1

Lemaitre reference frame
Doran coordinate

“Black holes” with N=N(t)?

- Schwarzschild BH in PG coordinate

$$ds^2 = -dt_P^2 + \left(dr \pm \sqrt{\frac{2m}{r}} dt_P \right)^2 + r^2 d\Omega^2$$

exact sol for $\lambda = 1$

- Gaussian normal coordinate

$$ds^2 = -dt_G^2 + \cdots$$

approx sol for $\lambda = 1$

Lemaitre reference frame

Doran coordinate

Q.

Is the $\lambda \rightarrow 1$ limit continuous or discontinuous?

Physical d.o.f.

- (6 + 3) – 3 – 3 = 3
  g_{ij} : 6 components
  N^i : 3 components
  $x^i \rightarrow x'^i(t,x)$: 3 gauge d.o.f.
  $\delta I/\delta N^i=0$: 3 constraints
- 3 = 2 + 1
  tensor graviton: 2 d.o.f.
  scalar graviton: 1 d.o.f.

Linear instability of scalar graviton

- Sign of (time) kinetic term $(\lambda-1)/(3\lambda-1) > 0$.
- The dispersion relation in flat background
  $\omega^2 = c_s{}^2 k^2 \times [1+ O(k^2/M^2)]$ with $c_s{}^2 = -(\lambda-1)/(3\lambda-1) < 0$
  → IR instability in linear level
  (Wang&Maartens; Blas,et.al.; Koyama&Arroja 2009)
- Slower than Jeans instability if
  $t_J \sim (G_N\rho)^{-1/2} < t_L \sim L/|c_s|$.
- Tamed by Hubble friction or/and $O(k^2/M^2)$ terms if
  $H^{-1} < t_L$ or/and $L < 1/M$.
- Thus, the linear instability does not show up if
  $|c_s| = |(\lambda-1)/(3\lambda-1)|^{1/2} < \mathrm{Max}\,[|\Phi|^{1/2}, HL]$. $(\Phi \sim -G_N\rho L^2)$
  for $L > \mathrm{Max}[0.01\mathrm{mm}, 1/M]$
  (Shorter scales → similar to spacetime foam)
- Phenomenological constraint on properties of RG flow.

Perturbative vs non-perturbative regimes

$$N = 1, \quad N_i = \partial_i B + n_i, \quad g_{ij} = a^2 e^{2\zeta_T} \left(e^h\right)_{ij}$$

$$\zeta_T = O(q), \quad h_{ij} = O(q), \quad \boxed{B = O(q^0), \quad n_i = O(q^0)}$$

Momentum constraint

$$\Rightarrow \quad B = \frac{O(1)}{\boxed{O(\lambda - 1) + O(q)}} \partial_t \zeta_T$$

- Perturbative regime: q << (λ-1)
  breakdown in the λ → 1 limit
- Non-perturbative regime: (λ-1) << q << 1
  responsible for recovery of GR

Vainshtein effect in massive gravity

- Linearized analysis results in vDVZ discontinuity of the massless limit.
- However, perturbative expansion breaks down in this limit and cannot be trusted.
- Non-perturbative analysis shows continuity and GR is recovered in the massless limit.
- Continuity is not uniform w.r.t. distance. (e.g. 1/r expansion does not work.) However, Vainshtein radius can be pushed to infinity in the massless limit.

Analogue of Vainshtein effect (mukohyama 2010)

- Spherically symmetric, static ansatz

$$N = 1, \quad N_i dx^i = \beta(x)dx, \quad g_{ij}dx^i dx^j = dx^2 + r(x)^2 d\Omega_2^2$$

$\Downarrow$ $R \equiv \beta^{(\lambda-1)/(2\lambda)} r$ without z>1 terms

$$R'' + \frac{\lambda-1}{\lambda}\left[\frac{(3\lambda-1)(\beta')^2 R}{4\lambda^2\beta^2} + \frac{(\lambda-1)\beta' R'}{\lambda\beta} - \frac{(R')^2}{R}\right] = 0$$

$$\frac{\beta'}{\beta} - \frac{(\lambda-1)R}{4\lambda R'}\left(\frac{\beta'}{\beta}\right)^2 + \frac{\lambda}{RR'}\frac{\beta^{(\lambda-1)/\lambda} + [(2\lambda-1)\beta^2 - 1](R')^2}{\boxed{(3\lambda-1)\beta^2 + (\lambda-1)}} = 0$$

- Two branches

$$\frac{\beta'}{\beta} = \frac{1 \pm \sqrt{1+4AB}}{2A},$$

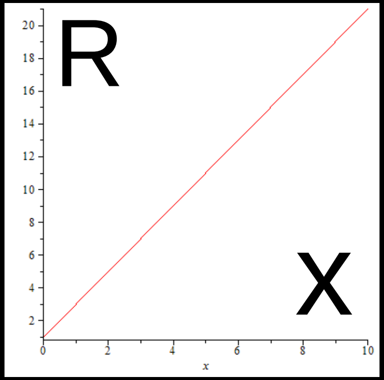
$$A \equiv \frac{(\lambda-1)R}{4\lambda R'}, \quad B \equiv \frac{\lambda}{RR'}\frac{\beta^{(\lambda-1)/\lambda} + [(2\lambda-1)\beta^2 - 1](R')^2}{\boxed{(3\lambda-1)\beta^2 + (\lambda-1)}}$$

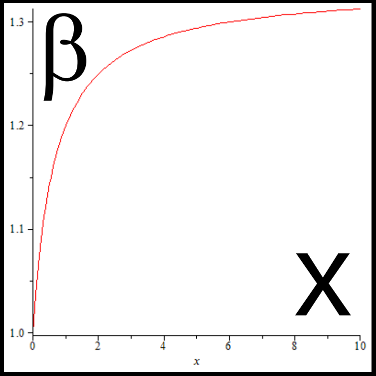
- "-" branch recovers GR in the λ → 1 limit

Analogue of Vainshtein effect

- Numerical integration in the "-" branch with β(x=0)=1, r(x=0)=1, r'(x=0) given

for
$\lambda-1=10^{-6}$
$r'(x=0)=2$



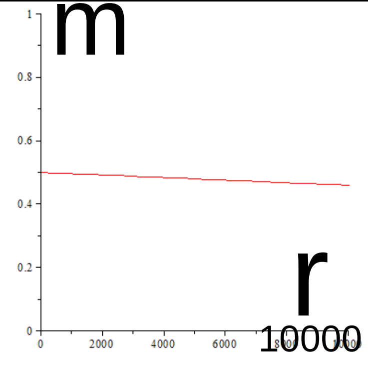


- Misner-Sharp energy

$$m \equiv \frac{r}{2}\left[1-\left(1-\beta^2\right)\left(r'\right)^2\right]$$

almost constant

GR is recovered!



Analogue of Vainshtein effect (mukohyama 2010)

$$\frac{\beta'}{\beta} = \frac{1 \pm \sqrt{1+4AB}}{2A},$$

choose the "-" branch

$$A \equiv \frac{(\lambda-1)R}{4\lambda R'}, \quad B \equiv \frac{\lambda}{RR'} \frac{\beta^{(\lambda-1)/\lambda} + [(2\lambda-1)\beta^2 - 1](R')^2}{(3\lambda-1)\beta^2 + (\lambda-1)}$$

- $(3\lambda-1)\beta^2 << (\lambda-1)$
  perturbative regime, 1/r expansion
- $(3\lambda-1)\beta^2 >> (\lambda-1)$
  non-perturvative regime, recovery of GR
- $(3\lambda-1)\beta^2 \sim (\lambda-1)$ with $\beta^2 \sim r_g/r$ → $r \sim r_g/(\lambda-1)$
  analogue of Vainshtein radius

Izumi & Mukohyama 2009
"Steller center is dynamical"

Fate of scalar graviton

$$L = \left[f\left(\frac{\zeta_T}{\lambda - 1}\right) + g(\zeta_T, \lambda) \right] \frac{M_{Pl}^2 \dot{\zeta}_T^2}{\lambda - 1} - V(\zeta_T, D_i)$$

subleading

Independent of λ
No time derivative

Local in time, no time derivative
Non-local in space, each term has the same # of spatial derivatives in denominator and numerator

λ → 1

$$L \sim \dot{\zeta}_c^2$$

- Looks like a minimally coupled FREE field with sound speed = 0
- **Scalar Graviton → "Dark Matter"**

Nonlinear cosmological perturbation and $\lambda \to 1$

arXiv: 1105.0246 [hep-th] with K.Izumi
arXiv: 1109.2609 [hep-th] with E.Gumruhcuglu & A.Wang

- HL gravity + a scalar matter field
- Flat FRW background
- **Nonlinear cosmological perturbation**
- Gradient expansion up to any order
- **Regular and continuous in the $\lambda \to 1$ limit**
- **Recovers GR+DM+scalar field in the $\lambda \to 1$ limit**

Summary

- Horava-Lifshitz gravity is power-counting renormalizable and can be a candidate theory of quantum gravity.
- The z=3 scaling solves horizon problem and leads to scale-invariant cosmological perturbations for $a \sim t^p$ with $p>1/3$.
- HL gravity in the $\lambda \rightarrow 1$ limit exhibits analogue of Vainshtein effect: GR (+DM) is recovered non-perturbatively at least in some simple cases.
  1. spherically-symmetric, static, vacuum configurations
  2. superhorizon cosmological perturbations
- In the $\lambda \rightarrow 1$ limit, Schwarzshild BH is an exact solution and large Kerr BH is an approximate solution.
- **Scalar graviton → Dark matter**
- **HL gravity at low-E can mimic GR+DM**

Future works

- Renormalizability beyond power-counting
- RG flow: is $\lambda = 1$ an IR fixed point ? Does it satisfy the stability condition for the scalar graviton?
($|c_s| < \mathrm{Max}\,[|\Phi|^{1/2}, HL]$ for $L > \mathrm{Max}[M^{-1}, 0.01\mathrm{mm}]$)
- Can we get a common "limit of speed" ?
(i) $M_{z=3} << M_{pl}$, (ii) supersymmetry, (iii) other ideas?
- How generic is 'Vainshtein effect'?
- How generic is caustic avoidance, (perhaps with $\lambda \to \infty$ & $M_{Pl}/M_{z=3} \to \infty$) ?
- Micro & macro behavior of "DM"
- Adiabatic initial condition for "DM" from the z=3 scaling
- Spectral tilt from anomalous dimension

Structure of HL gravity

- Foliation-preserving diffeomorphism
  = 3D spatial diffeomorphism
  + space-independent time reparametrization
- 3 local constraints + 1 global constraint
  = 3 momentum @ each time @ each point
  + 1 Hamiltonian @ each time integrated
- Constraints are preserved by dynamical equations.
- We can solve dynamical equations, provided that constraints are satisfied at initial time.

FRW spacetime in HL gravity

- Approximates overall behavior of our patch of the universe inside the Hubble horizon.
- No “local” Hamiltonian constraint
  E.o.m. of matter
  → conservation eq.

$$\dot{\rho}_i + 3\frac{\dot{a}}{a}(\rho_i + P_i) = 0$$

- Dynamical eq
  can be integrated to give
  Friedmann eq with
  “dark matter as
  integration constant”

$$-2\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2} = 8\pi G_N \sum_{i=1}^{n} P_i$$

$$3\frac{\dot{a}^2}{a^2} = 8\pi G_N \left(\sum_{i=1}^{n} \rho_i + \frac{C}{a^3} \right)$$

IR limit of HL gravity

$$\frac{1}{16\pi G_N}\int N dt \sqrt{g}\, d^3x \left(K_{ij}K^{ij} - \lambda K^2 + R - 2\Lambda\right)$$

- Looks like GR iff λ = 1. So, we assume that λ = 1 is an IR fixed point of RG flow.
- Global Hamiltonian constraint

$$\int d^3x \sqrt{g}\,(G^{(4)}_{\mu\nu} + \Lambda g^{(4)}_{\mu\nu} - 8\pi G_N T_{\mu\nu}) n^\mu n^\nu = 0$$

$$n_\mu dx^\mu = -N dt, \quad n^\mu \partial_\mu = \frac{1}{N}(\partial_t - N^i \partial_i)$$

- Momentum constraint & dynamical eq

$$(G^{(4)}_{i\mu} + \Lambda g^{(4)}_{i\mu} - 8\pi G_N T_{i\mu}) n^\mu = 0$$

$$G^{(4)}_{ij} + \Lambda g^{(4)}_{ij} - 8\pi G_N T_{ij} = 0$$

Dark matter as integration constant

- Def. $T^{HL}_{\mu\nu}$ $$G^{(4)}_{\mu\nu} + \Lambda g^{(4)}_{\mu\nu} = 8\pi G_N \left(T_{\mu\nu} + T^{HL}_{\mu\nu}\right)$$
- General solution to the momentum constraint and dynamical eq.

$$T^{HL}_{\mu\nu} = \rho^{HL} n_\mu n_\nu \qquad n^\mu \nabla_\mu n_\nu = 0$$

- Global Hamiltonian constraint

$$\int d^3x \sqrt{g} \rho^{HL} = 0$$

ρ^{HL} can be positive everywhere in our patch of the universe inside the horizon.

- Bianchi identity → (non-)conservation eq

$$\partial_\perp \rho^{HL} + K \rho^{HL} = n^\nu \nabla^\mu T_{\mu\nu}$$