ASPECTS OF HORAVA-LIFSHITZ COSMOLOGY

Emmanuel N. Saridakis

Physics Department, National and Technical University of Athens, Greece

Physics Department, Baylor University, Texas, USA

Goal

- We investigate cosmological scenarios in a universe governed by Horava-Lifshitz gravity

- Note:

  A consistent or interesting cosmology is not a proof for the consistency of the underlying gravitational theory

Talk Plan

- 1) Introduction: Horava-Lifshitz gravity and cosmology
- 2) Phase-space analysis and late-time cosmological behavior
- 3) Bouncing solutions and cyclic behavior
- 4) Observational Constraints
- 5) Thermodynamic aspects
- 6) Perturbative instabilities
- 7) Conclusions-Prospects

Introduction

- **Horava-Lifshitz** gravity: power-counting renormalizable, UV complete
- IR fixed point: General Relativity
- Good UV behavior: Anisotropic, Lifshitz scaling between time and space

[Horava, PRD 79]

- Theoretical and conceptual problems (instabilities etc)? Open subject.

Introduction: Horava-Lifshitz gravity

$$ds^2 = -N^2 dt^2 + g_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$$

$$t \to l^3 t, \;\; x^i \to l x^i$$

$$S_g = \int dt d^3x \sqrt{g} N \{ \frac{2}{\kappa^2}(K_{ij}K^{ij} - \lambda K^2)$$
$$+ \frac{\kappa^2}{2w^4} C_{ij}C^{ij} - \frac{\kappa^2 \mu \varepsilon^{ijk}}{2w^2\sqrt{g}} R_{il}\nabla_j R^l_k + \frac{\kappa^2\mu^2}{8} R_{il}R^{ij}$$
$$+ \frac{\kappa^2\mu^2}{8(1-3\lambda)}\left[\frac{1-4\lambda}{4}R^2 + \Lambda R - 3\Lambda^2\right]\}$$

(detailed-balanced)

$$K_{ij} = \frac{1}{2N}\left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i\right)$$ (extrinsic curvature)

$$C^{ij} = \frac{\varepsilon^{ijk}}{\sqrt{g}}\nabla_k\left(R^j_i - \frac{1}{4}R\delta^j_i\right)$$ (Cotton tensor)

[Kiritsis, Kofinas, NPB 821]

Introduction: Horava-Lifshitz cosmology

- Cosmological framework:

$$N = 1 \ , \quad g_{ij} = a^2(t)\gamma_{ij} \ , \quad N^i = 0$$

(projectability)

$$\gamma_{ij}dx^i dx^j = \frac{dr^2}{1-kr^2} + r^2 d\Omega_2^2$$

- Matter content:

$$S_M = \int dt d^3x \sqrt{g} N \left[\frac{1}{2}\partial_\mu \varphi \partial^\mu \varphi - V(\varphi) \right]$$

$$\varphi = \varphi(t)$$

$$\rho_M = \frac{\dot{\varphi}^2}{2} + V(\varphi) \ , \quad p_M = \frac{\dot{\varphi}^2}{2} - V(\varphi)$$

[Kiritsis, Kofinas, NPB 821]

Introduction: Horava-Lifshitz cosmology

- Friedmann Equations (under detailed balance):

$$H^2 = \frac{\kappa^2}{6(3\lambda-1)}\left(\rho_M + \frac{3\kappa^2\mu^2 k^2}{8(3\lambda-1)a^4} + \frac{3\kappa^2\mu^2\Lambda^2}{8(3\lambda-1)}\right) - \frac{\kappa^4\mu^2\Lambda}{8(3\lambda-1)^2}\frac{k}{a^2}$$

$$\dot{H} + \frac{3}{2}H^2 = -\frac{\kappa^2}{4(3\lambda-1)}\left(w_M\rho_M + \frac{\kappa^2\mu^2 k^2}{8(3\lambda-1)a^4} - \frac{3\kappa^2\mu^2\Lambda^2}{8(3\lambda-1)}\right) - \frac{\kappa^4\mu^2\Lambda}{16(3\lambda-1)^2}\frac{k}{a^2}$$

and $\dot{\rho}_M + 3H(\rho_M + p_M) = 0$

[Kiritsis, Kofinas, NPB 821]

Introduction: Horava-Lifshitz cosmology

- Friedmann Equations (under detailed balance):

$$H^2 = \frac{\kappa^2}{6(3\lambda-1)}\left(\rho_M + \frac{3\kappa^2\mu^2k^2}{8(3\lambda-1)a^4} + \frac{3\kappa^2\mu^2\Lambda^2}{8(3\lambda-1)}\right) - \frac{\kappa^4\mu^2\Lambda}{8(3\lambda-1)^2}\frac{k}{a^2}$$

$$\dot{H} + \frac{3}{2}H^2 = -\frac{\kappa^2}{4(3\lambda-1)}\left(w_M\rho_M + \frac{\kappa^2\mu^2k^2}{8(3\lambda-1)a^4} - \frac{3\kappa^2\mu^2\Lambda^2}{8(3\lambda-1)}\right) - \frac{\kappa^4\mu^2\Lambda}{16(3\lambda-1)^2}\frac{k}{a^2}$$

and $\dot{\rho}_M + 3H(\rho_M + p_M) = 0$

[Kiritsis, Kofinas, NPB 821]

- Effective dark energy:

$$\rho_{DE} = \frac{3\kappa^2\mu^2k^2}{8(3\lambda-1)a^4} + \frac{3\kappa^2\mu^2\Lambda^2}{8(3\lambda-1)}$$

$$p_{DE} = \frac{\kappa^2\mu^2k^2}{8(3\lambda-1)a^4} - \frac{3\kappa^2\mu^2\Lambda^2}{8(3\lambda-1)}$$

$$\Rightarrow \dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) = 0$$

$$G \equiv \frac{\kappa^2}{16\pi(3\lambda-1)}$$

$$\frac{\kappa^4\mu^2\Lambda}{8(3\lambda-1)^2} \equiv 1$$

[Leon, Saridakis, JCAP 0911]

Introduction: Horava-Lifshitz cosmology

- Friedmann Equations (beyond detailed balance):

$$H^2 = \frac{2\sigma_0}{(3\lambda-1)}\left(\rho_M + \frac{\sigma_1}{6} + \frac{\sigma_3 k^2}{6a^4} + \frac{\sigma_4 k}{6a^6}\right) + \frac{\sigma_2}{3(3\lambda-1)}\frac{k}{a^2}$$

$$\dot{H} + \frac{3}{2}H^2 = -\frac{3\sigma_0}{(3\lambda-1)}\left(w_M\rho_M - \frac{\sigma_1}{6} + \frac{\sigma_3 k^2}{18a^4} + \frac{\sigma_4 k}{6a^6}\right) + \frac{\sigma_2}{6(3\lambda-1)}\frac{k}{a^2}$$

[Elizalde et al, CQG 27]

- Effective dark energy:

$$\rho_{DE} = \frac{\sigma_1}{6} + \frac{\sigma_3 k^2}{6a^4} + \frac{\sigma_4 k}{6a^6}$$

$$p_{DE} = -\frac{\sigma_1}{6} + \frac{\sigma_3 k^2}{18a^4} + \frac{\sigma_4 k}{6a^6}$$

$$\Rightarrow \dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) = 0$$

$$G \equiv \frac{6\sigma_0}{8\pi(3\lambda-1)}$$

$$-\frac{\sigma_2}{3(3\lambda-1)} \equiv 1$$

[Leon, Saridakis, JCAP 0911]

Phase-space analysis

- Transform cosmological system to its autonomous form:

$$x=\frac{\kappa\dot{\varphi}}{2\sqrt{6}\mathrm{H}},\quad y=\frac{\kappa\sqrt{V(\varphi)}}{\sqrt{6}\mathrm{H}\sqrt{3\lambda-1}},\quad z=\frac{\kappa^2\mu}{4(3\lambda-1)a^2H},\quad u=\frac{\kappa^2\Lambda\mu}{4(3\lambda-1)H}$$

$$\Rightarrow \Omega_M \equiv \frac{\rho_{\mathrm{M}}}{3\mathrm{H}^2}=x^2+y^2,\qquad w_M=\frac{x^2-y^2}{x^2+y^2},$$

$$\Omega_{DE} \equiv \frac{\rho_{DE}}{3\mathrm{H}^2}=-k^2z^2-u^2\qquad w_{DE}=\frac{k^2z^2-3u^2}{3k^2z^2+3u^2}$$

[Leon, Saridakis, JCAP 0911]

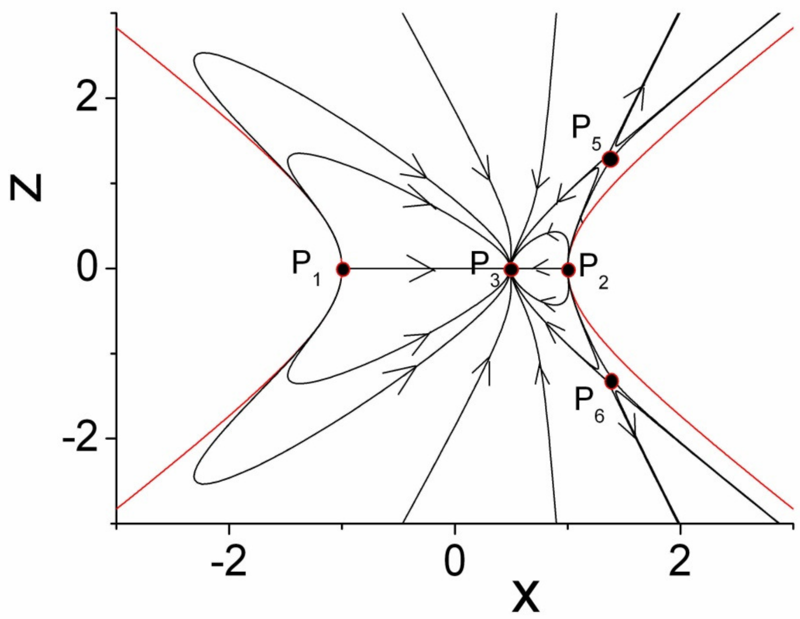
$$\Rightarrow X'=f(X)\qquad X'|_{X=X_C}=0$$

- Linear Perturbations: $X=X_C+U \qquad \Rightarrow U'=QU$
- Eigenvalues of Q determine type and stability of C.P

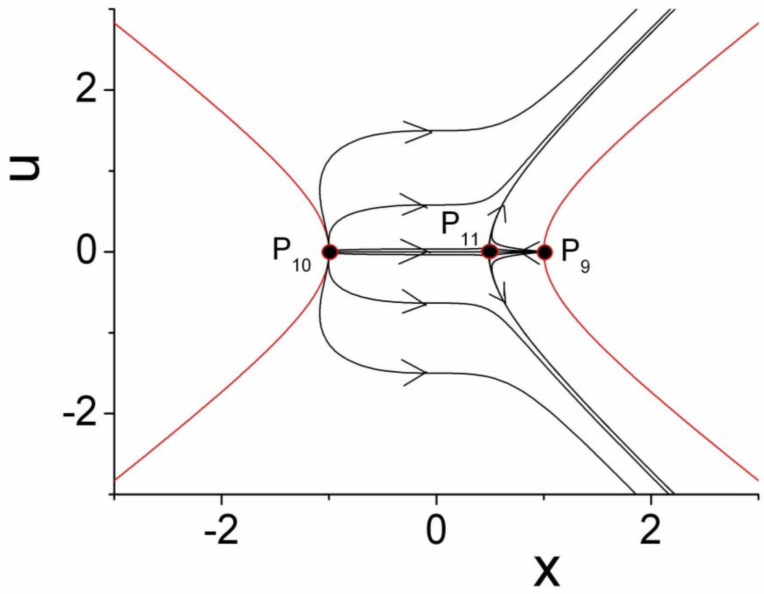
Phase-space analysis

Detailed balance

$k \neq 0, \ \Lambda = 0$



$k = 0, \ \Lambda \neq 0$



- P3: Stable with $\Omega_{DE} = 0$

P11: Saddle with $\Omega_{DE} = 0$

[Leon, Saridakis, JCAP 0911]

Phase-space analysis

- Beyond Detailed Balance (4D problem)

$$x_1 = \frac{\sigma_1}{3(3\lambda - 1)H^2},\quad x_2 = \frac{k\sigma_2}{3(3\lambda - 1)a^2H^2},\quad x_3 = \frac{\sigma_3}{3(3\lambda - 1)a^4H^2},\quad x_4 = \frac{2k\sigma_4}{3(3\lambda - 1)a^6H^2}$$

- Stable solution with $\Omega_{DE} = 1$ and $w_{DE} = -1$ (eternally expanding)

- Small probability (non-hyperbolid C.P) for an Oscillating solution (The a^{-4}, a^{-6} terms responsible for the bounce, and the c.c responsible for the turnaround)

[Leon, Saridakis, JCAP 0911]

Bounce and Cyclic behavior

- Contracting ($H<0$), bounce ($H=0$), expanding ($H>0$)
  near and at the bounce $\boxed{\dot{H}>0}$

- Expanding ($H>0$), turnaround ($H=0$), contracting ($H<0$)
  near and at the turnaround $\boxed{\dot{H}<0}$

Bounce and Cyclic behavior

- Contracting ($H<0$), bounce ($H=0$), expanding ($H>0$)
  near and at the bounce $\dot{H}>0$

- Expanding ($H>0$), turnaround ($H=0$), contracting ($H<0$)
  near and at the turnaround $\dot{H}<0$

$$H^2=\frac{2\sigma_0}{(3\lambda-1)}\left(\rho_M+\frac{\sigma_1}{6}+\frac{\sigma_3 k^2}{6a^4}+\frac{\sigma_4 k}{6a^6}\right)+\frac{\sigma_2}{3(3\lambda-1)}\frac{k}{a^2}$$

$$\dot{H}+\frac{3}{2}H^2=-\frac{3\sigma_0}{(3\lambda-1)}\left(w_M\rho_M-\frac{\sigma_1}{6}+\frac{\sigma_3 k^2}{18a^4}+\frac{\sigma_4 k}{6a^6}\right)+\frac{\sigma_2}{6(3\lambda-1)}\frac{k}{a^2}$$

- Bounce and cyclicity can be easily obtained
  [Brandenberger, PRD 80] [Cai, Saridakis, JCAP 0910]

Bounce and Cyclic behavior

- Input: $a(t)$ oscillatory

- Output: $$\varphi(t)=\pm\int^{t}dt'\sqrt{\frac{2k}{a(t')^2}-2\dot{H}(t')-\left(\frac{2\sigma_3 k^2}{9a(t')^4}+\frac{\sigma_4 k}{3a(t')^6}\right)}$$

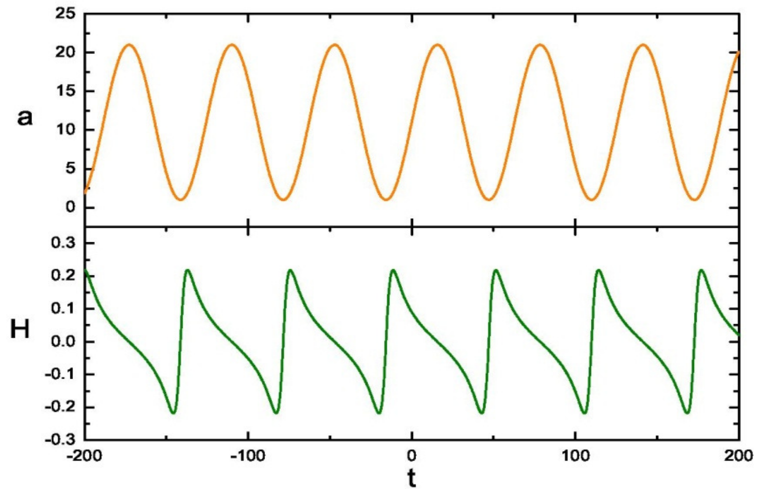
$$V(t)=3H(t)^2+\frac{2k}{a(t)^2}+\dot{H}(t)-\left(\frac{\sigma_1}{6}+\frac{\sigma_3 k^2}{18a(t')^4}\right)$$

- $\Rightarrow$ Reconstructed $V(\varphi)$

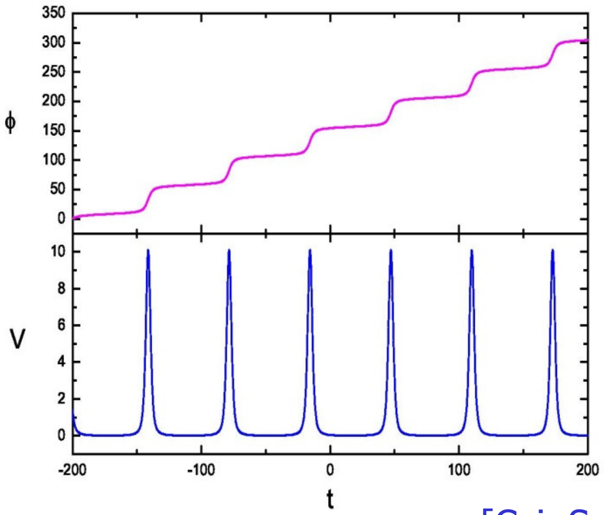
[Cai, Saridakis, JCAP 0910]

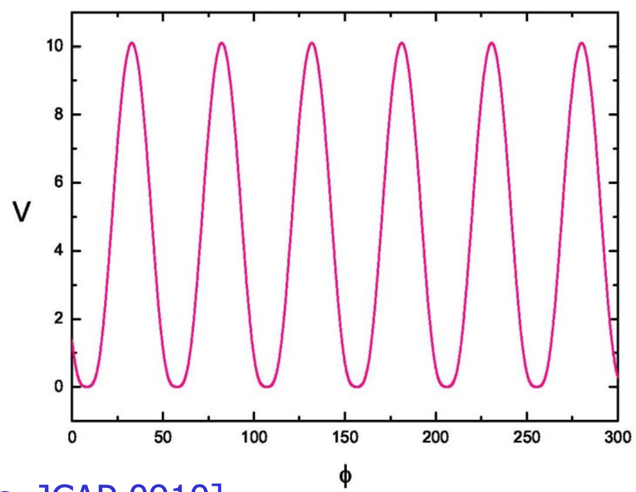
Bounce and Cyclic behavior

- Input: $a(t) = A\sin(\omega t) + a_c$



- Output:



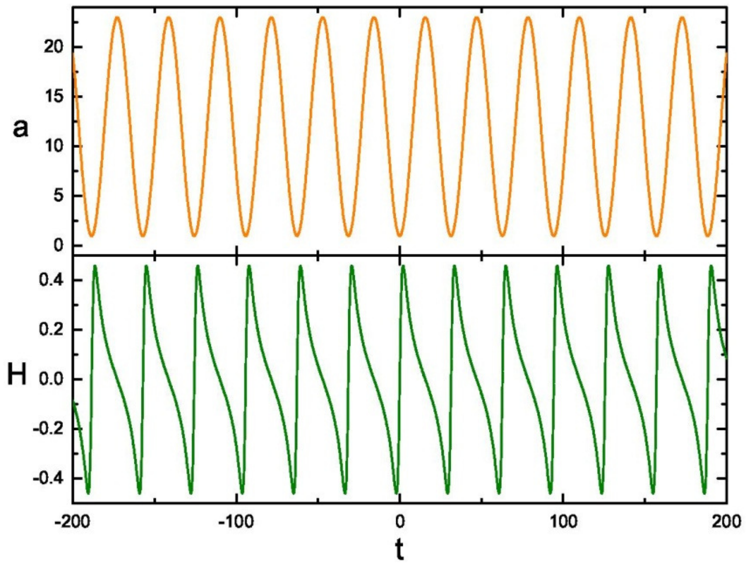


[Cai, Saridakis, JCAP 0910]

Bounce and Cyclic behavior

- Input: $V(\varphi) = V_0 \sin(\omega_V \varphi) + V_c$

- Output:



[Cai, Saridakis, JCAP 0910]

- Important: Processing of perturbations

[Brandenberger, PRD 80,b]

A more realistic dark energy

- In all the above discussion $w_{DE} \geq -1$
- Observational indications that $w_{DE} < -1$ today
- Possible solution: Insert a new scalar (canonical) field

$$S_h = \int dt d^3x \sqrt{g} N \left[\frac{1}{2} \partial_\mu h \partial^\mu h - V(h) \right], \qquad \rho_h = \frac{\dot{h}^2}{2} + V(h), \quad p_h = \frac{\dot{h}^2}{2} - V(h)$$

$$\Rightarrow \quad w_{DE,tot} = \frac{\frac{(3\lambda-1)\dot{h}^2}{4} - V(h) - \frac{\sigma_1}{6} + \frac{\sigma_3 k^2}{18a^4} + \frac{\sigma_4 k}{6a^6}}{\frac{(3\lambda-1)\dot{h}^2}{4} + V(h) + \frac{\sigma_1}{6} + \frac{\sigma_3 k^2}{6a^4} + \frac{\sigma_4 k}{6a^6}}$$

- Quintessence, Phantom and Quintom Cosmology easily acquired

[Saridakis, EJPC 65]

(see also f(R) Horava-Lifshitz cosmology [Nojiri, Odintsov, CQG27])

Observational constraints (detailed-balance)

- Use observational data (SNIa, BAO, CMB, BBN) to constrain the parameters of the theory

- Include matter and standard radiation hydrodynamically:
  $\rho_M = \rho_{M0}/a^3,\ \rho_r = \rho_{r0}/a^4,\ 1+z = 1/a$

- Fix $\boxed{\lambda = 1}$. Units $8\pi G = 1 \ \Rightarrow\ \kappa^2 = 4,\ \mu^2 \Lambda = 2$

Observational constraints (detailed-balance)

- Use observational data (SNIa, BAO, CMB, BBN) to constrain the parameters of the theory

- Include matter and standard radiation hydrodynamically:
  $\rho_M = \rho_{M0}/a^3,\ \rho_r = \rho_{r0}/a^4,\ 1+z = 1/a$

- Fix $\boxed{\lambda = 1}$. Units $8\pi G = 1 \ \Rightarrow\ \kappa^2 = 4,\ \mu^2\Lambda = 2$

$$\Rightarrow H^2 = H_0^2\left\{\Omega_{M0}(1+z)^3 + \Omega_{r0}(1+z)^4 + \Omega_{K0}(1+z)^2 + \left[\omega + \frac{\Omega_{K0}^2}{4\omega}(1+z)^4\right]\right\}$$

$$\Omega_{M0} = \frac{\rho_{M0}}{3H_0^2},\ \ \Omega_{r0} = \frac{\rho_{r0}}{3H_0^2}$$

$$\Omega_{K0} = -\frac{k}{H_0^2},\ \ \omega = \frac{\Lambda}{2H_0^2}$$

- 4 dimensionless parameters to be fitted: $\boxed{\Omega_{M0},\Omega_{K0},\Omega_{r0},\omega}$
  (we fix H_0 at its WAMP5 best fit value)

[Dutta, Saridakis, JCAP 1001]

Observational constraints (detailed-balance)

- At present: $\Omega_{M0}+\Omega_{r0}+\Omega_{K0}+\omega+\frac{\Omega_{K0}^2}{4\omega}=1$
- Total radiation (standard plus "dark") at Nucleosynthesis: $\frac{\Omega_{K0}^2}{4\omega}=0.135\Delta N_\nu\Omega_{r0}$

ΔN_ν: effective neutrino species. $-1.7\leq\Delta N_\nu\leq 2.0$

[Olive,et al, Phys. Rept. 333]

- Thus, 4 dimensionless parameters to be fitted $\Omega_{M0},\Omega_{K0},\omega,\Delta N_\nu$

  (we fix Ω_{r0} in terms of Ω_{M0},H_0)

Observational constraints (detailed-balance)

- At present: $\Omega_{M0}+\Omega_{r0}+\Omega_{K0}+\omega+\frac{\Omega_{K0}^2}{4\omega}=1$
- Total radiation (standard plus "dark") at Nucleosynthesis: $\frac{\Omega_{K0}^2}{4\omega}=0.135\Delta N_\nu \Omega_{r0}$

ΔN_ν: effective neutrino species. $-1.7\leq \Delta N_\nu \leq 2.0$

[Olive,et al, Phys. Rept. 333]

- Thus, 4 dimensionless parameters to be fitted $\Omega_{M0}, \Omega_{K0}, \omega, \Delta N_\nu$

  (we fix Ω_{r0} in terms of Ω_{M0}, H_0)

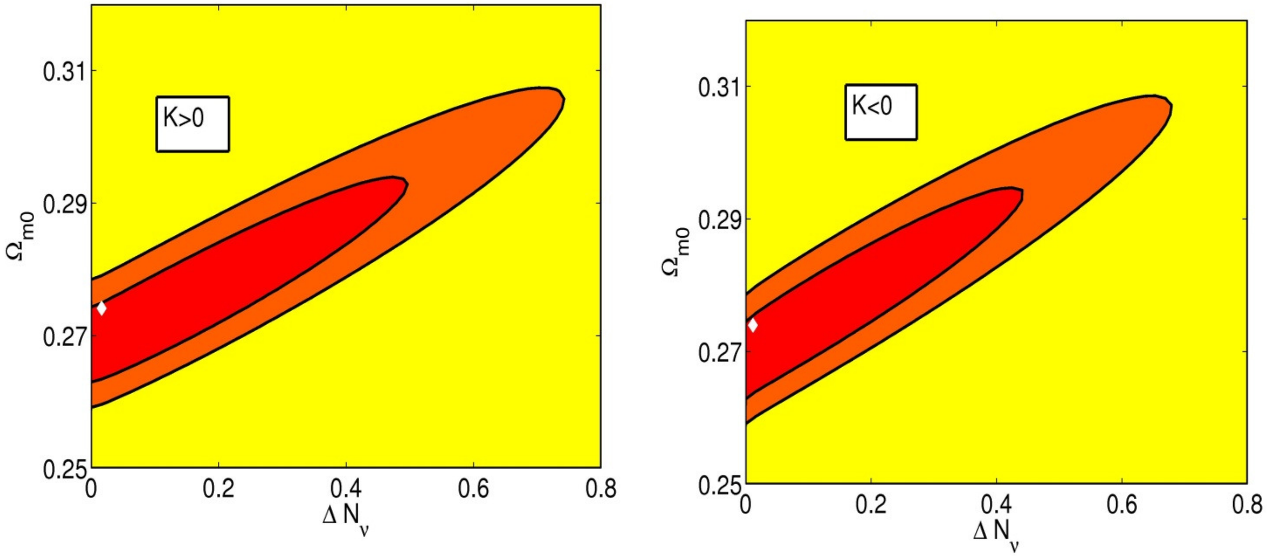
$$\Rightarrow \omega=1-\Omega_{M0}-(1-\Delta N_\nu)\Omega_{r0}-0.73k\sqrt{\Delta N_\nu}\sqrt{\Omega_{r0}(1-\Omega_{M0}-\Omega_{r0})}$$

$$\Rightarrow |\Omega_{K0}|=\sqrt{0.54\Delta N_\nu \Omega_{r0}\omega}$$

- 2 free parameters: $\Omega_{M0}, \Delta N_\nu$ [Dutta, Saridakis, JCAP 1001]

Observational constraints (detailed-balance)

- So:



- And thus in 1σ:

Ω_{K0}	Λ/H_0^2	$H_0\mu$
(0, 0.0038)	(0, 1.4189)	(1.1872, ∞)
(-0.0039, 0)	(0. 1.4063)	(1.1925, ∞)

[Dutta, Saridakis, JCAP 1001]

Observational constraints (beyond detailed-balance)

- $$H^2 = H_0^2\left\{\Omega_{M0}(1+z)^3 + \Omega_{r0}(1+z)^4 + \Omega_{K0}(1+z)^2 + \left[\omega_1 + \omega_3(1+z)^4 + \omega_4(1+z)^6\right]\right\}$$

$$\omega_1 = \frac{\sigma_1}{6H_0^2},\quad \omega_3 = \frac{\sigma_3 H_0^2 \Omega_{K0}^2}{6},\quad \omega_4 = -\frac{\sigma_4 \Omega_{K0}}{6}$$

- We fix Ω_{M0}, H_0 at their WAMP5 best fit values and Ω_{r0} is given in terms of them
- So 4 dimensionless parameters to be fitted: $\Omega_{K0}, \omega_1, \omega_3, \omega_4$

$$\Omega_{M0} + \Omega_{r0} + \Omega_{K0} + \omega_1 + \omega_3 + \omega_4 = 1 \quad \text{(at present)}$$

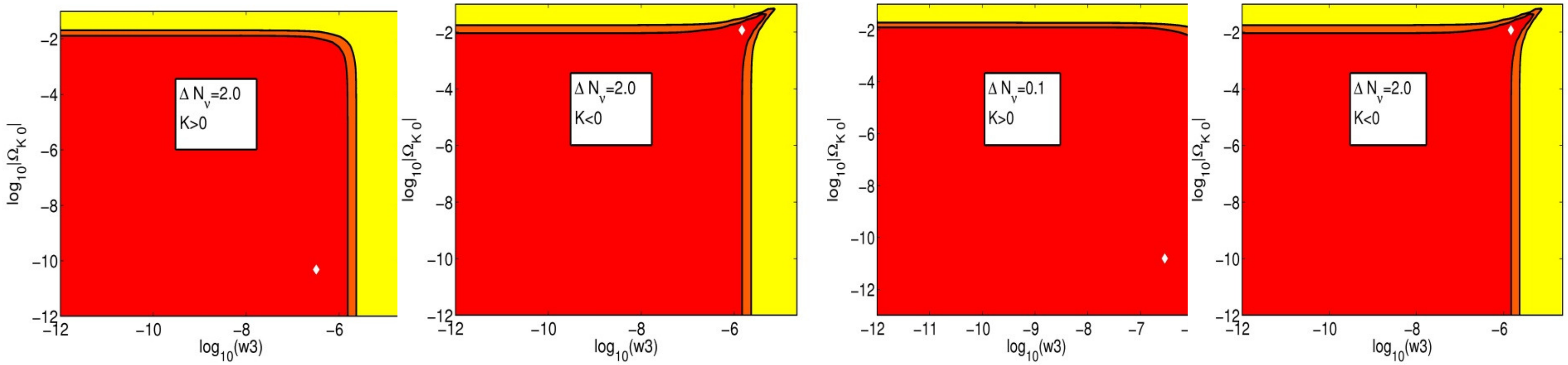
$$\omega_3 + \omega_4(1+z_{BBN})^2 = 0.135\,\Delta N_\nu \Omega_{r0} \quad \text{(Nucleosynthesis)}$$

- 2 free parameters: ω_3, Ω_{K0} for given values of ΔN_ν

[Dutta, Saridakis, JCAP 1001]

Observational constraints (beyond detailed-balance)

- So:



- And thus in 1σ:

ΔN_ν	Ω_{K0}	σ_1 / H_0^2	$\sigma_3 H_0^2$	σ_4
0.1	(0, 0.01)	(4.29, 4.33)	(0, 0.03)	$(-9\times10^{-22},\ 0)$
0.1	(-0.01, 0)	(4.40, 4.45)	(0, 0.81)	$(0,\ 6\times10^{-22})$
2.0	(0, 0.04)	(4.13, 4.45)	(0, 0.01)	$(-2\times10^{-20}, -3\times10^{-21})$
2.0	(-0.01, 0)	(4.40, 4.45)	(0, 0.23)	$(-3\times10^{-20}, -1\times10^{-20})$

[Dutta, Saridakis, JCAP 1001]

Observational constraints on λ

- Concerning cosmological observations λ is expected to be very close to its IR value 1.
- We perform an overall observational fitting, allowing λ to vary along with the other parameters of the theory.
- Detailed balance:

$$H^2 = H_0^2\left\{\Omega_{K0}(1+z)^2 + \left[\omega + \frac{\Omega_{K0}^2}{4\omega}(1+z)^4\right] + \frac{2}{3\lambda-1}\left[\Omega_{M0}(1+z)^3 + \Omega_{r0}(1+z)^4\right]\right\}$$

- Beyond detailed balance:

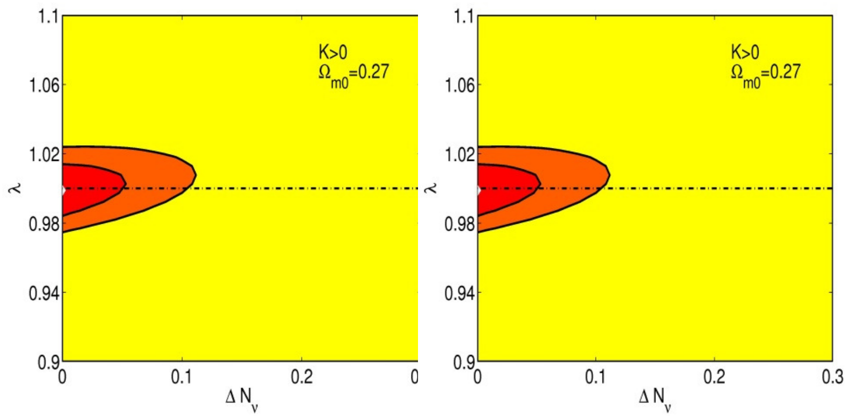
$$H^2 = H_0^2\left\{\Omega_{K0}(1+z)^2 + \frac{2}{3\lambda-1}\left\{\Omega_{M0}(1+z)^3 + \Omega_{r0}(1+z)^4 + \left[\omega_1 + \omega_3(1+z)^4 + \omega_4(1+z)^6\right]\right\}\right\}$$

- Repeat the aforementioned procedure.

[Dutta, Saridakis, JCAP 1005]

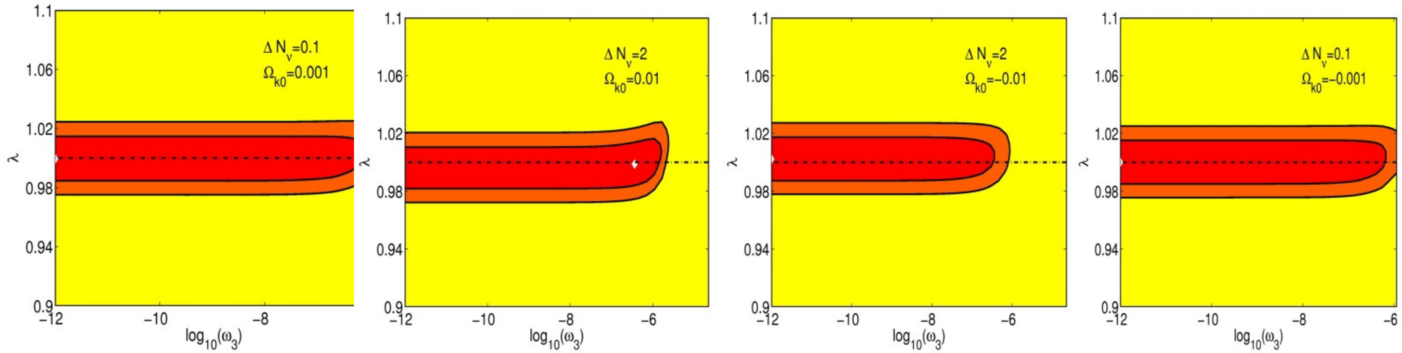
Observational constraints on λ

- Detailed balance:



$$\lambda \in (0.98,\ 1.01)$$

- Beyond detailed balance



$$\lambda \in (0.98,\ 1.02)$$

$$\left|\lambda_{b.f} - 1\right| \approx 0.0006$$

[Dutta, Saridakis, JCAP 1005]

Thermodynamic Aspects

- Known connection between gravity and thermodynamics.
- Field Equations $\Longrightarrow$ First Law of Thermodynamics.

- For a universe bounded by the apparent horizon $$r_A = \frac{1}{\sqrt{H^2 + k/a^2}}$$

  one calculates the entropy of the universe content, plus that of the horizon itself. Furthermore, all the "fluids" inside the universe have the same temperature with horizon.

Thermodynamic Aspects

- Known connection between gravity and thermodynamics.
- Field Equations $\Rightarrow$ First Law of Thermodynamics.
- For a universe bounded by the apparent horizon $r_A = \frac{1}{\sqrt{H^2 + k/a^2}}$

  one calculates the entropy of the universe content, plus that of the horizon itself. Furthermore, all the "fluids" inside the universe have the same temperature with horizon.
- In an FRW universe in GR: $dE = -4\pi r_A^3 H(\rho + p)dt$, $S_h = \frac{4\pi r_A^2}{4G}$, $T_h = \frac{1}{2\pi r_A}$

$$\Rightarrow -dE = TdS \quad \Rightarrow \quad \dot{H} - \frac{k}{a^2} = -4\pi G(\rho + p)$$

[R.G.Cai, Kim, JHEP 0502]

Thermodynamic Aspects

- Known connection between gravity and thermodynamics.
- Field Equations $\Rightarrow$ First Law of Thermodynamics.
- For a universe bounded by the apparent horizon $r_A = \dfrac{1}{\sqrt{H^2 + k/a^2}}$

  one calculates the entropy of the universe content, plus that of the horizon itself. Furthermore, all the "fluids" inside the universe have the same temperature with horizon.
- In an FRW universe in GR: $dE = -4\pi r_A^3 H(\rho + p)dt$, $S_h = \dfrac{4\pi r_A^2}{4G}, \quad T_h = \dfrac{1}{2\pi r_A}$

$$\Rightarrow -dE = TdS \quad \Rightarrow \quad \dot{H} - \frac{k}{a^2} = -4\pi G(\rho + p)$$

[R.G.Cai, Kim, JHEP 0502]

- In the same lines for the Generalized Second Law (GSL) of Thermodynamics (entropy time-variation of the universe content plus that of the horizon to be non-negative)

GSL in Horava-Lifshitz cosmology (detailed balance)

- The universe contains only matter. For its entropy time-variation:

$$dS_M = \frac{1}{T}\left(P_M\, dV + dE_M\right) \quad \text{with} \quad V = 4\pi r_A^3/3. \quad \Rightarrow \dot{S}_M = \frac{1}{T}\left(P_M\, 4\pi r_A^2 \dot{r}_A + \dot{E}_M\right)$$

with $E_M = 4\pi r_A^3 \rho_M / 3,\ P_M = w_M \rho_M$

and $$\dot{r}_A = H r_A^3 \left[4\pi G(1+w_M)\rho_M + \frac{k^2}{\Lambda a^4}\right]$$

- So: $$\dot{S}_M = \frac{1}{T}(1+w_M)\rho_M\, 4\pi r_A^2 \left(\dot{r}_A - H r_A\right)$$

GSL in Horava-Lifshitz cosmology (detailed balance)

- The universe contains only matter. For its entropy time-variation:

$$dS_M = \frac{1}{T}\left(P_M\, dV + dE_M\right)$$ with $V = 4\pi r_A^3/3$. $\Rightarrow \dot{S}_M = \frac{1}{T}\left(P_M\, 4\pi r_A^2 \dot{r}_A + \dot{E}_M\right)$

with $E_M = 4\pi r_A^3 \rho_M / 3,\ \ P_M = w_M \rho_M$

and $$\dot{r}_A = H r_A^3 \left[4\pi G\,(1 + w_M)\rho_M + \frac{k^2}{\Lambda a^4}\right]$$

- So: $$\dot{S}_M = \frac{1}{T}(1 + w_M)\rho_M\, 4\pi r_A^2\left(\dot{r}_A - H r_A\right)$$

- The temperature of the universe content is equal to that of the horizon:

$$T = T_h = \frac{1}{2\pi r_A}$$ (depends only on the universe geometry)

- The entropy of the horizon equals that of a black hole, with r_A as a horizon:

$$S_h = \frac{4\pi r_A^2}{4G} + \frac{\pi}{G\Lambda} k \ln\left(\Lambda r_A^2\right)$$ $$\Rightarrow \dot{S}_h = \frac{2\pi}{G}\left(r_A + \frac{k}{\Lambda r_A}\right)\dot{r}_A$$

[R.G.Cai, Ohta PRD 81] [Kehagias, Sfetsos PLB 678] [Jamil, Saridakis, Setare, JCAP 1011]

GSL in Horava-Lifshitz cosmology

- In total:

$$\dot{S}_{tot} = \dot{S}_M + \dot{S}_h =$$
$$= r_A^3 H \left[8\pi^2 r_A^3 (1 + w_M)\rho_M + \frac{2\pi k}{G\Lambda r_A} \right] \left[4\pi G (1 + w_M)\rho_M + \frac{k^2}{\Lambda a^4} \right] + \frac{2\pi k^2 H r_A^4}{G\Lambda a^4}$$

[Jamil, Saridakis, Setare , JCAP 1011]

- Clearly GSL is conditionally violated. Things are worse beyond detail balance, where the correction has not a standard sign.

GSL in Horava-Lifshitz cosmology

- In total:

$$\dot{S}_{tot} = \dot{S}_M + \dot{S}_h =$$
$$= r_A^3 H \left[8\pi^2 r_A^3 (1 + w_M)\rho_M + \frac{2\pi k}{G\Lambda r_A} \right] \left[4\pi G (1 + w_M)\rho_M + \frac{k^2}{\Lambda a^4} \right] + \frac{2\pi k^2 H r_A^4}{G\Lambda a^4}$$

[Jamil, Saridakis, Setare , JCAP 1011]

- Clearly GSL is conditionally violated. Things are worse beyond detail balance, where the correction has not a standard sign.
- Should we take other horizon? Can we define temperature, entropy or the horizon itself in HL cosmology? [Kiritsis, Kofinas, JHEP 1001]
- Or another "sign" against HL gravity?
- Interesting and Open Issues.

Superluminal neutrinos in Horava-Lifshitz cosmology

- Neutrinos motion in earth's gravitational field:

$$ds^2 = -N(r)^2 dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

$$N(r)^2 = f(r) = 1 + \frac{\Lambda r^2}{1-\varepsilon^2} - \frac{\sqrt{\alpha^2\left(1-\varepsilon^2\right)\sqrt{\Lambda}\, r + \varepsilon^2\Lambda^2 r^4}}{1-\varepsilon^2}$$

$$e_a^{\ \mu} = diag\left(\frac{1}{\sqrt{f}}, \sqrt{f}, \frac{1}{r}, \frac{1}{r\sin\theta}\right)$$

Superluminal neutrinos in Horava-Lifshitz cosmology

- Neutrinos motion in earth's gravitational field:

$$ds^2 = -N(r)^2 dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)$$

$$N(r)^2 = f(r) = 1 + \frac{\Lambda r^2}{1-\varepsilon^2} - \frac{\sqrt{\alpha^2\left(1-\varepsilon^2\right)\sqrt{\Lambda}\, r + \varepsilon^2\Lambda^2 r^4}}{1-\varepsilon^2}$$

$$e_a^{\ \mu} = diag\left(\frac{1}{\sqrt{f}}, \sqrt{f}, \frac{1}{r}, \frac{1}{r\sin\theta}\right)$$

- Dirac Eq.: $\left[\gamma^a e_a^{\ \mu}\left(\partial_\mu + \Gamma_\mu\right) + \frac{m}{\hbar}\right]\Psi = 0 \;\Rightarrow\; \left[\frac{\gamma^0}{\sqrt{f(r)}}\partial_t + \sqrt{f(r)}\gamma_1\partial_r + \ldots\right]\Psi = 0$

$$\Rightarrow v(r) \approx f(r)$$

- So: $v(R_\oplus) - 1 \approx 10^{-5} \;\Rightarrow\; 1-\varepsilon^2 \approx 10^{-15}$ [Saridakis [1110.0697]]

Perturbative instabilities?

- So far we discussed about HL cosmology. A consistent cosmology is not a proof for the consistency of the underlying gravitational theory. (It is necessary but not sufficient)
- Is HL gravity robust?

Perturbative instabilities?

- So far we discussed about HL cosmology. A consistent cosmology is not a proof for the consistency of the underlying gravitational theory. (It is necessary but not sufficient)
- Is HL gravity robust?

- Perturbations before analytic continuation:

$$\delta g_{00} = -2a^2\phi$$

$$\delta g_{0i} = a^2\left(\partial_i B + Q_i\right)$$ vector modes transverse ($\partial_i W^i = \partial_i Q^i = 0$)

$$\delta g_{ij} = a^2\left[h_{ij} - \left(\partial_i W_j + \partial_j W_i\right) - 2\psi\delta_{ij} + 2\partial_i\partial_j E\right]$$ tensor mode transverse and traceless ($\partial_i h^{ij} = \delta^{ij} h_{ij} = 0$)

- In "synchronous" gauge:

$$\delta N = \delta N_i = 0$$

$$\delta g_{ij} = h_{ij} - 2\psi\delta_{ij} + 2\partial_i\partial_j E - \left(\partial_i W_j + \partial_j W_i\right)$$

- Degrees of freedom: ψ, E (scalar), W_i (vector), h_{ij} (tensor)

[Bogdanos, Saridakis, CQG 27]

Perturbative instabilities?

- Fourier transforming, the dispersion relation for ψ at low k: $\omega^2 = -\frac{9\kappa^4\mu^2\Lambda^2}{32(3\lambda-1)^2}$

  at high k: $\omega^2 = \frac{\kappa^4\mu^2(\lambda-1)^2}{16(3\lambda-1)^2}k^4$

  For tensor mode we get: $\omega^2 = c^2k^2 + \frac{\kappa^4\mu^2}{16}k^4 \pm \frac{\kappa^4\mu}{4w^2}k^5 + \frac{\kappa^4}{4w^4}k^6$

- Beyond detail balance (assume $\delta S_{new} = \eta\int dtd^3x\left(-\frac{1}{4}h_{ij}\nabla^6h^{ij} - 6\psi\nabla^6\psi\right)$) we get:

  for scalar modes in the UV: $\omega^2 = \frac{\kappa^2(\lambda-1)^2}{16(3\lambda-1)^2}k^4 - \frac{3\kappa^2(\lambda-1)}{2(3\lambda-1)}\eta k^6$

  tensor modes: $\omega^2 = c^2k^2 + \frac{\kappa^4\mu^2}{16}k^4 \pm \frac{\kappa^4\mu}{4w^2}k^5 + \left(\frac{\kappa^4}{4w^4} - \frac{\kappa^2\eta}{2}\right)k^6$

Perturbative instabilities?

- Fourier transforming, the dispersion relation for ψ at low k: $\omega^2 = -\frac{9\kappa^4\mu^2\Lambda^2}{32(3\lambda-1)^2}$

  at high k: $\omega^2 = \frac{\kappa^4\mu^2(\lambda-1)^2}{16(3\lambda-1)^2}k^4$

  For tensor mode we get: $\omega^2 = c^2k^2 + \frac{\kappa^4\mu^2}{16}k^4 \pm \frac{\kappa^4\mu}{4w^2}k^5 + \frac{\kappa^4}{4w^4}k^6$

- Beyond detail balance (assume $\delta S_{new} = \eta\int dtd^3x\left(-\frac{1}{4}h_{ij}\nabla^6 h^{ij} - 6\psi\nabla^6\psi\right)$) we get:

  for scalar modes in the UV: $\omega^2 = \frac{\kappa^2(\lambda-1)^2}{16(3\lambda-1)^2}k^4 - \frac{3\kappa^2(\lambda-1)}{2(3\lambda-1)}\eta k^6$

  tensor modes: $\omega^2 = c^2k^2 + \frac{\kappa^4\mu^2}{16}k^4 \pm \frac{\kappa^4\mu}{4w^2}k^5 + \left(\frac{\kappa^4}{4w^4} - \frac{\kappa^2\eta}{2}\right)k^6$

- Cannot fix everything with analytic continuation: $\mu \to i\mu,\ \ w^2 \to -iw^2$
  (apart from the fact that this could radically change the renormalizability properties of the theory)
- One could take Λ=0 but what about the light speed?

[Bogdanos, Saridakis, CQG 27] [Charmousis, et al, JHEP 0908]

Healthy extension of Horava-Lifshitz gravity?

- So, one should search for extended versions of Horava-Lifshitz gravity:

$$S = S_k + S_1 + S_2 + S_{new}$$

$$S_k = \alpha \int dt d^3x \sqrt{g} N (K_{ij} K^{ij} - \lambda K^2)$$

$$S_1 = \int dt d^3x \sqrt{g} N \left[\gamma \frac{\varepsilon^{ijk}}{\sqrt{g}} R_{il} \nabla_j R^l_k + \zeta R_{il} R^{ij} + \eta R^2 + \xi R + \sigma \right]$$

$$S_2 = \int dt d^3x \sqrt{g} N \left[\beta C_{ij} C^{ij} + \beta_1 R \Diamond R + \beta_2 R^3 + \beta_3 R R_{il} R^{ij} + \beta_4 R_{il} R^{ik} R^j_k \right]$$

$$S_{new} = \int dt d^3x \sqrt{g} N \{ a_1 (a_i a^i) + a_2 (a_i a^i)^2 + a_3 R^{ij} a_i a_j + $$
$$+ a_4 R \nabla_i a^i + a_5 \nabla_i a_j \nabla^i a^j + a_6 \nabla^i a_i (a_i a^i) + \ldots \}$$

[Kiritsis, PRD 81] [R.G.Cai, Zhang PRD 83]

with $a_i = \dfrac{\partial_i N}{N}$ [Blas, et al, PRL 104]

Conclusions

- i) Horava-Lifshitz gravity applied as a cosmological framework
  $\Rightarrow$ Horava-Lifshitz cosmology. Very interesting.
- ii) Interesting late-time solution sub-classes, revealed by phase-space analysis. Amongst them an eternally expanding DE dominated universe.
- iii) We can obtain bouncing and cyclic behavior
- iv) We can use observations to constrain the model parameters. λ is constrained in $|\lambda - 1| \leq 0.02$
- v) The generalized second law of thermodynamics is not valid
- vi) However, there may be problems at Horava-Lifshitz gravity itself.
  Perturbative instabilities, that cannot be easily cured.
- vii) Search for healthy extensions

Outlook

- Many cosmological subjects are open. Amongst them:
- i) Calculate the Parametrized-Post-Newtonian (PPN) parameters for HL cosmology.
- ii) Constrain observationally the minimal extended version
- iii) Examine the generalized second law in the extended version
- iv) And of course provide clues, arguments, indications and proofs that Horava-Lifshitz gravity is indeed the underlying theory of gravity.

THANK YOU!