The quantum-to-classical transition of primordial cosmological perturbations

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Problem of the quantum-to-classical transition

According to inflation theory the large scale structure arises from quantum vacuum fluctuations.

→ How do the quantum fluctuations become classical fluctuations?

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→ Is especially severe in cosmological context! Which processes count as measurement in the early universe?

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Possible solutions:

collapse theories (Sudarsky!), many worlds, de Broglie-Bohm theory

Outline

- Introduction to de Broglie-Bohm
- Illustration of problem: inverted harmonic oscillator
- Discussion of cosmological perturbations.

Non-relativistic de Broglie-Bohm theory

(a.k.a. pilot-wave theory, Bohmian mechanics, . . .)

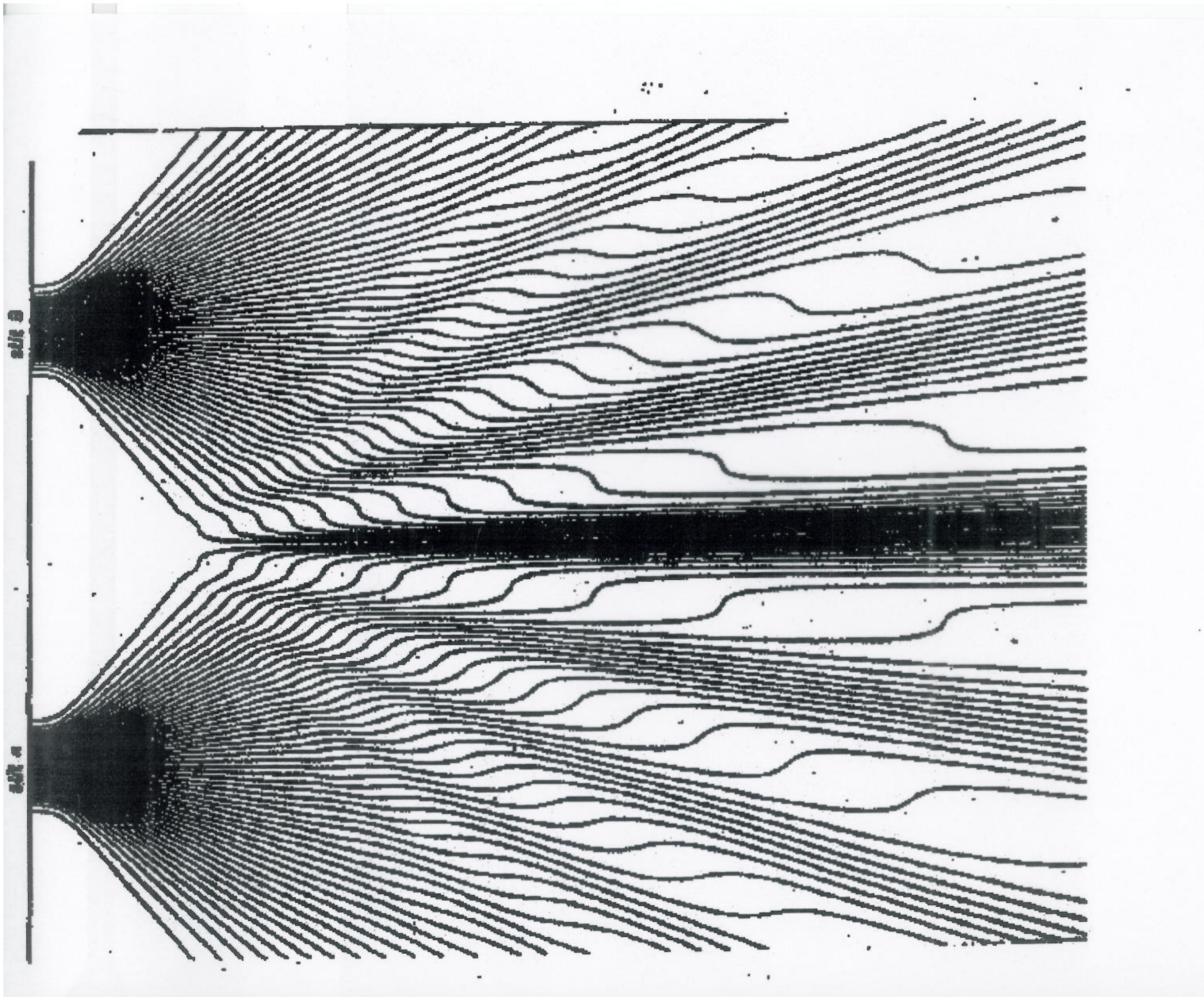
- De Broglie (1927), Bohm (1952)
- Point particles guided by wave function.
- Dynamics:
  - Wave $\psi(\mathbf{x}_1, \dots, \mathbf{x}_n, t)$:

$$\mathrm{i}\hbar\partial_t\psi = \left(-\sum_{k=1}^{n} \frac{\hbar^2}{2m_k}\nabla_k^2 + V \right)\psi$$

  - Particles positions $\mathbf{x}_1(t), \dots, \mathbf{x}_n(t)$:

$$\frac{d\mathbf{x}_k}{dt} = \mathbf{v}_k^{\psi}(\mathbf{x}_1, \dots, \mathbf{x}_n, t) = \frac{1}{m_k}\boldsymbol{\nabla}_k S(\mathbf{x}_1, \dots, \mathbf{x}_n, t), \qquad \psi = |\psi|e^{\mathrm{i}S/\hbar}$$

- **Double Slit experiment:**



- **Quantum equilibrium:**

  Consider an ensemble of systems with wave function ψ,

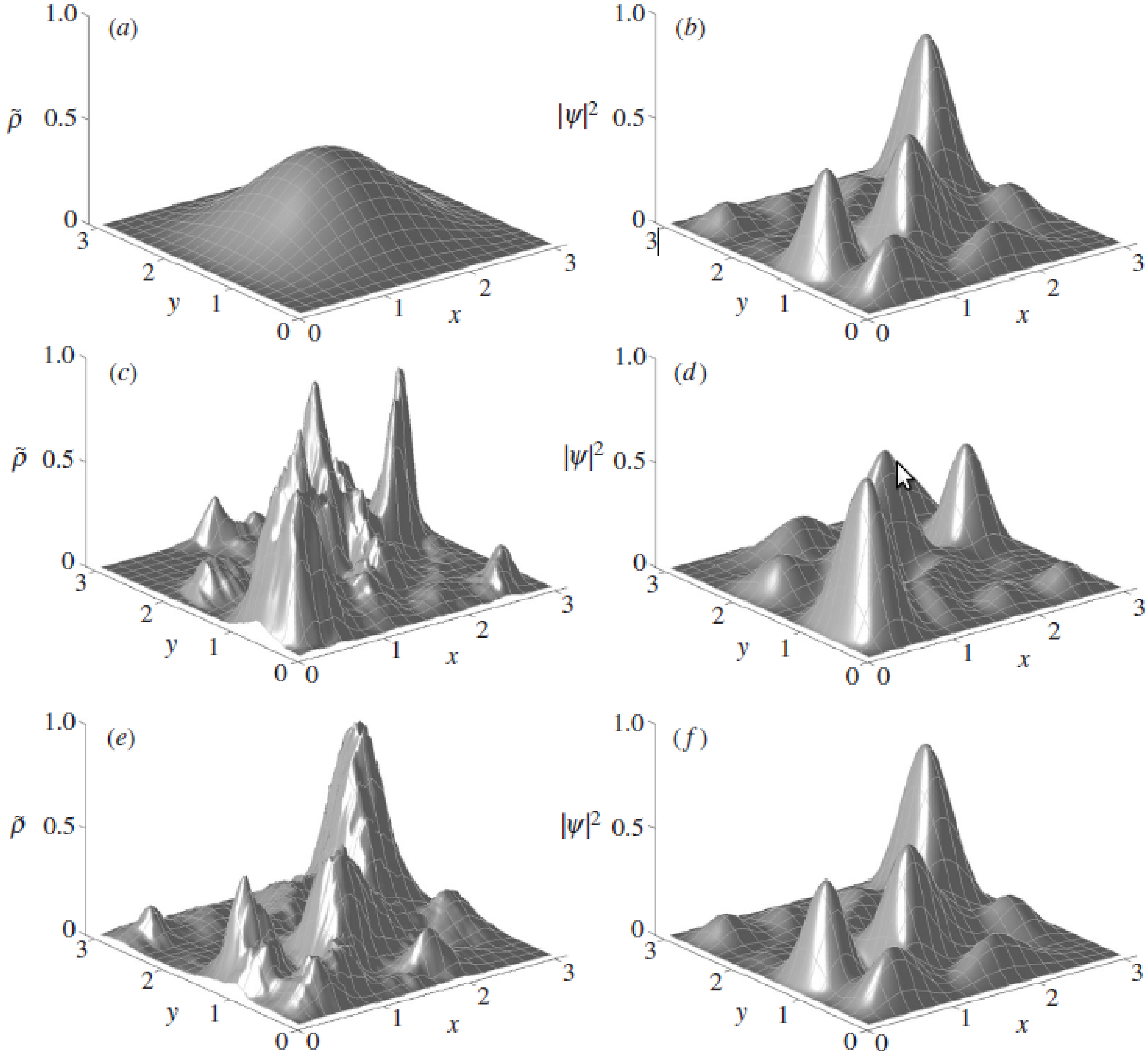
  and particle distribution $\rho(x)$.

  Quantum equilibrium if $\rho(x) = |\psi(x)|^2$.

  $\rightarrow$ Standard quantum theory emerges in quantum equilibrium.

  $\rightarrow$ Deviations from standard quantum theory in non-equilibrium ($\rho(x) \neq |\psi(x)|^2$).

Relaxation, $\rho(x) \to |\psi(x)|^2$ (Valentini & Westman, 2004):



- Quantum equilibrium is preserved by the particle motion because it satisfies the continuity equation:

$$\partial_t |\psi|^2 + \sum_{k=1}^{n} \boldsymbol{\nabla}_k \cdot (\mathbf{v}_k^{\psi} |\psi|^2) = 0$$

→ For other Schrödinger equations, the continuity equation of $|\psi|^2$ may be used to find a suitable guidance law.

That is

$$\partial_t |\psi|^2 + \mathrm{div} j^{\psi} = 0$$

suggests the guidance law

$$\dot{X} = \frac{j^{\psi}}{|\psi|^2}$$

(treatment of arbitrary Hamiltonians: Struyve & Valentini (2009))

• Classical limit:

$$\dot{\mathbf{x}} = \frac{1}{m}\boldsymbol{\nabla} S \qquad \Rightarrow \qquad m\ddot{\mathbf{x}} = -\boldsymbol{\nabla}(V + Q)$$

$$\psi = |\psi| e^{\mathrm{i}S/\hbar}, \qquad Q = -\frac{\hbar^2}{2m}\frac{\nabla^2|\psi|}{|\psi|} = \text{quantum potential}$$

Classical trajectories when $|\boldsymbol{\nabla} Q| \ll |\boldsymbol{\nabla} V|$.

- **Emergence of time:**

  Suppose

$$\widehat{H}\Psi(x_1, x_2) = 0$$

  and de Broglie-Bohm trajectories $x_1(t)$, $x_2(t)$.

  Wave function for system 1:

$$\psi(x_1, t) = \Psi(x_1, x_2(t))$$

$\rightarrow$ may have non-trivial time dependence

$\rightarrow$ may satisfy time dependent Schrödinger equation.

(see e.g. work by P. Peter, N. Pinto-Neto, ...)

Quantum field theory

- De Broglie-Bohm theory can be extended to quantum fields (see Struyve (2010), (2011) for reviews)
- E.g. scalar field:

Hamtonian:

$$\widehat{H} = \frac{1}{2}\int d^3x \left(\widehat{\pi}^2 + \boldsymbol{\nabla}\widehat{\phi}^2 + m^2\widehat{\phi}^2\right)$$

Functional Schrödinger picture:

$$\widehat{\phi} \to \phi\,, \qquad \widehat{\pi} \to -\mathrm{i}\frac{\delta}{\delta\phi}\,, \qquad \Psi(\phi, t) = \langle\phi|\Psi(t)\rangle$$

$$\mathrm{i}\frac{\partial\Psi}{\partial t} = \frac{1}{2}\int d^3x \left(-\frac{\delta^2}{\delta\phi^2} + \boldsymbol{\nabla}\phi^2 + m^2\phi^2\right)\Psi$$

Continuity equation:

$$\frac{\partial|\Psi|^2}{\partial t} + \int d^3x \frac{\delta}{\delta\phi(\mathbf{x})}\left(\frac{\delta S}{\delta\phi(\mathbf{x})}|\Psi|^2\right) = 0\,, \qquad \Psi = |\Psi|e^{\mathrm{i}S}\,.$$

Guidance equation:

$$\dot{\phi}(\mathbf{x}) = \frac{\delta S}{\delta\phi(\mathbf{x})}$$

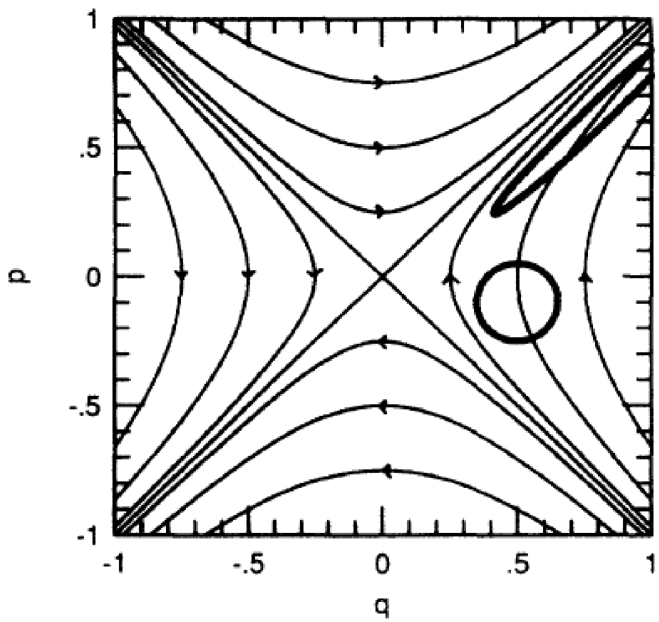
Inverted harmonic oscillator (e.g. Albrecht et al. 1994)

$$H = \frac{p^2}{2} - \frac{q^2}{2} \tag{1}$$

Classical trajectories:

$$q = A\mathrm{e}^{t} + B\mathrm{e}^{-t}, \qquad p = A\mathrm{e}^{t} - B\mathrm{e}^{-t}$$

$$q \approx p \approx A\mathrm{e}^{t} \qquad \text{for} \qquad t \gg 1 \qquad \Rightarrow \quad \text{squeezing}$$



Quantum mechanics

Squeezed state:

$$\psi(q,t) = N \exp\left(-\frac{(B - \mathrm{i}C)}{2} q^2 - \mathrm{i}\frac{B}{2} t \right)$$

$$N = \sqrt{\frac{B}{\pi}}\,, \qquad B = \frac{1}{\cosh 2t}\,, \qquad C = \tanh 2t$$

Note

$$\Delta q^2 = \frac{1}{2B}\,, \qquad \Delta p^2 = \frac{B}{2} + \frac{C^2}{2B}$$

$$\text{For } \; t = 0: \qquad \Delta q^2 = \frac{1}{2}\,, \qquad \Delta p^2 = \frac{1}{2}$$

$$\text{For } \; t \gg 1: \qquad \Delta q^2 \gg 1\,, \qquad \Delta p^2 \gg 1$$

→ Initially minimum uncertainty in q and p. However, both spread in time!

→ The wave function is not peaked around a classical trajectory!

How can it correspond to a classical system?

Common classicality arguments

1. Commuting observables

Heisenberg operators:

$$\widehat{q}(t) = \widehat{q}(0)\cosh t + \widehat{p}(0)\sinh t\,, \qquad \widehat{p}(t) = \widehat{q}(0)\sinh t + \widehat{p}(0)\cosh t$$

For $t \gg 1$:

$$\widehat{q}(t) \approx \widehat{p}(t) \approx \frac{1}{2}(\widehat{q}(0) + \widehat{p}(0))\mathrm{e}^t$$

Hence

$$[\widehat{q}(t), \widehat{p}(t)] \approx 0 \qquad \Rightarrow \qquad \text{Classicality}$$

However

$$[\widehat{q}(t), \widehat{p}(t)] = \mathrm{i} \not\approx 0$$

Similarly: free particle

Heisenberg operators:

$$\widehat{x}(t) = \widehat{x}(0) + \frac{\widehat{p}(0)}{m} t \,, \qquad \widehat{p}(t) = \widehat{p}(0)$$

For $t \gg 1$:

$$\widehat{x}(t) \approx \frac{\widehat{p}(0)}{m} t$$

Hence

$$[\widehat{x}(t), \widehat{p}(t)] \approx 0 \qquad \Rightarrow \qquad \text{Classicality}$$

However

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Better:

$$\begin{aligned} \Delta x(t)^2 &= \Delta x(0)^2 + \frac{t}{m}\Big(\langle \{ \widehat{x}(0), \widehat{p}(0) \} \rangle - \langle \widehat{x}(0) \rangle \langle \widehat{p}(0) \rangle \Big) + \frac{t^2}{m^2} \Delta p(0)^2 \\ &\approx \Delta x(0)^2 \qquad \text{for} \qquad \frac{t}{m} \ll 1 \end{aligned}$$

$\Rightarrow$ No spreading for a very massive particle for short enough times.

2. Wigner distribution:

$$\rho(q,p,t) = \frac{1}{\sqrt{\pi B}} |\psi(q,t)|^2 \exp\left(- \frac{(p - Cq)^2}{B} \right)$$
$$\to |\psi(q,t)|^2 \delta(p - q) \qquad \text{for} \qquad t \gg 1 \tag{2}$$

→ Is not peaked around a classical trajectory

→ But:

- satisfies Liouville equation $d\rho/dt = 0$
- and quantum mechanical expectation values equal classical averages over ρ

However, this does not mean classical limit is achieved!

3. WKB limit

With $\psi = |\psi| \mathrm{e}^{\mathrm{i}S}$:

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2} + V + Q = 0\,,$$

$$V = -\frac{q^2}{2}\,, \qquad Q = \frac{B}{2}(1 - Bq^2)$$

For $t \gg 1$:

$$\frac{\partial S}{\partial t} + \frac{(\nabla S)^2}{2} + V \approx 0\,,$$

$\rightarrow$ Formally same as classical Hamilton-Jacobi equation

But:

Does not imply we can assume a classical trajectory

4. Decoherence

Decoherence due to coupling with other degrees of freedom may yield decomposition of ψ into "classical wave packets". Collapse may select one of these.

De Broglie-Bohm description description of the inverted oscillator

$$\dot{q} = \nabla S \qquad \Rightarrow \qquad \ddot{q} = F_C + F_Q$$

Classical force: $F_C = q$

Quantum force: $F_Q = qB^2$

Ratio:

$$\frac{F_Q}{F_C} = B^2 \quad \to 0 \quad \text{for} \quad t \gg 1 \qquad \to \textbf{classical behaviour}$$

More precisely:

$$q(t) \sim \sqrt{\cosh 2t}$$
$$\sim \mathrm{e}^t \quad \text{for} \quad t \gg 1$$

Cosmological perturbations

Inflaton field: $\varphi(\mathbf{x},\eta) = \varphi_0(\eta) + \delta\varphi(\mathbf{x},\eta)$

Metric with scalar perturbations, in the longitudinal gauge:

$$\mathrm{d}s^2 = a^2(\eta)\left\{[1+2\phi(\eta,\mathbf{x})]\,\mathrm{d}\eta^2 - [1-2\phi(\eta,\mathbf{x})]\,\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\right\},$$

Gauge invariant Mukhanov-Sasaki variable:

$$y \equiv a\left[\delta\varphi + \frac{\varphi'}{\mathcal{H}}\phi\right],$$

where $\mathcal{H} = \frac{a'}{a}$ is comoving Hubble parameter.

Fourier modes:

$$y(\eta,\mathbf{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} y_{\mathbf{k}}(\eta)\mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}},$$

$$H = \int_{\mathbb{R}^{3+}} d^3k\left[p_{\mathbf{k}}p_{\mathbf{k}}^* + k^2 y_{\mathbf{k}}y_{\mathbf{k}}^* + \frac{z'}{z}(p_{\mathbf{k}}y_{\mathbf{k}}^* + y_{\mathbf{k}}p_{\mathbf{k}}^*)\right], \qquad z = a\varphi'/\mathcal{H}$$

Classical mode equation:

$$y''_{\mathbf{k}} + \left(k^2 - \frac{z''}{z}\right) y_{\mathbf{k}} = 0.$$

Physical modes, initially well inside the Hubble radius, i.e. $k|\eta| \gg 1$ or $k^2 \gg z''/z$ or :

$$y_{\mathbf{k}}(\eta) \sim \mathrm{e}^{-\mathrm{i}k\eta}\left(1 + \frac{A_k}{\eta} + \ldots\right).$$

At late times, modes outside the Hubble radius, i.e. $k|\eta| \ll 1$ or $k^2 \ll z''/z$:

$$y_{\mathbf{k}}(\eta) \sim \underbrace{A_k^d \eta^{\alpha_d}}_{\alpha_d > 0} + \underbrace{A_k^g \eta^{\alpha_g}}_{\alpha_g < 0} \approx A_k^g \eta^{\alpha_g}$$

decaying mode — growing mode

Quantum

For product wave functional $\Psi(y,\eta) = \Pi_{\mathbf{k}\in\mathbb{R}^{3+}} \Psi_{\mathbf{k}}(y_{\mathbf{k}}, y^*_{\mathbf{k}}, \eta)$:

$$\mathrm{i}\frac{\partial \Psi_{\mathbf{k}}}{\partial \eta} = \left[-\frac{\partial^2}{\partial y^*_{\mathbf{k}} \partial y_{\mathbf{k}}} + k^2 y^*_{\mathbf{k}} y_{\mathbf{k}} - \mathrm{i}\frac{z'}{z}\left(\frac{\partial}{\partial y^*_{\mathbf{k}}} y^*_{\mathbf{k}} + y_{\mathbf{k}} \frac{\partial}{\partial y_{\mathbf{k}}} \right) \right] \Psi_{\mathbf{k}}.$$

Ground state:

$$\Psi_{\mathbf{k}} = \frac{1}{\sqrt{2\pi}|f_k(\eta)|} \exp\left\{ -\frac{1}{2|f_k(\eta)|^2}|y_{\mathbf{k}}|^2 + \mathrm{i}\left[\left(\frac{|f_k(\eta)|'}{|f_k(\eta)|} - \frac{z'}{z} \right) |y_{\mathbf{k}}|^2 - \int^{\eta} \frac{d\tilde{\eta}}{2|f_k(\tilde{\eta})|^2} \right] \right\},$$

f_k a solution to the classical mode equation.

→ **Is two-mode squeezed state.**

→ **Is translationally and rotationally invariant.**

De Broglie-Bohm

Guidance equation:

$$y'_{\mathbf{k}} = \frac{\partial S_{\mathbf{k}}}{\partial y^*_{\mathbf{k}}} + \frac{z'}{z} y_{\mathbf{k}} \,, \qquad \Psi_{\mathbf{k}} = |\Psi_{\mathbf{k}}| \mathrm{e}^{\mathrm{i}S_{\mathbf{k}}}$$

For ground state:

$$y_{\mathbf{k}}(\eta) \sim |f_k(\eta)|$$

→ Is in general not translationally or rotationally invariant!

For physical modes, at early times ($k^2 \gg z''/z$):

$$y_{\mathbf{k}}(\eta) \sim \left(1 + \frac{\mathrm{Re} A_k}{\eta} + \dots \right) .$$

→ Nearly stationary

At late times, $k^2 \ll z''/z$:

$$y_{\mathbf{k}}(\eta) \sim \eta^{\alpha_g}$$

→ Behaves classically at lates time $k^2 \ll z''/z$

Can also be seen from

$$y_{\mathbf{k}}' = \frac{\partial S_{\mathbf{k}}}{\partial y_{\mathbf{k}}^*} + \frac{z'}{z} y_{\mathbf{k}}\,, \qquad \Rightarrow \qquad y_{\mathbf{k}}'' + \left(k^2 - \frac{z''}{z}\right) y_{\mathbf{k}} = -\frac{\partial Q_{\mathbf{k}}}{\partial y_{\mathbf{k}}^*}\,,$$

$$Q_{\mathbf{k}} = -\frac{1}{|\Psi_{\mathbf{k}}|} \frac{\partial^2 |\Psi_{\mathbf{k}}|}{\partial y_{\mathbf{k}}^* \partial y_{\mathbf{k}}}$$

Ratio quantum force $F_{Q,\mathbf{k}}$ and classical force $F_{C,\mathbf{k}}$:

$$\frac{F_{Q,\mathbf{k}}}{F_{C,\mathbf{k}}} = -\frac{1}{4|f_k|^4 \left(k^2 - \frac{z''}{z}\right)} \to 0 \quad \text{for} \quad k^2 \ll z''/z \qquad \to \textbf{classical behaviour}$$

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→ **No appeal to decoherence.**

→ **Decoherence in the field basis will not alter the classicality.**

Two-point correlation function

In quantum equilibrium ($\rho(y) = |\Psi(y)|^2$):

$$\langle y(\eta, \mathbf{x}) y(\eta, \mathbf{x} + \mathbf{r}) \rangle_{\text{dBB}} = \int \mathcal{D}y |\Psi(y, \eta)|^2 y(\mathbf{x}) y(\mathbf{x}+\mathbf{r}) = \frac{1}{2\pi^2} \int dk \frac{\sin kr}{r} k |f_k(\eta)|^2$$

Is usual expression. (It will correspond to a spatial average under the ergodic assumption.)