Second-order Boltzmann Code and CMB bispectrum from recombination

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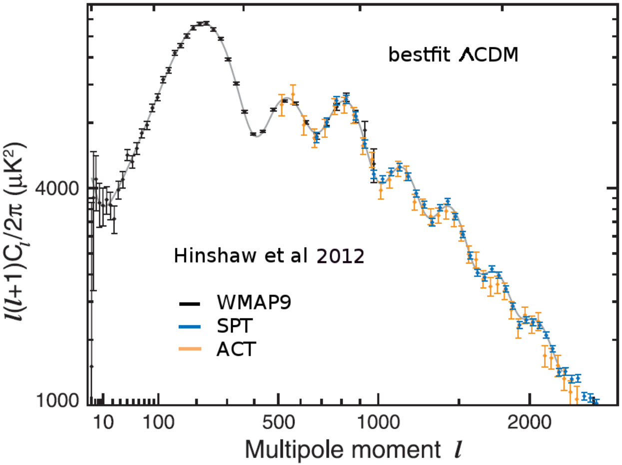
Outline

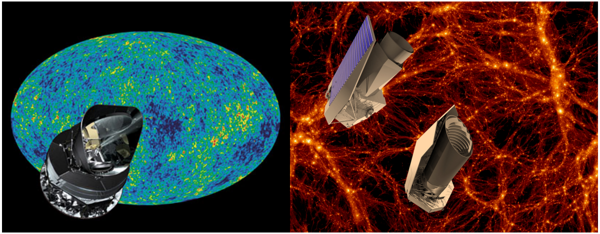
ΛCDM and CMB two-point statistics



primordial non-Gaussianity

$$\Phi = \Phi_G + f_{\rm NL}^{\rm local}\left(\Phi_G^2 - \langle\Phi_G^2\rangle\right)$$

WMAP7: $-10 < f_{\rm NL}^{\rm local} < 74$
SDSS-DR9: $-45 < f_{\rm NL}^{\rm local} < 195$
Planck Target: $\Delta f_{\rm NL}^{\rm local} \sim 5$
Planck + EUCLID-like: $\Delta f_{\rm NL}^{\rm local} \approx 3$ (Giannantonio et al)



primordial non-Gaussianity

Once we care about $O(1)\Phi^2$, we need to take into account:
Early-time effects:

Non-linearity of General relativity (GR);

Inhomogeneous recombination;

Late-time effects:

Non-linearity of GR;

Inhomogeneous reionization;

Secondary effects lensing, SZ.

For a realistic experiment, many other things to worry about: foreground, detector-induced systematics ...

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The debate about effective $f_{\rm NL}^{\rm local}$

Khatri et al 2009 : -1

Nitta et al 2009 (only quadratic terms): 1

Pitrou et al 2010: ~ 5

Senatore et al 2010: -3.5 (*)

Creminelli et al 2011: 0.94

Bartolo et al. 2011: $O(1)$

Su et al. 2012: 0.88

Second-order Boltzmann code

Linear-order perturbations

$$\frac{d\delta X_i}{d\eta} + A_{ij}(\eta)\delta X_i = 0$$

δX_i $(i = 1, 2, \ldots)$ are linear-order perturbations and $A_{ij}(\eta)$ are known background functions.

Perturbations include: baryon (density & velocity), CDM (density & velocity), neutrinos (phase-space distribution), radiation (phase-space distribution), and also DE if not a cosmological constant.

codes: CMBfast, CAMB, CMBEasy, CLASS, CosmoLib, CMBquick, ...

Second-order perturbations

$$\frac{d\delta X_i^{(2)}}{d\eta} + A_{ij}(\eta)\delta X_j^{(2)} = S_i$$

$\delta X_i^{(2)}$ $(i = 1, 2, \ldots)$ are second-order perturbations (in Fourier space), $A_{ij}(\eta)$ remain the same, and the sources S_i are convolutions of linear order perturbations.

Bruni *et al* 97; Pitrou *et al* 09, 10; Beneke & Fidler 10; Christopherson *et al* 08, 09, 11; Bartolo 07, 11; Senatore 08; Nitta *et al* 09; Khatri *et al* 09; Creminelli, Pitrou, and Vernizzi 11; Lewis 12 . . .

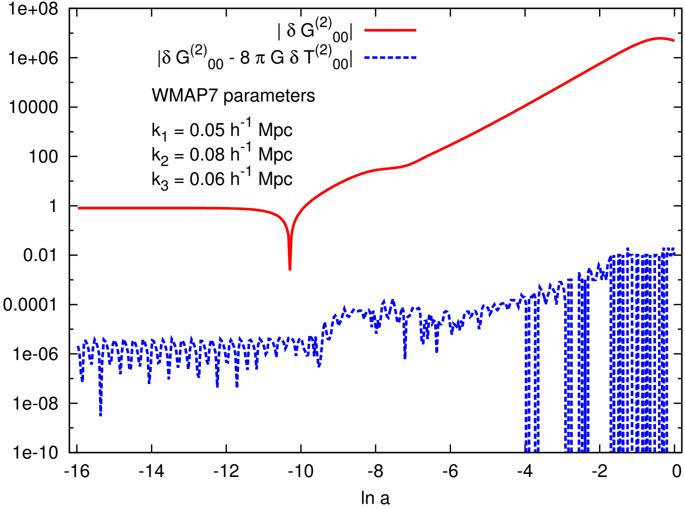
mathematica code CMBquick2 by Cyril Pitrou

Fortran code CosmoLib$^{\rm 2nd}$

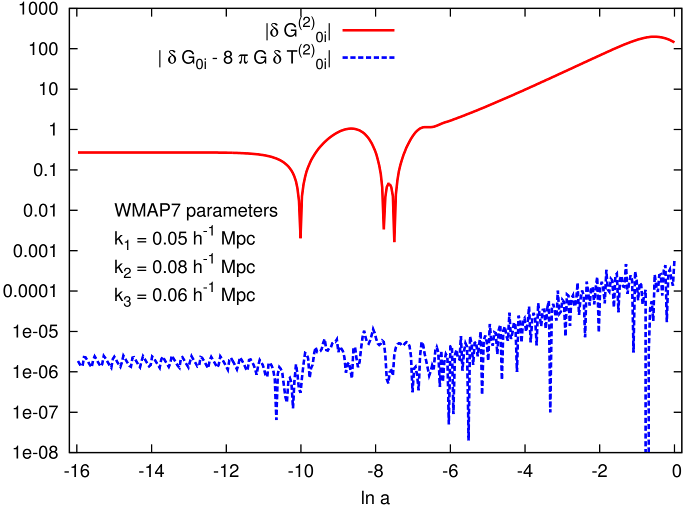
Comparison with CMBQuick2 by Cyril Pitrou

- Written in Fortran, no license constraint.
- Faster and parallelized.
- Much more accurate (energy and momentum constraint $\sim 10^{-6}$).
- Consistent treatment of perturbed RECFast (including Helium)
- Better scheme to integrate the CMB bispectrum (truncation error reduced by a nonlinear transformation).
- Full-sky bispectrum.

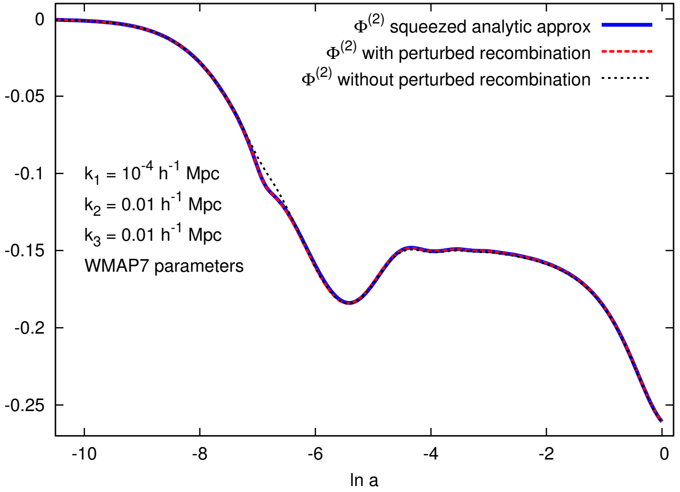
Einstein equations: energy constraint



Einstein equations: momentum constraint

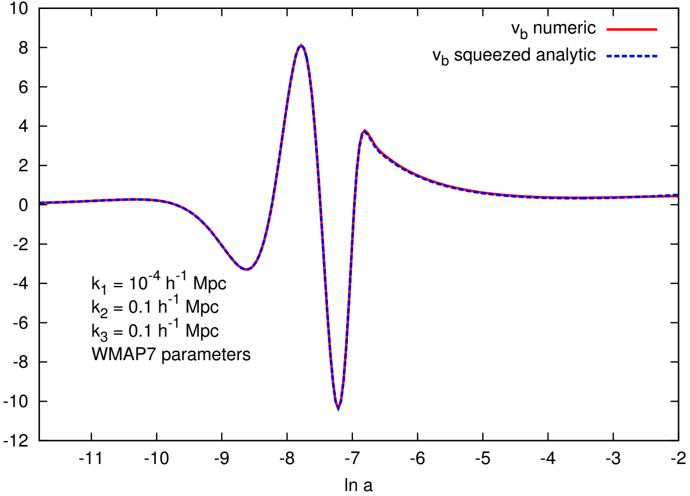


Squeezed limit: gravitational potential

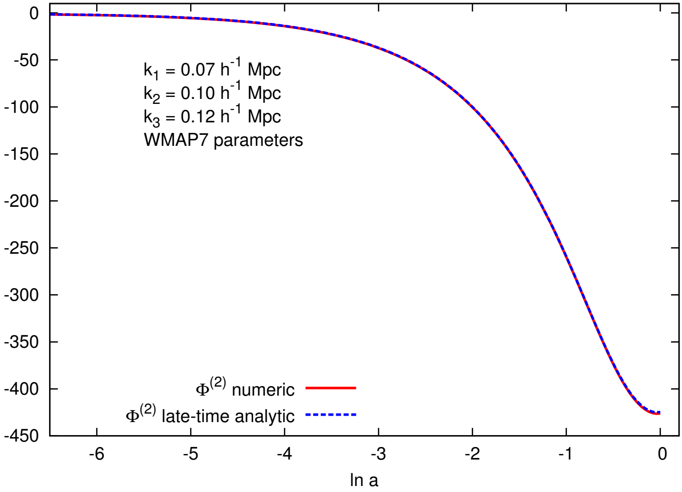


theoretical paper: Creminelli, Pitrou & Vernizzi, 2011

Squeezed limit: baryon velocity



Late-time exact solution



theoretical paper: Boubekeur, Creminelli, Norena, & Vernizzi, 2008

CMB bispectrum

The bispectrum: definition

$$\frac{\Delta T}{T}(\mathbf{n}) = \sum_{l,m} a_{lm} Y_{lm}(\mathbf{n}) \, .$$

$$\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle = B_{l_1 l_2 l_3} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \, .$$

$$B_{l_1 l_2 l_3} = b_{l_1 l_2 l_3} \sqrt{\frac{(2l_1+1)(2l_2+1)(2l_3+1)}{4\pi}} \begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix}$$

Primordial non-Gaussianity: a difficult integral

Primordial bispectrum ⇒ CMB angular bispectrum

$$b_{l_1 l_2 l_3} = \int dx dk_1 dk_2 dk_3 (x k_1 k_2 k_3)^2 B_{\rm prim}(k_1, k_2, k_3) \Delta_{l_1}(k_1) \Delta_{l_2}(k_2) \Delta_{l_3}(k_3) j_{l_1}(k_1 x) j_{l_2}(k_2 x) j_{l_3}(k_3 x)$$

Solution: Reduce the 4D integral into products of 1D integrals by factorizing the primordial bispectrum $B_{\rm prim}(k_1, k_2, k_3) = \sum_i X_i(k_1) Y_i(k_2) Z_i(k_3)$.

See e.g. Fergusson *et al.* 09.

Second-order perturbs: a more difficult integral

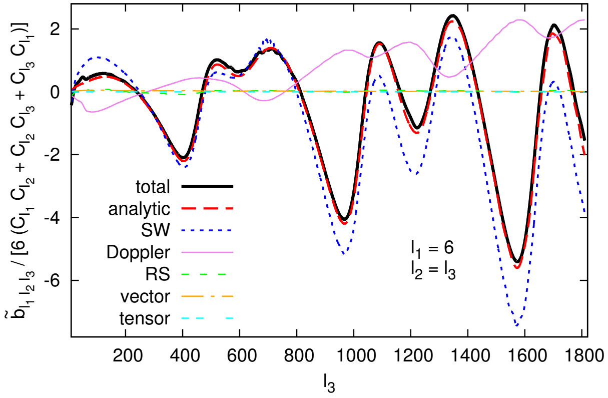
$$b_{l_1 l_2 l_3} \propto \sum_{m_3} \int dk_1 dk_2 d\mu d\eta S_{l_3' m_3}(k_1, k_2, \mu, \eta) j_{l_3}^{(l_3' m_3)}[k(\eta_0 - \eta)] \sum_{m_1 m_2} Y^*_{l_1 m_1} Y^*_{l_2 m_2} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \ldots + \text{perms.}$$

- The factorization method is not necessarily a winner (need to factorize at each time step).
- Evaluation of billions of spherical harmonics and a lot of 3-j symbols.
- Evaluation of $j_{l_3}^{(l_3' m_3)}$ with large l_3' is numerically expensive.
- Too many lensing source terms (need to compute source up to $l_3' \sim l_3$.)

However, if we only care about early-time effects...

- Source terms with large l_3' can be ignored (this is only true with a proper choice of variables, see our paper arXiv:1212.3573).
- The geometrical factors (Y_{lm} and 3-j symbols) can be precomputed and saved.
- Only needs $j_{l_3}^{(l_3' m_3)}$ with small l_3'.

Checking against the squeezed-limit theoretical formula



$$b_{l_S l_L l_L} = 2C_{l_L}^{TT} C_{l_S}^{TT} - C_{l_L}^{T\zeta} C_{l_S}^{TT} \frac{d \ln \left[\left(l_S + \frac{1}{2} \right)^2 C_{l_S}^{TT} \right]}{d \ln \left(l_S + \frac{1}{2} \right)}, \; l_L \ll l_S \text{ and } l_L \ll 60$$

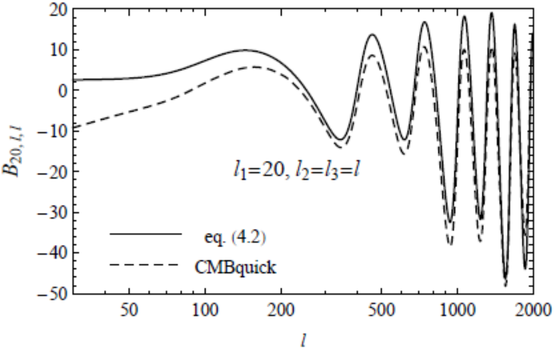
Physical meaning of the source terms are well understood.

In the line-of-sight-integral approach, late-time effects can be explicitly split out.
With late-time effects removed and in the squeezed-limit, the line-of-sight source can be explicitly written as product of first-order perturbations,

$$S^{(2)}(k_1, k_2, k_3) = -\zeta^{(1)}(k_1)\frac{\partial S^{(1)}(k_2)}{\partial \ln k_2}.$$

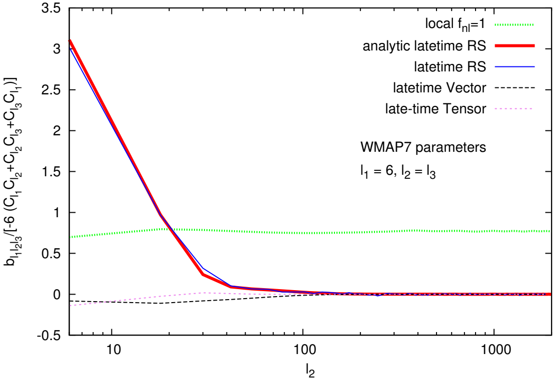
Using properties of $j_l^{(l'm)}$ we can explicitly reproduce the squeezed-limit formula of $b_{l_1 l_2 l_3}$ using the line-of-sight-integral method. (Huang & Vernizzi in preparation).

Comparison: CMBquick squeezed limit check

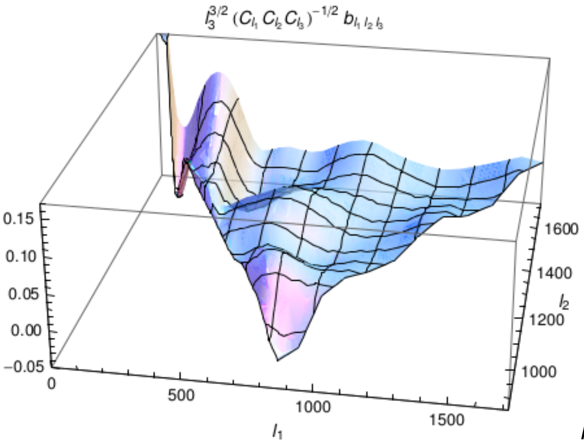


enhanced f_{NL} contamination due to the "offset"

Late-time Rees-Sciama contribution



More bispectrum...



$l_3 = 1720$ fixed.

The contamination to $f_{\rm NL}^{\rm local}$ measurement

For an ideal experiment, if we measure $f_{\rm NL}^{\rm local}$ without removing this effect, $f_{\rm NL}^{\rm local}$ will be biased by
0.82 (for $\ell_{\rm max} = 2000$) or 1.27 (for $\ell_{\rm max} = 2500$)

Though the contamination is small, it is important to remove it in order to get an unbiased result.

Can we see the early-time effects in the data?

For $\ell_{\max} = 2000$:
$S/N = 0.47$.
For $\ell_{\max} = 2500$:
$S/N = 0.71$.

Not likely to see it without including the polarization...

Conclusion

We did a very complicated calculation and we found nothing important...

Conclusions and Outlook

1. We found that the contamination of $f_{\mathrm{NL}}^{\mathrm{local}}$ due to non-linear recombination is ≈ 1, which can be removed using our template.
2. For future polarization experiments, the contamination can be important. (TO BE DONE)
3. Late-time effects need to be included, but likely need a new method for the bispectrum integration. (have some clue but still working on that...)