Velocity Dispersion Effects in the Linear Growth of Cosmic Structures

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Outline

Collaboration

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CIÊNCIA SEM FRONTEIRAS

Standard Cosmological Model

Comparison of *Planck*-only and *WMAP*-only Six-Parameter ΛCDM Fits[a]

Parameter	*Planck* ("CMB+Lens")	*WMAP* (9-year)	Difference value	Difference *WMAP* σ
$\Omega_b h^2$	0.02217 ± 0.00033	0.02264 ± 0.00050	-0.00047	0.9
$\Omega_c h^2$	0.1186 ± 0.0031	0.1138 ± 0.0045	0.0048	1.1
Ω_Λ	0.693 ± 0.019	0.721 ± 0.025	-0.028	1.1
τ	0.089 ± 0.032	0.089 ± 0.014	0	0
t_0 (Gyr)	13.796 ± 0.058	13.74 ± 0.11	56 Myr	0.5
H_0 (km s^{-1}Mpc^{-1})	67.9 ± 1.5	70.0 ± 2.2	-2.1	1.0
σ_8	0.823 ± 0.018	0.821 ± 0.023	0.002	0.1
Ω_b	0.0481[b]	0.0463 ± 0.0024	0.0018	0.7
Ω_c	0.257[b]	0.233 ± 0.023	0.024	1.0

[a]The new *Planck* results strongly favor the standard six-parameter ΛCDM model with parameter values that are consistent with *WMAP* parameters, as shown in this table which compares results derived entirely from *Planck* data with those derived entirely from *WMAP* data.

[b]Parameters derived from quoted values. No error estimate is given for this data/model combination.

http://lambda.gsfc.nasa.gov/

Issues of ΛCDM

Fundamental:

- The nature of Dark Matter;
- The Cosmological Constant problem (*Weinberg, 1989*);
- The Cosmic Coincidence Conundrum;
- The nature of Dark Energy;
- ...

From Cosmological Simulations:

- Core/Cusp problem (*de Blok, 2009*);
- Missing Satellites problem (*Bullock, 2010*);
- ...

Necessity of Dark Matter

- Primordial Nucleosynthesis;
- DM drives baryons in forming galaxies;
- Velocity curves of galaxies;
- Gravitational lensing;
- CMB peak structure;

Modelling Dark Matter

Phenomenologically:

- Perfect fluid $T^{\mu\nu} = \rho u^\mu u^\nu$, vanishing pressure;
- Fluctuations: $\delta p = 0$;
- If $p = w\rho$, observation requires $|w| \lesssim 10^{-3}$ (*Müller, 2005*).

Or:

- Particles;
- Modification of the gravitational theory;

Dark Matter Particles Candidates

Bertone, Hooper and Silk, 2005

- Sterile Neutrino ($m \gtrsim 10$ keV);
- Axion ($m \lesssim 0.01$ eV, $s = 0$);
- Neutralino (MSSM) ($m \gtrsim 100$ GeV);

Operating Experiments:

- LHC;
- DAMA/LIBRA;
- CoGent;
- CRESS-II;
- XENON100;

Our Purpose

Assess how velocity dispersion affects DM particles evolution.
References:

- *Hofmann, Schwarz, Stocker, 2001*
- *Green, Hofmann, Schwarz, 2005*
- *Peirani, Durier, De Freitas Pacheco, 2006*
- *Boyanovsky, Vega, Sanchez, 2008*;
- *Vass, Valluri, Kravtsov, Kazantzidis, 2009*
- *Vega, Sanchez, 2009*;
- *Vega, Sanchez, 2010*;
- *Vega, Salucci, Sanchez, 2010*;
- *Vega, Sanchez, 2013*;

DM Particles Primordial Evolution

Bringmann and Hofmann, 2007

Important energy scales and events:

- $T_{\rm nr} \approx m$: DM particles become non-relativistic;
- $\Gamma_{\rm X+\bar{X}\leftrightarrow f+\bar{f}} \approx \Gamma_{\rm H}$: chemical decoupling ($T = T_{\rm cd}$);
- $\Gamma_{X+f\leftrightarrow X+f} \approx \Gamma_{\rm H}$: kinetic decoupling ($T = T_{\rm kd}$).

For WIMPS $T_{\rm kd} < T_{\rm cd} < T_{\rm nr}$. In general, the precise hierarchy depends on the DM particles model under investigation.

For the neutralino, $T_{\rm cd} \approx 4$ GeV ($z \approx 10^{13}$) and $T_{\rm kd} \approx 25$ MeV ($z \approx 10^{11}$).
We consider the evolution after kinetic decoupling.

Vlasov-Einstein Equation

Collisionless Boltzmann equation coupled to GR. Given $f\left(t, x^i, P^i = m\frac{dx^i}{d\tau}\right)$:

$$\frac{df}{d\tau} = 0 \, , \Rightarrow \frac{df}{dt} = 0 \, ,$$

i.e.

$$\frac{\partial f}{\partial t} + \frac{dx^i}{dt}\frac{\partial f}{\partial x^i} + \frac{dP^i}{dt}\frac{\partial f}{\partial P^i} = 0 \, .$$

Since $g_{\mu\nu}P^\mu P^\nu = -m^2$, P^0 is not an independent variable. Metric enters via geodesic equation

$$\frac{dP^i}{d\tau} + \Gamma^i_{\mu\nu}P^\mu P^\nu = 0 \, .$$

Vlasov-Einstein System in Cosmology

Bernstein, 1988

Flat FLRW metric:

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a(t)^2\delta_{ij}dx^i dx^j \ ,$$

Proper momentum:

$$p^2 = a(t)^2\delta_{ij}P^i P^j \ ,$$

so

$$P^i = \frac{p}{a}\hat{n}^i \ ,$$

where $\delta_{ij}\hat{n}^i\hat{n}^j = 1$. VE becomes

$$\frac{\partial f}{\partial t} - H\frac{dp}{dt}\frac{\partial f}{\partial p} = 0 \ .$$

Because of isotropy, no dependence on x^i and $\hat{n}^i$.

Solution and Important Quantities

Solution:

$$f = f(ap) \; .$$

Particles number density:

$$n = \int d^3p f(ap) = \frac{4\pi}{a^3} \int_0^\infty dx \; x^2 f(x) \equiv \frac{4\pi}{a^3} I_2 \; ,$$

always scales as a^{-3}. Introducing the proper velocity

$$v^i \equiv a \frac{dx^i}{dt} = a \frac{P^i}{P^0} = \frac{p\hat{n}^i}{E} \; .$$

Velocity dispersion:

$$\sigma^2 = \frac{1}{3n} \int d^3p f(ap) \frac{p^2}{E^2} \; .$$

Velocity Dispersion and Constancy of Q

Since $E^2 = p^2 + m^2$,

$$\sigma^2 = \frac{1}{3n} \int d^3p f(ap) \frac{p^2}{p^2 + m^2} \ .$$

Neglecting $O\left(\frac{p^4}{m^4}\right)$ terms (non-relativistic particles):

$$\sigma^2 \approx \frac{4\pi}{3nm^2a^5} \int_0^\infty dx f(x) x^4 \equiv \frac{4\pi}{3nm^2a^5} I_4 \ .$$

Phase-space density Q:

$$Q \equiv \frac{nm}{\sigma^3} \approx 4\pi\sqrt{27} m^4 I_2^{5/2} I_4^{-3/2} \ .$$

It is constant for non-relativistic particles.

Momenta of Vlasov Equation

Neglecting again $O\left(\frac{p^4}{m^4}\right)$ terms ($p^4/m^4 \sim 10^{-6}$ for the neutralino).

Zero momentum:

$$\int d^3p \left(\frac{\partial f}{\partial t} - H \frac{dp}{dt} \frac{\partial f}{\partial p} \right) = 0 \,, \quad \Rightarrow \quad \frac{\partial n}{\partial t} + 3Hn = 0 \,.$$

Second momentum:

$$\frac{\partial}{\partial t} \int d^3p f \frac{p^2}{E^2} \hat{n}^i \hat{n}^j - H \int d^3p p \frac{\partial f}{\partial p} \frac{p^2}{E^2} \hat{n}^i \hat{n}^j = 0 \,,$$

i.e.

$$\frac{\partial \sigma^2}{\partial t} + 2H\sigma^2 = 0 \,,$$

which gives the known result $\sigma^2 \propto a^{-2}$.

Modification of the Energy Density Evolution

Defining the energy density as:

$$\varepsilon \equiv \int d^3 p E f \ ,$$

multiplying Vlasov equation by $d^3 pE$ and integrating over the momenta gives

$$\frac{\partial \varepsilon}{\partial t} + 3H \left(\varepsilon + \frac{1}{3} m n \sigma^2 \right) = 0 \ ,$$

which has solution

$$\varepsilon = m n_{\mathrm{kd}} \left(\frac{a_{\mathrm{kd}}}{a} \right)^3 + \frac{m n_{\mathrm{kd}} \sigma_{\mathrm{kd}}^2}{2} \left(\frac{a_{\mathrm{kd}}}{a} \right)^5 \ .$$

For the neutralino $\sigma_{\mathrm{kd}}^2 \approx 10^{-3}$.

Perturbations

In the metric:

$$ds^2 = -(1+2\Psi)dt^2 + a(t)^2\delta_{ij}(1+2\Phi)dx^i dx^j \ .$$

In the distribution function:

$$f(t, x^i, P^i) = f^{(0)}(t, p) + f^{(1)}(t, x^i, P^i) \ .$$

Perturbed Vlasov-Einstein equation:

$$\frac{\partial f^{(1)}}{\partial t} + \frac{p}{aE}\hat{n}^i \frac{\partial f^{(1)}}{\partial x^i} - Hp\frac{\partial f^{(1)}}{\partial p} - \left(p\frac{\partial \Phi}{\partial t} + \frac{E\hat{n}^i}{a}\frac{\partial \Psi}{\partial x^i}\right)\frac{\partial f^{(0)}}{\partial p} = 0 \ .$$

Perturbed Quantities

Particle number density:

$$\int d^3pf = \int d^3pf^{(0)} + \int d^3pf^{(1)} \to n = n^{(0)} + n^{(1)} \ .$$

Velocity now gains a correction:

$$v^i = a\frac{dx^i}{dt} = a\frac{P^i}{P^0} = (1 - \Phi + \Psi)\frac{p}{E}\hat{n}^i \ .$$

Bulk velocity:

$$V^i = \frac{1}{n^{(0)}}\int d^3p\left(\frac{p}{E}\hat{n}^i\right)f^{(1)} \ .$$

It is a pure first-order quantity.

Momenta of the Perturbed VE Equation

Zero momentum:

$$\frac{\partial n^{(1)}}{\partial t} + \frac{1}{a}\frac{\partial (n^{(0)} V^i)}{\partial x^i} + 3Hn^{(1)} + 3n^{(0)}\frac{\partial \Phi}{\partial t} = 0 \; .$$

With $\delta \equiv n^{(1)}/n^{(0)}$ ($\approx \varepsilon^{(1)}/\varepsilon^{(0)}$):

$$\dot{\delta} + \frac{1}{a}\partial_i V^i + 3\dot{\Phi} = 0 \; .$$

First momentum:

$$\ddot{\delta} + 2H\dot{\delta} + 6H\dot{\Phi} + 3\ddot{\Phi} - \frac{1}{a^2}\nabla^2\Psi + \frac{1}{a^2}\partial_i\partial_j\omega^{ij} = 0 \; ,$$

where

$$\omega^{ij} \equiv \frac{1}{n^{(0)}}\int d^3p \frac{p^2}{E^2}\hat{n}^i\hat{n}^j f^{(1)} \; .$$

Coupling to Einstein Equations

Assuming:

1. DM domination;
2. negligible DM anisotropic stresses, i.e. $\Phi = -\Psi$;

$$3H^2\Psi + 3H\dot{\Psi} + \frac{k^2}{a^2}\Psi = -4\pi G\rho_{\rm dm}\delta \ ,$$

$$\ddot{\Psi} + 4H\dot{\Psi} + \left(3H^2 + 2\dot{H}\right)\Psi = -4\pi G\delta p_{\rm dm} \ .$$

Assuming again

1. Negligible effective pressure;
2. Negligible effective $\delta p_{\rm dm}$;

$$\ddot{\Psi} + 4H\dot{\Psi} \simeq 0 \ .$$

Jeans Length and Jeans Mass

More assumptions:

1. Shear-free velocity field, $\omega^{ij} = v_1^2 \delta^{ij}$ and $v_1^2 = \sigma_{(0)}^2 \delta + \sigma_{(1)}^2$;
2. Q constant $\Rightarrow \sigma_{(1)}^2 = (2/3)\sigma_{(0)}^2 \delta \Rightarrow v_1^2 = (5/3)\sigma_{(0)}^2 \delta$.

For $k >> Ha$,

$$\ddot{\delta} + 2H\dot{\delta} - \left(4\pi G \rho_{\rm dm} - \frac{5}{3}\frac{k^2}{a^2}\sigma_{(0)}^2\right)\delta = 0 \; .$$

Critical Jeans length (physical)

$$\lambda_{\rm J}^2 = \frac{5\pi}{G}\rho_{\rm dm}^{-1/3} Q^{-2/3} \; ,$$

For $m = 100$ GeV, $\lambda_{\rm J} \approx 1$ pc at matter-radiation equality.

Jeans mass:

$$M_{\rm J} = \frac{\pi^3}{2}\left(\frac{H_0^2 \Omega_{\rm dm}}{k_{\rm J}^3 G}\right) \; .$$

For $m = 100$ GeV, $M_{\rm J} \approx 10^{-6}\ M_\odot$ at matter-radiation equality.

Physical Interpretation of the Jeans Length

Suppose a spherical homogeneous perturbation of radius R and density ρ:

$$W = -\frac{3GM^2}{5R} \ .$$

Upon a contraction:

$$\Delta W \propto -GMR^2\Delta\rho = -GMR^2\delta \ .$$

Variation of the potential energy goes into bulk and internal motions:

$$\Delta\sigma^2 \lesssim G\rho R^2\delta \ .$$

If Q remains constant (isentropic process) then $\Delta\sigma^2 \propto \rho^{2/3}Q^{-2/3}\delta$, i.e.

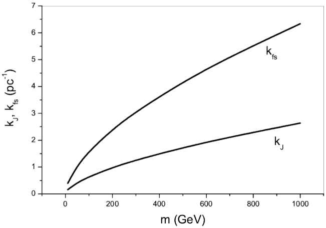
$$R^2 \gtrsim \rho^{-1/3}Q^{-2/3}/G \ .$$

Comparison with Free-Streaming Length

Green, Hofmann and Schwarz, 2005
Comoving wavenumbers:

$$k_{\rm fs} \propto \frac{1}{l_{\rm fs}} \,, \qquad l_{\rm fs} \approx \bar{v}_{\rm kd} a_{\rm kd} \int_{\eta_{\rm kd}}^{\eta} \frac{d\eta'}{a(\eta')} \,, \qquad \bar{v}_{\rm kd} = \sqrt{3gT_{\rm kd}/m} \,.$$

$$k_{\rm J}^2 = \frac{12\pi G}{5} \rho_{\rm dm}^{1/3} a^2 Q^{2/3} = \frac{12\pi G}{5} n_{\rm dm0}^{1/3} m^{1/3} a Q^{2/3} \,.$$



Values at matter-radiation equality.

Including Radiation and Baryons

Assumptions:

1. We neglect all multipoles $l \geq 2$ (this also allows us to set $\Phi = -\Psi$);
2. We assume baryons tight coupled to radiation;

For radiation:

$$\dot{\Theta}_{r,0} + \frac{k}{a}\Theta_{r,1} = -\dot{\Phi} \ ,$$

$$\dot{\Theta}_{r,1} + H\frac{R}{1+R}\Theta_{r,1} - \frac{k}{3a(1+R)}\Theta_{r,0} = -\frac{k}{3a}\Phi \ ,$$

where $\Theta \equiv \delta T/T$ ($\delta_r = 4\Theta$), and $\Theta_{r,0}$ and $\Theta_{r,1}$ monopole and dipole respectively and

$$R \equiv \frac{3\rho_{\rm b}}{4\rho_{\rm r}} = \frac{3\Omega_{\rm b0}}{4\Omega_{\rm r0}}a \ ,$$

is the baryon-to-photon ratio.

System to solve up to Recombination

$$\delta' + \frac{ik}{Ha^2} V = -3\Phi' \ ,$$

$$V' + \frac{1}{a} V = \frac{ik}{Ha^2}\Phi - \frac{5}{3}\frac{ik}{Ha^2}\frac{\sigma_{\rm kd}^{(0)2} a_{\rm kd}^2}{a^2}\delta \ ,$$

$$\delta_{\rm b}' + \frac{3k}{Ha^2}\Theta_{r,1} = -3\Phi' \ ,$$

$$\Theta_{r,0}' + \frac{k}{Ha^2}\Theta_{r,1} = -\Phi' \ ,$$

$$\Theta_{r,1}' + \frac{R}{a(1+R)}\Theta_{r,1} - \frac{k}{3Ha^2(1+R)}\Theta_{r,0} = -\frac{k}{3Ha^2}\Phi \ ,$$

$$\frac{k^2}{H^2a^2}\Phi + 3a\left(\Phi' + \frac{1}{a}\Phi\right) = \frac{3H_0^2}{2H^2}\left(\frac{\Omega_{m0}}{a^3}\delta + \frac{\Omega_{b0}}{a^3}\delta_{\rm b} + 4\frac{\Omega_{r0}}{a^4}\Theta_{r,0}\right) \ .$$

Initial Scale Factor and Mass Dependence

We use the following formula for the kinetic decoupling temperature (*Green, Hofmann and Schwarz, 2005*):

$$T_{\rm kd} \approx 25.5 \left(\frac{m}{100\ {\rm GeV}}\right)^{0.23} {\rm MeV}\ ,$$

therefore

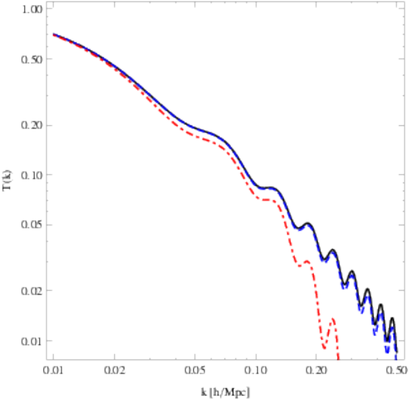
$$a_{\rm kd} = \frac{T_0}{T_{\rm kd}} = 2.64 \times 10^{-11} \left(\frac{\rm GeV}{m}\right)^{0.23} \quad .$$

For the initial velocity dispersion, since the particles decouple non relativistically:

$$\sigma_{\rm kd}^{(0)2} \approx \frac{3T_{\rm kd}}{m} \approx 2.2 \times 10^{-1} \left(\frac{\rm GeV}{m}\right)^{1.23} \quad .$$

Transfer Function

From bottom to top, $m = 0.1, 1$ keV and the zero velocity dispersion case.



Qualitative constraint: $m \gtrsim 1$ keV,
in agreement with *de Vega and Sanchez, 2013*.

Summary and Conclusions

1. DM as a system of collisionless particles;
2. $Q = nm/\sigma^3$ remains constant during the expansion of the universe for non-relativistic particles prior to structure formation;
3. Corrections to the energy density of DM particles coming from their velocity dispersions: kinetic term scaling as a^{-5} which acts as an effective pressure;
4. Physical Jeans length $\lambda_{\rm J} = (5\pi/G)^{1/2} Q^{-1/3} \rho_{\rm dm}^{-1/6}$;
5. Jeans mass scale $M_{\rm J} = \frac{\pi^3}{2}\left(\frac{H_0^2\Omega_{\rm dm}}{k_{\rm J}^3 G}\right)$.
6. Including radiation and baryons, $m \gtrsim 1$ keV.

Problems, Perspectives and Improvements

1. For $m = 1$ keV, DM particles decouple while relativistic;
2. If the particles are relativistic, Q does not conserve;
3. We should accordingly correct the equations;
4. Including spatial curvature;
5. Differences between baryons and DM transfer functions;
6. Application to the non-linear regime of evolution;

Obrigado!