The Effective Field Theory of Dark Energy

Filippo Vernizzi
IPhT, CEA/Saclay

with Giulia Gubitosi, Federico Piazza, 1210.0201, JCAP

IAP, 11 February 2013

Outline

- Motivations
- The Action
- Examples
- Mixing with gravity, stability and speed of sound
- The zero sound speed limit
- Conclusions

Outline

- Motivations
- The Action
- Examples
- Mixing with gravity, stability and speed of sound
- The zero sound speed limit
- Conclusions

Motivations

- Future surveys (EUCLID, LSST, BigBoss, etc.) will be sensitive to dynamical properties of dark energy and modified gravity (DE), observable in the power spectra and higher-order correlation functions

- Many models of DE, each one with its own motivations, physical effects, etc...

- Democratic view: look for a unifying (many models) and effective (agnostic to motivations) treatment of DE to test models against the data

Ideally...

- Description in terms of limited number of effective operators, each one responsible for an observable dynamical feature (e.g. flavor-changing neutral currents in physics beyond Standard Model)

Another acceleration

- Common feature of many DE models: gravity + single scalar degree of freedom (in some regime)

- Similar to inflation, where scalar field is needed to break de Sitter: clock

- Models of Inflation/DE share the same motivations and problems

- Two types of Effective Field Theory approaches to inflation: “covariant” (*à la* Weinberg) and “geometrical” (Creminelli et al. ’06, Cheug et al. ’07, deals directly with cosmological perturbations)

EFT: the covariant approach

Many inflation/DE models reduce, in their relevant regimes, to scalar tensor-theories

$$S=\int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(\phi)R-\frac{1}{2}(\partial\phi)^2-V(\phi)+\mathcal{F}[\phi,g^{\mu\nu}]\right]$$

One possible strategy: (Weinberg `08, Park, Zurek and Watson `10, Bloomfield and Flanagan `11)

Apply covariant EFT to explore $\mathcal{F}[\phi,g^{\mu\nu}]$: field/derivative expansion

EFT: the covariant approach

Many inflation/DE models reduce, in their relevant regimes, to scalar tensor-theories

$$S = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(\phi)R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{F}[\phi, g^{\mu\nu}]\right]$$

One possible strategy: (Weinberg `08, Park, Zurek and Watson `10, Bloomfield and Flanagan `11)

Apply covariant EFT to explore $\mathcal{F}[\phi, g^{\mu\nu}]$: field/derivative expansion

However:

1) Expansion in number of fields is not necessarily meaningful

EFT: the covariant approach

Many inflation/DE models reduce, in their relevant regimes, to scalar tensor-theories

$$S = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(\phi)R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{F}[\phi, g^{\mu\nu}]\right]$$

One po

Apply c

$$\begin{aligned} V &= V_1\phi + V_2\phi^2 + V_3\phi^3 + V_4\phi^4 \\ &= V_2\delta\phi^2 + V_3\phi_0(t)\delta\phi^2 + 6V_4\phi_0^2(t)\delta\phi^2 \end{aligned}$$

All terms potentially important in cosmological perturbation theory!

Howeve

1) Expansion in number of fields is not necessarily meaningful

EFT: the covariant approach

Many inflation/DE models reduce, in their relevant regimes, to scalar tensor-theories

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} f(\phi) R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{F}[\phi, g^{\mu\nu}] \right]$$

One possible strategy: (Weinberg `08, Park, Zurek and Watson `10, Bloomfield and Flanagan `11)

Apply covariant EFT to explore $\mathcal{F}[\phi, g^{\mu\nu}]$: field/derivative expansion

However:

1) Expansion in number of fields is not necessarily meaningful
2) Naively "perturbations" but not always so...

EFT: the covariant approach

Many inflation/DE models reduce, in their relevant regimes, to scalar tensor-theories

$$S=\int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(\phi)R-\frac{1}{2}(\partial\phi)^2-V(\phi)+\mathcal{F}[\phi,g^{\mu\nu}]\right]$$

One possible strategy: (Weinberg `08, Park, Zurek and Watson `10, Bloomfield and Flanagan `11)

Apply covariant EF ... ative expansion

$$\partial\phi^4,\ \Box\phi\partial\phi^2,\ \text{etc.}$$

However:

1) Expansion in number of fields is not necessarily meaningful
2) Naively "perturbations" but not always so...

EFT: the covariant approach

Many inflation/DE models reduce, in their relevant regimes, to scalar tensor-theories

$$S=\int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(\phi)R-\frac{1}{2}(\partial\phi)^2-V(\phi)+\mathcal{F}[\phi,g^{\mu\nu}]\right]$$

One possible strategy: (Weinberg \`08, Park, Zurek and Watson \`10, Bloomfield and Flanagan \`11)

Apply covariant EF ... ative expansion

$$\partial\phi^4,\ \Box\phi\partial\phi^2,\ \text{etc.}$$

Ghost Condensate

However:

1) Expansion in number of fields is not necessarily meaningful
2) Naively "perturbations" but not always so...

EFT: the covariant approach

Many inflation/DE models reduce, in their relevant regimes, to scalar tensor-theories

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} f(\phi) R - \frac{1}{2} (\partial\phi)^2 - V(\phi) + \mathcal{F}[\phi, g^{\mu\nu}] \right]$$

One possible strategy: (Weinberg `08, Park, Zurek and Watson `10, Bloomfield and Flanagan `11)

Apply covariant EF ... ative expansion

$$\partial\phi^4, \quad \Box\phi\partial\phi^2, \quad \text{etc.}$$

Ghost Condensate Galileons

However:

1) Expansion in number of fields is not necessarily meaningful
2) Naively "perturbations" but not always so...

EFT: the covariant approach

Many inflation/DE models reduce, in their relevant regimes, to scalar tensor-theories

$$S = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(\phi)R - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{F}[\phi, g^{\mu\nu}]\right]$$

One possible strategy: (Weinberg `08, Park, Zurek and Watson `10, Bloomfield and Flanagan `11)

Apply covariant EFT to explore $\mathcal{F}[\phi, g^{\mu\nu}]$: field/derivative expansion

However:

1) Expansion in number of fields is not necessarily meaningful
2) Naively "perturbations" but not always so...
3) Only halfway through the work to be done (background first + expand..)

EFT: a theory for the relevant low-energy d.o.f.

Examples:

1) QCD: quarks and gluons → nucleons and pions at low energies

EFT: a theory for the relevant low-energy d.o.f.

Examples:

1) QCD: quarks and gluons → nucleons and pions at low energies

2) EW theory: 4 massless vector bosons, 2 complex scalars etc.

UNITARY GAUGE ↓

1 massless and 3 massive vector bosons, 1 massive ``Higgs'' field etc.

EFT: a theory for the relevant low-energy d.o.f.

Examples:

1) QCD: quarks and gluons ⟶ nucleons and pions at low energies

2) EW theory: 4 massless vector bosons, 2 complex scalars etc.

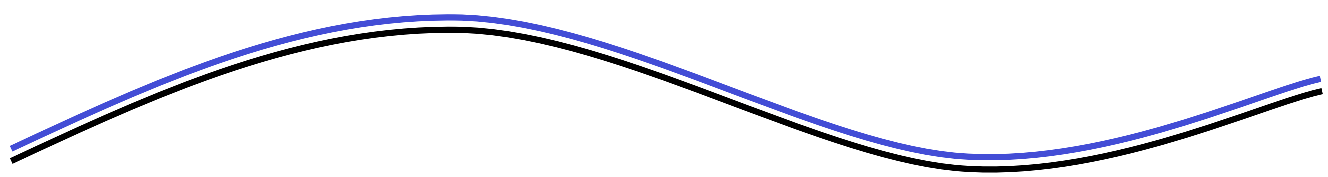
UNITARY GAUGE ⟶

1 massless and 3 massive vector bosons, 1 massive ``Higgs'' field etc.

3) Cosmology: ...Cosmological Perturbations!

The Effective Field Theory of Inflation

Unitary gauge action: (Creminelli et al. `06, Cheung et al. `07)



- Main idea: scalar degree of freedom is "eaten" by the metric. Ex:

$$\phi(t,\vec{x}) \to \phi_0(t) \quad (\delta\phi = 0) \qquad\qquad -\frac{1}{2}(\partial\phi)^2 \to -\frac{1}{2}\dot{\phi}_0^2(t)\; g^{00}$$

- Action contains all operators invariant under spatial diffeomorphisms

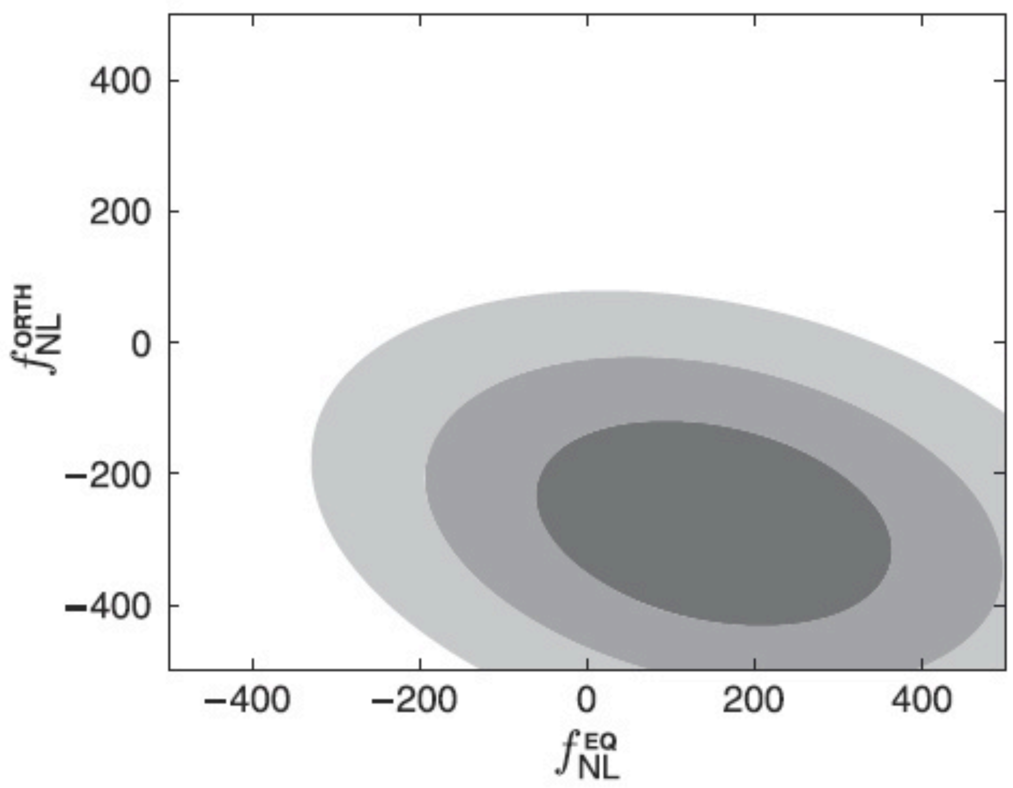
$$\int d^4x\sqrt{-g}\left[\frac{M_{\rm Pl}^2 R}{2} + \dot{H}(t) M_{\rm Pl}^2 g^{00} - 3H^2(t) - \dot{H}(t) + \frac{M_2^4(t)}{2}(\delta g^{00})^2 - \frac{\bar{M}_1^3(t)}{2}\delta g^{00}\delta K + \dots\right]$$

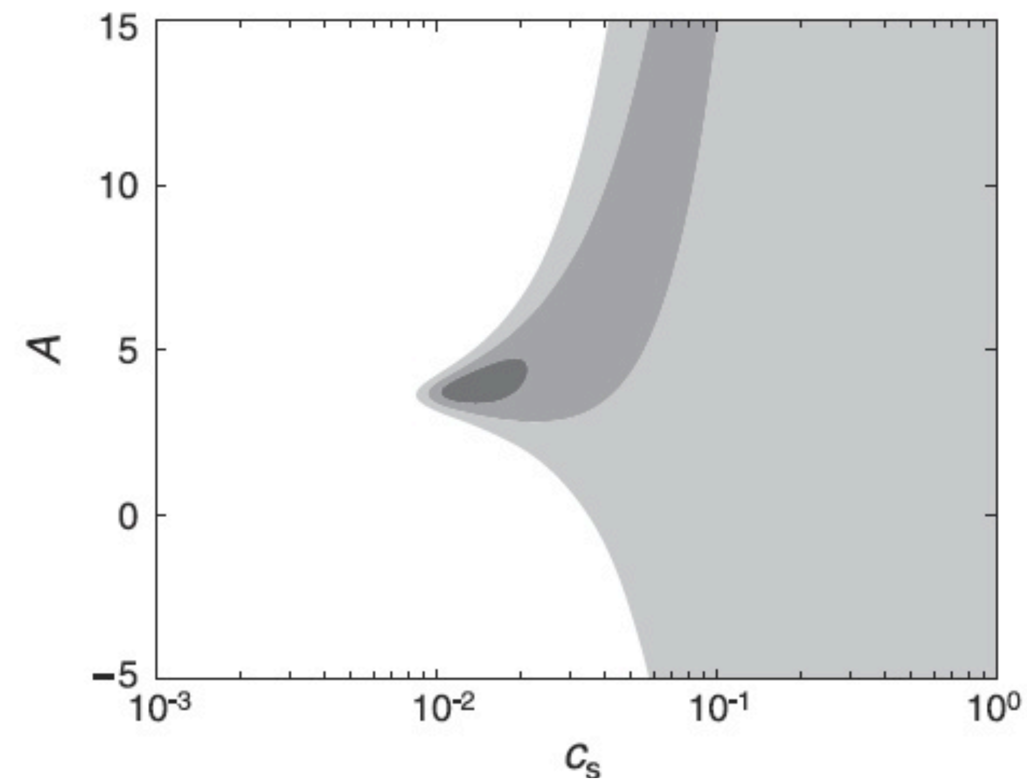
- Dictionary between operators and observables, i.e. shape and amplitude of non-Gaussianity constrained by WMAP and Planck

EFT of Inflation and non-Gaussianity

(Bennett et al, 2012 - Final WMAP paper)

$$S = \int d^4x \sqrt{-g} \left[-\frac{M_{\rm Pl}^2 \dot{H}}{c_s^2} \left(\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2} \right) + (M_{\rm Pl}^2 \dot{H}) \frac{1 - c_s^2}{c_s^2} \left(\frac{\dot{\pi}(\partial_i \pi)^2}{a^2} + \frac{A}{c_s^2} \dot{\pi}^3 \right) + \cdots \right]$$





$$f_{NL}^{\rm eq} = \frac{1 - c_s^2}{c_s^2} (-0.276 + 0.0785A)$$

$$f_{NL}^{\rm orth} = \frac{1 - c_s^2}{c_s^2} (0.0157 - 0.0163A)$$

... and DE?

- Obvious difference: energy scales and presence of different species (baryons, CDM, photons, neutrinos, etc) and thus different couplings, in the DE case

- $\Rightarrow$ Minimally coupled DE: Effective Field Theory of Quintessence: stability and zero sound speed limit (with Creminelli et al. 2008)

... and DE?

- Obvious difference: energy scales and presence of different species (baryons, CDM, photons, neutrinos, etc) and thus different couplings, in the DE case

- ⇒ Minimally coupled DE: Effective Field Theory of Quintessence: stability and zero sound speed limit (with Creminelli et al. 2008)

Our Recipe for Dark Energy: (with Gubitosi, Piazza, 2012)

1) Assume WEP (universally coupled metric $S_m[g_{\mu\nu}, \Psi_i]$): Jordan frame clock.

2) Write the most generic action for $g_{\mu\nu}$ compatible with the residual un-broken symmetries (3-diff).

Outline

- Motivations
- The Action
- Examples
- Mixing with gravity, stability and speed of sound
- The zero sound speed limit
- Conclusions

The (Jordan frame) Action

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} \right] + S_{DE}^{(2)}$$

The most generic action written in terms of $g_{\mu\nu}$ compatible with the residual symmetry of spatial diffeomorphisms

The (Jordan frame) Action

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} \right] + S_{DE}^{(2)}$$

The most generic action written in terms of $g_{\mu\nu}$ compatible with the residual symmetry of spatial diffeomorphisms

Genuine 4-dim covariant terms are still allowed, but will in general be multiplied by functions of time because time translations are broken

The (Jordan frame) Action

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} \right] + S_{DE}^{(2)}$$

The most generic action written in terms of $g_{\mu\nu}$ compatible with the residual symmetry of spatial diffeomorphisms

Genuine 4-dim covariant terms are still allowed, but will in general be multiplied by functions of time because time translations are broken

The function $f(t)$ cannot be set to unity by a metric redefinition ≠ EFT of Inflation

$$g_{\mu\nu} \to f^{1/2} g_{\mu\nu}$$

The (Jordan frame) Action

$$S = \int d^4x \sqrt{-g}\left[\frac{M^2}{2} f(t) R - \Lambda(t) - c(t) g^{00}\right] + S_{DE}^{(2)}$$

The most generic action written in terms of $g_{\mu\nu}$ compatible with the residual symmetry of spatial diffeomorphisms

General functions of time are allowed

The (Jordan frame) Action

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} \right] + S_{DE}^{(2)}$$

The most generic action written in terms of $g_{\mu\nu}$ compatible with the residual symmetry of spatial diffeomorphisms

...as well as tensors with “0” indices

Essentially: contractions with $$n_\mu = -\frac{\partial_\mu \phi}{\sqrt{-(\partial\phi^2)}}$$

The Action: main message

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} \right] + S_{DE}^{(2)}$$

Any arbitrarily complicate action with one scalar d.o.f. will reduce to this in Unitary gauge, plus terms that start explicitly quadratic in the perturbations

The Action: main message

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} \right] + S^{(2)}_{DE}$$

Any arbitrarily complicate action with one scalar d.o.f. will reduce to this in Unitary gauge, plus terms that start explicitly quadratic in the perturbations

Example:

$$(\partial\phi)^2 R \;=\; \dot{\phi}_0^2(-1 + \delta g^{00})(R^{(0)} + \delta R) \;=\; \dot{\phi}_0^2 \left[-R + R^{(0)}(t) + R^{(0)}(t) g^{00} + \delta g^{00} \delta R \right]$$

The Action: background

$$S = \int d^4x \sqrt{-g}\left[\frac{M^2}{2} f(t) R - \Lambda(t) - c(t) g^{00}\right] + S_{DE}^{(2)}$$

Enough for background equations:

$$M^2(f G_{\mu\nu} - \nabla_\mu \nabla_\nu f + g_{\mu\nu} \Box f) + (c g^{00} + \Lambda) g_{\mu\nu} - 2c \delta^0_\mu \delta^0_\nu = T^{(\mathrm{m})}_{\mu\nu}$$

$$\begin{aligned} c &= \frac{1}{2}(-\ddot{f} + H\dot{f})M^2 + \frac{1}{2}(\rho_D + p_D) \\ \Lambda &= \frac{1}{2}(\ddot{f} + 5H\dot{f})M^2 + \frac{1}{2}(\rho_D - p_D) \end{aligned}$$

The Action: background

$$S = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(t)R - \Lambda(t) - c(t)g^{00}\right] + S_{DE}^{(2)}$$

Enough for background equations:

$$M^2(fG_{\mu\nu} - \nabla_\mu\nabla_\nu f + g_{\mu\nu}\Box f) + (cg^{00} + \Lambda)g_{\mu\nu} - 2c\delta_\mu^0\delta_\nu^0 = T_{\mu\nu}^{(\mathrm{m})}$$

$$c = \frac{1}{2}(-\ddot{f} + H\dot{f})M^2 + \frac{1}{2}(\rho_D + p_D)$$

$$\Lambda = \frac{1}{2}(\ddot{f} + 5H\dot{f})M^2 + \frac{1}{2}(\rho_D - p_D)$$

$$H^2 = \frac{1}{3fM^2}(\rho_m + \rho_D)$$

$$\dot{H} = -\frac{1}{2fM^2}(\rho_m + \rho_D + p_m + p_D)$$

Generally Related to post-newtonian parameters

“Bare” Planck Mass

Defined by the modified Friedman equations

The Action: perturbations

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} \right] + S_{DE}^{(2)}$$

Explicitly quadratic in the perturbations:

$$S_{DE}^{(2)} = \frac{M_2^4}{2} (\delta g^{00})^2 - \frac{\bar{m}_1^3}{2} \delta g^{00} \delta K - \frac{\bar{M}_2^2}{2} \delta K^2 - \frac{\bar{M}_3^2}{2} \delta K_\mu{}^\nu \delta K^\mu{}_\nu + \ldots$$

The Action: perturbations

$$S = \int d^4x\sqrt{-g}\left[\frac{M^2}{2}f(t)R - \Lambda(t) - c(t)g^{00}\right] + S^{(2)}_{DE}$$

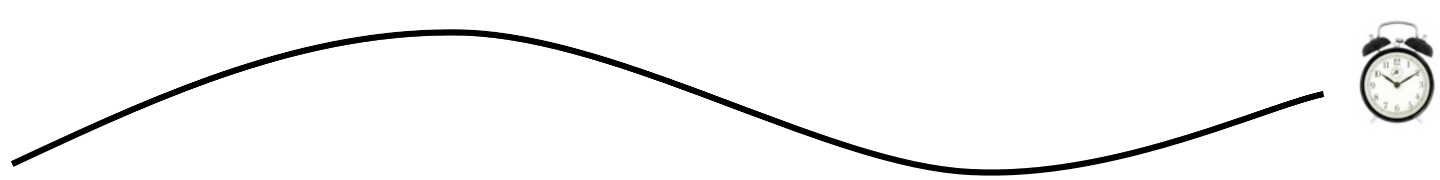
Explicitly quadratic in the perturbations:

$$S^{(2)}_{DE} = \frac{M_2^4}{2}(\delta g^{00})^2 - \frac{\bar{m}_1^3}{2}\,\delta g^{00}\delta K - \frac{\bar{M}_2^2}{2}\,\delta K^2 - \frac{\bar{M}_3^2}{2}\,\delta K_\mu{}^\nu \delta K^\mu{}_\nu + \ldots$$

Extrinsic curvature:

$$n_\mu = -\frac{\partial_\mu \phi}{\sqrt{-(\partial\phi^2)}} \qquad h_{\mu\nu} \equiv g_{\mu\nu} + n_\mu n_\nu$$

$$K_{\mu\nu} = h_\mu{}^\sigma \nabla_\sigma n_\nu \qquad \delta K_{\mu\nu} = K_{\mu\nu} - H h_{\mu\nu}$$



The Action: perturbations

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} \right] + S_{DE}^{(2)}$$

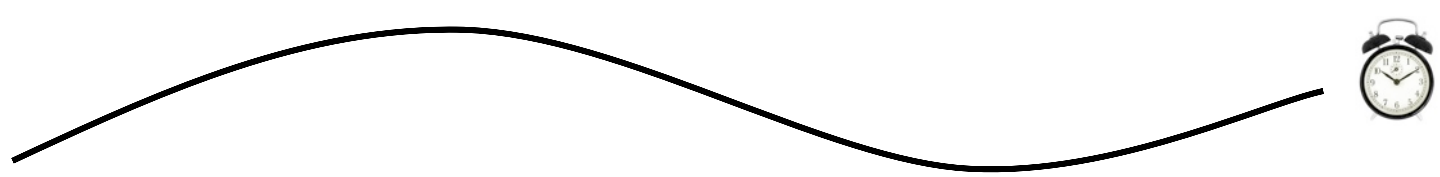
Explicitly quadratic in the perturbations:

$$S_{DE}^{(2)} = \frac{M_2^4}{2} (\delta g^{00})^2 - \frac{\bar{m}_1^3}{2} \delta g^{00} \delta K - \frac{\bar{M}_2^2}{2} \delta K^2 - \frac{\bar{M}_3^2}{2} \delta K_\mu{}^\nu \delta K^\mu{}_\nu + \ldots$$

3-curvature terms:

$$+ \frac{\tilde{m}_1}{2} \delta g^{00}\, {}^{(3)}R + \frac{\tilde{M}_1}{2} \delta K_\mu{}^\nu\, {}^{(3)}R_\nu{}^\mu + \ldots$$

In EFT of Inflation these terms can be eliminated by a metric redefinition



The Action: perturbations

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} f(t) R - \Lambda(t) - c(t) g^{00} \right] + S_{DE}^{(2)}$$

Explicitly quadratic in the perturbations:

$$S_{DE}^{(2)} = \frac{M_2^4}{2} (\delta g^{00})^2 - \frac{\bar{m}_1^3}{2} \delta g^{00} \delta K - \frac{\bar{M}_2^2}{2} \delta K^2 - \frac{\bar{M}_3^2}{2} \delta K_\mu{}^\nu \delta K^\mu{}_\nu + \ldots$$

$$+ \frac{\tilde{m}_1}{2} \delta g^{00}\, {}^{(3)}R + \frac{\tilde{M}_1}{2} \delta K_\mu{}^\nu\, {}^{(3)}R_\nu{}^\mu + \ldots$$

Action in “standard form” (no ambiguities, field redefinitions)

Outline

- Motivations
- The Action
- Examples
- Mixing with gravity, stability and speed of sound
- The zero sound speed limit
- Conclusions

Examples

$$S = \int \sqrt{-g} \left(\frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2} (\delta g^{00})^2 - \frac{\bar{m}_1^3}{2} \delta g^{00} \delta K \ + \frac{M_3^4}{6} (\delta g^{00})^3 + \ldots \right)$$

Examples

$$S = \int \sqrt{-g}\left(\frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2}(\delta g^{00})^2 - \frac{\bar{m}_1^3}{2}\delta g^{00}\delta K + \frac{M_3^4}{6}(\delta g^{00})^3 + \dots\right)$$

Non-minimally coupled scalar field

$$S = \int d^4x \sqrt{-g}\left[\frac{M^2}{2}F(\phi)R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

$$f(t) = F(\phi_0(t))\ , \quad \Lambda(t) = V(\phi_0(t))\ , \quad c(t) = \dot{\phi}_0^2(t)$$

Examples

$$S = \int \sqrt{-g} \left(\frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2} (\delta g^{00})^2 - \frac{\bar{m}_1^3}{2} \delta g^{00} \delta K + \frac{M_3^4}{6} (\delta g^{00})^3 + \ldots \right)$$

K-essence

(Amendariz-Picon et al., 2000)

$$S = \int d^4x \sqrt{-g} P(\phi, X) \qquad X \equiv g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

Expansion: $X = \dot{\phi}_0^2(t)(-1 + \delta g^{00})$

$$\Lambda(t) = c(t) - P(\phi_0(t), \dot{\phi}_0^2(t)) \,, \quad c(t) = \left. \frac{\partial P}{\partial X} \right|_{\phi=\phi_0,\, X=\dot{\phi}_0^2} \,,$$

$$M_n^4(t) = \left. \frac{\partial^n P}{\partial X^n} \right|_{\phi=\phi_0,\, X=\dot{\phi}_0^2} \qquad (n \geq 2)$$

Examples

$$S = \int \sqrt{-g} \left(\frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2} (\delta g^{00})^2 - \frac{\bar{m}_1^3}{2} \delta g^{00} \delta K + \frac{M_3^4}{6} (\delta g^{00})^3 + \dots \right)$$

"Galilean Cosmology" (Chow and Khoury, 2009)

$$S = \int d^4x \sqrt{-g} \left[\frac{M^2}{2} e^{-2\phi/M} R - \frac{r_c^2}{M} (\partial\phi)^2 \Box\phi \right]$$

$$f(t) = e^{-2\frac{\phi_0}{M}} \quad , \quad \Lambda(t) = -\frac{r_c^2}{M} \dot{\phi}_0^2 (\ddot{\phi}_0 + 3H\dot{\phi}_0) \, , \quad c(t) = \frac{r_c^2}{M} \dot{\phi}_0^2 (\ddot{\phi}_0 - 3H\dot{\phi}_0) \, ,$$

$$M_2^4(t) = -\frac{r_c^2}{2M} \dot{\phi}_0^2 (\ddot{\phi}_0 + 3H\dot{\phi}_0) \, , \quad M_3^4(t) = -\frac{3r_c^2}{4M} \dot{\phi}_0^2 (\ddot{\phi}_0 + H\dot{\phi}_0) \, , \quad \bar{m}_1^3(t) = -\frac{r_c^2}{M} 2\dot{\phi}_0^3 \, ,$$

Outline

- Motivations
- The Action
- Examples
- Mixing with gravity, stability and speed of sound
- The zero sound speed limit
- Conclusions

Mixing with gravity:

$$S = \int \sqrt{-g}\left(\frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2}(\delta g^{00})^2 - \frac{\bar{m}_1^3}{2}\delta g^{00}\delta K + \frac{1}{2}T^{\mu\nu}\delta g_{\mu\nu}\right)$$

Apply "Stueckelberg trick" and go to Newtonian Gauge

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\delta_{ij}dx^i dx^j$$

Mixing with gravity:

$$S = \int \sqrt{-g}\left(\frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2}(\delta g^{00})^2 - \frac{\bar{m}_1^3}{2}\delta g^{00}\delta K + \frac{1}{2}T^{\mu\nu}\delta g_{\mu\nu}\right)$$

Apply "Stueckelberg trick" and go to Newtonian Gauge

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\delta_{ij}dx^i dx^j$$

The scalar d.o.f. can be made explicit by forcing a time-diff on the action:

$$t \to t + \pi(x)$$

and by promoting the parameter of diffeomorphism to a field:

$$\begin{aligned} c(t) &\to c(t+\pi) = c(t) + \dot{c}(t)\,\pi + \frac{1}{2}\ddot{c}(t)\,\pi^2 + \dots \\ g^{00} &\to g^{\mu\nu}\partial_\mu(t+\pi)\partial_\nu(t+\pi) = g^{00} + 2g^{0\mu}\partial_\mu\pi + g^{\mu\nu}\partial_\mu\pi\partial_\nu\pi\ , \\ \delta K &\to \delta K - 3\dot{H}\pi - a^{-2}\nabla^2\pi \end{aligned}$$

Mixing with gravity:

$$S = \int \sqrt{-g} \left(\frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2} (\delta g^{00})^2 - \frac{\bar{m}_1^3}{2} \delta g^{00} \delta K + \frac{1}{2} T^{\mu\nu} \delta g_{\mu\nu} \right)$$

Apply "Stueckelberg trick" and go to Newtonian Gauge

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\delta_{ij} dx^i dx^j$$

Expand at quadratic order and retain only kinetic operators (2 derivatives):

$$\dot{\Psi}^2 \,, \quad (\vec{\nabla}\Psi)^2 \,, \quad \text{etc.}$$

Modified Gravity $\approx$ Kinetic mixing $\quad \dot{\Psi}\dot{\pi} \,, \quad \vec{\nabla}\Psi\vec{\nabla}\pi \,, \quad \text{etc.}$

One less derivative in couplings $\quad \dot{\Psi}\pi \quad \approx$ Jeans length (in progress)

Mixing with gravity 1: Brans-Dicke

$$S = \int \sqrt{-g}\left(\frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2}(\delta g^{00})^2 - \frac{\bar{m}_1^3}{2}\delta g^{00}\delta K + \frac{1}{2}T^{\mu\nu}\delta g_{\mu\nu}\right)$$

Apply Stueckelberg and go to Newtonian Gauge

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\delta_{ij}dx^i dx^j$$

$$S \overset{\text{kinetic}}{=} \int M^2 f \left[-3\dot{\Psi}^2 - 2\vec{\nabla}\Phi\vec{\nabla}\Psi + (\vec{\nabla}\Psi)^2 + c\,\dot{\pi}^2 - c(\vec{\nabla}\pi)^2 + 3(\dot{f}/f)\dot{\Psi}\dot{\pi} + (\dot{f}/f)\vec{\nabla}\pi(\vec{\nabla}\Phi - 2\vec{\nabla}\Psi)\right]$$

Mixing with gravity 1: Brans-Dicke

$$S = \int \sqrt{-g}\left(\frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2}(\delta g^{00})^2 - \frac{\bar{m}_1^3}{2}\delta g^{00}\delta K + \frac{1}{2}T^{\mu\nu}\delta g_{\mu\nu}\right)$$

Apply Stueckelberg and go to Newtonian Gauge

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\delta_{ij}dx^i dx^j$$

$$S^{\text{kinetic}} = \int M^2 f\left[-3\dot{\Psi}^2 - 2\vec{\nabla}\Phi\vec{\nabla}\Psi + (\vec{\nabla}\Psi)^2 + c\,\dot{\pi}^2 - c(\vec{\nabla}\pi)^2 + 3(\dot{f}/f)\dot{\Psi}\dot{\pi} + (\dot{f}/f)\vec{\nabla}\pi(\vec{\nabla}\Phi - 2\vec{\nabla}\Psi)\right]$$

1 propagating d.o.f.

$$\det\mathcal{L} = k^4(\omega^2 - k^2)$$

De-mixing = conformal transformation

$$\Phi_E = \Phi + \frac{1}{2}(\dot{f}/f)\pi$$

$$\Psi_E = \Psi - \frac{1}{2}(\dot{f}/f)\pi$$

Newtonian limit $\quad \partial_t \ll \vec{\nabla}$

$$S^{\text{kinetic}} = \int M^2 f \left[-3\dot{\Psi}^2 - 2\vec{\nabla}\Phi\vec{\nabla}\Psi + (\vec{\nabla}\Psi)^2 + c\dot{\pi}^2 - c(\vec{\nabla}\pi)^2 + 3(\dot{f}/f)\dot{\Psi}\pi + (\dot{f}/f)\vec{\nabla}\pi(\vec{\nabla}\Phi - 2\vec{\nabla}\Psi) \right]$$

$$-\Phi\delta\rho_m$$

$$1 - \gamma \equiv \frac{\Phi - \Psi}{\Phi} = \frac{M^2\dot{f}^2/f}{2(c + M^2\dot{f}^2/f)}$$

$$\nabla^2\pi = \frac{-M^2\dot{f}}{2(c + M^2\dot{f}^2/f)}\nabla^2\Phi$$

$$\nabla^2\Phi = 4\pi G_{\text{eff}}\delta\rho_m$$ Poisson equation

$$G_{\text{eff}} = \frac{1}{8\pi M^2 f}\frac{c + M^2\dot{f}^2/f}{c + \frac{3}{4}M^2\dot{f}^2/f}$$ "dressed" Newton constant

Mixing with gravity 2:

(see also Creminelli et al. 2006 & 2008)

$G(\phi, X)\Box\phi$ (Cf. braiding: Deffayet et al., 2010)

$f(t) = 1$

$$S = \int \sqrt{-g}\left(\frac{M^2}{2} fR - \Lambda - cg^{00} + \frac{M_2^4}{2}(\delta g^{00})^2 - \frac{\bar{m}_1^3}{2}\delta g^{00}\delta K + \frac{1}{2}T^{\mu\nu}\delta g_{\mu\nu}\right)$$

Apply Stueckelberg and go to Newtonian Gauge

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\delta_{ij}dx^i dx^j$$

$$S \stackrel{\text{kinetic}}{=} \int M^2\left[-3\dot{\Psi}^2 - 2\vec{\nabla}\Phi\vec{\nabla}\Psi + (\vec{\nabla}\Psi)^2\right] + c\dot{\pi}^2 - c(\vec{\nabla}\pi)^2 - 3\bar{m}_1^3\dot{\Psi}\dot{\pi} - \bar{m}_1^3\vec{\nabla}\Phi\vec{\nabla}\pi$$

Mixing with gravity 2:

(see also Creminelli et al. 2006 & 2008)

$G(\phi, X)\Box\phi$ (Cf. braiding: Deffayet et al., 2010)

$f(t) = 1$

$$S = \int \sqrt{-g}\left(\frac{M^2}{2} fR - \Lambda - cg^{00} + \frac{M_2^4}{2}(\delta g^{00})^2 - \frac{\bar{m}_1^3}{2}\delta g^{00}\delta K + \frac{1}{2}T^{\mu\nu}\delta g_{\mu\nu}\right)$$

Apply Stueckelberg and go to Newtonian Gauge

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\delta_{ij}dx^i dx^j$$

$$S \overset{\text{kinetic}}{=} \int M^2\left[-3\dot{\Psi}^2 - 2\vec{\nabla}\Phi\vec{\nabla}\Psi + (\vec{\nabla}\Psi)^2\right] + c\dot{\pi}^2 - c(\vec{\nabla}\pi)^2 - 3\bar{m}_1^3\dot{\Psi}\dot{\pi} - \bar{m}_1^3\vec{\nabla}\Phi\vec{\nabla}\pi$$

1 propagating d.o.f.

$$\det\mathcal{L} = k^4(\omega^2 - c_s^2 k^2)$$

$$c_s^2 = \frac{c + \frac{1}{2}(H\bar{m}_1^3 + \dot{\bar{m}}_1^3) - \frac{1}{4}\bar{m}_1^6/M^2}{c + \frac{3}{4}\bar{m}_1^6/M^2}$$

De-mixing ≠ conformal transformation

$$\Phi_E = \Phi + \frac{\bar{m}_1^3}{2M^2}\pi$$

$$\Psi_E = \Psi + \frac{\bar{m}_1^3}{2M^2}\pi$$

Newtonian limit $\partial_t \ll \vec{\nabla}$

$$S \overset{\text{kinetic}}{=} \int M^2 \left[-3\dot{\Psi}^2 - 2\vec{\nabla}\Phi\vec{\nabla}\Psi + (\vec{\nabla}\Psi)^2\right] + c\dot{\pi}^2 - c(\vec{\nabla}\pi)^2 - 3\bar{m}_1^3\dot{\Psi}\dot{\pi} - \bar{m}_1^3\vec{\nabla}\Phi\vec{\nabla}\pi$$

$$-\Phi\delta\rho_m$$

$\Phi = \Psi$ unlike Brans-Dicke theories $\gamma = 1$

$$\nabla^2\pi = -\frac{\bar{m}_1^3}{2c}\nabla^2\Phi$$

$$\nabla^2\Phi = 4\pi G_{\rm eff}\delta\rho_m$$ Poisson equation

$$G_{\rm eff} = \frac{1}{8\pi M^2 f}\left(1 - \frac{\bar{m}_1^3}{4cM^2}\right)^{-1}$$ “dressed” Newton constant

Model building v.s. General treatment (with Gubitosi, Piazza, 2012)

$$S = \int \sqrt{-g} \left(\frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2} (\delta g^{00})^2 - \frac{\bar{m}_1^3}{2} \delta g^{00} \delta K \ + \frac{M_3^4}{6} (\delta g^{00})^3 + \dots \right)$$

Find, once and for all, the action for the scalar degree of freedom:

$$S_\pi \overset{\text{kinetic}}{=} \int a^3 \left\{ \left[c + 2M_2^4 + \frac{3}{4} \frac{\dot{f}^2}{f} M^2 - \frac{3}{2} \bar{m}_1^3 \frac{\dot{f}}{f} + \frac{3}{4} \frac{\bar{m}_1^6}{M^2} \right] \dot{\pi}^2 \right.$$

$$\left. - \left[c + \frac{3}{4} \frac{\dot{f}^2}{f} M^2 - \frac{1}{2} \bar{m}_1^3 \frac{\dot{f}}{f} - \frac{1}{4} \frac{\bar{m}_1^6}{M^2} + \frac{1}{2} \left(\dot{\bar{m}}_1^3 + H \bar{m}_1^3 \right) \right] \frac{(\vec{\nabla} \pi)^2}{a^2} \right\}$$

And address, once and for all, all questions of stability, speed of sound and deviations from GR:

$$1 - \gamma = \frac{1}{2} \frac{(M^2 \dot{f}^2 + \bar{m}_1^3 \dot{f})/f}{c + M^2 \dot{f}^2/f + \frac{1}{2}(\dot{\bar{m}}_1^3 + H \bar{m}_1^3)}$$

$$G_{\text{eff}} = \frac{1}{8\pi M^2 f} \frac{c + M^2 \dot{f}^2/f + \frac{1}{2}(\dot{\bar{m}}_1^3 + H \bar{m}_1^3)}{c + \frac{3}{4} M^2 \dot{f}^2/f - \frac{1}{2} \bar{m}_1^3 \dot{f}/f - \frac{1}{4} \bar{m}_1^6/M^2 + \frac{1}{2}(\dot{\bar{m}}_1^3 + H \bar{m}_1^3)}$$

Outline

- Motivations
- The Action
- Examples
- Mixing with gravity, stability and speed of sound
- The zero sound speed limit
- Conclusions

The zero sound speed limit of quintessence

$$S = \int \sqrt{-g} \left(\frac{M^2}{2} f R - \Lambda - c g^{00} + \frac{M_2^4}{2} (\delta g^{00})^2 - \frac{\bar{m}_1^3}{2} \delta g^{00} \delta K + \frac{M_3^4}{6} (\delta g^{00})^3 + \dots \right)$$

- Consider a minimally coupled field, $\dot{f} = 1$, and the limit $c = \frac{1}{2}(\rho_D + p_D) \ll M_2^4$ with $M_2 \simeq \bar{m}_1$

- The action reads

$$S_\pi \overset{\text{kinetic}}{=} \int a^3 \left\{ 2 M_2^4 \dot{\pi}^2 - \left[c - \frac{1}{4} \frac{M_2^6}{M_{\rm Pl}^2} + \frac{1}{2} \left(\dot{M}_2^3 + H M_2^3 \right) \right] \frac{(\vec{\nabla}\pi)^2}{a^2} \right\}$$

- The speed of sound of fluctuations vanishes

$$c_s^2 = \frac{c}{2M_2^4} - \frac{1}{8} \frac{M_2^2}{M_{\rm Pl}^2} + \frac{3\dot{M}_2}{8M_2^2} + \frac{H}{8M_2} \ll 1$$

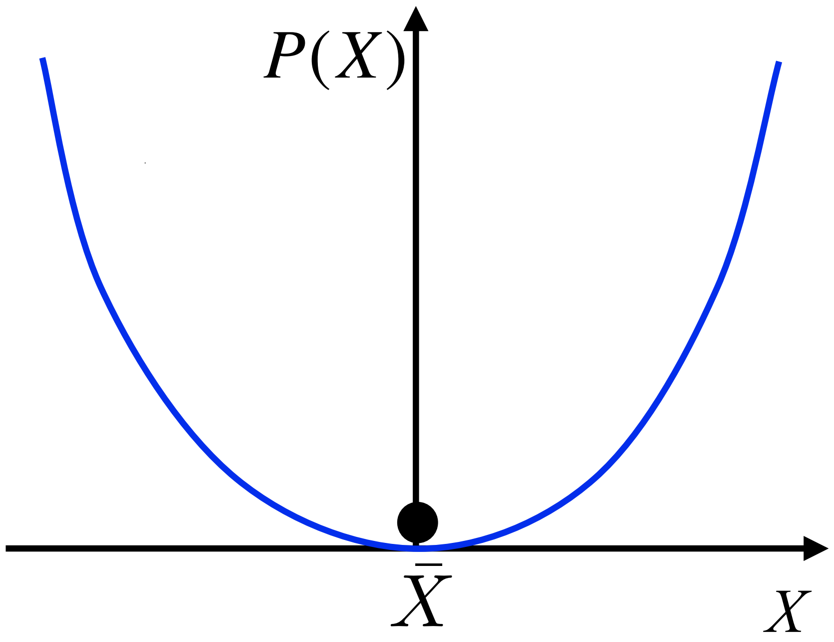
Motivations

- <u>Shift symmetry invariance</u>: $\phi \rightarrow \phi + \lambda \quad \Rightarrow \quad \mathcal{L} = P(X)$

EOM in expanding Universe: $\partial_t (a^3 \dot{\phi} P_{,X}) = 0$

- Solution with $\dot{\phi} = \text{const} \quad \Rightarrow \quad \bar{X} = \text{const}^2$

and $P_{,X} \rightarrow 0 \quad \Longrightarrow \quad w \rightarrow -1 \;\text{ and }\; c_s^2 \rightarrow 0$



Motivations

- Shift symmetry invariance: $\phi \to \phi + \lambda \quad \Rightarrow \quad \mathcal{L} = P(X)$

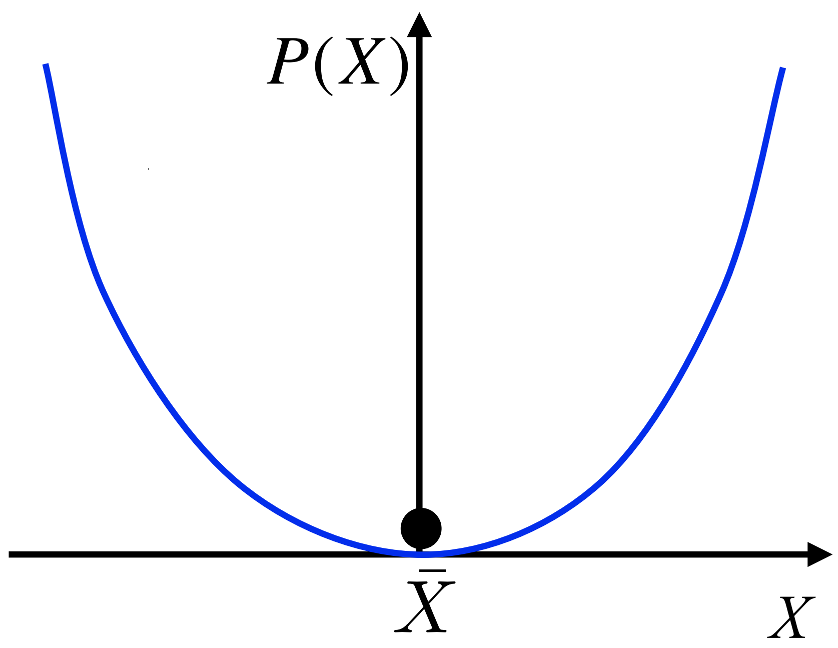
EOM in expanding Universe: $\partial_t(a^3 \dot\phi P_{,X}) = 0$

- Solution with $\dot\phi = \text{const} \quad \Rightarrow \quad \bar X = \text{const}^2$

and $P_{,X} \to 0 \quad \Longrightarrow \quad w \to -1 \;\; \text{and} \;\; c_s^2 \to 0$

Ghost condensate theory: [Arkani-Hamed et al., '03, '05]

$$P(X) = \bar P + \frac{1}{2} P_{,XX}(X - \bar X)^2 + \text{higher der.}$$



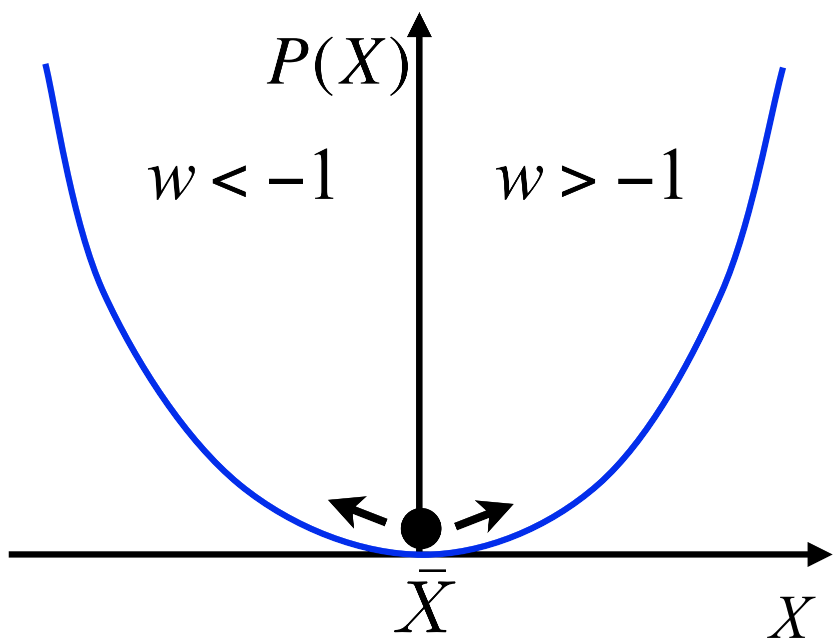
Motivations

- <u>Shift symmetry invariance</u>: $\phi \to \phi + \lambda \quad \Rightarrow \quad \mathcal{L} = P(X)$

EOM in expanding Universe: $\partial_t(a^3 \dot\phi P_{,X}) = 0$

- Solution with $\dot{\bar\phi} = \text{const} \quad \Rightarrow \quad \bar X = \text{const}^2$

and $P_{,X} \to 0 \quad \Rightarrow \quad w \to -1 \;\; \text{and} \;\; c_s^2 \to 0$



Ghost condensate theory: [Arkani-Hamed et al., '03, '05]

$$P(X) = \bar P + \frac{1}{2} P_{,XX} (X - \bar X)^2 + \text{higher der.}$$

- Tiny breaking of shift symmentry: $c \ll M_2^4 \quad \Leftrightarrow \quad \bar P_{,X} \ll \bar P_{,XX} \bar X$

$$P(\phi, X) = -V(\phi) + \bar P_{,X}(\phi, X)(X - \bar X) + \frac{1}{2} \bar P_{,XX}(\phi, X)(X - \bar X)^2 + \ldots$$

Pressure gradients suppressed wrt density gradients:

$$\delta P|_{\delta\phi=0} \sim \bar P_{,X} \cdot \delta X$$

$$\delta\rho|_{\delta\phi=0} \sim \bar P_{,XX} \bar X \cdot \delta X$$

- Stable model even for *w<-1* (higher derivatives operators) [Arkani-Hamed et al., '05; Creminelli et al., '06]

Clustering quintessence

- Euler equation: $\dot{\vec{v}} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{\rho+p}\left[\vec{\nabla}p + \vec{v}\frac{\partial p}{\partial t}\right] - \vec{\nabla}\Phi$

[Creminelli et al. '10; see also Lim et al. '10]

Clustering quintessence

- Euler equation: $\dot{\vec{v}} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{\rho + p}\left[\vec{\nabla}p + \vec{v}\frac{\partial p}{\partial t}\right] - \vec{\nabla}\Phi$ [Creminelli et al. '10; see also Lim et al. '10]

For $c_s^2 = 0$ pressure gradients (orthogonal to the fluid 4-velocity) vanish!

($u^\mu \nabla_\mu u^\nu = 0$ if $c_s^2 = 0$)

➡ **Geodesic motion**: quintessence remains comoving with dark matter (also nonlinearly)

Clustering quintessence

- Euler equation: $\dot{\vec{v}} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\frac{1}{\rho+p}\left[\vec{\nabla}p + \vec{v}\frac{\partial p}{\partial t}\right] - \vec{\nabla}\Phi$ [Creminelli et al. '10; see also Lim et al. '10]

For $c_s^2 = 0$ pressure gradients (orthogonal to the fluid 4-velocity) vanish!

($u^\mu \nabla_\mu u^\nu = 0$ if $c_s^2 = 0$)

➡ **Geodesic motion**: quintessence remains comoving with dark matter (also nonlinearly)

- Continuity equation: $\dot{\rho}_Q + \vec{\nabla}[(\rho_Q + p_Q)\vec{v}] = 0$

No pressure gradients but pressure is important!

No conserved particle number or current

$$\bar{\rho}_m \propto \frac{1}{a^3}; \quad \bar{\rho}_Q \propto \frac{1}{a^{3(1+w)}}$$

Clustering quintessence

- Linearized continuity equations:

$$\dot{\delta}_m + \frac{1}{a}\vec{\nabla}\cdot\vec{v} = 0$$

$$\dot{\delta}_Q - 3w\frac{\dot{a}}{a}\delta_Q + (1+w)\frac{1}{a}\vec{\nabla}\cdot\vec{v} = 0$$

$$\Rightarrow$$

During dark matter dominance:

$$\delta_Q = \frac{1+w}{1-3w}\delta_{\rm DM}$$

Clustering quintessence

- Linearized continuity equations:

$$\dot{\delta}_m + \frac{1}{a}\vec{\nabla}\cdot\vec{v} = 0$$

$$\dot{\delta}_Q - 3w\frac{\dot{a}}{a}\delta_Q + (1+w)\frac{1}{a}\vec{\nabla}\cdot\vec{v} = 0$$

$$\Rightarrow$$

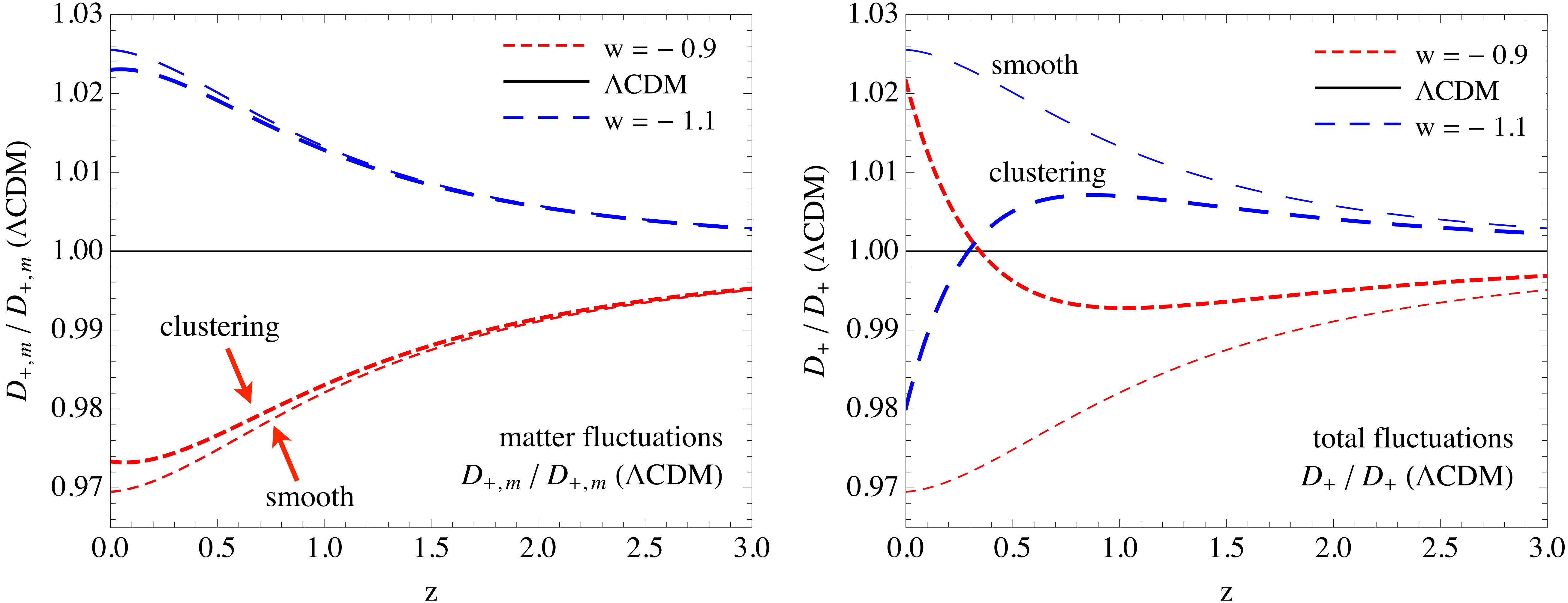
During dark matter dominance:

$$\delta_Q = \frac{1+w}{1-3w}\delta_{\rm DM}$$

- Linearized Euler + Poisson equations:

$$\dot{\vec{v}} + \frac{\dot{a}}{a}\vec{v} = -\vec{\nabla}\Phi$$

$$\nabla^2\Phi = 4\pi G a^2 \bar{\rho}_m \left(\delta_m + \frac{\Omega_Q}{\Omega_m}\delta_Q\right)$$



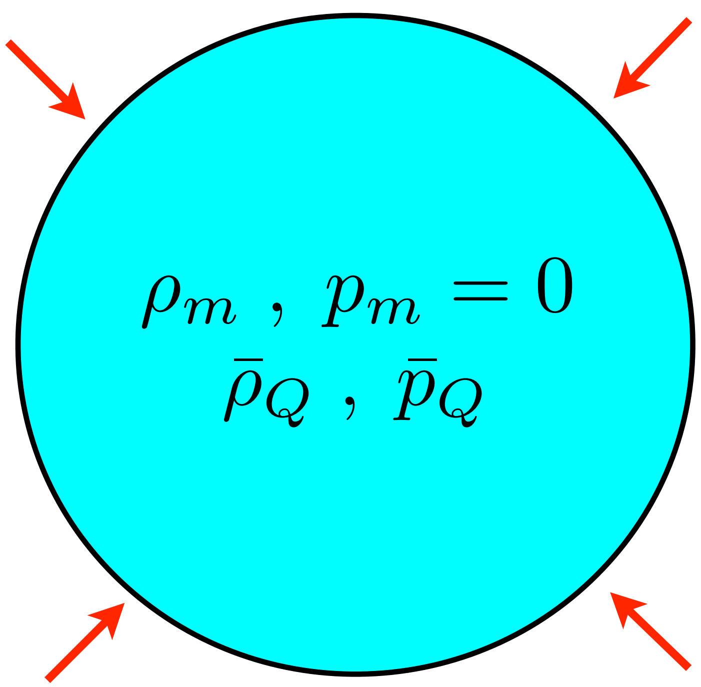
Spherical collapse

- Quintessence affects the spherical collapse model:

Spherical collapse

- Quintessence affects the spherical collapse model:

- Both density and pressure remain homogeneous and follow the outside Hubble flow:



$$c_s^2 = 1$$

$$\bar{\rho}_m \ , \ \bar{p}_m = 0$$
$$\bar{\rho}_Q \ , \ \bar{p}_Q$$

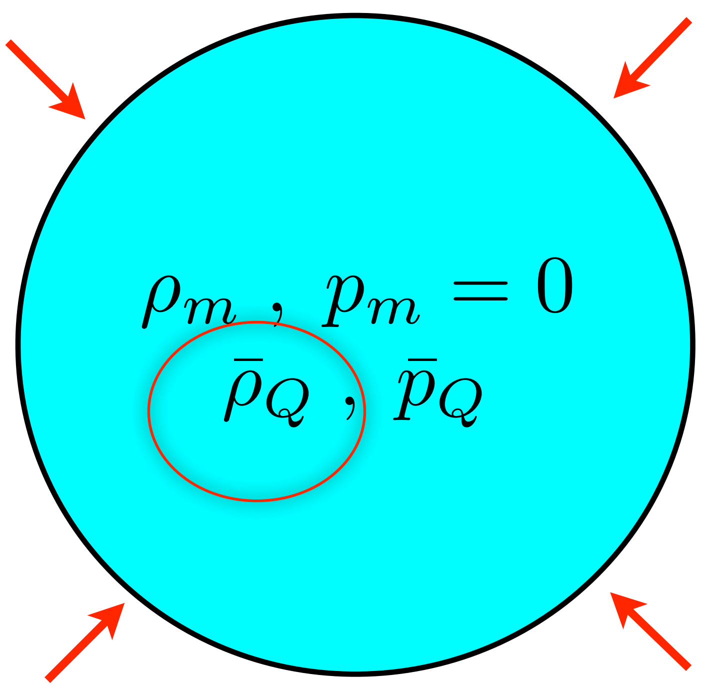
$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho_m + \bar{\rho}_Q + 3\bar{p}_Q)$$

No FRW universe inside [Wang & Steinhardt '98]

Spherical collapse

- Quintessence affects the spherical collapse model:

- <u>Both</u> density and pressure remain homogeneous and <u>follow the outside</u> Hubble flow:



$$c_s^2 = 1$$

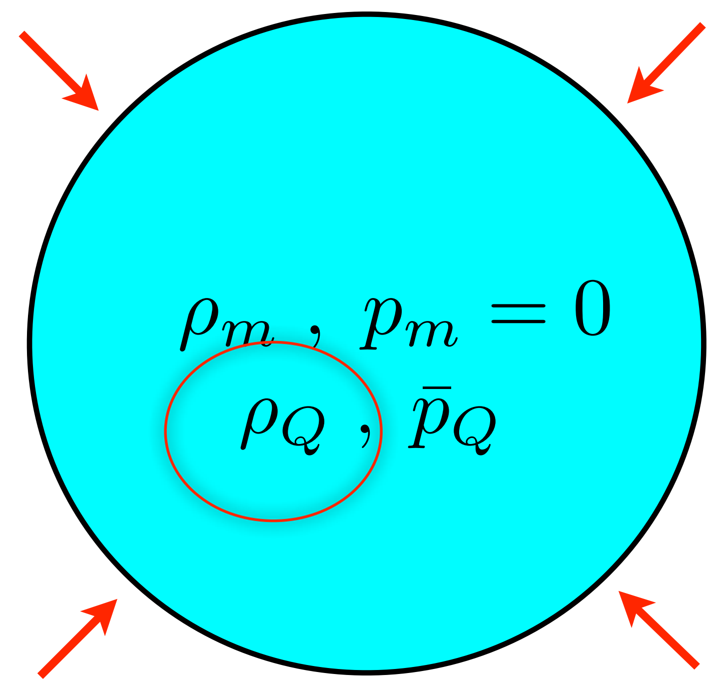
$\bar{\rho}_m \; , \; \bar{p}_m = 0$

$\bar{\rho}_Q \; , \; \bar{p}_Q$

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho_m + \bar{\rho}_Q + 3\bar{p}_Q)$$

No FRW universe inside [Wang & Steinhardt '98]

- Quintessence density follows dark matter flow but <u>pressure remains as outside</u>:



$$c_s^2 = 0$$

$\bar{\rho}_m \; , \; \bar{p}_m = 0$

$\bar{\rho}_Q \; , \; \bar{p}_Q$

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3}(\rho_m + \rho_Q + 3\bar{p}_Q)$$

Exact FRW universe inside!

Quintessence mass

- Evolution equation inside the overdensity: $\dot{\rho}_Q + 3\frac{\dot{R}}{R}(\rho_Q + \bar{p}_Q) = 0$
- Large overdensities behave as DM:

$$\delta_Q \gg |1+w| \quad \Rightarrow \quad \delta\dot{\rho}_Q + 3\frac{\dot{R}}{R}\delta\rho_Q \approx 0$$

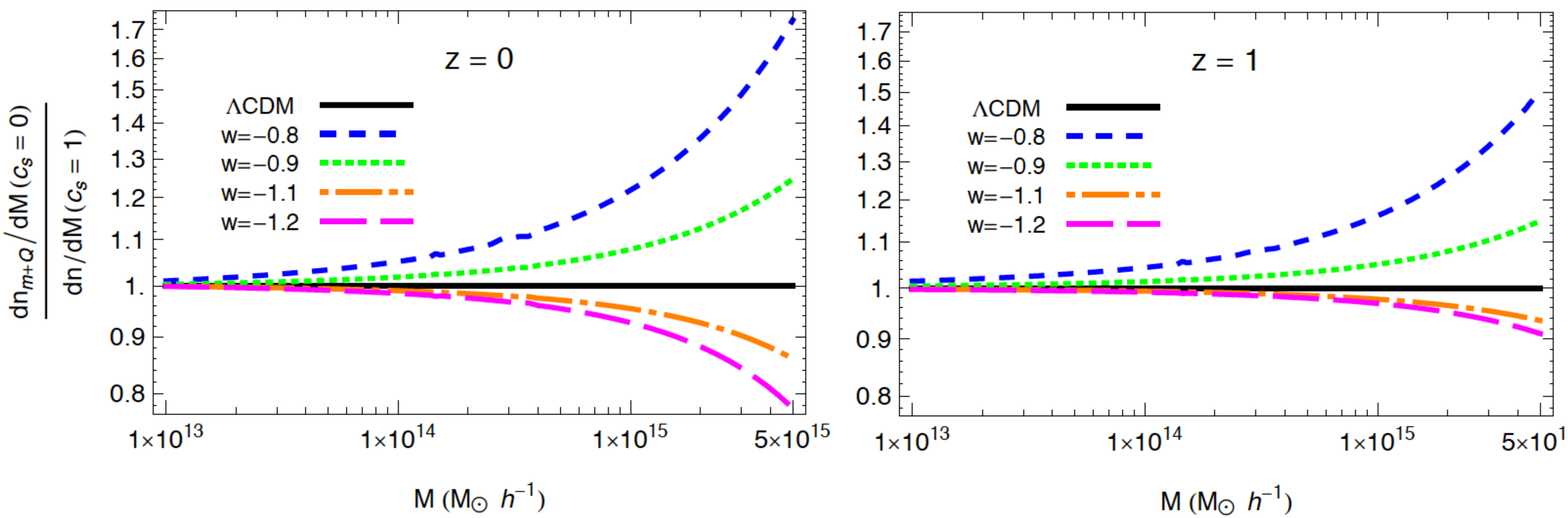
Quintessence mass

- Evolution equation inside the overdensity: $$\dot{\rho}_Q + 3\frac{\dot{R}}{R}(\rho_Q + \bar{p}_Q) = 0$$
- Large overdensities behave as DM:

$$\delta_Q \gg |1+w| \quad \Rightarrow \quad \dot{\delta\rho_Q} + 3\frac{\dot{R}}{R}\delta\rho_Q \approx 0$$

Conserved quintessence mass inside halos!

$$M_Q = \frac{4\pi R^3}{3}\delta\rho_Q \approx (1+w)\left.\frac{\Omega_Q}{\Omega_{\mathrm{DM}}}\right|_{z_{\mathrm{coll}}} \cdot M_{\mathrm{DM}}$$



Conclusion

- Unifying framework for dark energy/modified gravity
- Effective language: cosmological perturbations as the relevant d.o.f.
- Extension of EFT of inflation and quintessence to non-minimal couplings
- Unambiguous way to address mixing, stability, speed of sound etc.
- See also Bloomfield et al. 1211.7054. Much work in progress to consider effects of coupling to matter
- Quintessence can have zero sound speed! Simplest phenomenological alternative to the smooth case
- New phenomenology: 1) nonlinear corrections to PS and bispectrum; 2) Quintessence mass in virialized objects