Gravitational Waves from Spin Precessing, Compact Binary Inspirals

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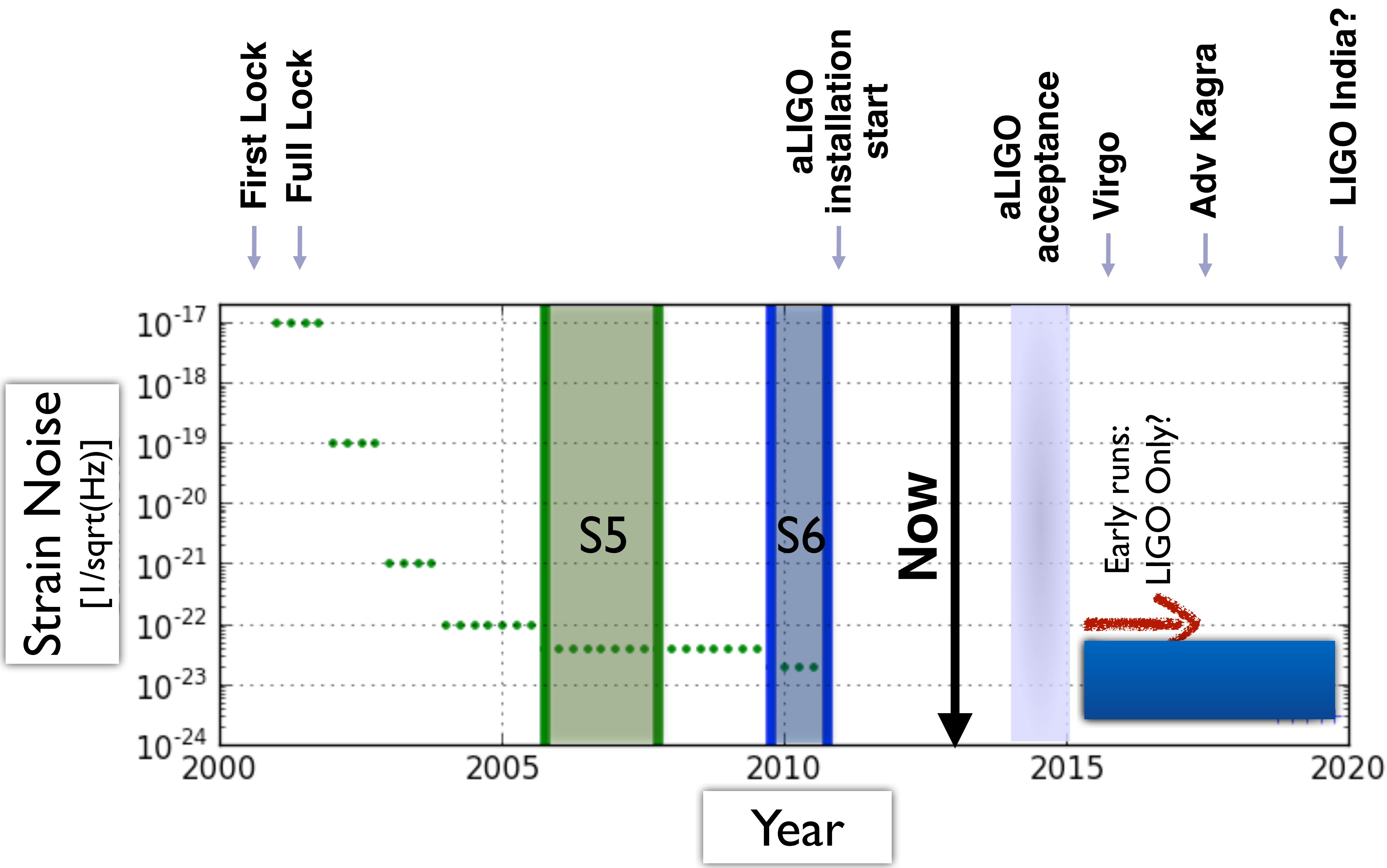
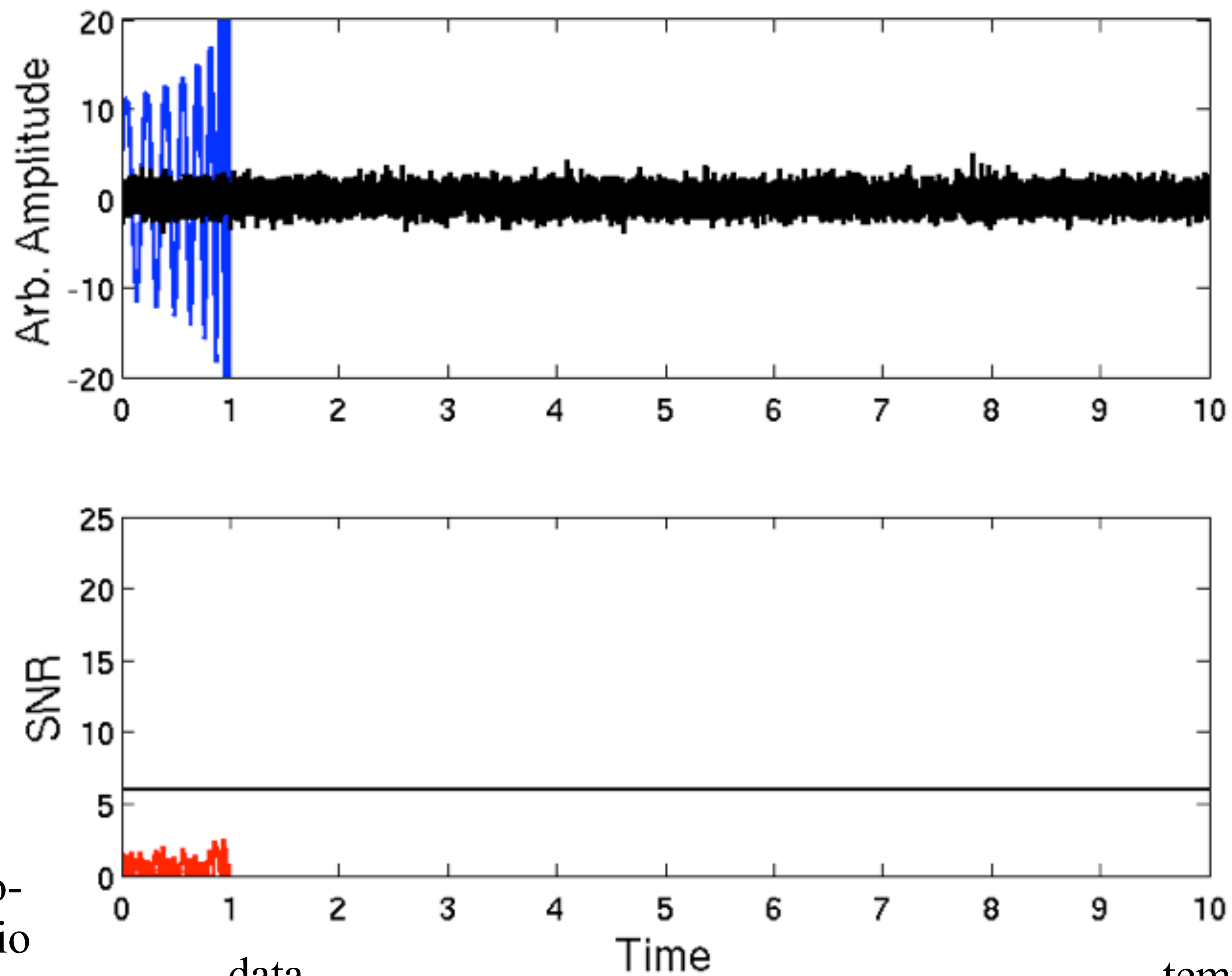
arXiv: ????.???? (to be submitted)

At our doorstep...



Data Analysis and Parameter Estimation

C. Hanna, LSC/PI



Matched Filtering: Maximize the SNR over all template parameters

$$\rho^2 \sim \int \frac{\tilde{s}(f)\tilde{h}(f,\lambda^\mu)}{S_n(f)} df$$

signal-to-noise ratio (SNR)

data

template param that characterize system

detector noise (spectral noise density)

template (projection of GW metric perturbation)

Return on Investment

What information we get from detecting an inspiral?

- Location Parameters: right ascension, declination, luminosity distance.

- Spin-Independent Parameters: chirp mass, symmetric mass ratio, inclination angle, polarization angle, time and phase of coalescence.

- Spinning Parameters: 3 components of spin 1 + 3 components of spin 2.

Provided we have waveforms that can accurately model the inspiral with full dependence on these parameters.

Road Map

I. Template Construction Driven By Systems

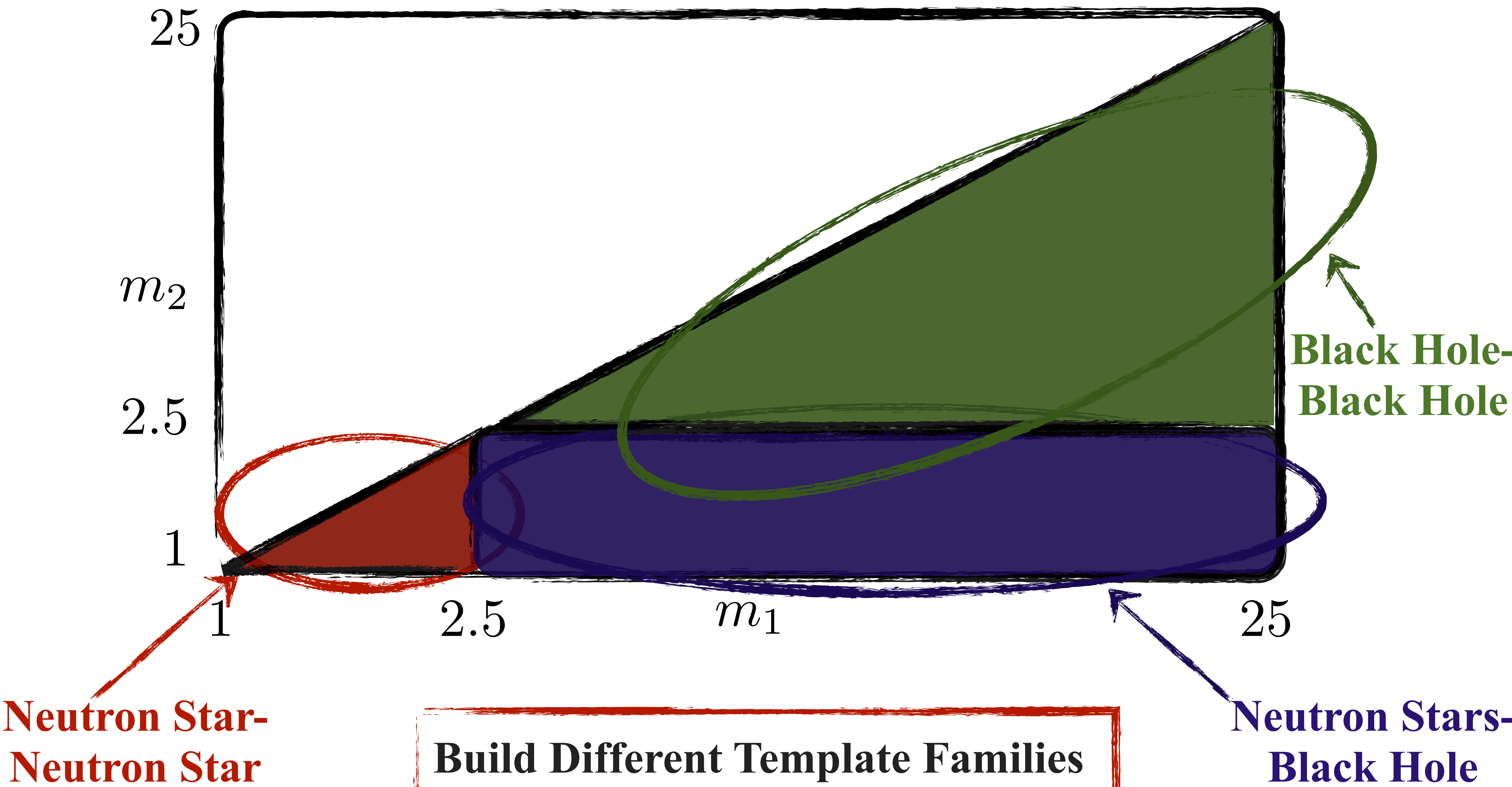
II. Analytic Construction of Precessing Waveforms

III. Performance of Analytic Waveforms

Road Map

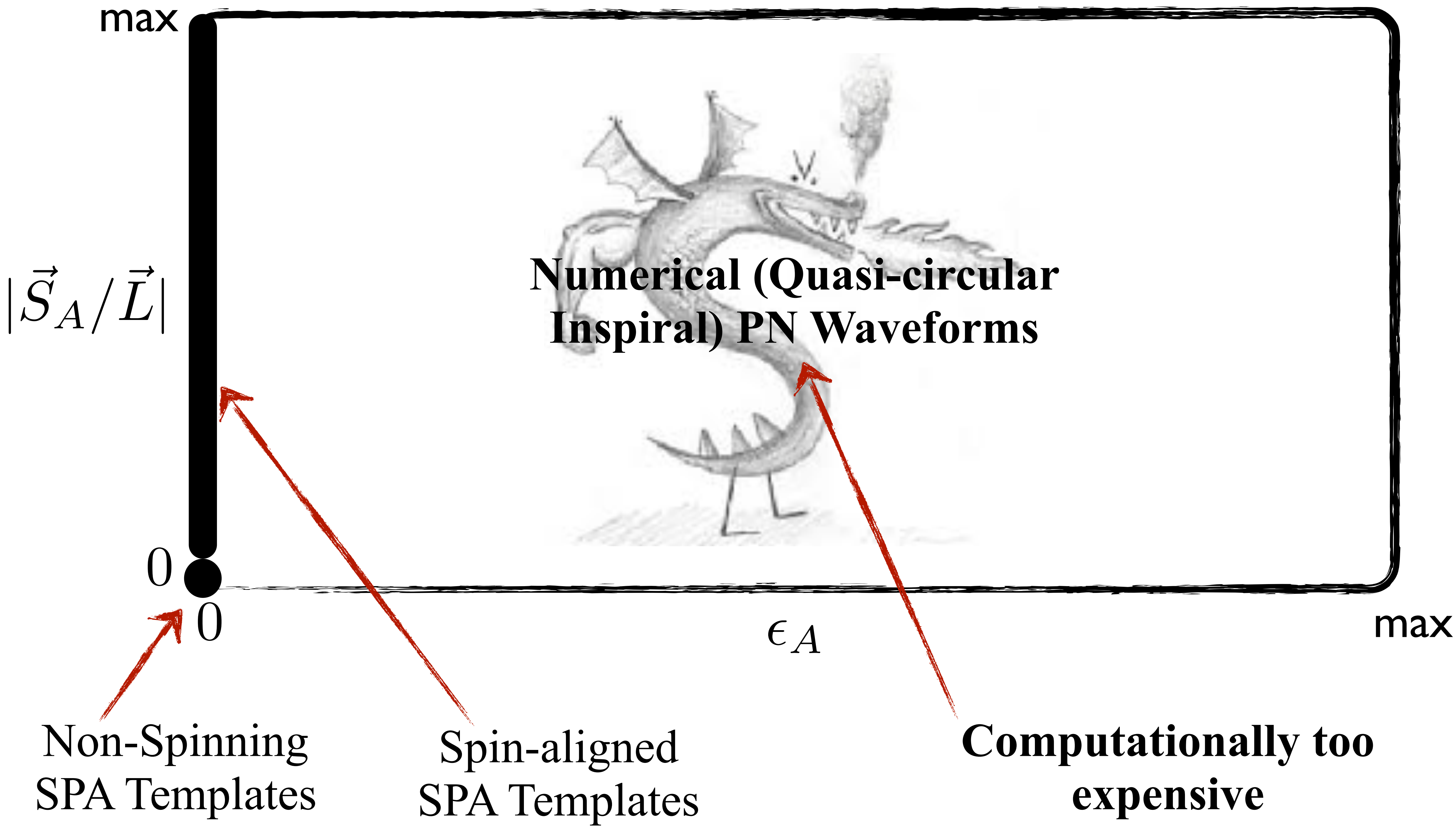
I. Template Construction Driven By Systems

System-Driven Templates



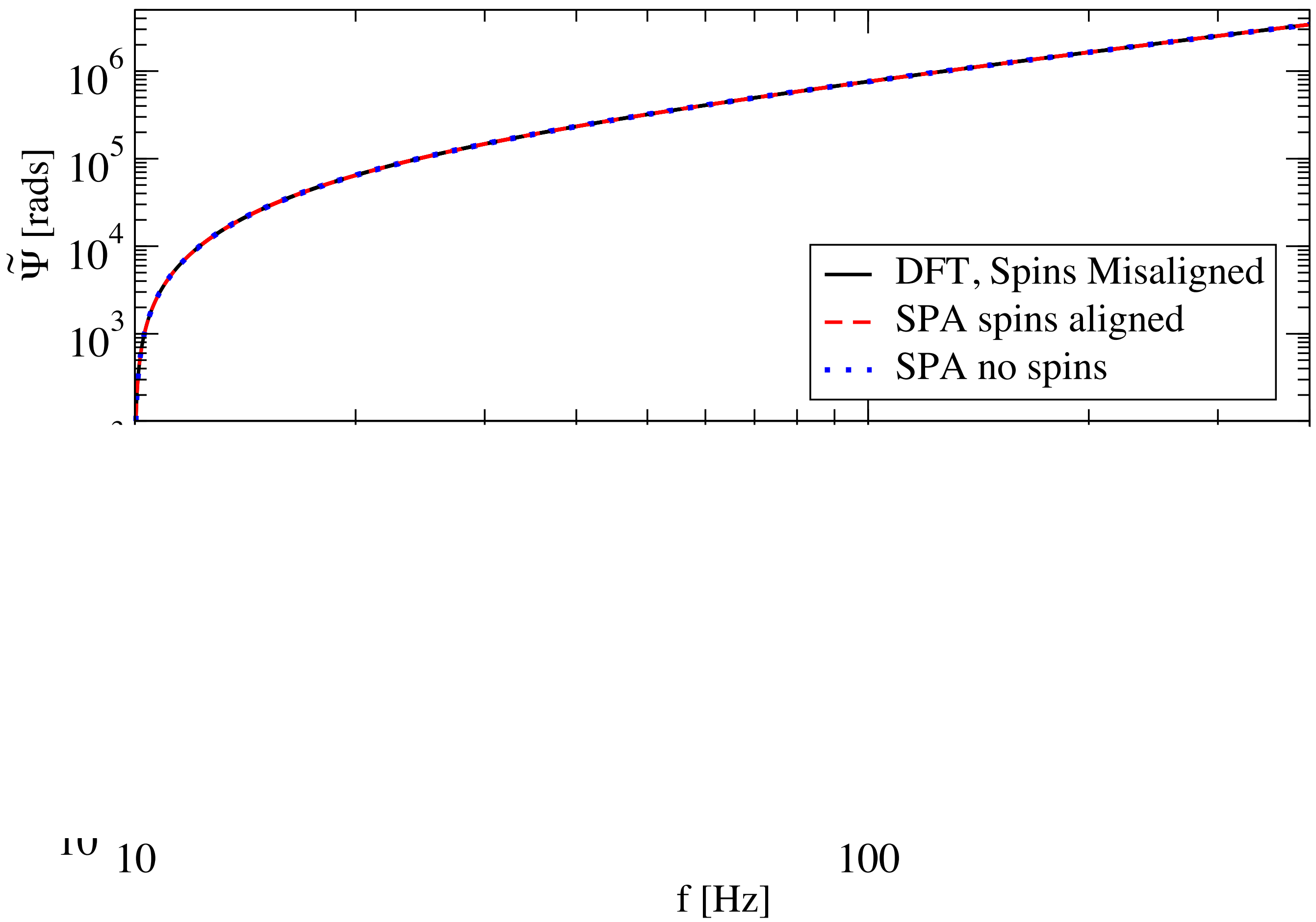
Build Different Template Families for Different Systems

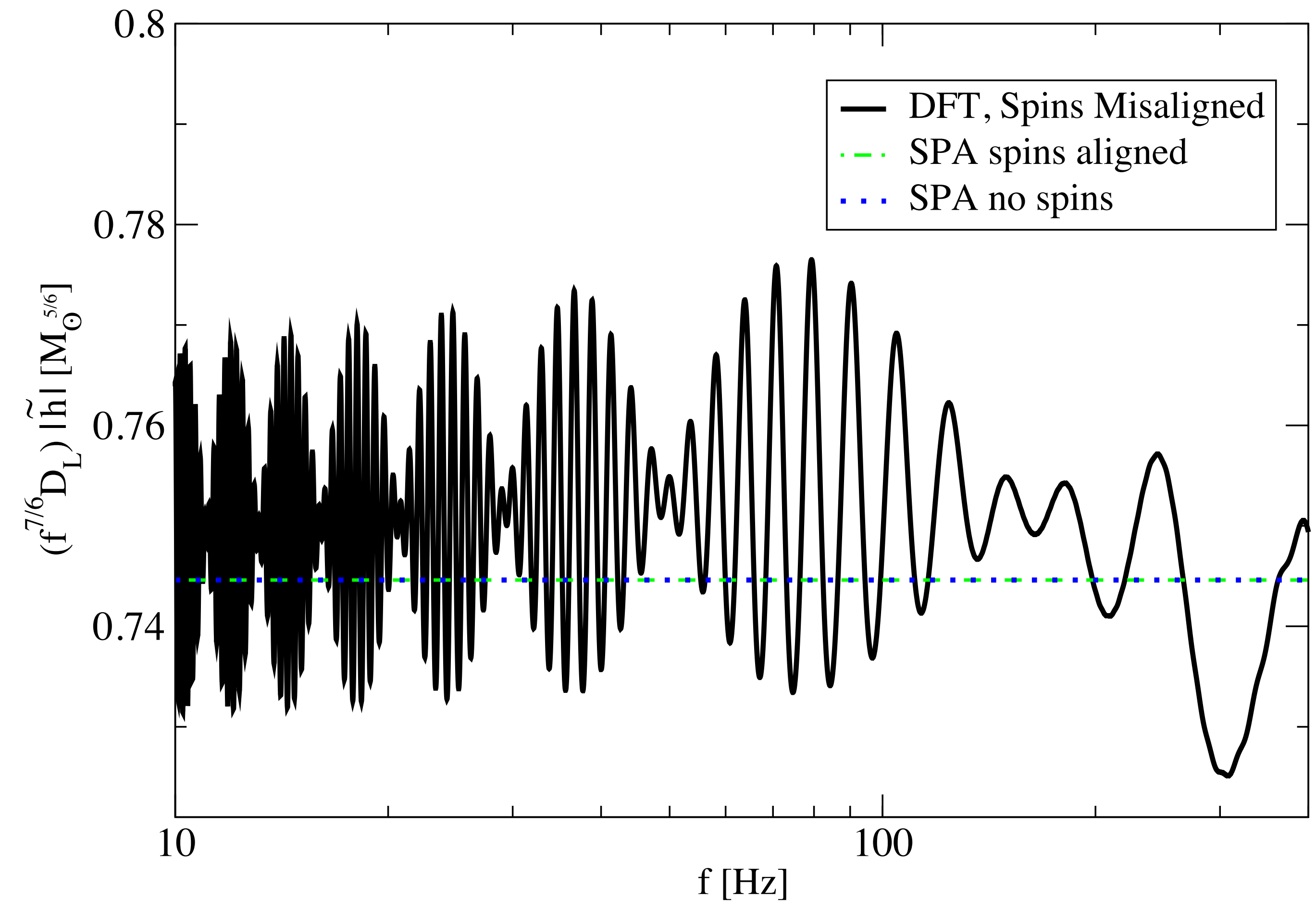
What templates do we have today?



Are these templates enough?

Yes for detection, but no for parameter estimation.



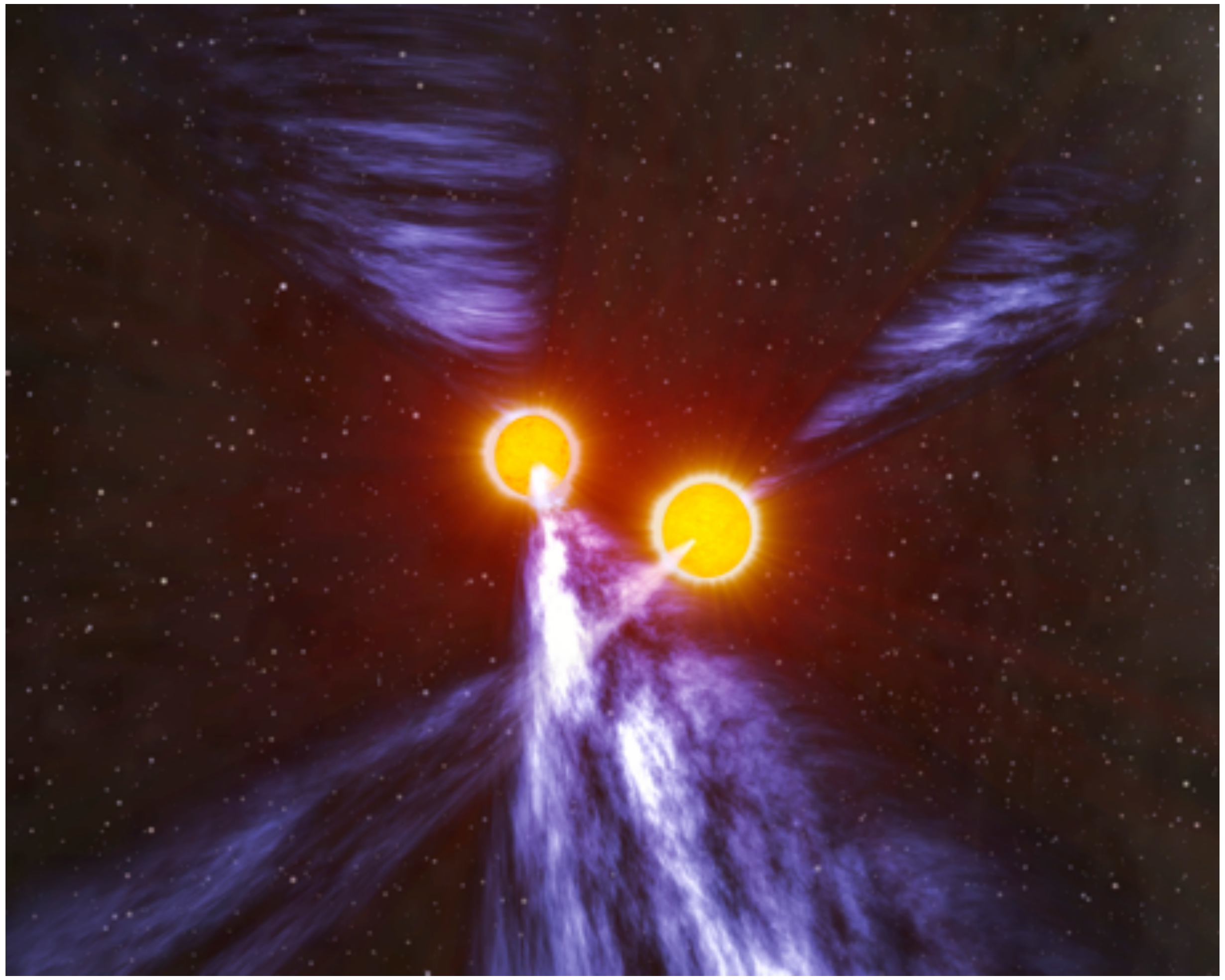


$$(M_1, M_2) = (1.4, 1.6) M_\odot \quad |\vec{S}_1/M_1^2| = 0.1 = |\vec{S}_2/M_2^2| \quad (\epsilon_1, \epsilon_2) = (60^\circ, 45^\circ)$$

Precession modifies the waveform dramatically

$$\tilde{h}(f) = \tilde{\mathcal{A}}(f)\, e^{i\Psi(f)}$$

Physical Scenarios



Quasi-circular inspiral,
with spins misaligned with L

↓

Precession of Orbital Plane

↓

Waveform Modulation

1) Neutron Star binaries in LIGO band will have randomly oriented spins, but small spin magnitude. → $|\vec{S}_A| \ll \vec{L}$

2) Black hole binaries in gas-rich galaxies (or due to PN evolution) will have random spin magnitudes, but spins nearly aligned with L. → $\epsilon_A \equiv \arccos\left(\hat{S}_A \cdot \hat{L}\right) \ll 1$

Road Map

II. Analytic Construction of Precessing Waveforms

Time Domain

Constructing Spin-Precessing Waveforms

$$h_+ \sim \frac{\eta M}{D_L}(M\omega)^{2/3}\left(1+\cos^2\iota\right)\cos 2\Phi^{\rm orb}$$

But the inclination angle depends on time, as the orbital plane precesses about the total angular momentum.

↓

You must solve the Spin-Precession Equations

$$\dot{\vec{L}} = \omega^2 C_1\left(\vec{S}_1\times\vec{L}\right) + \omega^2 C_2\left(\vec{S}_1\cdot\hat{L}\right)\left(\vec{S}_2\times\hat{L}\right) - k\vec{L} + 1 \leftrightarrow 2$$

$$\dot{\vec{S}}_1 = \omega^2 C_1\left(\vec{L}\times\vec{S}_1\right) + \omega^2 C_2\left(\vec{S}_2\cdot\hat{L}\right)\left(\hat{L}\times\vec{S}_1\right) + \omega^2 C_3\left(\vec{S}_2\times\vec{S}_1\right)$$

Multiple Scale Analysis: Simple Example

$$\ddot{y} + y + \epsilon y^3 = 0 \qquad [y(0) = 0,\ \dot{y}(0) = 1]$$

- Try Perturbation Theory: $$y(t) = \sum_{n=0}^{\infty} \epsilon^n \, y_n(t) \qquad \epsilon \ll 1$$

Solution

$$y(t) = \cos t + \epsilon \left[\frac{1}{32} \cos 3t - \frac{1}{32} \cos t - \frac{3}{8} t \sin t \right] + \mathcal{O}(\epsilon^2)$$

Diverges for large t

Perturbation Theory breaks down at finite time t ~ 8/(3e)

Multiple Scale Analysis: Simple Example (Cont'd)

- Try Multiple Scale Analysis:

Induces an unphysical resonance !!

1. Introduce new independent variable that varies on different timescale

$$y(t) = \sum_{n=0}^{\infty} \epsilon^n Y_n(t,\tau)\,, \qquad \tau = \mathcal{O}(\epsilon t)$$

2. Obtain PDEs

$$n=0: \quad \frac{\partial^2 Y_0}{\partial t^2} + Y_0 = 0\,,$$

$$n=1: \quad \frac{\partial^2 Y_1}{\partial t^2} + Y_1 = -Y_0^3 - 2\frac{\partial^2 Y_0}{\partial t \partial \tau}\,,$$

3. Solve PDEs

$$Y_0 = A(\tau) e^{it} + \text{c.c.}$$

$$\frac{\partial^2 Y_1}{\partial t^2} + Y_1 = e^{it}\left[3A^2A^* - 2i\frac{\partial A}{\partial \tau}\right] - e^{3it}A^3 + \text{c.c.}$$

4. Remove unphysical resonances

$$3A^2A^* - 2i\frac{\partial A}{\partial \tau} = 0$$

$$A(\tau) = A_0 e^{iA_1 + 3iA_0^2\tau/2}$$

5. Resum the full solution

$$y(t) = \cos\left[t\left(1 + \frac{3}{8}\epsilon t\right)\right] + \mathcal{O}(\epsilon)$$

Multiple Scale Analysis produces an accurate solution for all times without unphysical divergences

Solving Precession via Multiple Scale Analysis

$$\dot{\vec{L}} = \omega^2 C_1 \left(\vec{S}_1 \times \vec{L}\right) + \omega^2 C_2 \left(\vec{S}_1 \cdot \hat{L}\right)\left(\vec{S}_2 \times \hat{L}\right) - k\vec{L} + 1 \leftrightarrow 2$$

$$\dot{\vec{S}}_1 = \omega^2 C_1 \left(\vec{L} \times \vec{S}_1\right) + \omega^2 C_2 \left(\vec{S}_2 \cdot \hat{L}\right)\left(\hat{L} \times \vec{S}_1\right) + \omega^2 C_3 \left(\vec{S}_2 \times \vec{S}_1\right)$$

For small spins or small misalignment angle, the scales separate, so expand in their ratio

$$t_{\rm rad.reac} \gg t_{\rm prec} \gg t_{\rm orb}$$

Promote all momenta to functions of 2 independent variables

$$\text{time}: \quad t$$
$$\text{new time}: \quad \tau = \mathcal{O}(t_{\rm prec}/t_{\rm rad.reac})$$

Precession equations become PDEs and we demand no resonances are present.

$$\tau = f(t)$$
$$L(t,\tau), S_1(t,\tau), S_2(t,\tau)$$

Eg. Small spins (BNS)

$$L_x(t,\tau) \sim L_{c,1}\cos\phi_1 + L_{s,1}\sin\phi_1 + 1 \rightarrow 2$$

$$L_z = \frac{M^2\eta}{\xi} \qquad \phi_{1,2} = C_{1,2}\sum_{n=-3}\phi_n \xi^n \qquad \xi = (M\omega)^{1/3}$$

Road Map

II. Analytic Construction of Precessing Waveforms

Frequency Domain

Stationary Phase Approximation: Simple Example

$$\tilde{h}(f) = \int h(t) e^{2\pi i f t} dt = \int A(t) e^{i[2\pi f t - \Phi(t)]} dt$$

$\phi(t)$

For any given frequency, the integral is dominated by the regime where the phase is varying slowly

1. Expand the phase about the stationary point $\dot{\phi}(t_{\rm SP}) = 0$

$$\phi(t) = \phi(t_{\rm SP}) + \frac{1}{2}\ddot{\phi}(t_{\rm SP})(t - t_{\rm SP})^2$$

2. The Fourier Integral becomes a Gaussian Integral so solve it.

$$\tilde{h}(f) = \left[\frac{2}{|\ddot{\Phi}(t_{\rm SP})|}\right]^{1/2} A(t_{\rm SP}) \Gamma(1/2) e^{2\pi i f t_{\rm SP} - \Phi(t_{\rm SP}) - \pi/4}$$

Fourier Transform via the Stationary Phase Approx.

$$h_+ \sim \frac{\eta M}{D_L} (M\omega)^{2/3} \left(1 + \cos^2 \iota\right) \cos 2\Phi^{\rm orb}$$

Rewrite response function as slowly-varying amplitude times rapidly-varying phase

$$h(t) \sim \frac{\eta M}{D_L} (M\omega)^{2/3} \sum_{n,k,m} \mathcal{A}_{n,k,m} \, e^{i\Psi^{\rm GW}_{n,k,m}(t)}$$

Precession induces rapidly-varying amplitude terms that should be promoted to the phase.

$$\Psi_{n,k,m}(t) = n\Phi^{\rm orb}(t) + n\delta\phi(t) + k\iota(t) + m\psi(t)$$

Expand generalized Fourier integral about stationary point (where integral accumulates most)

$$\tilde{h}(f) \sim \sum_{n,k,m} \frac{1}{\sqrt{\ddot{\Psi}^{\rm GW}_{n,k,m}[t(f)]}} \, e^{i\left\{2\pi f t(f) - \Psi^{\rm GW}_{n,k,m}[t(f)]\right\}}$$

Fourier Transform via Uniform Asymptotics

$$\tilde{h}(f) \sim \sum_{n,k,m} \frac{1}{\sqrt{\ddot{\Psi}^{\rm GW}_{n,k,m}[t(f)]}} \sim \sum_{n\geq 0} \sum_{k\in\mathbb{Z}} \sum_{m=\pm 2} \frac{1}{\sqrt{n\ddot{\Phi}^{\rm orb} + n\delta\ddot{\phi} + k\ddot{i} + m\ddot{\psi}}}$$

For large enough spin, large precession could force the denominator to vanish and the stationary phase approx. to break down.

1) Rewrite the time-domain waveform

$$h(t) \sim \frac{\eta M}{D_L} (M\omega)^{2/3} \sum_{n,k,m} \mathcal{A}_{n,k,m} e^{-i[\Phi_C(t) + \alpha(t)\cos\beta(t)]}$$

2) Bessel functions

$$h(t) \sim \frac{\eta M}{D_L} (M\omega)^{2/3} \sum_{n,k,m} \mathcal{A}_{n,k,m} e^{-i\Phi_C(t)} \sum_{j=-\infty}^{\infty} (-i)^j J_j[\alpha(t)] e^{-ij\beta(t)}$$

3) SPA

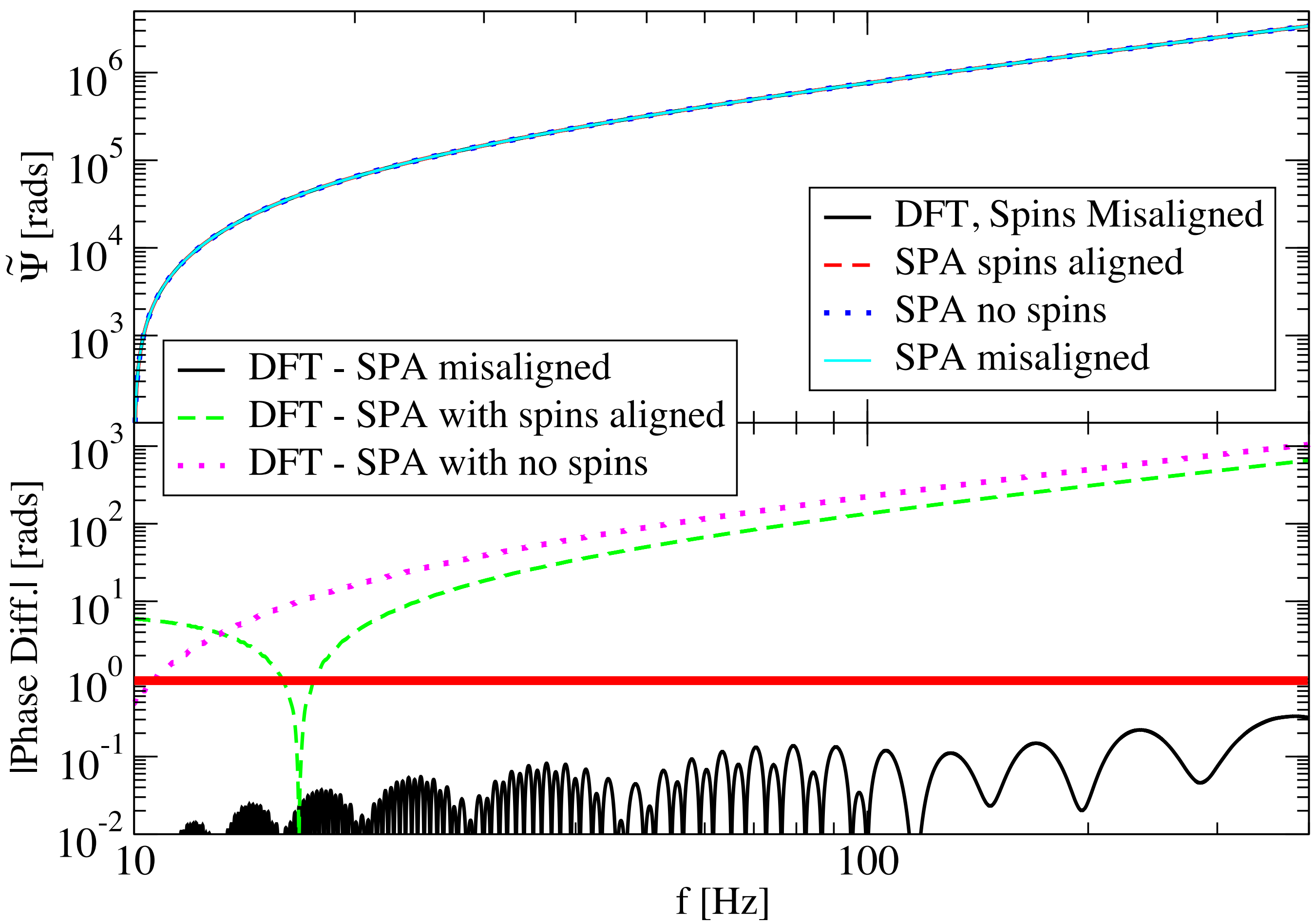
$$\tilde{h}(f) \sim \sum_{n,k,m,j} \frac{1}{\sqrt{\ddot{\Phi}^C + j\ddot{\beta}}} e^{i\left\{2\pi f t(f) - \Phi^C[t(f)] - j\beta[t(f)]\right\}}$$

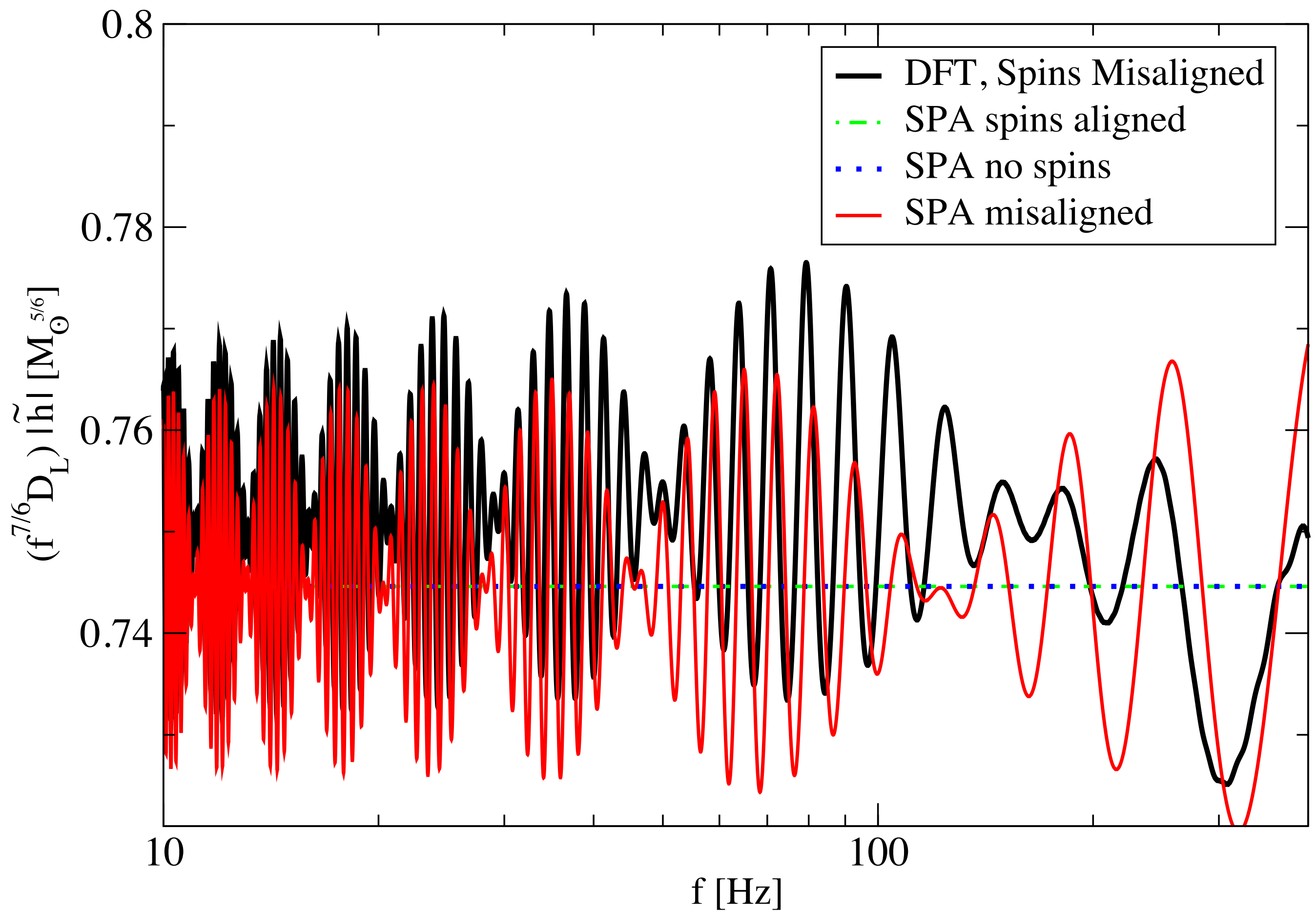
Does not blow up!!

Road Map

III. Performance of Analytic Waveforms

Results: Fourier Amplitude and Phase





$$(M_1, M_2) = (1.4, 1.6) M_\odot \quad |\vec{S}_1/M_1^2| = 0.1 = |\vec{S}_2/M_2^2| \quad (\epsilon_1, \epsilon_2) = (60^\circ, 45^\circ)$$

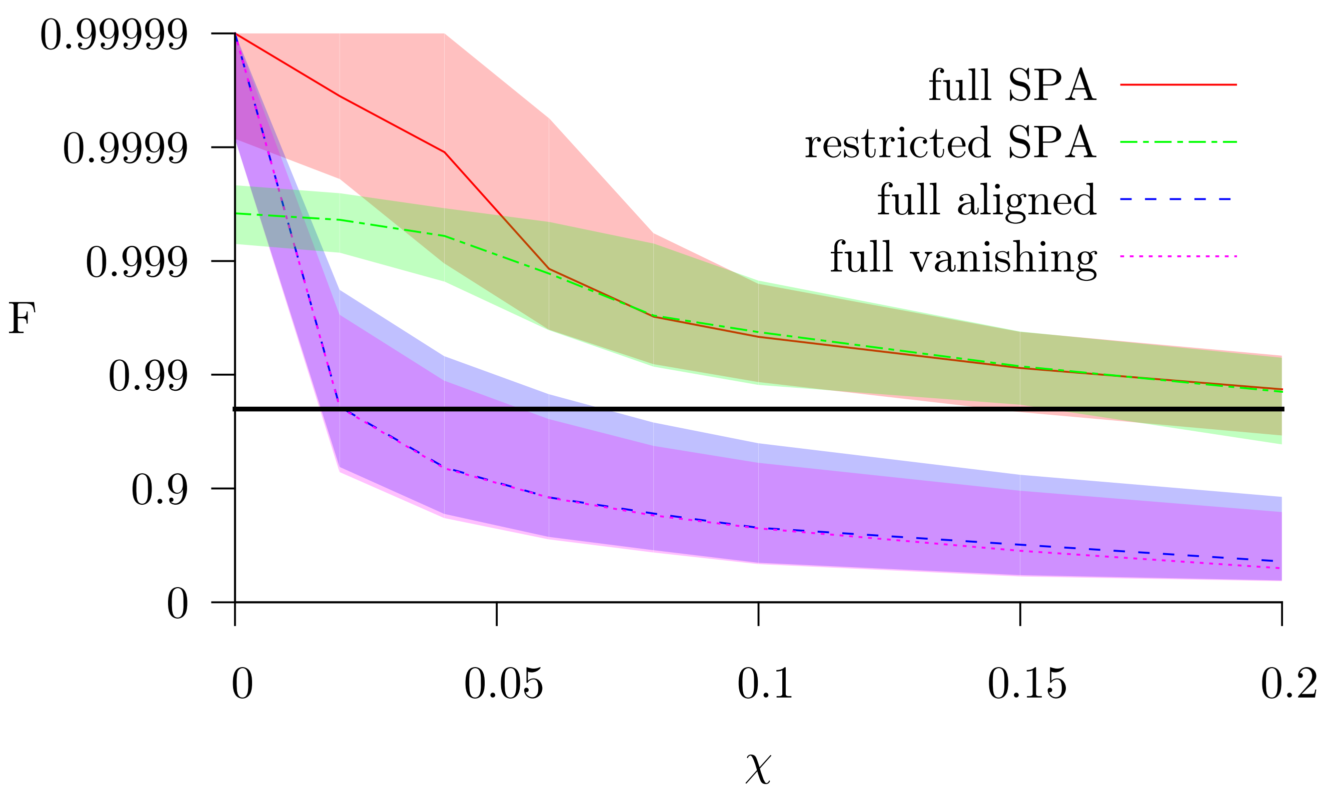
Phase Error is Tiny

SPA Amplitude reproduces DFT

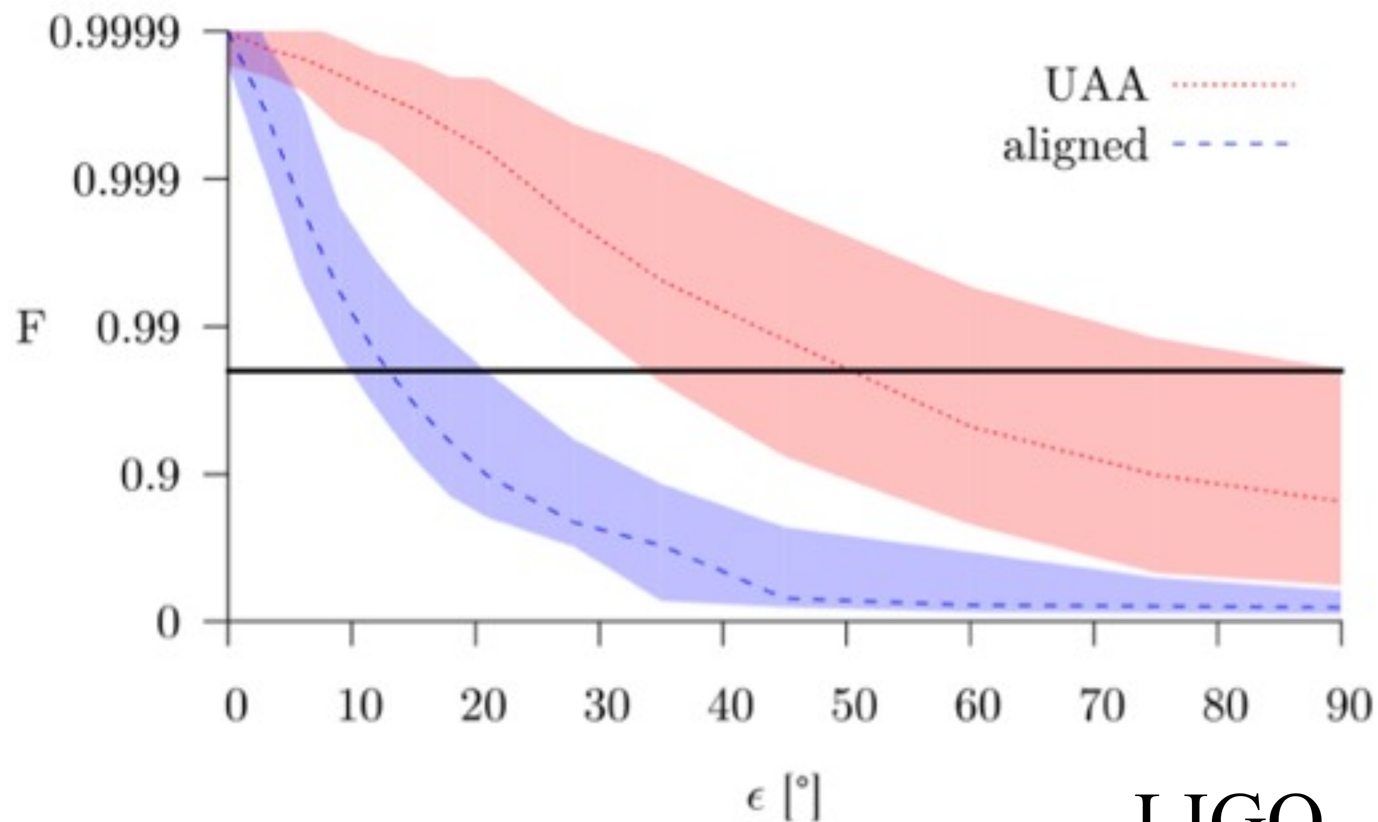
(and we have not adjusted the physical parameters)

Results: Faithfulness

NS/NS



BH/BH



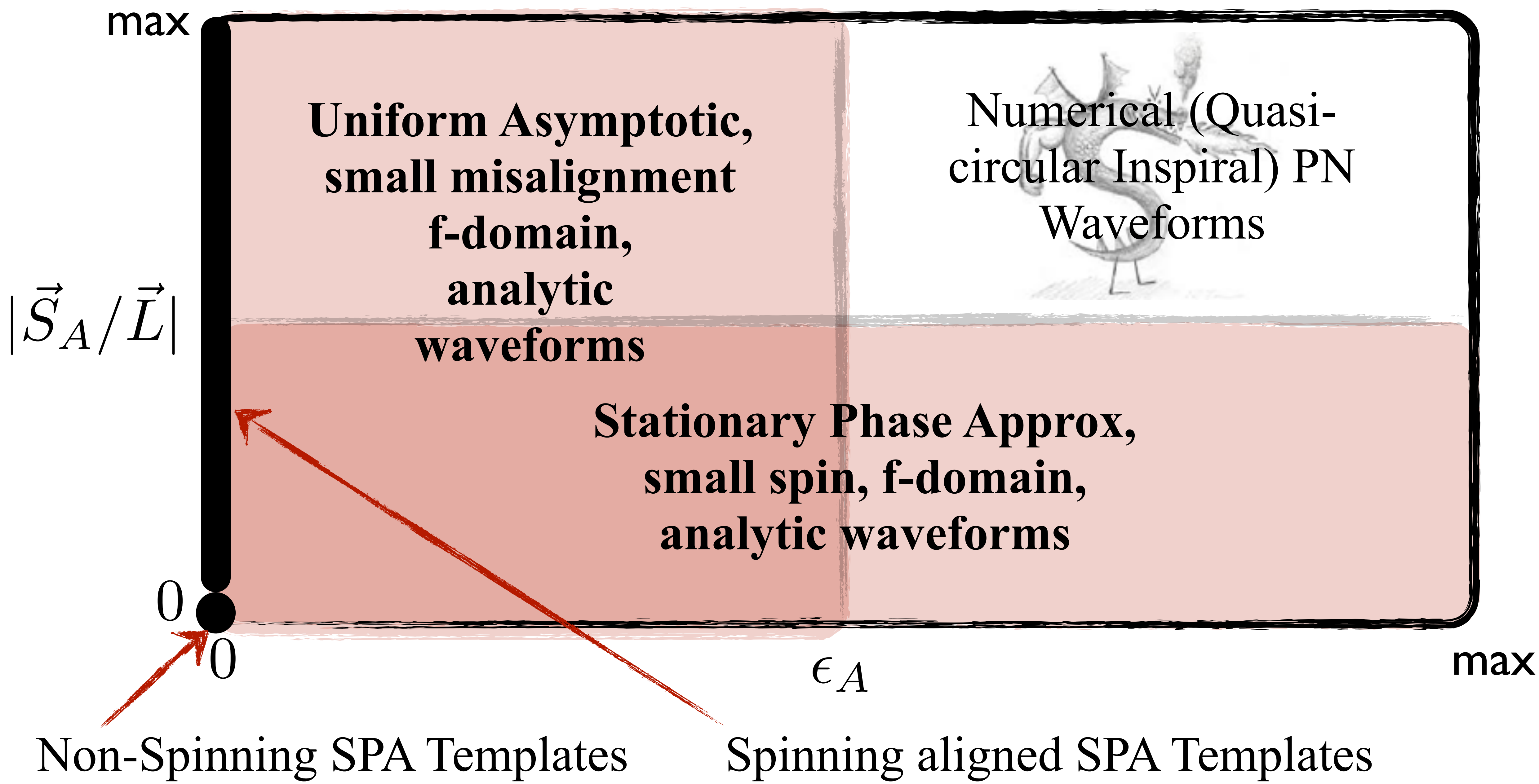
LIGO

Overlap between DFT and spin-aligned SPA or uniform asymptotic waveform, for 1000 systems with random parameters (mass, mass ratio, spins, sky position, etc.) **without** adjusting the parameters to maximize the overlap (faithfulness).

New Analytic f-domain waveforms for spin-precessing binary inspirals are sufficiently accurate for detection and parameter estimation.

Conclusions

What templates do we have now?

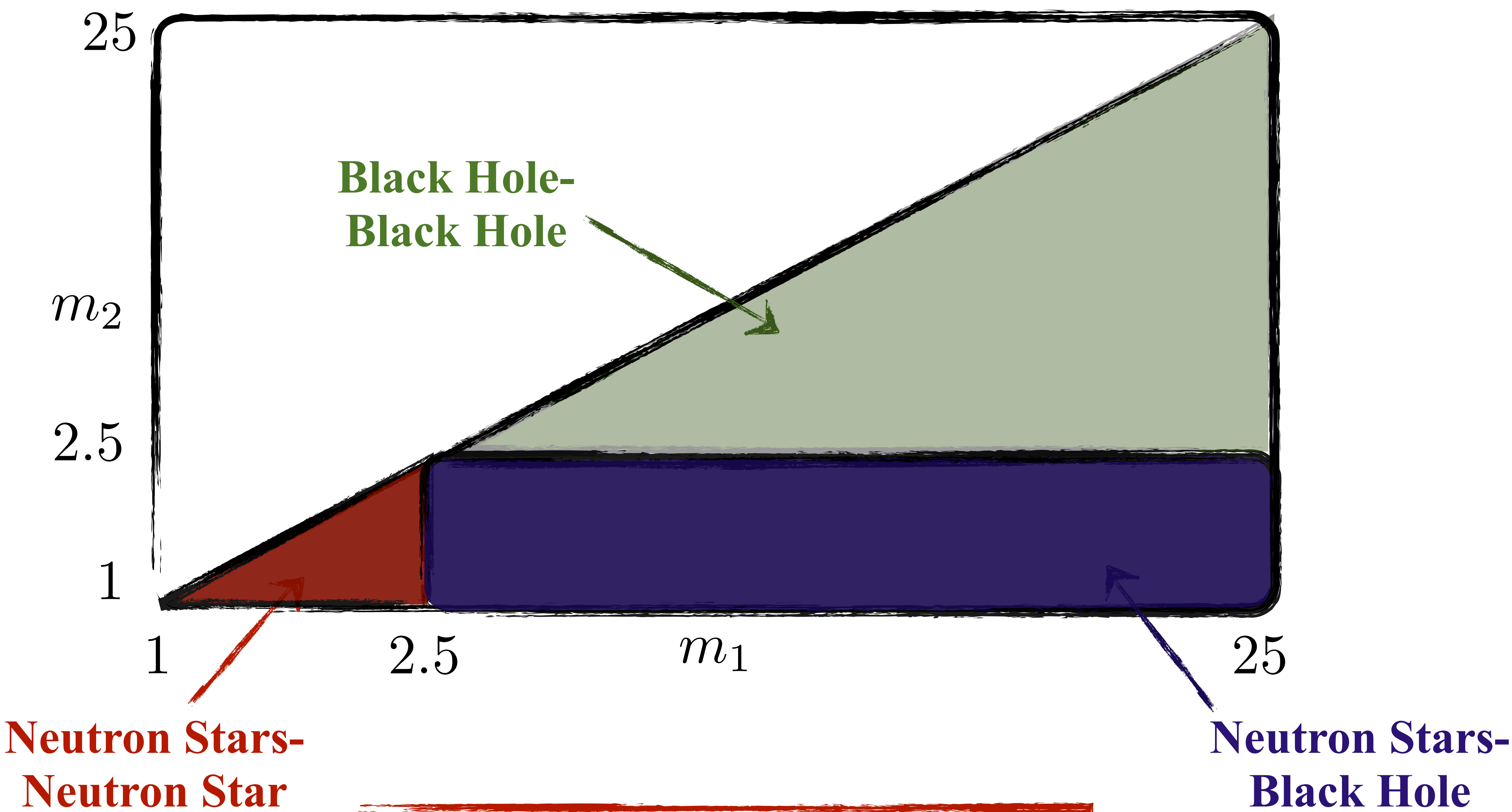


New analytic waveforms push the dragons to a corner of parameter space

Order Counting

System/ Domain	Time Domain	Frequency Domain
Neutron Star Binary	• Lx, Ly to first-order in spin. • h(t) to all orders in spin. • Lz evolution includes ✤ 3.5PN complete RR, ✤ 8PN pt-ptcle scri+ RR, • Lz expanded to 8PN order.	• Standard SPA • SPA to NLO up to 8PN. • Restricted SPA uses only dominant PN amplitude.
Black Hole Binary	• Lx, Ly to 1st order in angle. • h(t) to all orders in angle. • Lz evolution includes ✤ 3.5PN complete RR, ✤ 8PN pt-ptcle scri+ RR, ✤ 5.5PN pt-ptcle Hor RR, • Lz expanded to 8PN order.	• Uniform Asymptotics. • SPA to NLO up to 6.5PN. • Restricted SPA uses only dominant PN amplitude.

System-Driven Templates



We can cover large sectors of parameter space analytically.