Hawking radiation in acoustic black and white holes

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14 April, 2014

Collaborations with R. Parentani, S. Finazzi, A. Fabbri

Introduction

Hawking Radiation context

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  - Spectrum is **thermal**

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  - Temperature $T_H = \kappa/2\pi$, κ the surface gravity

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- Unruh, Wald (P.R.D. 82)
  Necessary for consistency of **generalized second law**

Transplanckian question

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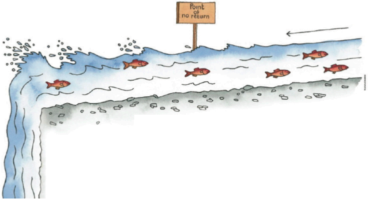
- Since $\sim$ 2010, first experiments (Water, BEC, optical fibers)
  - Water (Weinfurtner, Tedford, Penrice, Unruh, Lawrence, *PRL* 2010)

Plan of the talk

Outline



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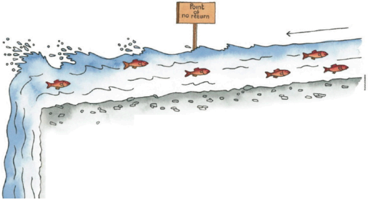
- Sound waves of celerity c_S on moving fluid
- Current velocity $v(x)$



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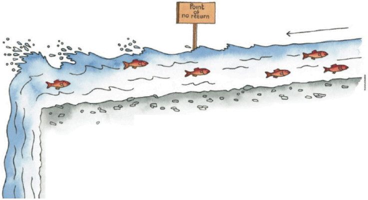

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- Geometry $ds^2 = c_S^2 dt^2 - (dx - v(x)dt)^2$

(Convention: $c_S = 1$)

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  - Co-moving $\Omega = \omega - v(x)k$

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Short distance physics $\longrightarrow$ dispersion (Jacobson, *PRD* 1991)

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group velocity **increase** (+) or **decrease** (-) with k

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   - Einstein-Aether theory
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    - BEC
    - Water surface waves
    - Optical fibers
    - ...

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3. Understand Lorentz invariance better !

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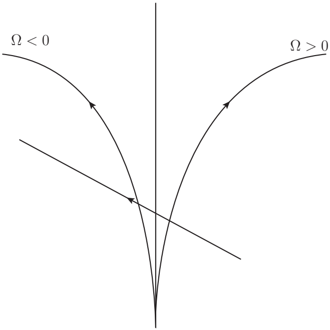
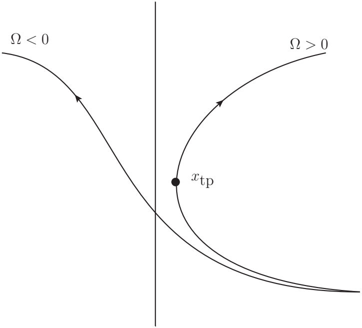
1. Analog gravity
2. Robustness of Black Hole radiation
   - Characteritics
   - Mode mixing
3. Undulations in White Holes
   - Zero-mode
   - Classical scattering
   - Quantum and thermal noise
4. Conclusion



- Infinite focusing on the horizon

$$x = x_0 e^{\kappa t}$$
$$p = p_0 e^{-\kappa t}$$

- v-modes fall in and are regular (Hence ignored)

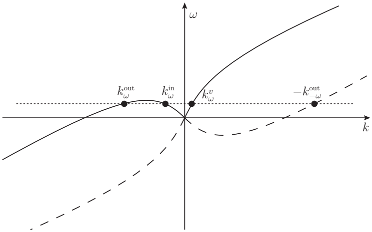


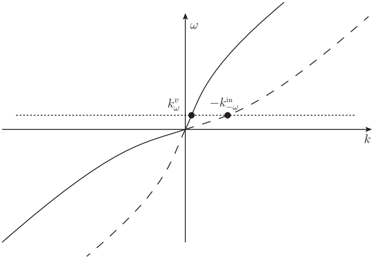
- Finite time redshift $\Delta t = \ln(p_{in}/p_{out})$

$$\begin{aligned} x &= x_0 e^{\kappa t} + \frac{p_0^3}{2\Lambda^2 \kappa} e^{-3\kappa t} \\ p &= p_0 e^{-\kappa t} \end{aligned}$$

Outline

Homogeneous flow → plane wave solutions: $e^{-i(\omega t - k_\omega x)}$





$$\omega = vk_\omega \pm \sqrt{k\Lambda \tanh(k/\Lambda)}$$

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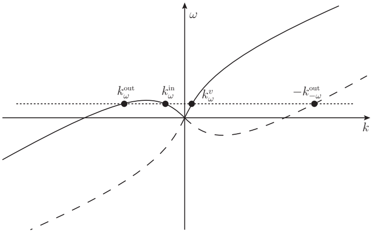

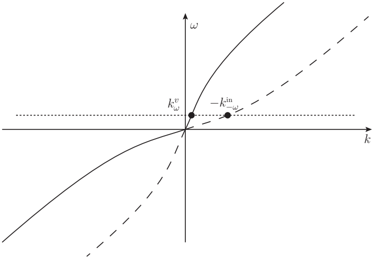

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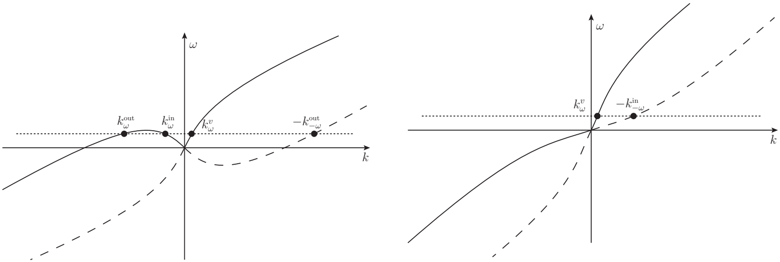

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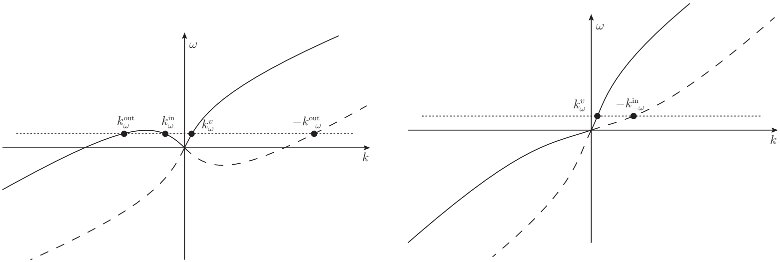

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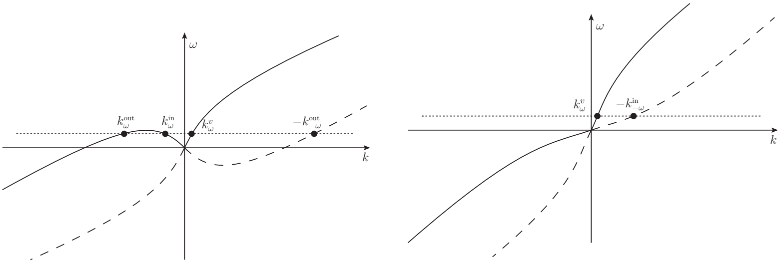

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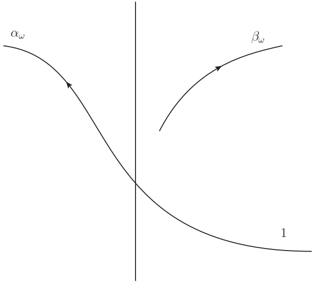
- $k_-^{\rm out}$ is dragged by the flow

Inhomogeneous flow: mode mixing



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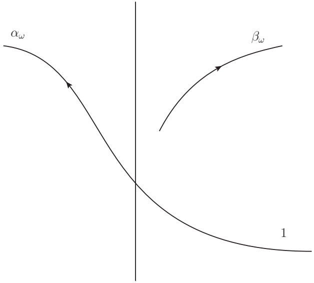

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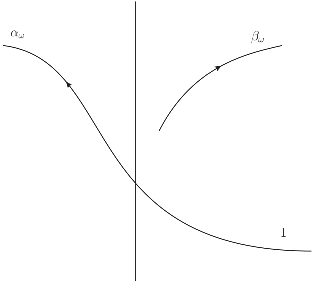

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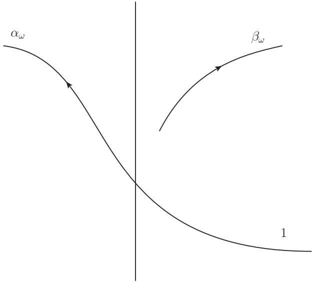

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2nd quantization:

- Mixing of $\hat{a}_\omega$ and $\hat{a}_\omega^\dagger$

$$\langle 0_{in}|\hat{a}_\omega^\dagger \hat{a}_\omega|0_{in}\rangle = |\beta_\omega|^2 \neq 0$$

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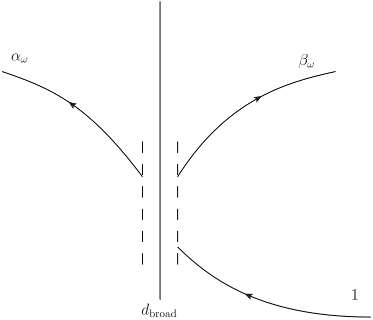
- Spontaneous emission
- **Direct** and **necessary** consequence of the **classical** field equation



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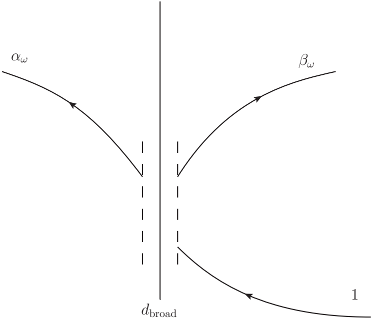

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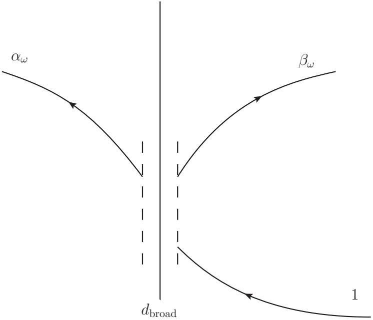
- Near horizon region: $v(x) = -1 + \kappa x$



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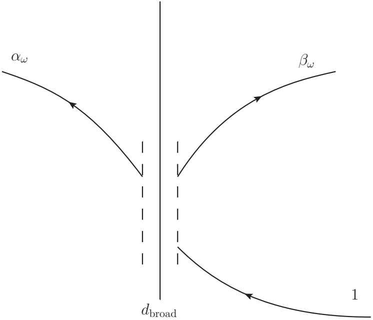

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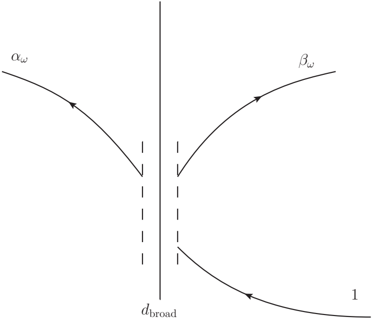

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Mode mixing occurs on a finite region around horizon

$$d_{\rm br} = \kappa^{-1/3}\Lambda^{-2/3}$$

Condition to recover HR

$$d_{\rm br} \ll L_{\rm lin} \tag{3}$$

$$x_{\rm tp}(\omega) \ll L_{\rm lin} \tag{4}$$

(AC, R.Parentani, S.Finazzi PRD 2012)

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Total S-matrix

Outside NHR, UV-modes **decouple**

$$S = S_{\mathrm{int}} \cdot S_{\mathrm{ext}} \cdot S_{\mathrm{NHR}}$$

- S_{NHR}: HR, involve UV modes
- S_{int} and S_{ext}: **relativistic** scattering with k_ω^v (greybody factors)

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Correct interpretation: **Broadened horizon**

(AC, R.Parentani, preprint arXiv:1402.2514)

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   - Characteritics
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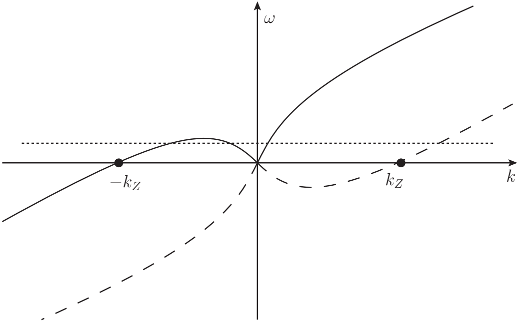

- Ultra-low frequencies: **Zero-mode**

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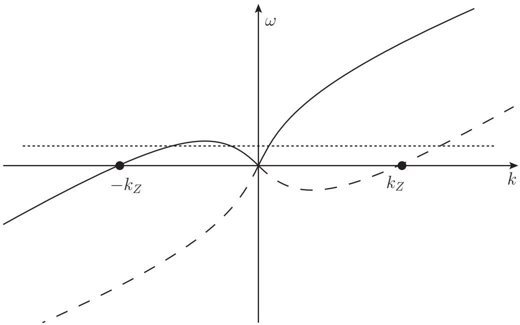

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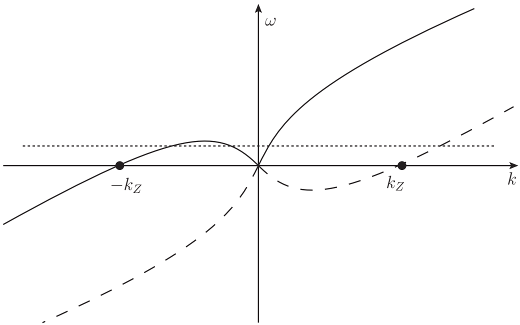
- This modes can be produced at **0-energy cost**



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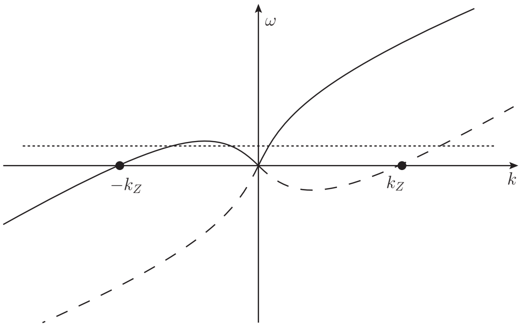

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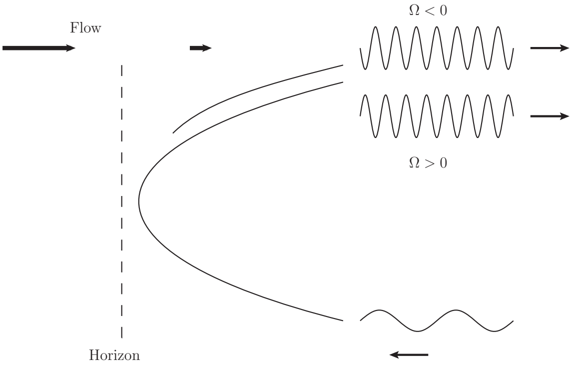
Aparte

- White holes **very** relevant for analog experiment
- White holes (acoustic) stability has been **debated** (Leonhardt, Ohberg, 02 - Macher, Parentani, 09)

Outline



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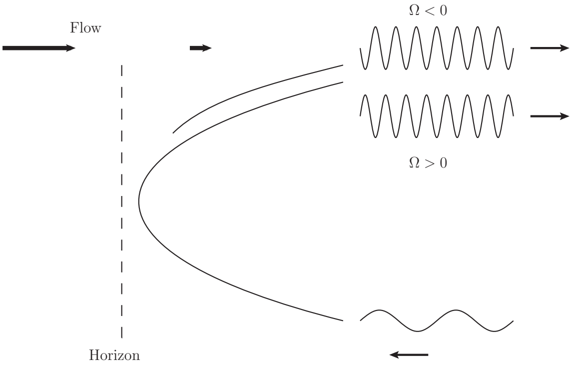

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- Investigate $\omega \longrightarrow 0$ limit
  AC, R. Parentani, P.o.F. 2014

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- **Merging** of 2 out-going waves

$$\alpha_\omega e^{-i(\omega t - k_\omega^{\rm out} x)} + \beta_\omega e^{-i(\omega t + k_{-\omega}^{\rm out} x)} \underset{\omega \to 0}{\sim} 2|\alpha_\omega| \underbrace{\mathrm{Re}\left\{ e^{i(k_Z x + \theta)} \right\}}_{\text{Undulation}} \underbrace{e^{-i\omega(t - x/v_g^Z)}}_{\text{modulation}}$$

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- Incoming noise, easily excite $\omega \to 0$ waves

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- **Thermal** state: **same** phenomenon, with different growing law ($\propto t$)

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- This undulation has been **Observed**
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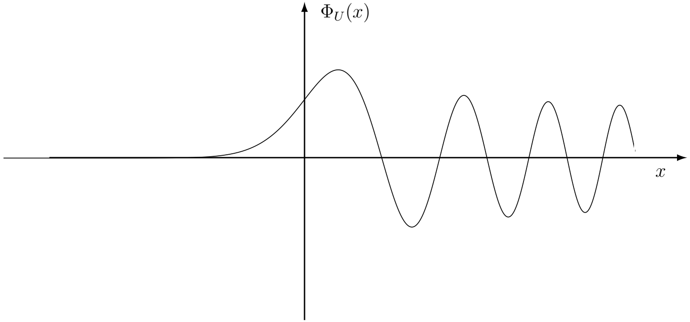

Near horizon region → **analytical control** of the profile (Airy-like)

- Nodes at definite locations
- Wavelength $\sim d_{\rm br}^{-1}$
- Could be confirm/infirm experimentally

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  (A.C., A.Fabbri, R.Parentani, R.Balbinot, P.R.Anderson **PRD** 2012)

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Results

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Acoustic BH evaporation

- Backreaction in BEC $\longrightarrow$ unitarity restoration

Thank you.