A NEW ANALYTICAL APPROACH TO STUDY NEUTRINOS BEYOND THE LINEAR REGIME

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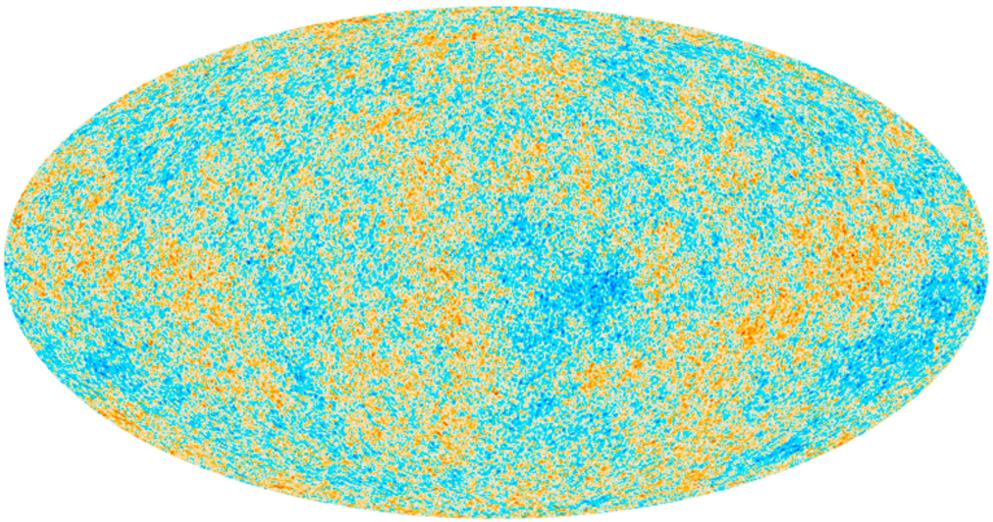
Cosmological Perturbation Theory

- According to the cosmological principle, the universe is spatially homogeneous and isotropic on very large scales (> 100 Mpc).
- But the real universe is not perfect!



Temperature fluctuations of the CMB as seen by PLANCK

- Inhomogeneities are small ($\sim 10^{-5}$).

➡ They can be treated as perturbations.

Why care about neutrinos?

- Neutrinos interact through weak interaction.

→ In the primordial universe, they interact continually.

→ Primordial cosmology is influenced by neutrinos.

- Neutrinos are massive particles.

→ Neutrinos interact through gravity.

→ They affect the formation of the large-scale structure.

- In the linear regime, neutrinos have been proven to slow down the growth of structure.

- What about the nonlinear regime?

The standard description of neutrinos

- Neutrinos are considered as a non-cold fluid.
- The key equation is the Vlasov equation: $\frac{\mathrm{d}f(x,q,\eta)}{\mathrm{d}\eta} = 0.$
- $f = f_0(1+\Psi).$

background — first order perturbation

- The expanded form of the Vlasov equation gives (in the conformal Newtonian gauge)

$$\partial_\eta \Psi + \frac{q^i}{a\epsilon}\partial_i \Psi + \frac{\mathrm{d}\log f_0(q)}{\mathrm{d}\log q}\left(\partial_\eta \phi - \frac{a\epsilon}{q^2} q^i \partial_i \psi\right) = 0.$$

The standard description of neutrinos

- The quantity $\tilde{\Psi} \equiv \left(\frac{\mathrm{d}\log f_0(q)}{\mathrm{d}\log q}\right)^{-1} \Psi$ is decomposed into

Legendre polynomials: $\tilde{\Psi} = \sum_{\ell} (-\mathrm{i})^{\ell} \tilde{\Psi}_{\ell} \, P_{\ell}(\alpha).$

- It leads to the linear Boltzmann hierarchy

$$\partial_\eta \tilde{\Psi}_0(\eta, q) = -\frac{qk}{3a\epsilon} \tilde{\Psi}_1(\eta, q) - \partial_\eta \phi(\eta)$$

$$\partial_\eta \tilde{\Psi}_1(\eta, q) = \frac{qk}{a\epsilon} \left(\tilde{\Psi}_0(\eta, q) - \frac{2}{5} \tilde{\Psi}_2(\eta, q) \right) - \frac{a\epsilon k}{q} \psi(\eta),$$

$$\partial_\eta \tilde{\Psi}_\ell(\eta, q) = \frac{qk}{a\epsilon} \left[\frac{\ell}{2\ell - 1} \tilde{\Psi}_{\ell-1}(\eta, q) - \frac{\ell + 1}{2\ell + 3} \tilde{\Psi}_{\ell+1}(\eta, q) \right] \quad (\ell \geq 2).$$

The standard description of neutrinos

- From the Boltzmann hierarchy, one can compute the multipole energy distribution

$$\rho^{(1)}(\eta) = 4\pi \int q^2 \mathrm{d}q \frac{\epsilon f_0(q)}{a^3} \frac{\mathrm{d}\log f_0(q)}{\mathrm{d}\log q}\, \tilde{\Psi}_0(\eta, q)$$

$$(\rho^{(0)} + P^{(0)})\theta(\eta) = \frac{4\pi}{3} \int q^2 \mathrm{d}q \frac{\epsilon f_0(q)}{a^3} \frac{\mathrm{d}\log f_0(q)}{\mathrm{d}\log q}\, \frac{q}{a\epsilon} \tilde{\Psi}_1(\eta, q)$$

$$(\rho^{(0)} + P^{(0)})\sigma(\eta) = \frac{8\pi}{15} \int q^2 \mathrm{d}q \frac{\epsilon f_0(q)}{a^3} \frac{\mathrm{d}\log f_0(q)}{\mathrm{d}\log q}\, \left(\frac{q}{a\epsilon}\right)^2 \tilde{\Psi}_2(\eta, q).$$

Generalizable to the nonlinear regime?

- The nonlinear moments of the phase-space distribution function,

$$A^{ij\ldots k} \equiv \int \mathrm{d}^3\mathbf{q} \left[\frac{q^i}{a\epsilon} \frac{q^j}{a\epsilon} \cdots \frac{q^k}{a\epsilon} \right] \frac{\epsilon f}{a^3},$$

obey (in the conformal Newtonian gauge) the equation

$$\partial_\eta A^{i_1\ldots i_n} + (\mathcal{H} - \partial_\eta \phi) \left[(n+3) A^{i_1\ldots i_n} - (n-1) A^{i_1\ldots i_n jj} \right]$$
$$+ \sum_{m=1}^{n} (\partial_{i_m} \psi) A^{i_1\ldots i_{m-1} i_{m+1} \ldots i_n} + \sum_{m=1}^{n} (\partial_{i_m} \phi) A^{i_1\ldots i_{m-1} i_{m+1} \ldots i_n jj}$$
$$+ (1 + \phi + \psi) \partial_j A^{i_1\ldots i_n j} + \left[(2-n) \partial_j \psi - (2+n) \partial_j \phi \right] A^{i_1 \ldots i_n j} = 0.$$

See the PhD thesis of Nicolas Van de Rijt: "*Signatures of the primordial universe in large-scale structure surveys*" (2012).

➡ The nonlinear Boltzmann hierarchy is difficult to manipulate.

The nonlinear description of CDM

- The Vlasov equation encodes the conservation of the number of particles:

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t}\frac{\partial f}{\partial \mathbf{x}} + \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t}\frac{\partial f}{\partial \mathbf{p}} = 0.$$

- By definition, $\dfrac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \dfrac{\mathbf{p}}{ma^2}$.
- In the Newtonian approximation, $\dfrac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = -m\nabla_{\mathbf{x}}\Phi,$

where the gravitational potential is given by the Poisson equation

$$\Delta_{\mathbf{x}}\Phi = 4\pi G\rho a^2 = \frac{4\pi G m \int f(\mathbf{x},\mathbf{p},t)\mathrm{d}^3\mathbf{p}}{a}.$$

The nonlinear description of CDM

- From the phase-space distribution function, macroscopic fields can be defined:

$$u_i(\mathbf{x},t) = \frac{1}{\int f(\mathbf{x},\mathbf{p},t)\mathrm{d}^3\mathbf{p}} \int \frac{p_i}{ma} f(\mathbf{x},\mathbf{p},t)\mathrm{d}^3\mathbf{p},$$

macroscopic velocity field = average of the phase-space velocities

$$u_i(\mathbf{x},t)u_j(\mathbf{x},t) + \sigma_{ij}(\mathbf{x},t) = \frac{1}{\int f(\mathbf{x},\mathbf{p},t)\mathrm{d}^3\mathbf{p}} \int \frac{p_i}{ma}\frac{p_j}{ma} f(\mathbf{x},\mathbf{p},t)\mathrm{d}^3\mathbf{p},$$

velocity dispersion

$$\delta(\mathbf{x},t) = \frac{\rho(\mathbf{x},t)}{\bar{\rho}} - 1.$$

density contrast

The nonlinear description of CDM

- The Vlasov-Poisson system leads to the continuity and Euler equations:

$$\frac{\partial \delta(\mathbf{x},t)}{\partial t} + \frac{1}{a}[(1+\delta(\mathbf{x},t))u_i(\mathbf{x},t)]_{,i} = 0,$$

$$\frac{\partial u_i(\mathbf{x},t)}{\partial t} + \frac{\dot{a}}{a}u_i(\mathbf{x},t) + \frac{1}{a}u_j(\mathbf{x},t)u_i(\mathbf{x},t)_{,j} = -\frac{1}{a}\Phi(\mathbf{x},t)_{,i} - \frac{(\rho(\mathbf{x},t)\sigma_{ij}(\mathbf{x},t))_{,j}}{a\rho(\mathbf{x},t)}$$

- Single-flow approximation: $\cancel{\dfrac{(\rho(\mathbf{x},t)\sigma_{ij}(\mathbf{x},t))_{,j}}{a\rho(\mathbf{x},t)}}.$

Illustration of the emergence of shell-crossing

The nonlinear description of CDM

- In the single-flow approximation, the Euler equation reads

$$\underbrace{a\frac{\partial u_i(\mathbf{x},t)}{\partial t} + \dot{a}u_i(\mathbf{x},t) + u_j(\mathbf{x},t)u_i(\mathbf{x},t)_{,j}}_{\dfrac{\mathrm{d}(au_i(\mathbf{x},t))}{\mathrm{d}t}} = -\Phi(\mathbf{x},t)_{,i}.$$

The velocity field is potential.

It is entirely characterized by its divergence

$$\theta(\mathbf{x},t) = 1/(aH)u_i(\mathbf{x},t)_{,i}\,.$$

In Fourier space, the system can be rewritten compactly with the variable $\Psi_a(\mathbf{k},\eta) \equiv (\delta(\mathbf{k},\eta), -\theta(\mathbf{k},\eta))$.

The nonlinear description of CDM

- The resulting equation is

$$\frac{\partial \Psi_a(\mathbf{k},\eta)}{\partial \eta} + \Omega_a{}^b(\eta)\Psi_b(\mathbf{k},\eta) = \gamma_a{}^{bc}(\mathbf{k}_1,\mathbf{k}_2)\Psi_b(\mathbf{k}_1,\eta)\Psi_c(\mathbf{k}_2,\eta),$$

with $\gamma_a{}^{bc}(\mathbf{k}_a,\mathbf{k}_b) = \gamma_a{}^{cb}(\mathbf{k}_b,\mathbf{k}_a)$,

$$\gamma_2{}^{22}(\mathbf{k}_1,\mathbf{k}_2) = \int \mathbf{d}^3\mathbf{k}_1\mathbf{d}^3\mathbf{k}_2\delta_{\mathrm{D}}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\frac{|\mathbf{k}_1+\mathbf{k}_2|^2(\mathbf{k}_1.\mathbf{k}_2)}{2\mathrm{k}_1^2\mathrm{k}_2^2},$$

$$\gamma_2{}^{21}(\mathbf{k}_1,\mathbf{k}_2) = \int \mathbf{d}^3\mathbf{k}_1\mathbf{d}^3\mathbf{k}_2\delta_{\mathrm{D}}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\frac{(\mathbf{k}_1+\mathbf{k}_2).\mathbf{k}_1}{2\mathrm{k}_1^2}$$

and $\gamma = 0$ otherwise.

The nonlinear description of CDM

- "Hard domain": $k_1 \sim k_2$.

- "Soft domain": $k_2 \ll k_1$.

$$\frac{\partial}{\partial z}\Psi_a(z,\mathbf{k}) + \Omega_a{}^b(z,\mathbf{k})\Psi_b(z,\mathbf{k}) - \Xi_a^b(z,\mathbf{k})\Psi_b(z,\mathbf{k})$$
$$= \left[\gamma_a{}^{bc}(\mathbf{k}_1,\mathbf{k}_2)\ \Psi_b(\mathbf{k}_1,z)\ \Psi_c(\mathbf{k}_2,z)\right]_{\mathcal{H}},$$

with $\Xi_a{}^b(\mathbf{k},z) \equiv 2\int_{\mathcal{S}} \mathrm{d}^3\mathbf{q}\ {}^{\mathrm{eik.}}\gamma_{\mathrm{a}}{}^{\mathrm{bc}}(\mathbf{k},\mathbf{q})\Psi_{\mathrm{c}}(\mathbf{q},\mathrm{z})$.

- Eikonal approximation: the hard domain is negligible.

One recovers the linear equation, corrected by a term coming from the soft domain.

The nonlinear description of CDM

- A formal solution of the compact equation of motion exists:

$$\Psi_a(\mathbf{k},z) = g_a{}^b(\mathbf{k},z,z_0)\ \Psi_b(\mathbf{k},z_0) + \int_{z_0}^{z} \mathrm{dz}'\ g_a{}^b(\mathbf{k},z,z')\ \gamma_b{}^{cd}(\mathbf{k}_1,\mathbf{k}_2)\Psi_c(\mathbf{k}_1,z')\Psi_d(\mathbf{k}_2,z')$$

initial time

Green function

All the physics is encoded in the Green operator.

Efforts are focused on the study of the properties of Green functions beyond the linear regime.

A relativistic generalization of the description of CDM

- A multifluid approach allows to get rid of velocity dispersion: in each fluid, the single-flow approximation holds.
- Two macroscopic fields are chosen as variables:

$$n_c(\eta, x^i) = \int \mathrm{d}^3 p_i \, f(\eta, x^i, p_i),$$

$$P_i(\eta, x^i),$$

(which satisfies $P_i(\eta, x^i) n_c(\eta, x^i) = \int \mathrm{d}^3 p_i \, p_i f(\eta, x^i, p_i)$).

A relativistic generalization of the description of CDM

- The Vlasov equation gives the first equation of motion:

$$\frac{\partial}{\partial \eta} n_c + \frac{\partial}{\partial x^i}\left(\frac{P^i}{P^0} n_c\right) = 0,$$

where $P^i = g^{ij} P_j$ and P^0 is defined so that $P^\mu P_\mu = -m^2$.

- The combination of two basic conservation laws (energy-momentum tensor $T^{\mu\nu}$ and four-current J^μ) gives the second equation of motion:

$$P^\nu P_{i,\nu} = \frac{1}{2} P^\sigma P^\nu g_{\sigma\nu,i},$$

since, in a single-flow fluid, $T^{\mu\nu} = -P^\mu J^\nu$.

- Those equations are very general, no perturbative calculations are involved and the metric is not specified.

A relativistic generalization of the description of CDM

- By definition (*see e.g. Ma & Bertschinger 1995*),

$$T_{\mu\nu}(\eta, x^i) = \int \mathrm{d}^3 p_i \, (-g)^{-1/2} \frac{p_\mu p_\nu}{p^0} \, f(\eta, x^i, p_i).$$

- In a single-flow fluid, $f^{\text{one-flow}}(\eta, x^i, p_i) = n_c(\eta, x^i)\delta_{\mathrm{D}}(p_i - P_i(\eta, x^i))$.

➡ With our variables, $T_{\mu\nu}^{\text{one-flow}}(\eta, x^i) = \dfrac{P_\mu(\eta, x^i) P_\nu(\eta, x^i)}{(-g)^{1/2} P^0(\eta, x^i)} n_c(\eta, x^i).$

➡ In our formalism, after recombination, the Einstein equation reads

$$G_{\mu\nu}(\eta, x^i) = 8\pi G \sum_{\text{species, flows}} \frac{P_\mu(\eta, x^i) P_\nu(\eta, x^i)}{(-g)^{1/2} P^0(\eta, x^i)} n_c(\eta, x^i).$$

A relativistic generalization of the description of CDM

- It is useful to work in a generic perturbed Friedmann-Lemaître metric:

$$\mathrm{d}s^2 = a^2(\eta)\left[-(1+2A)\mathrm{d}\eta^2 + 2B_i\mathrm{d}x^i\mathrm{d}\eta + (\delta_{ij}+h_{ij})\mathrm{d}x^i\mathrm{d}x^j\right].$$

- The second equation then reads

$$\frac{\partial P_i}{\partial\eta} + \frac{P^j}{P^0}\frac{\partial P_i}{\partial x^j} = a^2(\eta)\left[-P^0\partial_i A + P^j\partial_i B_j + \frac{1}{2}\frac{P^jP^k}{P^0}\partial_i h_{jk}\right].$$

- Focusing on subhorizon scales allows to simplify the equations while maintaining the relevant coupling terms.

A relativistic generalization of the description of CDM

- The equations of motion corresponding to the subhorizon scales are:

$$\mathcal{D}_\eta n_c + \partial_i (V_i n_c) = 0,$$

$$\mathcal{D}_\eta P_i + V_j \partial_j P_i = \tau_0 \partial_i A + \tau_j \partial_i B_j - \frac{1}{2}\frac{\tau_j \tau_k}{\tau_0}\partial_i h_{jk},$$

initial momentum of the flow

with $\tau_0 = -\sqrt{m^2 a^2 + \tau_i^2}$, $\mathcal{D}_\eta = \frac{\partial}{\partial \eta} - \frac{\tau_i}{\tau_0}\frac{\partial}{\partial x^i}$

and $V_i = -\frac{P_i - \tau_i}{\tau_0} + \frac{\tau_i}{\tau_0}\frac{\tau_j (P_j - \tau_j)}{(\tau_0)^2}.$

peculiar velocity

A relativistic generalization of the description of CDM

- The equations describing cold dark matter satisfy the extended Galilean invariance.

- The corresponding transformation laws are:

$$\tilde{x}^i = x^i + d_i(\eta),$$

$$\tilde{\eta} = \eta,$$

arbitrarily time-dependent uniform field

$$\tilde{\delta} = \delta,$$

$$\tilde{V}^i = V^i + \frac{\mathrm{d}}{\mathrm{d}\eta} d_i(\eta),$$

$$\tilde{A} = A - \mathcal{H}\frac{\mathrm{d}}{\mathrm{d}\eta} d_i(\eta) x^i - \frac{\mathrm{d}^2}{\mathrm{d}\eta^2} d_i(\eta) x^i.$$

A relativistic generalization of the description of CDM

- We generalized the extended Galilean invariance to the relativistic equations.

- For the general equations (NOT restricted to subhorizon scales), the corresponding transformation laws are:

$$\tilde{x}^i = x^i + d_i(\eta) + g_i(\eta),$$

$$\tilde{\eta} = \eta + v_i\, x^i, \text{ with } v_i = \dot{d}_i(\eta),$$

$$\tilde{n}_c = n_c \left(1 + v_i \frac{P^i}{P^0}\right), \quad \tilde{P}_i = -v_i P_0 + P_i,$$

$$\tilde{A} = A - \mathcal{H} v_i x^i - \dot{v}_i x^i, \ \tilde{h}_{ij} = h_{ij} - 2\mathcal{H}\delta_{ij} v_k x^k,$$

$$\tilde{B}_i = B_i - u_i, \text{ with } u_i = \dot{g}_i(\eta).$$

A relativistic generalization of the description of CDM

- On subhorizon scales, the accurate transformation laws are:

$$\tilde{x}^i = x^i + d_i(\eta),$$
$$\tilde{\eta} = \eta,$$
$$\tilde{\delta}_{\tau_i}(\eta, \tilde{x}^i) = \delta_{\tau_i}(\eta, x^i),$$
$$\tilde{P}_i(\eta, \tilde{x}^i) = P_i(\eta, x^i) - \tau_0 \partial_\eta d_i(\eta) - \frac{\tau_0}{\tau_0^2 - \tau_j \tau_j} \tau_i \tau_j \partial_\eta d_j(\eta).$$

- The metric perturbations remain unchanged.

A relativistic generalization of the description of CDM

- On subhorizon scales the curl field, defined as

$$\Omega_i = \epsilon_{ijk}\partial_k P_j \,,$$

Levi-Civita symbol

obeys the equation

$$\mathcal{D}_\eta \Omega_k + V_i \partial_i \Omega_k + \partial_i V_i \Omega_k - \partial_i V_k \Omega_i = 0.$$

➡ The curl field is only sourced by itself.

➡ For adiabatic initial conditions, the comoving momentum field is potential.

➡ It is entirely characterized by its divergence.

➡ It can be treated in a similar manner to the velocity field of cold dark matter.

A relativistic generalization of the description of CDM

- By analogy with cold dark matter, we introduce

$$\theta_{\tau_i}(\eta, x^i) = -\frac{P_{i,i}(\eta, x^i; \tau_i)}{ma\mathcal{H}}, \quad \delta_{\tau_i}(\eta, x^i) = \frac{n_c(\eta, x^i; \tau_i)}{n_c^{(0)}(\tau_i)} - 1.$$

- In Fourier space, it gives

$$\left(a\partial_a - \mathrm{i}\frac{\mu k \tau}{\mathcal{H}\tau_0}\right)\delta_{\tau_i}(\mathbf{k}) + \frac{ma}{\tau_0}\left(1 - \frac{\mu^2\tau^2}{\tau_0^2}\right)\theta_{\tau_i}(\mathbf{k}) = -\frac{ma}{\tau_0}\int \mathrm{d}^3\mathbf{k}_1 \mathrm{d}^3\mathbf{k}_2 \alpha_R(\mathbf{k}_1, \mathbf{k}_2; \tau_i)\delta_{\tau_i}(\mathbf{k}_1)\theta_{\tau_i}(\mathbf{k}_2),$$

$$\left(1 + a\frac{\partial_a \mathcal{H}}{\mathcal{H}} + a\partial_a - \mathrm{i}\frac{\mu k \tau}{\mathcal{H}\tau_0}\right)\theta_{\tau_i}(\mathbf{k}) - \frac{k^2}{ma\mathcal{H}^2}\mathcal{S}_{\tau_i}(\mathbf{k}) = -\frac{ma}{\tau_0}\int \mathrm{d}^3\mathbf{k}_1 \mathrm{d}^3\mathbf{k}_2 \beta_R(\mathbf{k}_1, \mathbf{k}_2; \tau_i)\theta_{\tau_i}(\mathbf{k}_1)\theta_{\tau_i}(\mathbf{k}_2).$$

A relativistic generalization of the description of CDM

- The source term is

$$\mathcal{S}_{\tau_i}(\mathbf{k}) = \tau_0 A(\mathbf{k}) + \vec{\tau}\cdot\vec{B}(\mathbf{k}) - \frac{1}{2}\frac{\tau_i\tau_j}{\tau_0}h_{ij}(\mathbf{k}).$$

- The kernel functions are

$$\alpha_R(\mathbf{k}_1,\mathbf{k}_2;\tau) = \delta_{\rm Dirac}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\frac{(\mathbf{k}_1+\mathbf{k}_2)}{k_2^2}\cdot\left[\mathbf{k}_2 - \vec{\tau}\frac{\mathbf{k}_2\cdot\vec{\tau}}{\tau_0^2}\right],$$

$$\beta_R(\mathbf{k}_1,\mathbf{k}_2;\tau) = \delta_{\rm Dirac}(\mathbf{k}-\mathbf{k}_1-\mathbf{k}_2)\frac{(\mathbf{k}_1+\mathbf{k}_2)^2}{2k_1^2k_2^2}\left[\mathbf{k}_1\cdot\mathbf{k}_2 - \frac{\mathbf{k}_1\cdot\vec{\tau}\mathbf{k}_2\cdot\vec{\tau}}{\tau_0^2}\right].$$

- Considering *N* flows, it is useful to introduce the 2*N*-uplet

$$\Psi_a(\mathbf{k}) = (\delta_{\tau_1}(\mathbf{k}),\theta_{\tau_1}(\mathbf{k}),\ldots,\delta_{\tau_n}(\mathbf{k}),\theta_{\tau_n}(\mathbf{k}))^T.$$

A relativistic generalization of the description of CDM

- The resulting equations is

$$\partial_\eta \Psi_a(\mathbf{k}) + \Omega_a{}^b \Psi_b(\mathbf{k}) = \gamma_a{}^{bc}(\mathbf{k}_1, \mathbf{k}_2)\Psi_b(\mathbf{k}_1)\Psi_c(\mathbf{k}_2).$$

The relativistic equation is formally the same as the equation of motion describing CDM.

It is possible to apply the eikonal approximation to the relativistic system.

- The non-zero elements of the vertex matrix are

$$\gamma_{2p-1}{}^{2p-1\,2p}(\mathbf{k}_1, \mathbf{k}_2) = -\frac{ma}{2\tau_0}\alpha_R(\mathbf{k}_1, \mathbf{k}_2, \tau_p),$$

$$\gamma_{2p}{}^{2p\,2p}(\mathbf{k}_1, \mathbf{k}_2) = -\frac{ma}{\tau_0}\beta_R(\mathbf{k}_1, \mathbf{k}_2, \tau_p).$$

A relativistic generalization of the description of CDM

- The eikonal limit of the vertex matrix is

$$^{\text{eik.}}\gamma_{2p}^{bc}(\mathbf{k},\mathbf{k}_2) = -\delta_{2p}^{b}\delta_{2p}^{c}\ \frac{ma}{2k_2^2(\tau_p)_0}\mathbf{k}\cdot\left(\mathbf{k}_2 - \frac{\mathbf{k}_2\cdot\vec{\tau_p}}{(\tau_p)_0^2}\vec{\tau_p}\right),$$

$$^{\text{eik.}}\gamma_{2p-1}^{bc}(\mathbf{k},\mathbf{k}_2) = -\delta_{2p-1}^{b}\delta_{2p}^{c}\ \frac{ma}{2k_2^2(\tau_p)_0}\mathbf{k}\cdot\left(\mathbf{k}_2 - \frac{\mathbf{k}_2\cdot\vec{\tau_p}}{(\tau_p)_0^2}\vec{\tau_p}\right).$$

→ Coupling terms differ from one flow to another.

- Large-scale modes (i.e. large modes of the soft domain) induce a displacement field:

$$\int_{z_0}^{z}\Xi_a{}^b(z',\mathbf{k})\mathrm{d}z' = \mathrm{i}\,\mathbf{k}.\mathbf{d}_p(z_0,z)\ \delta_a{}^b.$$

A relativistic generalization of the description of CDM

- The displacement field is necessarily

$$\mathbf{d}_p(z,z_0) = \mathrm{i}\int_{z_0}^{z}\mathrm{d}z'\int \mathrm{d}^3\mathbf{q}\,\frac{ma}{q^2(\tau_p)_0}\left(\mathbf{q}-\frac{\mathbf{q}\cdot\vec{\tau_p}}{(\tau_p)_0^2}\vec{\tau_p}\right)\Psi_{2p}(z',\mathbf{q}).$$

- In arXiv: 1005.2416 (D. Tseliakhovich and C. Hirata), it has been shown that, in baryon-CDM mixtures, large relative displacements between species damp the small-scale density fluctuations.

- Do the large-scale modes of the neutrino flows induce the same effect on the large-scale structure of the universe?

A relativistic generalization of the description of CDM

- To investigate the role of massive neutrinos on structure growth, we compute (in the conformal Newtonian gauge)

$$\langle(\mathbf{k}.\mathbf{d}_{\rm cdm})^2\rangle = 4\pi k^2 \int \mathrm{d}q\, P_\psi(q)\, \frac{1}{3}\, |D_{\rm cdm}(\mathbf{q})|^2,$$

$$\langle(\mathbf{k}.(\mathbf{d}_p - \mathbf{d}_{\rm cdm}))^2\rangle = 2\pi k^2 \int \mathrm{d}q\, P_\psi(q) \times$$

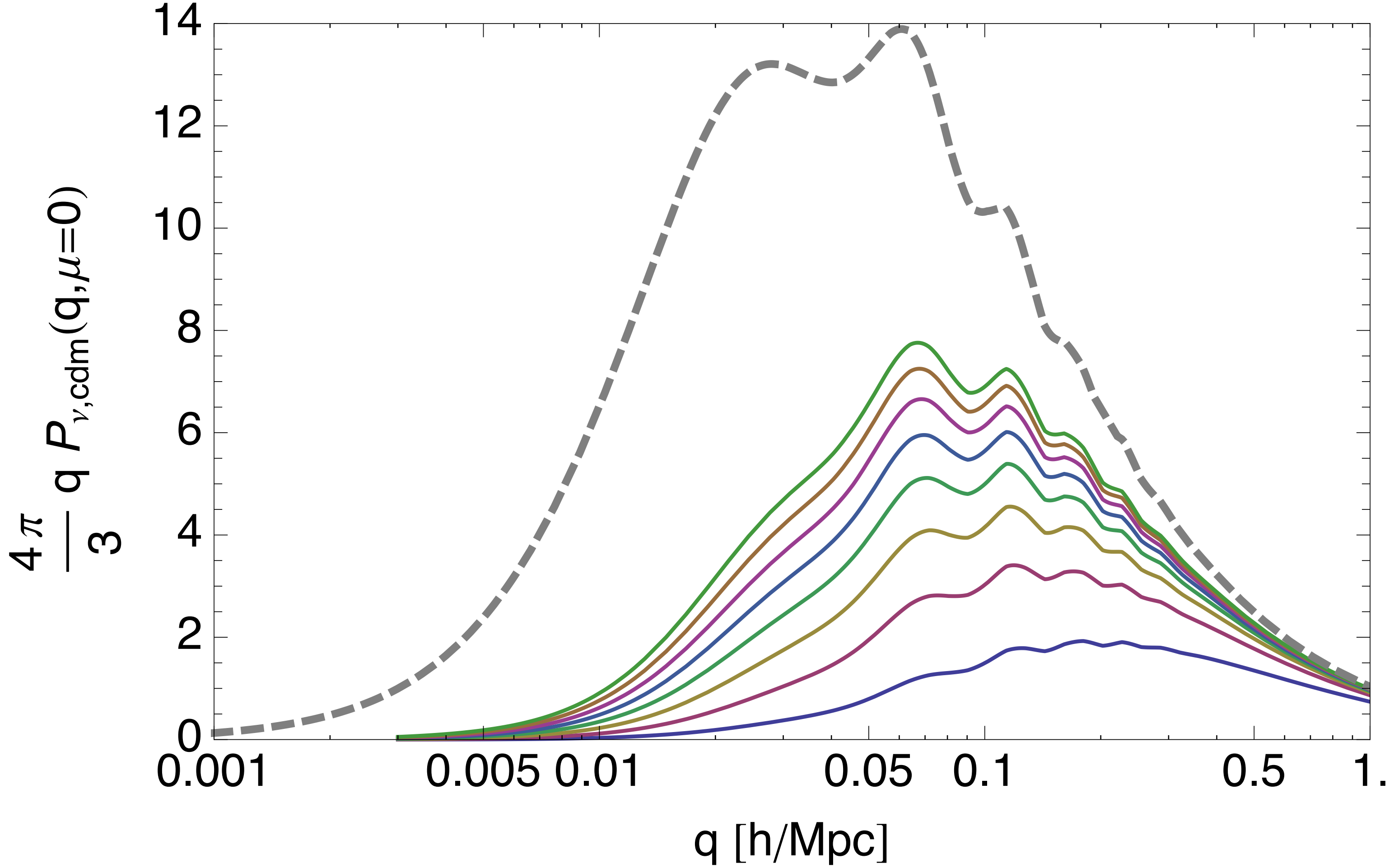
$$\int_{-1}^{1} \mathrm{d}\mu\, \left[\frac{1}{2}(1-\mu_k^2)(1-\mu^2)|D_p^{(0)}(\mathbf{q}) - D_{\rm cdm}(\mathbf{q}) - D_p^{(2)}(\mathbf{q})|^2 + \mu^2\mu_k^2|D_p^{(2)}(\mathbf{q})|^2\right],$$

with
$$\int_{z_0}^{z} \mathrm{d}z'\; \theta_{\rm cdm}(z',\mathbf{q}) = D_{\rm cdm}(z,z_0,\mathbf{q})\psi_{\rm init}(\mathbf{q}),$$

$$\int_{z_0}^{z} \mathrm{d}z' \frac{ma}{-(\tau_p)_0} \left(\frac{\tau_p}{(\tau_p)_0}\right)^{\alpha} \theta_{\tau_p}(z',\mathbf{q}) = D_p^{(\alpha)}(z,z_0,\mathbf{q})\psi_{\rm init}(\mathbf{q}),$$

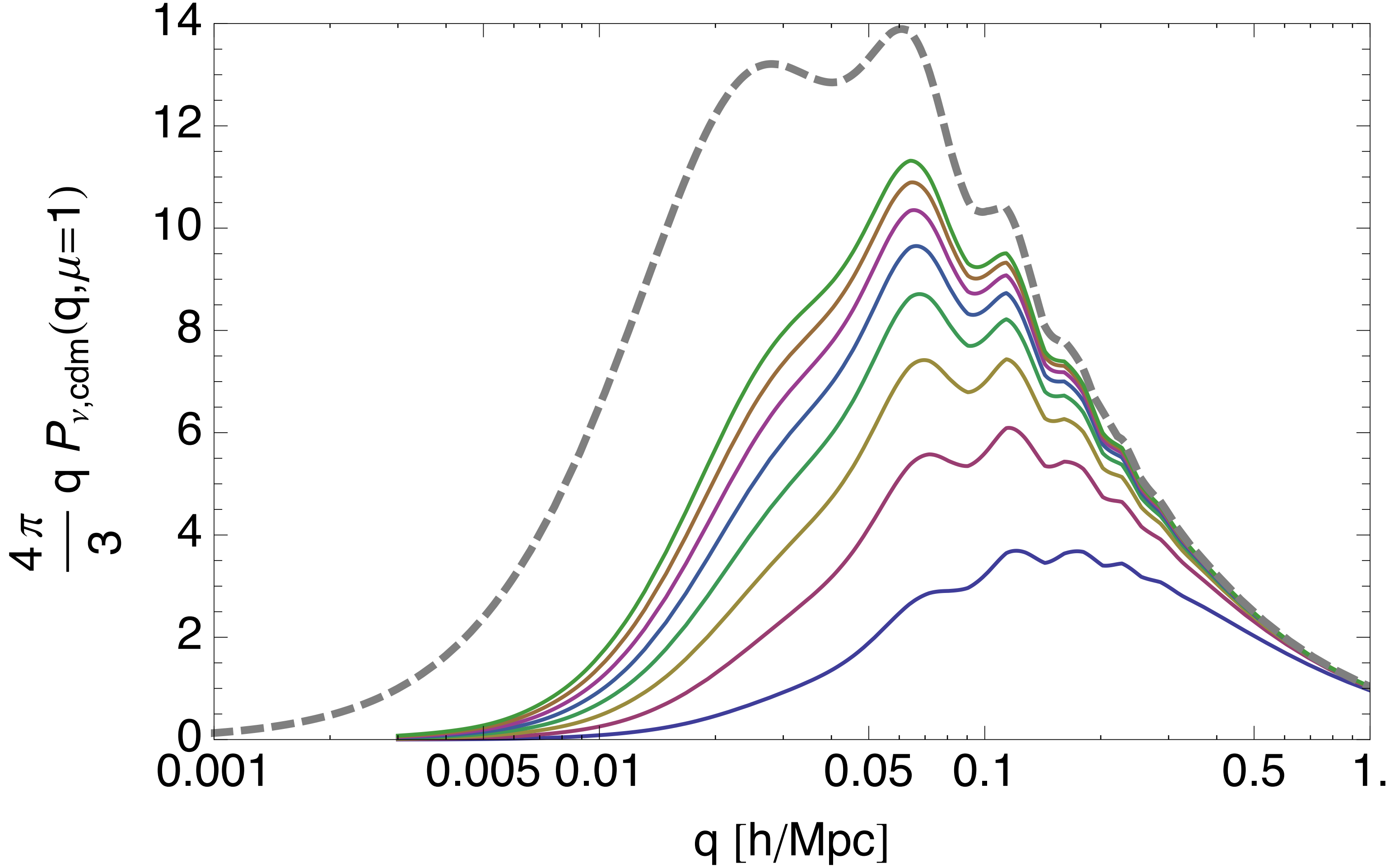
$$\langle\psi_{\rm init}(\mathbf{q})\psi_{\rm init}(\mathbf{q}')\rangle = (2\pi)^3\delta_{\rm Dirac}(\mathbf{q}+\mathbf{q}')\; P_\psi(q).$$

A relativistic generalization of the description of CDM



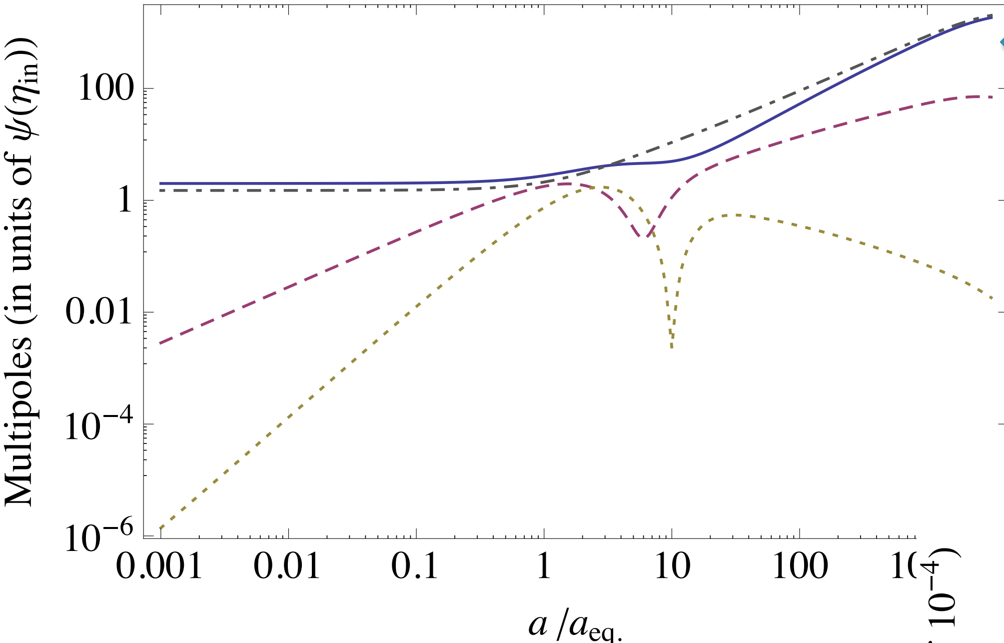
Resulting power spectra for an initial momentum orthogonal to the wave vector

A relativistic generalization of the description of CDM



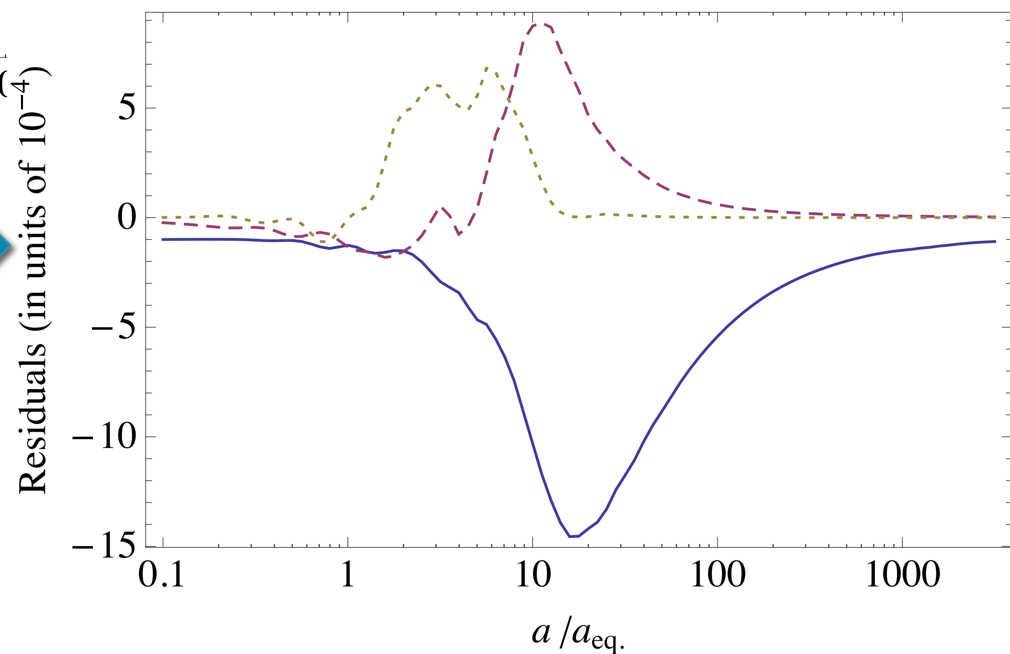
Resulting power spectra for an initial momentum along the wave vector

A relativistic generalization of the description of CDM



Time evolution of the energy multipoles computed using our multifluid description.
Solid line: energy density contrast.
Dashed line: velocity divergence.
Dotted line: shear stress.
Dot-dashed line: energy density contrast of the CDM component.

Comparison with the Boltzmann approach: relative differences between energy mulitpoles when they are computed in both approaches, in units of 10^{-4}.



A relativistic generalization of the description of CDM

- For explicit calculations, the initial conditions must be specified.

- For adiabatic initial conditions, the initial distribution function is

$$f\left(\eta_{\rm in},\mathbf{x},q\right)\propto\frac{1}{1+\exp\left[q/(ak_B(T+\delta T(\eta_{\rm in},\mathbf{x}))\right]}.$$

- With our variables, it reads in the linear regime

scalar modes of the metric perturbations

$$f\left(\eta_{\rm in},\mathbf{x},p_j\right)\propto\left(1+\exp\left[\frac{\tau-\frac{\tau_0^2}{\tau}\frac{\tau^2}{\tau_0{}^2}\left[\frac{1}{2}\mu^2 h+\gamma\left(3\mu^2-1\right)\right]+\frac{\tau_j}{\tau}p_j^{(1)}}{ak_B(T+\delta T(\eta_{\rm in},\mathbf{x}))}\right]\right)^{-1}.$$

Conclusions

- Relativistic species, and in particular massive neutrinos, can be studied with a multifluid approach.
- In the subhorizon limit, the implementation is formally the same as for cold dark matter.
- The use of the eikonal approximation allows to identify the scales at which nonlinear couplings involving neutrinos affect the formation of the large-scale structure.
- Such couplings can be significant for wavenumbers larger than about 0.2 h/Mpc for most of the neutrino streams.