From Black Holes to Cosmology, and back

Alexis HELOU

APC - AstroParticule et Cosmologie, Paris, France

alexis.helou@apc.univ-paris7.fr

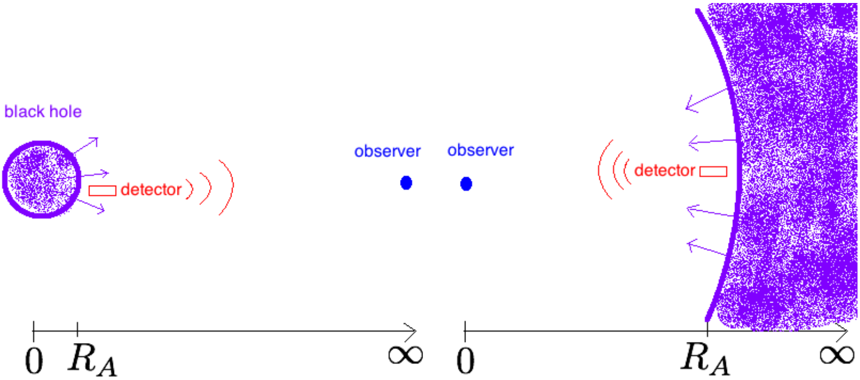
12^{th} January 2015

Reference

Part of this presentation is based on a work by
P. Binétruy & A. Helou:
The Apparent Universe,
arXiv:1406.1658 [gr-qc].

Introduction

- Usual depiction of Black Hole is the Schwarzschild one: static.
- But: black holes accrete matter, and may evaporate through Hawking Radiation $\rightarrow$ highly dynamical objects!
- We present a formalism for dynamical black holes, and show it applies perfectly to our cosmological horizon.
- With this common formalism in hands, we may now try to use our knowledge of the dynamical FLRW Universe to learn more about the dynamics of black holes.


black hole
detector
observer
observer
detector
0
R_A
∞
0
R_A
∞

Overview

Overview

Apparent Horizon versus Event Horizon

- Analogy between de Sitter Universe and Schwarzschild black hole:
  → de Sitter static metric:

$$ds^2 = -\left(1 - \frac{R^2}{R_H^2}\right) dt^2 + \left(1 - \frac{R^2}{R_H^2}\right)^{-1} dR^2 + R^2 d\Omega^2$$

  → Schwarzschild metric:

$$ds^2 = -\left(1 - \frac{R_S}{R}\right) dt^2 + \left(1 - \frac{R_S}{R}\right)^{-1} dR^2 + R^2 d\Omega^2$$

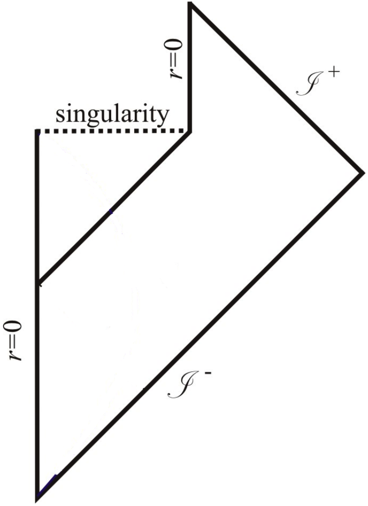
- Traditional definition of a black hole:

$$\mathcal{B} = \mathcal{M} - I^-(\mathcal{I}^+) \ . \tag{1}$$

- Traditional definition of an event horizon:

$$\mathcal{H} = \partial\mathcal{B} \ . \tag{2}$$

Static Black Hole

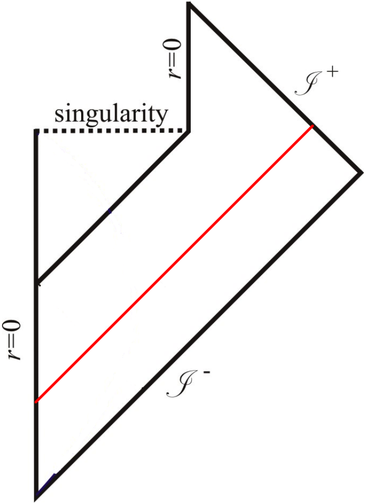


Event Horizon

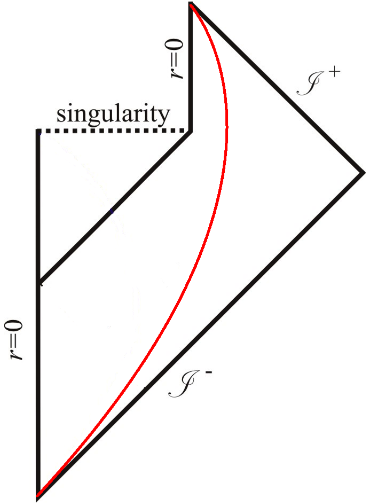
(http://backreaction.blogspot.fr)

Apparent Horizon versus Event Horizon

Safe photon trajectory

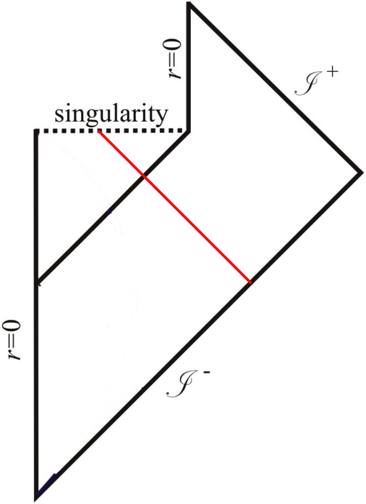


Safe matter trajectory

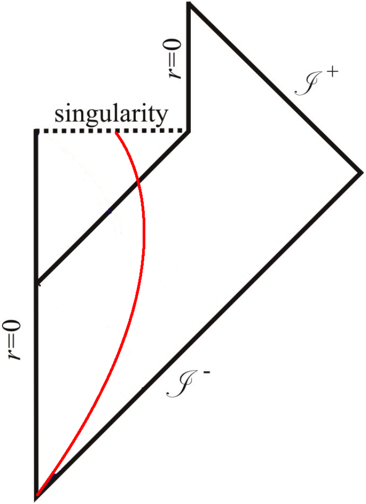


Apparent Horizon versus Event Horizon

Deadly photon trajectory



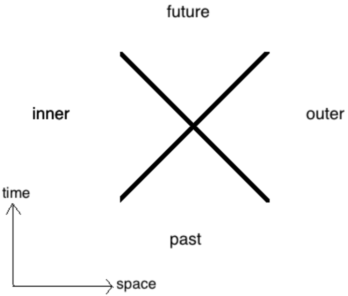
Deadly matter trajectory



Apparent Horizon versus Event Horizon

- At each point of a Penrose diagram, there are 4 null directions: future/past, outer/inner.

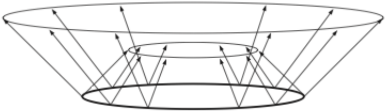
The 4 null directions



- Expansion of bundle of light rays, θ (enters Raychaudhuri equation).

Apparent Horizon versus Event Horizon

Normal Surface



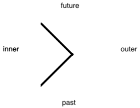
(Hayward, 2005)

Ingoing light-rays converge

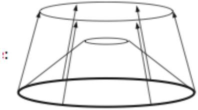
$$\theta_{in} \leq 0 \ .$$

Outgoing light-rays diverge

$$\theta_{out} \geq 0 \ .$$



Trapped Surface



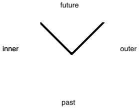
(Hayward, 2005)

Ingoing light-rays converge

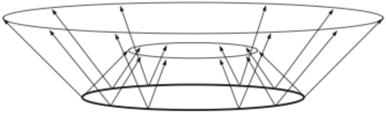
$$\theta_{in} \leq 0 \ .$$

Outgoing light-rays converge

$$\theta_{out} \leq 0 \ .$$



Normal Surface



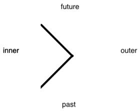
(Hayward, 2005)

Ingoing light-rays converge

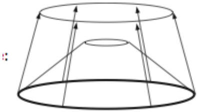
$$\theta_{in} \leq 0 \ .$$

Outgoing light-rays diverge

$$\theta_{out} \geq 0 \ .$$



Trapped Surface



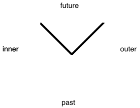
(Hayward, 2005)

Ingoing light-rays converge

$$\theta_{in} \leq 0 \ .$$

Outgoing light-rays converge

$$\theta_{out} \leq 0 \ .$$

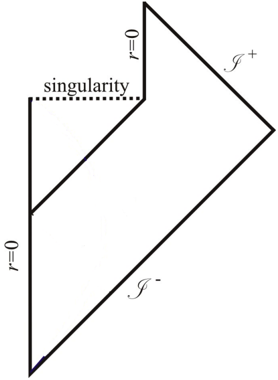


The Apparent Horizon

is the boundary between the two regions ($\theta = 0$).

Apparent Horizon versus Event Horizon

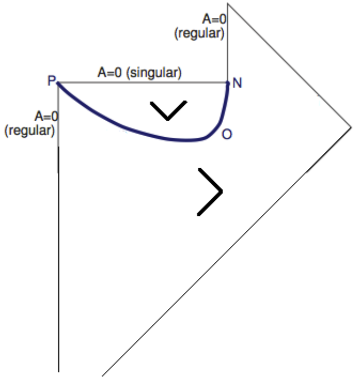
Static Black Hole



Event Horizon

(http://backreaction.blogspot.fr)

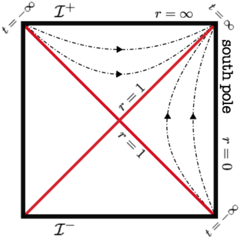
Dynamical Black Hole



Apparent Horizon

(Hayward, 2005)

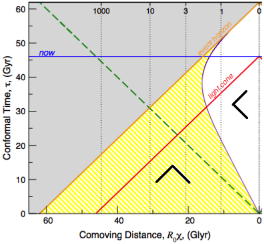
Static de Sitter



Event Horizon

(Anous, Freedman, Maloney, 2014)

Dynamical FLRW



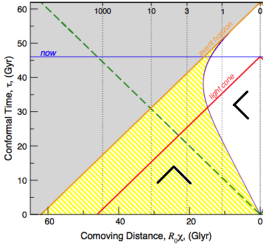
Apparent Horizon

(Davis, Lineweaver, 2003)

Apparent Horizon versus Event Horizon

- Example in the flat case (k=0): the apparent horizon is just the Hubble Sphere ($R_A = H^{-1}$).
- Inside: $v_{rec} \leqslant c$.
- Outside: $v_{rec} \geqslant c$.

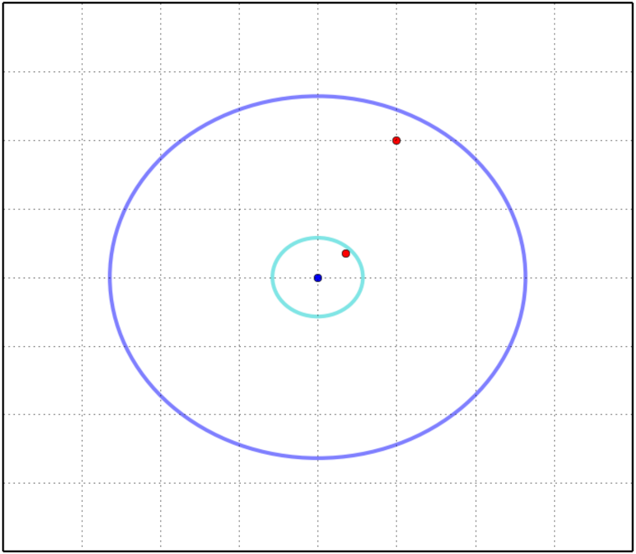
Dynamical FLRW



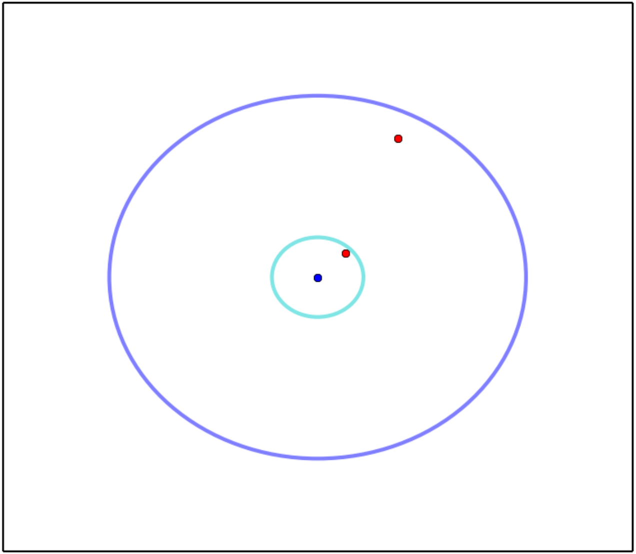
Apparent Horizon

(Davis, Lineweaver, 2003)

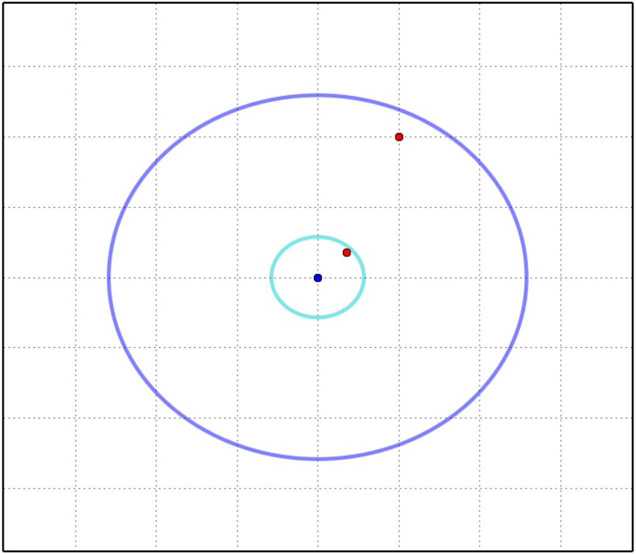
Apparent Horizon versus Event Horizon



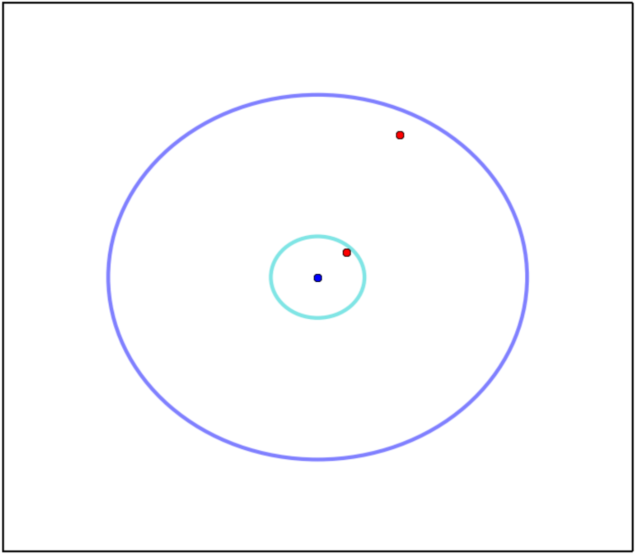
Apparent Horizon versus Event Horizon



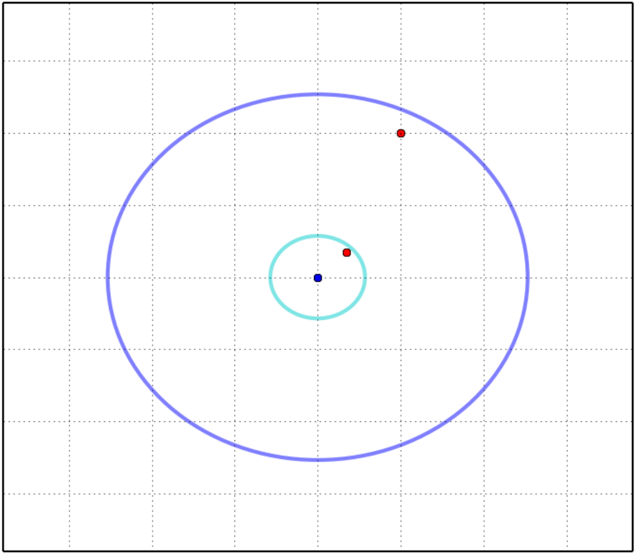
Apparent Horizon versus Event Horizon



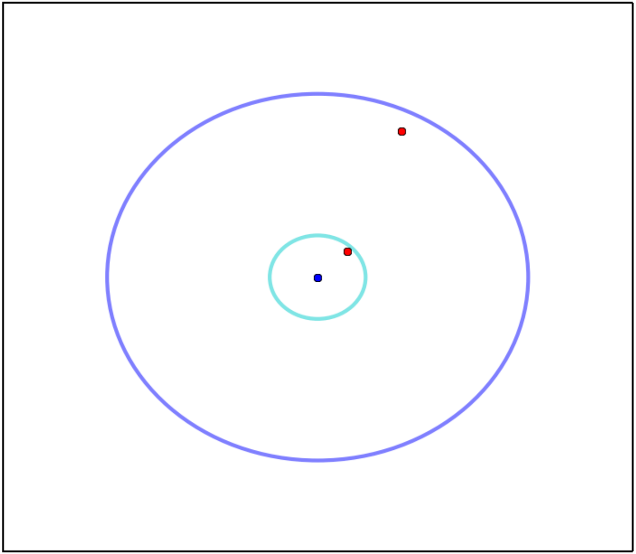
Apparent Horizon versus Event Horizon



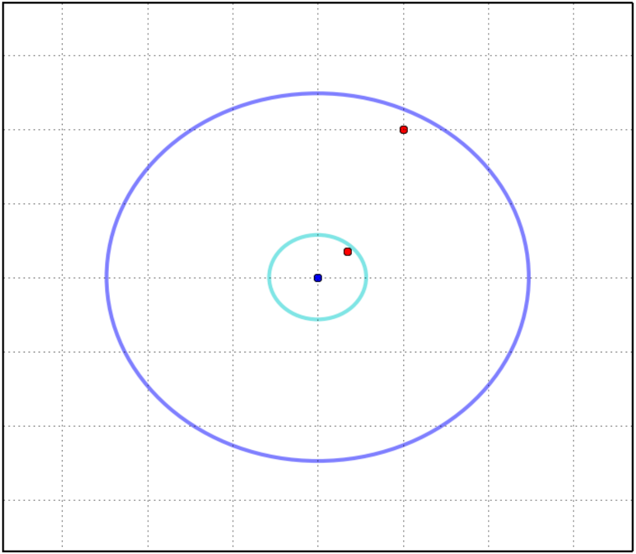
Apparent Horizon versus Event Horizon



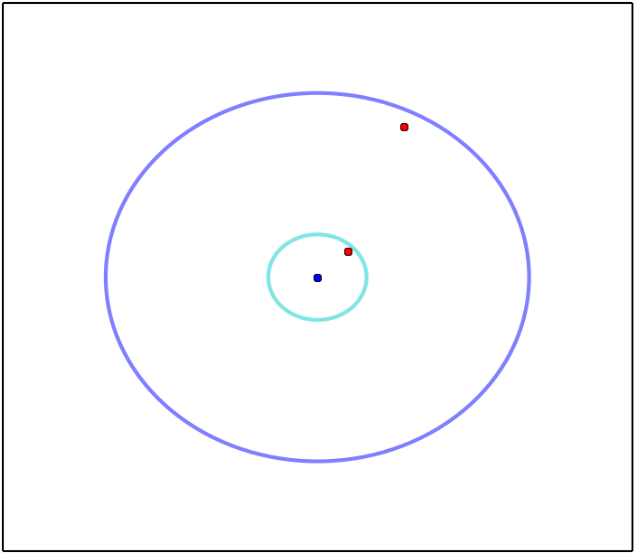
Apparent Horizon versus Event Horizon



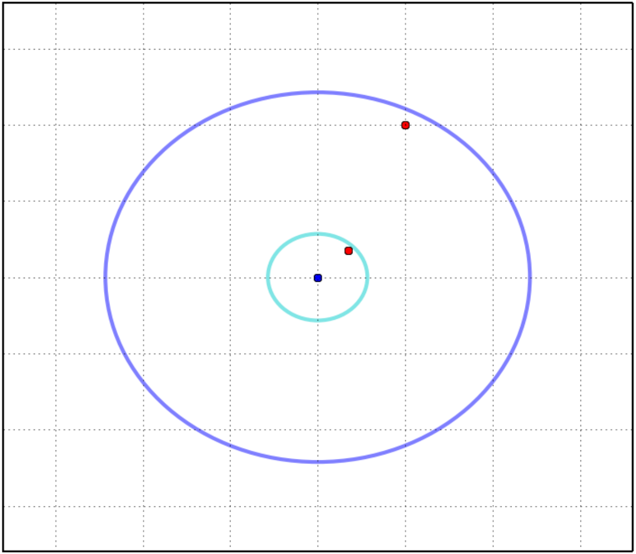
Apparent Horizon versus Event Horizon



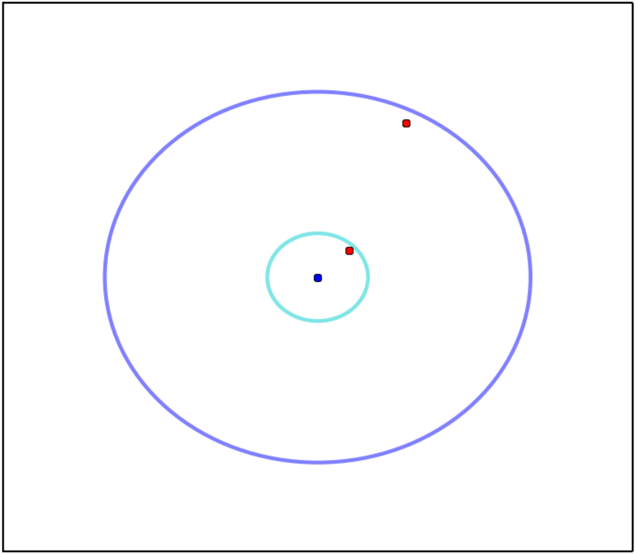
Apparent Horizon versus Event Horizon



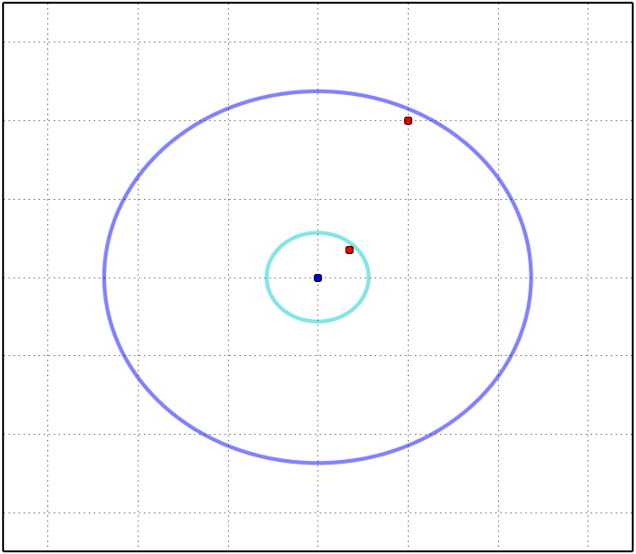
Apparent Horizon versus Event Horizon



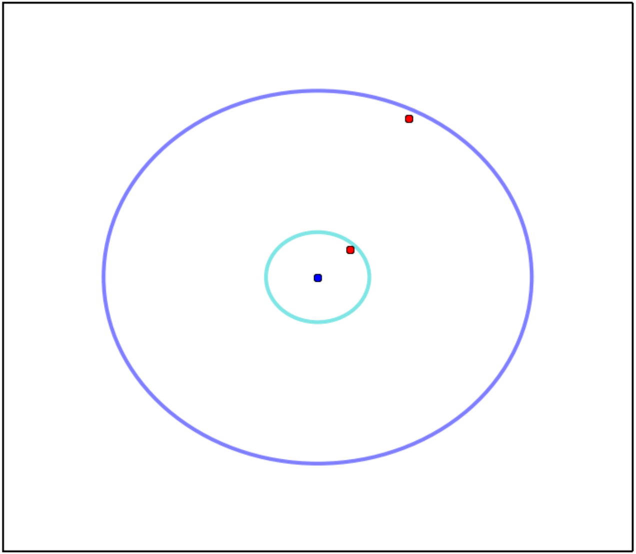
Apparent Horizon versus Event Horizon



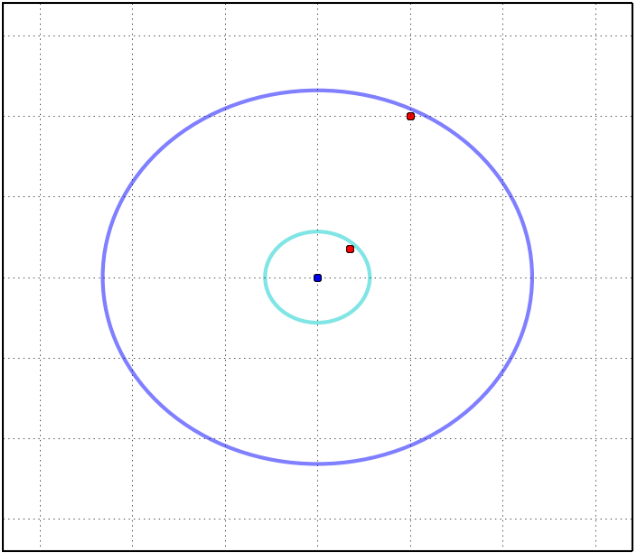
Apparent Horizon versus Event Horizon



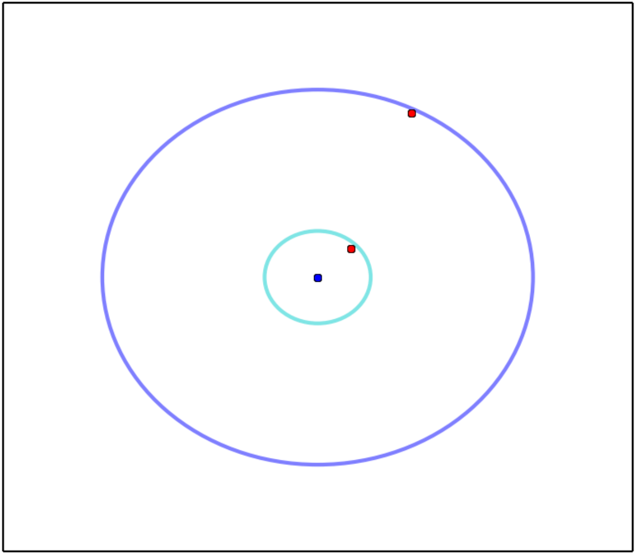
Apparent Horizon versus Event Horizon



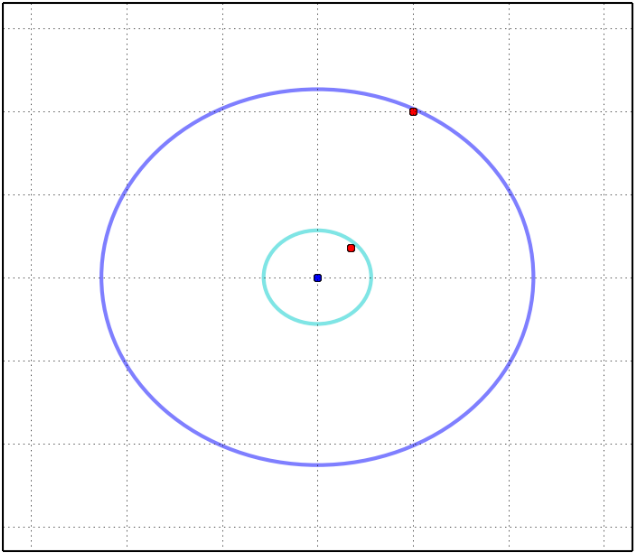
Apparent Horizon versus Event Horizon



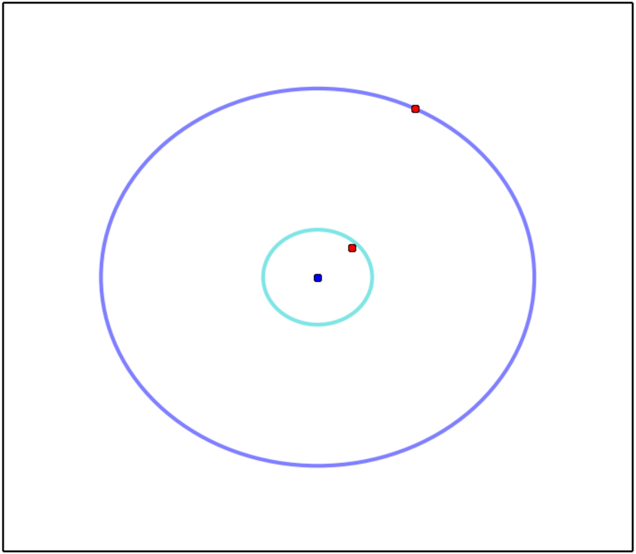
Apparent Horizon versus Event Horizon



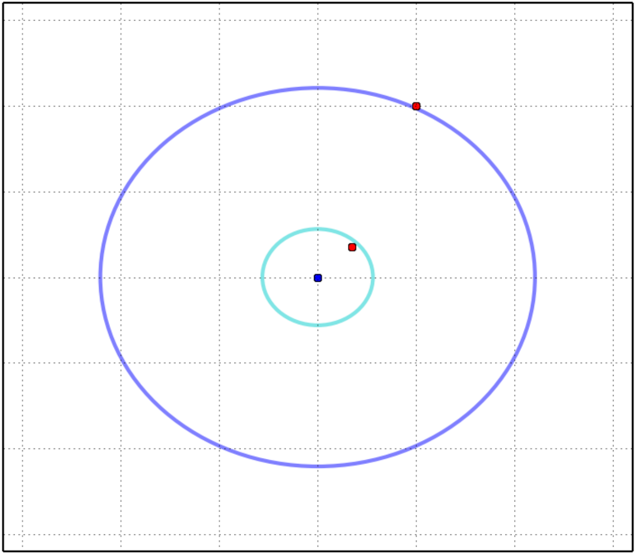
Apparent Horizon versus Event Horizon



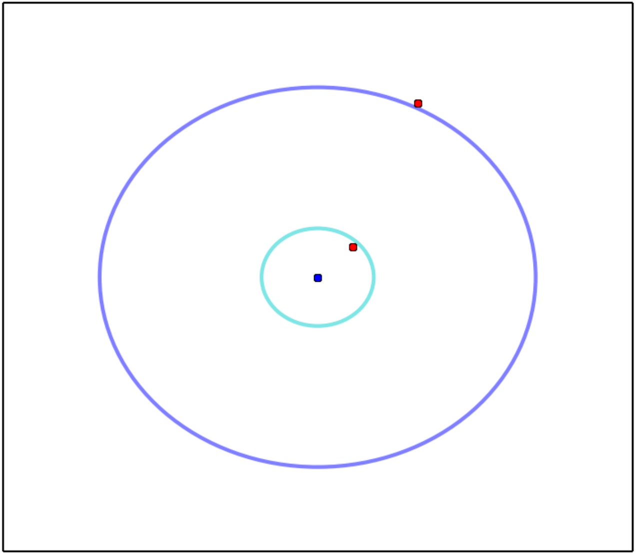
Apparent Horizon versus Event Horizon



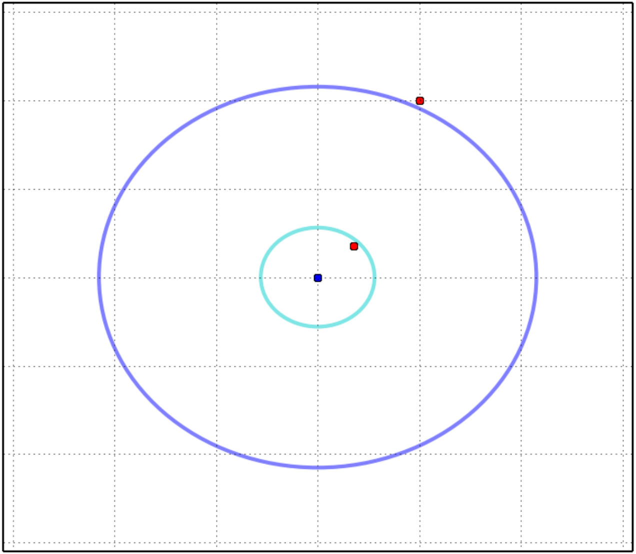
Apparent Horizon versus Event Horizon



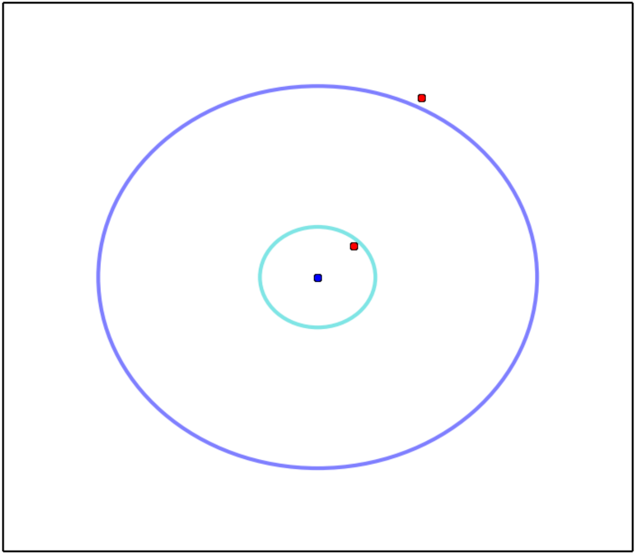
Apparent Horizon versus Event Horizon



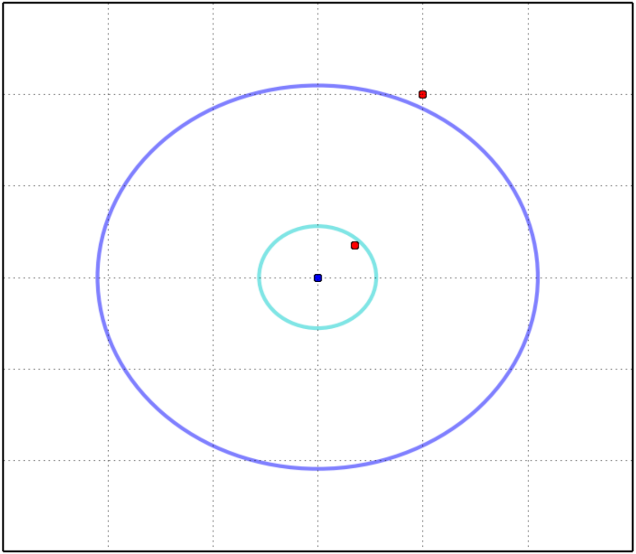
Apparent Horizon versus Event Horizon



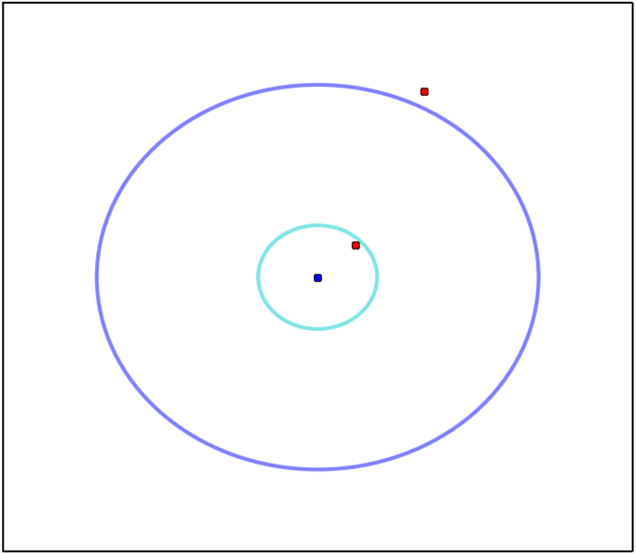
Apparent Horizon versus Event Horizon



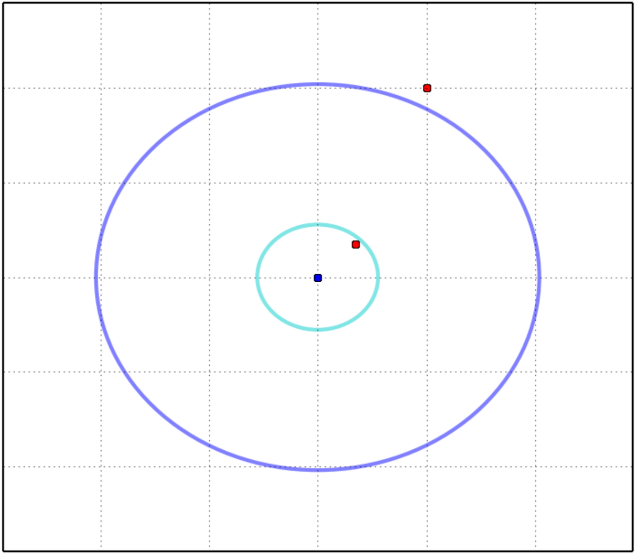
Apparent Horizon versus Event Horizon



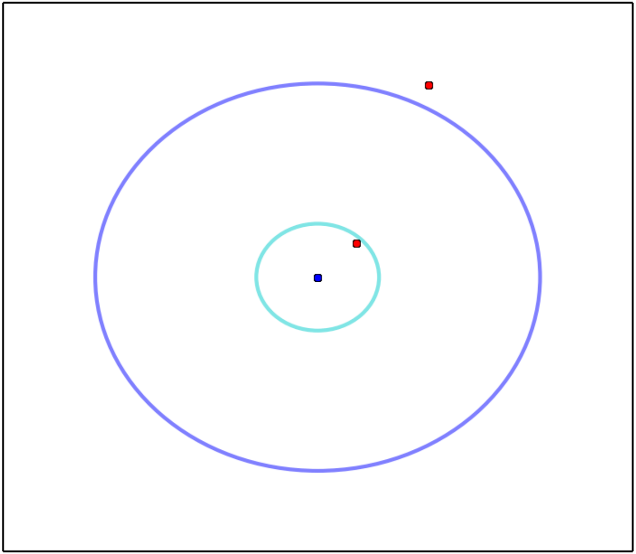
Apparent Horizon versus Event Horizon



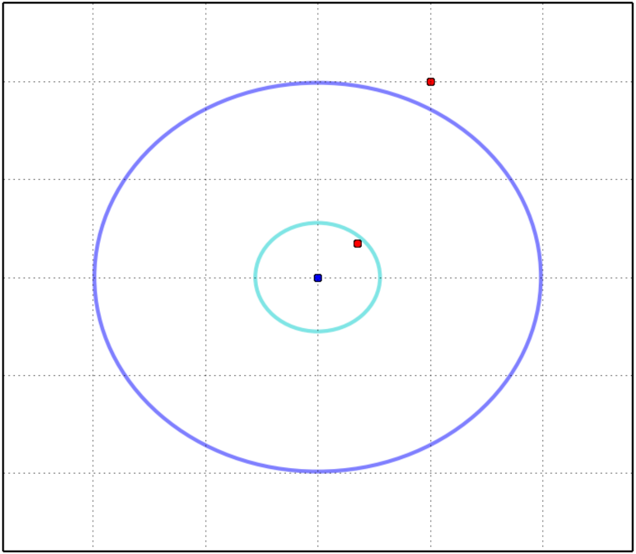
Apparent Horizon versus Event Horizon



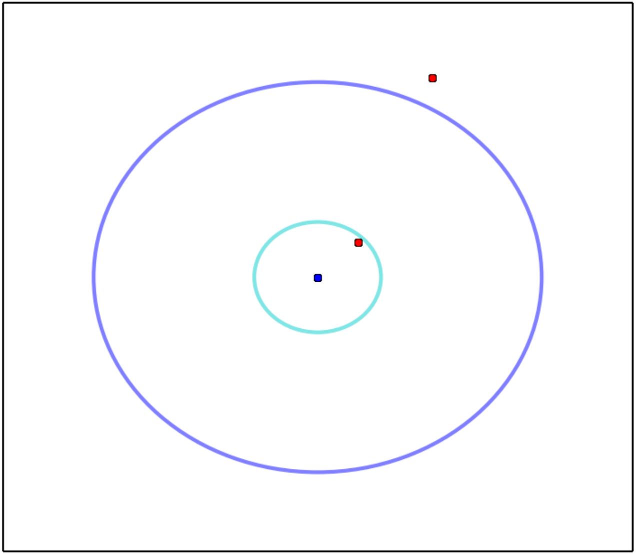
Apparent Horizon versus Event Horizon



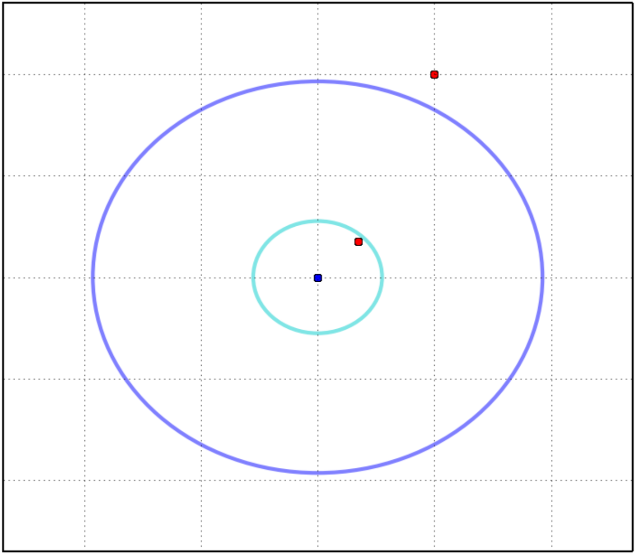
Apparent Horizon versus Event Horizon



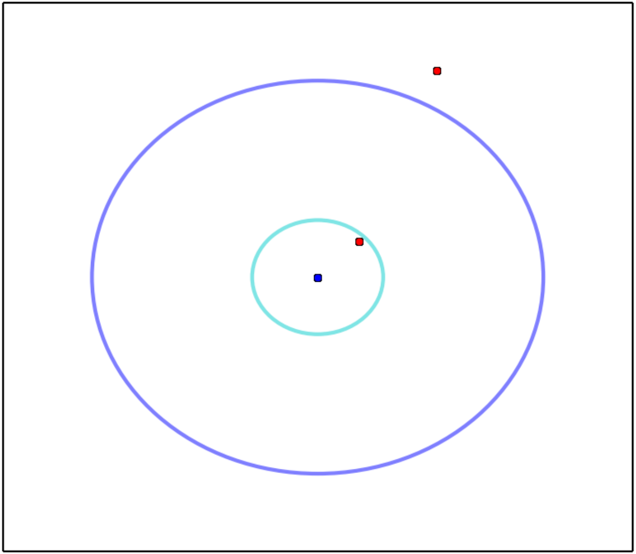
Apparent Horizon versus Event Horizon



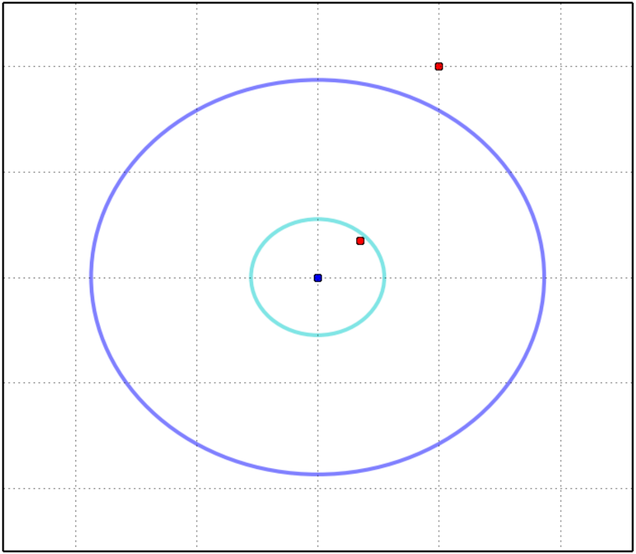
Apparent Horizon versus Event Horizon



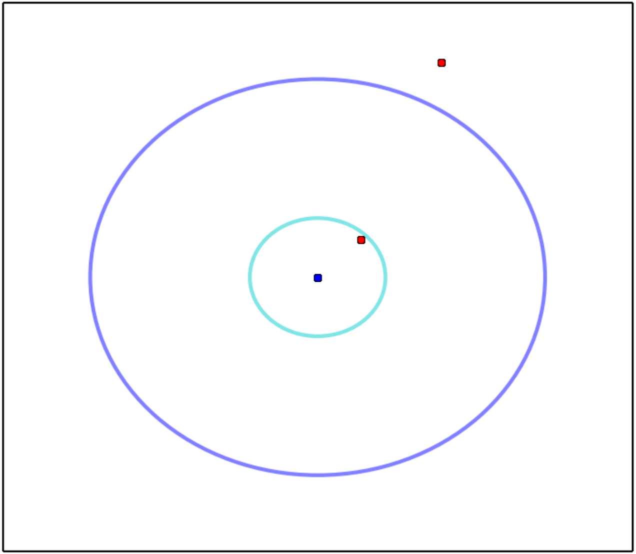
Apparent Horizon versus Event Horizon

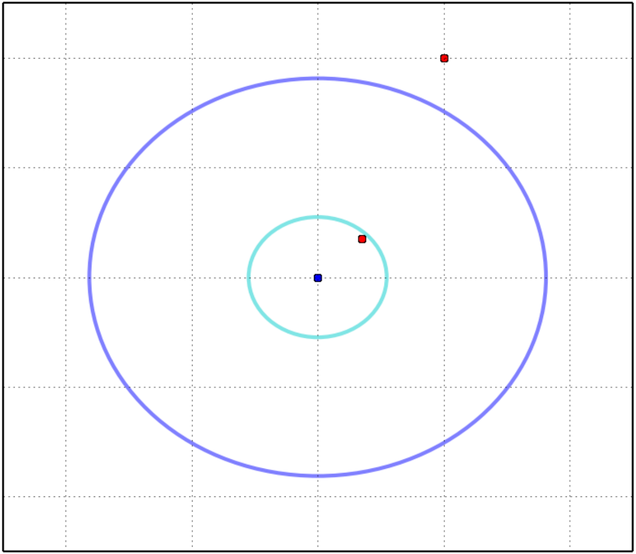


Apparent Horizon versus Event Horizon

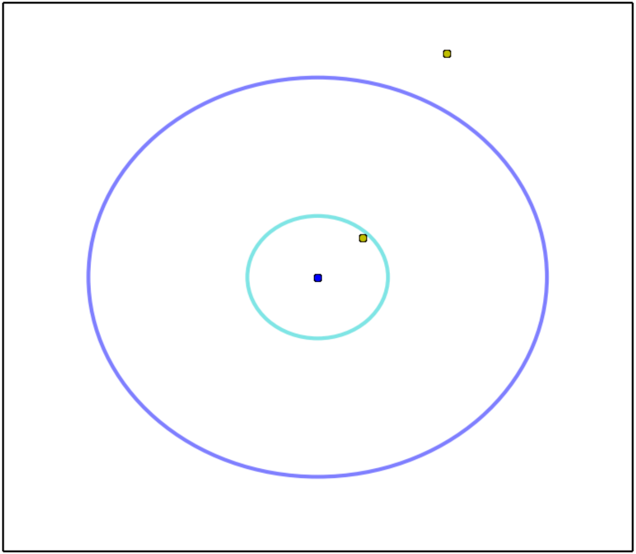


Apparent Horizon versus Event Horizon

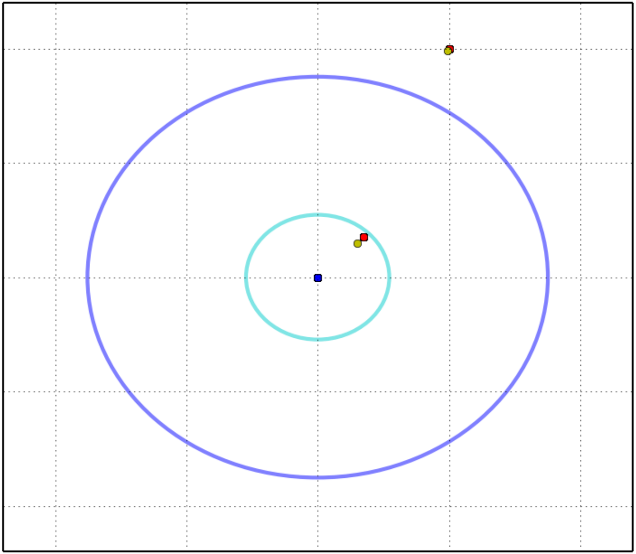




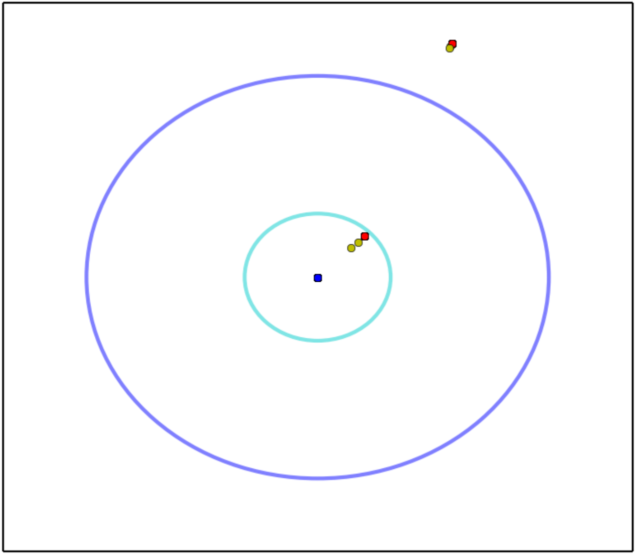
Apparent Horizon versus Event Horizon



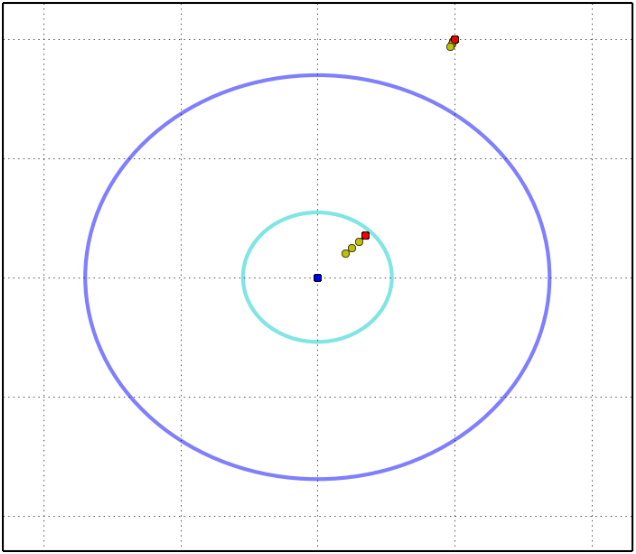
Apparent Horizon versus Event Horizon



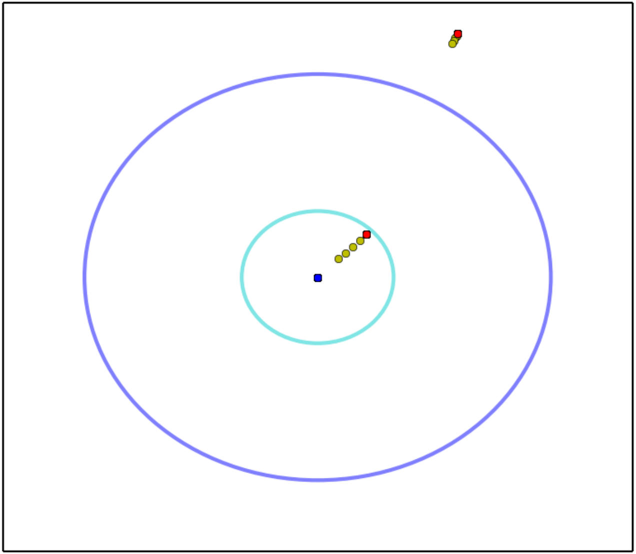
Apparent Horizon versus Event Horizon

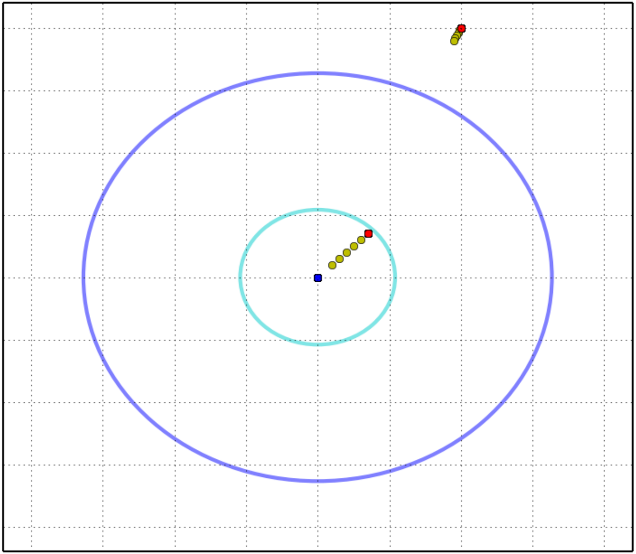


Apparent Horizon versus Event Horizon

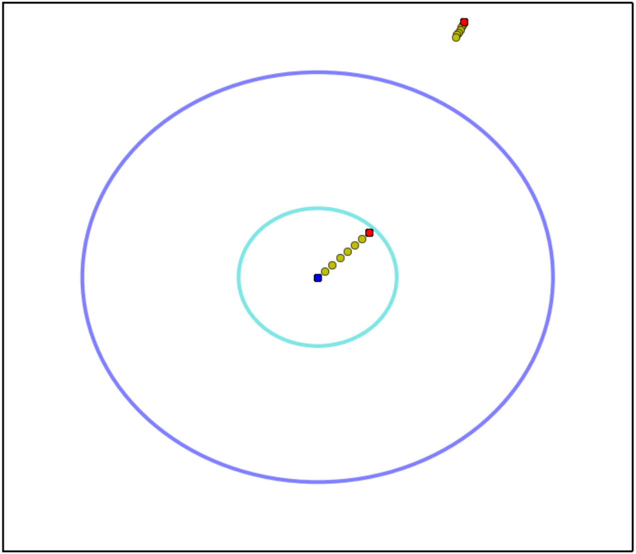


Apparent Horizon versus Event Horizon

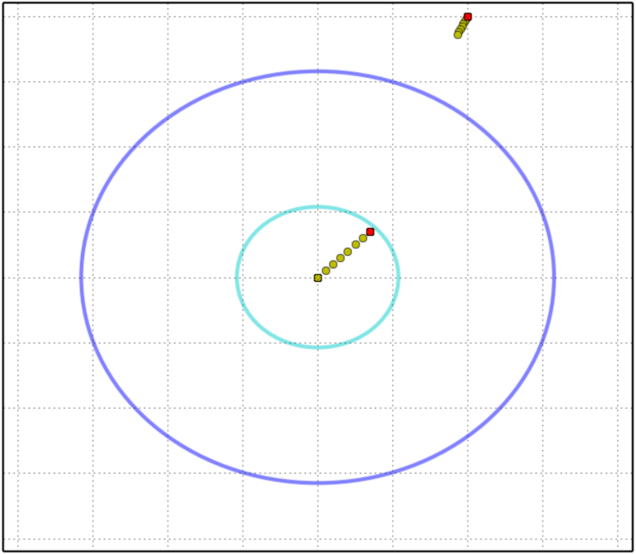




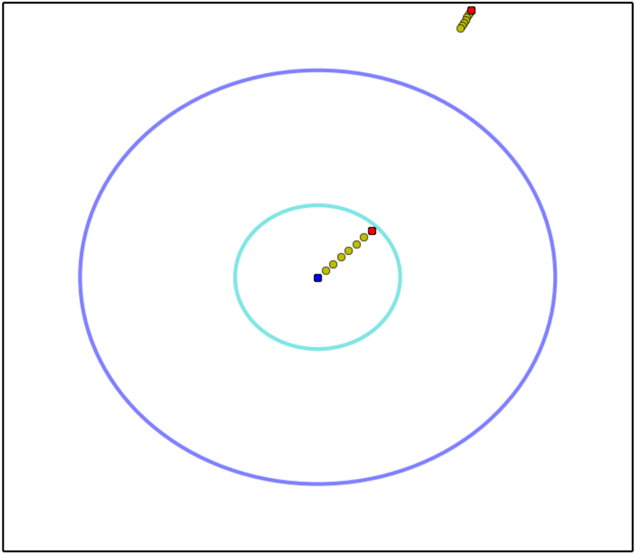
Apparent Horizon versus Event Horizon



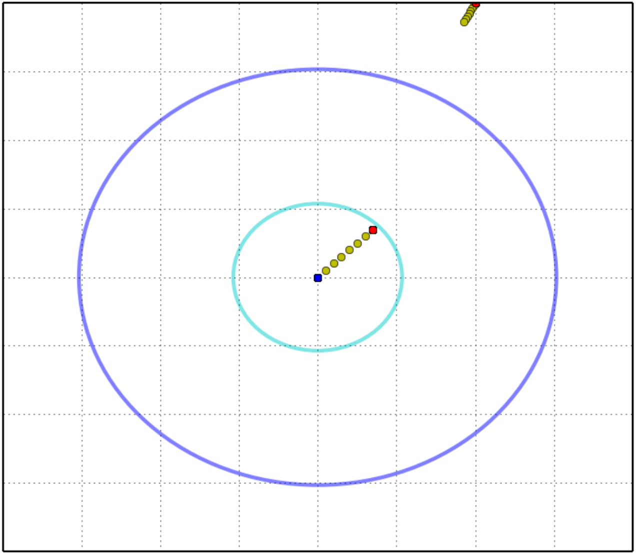
Apparent Horizon versus Event Horizon



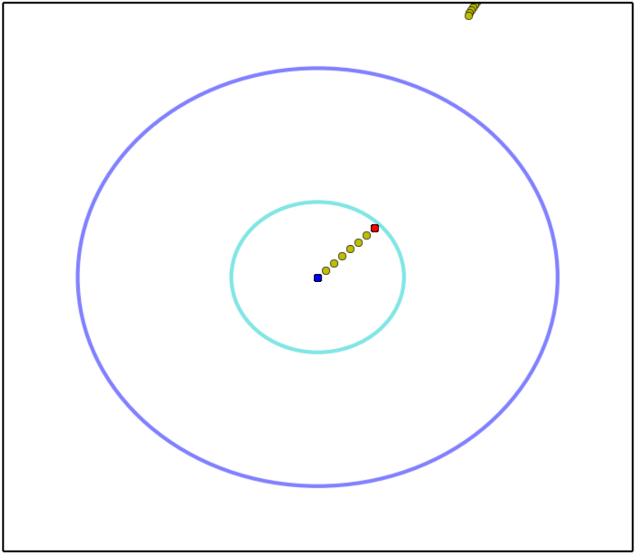
Apparent Horizon versus Event Horizon



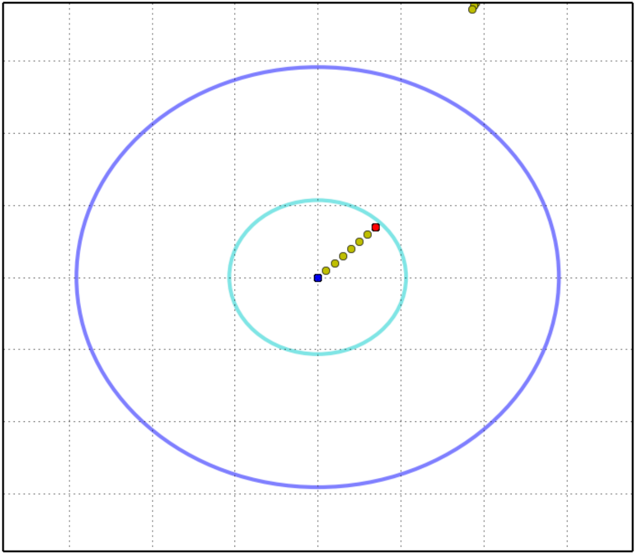
Apparent Horizon versus Event Horizon



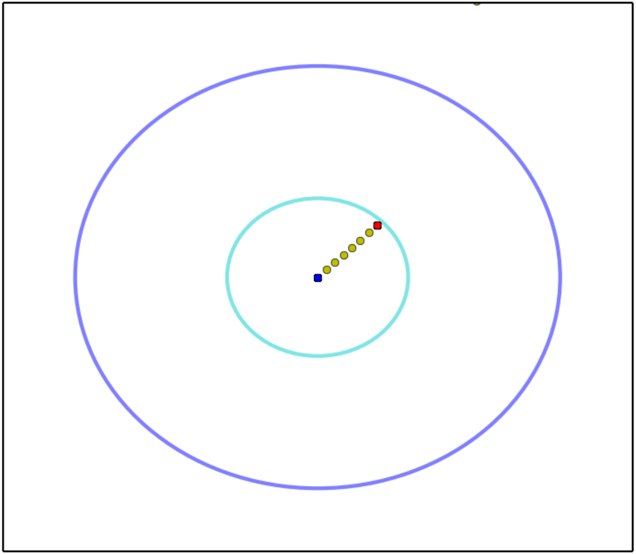
Apparent Horizon versus Event Horizon



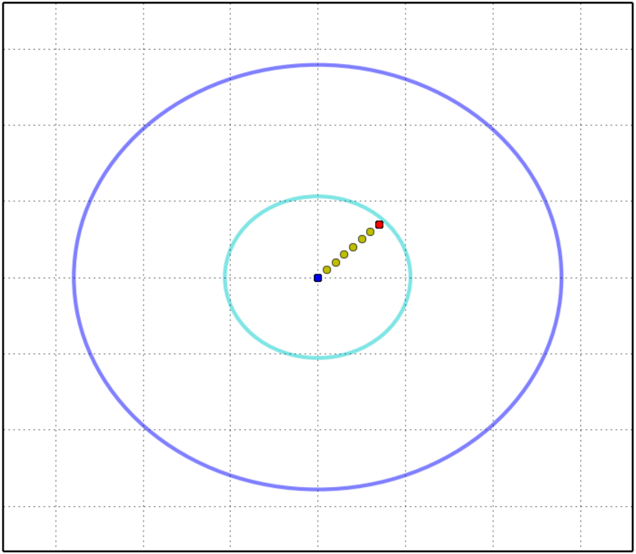
Apparent Horizon versus Event Horizon



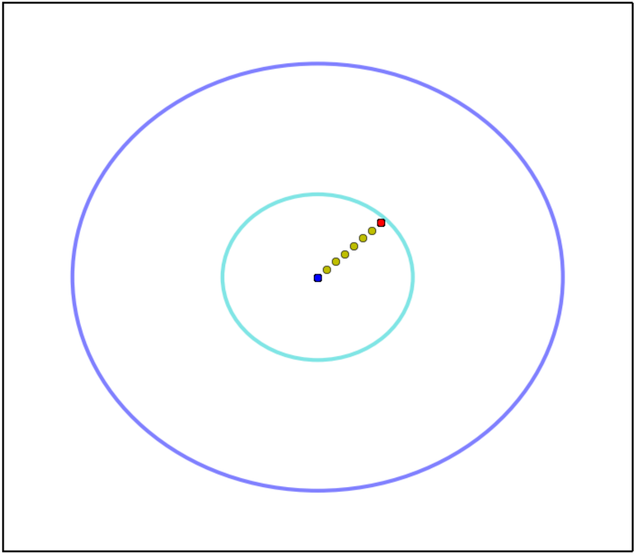
Apparent Horizon versus Event Horizon



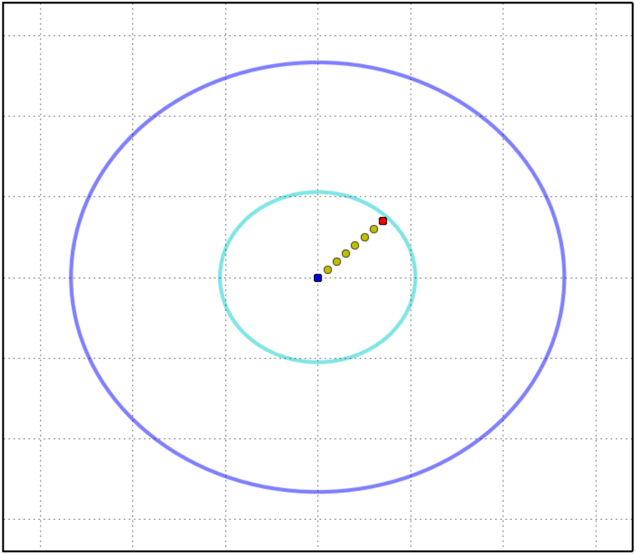
Apparent Horizon versus Event Horizon



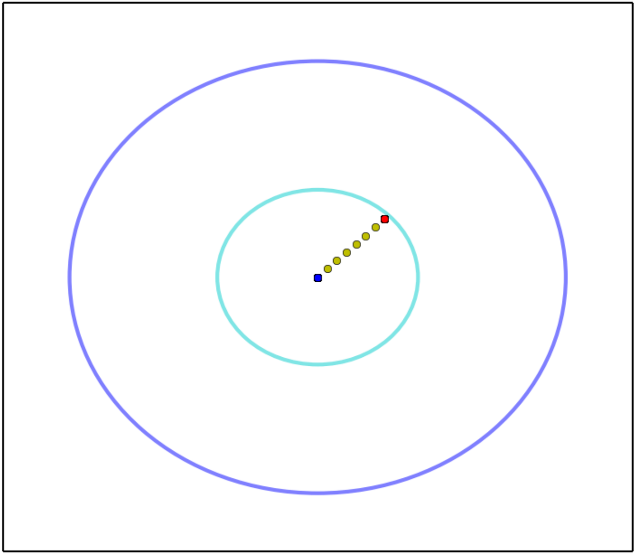
Apparent Horizon versus Event Horizon



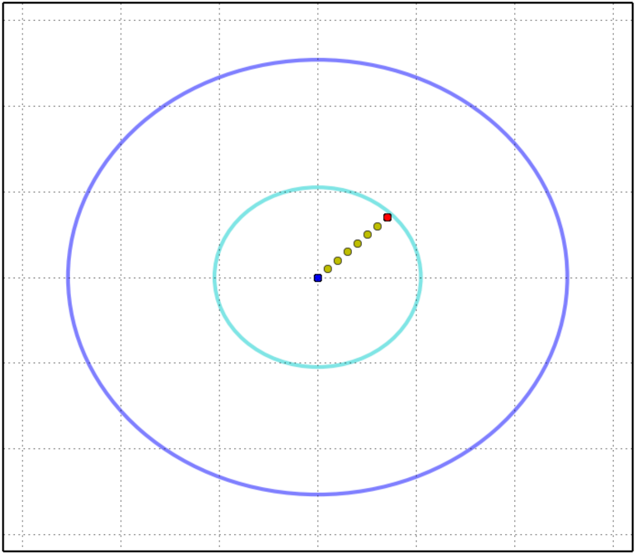
Apparent Horizon versus Event Horizon



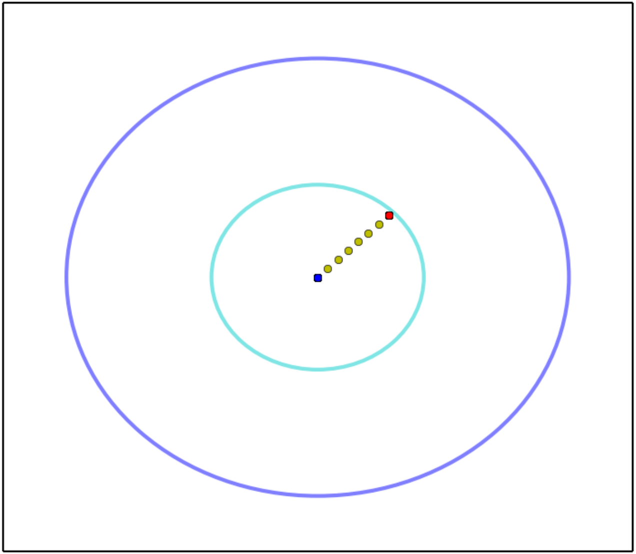
Apparent Horizon versus Event Horizon



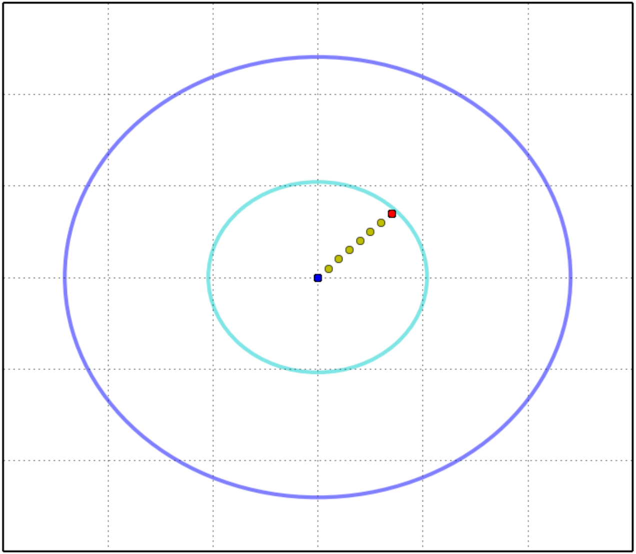
Apparent Horizon versus Event Horizon



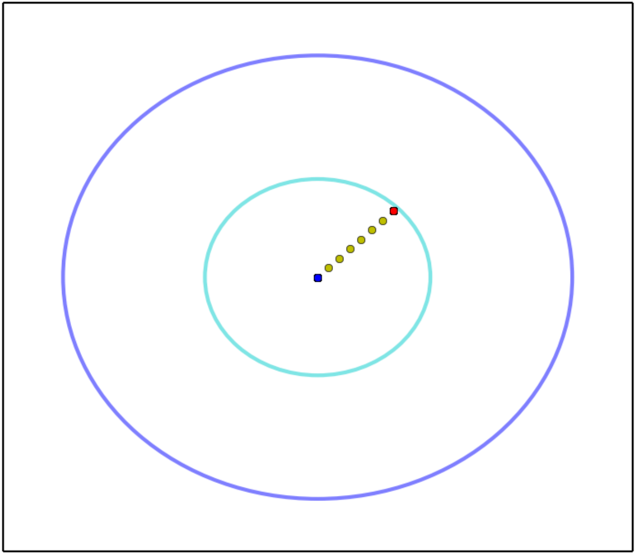
Apparent Horizon versus Event Horizon



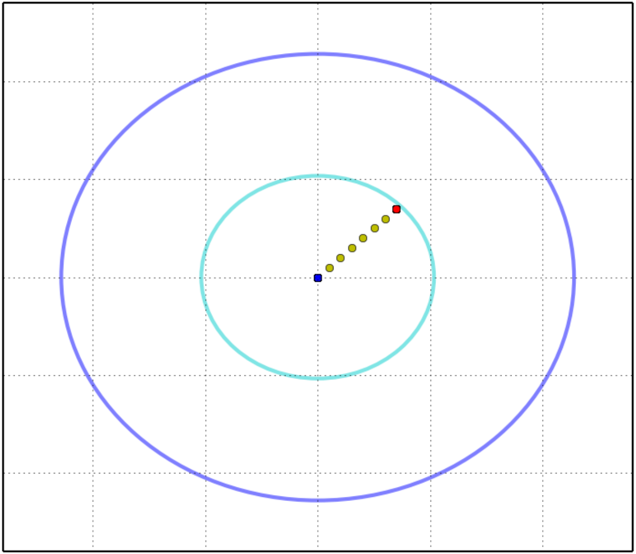
Apparent Horizon versus Event Horizon



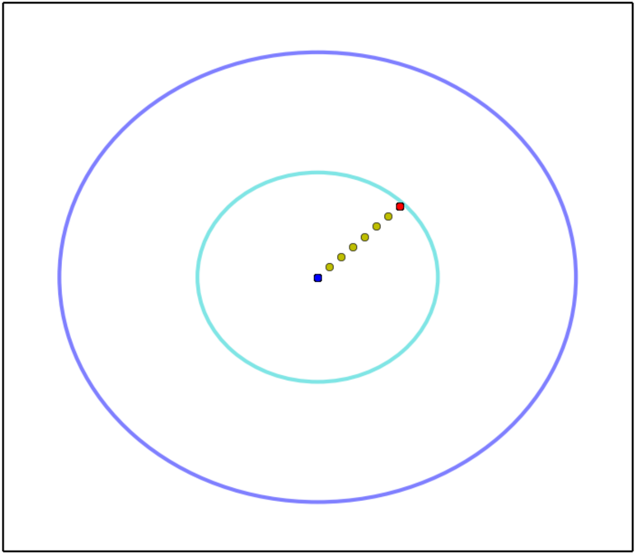
Apparent Horizon versus Event Horizon

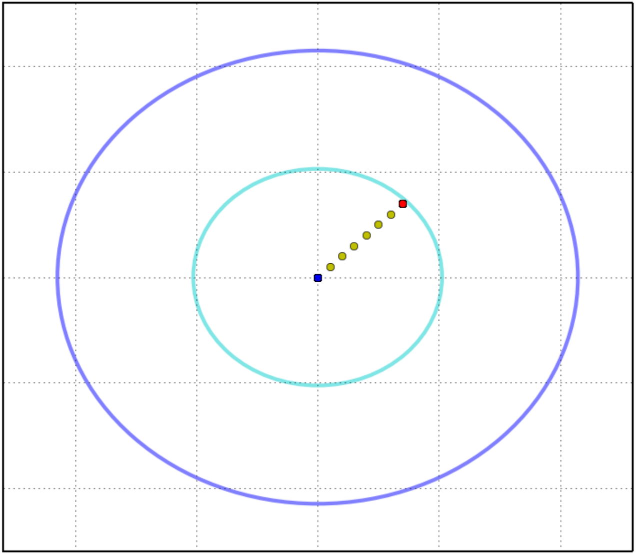


Apparent Horizon versus Event Horizon

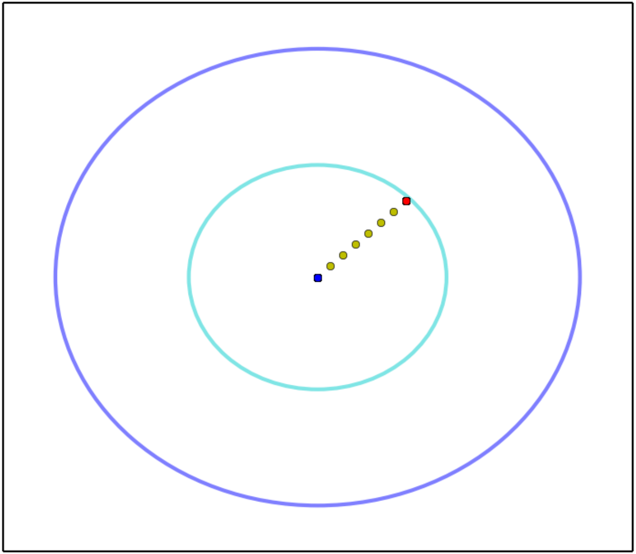


Apparent Horizon versus Event Horizon

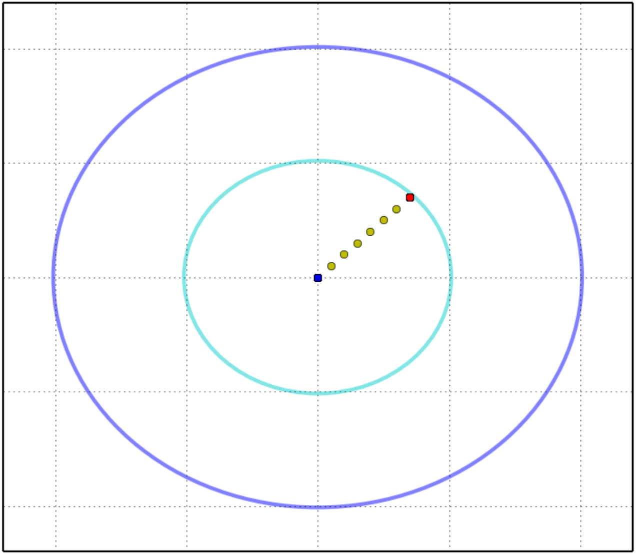




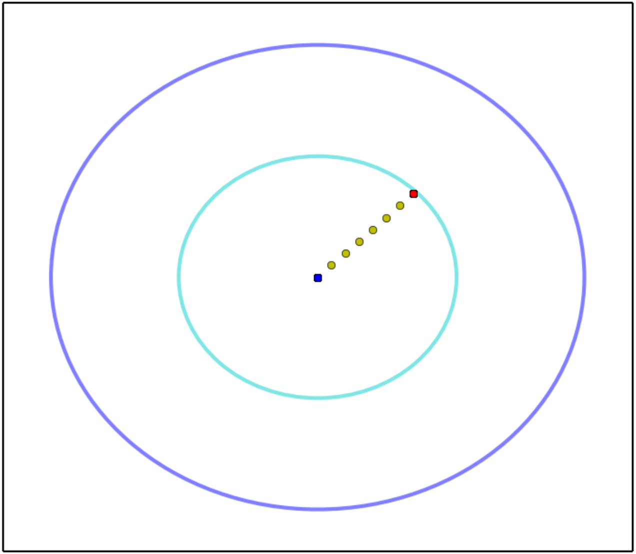
Apparent Horizon versus Event Horizon



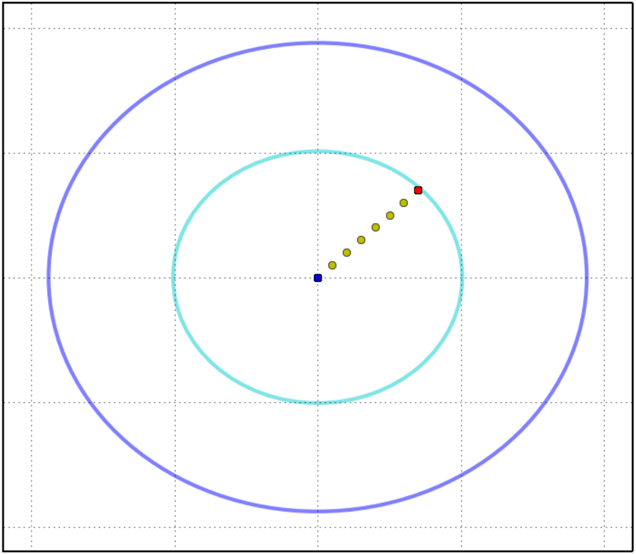
Apparent Horizon versus Event Horizon



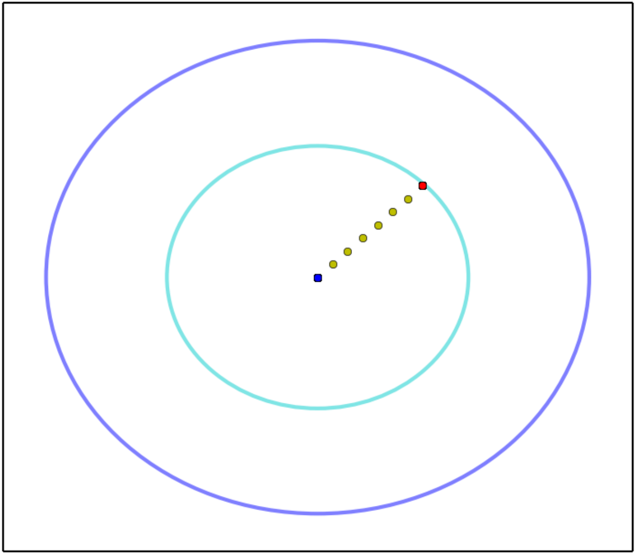
Apparent Horizon versus Event Horizon



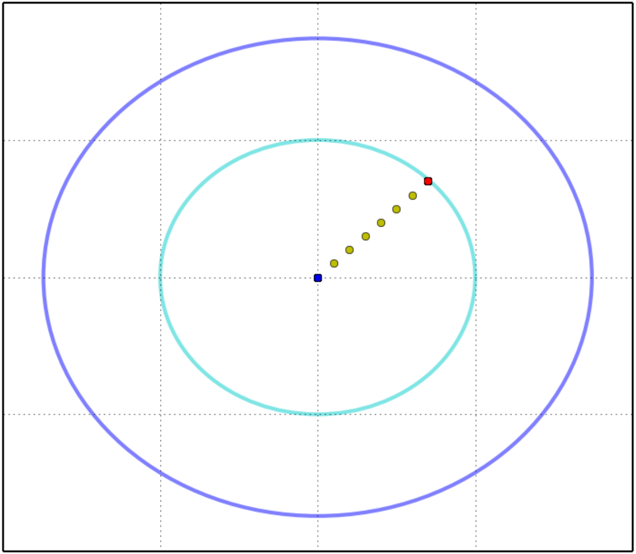
Apparent Horizon versus Event Horizon



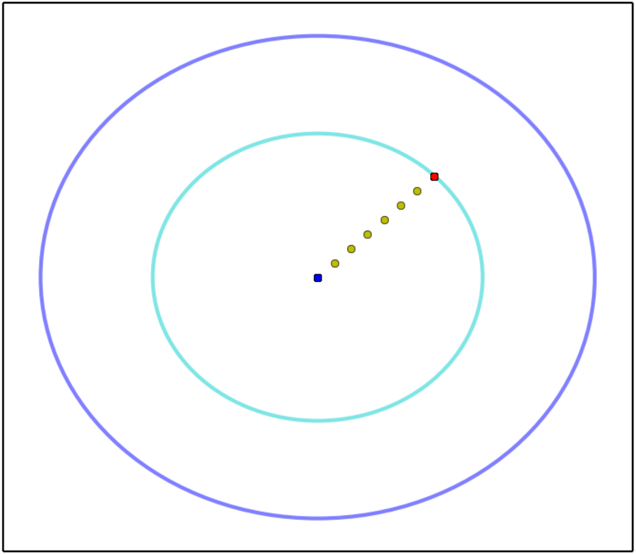
Apparent Horizon versus Event Horizon



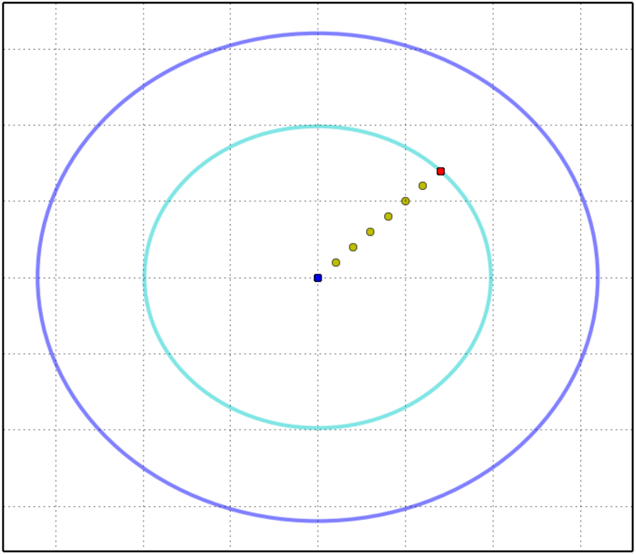
Apparent Horizon versus Event Horizon



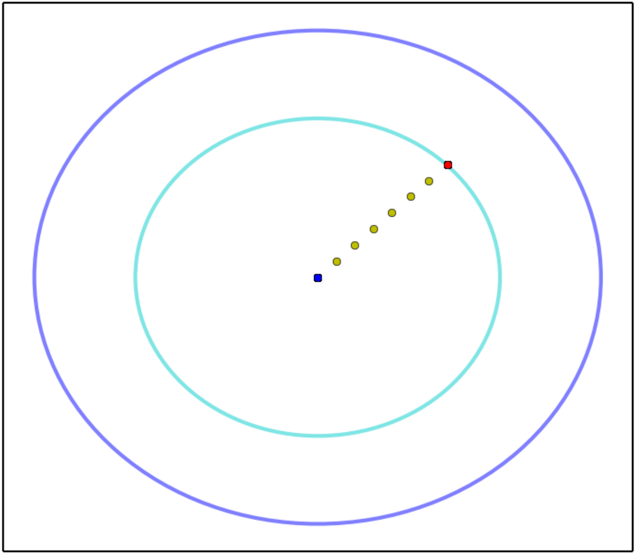
Apparent Horizon versus Event Horizon



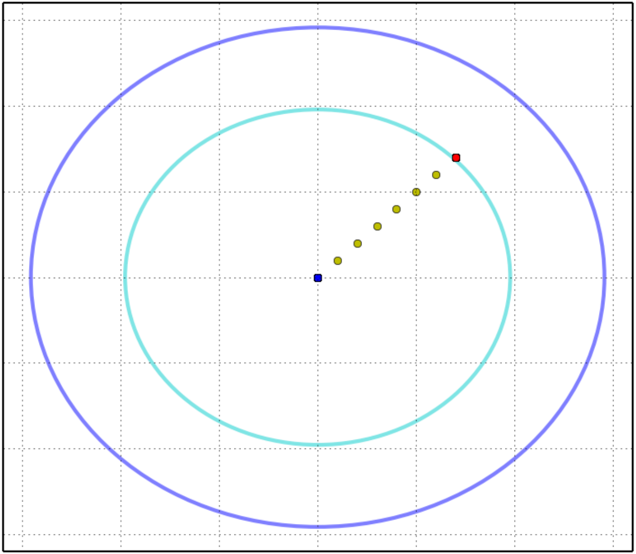
Apparent Horizon versus Event Horizon



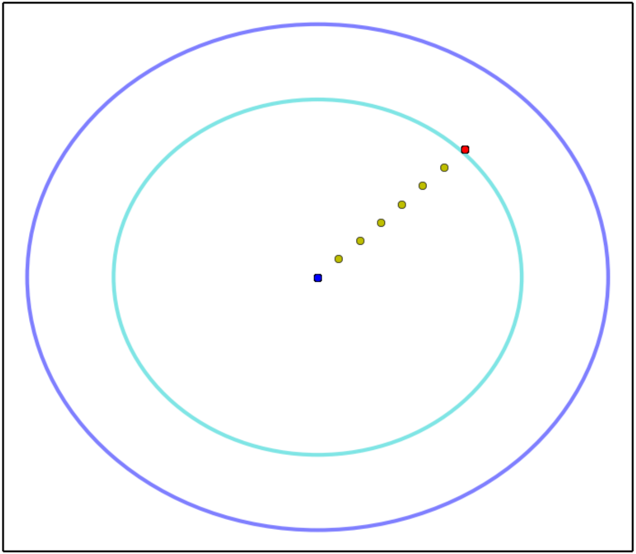
Apparent Horizon versus Event Horizon



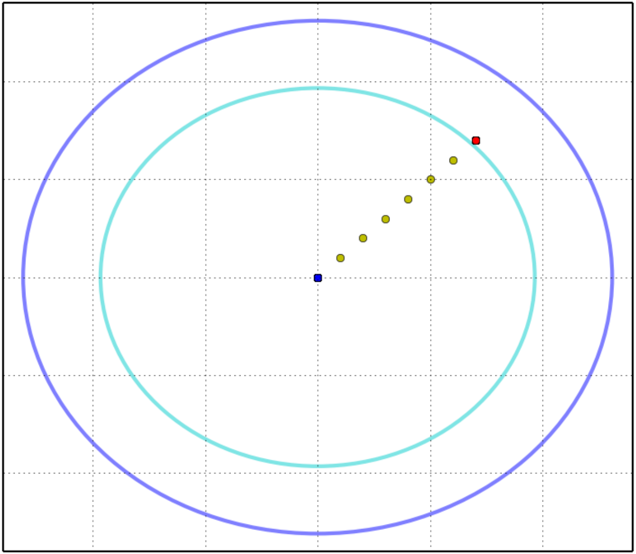
Apparent Horizon versus Event Horizon



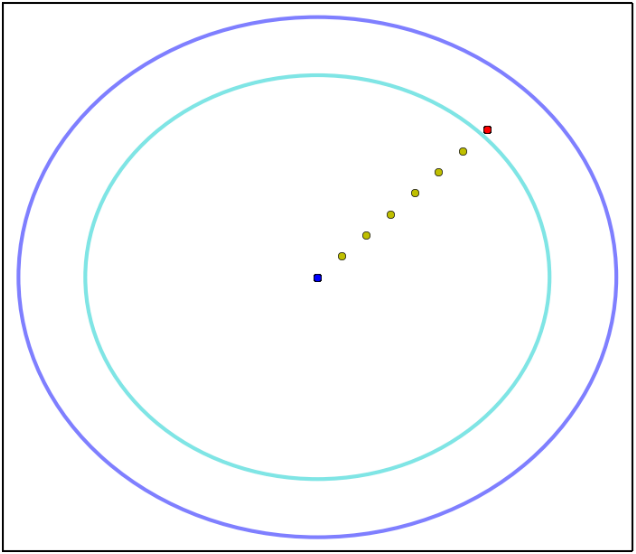
Apparent Horizon versus Event Horizon



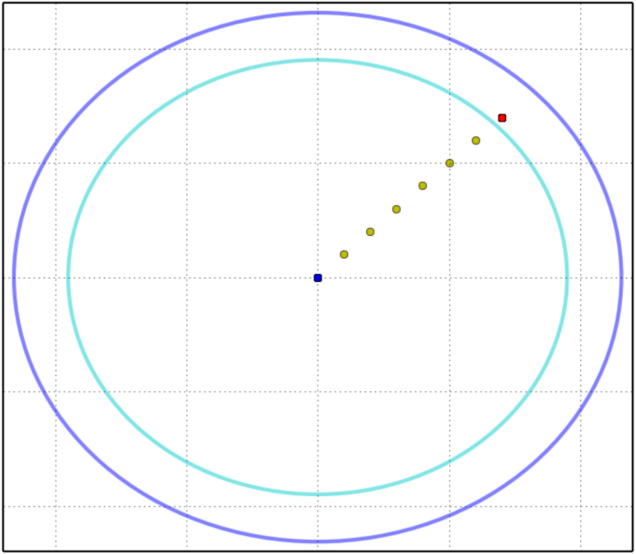
Apparent Horizon versus Event Horizon



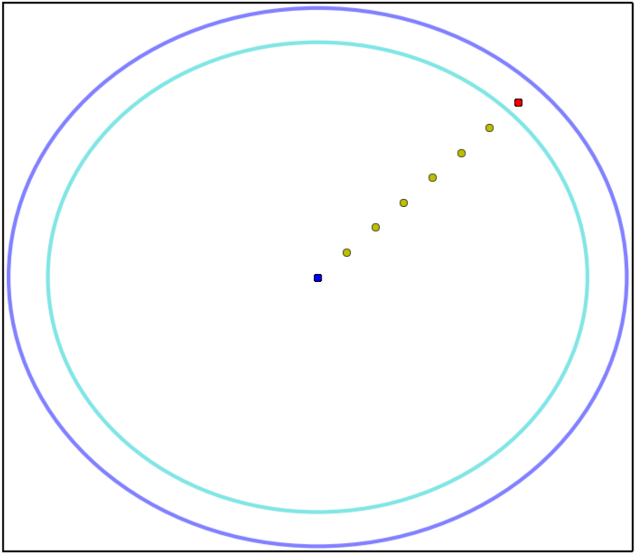
Apparent Horizon versus Event Horizon



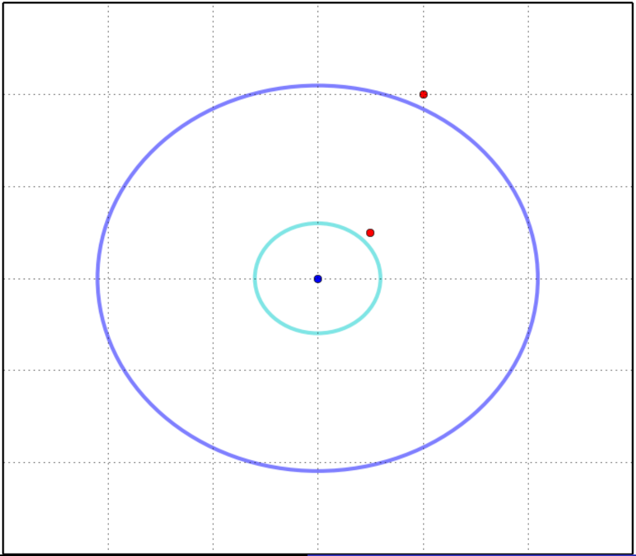
Apparent Horizon versus Event Horizon



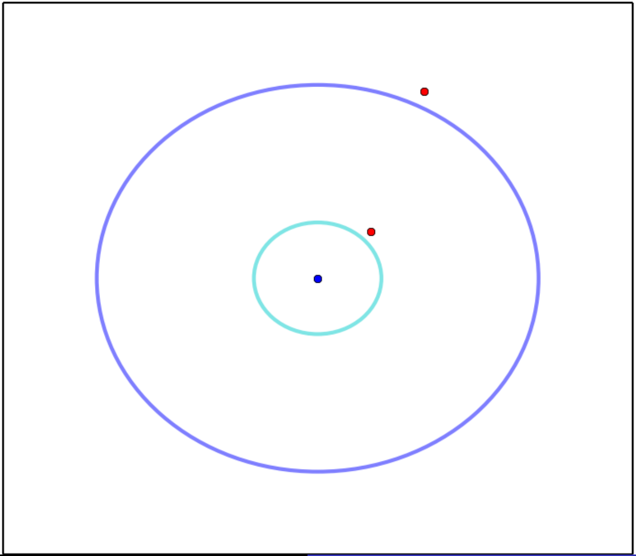
Apparent Horizon versus Event Horizon



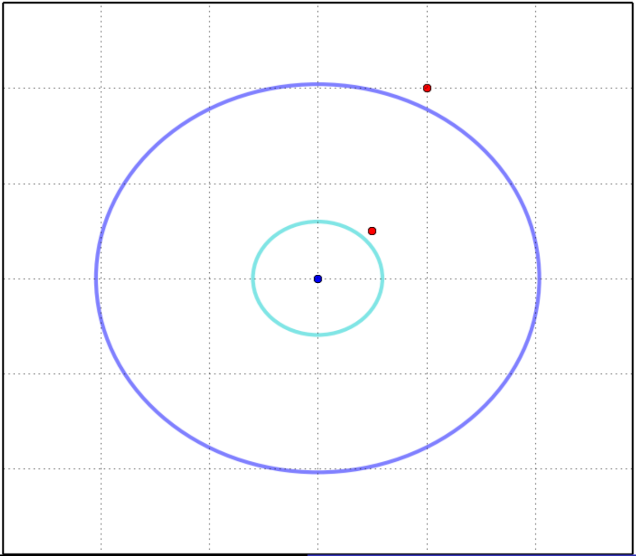
Apparent Horizon versus Event Horizon



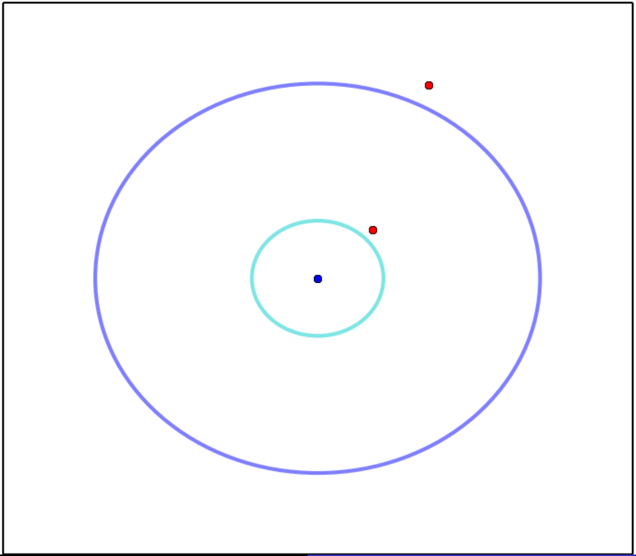
Apparent Horizon versus Event Horizon



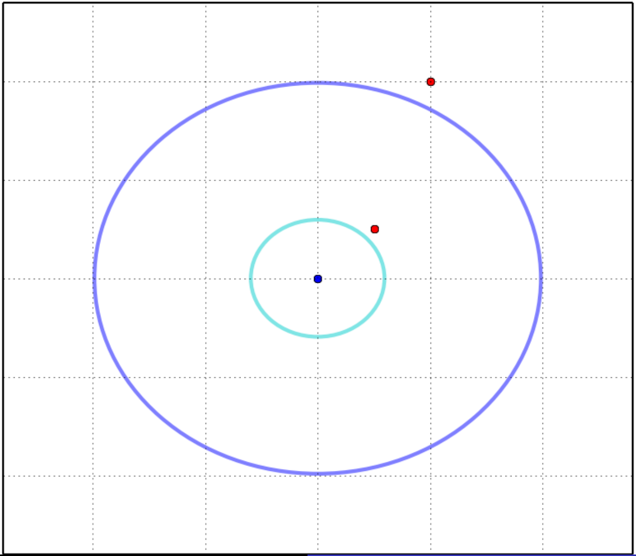
Apparent Horizon versus Event Horizon



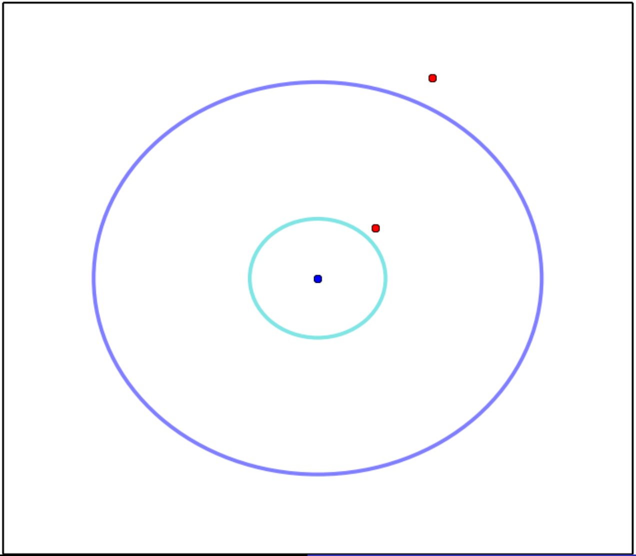
Apparent Horizon versus Event Horizon



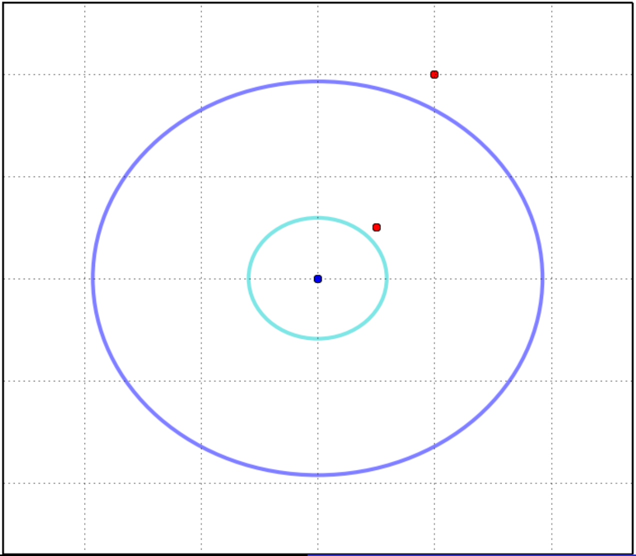
Apparent Horizon versus Event Horizon



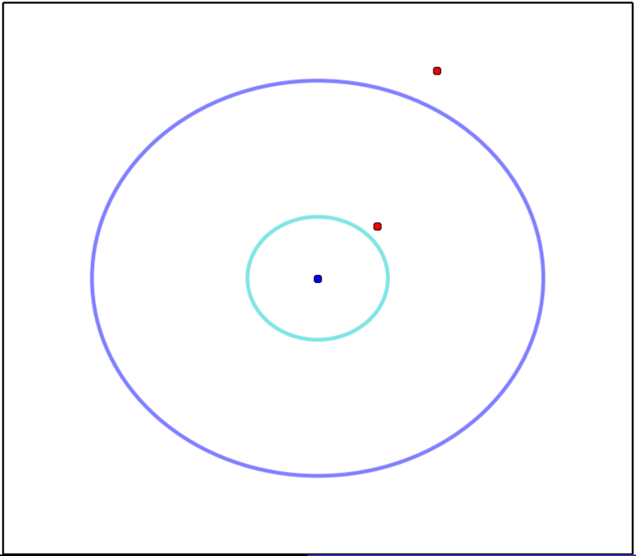
Apparent Horizon versus Event Horizon



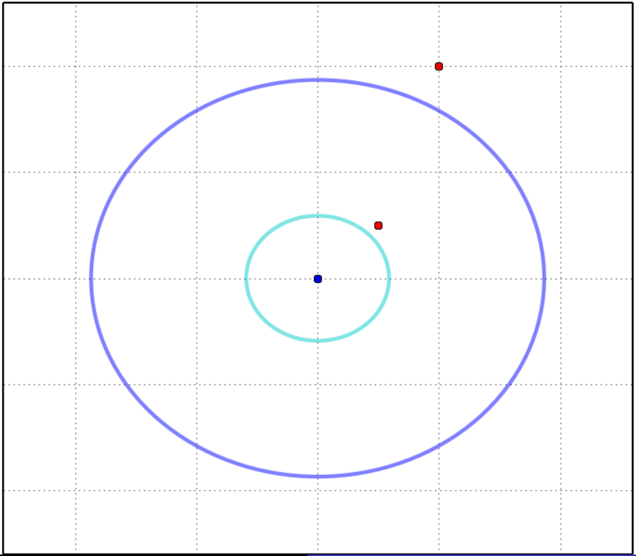
Apparent Horizon versus Event Horizon



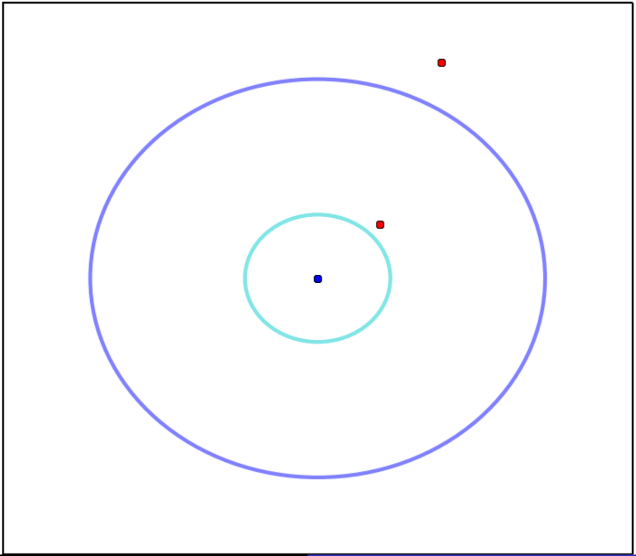
Apparent Horizon versus Event Horizon



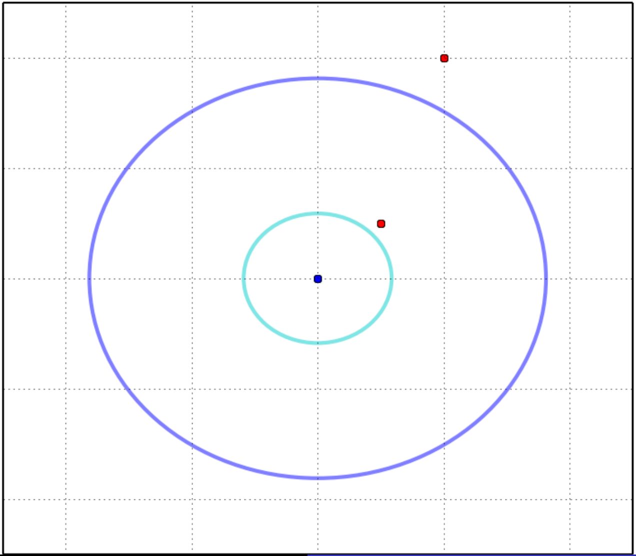
Apparent Horizon versus Event Horizon



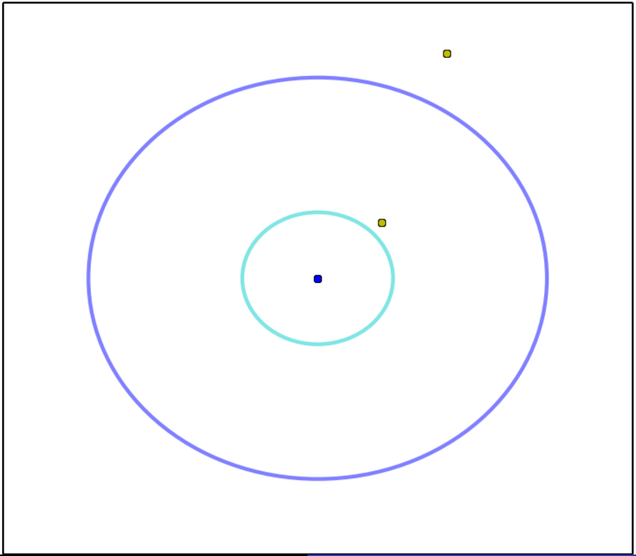
Apparent Horizon versus Event Horizon



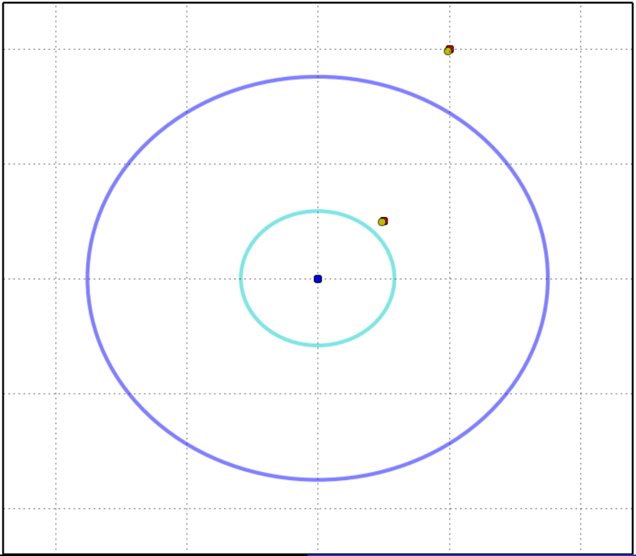
Apparent Horizon versus Event Horizon



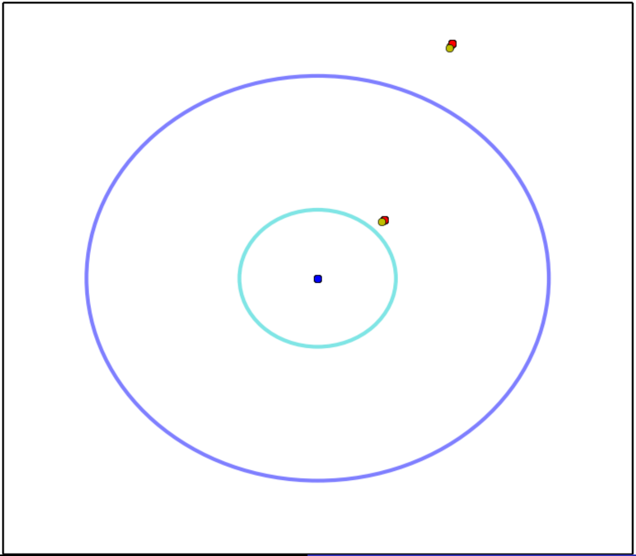
Apparent Horizon versus Event Horizon



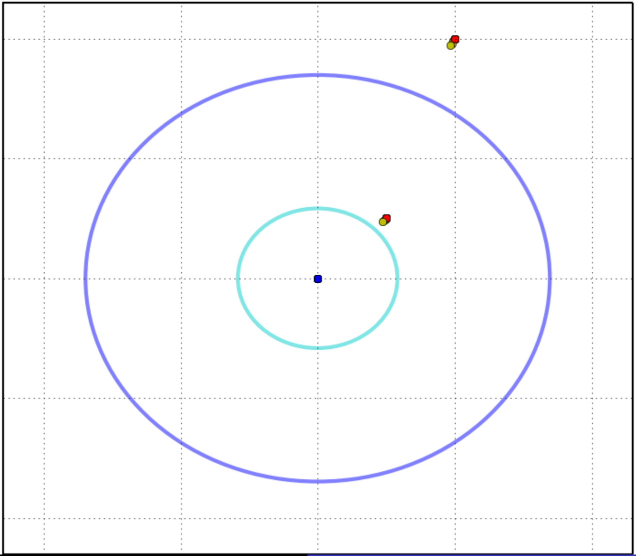
Apparent Horizon versus Event Horizon



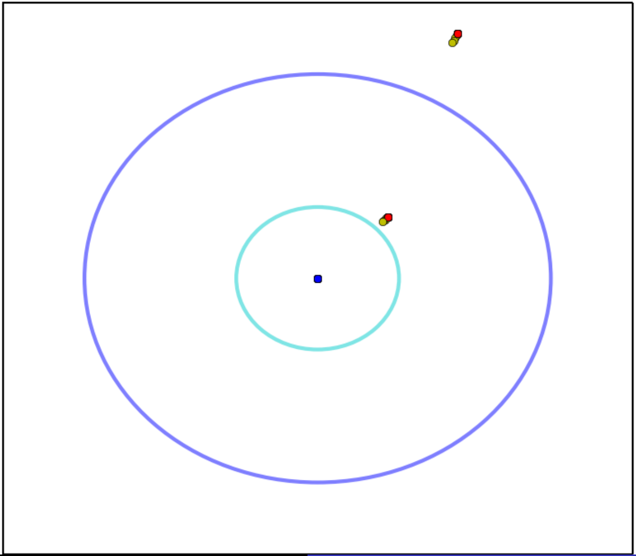
Apparent Horizon versus Event Horizon



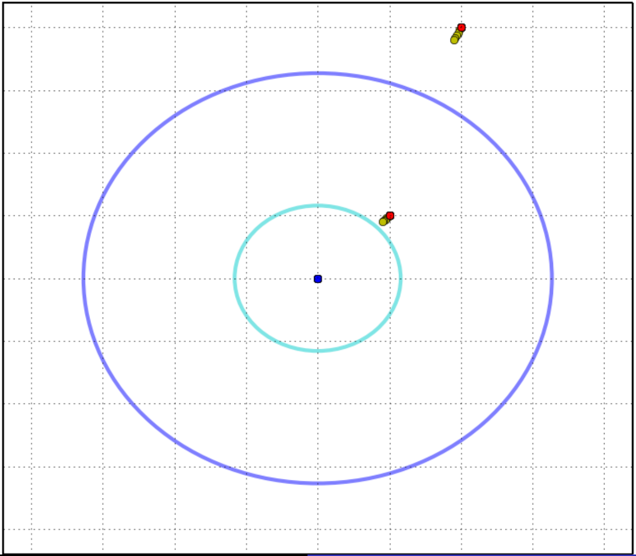
Apparent Horizon versus Event Horizon



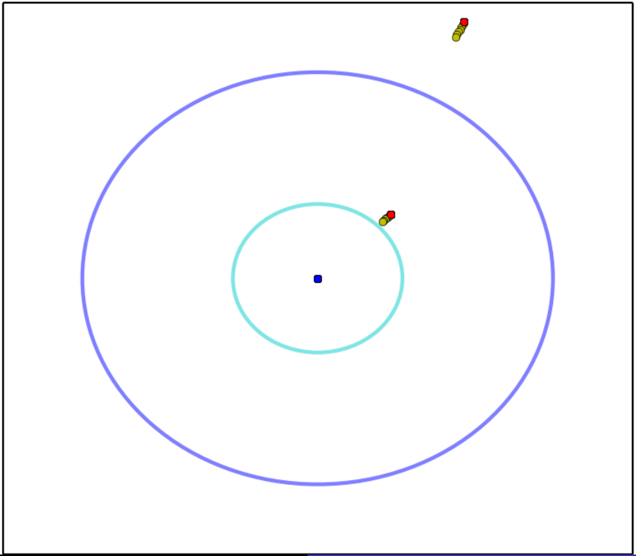
Apparent Horizon versus Event Horizon



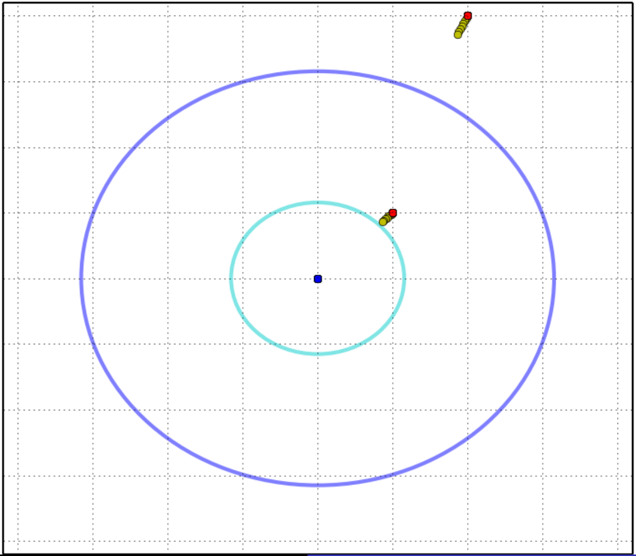
Apparent Horizon versus Event Horizon



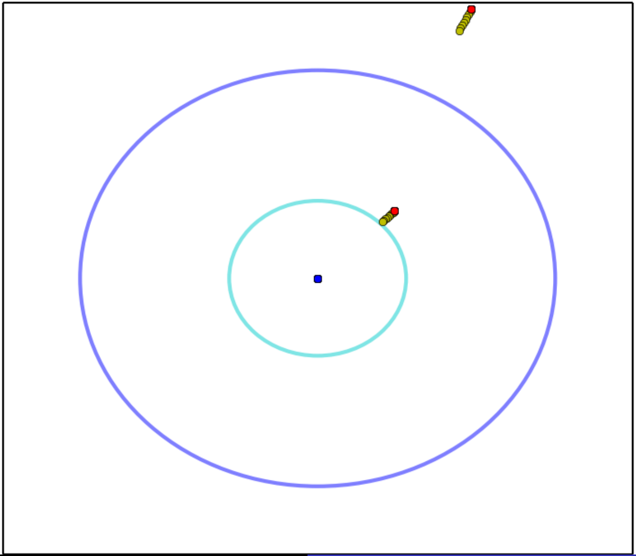
Apparent Horizon versus Event Horizon



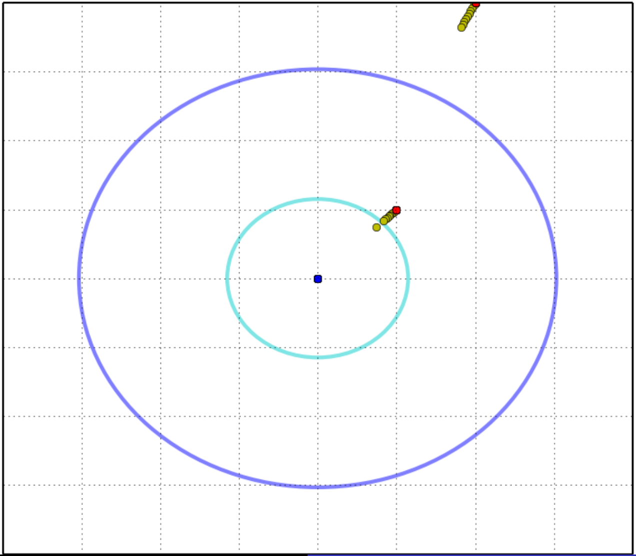
Apparent Horizon versus Event Horizon



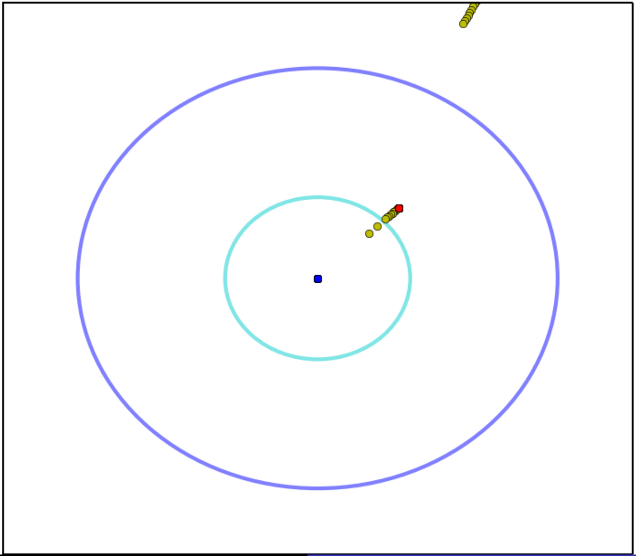
Apparent Horizon versus Event Horizon



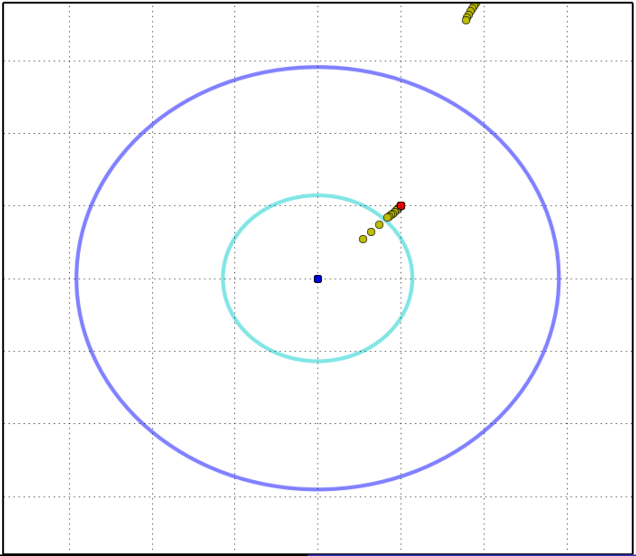
Apparent Horizon versus Event Horizon



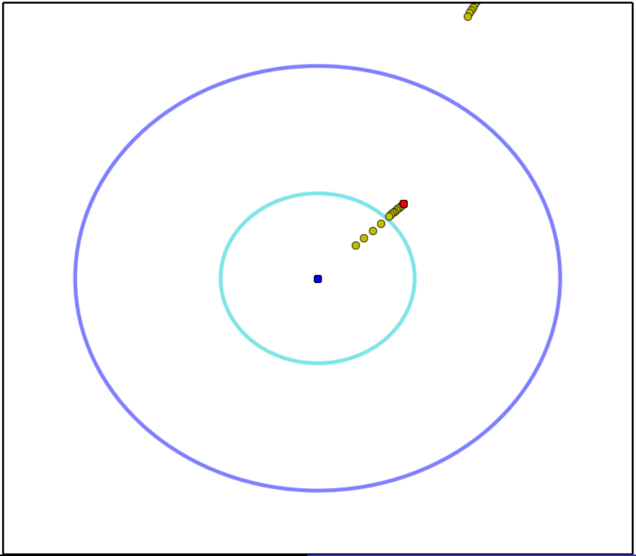
Apparent Horizon versus Event Horizon



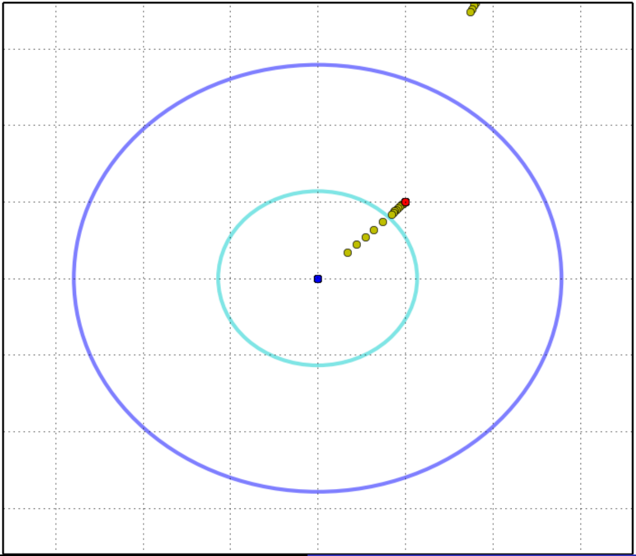
Apparent Horizon versus Event Horizon



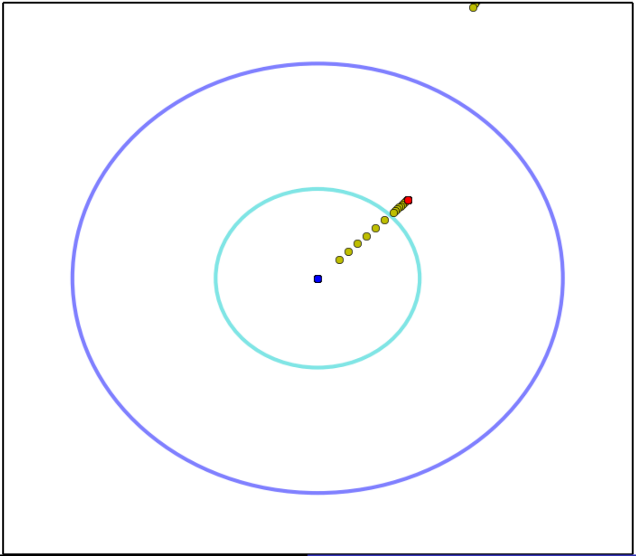
Apparent Horizon versus Event Horizon



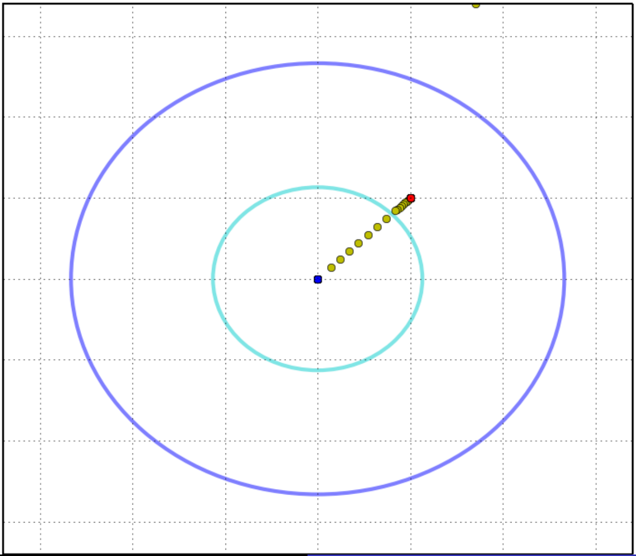
Apparent Horizon versus Event Horizon



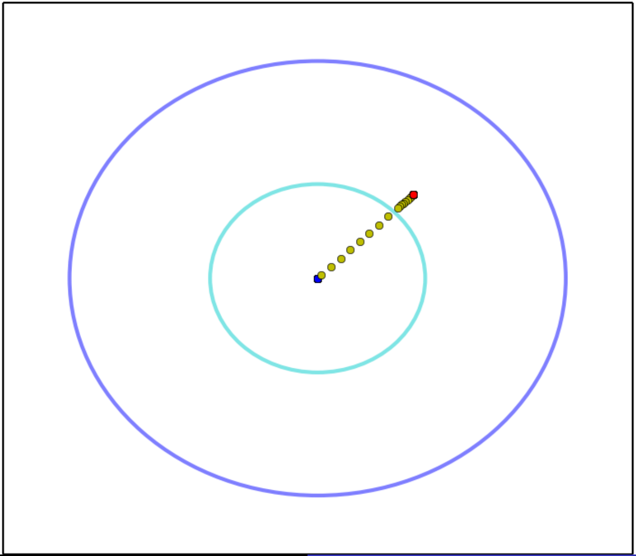
Apparent Horizon versus Event Horizon



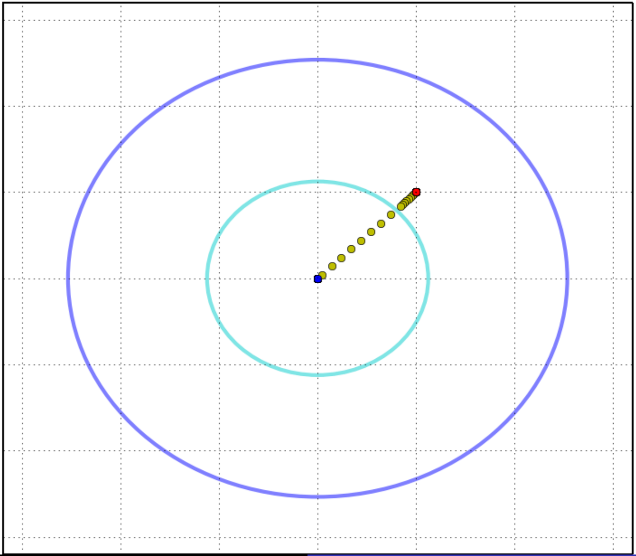
Apparent Horizon versus Event Horizon



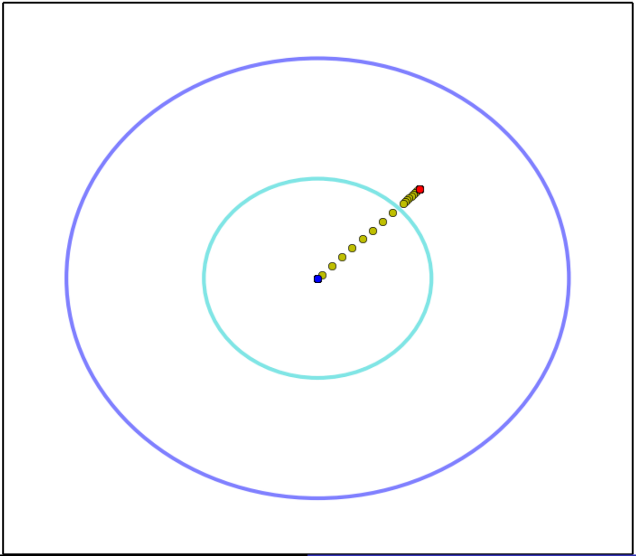
Apparent Horizon versus Event Horizon



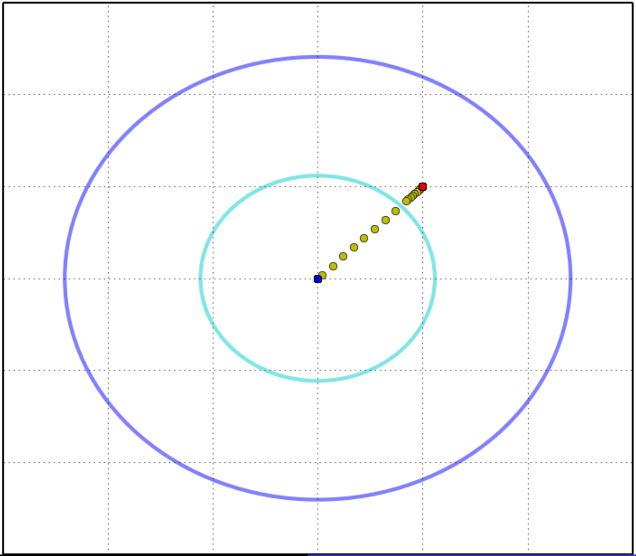
Apparent Horizon versus Event Horizon



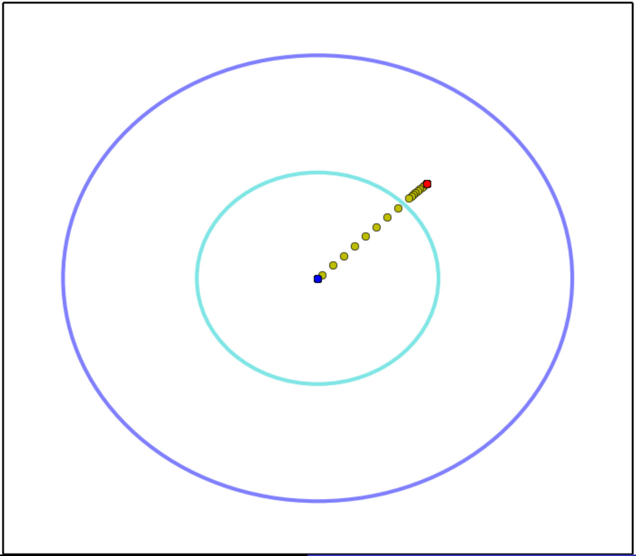
Apparent Horizon versus Event Horizon



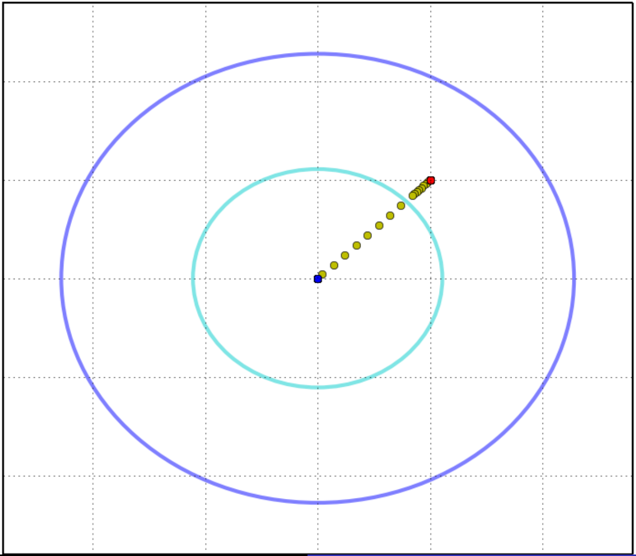
Apparent Horizon versus Event Horizon



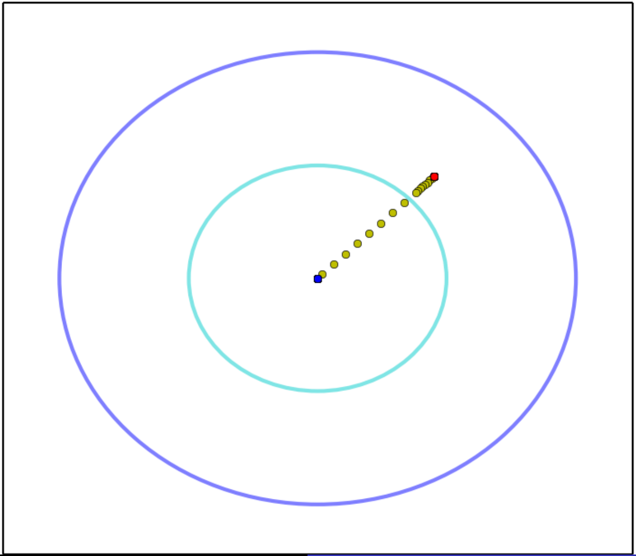
Apparent Horizon versus Event Horizon



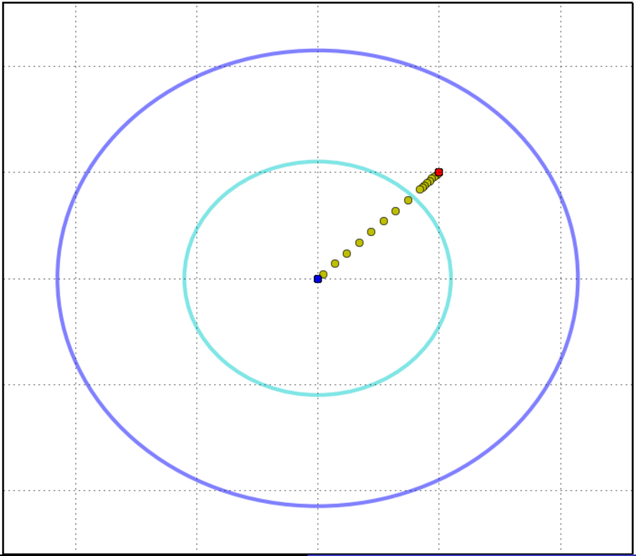
Apparent Horizon versus Event Horizon



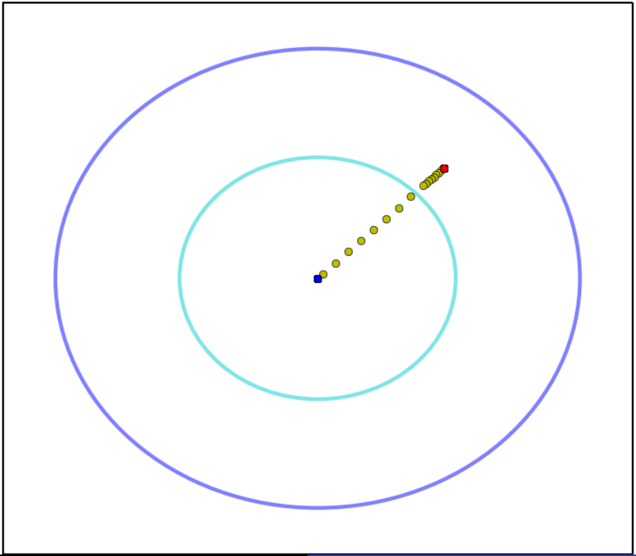
Apparent Horizon versus Event Horizon



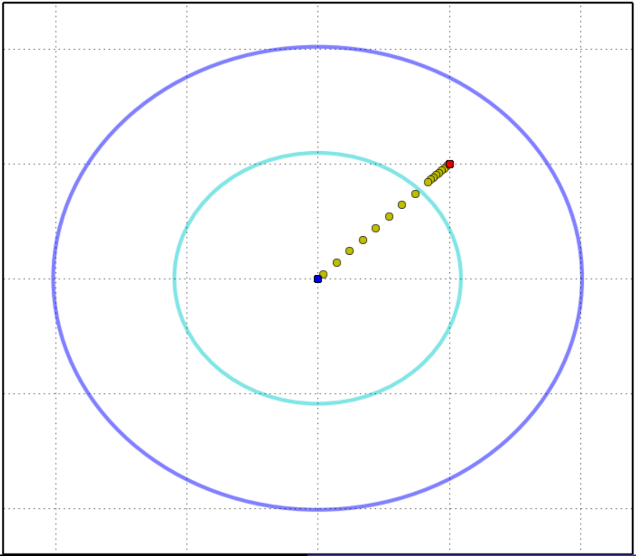
Apparent Horizon versus Event Horizon



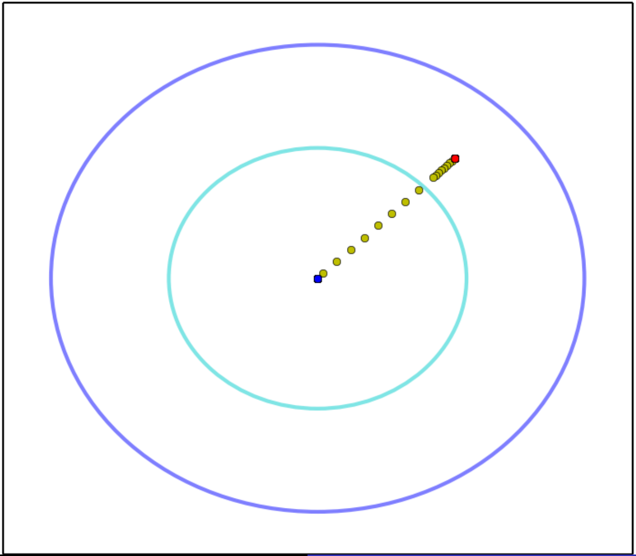
Apparent Horizon versus Event Horizon



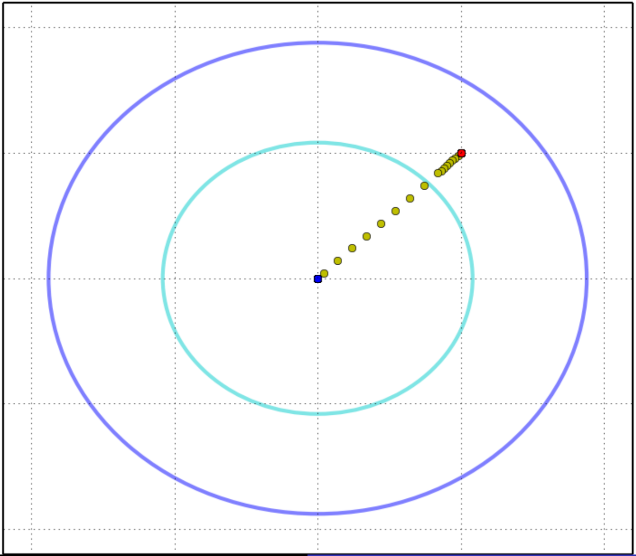
Apparent Horizon versus Event Horizon



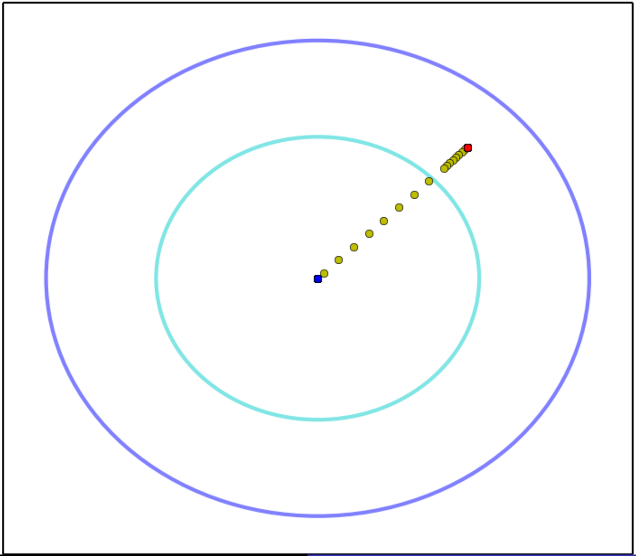
Apparent Horizon versus Event Horizon



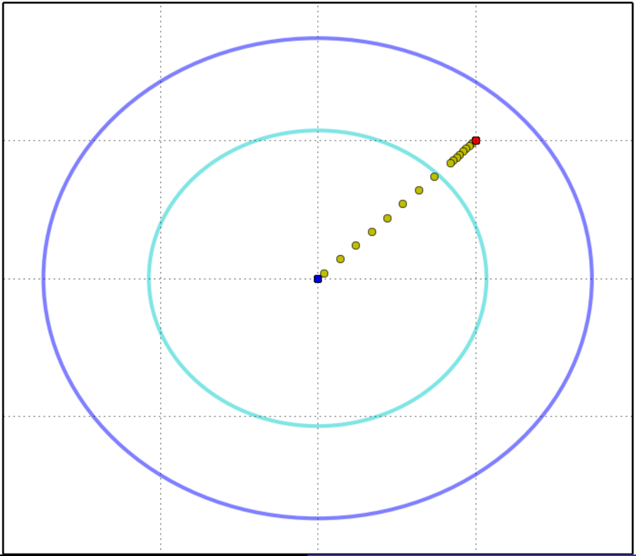
Apparent Horizon versus Event Horizon



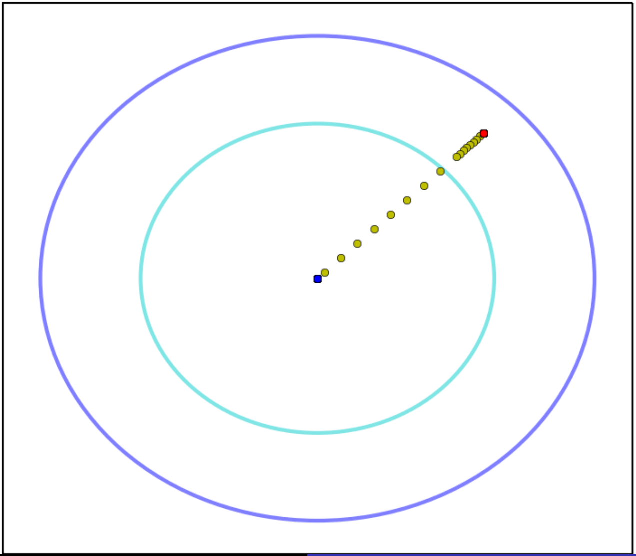
Apparent Horizon versus Event Horizon



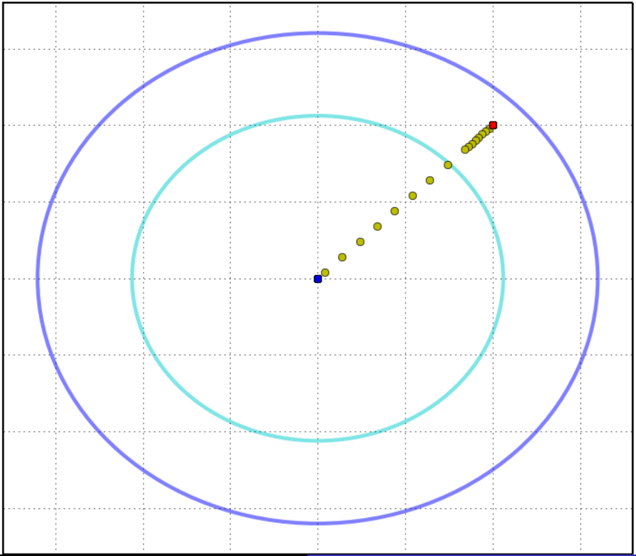
Apparent Horizon versus Event Horizon



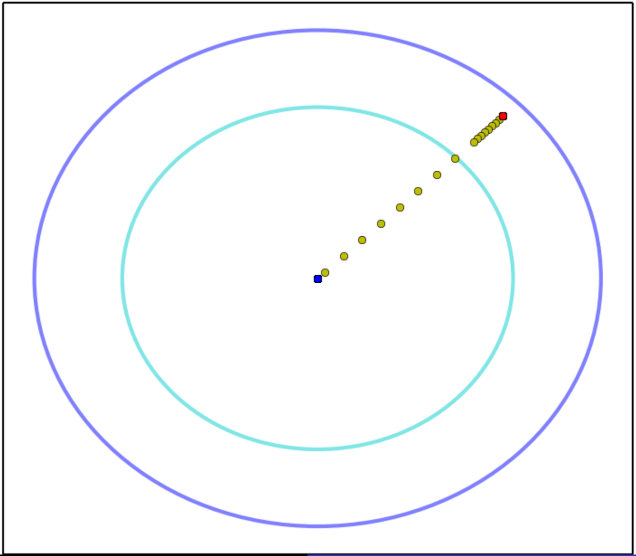
Apparent Horizon versus Event Horizon



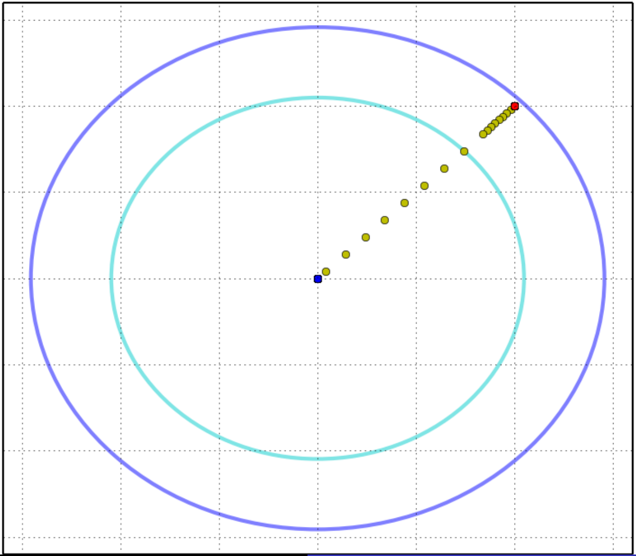
Apparent Horizon versus Event Horizon



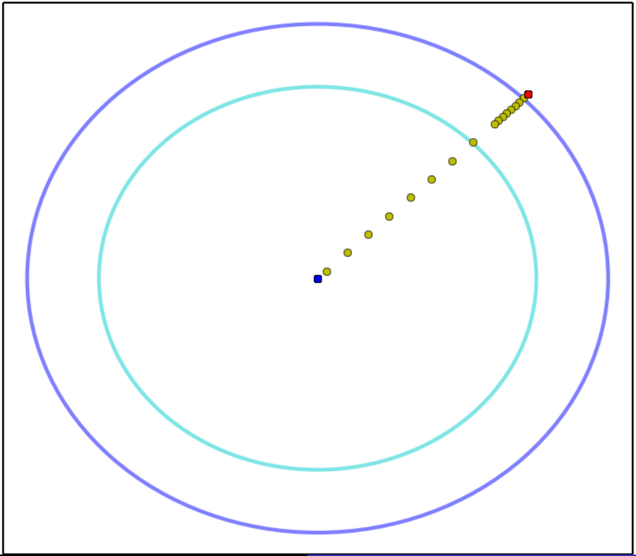
Apparent Horizon versus Event Horizon



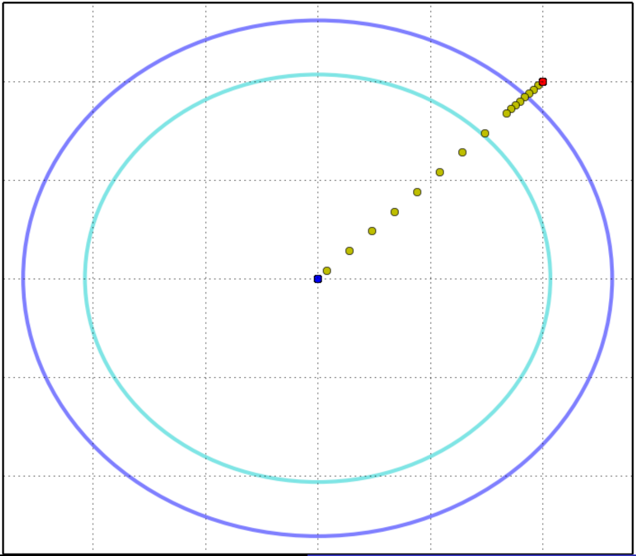
Apparent Horizon versus Event Horizon



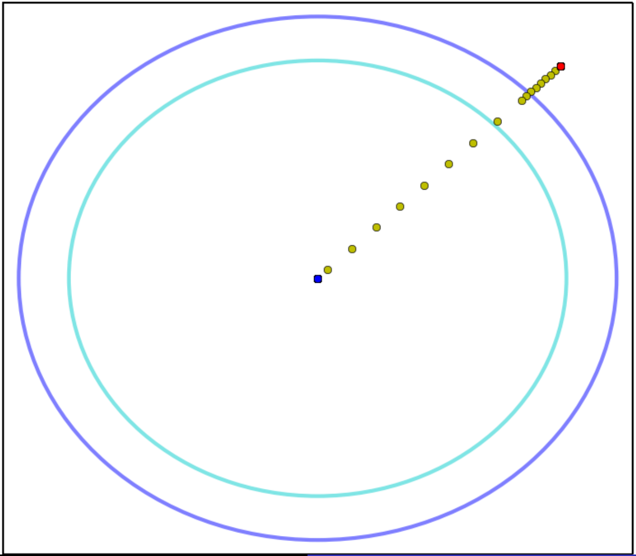
Apparent Horizon versus Event Horizon



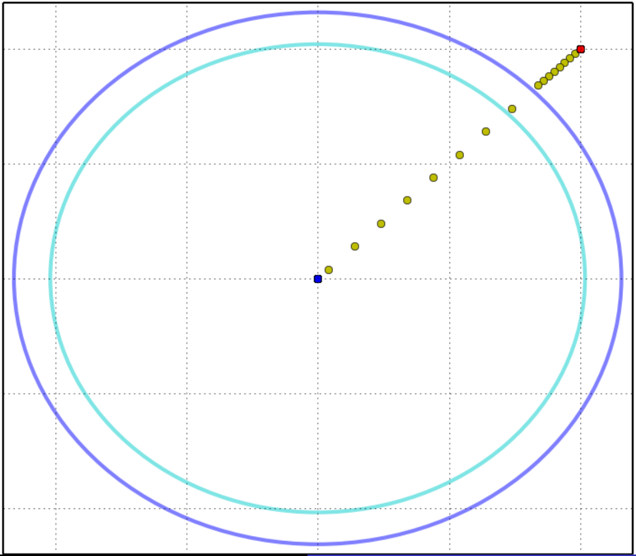
Apparent Horizon versus Event Horizon



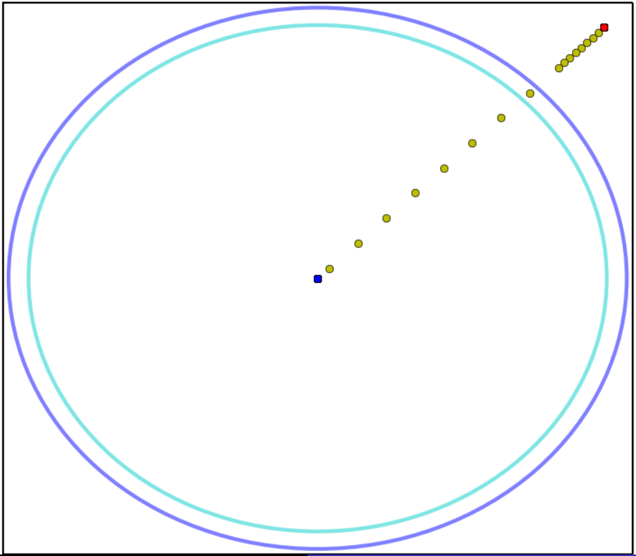
Apparent Horizon versus Event Horizon



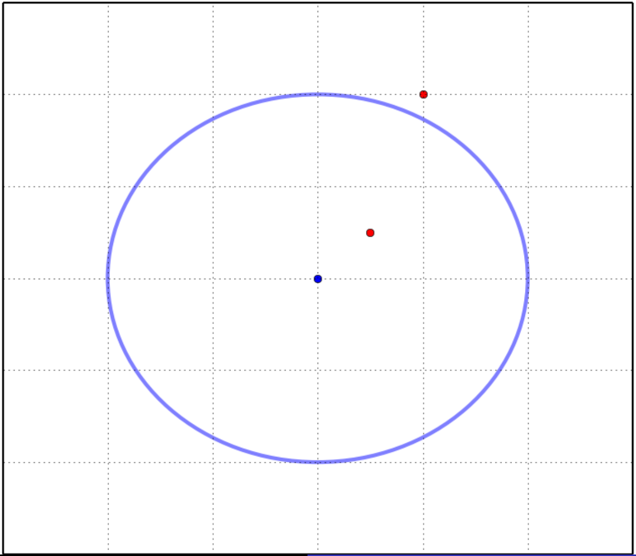
Apparent Horizon versus Event Horizon



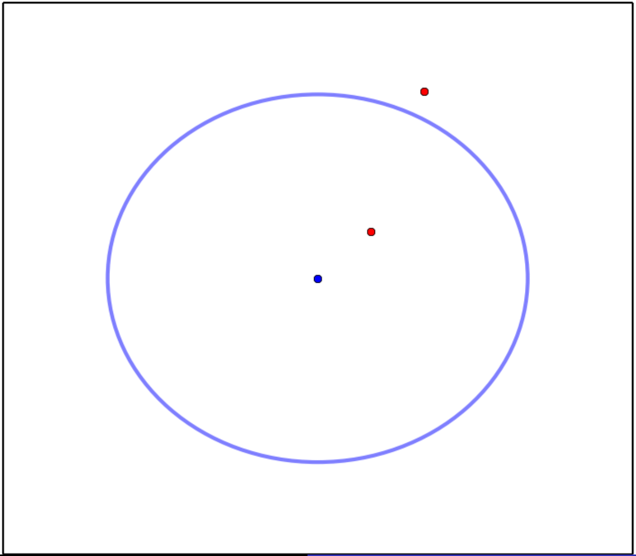
Apparent Horizon versus Event Horizon



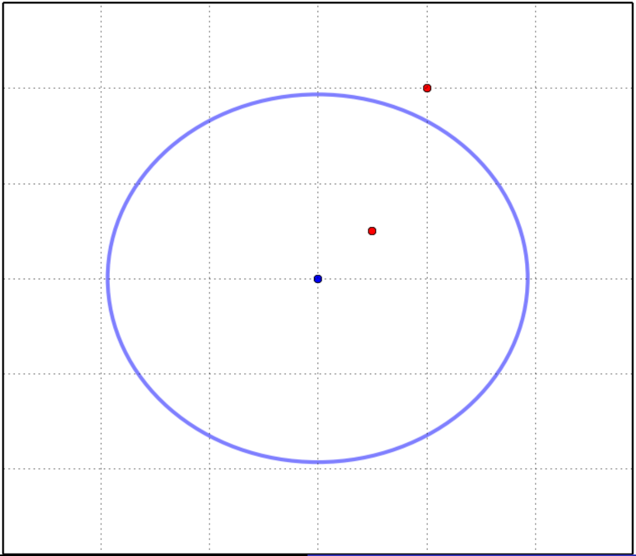
Apparent Horizon versus Event Horizon



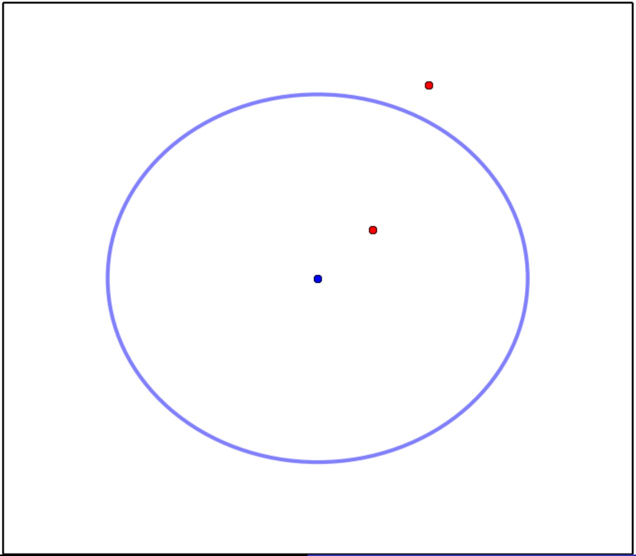
Apparent Horizon versus Event Horizon



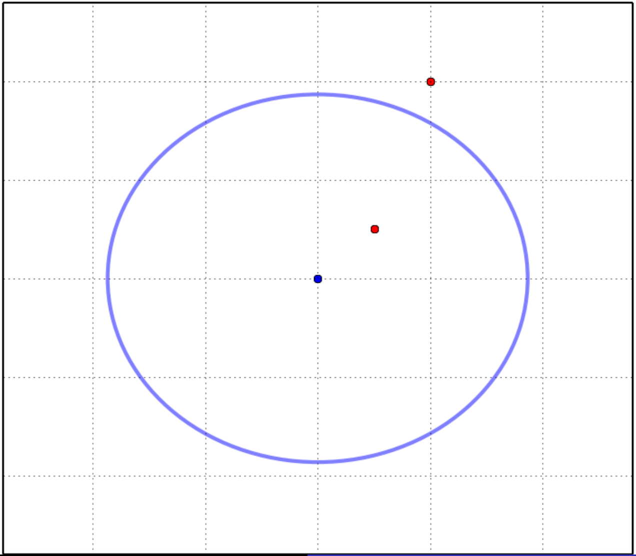
Apparent Horizon versus Event Horizon



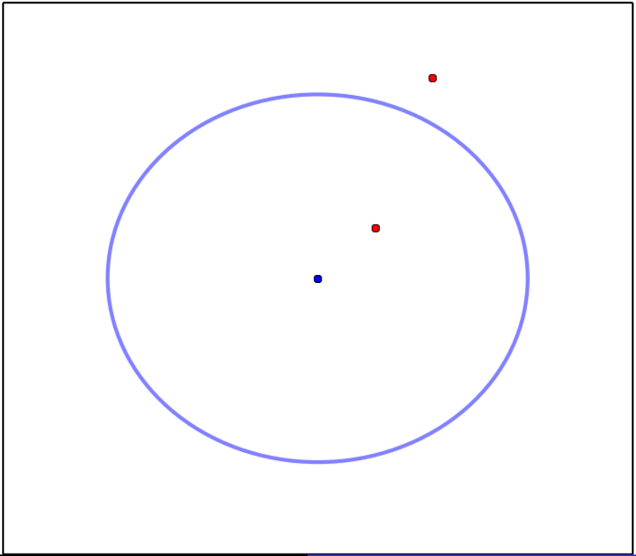
Apparent Horizon versus Event Horizon



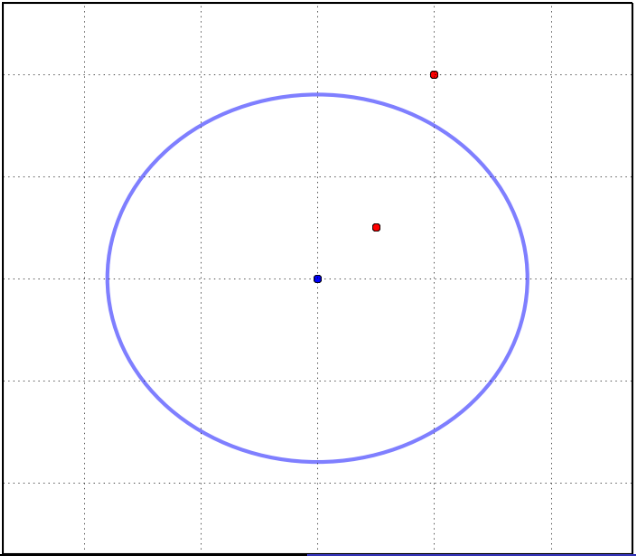
Apparent Horizon versus Event Horizon

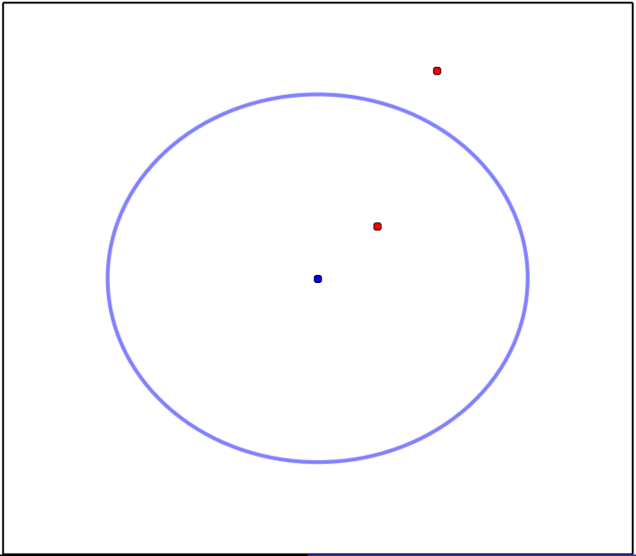


Apparent Horizon versus Event Horizon

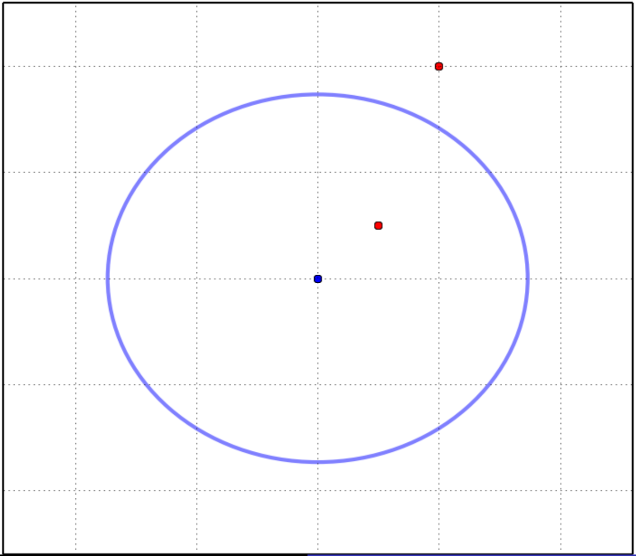


Apparent Horizon versus Event Horizon

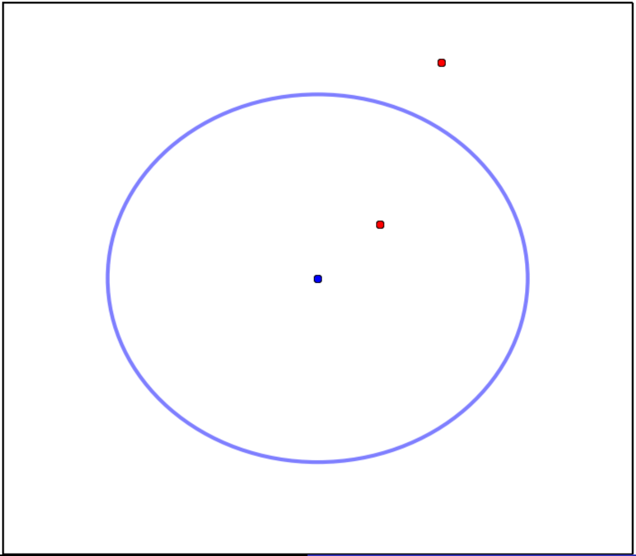




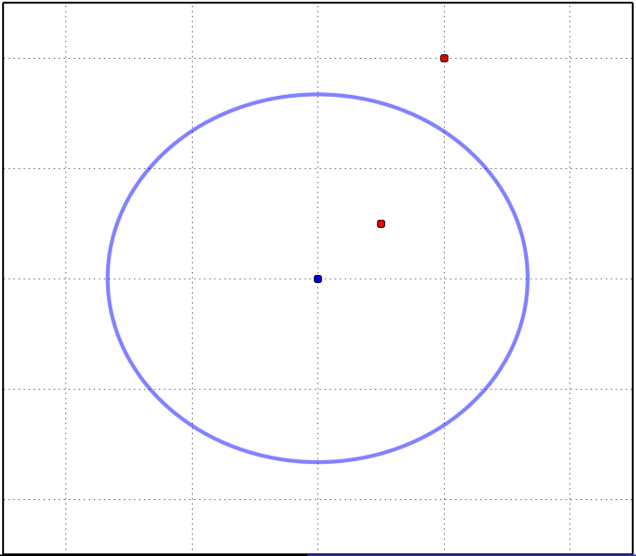
Apparent Horizon versus Event Horizon



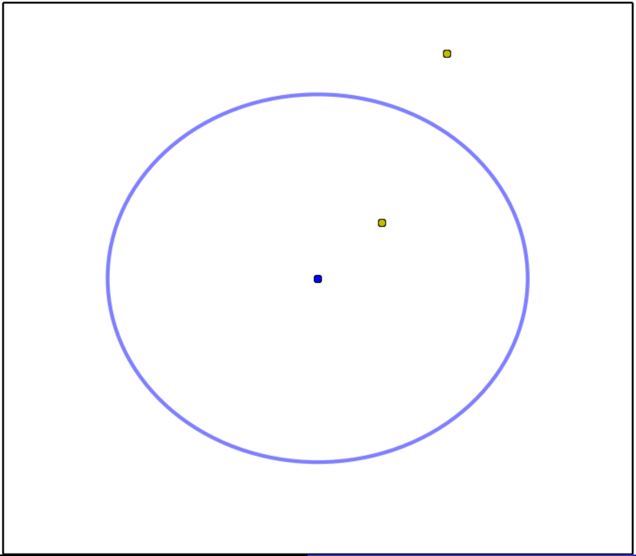
Apparent Horizon versus Event Horizon



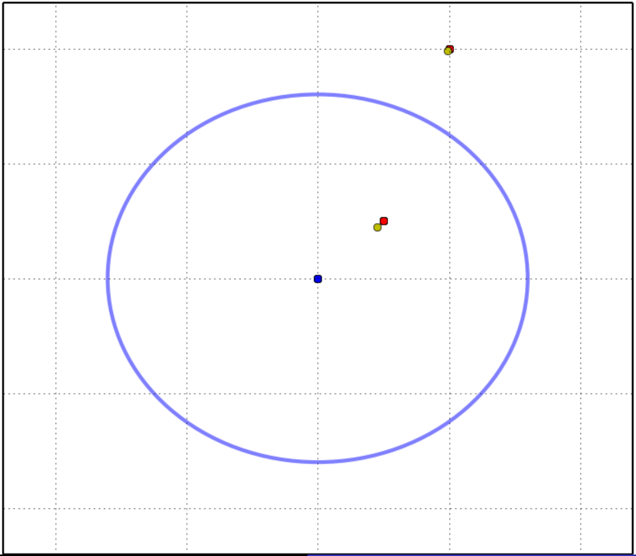
Apparent Horizon versus Event Horizon



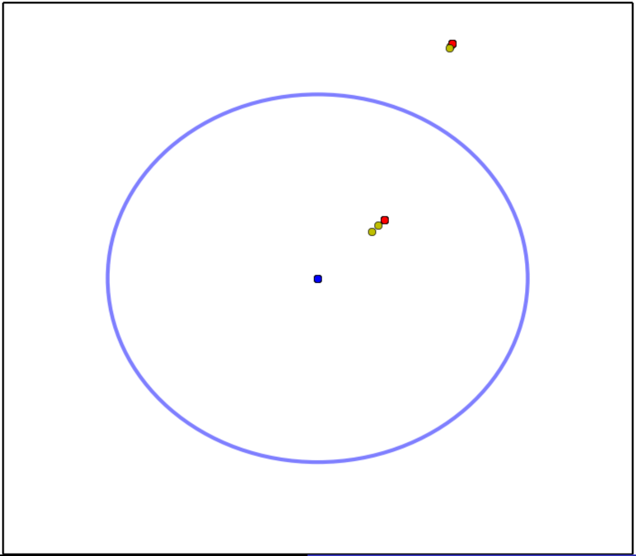
Apparent Horizon versus Event Horizon



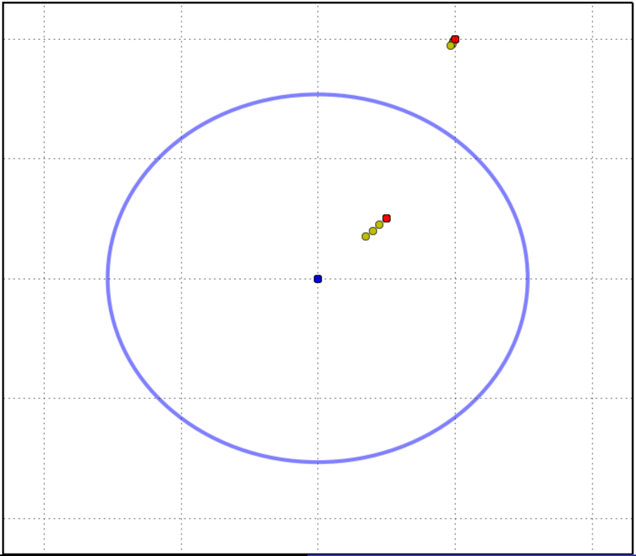
Apparent Horizon versus Event Horizon



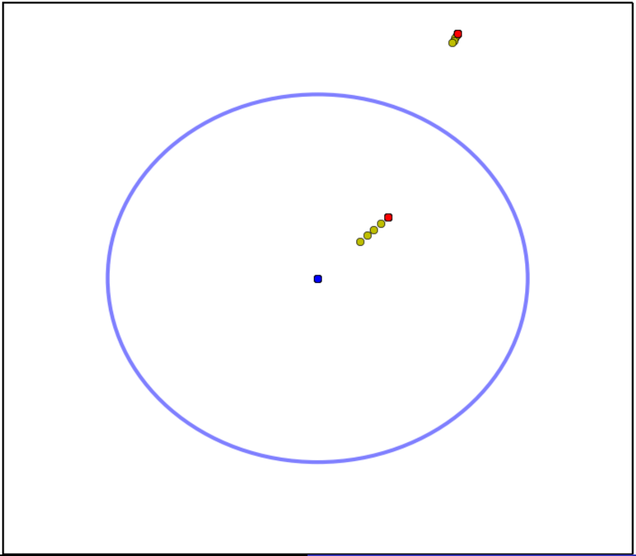
Apparent Horizon versus Event Horizon



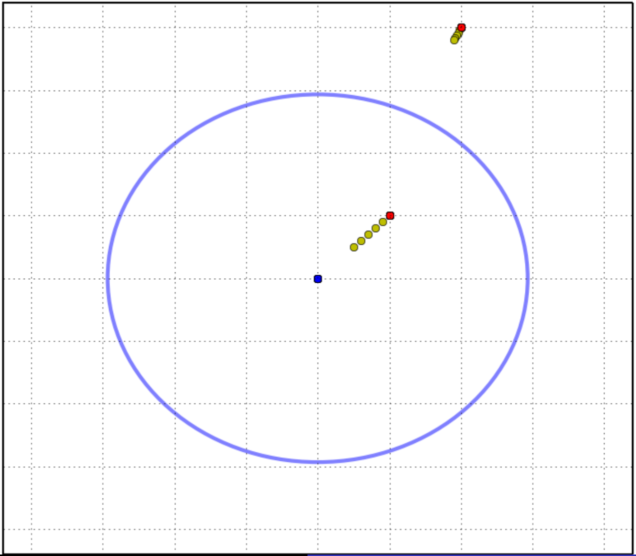
Apparent Horizon versus Event Horizon



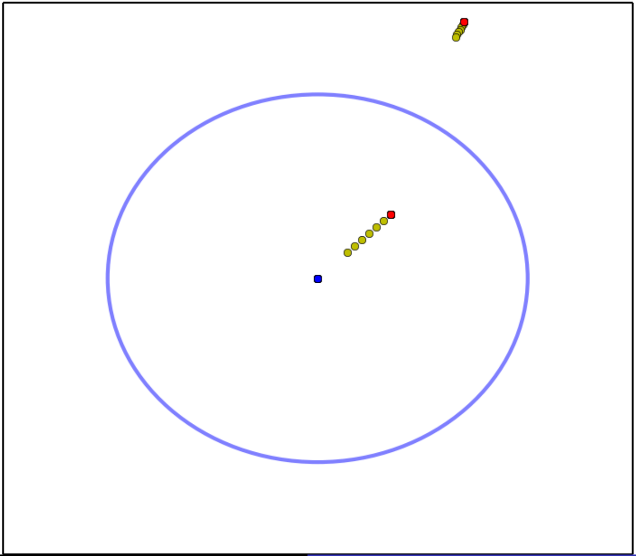
Apparent Horizon versus Event Horizon



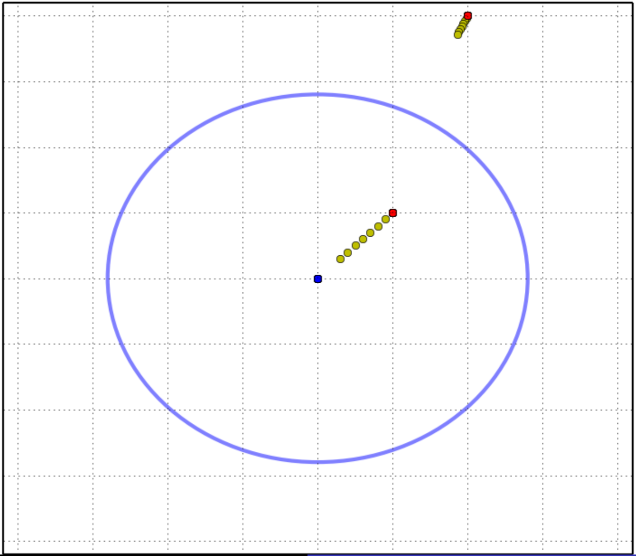
Apparent Horizon versus Event Horizon



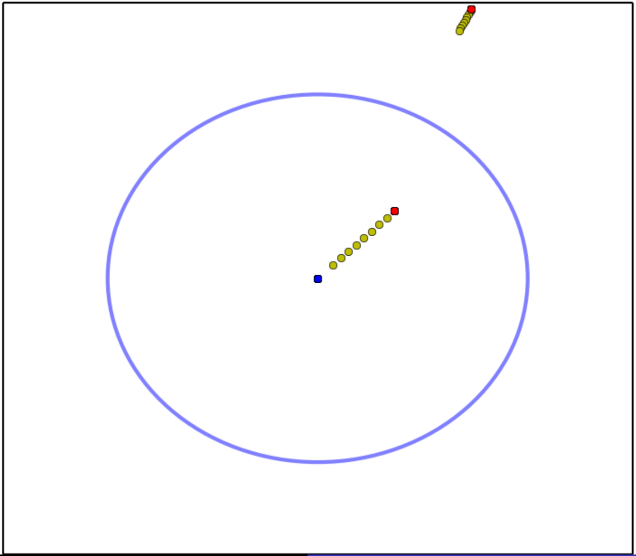
Apparent Horizon versus Event Horizon



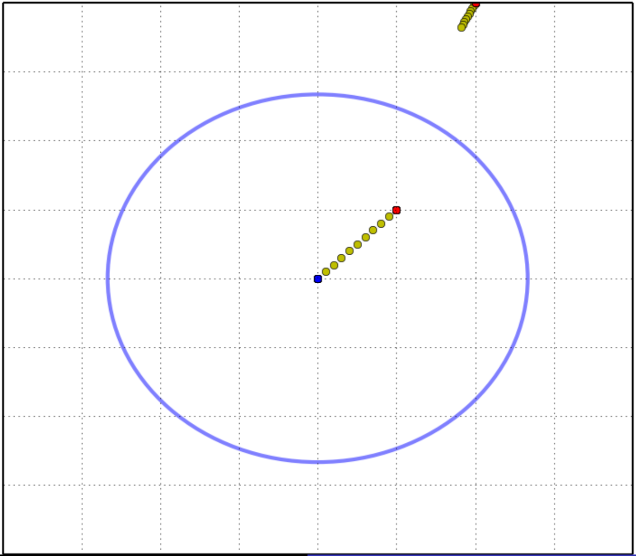
Apparent Horizon versus Event Horizon



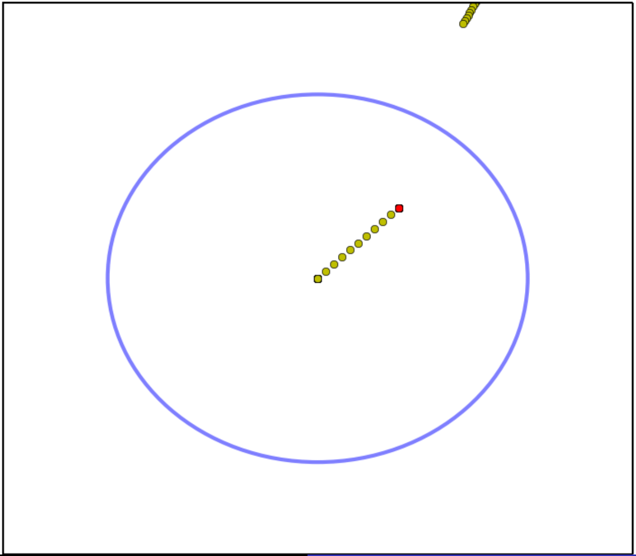
Apparent Horizon versus Event Horizon



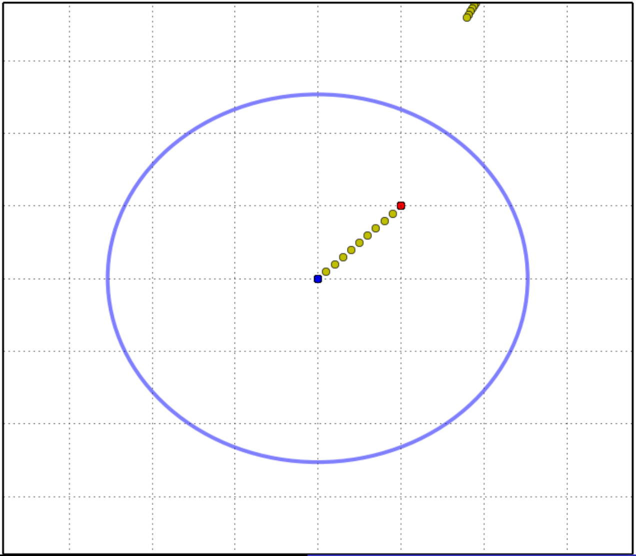
Apparent Horizon versus Event Horizon



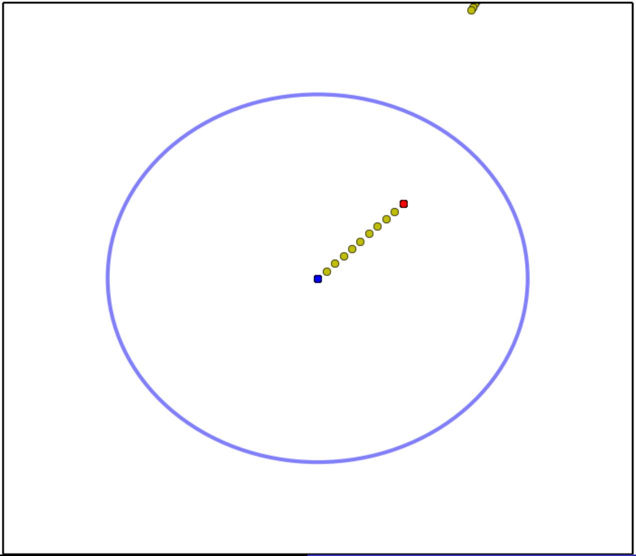
Apparent Horizon versus Event Horizon



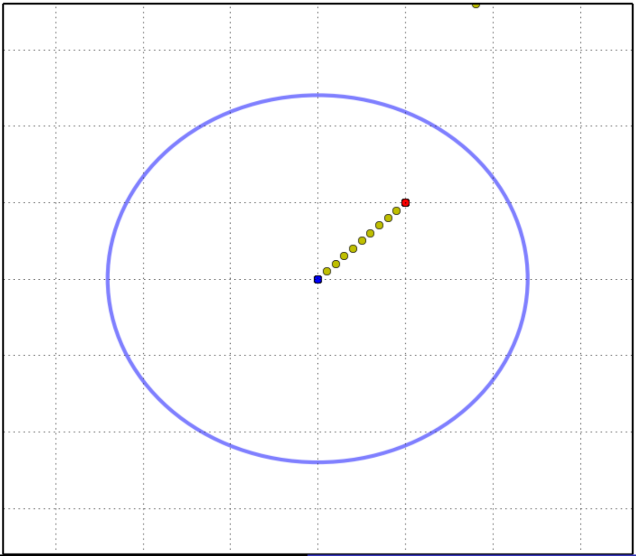
Apparent Horizon versus Event Horizon



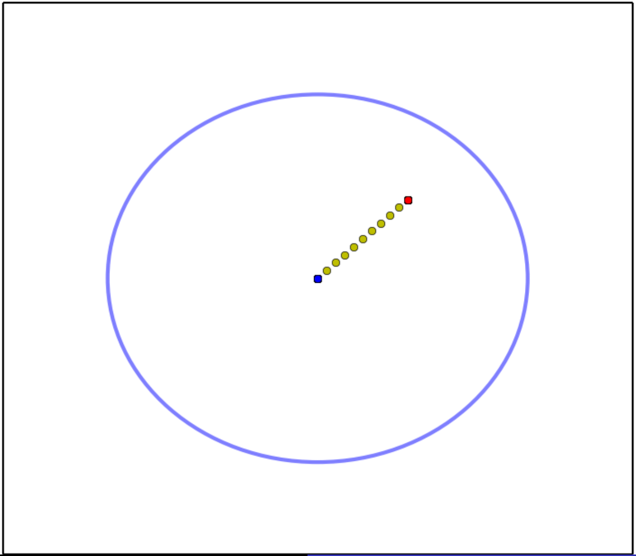
Apparent Horizon versus Event Horizon



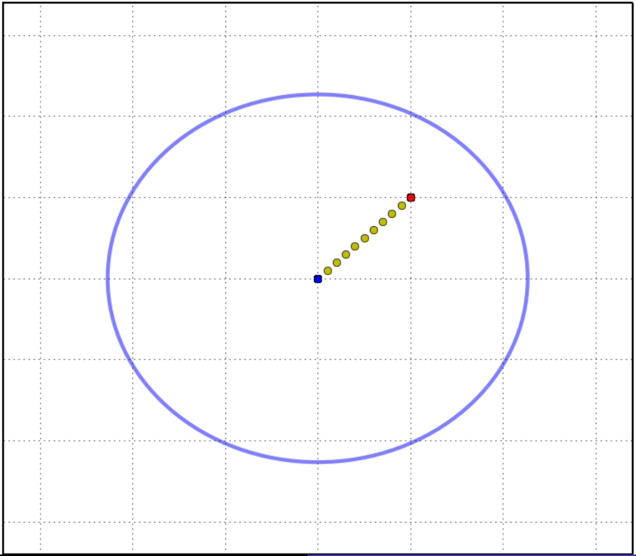
Apparent Horizon versus Event Horizon



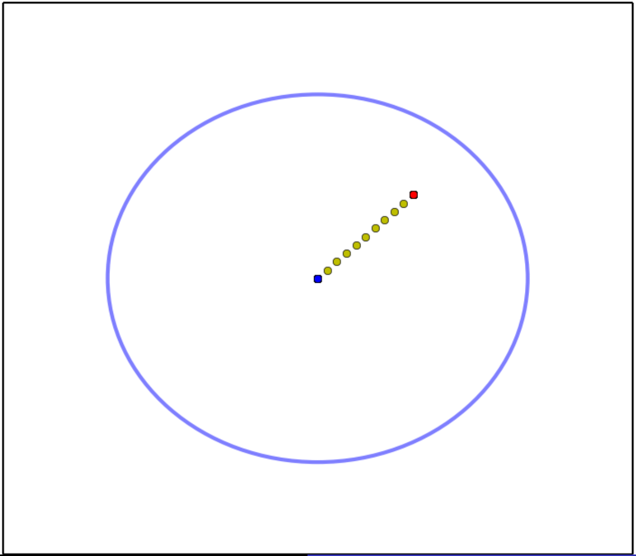
Apparent Horizon versus Event Horizon



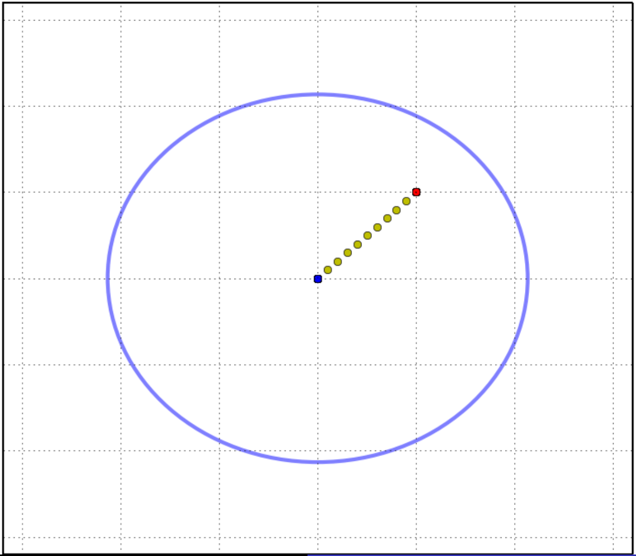
Apparent Horizon versus Event Horizon



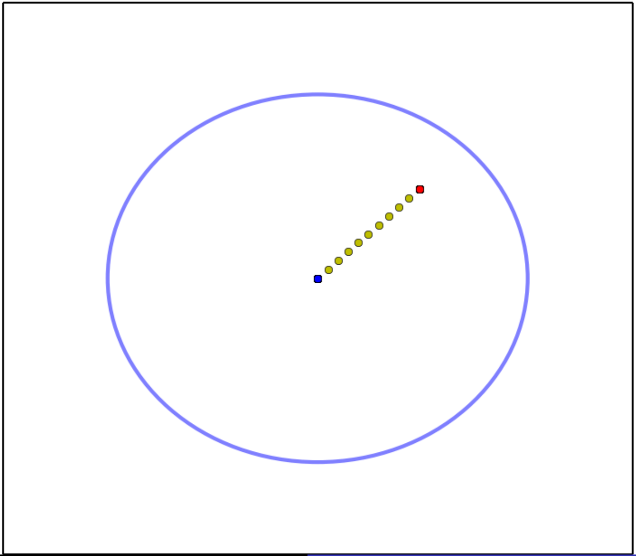
Apparent Horizon versus Event Horizon



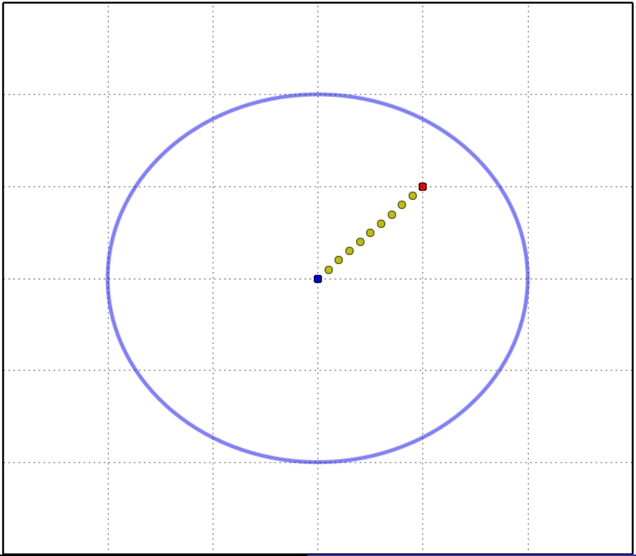
Apparent Horizon versus Event Horizon



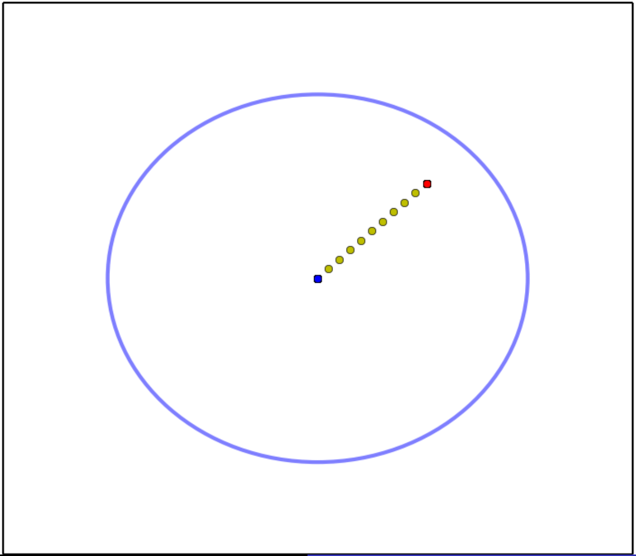
Apparent Horizon versus Event Horizon



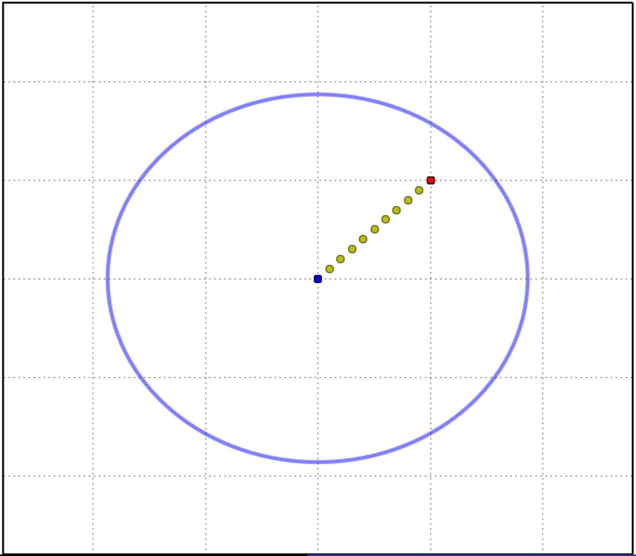
Apparent Horizon versus Event Horizon



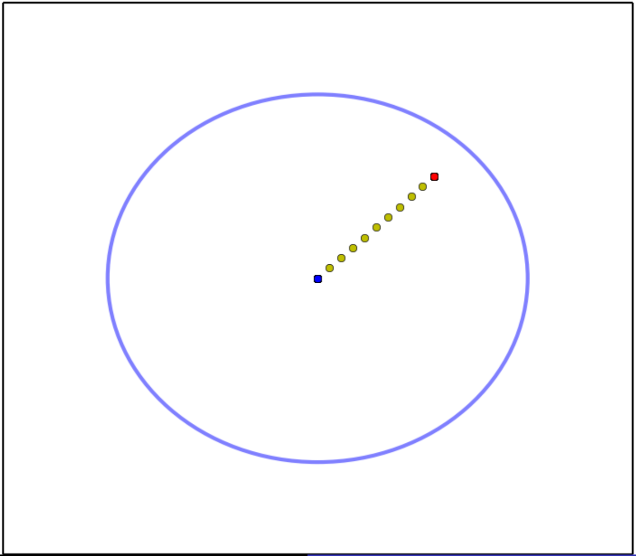
Apparent Horizon versus Event Horizon



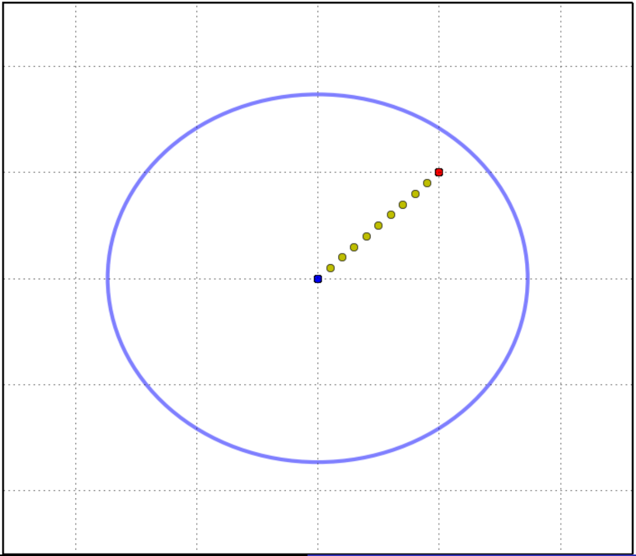
Apparent Horizon versus Event Horizon



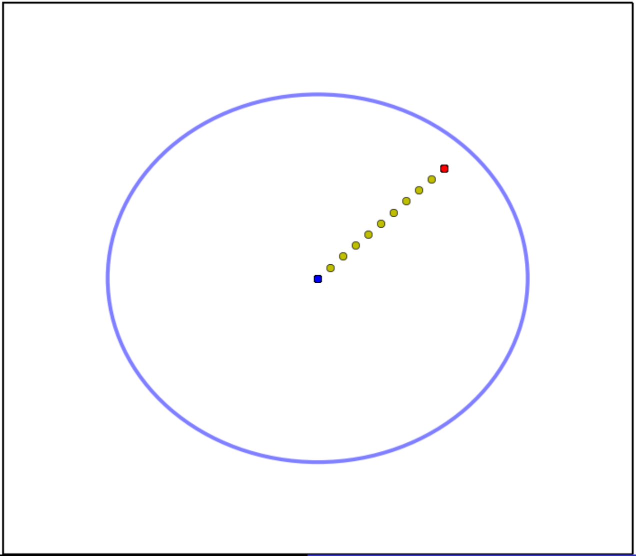
Apparent Horizon versus Event Horizon



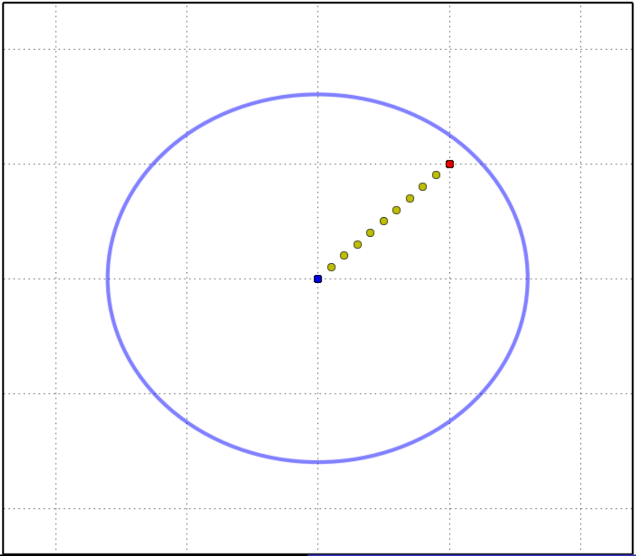
Apparent Horizon versus Event Horizon



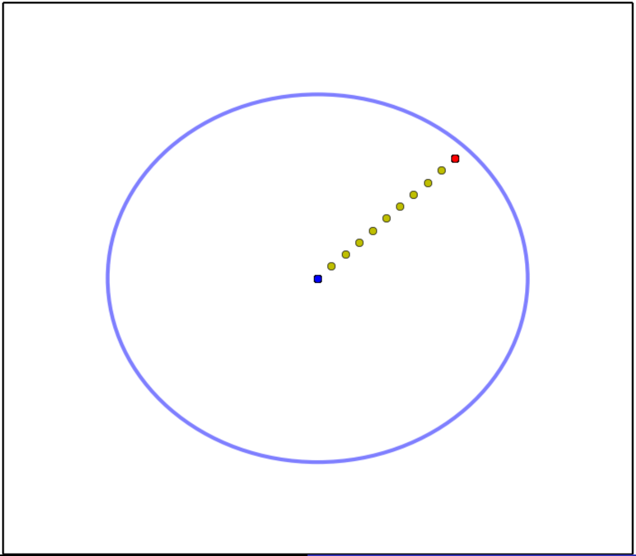
Apparent Horizon versus Event Horizon



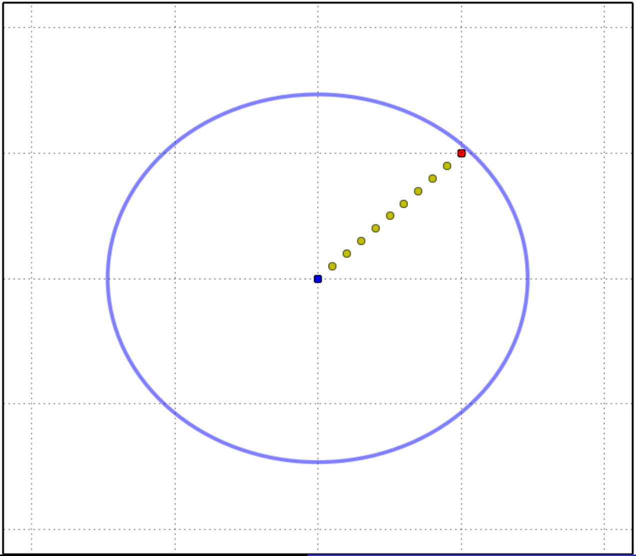
Apparent Horizon versus Event Horizon



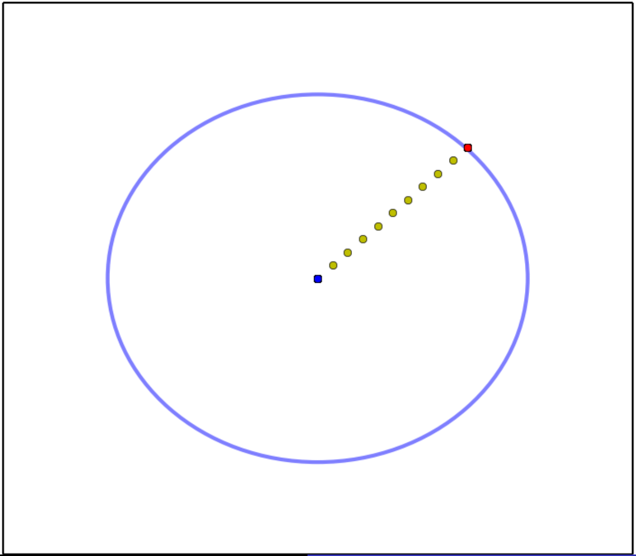
Apparent Horizon versus Event Horizon



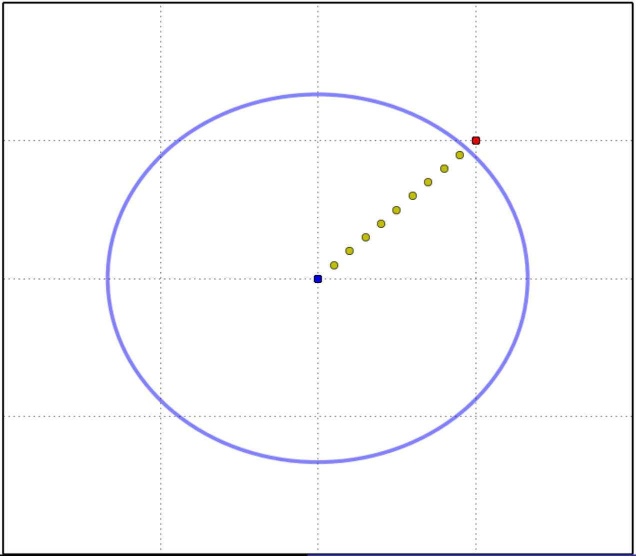
Apparent Horizon versus Event Horizon



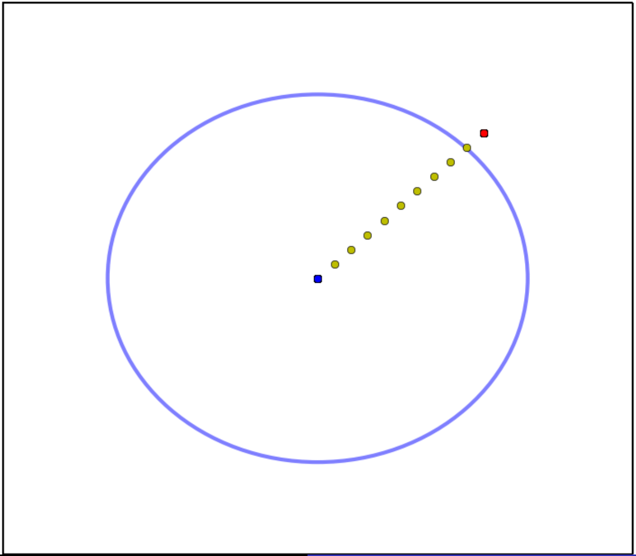
Apparent Horizon versus Event Horizon



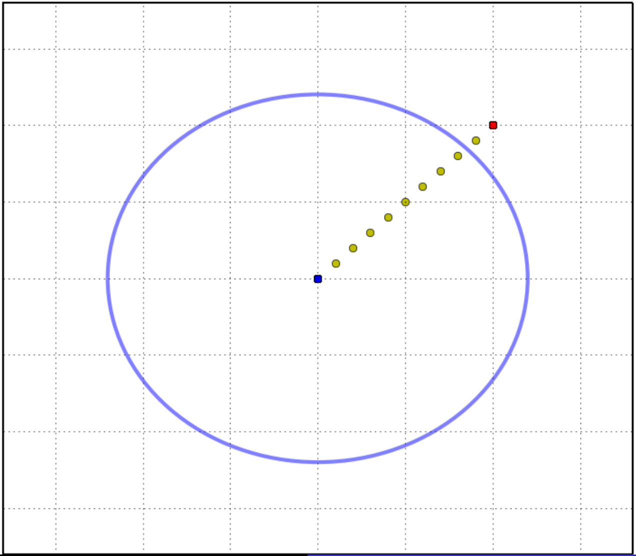
Apparent Horizon versus Event Horizon



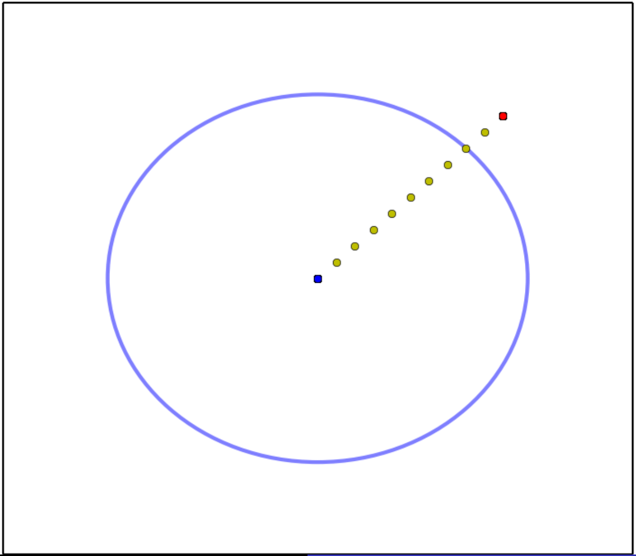
Apparent Horizon versus Event Horizon



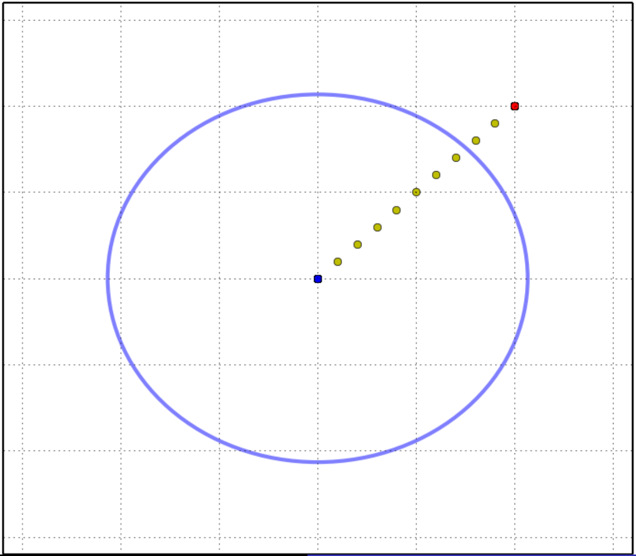
Apparent Horizon versus Event Horizon



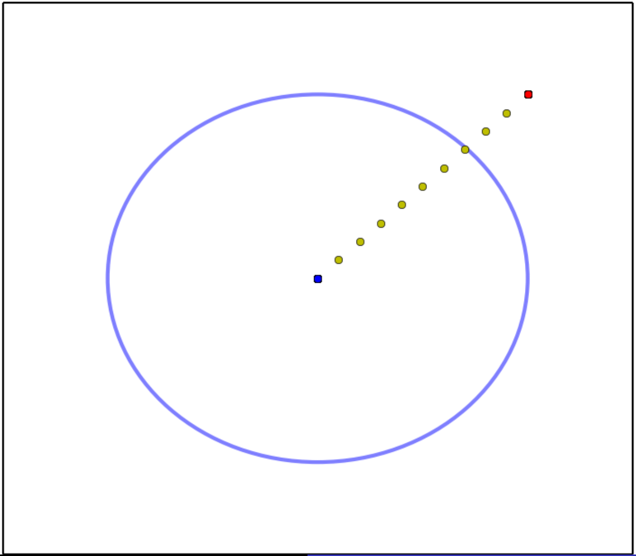
Apparent Horizon versus Event Horizon



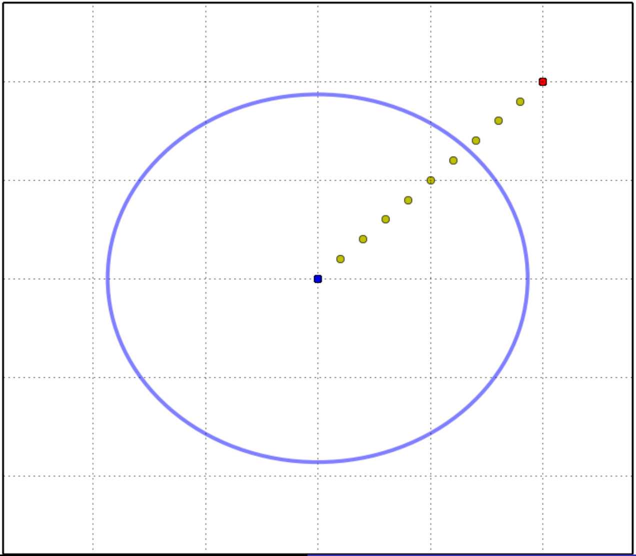
Apparent Horizon versus Event Horizon



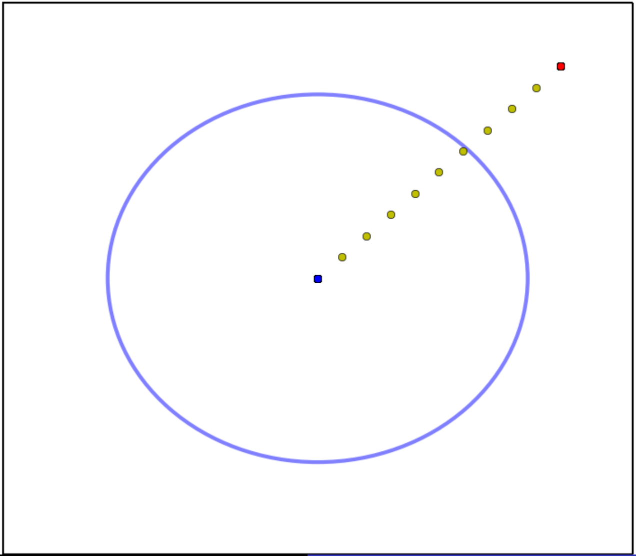
Apparent Horizon versus Event Horizon



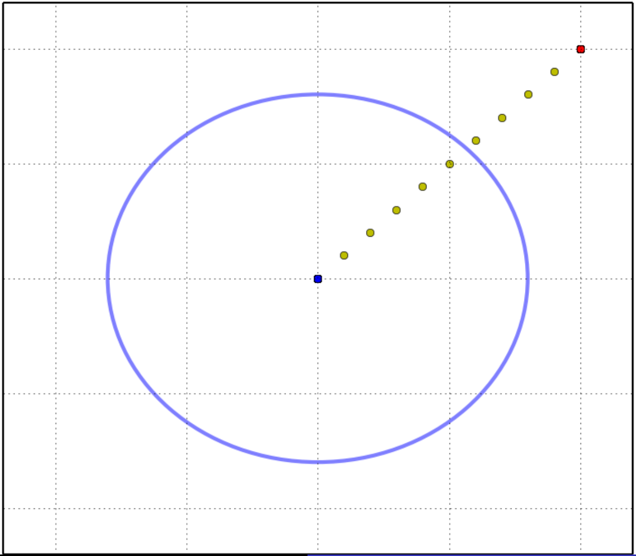
Apparent Horizon versus Event Horizon



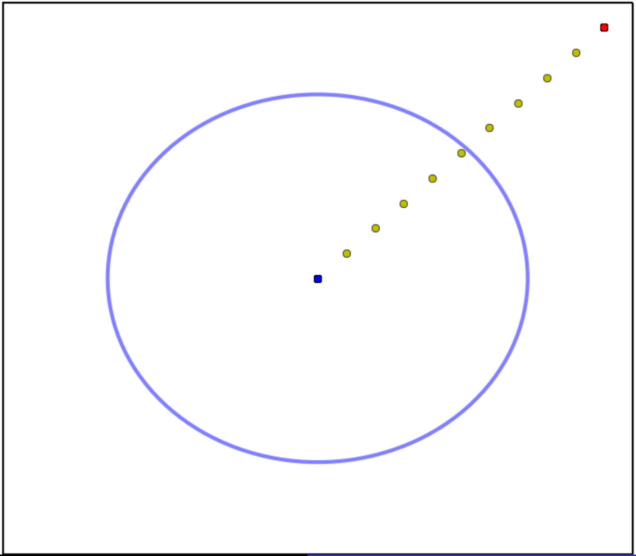
Apparent Horizon versus Event Horizon



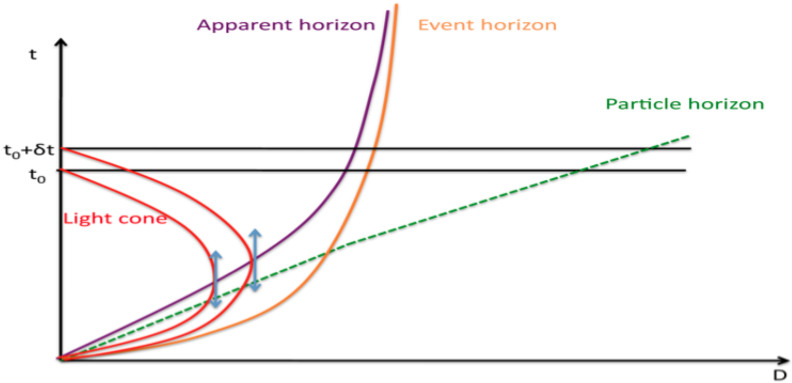
Apparent Horizon versus Event Horizon



Apparent Horizon versus Event Horizon



Dynamical FLRW



Overview

Dynamical Black Holes

How to describe a dynamical black hole?

Hayward's Machinery

- Spherical Symmetry: $ds^2 = \gamma_{ij}(x)dx^i dx^j + R^2(x)d\Omega^2$.
- Misner-Sharp energy:

$$E(R) \equiv \frac{R}{2G}\left(1 - \nabla^a R \nabla_a R\right) \ . \tag{3}$$

Why Misner-Sharp energy?
$\rightarrow$ A sphere is trapped/marginal/normal

if $\nabla^a R$ is timelike/null/spacelike.

$\rightarrow$ Thus on the Apparent Horizon: $E(R_A) = \frac{R_A}{2G}$.
This is reminiscent of the Schwarzschild Radius!
$\rightarrow$ Gravitational energy

Static spacetimes: Killing field

- Symmetries are encoded in the Killing vector fields ξ^a:

$$\nabla_a \xi_b + \nabla_b \xi_a = 0 \ . \tag{4}$$

  In particular, time-translational symmetry (for static spacetimes).

- From that we define the quantity κ, the surface gravity:

$$\xi^a \left(\nabla_a \xi_b - \nabla_b \xi_a\right) = 2\kappa \xi_b \ , \tag{5}$$

  It is this quantity that enters into the exponential and gives a thermal form to the probability distribution.
  $\rightarrow$ Hawking Radiation & Temperature: $T = \frac{\kappa}{2\pi}$.

Dynamical spacetimes: Kodama field

- If no time-translational Killing field, we can still define a Kodama field K^a:

$$K^a \equiv \epsilon_\perp^{ab} \nabla_b R \ , \tag{6}$$

which gives a preferred time direction.

- Killing's Equation is no more satisfied:

$$K^a \left(\nabla_a K_b + \nabla_b K_a \right) = 8\pi G R \psi_b \neq 0 \ . \tag{7}$$

$\rightarrow$ energy-supply vector ψ_b characterizes the departure of the Kodama from being a Killing.
$\rightarrow$ ψ_b is built on the energy-momentum tensor.

- Surface gravity (to compute the Hawking Temperature):

$$K^a \left(\nabla_a K_b - \nabla_b K_a \right) = 2\kappa \nabla_b R \tag{8}$$

$$\neq 2\kappa K_b \ . \tag{9}$$

Unified First Law

- Hayward's Unified First Law:

$$\nabla_a E = A\psi_a + \omega \nabla_a V \ . \tag{10}$$

$(A = 4\pi R^2 \ , \ V = \frac{4}{3}\pi R^3 \ , \omega \equiv -\frac{1}{2} T^{ij}\gamma_{ij} \ .)$

- Introduce a vector t^a tangent to the apparent horizon:

$$t^a . \nabla_a \left[\nabla^b R \nabla_b R \right] = 0 \ .$$

- Projecting the Unified First Law tangentially to the horizon:

$$dE = \frac{\kappa}{2\pi} dS + \omega dV \ . \tag{11}$$

- We identify the Clausius Relation:

$$\delta Q = TdS \ ,$$

with temperature $T = \frac{\kappa}{2\pi}$.

Dynamical Black Holes

- Hayward developed this formalism to describe dynamical black holes. Dynamical black holes are defined as the trapped regions of spacetime, bounded by the apparent horizon.
- From the Unified First Law projected tangentially to the apparent horizon, he recovers the first law of black hole thermodynamics.
- Since we also have an apparent horizon in cosmology, we can try to project the Unified First Law onto it: we expect to recover laws for the dynamics of our Universe, *i.e.* the Friedmann Equations.
- Let us therefore apply the previous formalism to our FLRW Universe.

Friedmann Cosmology

- FLRW metric:

$$ds^2 = -d\tau^2 + a^2(\tau)\frac{dr^2}{1 - kr^2} + R^2 d\Omega^2 \ . \tag{12}$$

- Expression for the apparent horizon radius:

$$R_A^2 = \frac{1}{H^2 + k/a^2} \ . \tag{13}$$

$\rightarrow$ reduces to the Hubble radius in the flat case (k=0).
$\rightarrow$ reduces to the event horizon radius in the de Sitter limit (k=0, H=cst).

Friedmann Cosmology

- Misner-Sharp energy:

$$E = \frac{R^3}{2G}\left[H^2 + \frac{k}{a^2}\right] \ . \tag{14}$$

- If one assumes space is flat (as seems common when using M-S energy), the expression of the energy density $\rho = E/V$ yields the First Friedmann Equation:

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho \ . \tag{15}$$

- Or the other way around, using Friedman Equation we get a flat spatial volume $V = \frac{4}{3}\pi R^3$.
$\rightarrow$ curvature information encoded in the full gravitational energy.

Friedmann Cosmology

- Kodama vector:

$$K^a = \sqrt{1 - kr^2}(1; -Hr; 0; 0)\ . \tag{16}$$

$\rightarrow$ reduces to the dilatational Killing vector in the de Sitter limit (k=0, H=cst).

- Surface gravity:

$$\kappa = -\frac{R}{2}\left[2H^2 + \dot{H} + \frac{k}{a^2}\right]\ . \tag{17}$$

This is valid for all R, but the physical meaning is clear only for $R = R_A$.
$\rightarrow$ we expect $\kappa|_{\mathcal{H}}$ to give the Hawking Temperature of our apparent horizon (and we will prove it).
$\rightarrow$ reduces to the inflationary temperature in the de Sitter limit: $T = \frac{|\kappa|}{2\pi} = \frac{H}{2\pi}$.

Overview

Dynamics at the horizon

- Now that we have computed the relevant quantities, we can try to project the Unified First Law onto the horizon: we will obtain the Clausius Relation, and from there we expect to get the Friedmann Equations.
- Gravity from Thermodynamics?...
- ...this is very reminiscent of a famous work by Jacobson:

Thermodynamics of Spacetime:
The Einstein Equation of State,

Ted Jacobson, Phys. Rev. Lett. 75: 1260-1263, 1995

- We use the same concepts as Jacobson, but in our global cosmological setting.

- The expansion is defined by: $\theta = h^{cd}\nabla_c k_d$, with h_{cd} the induced metric on the 2-spheres.
- In FLRW metric it yields:

$$\theta_{in} = H - \frac{1}{R}\sqrt{1 - kr^2}\ . \tag{18}$$

for the future-directed ingoing light-rays. As expected by definition, the apparent radius R_A cancels this expansion.

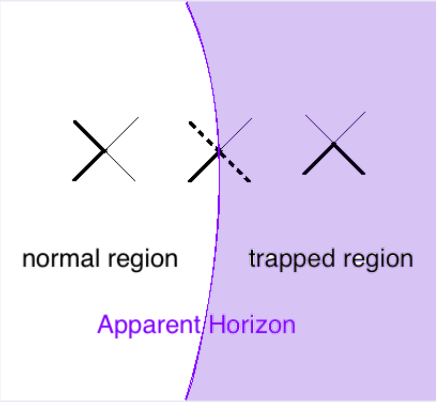
- And for the future-directed outgoing light-rays:

$$\theta_{out} = H + \frac{1}{R}\sqrt{1 - kr^2}\ . \tag{19}$$

which is non-zero on the horizon.

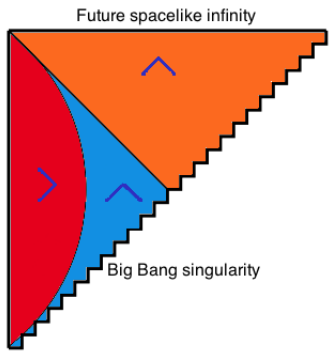
- On the apparent horizon $\theta_{in} = 0$ and $\theta_{out} \geqslant 0$. Thus the apparent horizon of a FLRW Universe is of the past-inner type.

Past-inner Horizon



- On the apparent horizon $\theta_{in} = 0$ and $\theta_{out} \geqslant 0$. Thus the apparent horizon of a FLRW Universe is of the past-inner type.

Past-inner Horizon



(Shaghoulian, 2014)

- As seen above, we need to project the Unified First Law tangentially to the horizon to get Clausius Relation:

$$t^a.(A\psi_a) \overset{\mathcal{H}}{=} t^a.\left(\frac{\kappa}{8\pi G}\nabla_a A\right) \ . \tag{20}$$

- We get for the left-hand side:

$$At^a\psi_a|_{\mathcal{H}} \quad = \quad 2\pi\kappa H R_A^4(p+\rho) \ .$$

- Now for the right-hand side:

$$\frac{\kappa}{8\pi G}\ t^a\nabla_a A|_{\mathcal{H}} = -\frac{1}{2G}\kappa H R_A^4\left(\dot{H} - \frac{k}{a^2}\right) \ . \tag{21}$$

- Equating RHS and LHS:

$$2\pi\kappa H R_A^4 (p+\rho) = -\frac{1}{2G}\kappa H R_A^4 \left(\dot{H} - \frac{k}{a^2}\right) , \tag{22}$$

we get the 2^{nd} Friedmann Equation:

$$\dot{H} - \frac{k}{a^2} = -4\pi G(p+\rho) . \tag{23}$$

- Idea: Einstein Equations obtained from Clausius Relation $\delta Q = TdS$ applied on a local Rindler horizon.

Left-hand side

- Link between δQ and energy-momentum tensor:

$$\delta Q = \int_{\mathcal{H}} T_{ab}\xi^a d\Sigma^b \ . \tag{24}$$

We need to equate this LHS expressed in terms of energy to a RHS expressed in terms of geometry (Ricci tensor).

Right-hand side

- Link between entropy S and area A, as is usual for horizons.
- Link between area A and expansion θ of the null congruence of horizon generators: $\delta A = \int \theta d\lambda dA$.
- Link between expansion θ and geometry R_{ab} using Raychaudhuri equation:

$$\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \sigma^2 - R_{ab}k^a k^b \ . \tag{25}$$

- Key point: working on a "local Rindler horizon", where the expansion is zero. This is just the local version of our global apparent horizon!

Overview

- Advanced Eddington-Finkelstein coordinates:

$$ds^2 = -e^{2\Psi} C dx_+^2 + 2e^{\Psi} dx_+ dR + R^2 d\Omega^2 \ .$$

(flat case: $x_+ = \eta + r$)

- Retarded Eddington-Finkelstein coordinates:

$$ds^2 = -e^{2\Psi} C dx_-^2 - 2e^{\Psi} dx_- dR + R^2 d\Omega^2 \ .$$

(flat case: $x_- = \eta - r$)

-Kodama vector: $K^a = (e^{-\Psi}; 0; 0; 0)$.

-Surface gravity: $\kappa_{\mathcal{H}} = \frac{1}{2} \partial_R C|_{\mathcal{H}}$.

-BKW approximation of tunneling probability:

$$\Gamma \propto exp\left(-2\frac{Im\mathcal{I}}{\hbar}\right) \ . \qquad (26)$$

Hawking Radiation at the horizon

First method: Hamilton-Jacobi

- EoM: $k(Ck - 2\omega) = 0$.
  $\rightarrow k = +2\omega/C$ (outgoing sol).
  $\rightarrow$ k=0 (ingoing solution).
- Outgoing solution contributes:

$$Im\mathcal{I} = +\frac{\pi\omega}{\kappa_{\mathcal{H}}} \; .$$

- EoM: $k(Ck + 2\omega) = 0$.
  $\rightarrow$ k=0 (outgoing solution).
  $\rightarrow k = -2\omega/C$ (ingoing sol).
- Ingoing solution contributes:

$$Im\mathcal{I} = -\frac{\pi\omega}{\kappa_{\mathcal{H}}} \; .$$

The tunneling probability takes a thermal form:

$$\Gamma \propto exp\left(-2Im\mathcal{I}\right) \propto exp(-\omega/T) \; ,$$

- with temperature:

$$T = +\frac{\kappa_{\mathcal{H}}}{2\pi} \geqslant 0 \; .$$

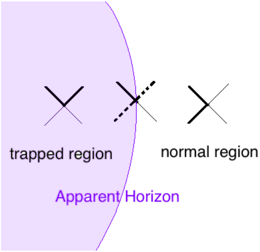
- with temperature:

$$T = -\frac{\kappa_{\mathcal{H}}}{2\pi} \geqslant 0 \; .$$

- Differences with black holes: $\kappa_{\mathcal{H}}$ negative & Hawking radiation aimed at the central observer, to the inside.
  $\rightarrow$ difference between future-outer and past-inner trapped!

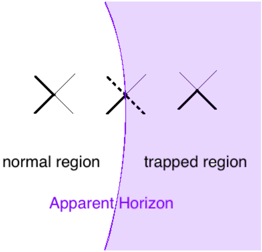
Future-outer trapped

$\theta_{out} = 0$



Past-inner trapped

$\theta_{in} = 0$

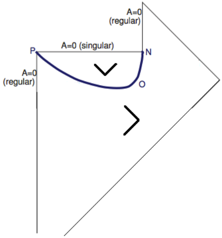


Hawking Radiation at the horizon

- Differences with black holes: $\kappa_{\mathcal{H}}$ negative & Hawking radiation aimed at the central observer, to the inside.
  $\rightarrow$ difference between future-outer and past-inner trapped!

Future-outer trapped

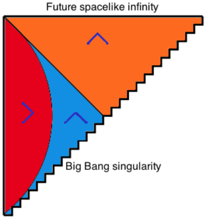
$\theta_{out} = 0$



(Hayward, 2005)

Past-inner trapped

$\theta_{in} = 0$



(Shaghoulian, 2014)

Acceleration and the Unruh effect

- Unruh effect: accelerated observer in Minkowski spacetime feels a thermal bath.
  $\rightarrow$ Unruh Temperature T_U .
- Detector hovering above the black hole sees radiation.
- To recover the Hawking temperature, lower the detector to the horizon, and redshift the measured temperature to an observer at infinity.

Hawking Radiation at the horizon
Second method: acceleration of a Kodama observer

- The 4-acceleration:

$$A^a = U^b \nabla_b U^a = U^b(\partial_b U^a + \Gamma^a{}_{bc} U^c) \ . \qquad (27)$$

- For a Kodama observer: $U^a \propto K^a$. Thus A^a is most easily computed in a coordinate system where K^a has only one component, and Γ are the simplest (diagonal metric). Such a frame is provided by the Kodama time $\flat$:

$$\mathrm{d}\flat = dx_- + \frac{e^{-\Psi}}{C} dR \ , \qquad (28)$$

yielding the metric:

$$ds^2 = -e^{2\Psi} C \mathrm{d}\flat^2 + \frac{1}{C} dR^2 + R^2 d\Omega^2 \ . \qquad (29)$$

- In this frame, a fiducial observer has $U^a = (e^{-\Psi}/\sqrt{C}; 0; 0; 0)$.
- The acceleration is:

$$A = \frac{1}{\sqrt{C}} \left(C\partial_R \Psi + \frac{1}{2}\partial_R C \right) \ . \tag{30}$$

- This is a diverging quantity at the horizon: $C|_{\mathcal{H}} = 0$. In order to obtain the Hawking temperature, we need to redshift the acceleration of a detector at the horizon ($R = R_A$), as seen by an observer on Earth (R'=0). We get:

$$T = T_{U,0}(R = R_A) = \frac{1}{2\pi} \left| \frac{1}{2}\partial_R C \right|_{\mathcal{H}} = \frac{|\kappa|}{2\pi} \ . \tag{31}$$

- Static Schwarzschild coord:

$$ds^2 = -Cdt^2 + C^{-1}dR^2$$
$$C = (1 - R_S/R)$$

- Advanced Eddington-Finkelstein:

$$ds^2 = -Cdv^2 + 2dvdR$$
$$v = t + R*$$

- Static de Sitter coordinates:

$$ds^2 = -Cdt^2 + C^{-1}dR^2$$
$$C = \left(1 - R^2/R_H^2\right)$$

- Retarded Eddington-Finkelstein:

$$ds^2 = -Cdu^2 - 2dudR$$
$$u = t - R*$$

Overall Picture
Dynamical Case

- Dynamical Black Hole:

$$ds^2 = A(\tau, r)d\tau^2 + B(\tau, r)dr^2$$

- Advanced null coord $x_+ = \eta + r$:

$$ds^2 = -e^{2\Psi}Cdx_+^2 + 2e^{\Psi}dx_+dR$$

- Going to Kodama time:

$$\mathrm{d}\flat = dx_+ - (e^{\Psi}C)^{-1}dR$$

- Dynamical FLRW:

$$ds^2 = -d\tau^2 + a^2(\tau)dr^2/(1 - kr^2)$$

- Retarded null coord $x_- = \eta - r$:

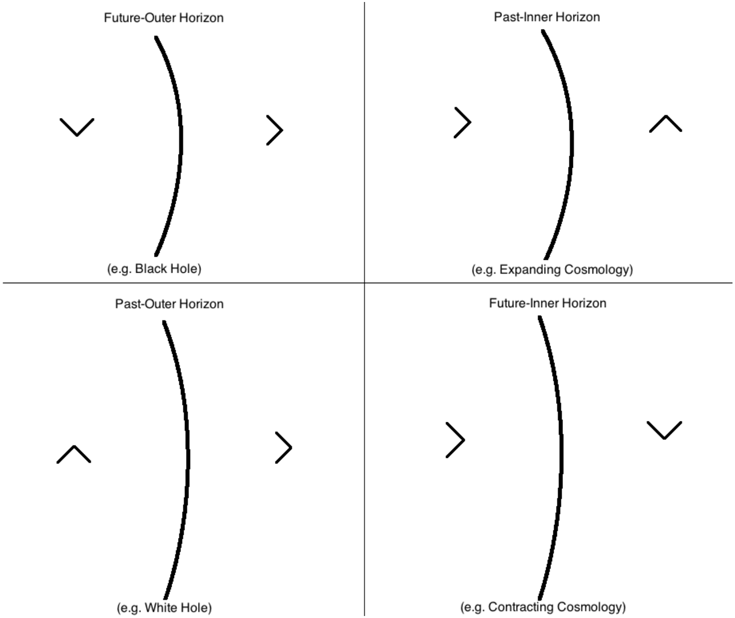
$$ds^2 = -e^{2\Psi}Cdx_-^2 - 2e^{\Psi}dx_-dR$$

- Going to Kodama time:

$$\mathrm{d}\flat = dx_- + (e^{\Psi}C)^{-1}dR$$

$\rightarrow$ Kodama-time metric:

$$ds^2 = -e^{2\Psi}C\mathrm{d}\flat^2 + \frac{1}{C}dR^2 + R^2d\Omega^2 \ . \tag{32}$$


Future-Outer Horizon
(e.g. Black Hole)
Past-Inner Horizon
(e.g. Expanding Cosmology)
Past-Outer Horizon
(e.g. White Hole)
Future-Inner Horizon
(e.g. Contracting Cosmology)

Overview

Conclusions

- Why Apparent Horizon?
  $\rightarrow$ chosen by Hayward as relevant boundary for dynamical black hole,
  $\rightarrow$ same formalism applies perfectly to our cosmological horizon: yields Friedmann Equations,
  $\rightarrow$ global version of Jacobson's local horizon,
  $\rightarrow$ chosen by R. Bousso to enforce his Covariant Entropy Bound,
  $\rightarrow$ chosen by S. Hawking: *Information Preservation and Weather Forecasting for Black Holes, 2014*.
- We used our formalism for black holes on our cosmological patch. Now that we have shown the strong parallel, we may use our knowledge of our visible Universe to better understand black holes!

Thank you!

Renormalization group-flow in AdS-CFT

- Variation of the apparent radius:

$$\dot{R}_A = -HR_A^3\left(\dot{H} - \frac{k}{a^2}\right) . \tag{33}$$

We will be interested in the quantity:

$$\beta^2 \equiv \frac{2\dot{R}_A}{HR_A} = -2\frac{\dot{H} - \frac{k}{a^2}}{H^2 + k/a^2} = 3\frac{\rho + P}{\rho}. \tag{34}$$

$\rightarrow$ beta-function of the renormalisation group flow in the AdS/CFT correspondence. We notice that de Sitter is a zero of the β function.
$\rightarrow$ beta characterizes the departure from conformal symmetry, the departure from de Sitter.
$\rightarrow$ de Sitter is a fixed point. Fluctuations will take the Universe away from dS, but the flow will eventually bring it back to $\beta = 0$.