Quantum reflection from the Casimir-Polder potential and the gravitational properties of antimatter

IAP theory group seminar

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Outline

1. Motivation : quantum reflection in GBAR
2. Casimir-Polder potential for real materials
3. Quantum reflection from the Casimir-Polder potential
4. Liouville transformations of the Schrödinger equation
5. Enhancing and using quantum reflection

The GBAR experiment

Gravitational Behavior of Antihydrogen at Rest

http://gbar.web.cern.ch

Test the equivalence principle for antimatter by timing the free fall of antihydrogen ($\overline{\mathrm{H}}$) dropped from $\sim$ 10 cm

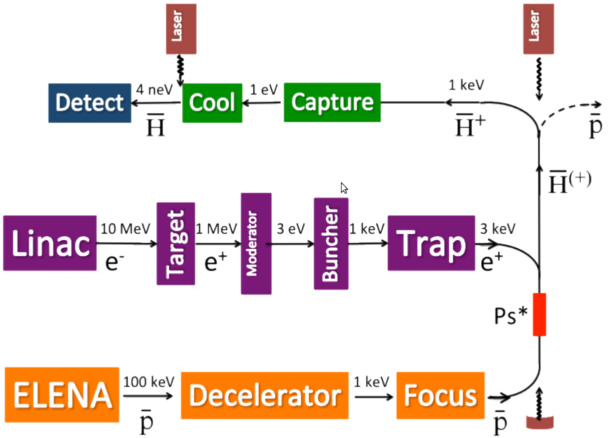
Experiment under construction at CERN

Current experimental bound on gravitational acceleration of $\overline{\mathrm{H}}$ from ALPHA experiment:

$$-65g \leq \bar{g} \leq +110g$$

ALPHA collab. *Nature Communications* 4 (2013) 1785

GBAR: overall scheme



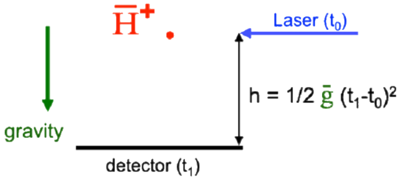
At LKB:

- Lasers for Ps excitation
- Sympathetic cooling of $\overline{\mathrm{H}}^{+}$ in ion traps
- Free fall and detection

P. Perez & Y. Sacquin, *Class. Quantum Grav. 29 (2012) 184008*

GBAR: free fall and detection

- initial state: $\overline{\mathrm{H}}^+$ in the ground state of a harmonic Paul trap
- start: the extra e^+ is photodetached
- freefall of $\overline{\mathrm{H}}$
- stop: $\overline{\mathrm{H}}$ annihilates on the detector



P. Perez & Y. Sacquin,
Class. Quantum Grav. 29 (2012) 184008

The free fall acceleration $\bar{g}$ of $\overline{\mathrm{H}}$ is deduced from the free fall time

Question: are there other forces than gravity acting on $\overline{\mathrm{H}}$?

Effect of the atom-detector interaction

Attractive Casimir-Polder interaction between atom and detector :

- no noticeable change in time of fall
- BUT part of the atomic wavepacket is reflected

Quantum reflection : classically forbidden reflection of a matter wave from an attractive potential

Need to estimate and master this bias in GBAR:

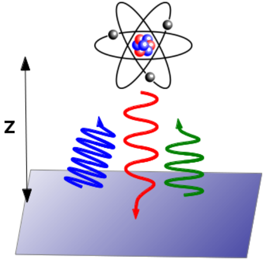
- How much quantum reflection can we expect?
- How does it depend on the atom's velocity?
- How is this affected by the materials used?
- Can it be used to improve the accuracy of the experiment?

The Casimir-Polder force

Electromagnetic (EM) modes are modified when the atom comes close to the detector:

$\Rightarrow$ the EM ground state (vacuum) energy changes

$\Rightarrow$ attractive Casimir-Polder force between atom and detector



Casimir & Polder 1948 : long-range interaction energy between an atom and a perfectly conducting mirror:

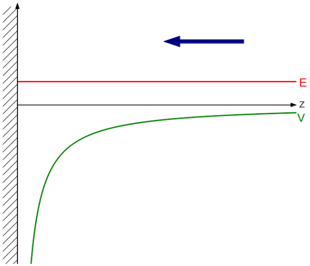
$$V^*(z) = -\frac{3\hbar c}{8\pi z^4}\frac{\alpha(0)}{4\pi\epsilon_0} = -\frac{C_4^*}{z^4}$$

For H and $\overline{\mathrm{H}}$, $\quad \dfrac{\alpha(0)}{4\pi\epsilon_0} = \dfrac{9}{2}a_0^3,$

$$C_4^* \approx 73.6\ E_h a_0^4 \approx 15.7\ \text{meV.nm}^4$$

Scattering on the Casimir-Polder potential

What happens when the atom scatters on this potential ?



Length scales :

- free fall height : $h \approx 10$ cm
- quantum gravitational scale : $\ell_{grav} = \left(\hbar^2/2m^2 g\right)^{1/3} \approx 6\ \mu$m
- Casimir-Polder scale : $\ell_{CP} = \sqrt{2mC_4}/\hbar \approx 30$ nm

We can decouple the free fall and the scattering on the potential: the incoming wavefunction is a plane wave with energy $E = mgh$

Examples of observation of quantum reflection

Shimizu 2001: Ne* on Silicon and BK7 glass, grazing incidence

VOLUME 86, NUMBER 6 PHYSICAL REVIEW LETTERS 5 FEBRUARY 2001

Specular Reflection of Very Slow Metastable Neon Atoms from a Solid Surface

Fujio Shimizu
Institute for Laser Science and CREST, University of Electro-Communications, Chofu-shi, Tokyo 182-8585, Japan
(Received 7 July 2000)

An ultracold narrow atomic beam of metastable neon in the $1s_3[(2s)^5 3p:{}^1P_0]$ state is used to study specular reflection of atoms from a solid surface at extremely slow incident velocity. The reflectivity on a silicon (1,0,0) surface and a BK7 glass surface is measured at the normal incident velocity between 1 mm/s and 3 cm/s. The reflectivity above 30% is observed at about 1 mm/s. The observed velocity dependence is explained semiquantitatively by the quantum reflection that is caused by the attractive Casimir–van der Waals potential of the atom-surface interaction.

DOI: 10.1103/PhysRevLett.86.987 PACS numbers: 34.50.Dy, 03.75.–b, 34.20.Cf

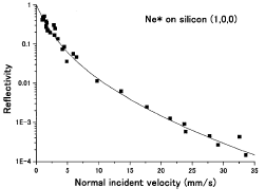


FIG. 3. The reflectivity vs the normal incident velocity on the Si(1,0,0) surface. The solid curve is the reflectivity calculated by using the potential Eq. (1) with $\lambda = 0.4\ \mu\text{m}$ and $C_4 = 6.8 \times 10^{-56}\ \text{J m}^4$, which corresponds to $\alpha = 2.0 \times 10^{-39}\ \text{F m}^2$ of Casimir's theory.

Examples of observation of quantum reflection

Pasquini et al. 2004: dilute BEC of Na on silicon, normal incidence

PRL **93**, 223201 (2004) PHYSICAL REVIEW LETTERS week ending 26 NOVEMBER 2004

Quantum Reflection from a Solid Surface at Normal Incidence

T. A. Pasquini, Y. Shin, C. Sanner, M. Saba, A. Schirotzek, D. E. Pritchard, and W. Ketterle*
Department of Physics, MIT-Harvard Center for Ultracold Atoms,
and Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Massachusetts, 02139, USA
(Received 15 June 2004; published 24 November 2004)

We observed quantum reflection of ultracold atoms from the attractive potential of a solid surface. Extremely dilute Bose-Einstein condensates of ^{23}Na, with peak density 10^{11}–10^{12} atoms/cm^3, confined in a weak gravitomagnetic trap were normally incident on a silicon surface. Reflection probabilities of up to 20% were observed for incident velocities of 1–8 mm/s. The velocity dependence agrees qualitatively with the prediction for quantum reflection from the attractive Casimir-Polder potential. Atoms confined in a harmonic trap divided in half by a solid surface exhibited extended lifetime due to quantum reflection from the surface, implying a reflection probability above 50%.

DOI: 10.1103/PhysRevLett.93.223201 PACS numbers: 34.50.Dy, 03.75.Be

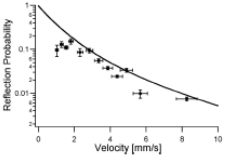


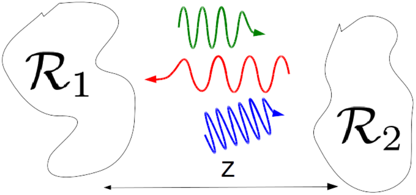
FIG. 3. Reflection probability vs incident velocity. Data were collected in a magnetic trap with trap frequencies $2\pi \times (3.3, 2.5, 6.5)$ Hz. Incident and reflected atom numbers were averaged over several shots. Vertical error bars show the standard deviation of the mean of six measurements. Horizontal error bars reflect the uncertainty in deducing $v_\perp$ from the applied magnetic field $B_\perp$. The solid curve is a numerical calculation for individual atoms incident on a conducting surface as described in the text.

Scattering approach to Casimir forces

Scattering formula for Casimir energy (here at $T = 0$):

$$V(z) = \hbar \int_0^\infty \frac{\mathrm{d}\xi}{2\pi} \mathrm{Tr} \log \left(1 - \mathcal{R}_1 e^{-\kappa z} \mathcal{R}_2 e^{-\kappa z}\right)$$

Objects described by EM reflection matrices $\mathcal{R}_1, \mathcal{R}_2$



- Tr : Trace on transverse wave vector $\vec{k}_\perp$ and polarisation
- $\omega = i\xi$: imaginary frequency
- $\kappa = \sqrt{k_\perp^2 + \xi^2/c^2}$: longitudinal wave vector

A. Lambrecht, P.A. Maia Neto, S. Reynaud, *New J. Phys. 8 (2006) 243*

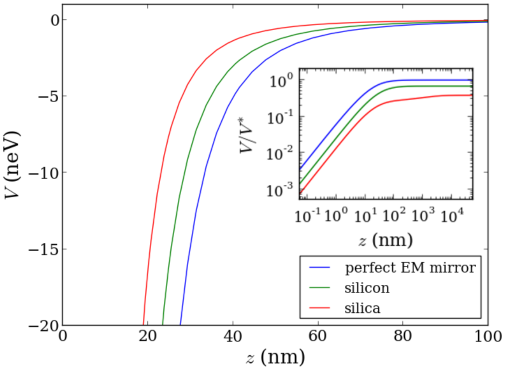
Casimir-Polder potential

- Atom treated in dipolar approximation, described by its dynamic polarizability $\alpha(i\xi)$
- Plane surface described by reflection coefficients ρ^{TE}, ρ^{TM} which depend on the mirror's permittivity $\epsilon(i\xi)$
- Weak reflection on atom: the log() can be expanded to 1st order (multiple reflections ignored)

$$V(z) = \frac{\hbar}{c^2} \int_0^\infty \mathrm{d}\xi \, \xi^2 \alpha(i\xi) \int \frac{\mathrm{d}^2 \vec{k}_\perp}{(2\pi)^2} \frac{e^{-2\kappa z}}{\kappa} \left[\rho^{TE} - \left(1 + \frac{2c^2 k_\perp^2}{\xi^2} \right) \rho^{TM} \right]$$

Calculation of the Casimir-Polder potential

Casimir-Polder potential above various semi-infinite media, numerical results (inset : normalized potential V/V^*):



- long distance (retarded regime): $V(z) \simeq -C_4/z^4$
- short distance (van der Waals regime): $V(z) \simeq -C_3/z^3$
- weaker potential for materials weakly coupled to the EM field

Reflection equations and boundary conditions

We want to solve the Schrödinger equation with the CP potential:

$$\psi''(z) + k(z)^2\psi(z) = 0 \ , \qquad \hbar k(z) = \sqrt{2m(E - V(z))}$$

Exact wavefunction written as a combination of semiclassical (WKB) waves which have a well defined direction of propagation:

$$\psi(z) = b_+(z)\psi^+_{\mathrm{WKB}}(z) + b_-(z)\psi^-_{\mathrm{WKB}}(z)$$

$$\psi^\pm_{\mathrm{WKB}}(z) = \frac{\exp(\pm i\phi(z))}{\sqrt{k(z)}} \ , \qquad \phi(z) = \int^z k(z')\mathrm{d}z'$$

The coefficients obey coupled equations:

$$b'_\pm(z) = \pm Q(z)\frac{k(z)}{2i}\left(b_\pm(z) + b_\mp(z)\exp(\mp 2i\phi(z))\right)$$

Badlands function: $Q(z) = \dfrac{k''(z)}{2k(z)^3} - \dfrac{3k'(z)^2}{4k(z)^4}$

Reflection and transmission probabilities

$b_\pm(z)$ become constant

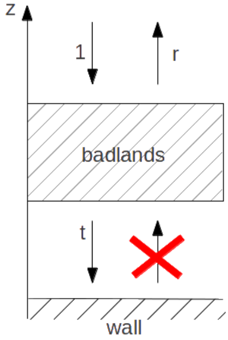
- as $z \to \infty$, where the potential goes to 0
- as $z \to 0$, where the classical momentum is large

$\Rightarrow$ reflection only occurs in an intermediate region, the "badlands"

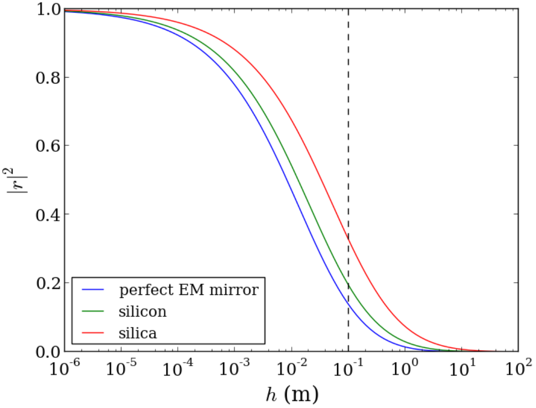
Annihilation of $\overline{\mathrm{H}}$ on the surface:
no reflected wave $b_+(z=0)=0$
$\Rightarrow$ different from matter atoms
& less sensitive to surface physics

reflection probability:
$R = |r|^2 = |b_+(\infty)/b_-(\infty)|^2$

transmission and annihilation probability:
$T = 1 - R = |t|^2 = |b_-(0)/b_-(\infty)|^2$



Reflection probability versus energy



- significant probability of reflection in GBAR
- bias : high energy atoms more likely to be detected
- weaker reflectors of EM field are better reflectors of atoms !

G.Dufour, A.Gérardin, R.Guérout, A.Lambrecht, V.V.Nesvizhevsky, S.Reynaud, A.Yu.Voronin, *Phys. Rev. A 87 (2013) 012901*

Liouville transformation

Schrödinger equations are invariant under the group of Liouville transformations:

$$z \to \tilde{z} = \tilde{z}(z) \ , \qquad \psi(z) \to \tilde{\psi}(\tilde{z}) = \sqrt{\tilde{z}'(z)}\psi(z) \ ,$$

$$k(z)^2 \to \tilde{k}(\tilde{z})^2 = \frac{1}{\tilde{z}'(z)^2}\left(k(z)^2 - \frac{1}{2}\{\tilde{z}, z\}\right)$$

Schwarzian derivative: $\{\tilde{z}, z\} = \dfrac{\tilde{z}'''(z)}{\tilde{z}'(z)} - \dfrac{3\tilde{z}''(z)^2}{2\tilde{z}'(z)^2}$

The transformation preserves Wronskians:

$$\psi_1(z)\psi_2'(z) - \psi_1'(z)\psi_2(z) = \tilde{\psi}_1(\tilde{z})\tilde{\psi}_2'(\tilde{z}) - \tilde{\psi}_1'(\tilde{z})\tilde{\psi}_2(\tilde{z})$$

$\Rightarrow$ it preserves scattering amplitudes:

$$r = \tilde{r} \ , \qquad\qquad t = \tilde{t}$$

G.Dufour, R.Guérout, A.Lambrecht, S.Reynaud, *EPL* 110 (2015) 30007

A special choice of coordinate

We use the WKB phase as the coordinate: $\tilde{z}(z) = \phi(z)$
The domain $z > 0$ is mapped onto the whole real axis
The transformed equation is

$$\tilde{\psi}''(\tilde{z}) + \left(1 - \tilde{Q}(\tilde{z})\right)\tilde{\psi}(\tilde{z}) = 0$$

$\tilde{Q}(\tilde{z}) = Q(z) = \dfrac{k''(z)}{2k(z)^3} - \dfrac{3k'(z)^2}{4k(z)^4}$ plays the role of a potential

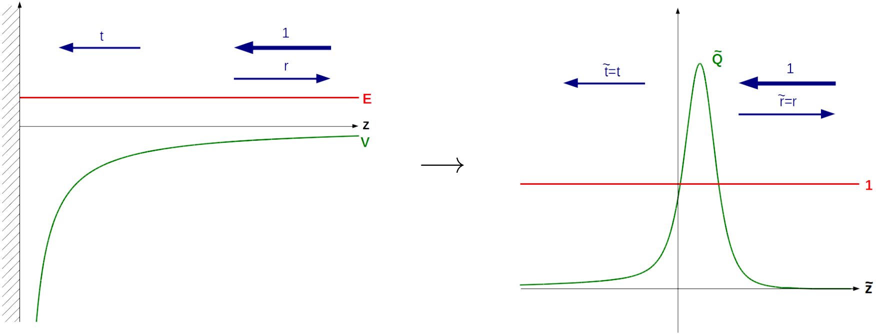
In regions where $Q(z) = \tilde{Q}(\tilde{z}) \simeq 0$:

$$\tilde{\psi}(\tilde{z}) \simeq e^{\pm i\tilde{z}} \qquad \text{so} \qquad \psi(z) \simeq \frac{1}{\sqrt{k(z)}} e^{\pm i\phi(z)} = \psi^{\pm}_{\mathrm{WKB}}(z)$$

Conversely when $Q(z) \neq 0$ the WKB approximation breaks down

Transformation from well to wall

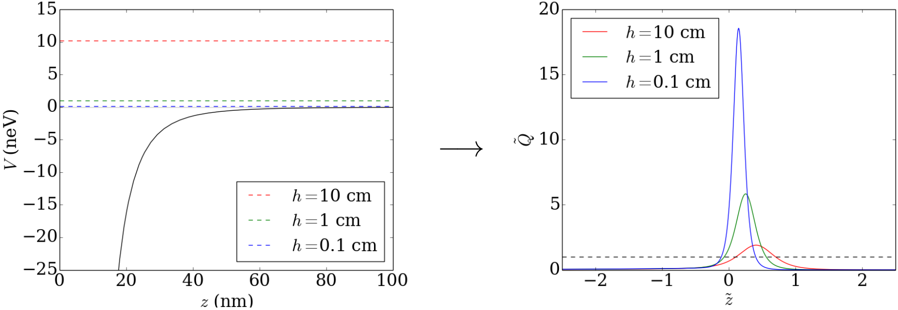
$Q(z)$ is a peaked function which vanishes both near the surface and far from it : quantum reflection on an attractive well is mapped onto reflection on a repulsive wall



The two problems correspond to very different semiclassical pictures but are equivalent from the point of view of scattering

Varying the energy

$\overline{\mathrm{H}}$ on silica:



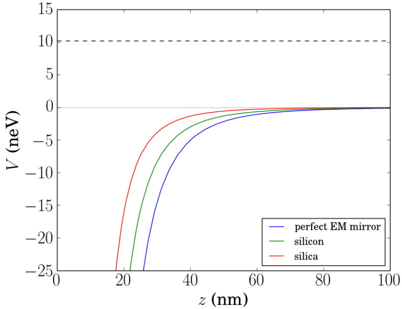
$h = 10$ cm
$R = 33\%$

$h = 1$ cm
$R = 67\%$

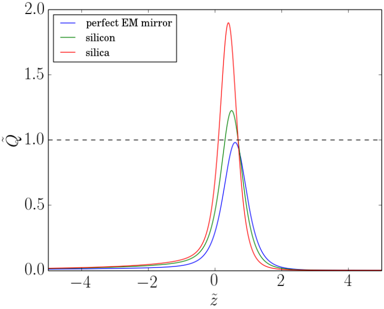
$h = 0.1$ cm
$R = 88\%$

Varying the potential

$\overline{\mathrm{H}}$ from $h = 10$ cm:



$\longrightarrow$



silica	silicon	perfect mirror
$R = 33\%$	$R = 19\%$	$R = 14\%$

Threshold behavior

At low energies, interaction with the surface is described by a single complex parameter, the scattering length a:

$$\text{when} \qquad \kappa = \sqrt{2mE}/\hbar \to 0 \ ,$$

$$r \simeq -\exp\left(-2i\kappa a\right) \ , \quad R \simeq \exp\left(-4\kappa b\right) \ , \quad b = -\text{Im}(a) > 0$$

For the pure retarded potential $V(z) = -C_4/z^4$: $a = -i\sqrt{2mC_4}/\hbar$

For the real CP potential, $\text{Re}(a) \neq 0$ and $b \neq \sqrt{2mC_4}/\hbar$

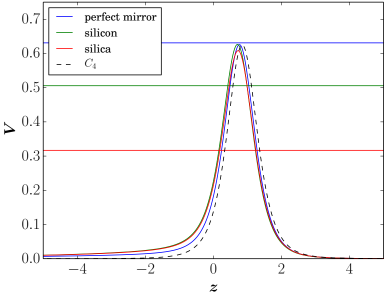
$\rightarrow$ the scattering length depends on the full CP potential and not only on its long-distance limit

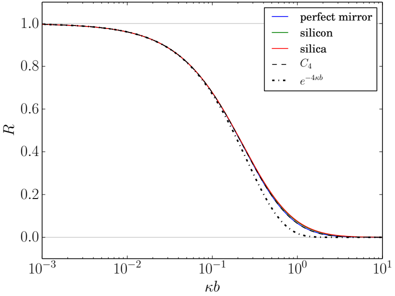
Scaling laws

Rescale the coordinate: $\tilde{z} = \phi(z)/\sqrt{\kappa b}$

$$\tilde{\psi}''(\tilde{z}) + (\kappa b - v(\tilde{z}))\,\tilde{\psi}(\tilde{z}) = 0\ , \qquad v(\tilde{z}) = \kappa b Q(z)$$

For the pure retarded potential, $v(\tilde{z})$ is a universal function independent of the parameters of the problem.





Interest and general idea

We can use our understanding of quantum reflection to enhance it:

- reduce the energy
- weaken the Casimir-Polder interaction

Increasing quantum reflection opens many possibilities:

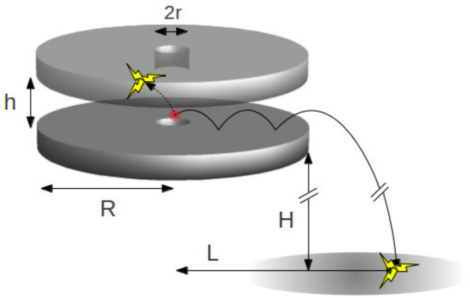
⇒ store and guide antimatter with material surfaces

⇒ study gravitationally bound states above the surface

A.Yu. Voronin, P. Froelich, V.V. Nesvizhevsky, *Phys. Rev. A 83 (2011) 032903*

Velocity selector for GBAR

GBAR resolution is limited by $\Delta v \sim 1$ m/s uncertainty on initial vertical velocity: filtering device to reduce the velocity spread



Output : $\Delta z \sim h$
and $\Delta v \sim \sqrt{2gh}$

Precision of GBAR experiment taken from 1% to 1‰

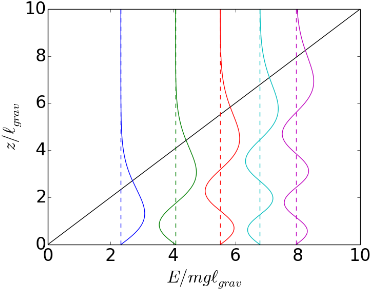
G.Dufour, P.Debu, A.Lambrecht, V.V.Nesvizhevsky, S.Reynaud, A.Yu.Voronin,
Eur. Phys. J. C 74 (2014) 2731

Gravitationally bound states above the surface

Eigenstates in a linear potential: $\psi(z) \propto \mathrm{Ai}\left(\frac{z}{\ell_{grav}} - \frac{E}{mg\ell_{grav}}\right)$

Dirichlet boundary condition: $\psi_n(0) = 0$

$\Rightarrow -E_n/mg\ell_{grav}$ is a zero of $\mathrm{Ai}(x)$



Matching with the solution in the CP potential (within scattering length approximation):

- $E_n \to E_n + mga$
- energy shift: $mg\mathrm{Re}(a)$
- decay rate: $2mg|\mathrm{Im}(a)| = 2mgb$

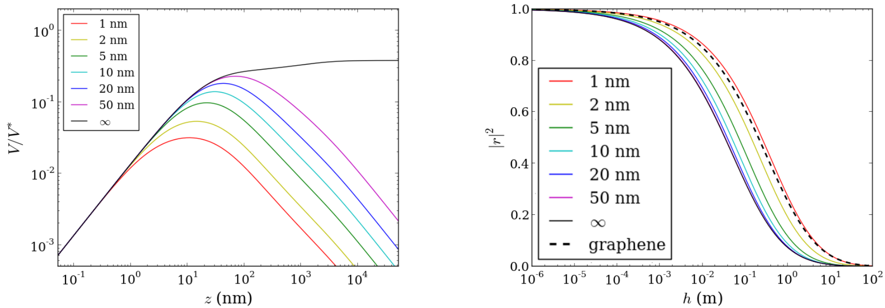
Transition frequencies are independent of the details of the interaction
$\to$ spectroscopic tests of WEP

Thin slabs and graphene

Thin slabs invisible to large wavelengths → reduction of potential

Graphene, using reflection coefficients from:

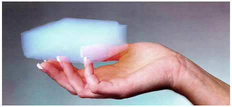
M. Bordag, I. V. Fialkovsky, D. M. Gitman, D. V. Vassilevich,
Phys. Rev. B 80 (2009) 245406



Notice $V \sim z^{-5}$ behavior as $z \to \infty$ for slabs

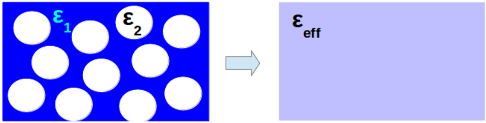
Nanoporous materials

Materials that incorporate a large fraction of gas or vacuum
Eg: silica aerogels, powders of nanodiamonds and porous silicon



NASA

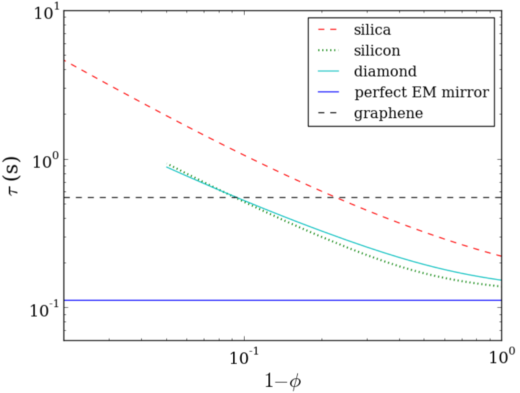
Pore size in the 10-100 nm range: if the atom is reflected far enough, we can use an effective medium approximation (Bruggeman model)



G.Dufour, R.Guérout, A.Lambrecht, V.V.Nesvizhevsky, S.Reynaud, A.Yu.Voronin, *Phys. Rev. A 87 (2013) 022506*

Lifetimes above surfaces

Lifetime of first gravitationally bound states: $\tau = \frac{\hbar}{2mgb}$



surface (porosity)	Lifetime (s)
perfect mirror	0.11
bulk silicon	0.14
bulk silica	0.22
5 nm silica slab	0.33
diamond powder (95%)	0.89
porous silicon (95%)	0.94
silica aerogel (98%)	4.6

Conclusions

- The Casimir-Polder interaction between $\overline{\mathrm{H}}$ and the detector can cause a significant amount of reflection
  - $\rightarrow$ lower statistics
  - $\rightarrow$ bias towards high energy atoms
- We can transform the quantum reflection problem into an equivalent problem of scattering on a barrier
- Counterintuitive dependence of the reflection probability on the energy and potential strength is well understood
- Quantum reflection can be increased by reducing the Casimir interaction : a new way to trap and study antimatter

The end

Thank you for your attention

Any questions?