Approximate No-hair Relations for Neutron Stars

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11th May 2015

Who is Kent Yagi???

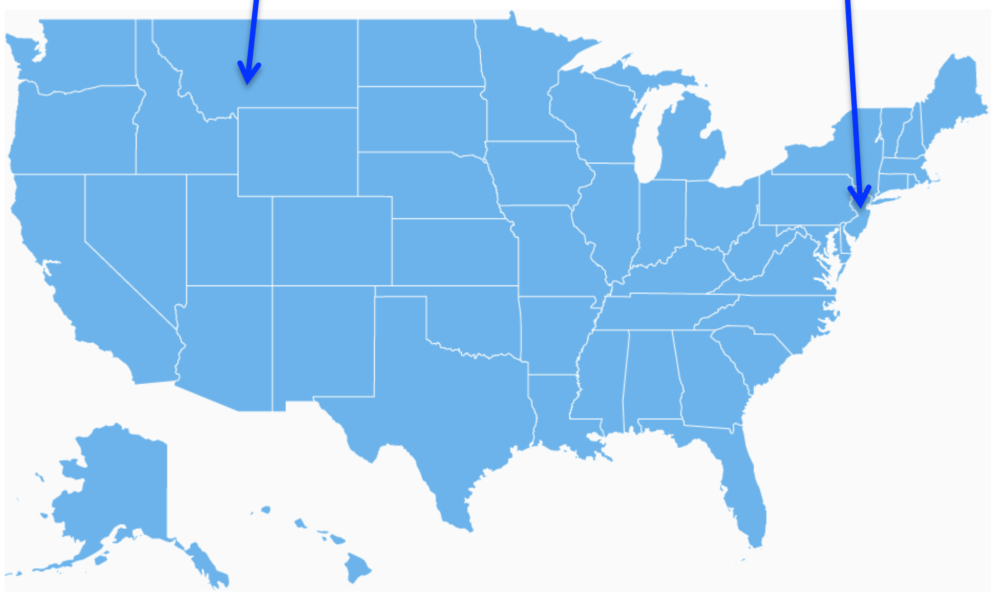
Kyoto U.
(Ph.D)

Tokyo

Chiba
(home town)

Montana State University
(postdoc)

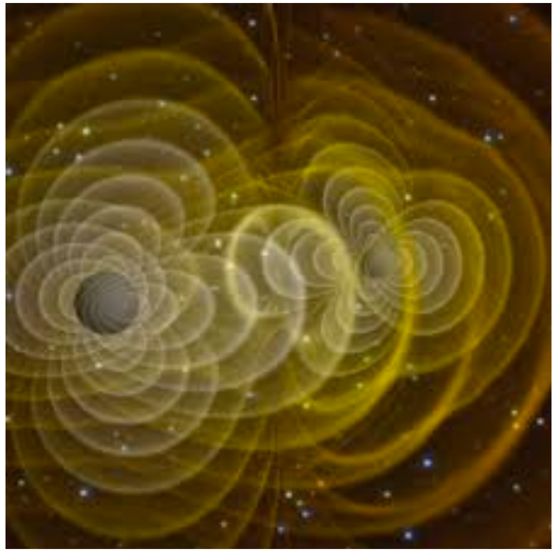
Princeton University
(JSPS fellow, from Sept.)



What does Kent work on?

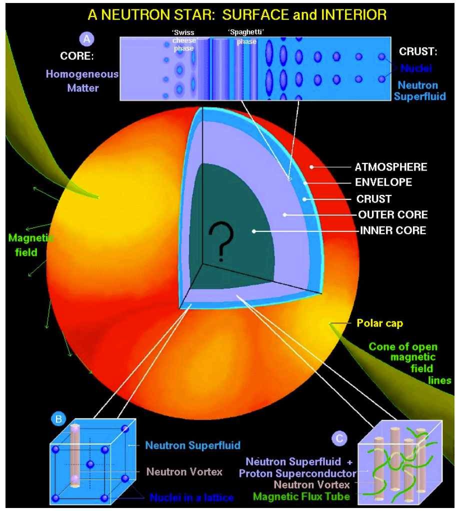
(I) Testing strong-field gravity with binary pulsar and gravitational wave observations

- -scalar-tensor theories
- -massive graviton theories
- -extra dimension theories
- -Einstein-dilaton Gauss-Bonnet gravity
- -dynamical Chern-Simons gravity
- -Lorentz-violating gravity



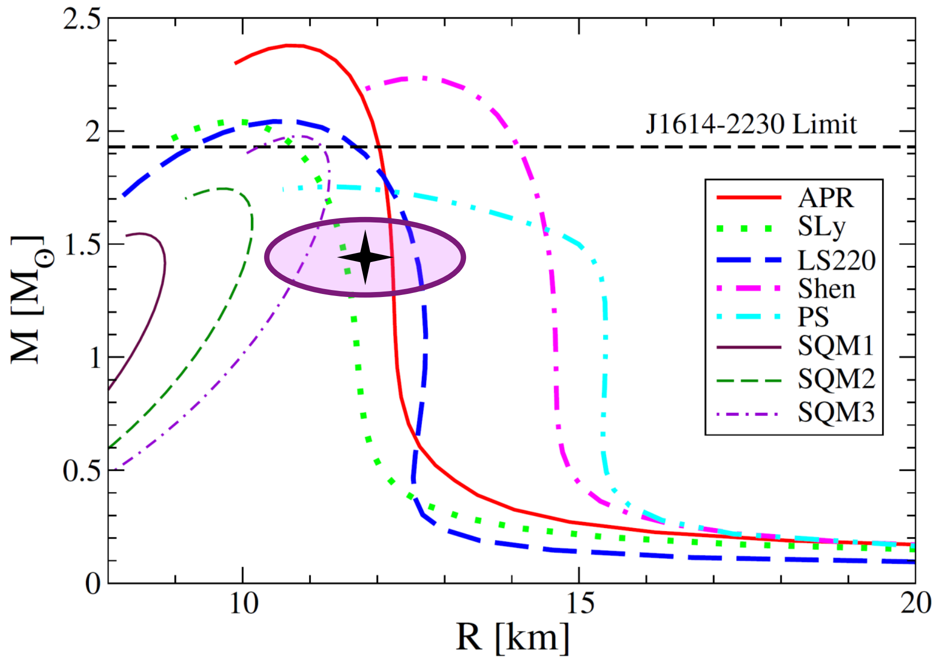
(II) Neutron star properties in General Relativity

- -universal relations among observables
- -approximate no-hair relations

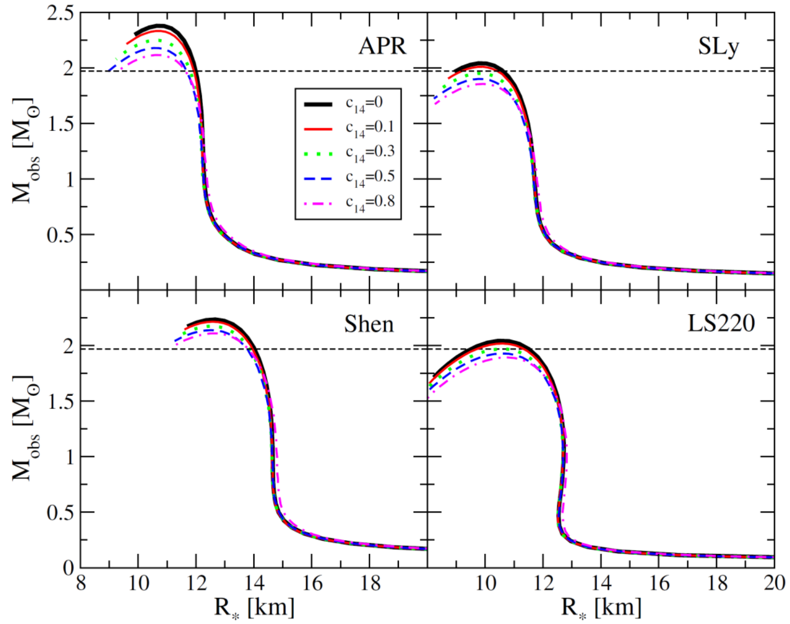


Why Neutron Stars?

(i) probing nuclear physics



(ii) probing strong-field gravity



Einstein-EAther theory
[KY, Blas, Barausse & Yunes 2014]

Problems

-Degeneracies among parameters

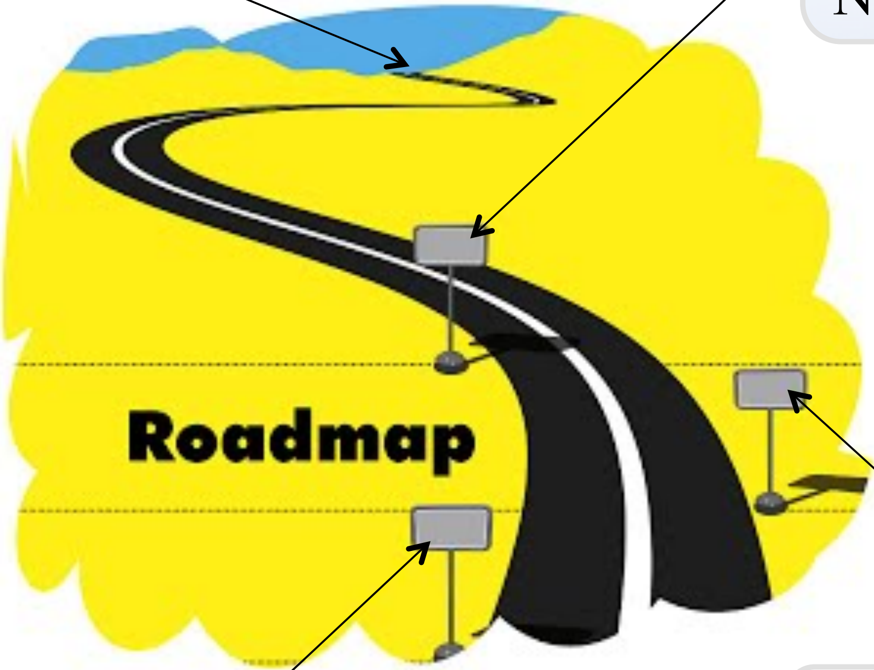
-Degeneracies between uncertainties in nuclear and gravitational physics

universal relations!!

Roadmap

(IV) Conclusions and Future work

(III) Relating follicly-challenged NSs to bald BHs



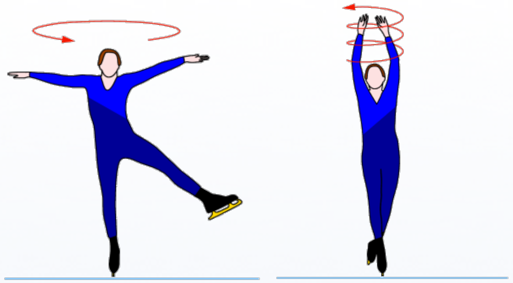
(II) Approximate No-hair Relations for NSs

(I) I-Love-Q Relations

I-Love-Q Relations

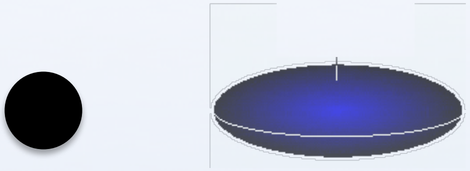
I-Love-Q

I moment of inertia



Q quadrupole moment

λ_2 tidal Love number
(tidal deformability)



$$\lambda_2 = \frac{\text{(tidally induced) Q}}{\text{tidal field}}$$

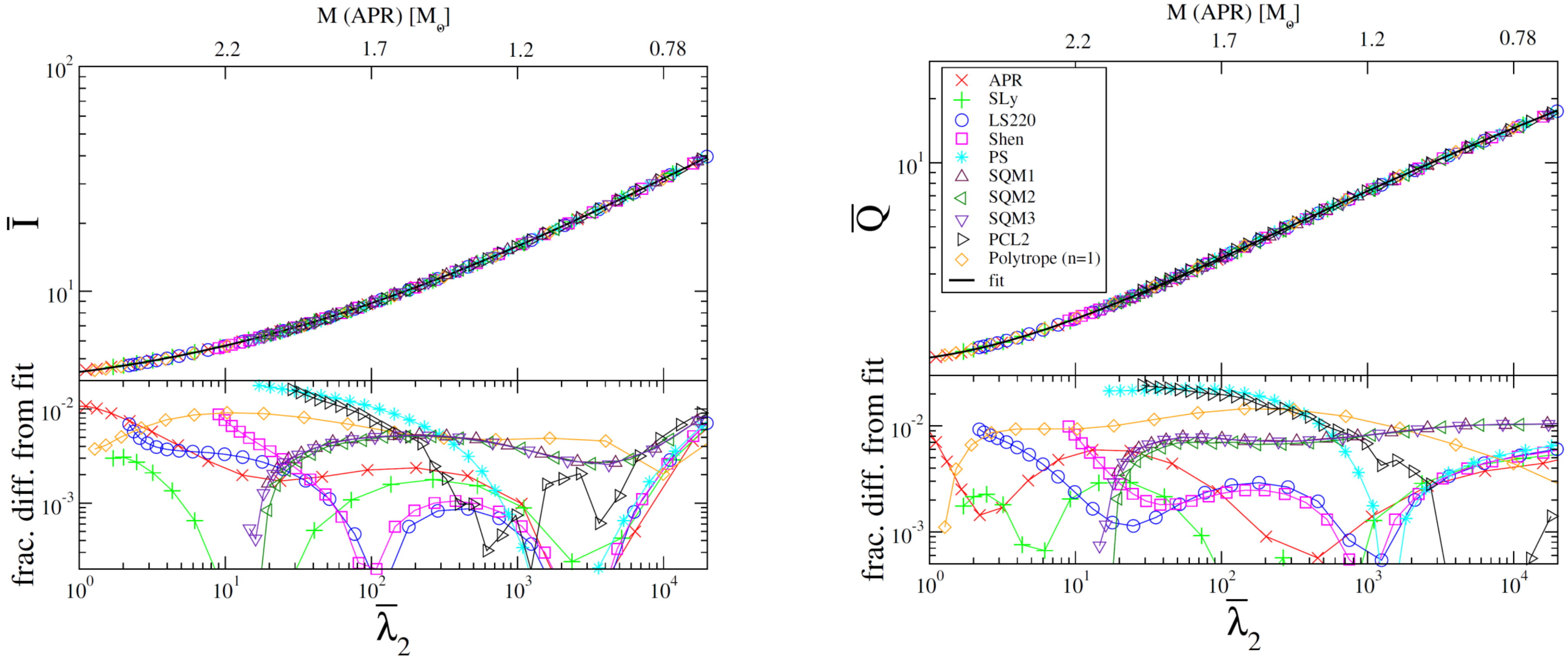
Construct slowly-rotating/tidally-deformed NS solutions by solving the Einstein equations numerically.

⇒ Extract I, Love & Q from the asymptotic behavior of the metric at spatial infinity.

I-Love-Q Relations

[KY & Yunes, Science, PRD (2013)]

- (i) small/static tidal deformation → Maselli+ (2013)
- (ii) unmagnetized → Haskell+ (2014)
- (iii) uniform/slow-rotation → Doneva+ (2013), Papps+ (2014), Chakrabarti+ (2014), KY+ (2014)
- (iv) barotropic, isotropic → Martinon+ (2014), KY & Yunes [2015]



$\bar{I} \equiv I/M_*^3 \qquad \bar{\lambda}_2 \equiv \lambda_2/M_*^5 \qquad \bar{Q} \equiv -Q/(M_*^3\chi^2) \qquad \chi \equiv S_1/M_*^2$

Applications (I): Nuclear Physics

Parameters:

$(M, R, I, \cancel{Q}, \cdots)$

I-Q relation

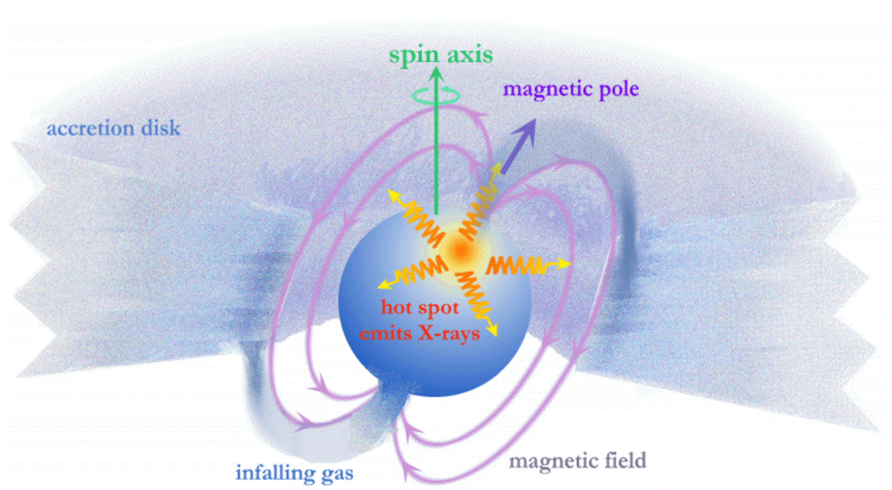
Strong degeneracies among parameters

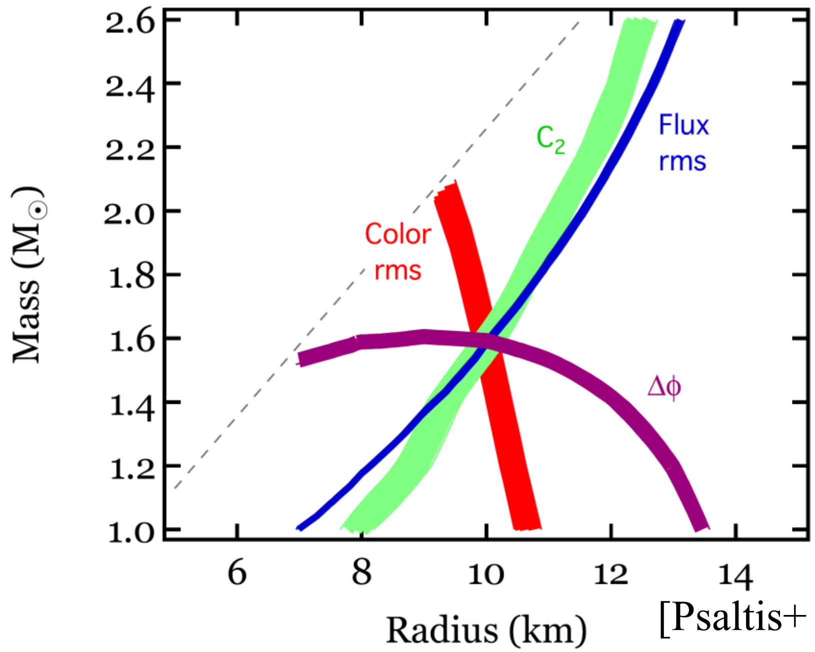
⬇ universal relations

Reduce the number of parameters

⬇

Allows one to measure mass and radius accurately!





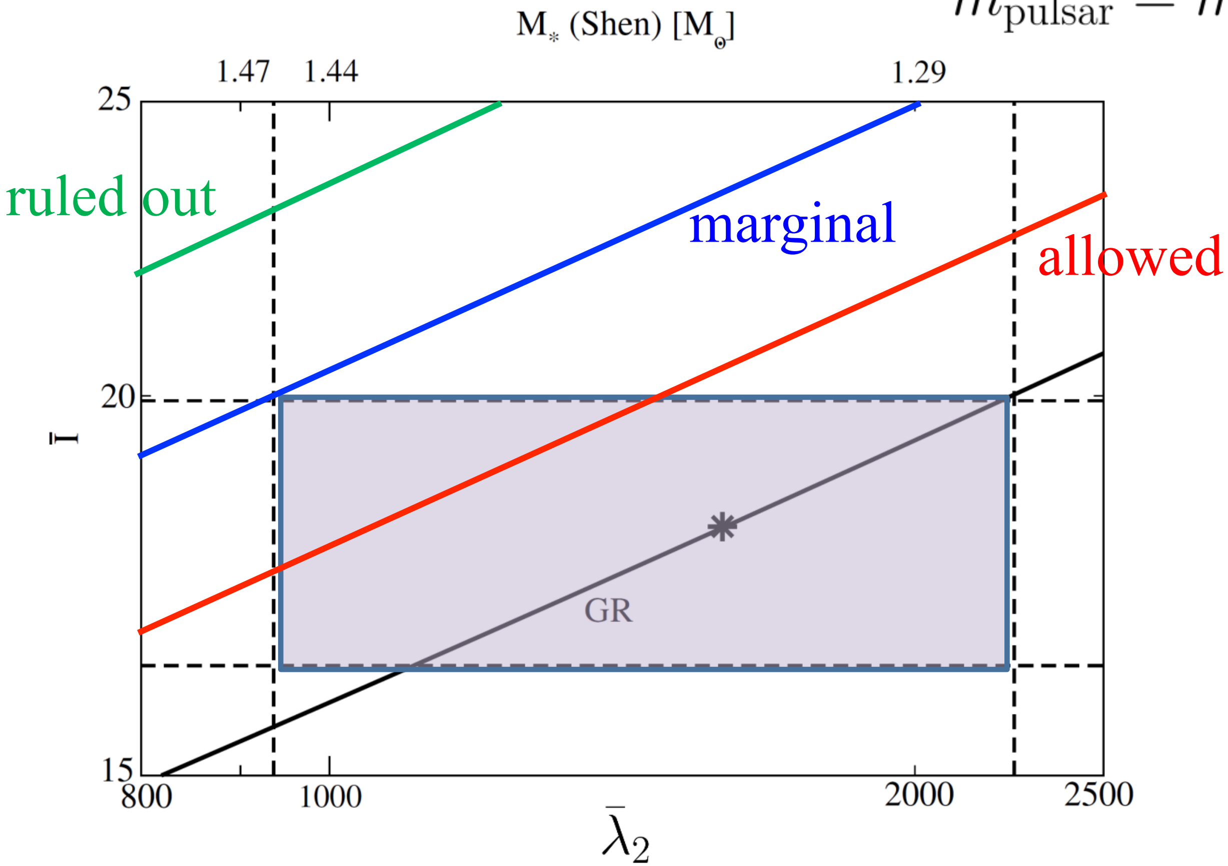
[Psaltis+ (2014)]

Applications (II): Gravitational Physics

-double binary pulsar ➡ $\Delta \bar{I}/\bar{I} = 10\%$

-gravitational waves ➡ $\Delta \bar{\lambda}_2/\bar{\lambda}_2 = 60\%$

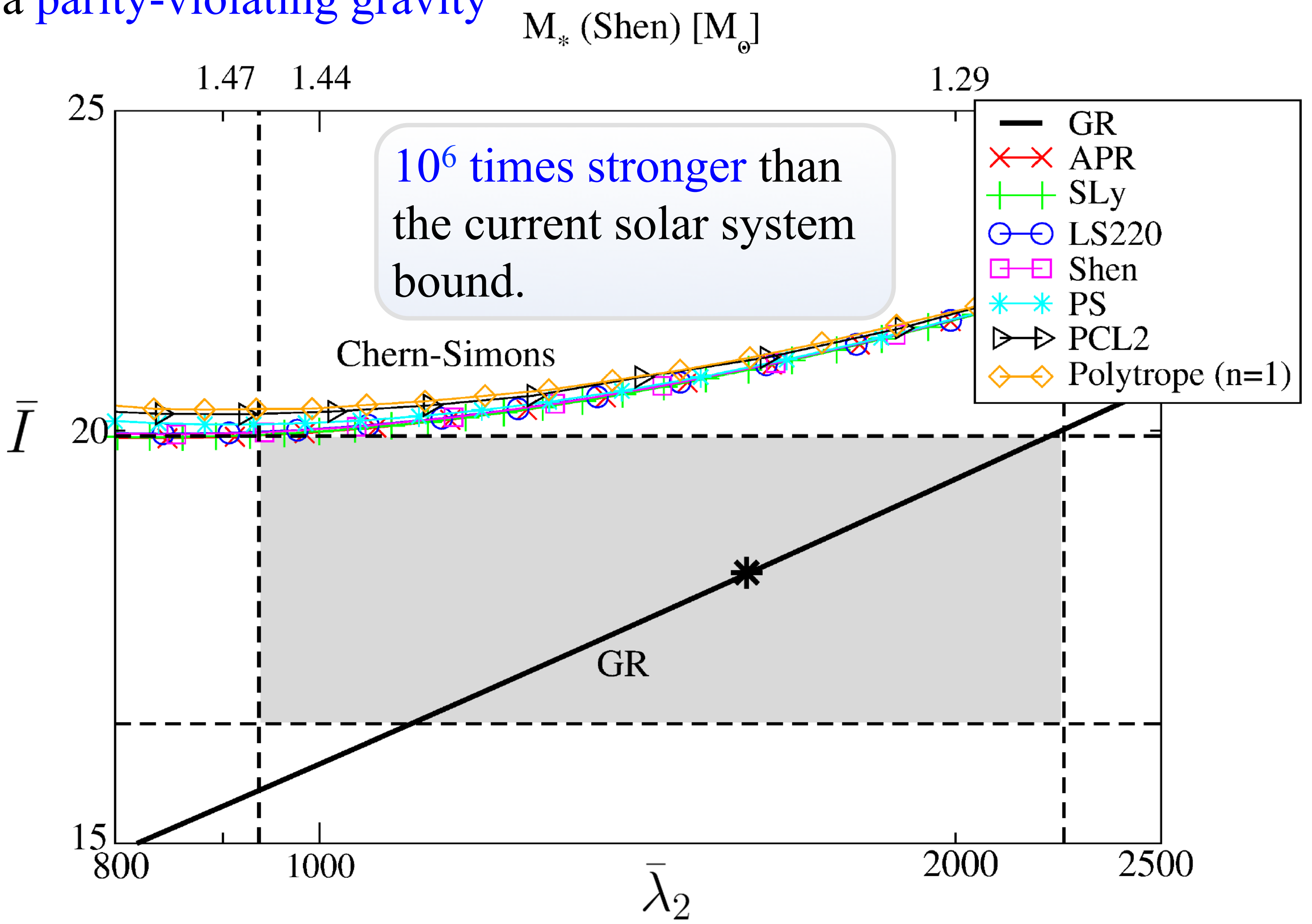
$m_{\text{pulsar}} = m_1 = m_2$



Strong-field Tests of Gravity

[KY & Yunes, Science, PRD (2013)]

Testing a parity-violating gravity

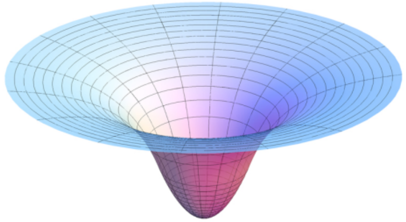


Approximate No-hair Relations for Neutron Stars

Multipole Moments (Gravity)

Exterior spacetime of an object is characterized by multipole moments

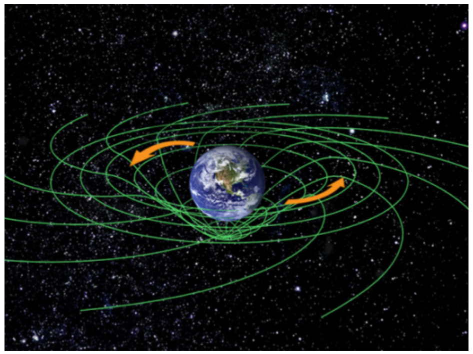
Gravitational potential



$$g_{tt} \sim -1 - 2 \sum_{\ell=0} \left[\frac{M_\ell}{r^{\ell+1}} + \mathcal{O}\left(\frac{1}{r^{\ell+2}} \right) \right] P_\ell(\cos\theta)$$

$$g_{t\phi} \sim -2 \sin^2\theta \sum_{\ell=1} \left[\frac{1}{\ell} \frac{S_\ell}{r^\ell} + \mathcal{O}\left(\frac{1}{r^{\ell+1}} \right) \right] P_\ell'(\cos\theta)$$

"spacetime" dragging



Black Holes are Bald

BHs only have two hairs, mass and spin

Black Hole No-hair Relation

$$M_\ell + i S_\ell = M (i a)^\ell$$

$a = S/M$ [Hansen (1974)]

Is there a similar relation for neutron stars?

Are Newtonian Stars also bald?

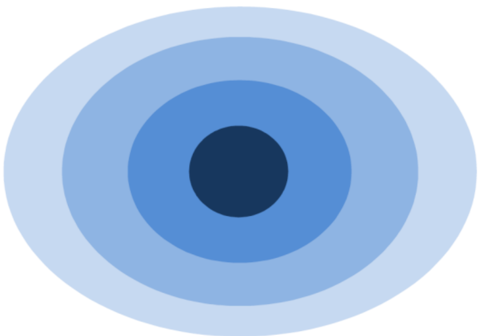
Mass multipole moment in the Newtonian limit [Ryan (1997)]

$$M_\ell = 2\pi \int_0^\pi \int_0^{R_*(\theta)} \underbrace{\rho(r,\theta)}_{\text{mass density}} P_\ell(\cos\theta)\, \sin\theta\, d\theta\, r^{\ell+2}\, dr$$

$$\left(\frac{1}{R_*^2(\theta)} = \frac{\sin^2\theta}{\underbrace{a_1^2}_{\text{semi-major}}} + \frac{\cos^2\theta}{\underbrace{a_3^2}_{\text{semi-minor}}} \right)$$

Integrals are not separable…

Elliptical Isodensity Approximation [Lai et al. (1993)]



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mass density

$$\left(\frac{1}{R_*^2(\theta)} = \frac{\sin^2\theta}{a_1^2} + \frac{\cos^2\theta}{a_3^2} \right)$$

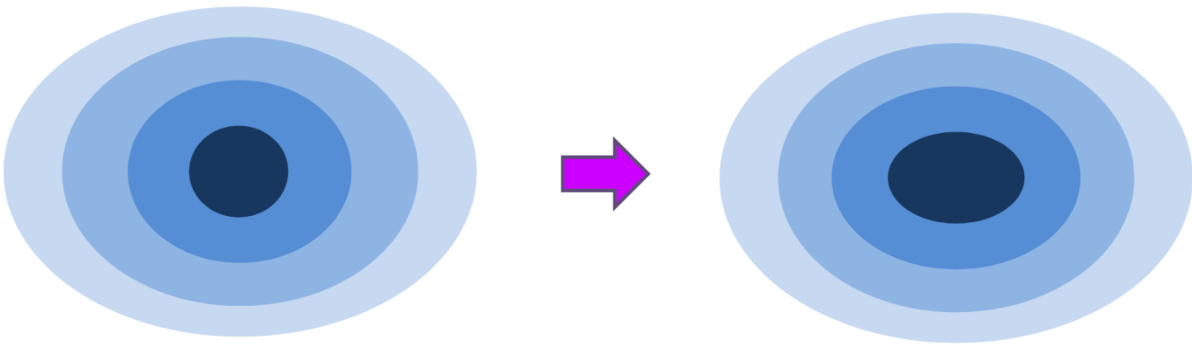
semi-major semi-minor

Integrals are not separable…

<u>Elliptical Isodensity Approximation</u> [Lai et al. (1993)]

reality approximation

isodensity contours



ellipticity is constant throughout

Decoupling the Integrals

Radial coordinate transformation → allows us to decouple the integrals

$$r \to \tilde{r} = \frac{r}{\Theta(\cos\theta)} \quad \left(\Theta(\cos\theta) = \frac{R_*(\theta)}{a_1}\right)$$

$$\boxed{M_\ell = 2\pi\, R_\ell\, I_{\ell,3}}$$

$$\left(R_\ell = \int_0^{a_1} \rho(\tilde{r})\, \tilde{r}^{\ell+2}\, d\tilde{r} \qquad I_{\ell,k} = \int_{-1}^{1} \Theta(\cos\theta)^{\ell+k} P_\ell(\cos\theta)\, d\cos\theta \right)$$

Rewrite in terms of Lane-Emden function

$$\rho = \rho_c\, (\vartheta_{\mathrm{LE}})^n$$

polytropic index

$$I_{\ell,3} = (-1)^{\ell/2} \frac{2}{\ell+1} \sqrt{1-e^2}\, e^\ell$$

ellipticity: $e^2 = 1 - \dfrac{a_3^2}{a_1^2}$

-Similar relation holds for S_ℓ

spin angular velocity

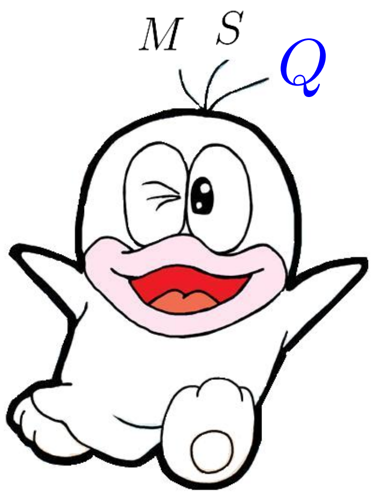
$$\boxed{S_\ell = \frac{4\pi\ell}{2\ell+1} \Omega\, R_{\ell+1} \left(I_{\ell-1,5} - I_{\ell+1,3}\right)}$$

3-Hair Relations for Newtonian Stars

$$M_\ell + i\frac{q}{a}S_\ell = \bar{B}_{n,\lfloor\frac{\ell-1}{2}\rfloor} M (iq)^\ell$$

[Stein, KY & Yunes (2014)]

$$\left(a = S/M \quad q^2 = -Q/M \right)$$



Q-taro

Once the polytropic index n is specified, all the higher moments can be expressed in terms of the first three.

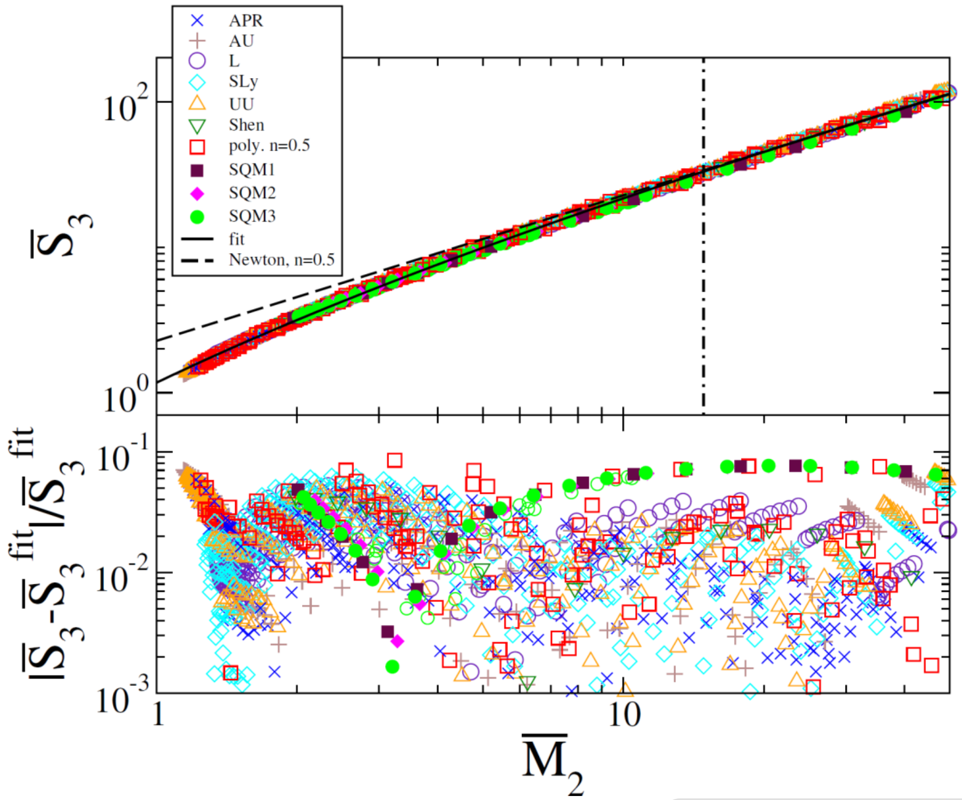
Coefficient is equation of state insensitive within ~5% for low-l modes.

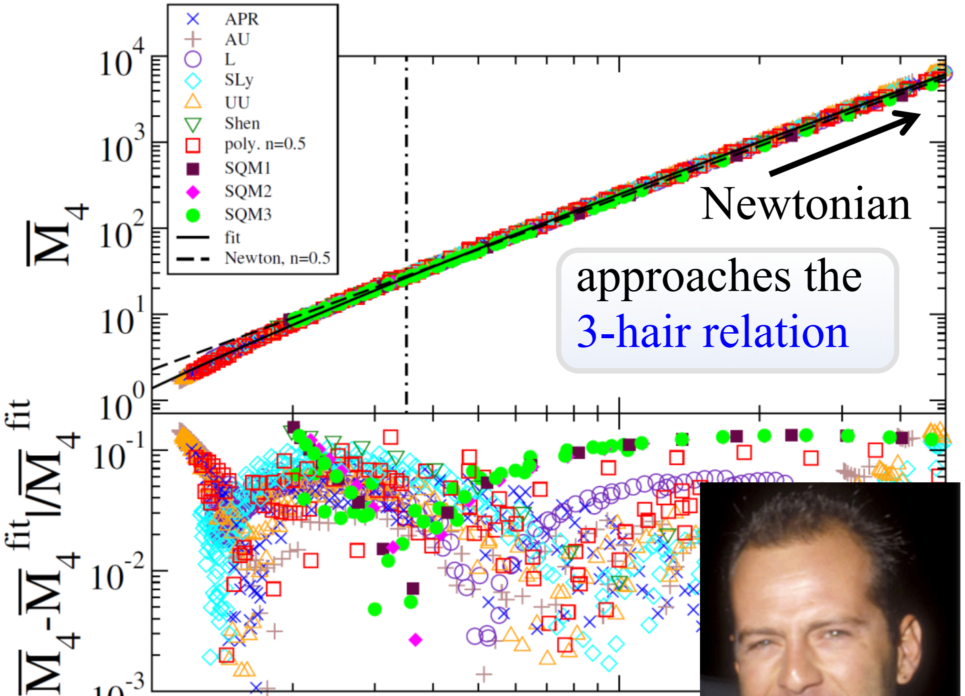
NSs are Follicly-challenged

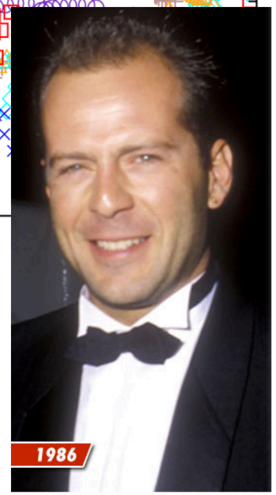
[KY+ (2014)]

Calculations in full GR with realistic EoSs

NS lower multipole moments can be given in terms of the first three.





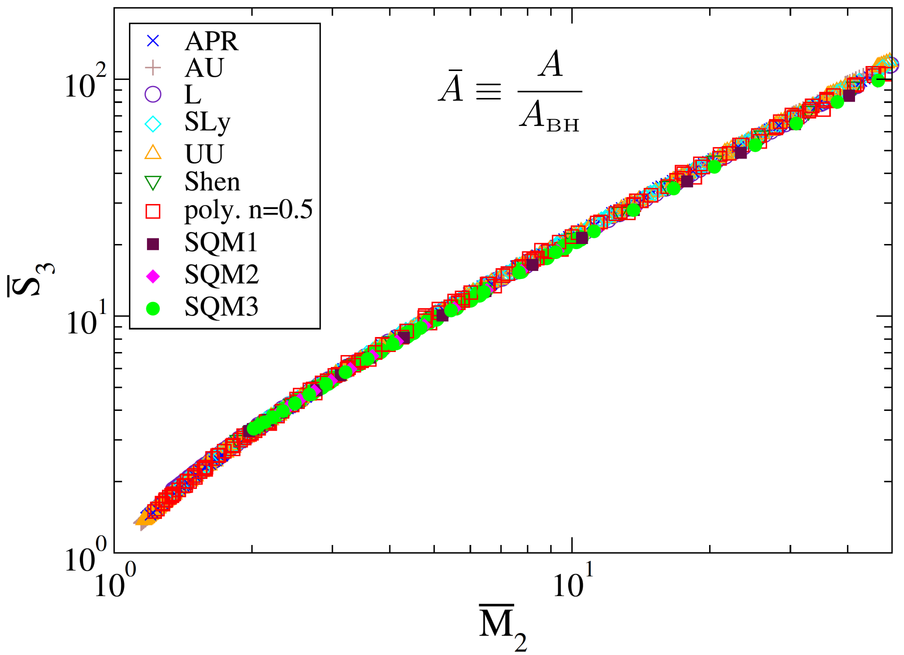


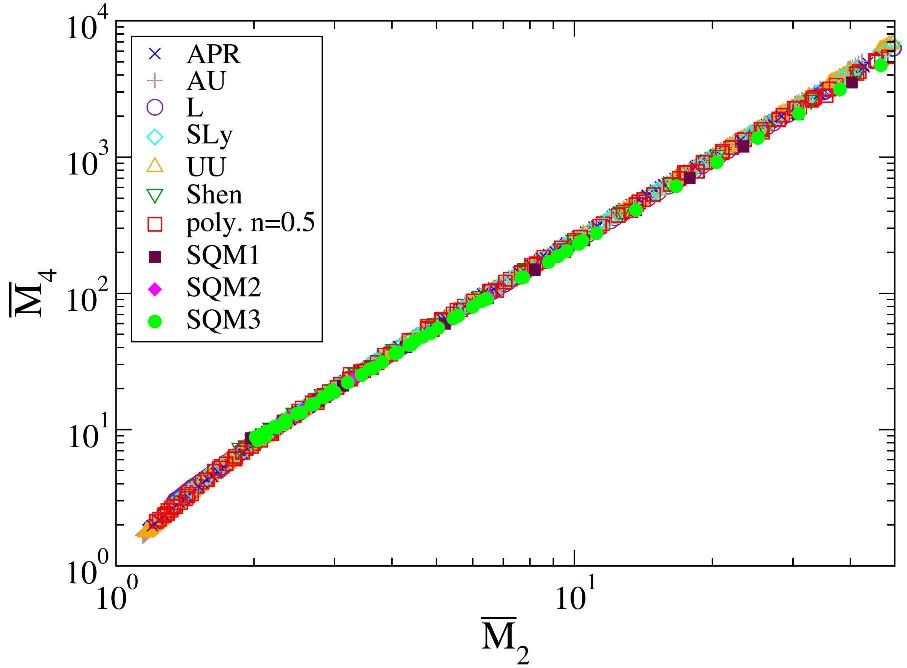
$$\bar{A} \equiv \frac{A}{A_{\rm BH}}$$

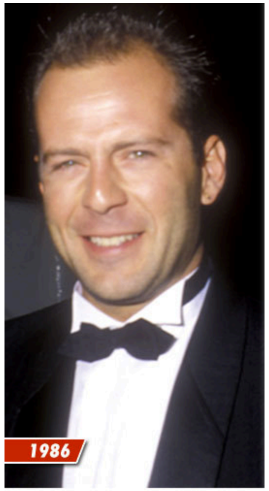
NSs are follicly challenged!

Relating Follicly-Challenged Neutron Stars to Bald Black Holes

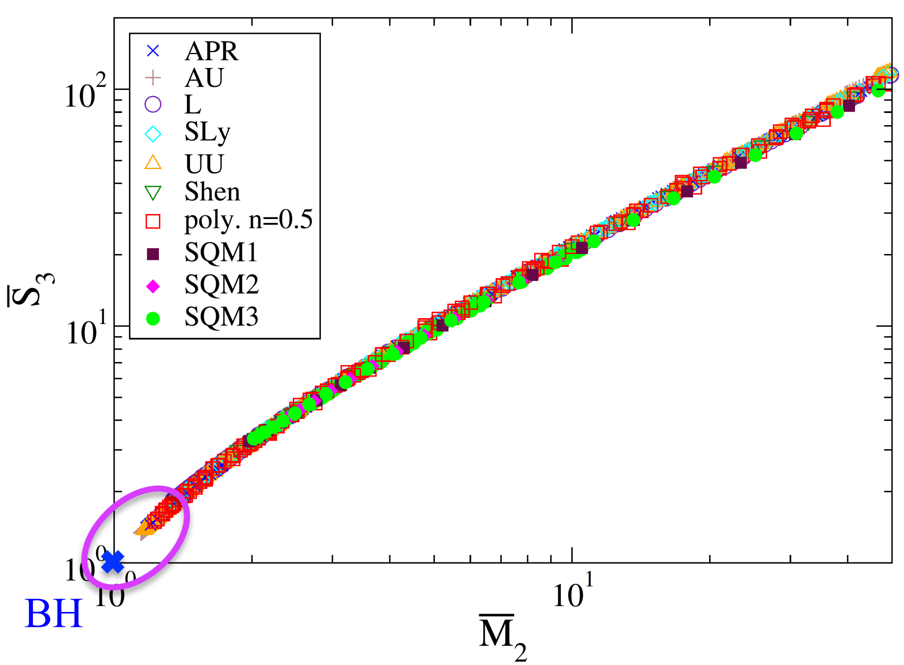
Filling the gap…?

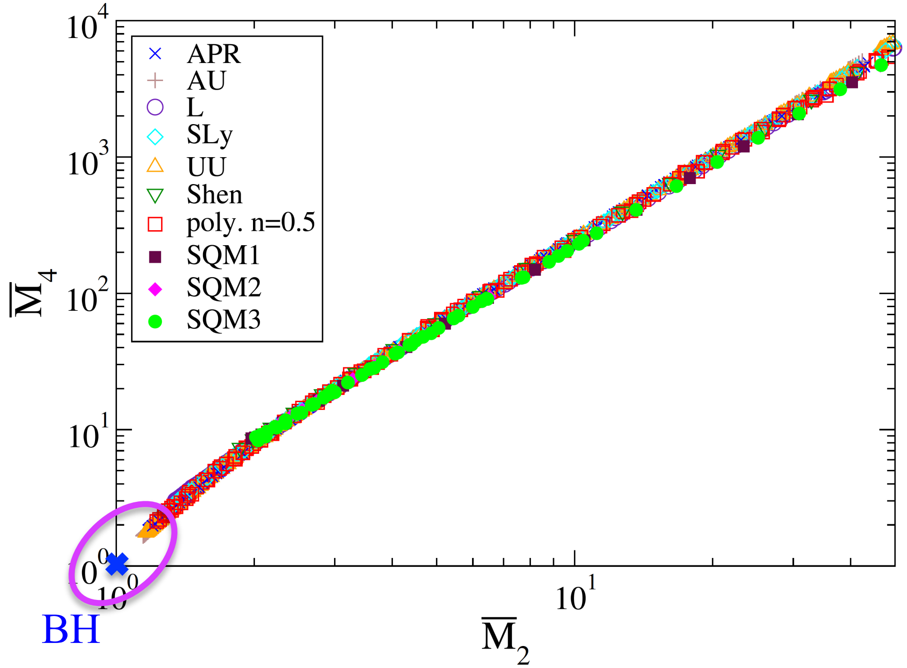






Filling the gap…?





Relation between follicly-challenged neutron stars and bald black holes is unclear

Is there a stellar sequence that can continuously reach the black hole limit?

→ Anisotropic neutron stars!
(radial pressure) $\neq$ (tangential pressure)

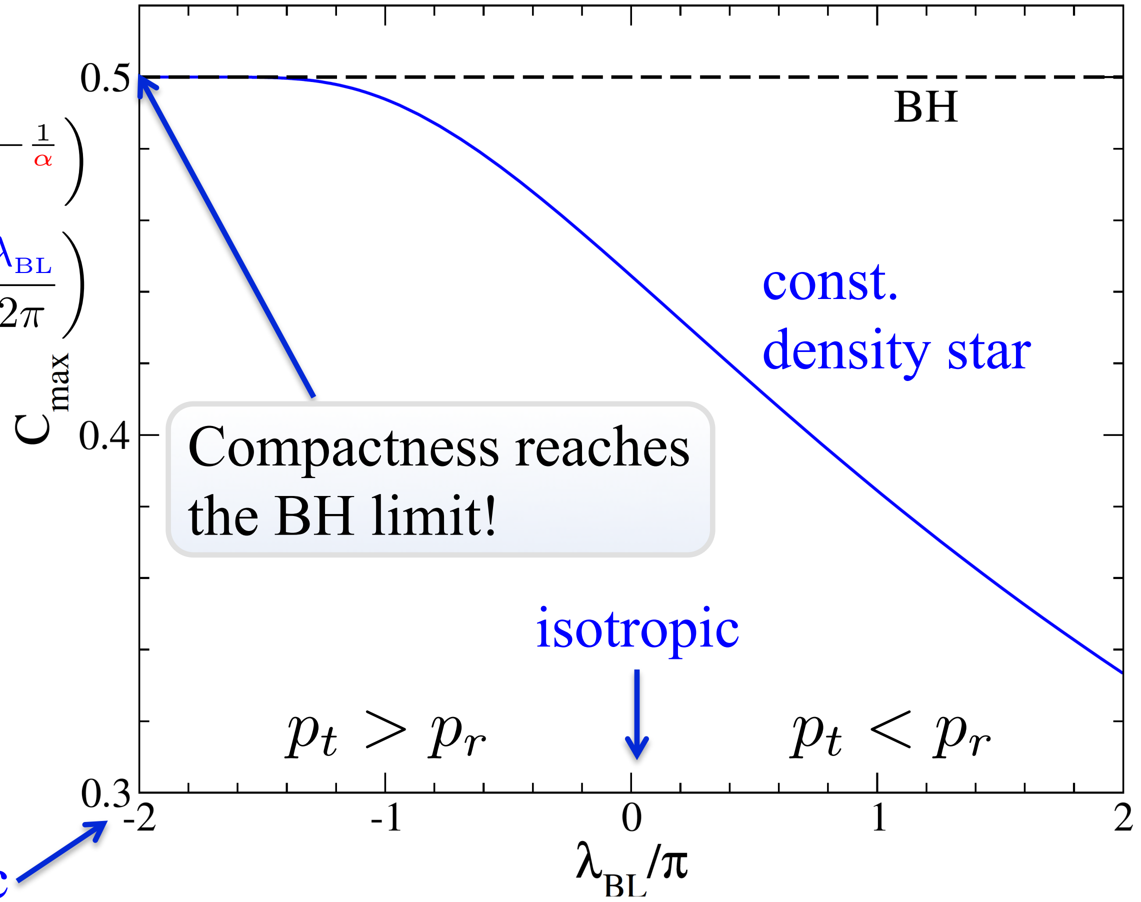
Max Compactness

Anisotropic model proposed by Bowers & Liang (1974)

$$\sigma(r) = p_r - p_t = \frac{\lambda_{\rm BL}}{3}(\rho + 3p_r)(\rho + p_r)\frac{r^3}{r - 2m} \qquad m(r) = 4\pi \int_0^r \rho r^2 dr$$

$$C_{\rm max} = \frac{1}{2}\left(1 - 3^{-\frac{1}{\alpha}}\right)$$

$$\alpha = \frac{1}{2}\left(1 + \frac{\lambda_{\rm BL}}{2\pi}\right)$$

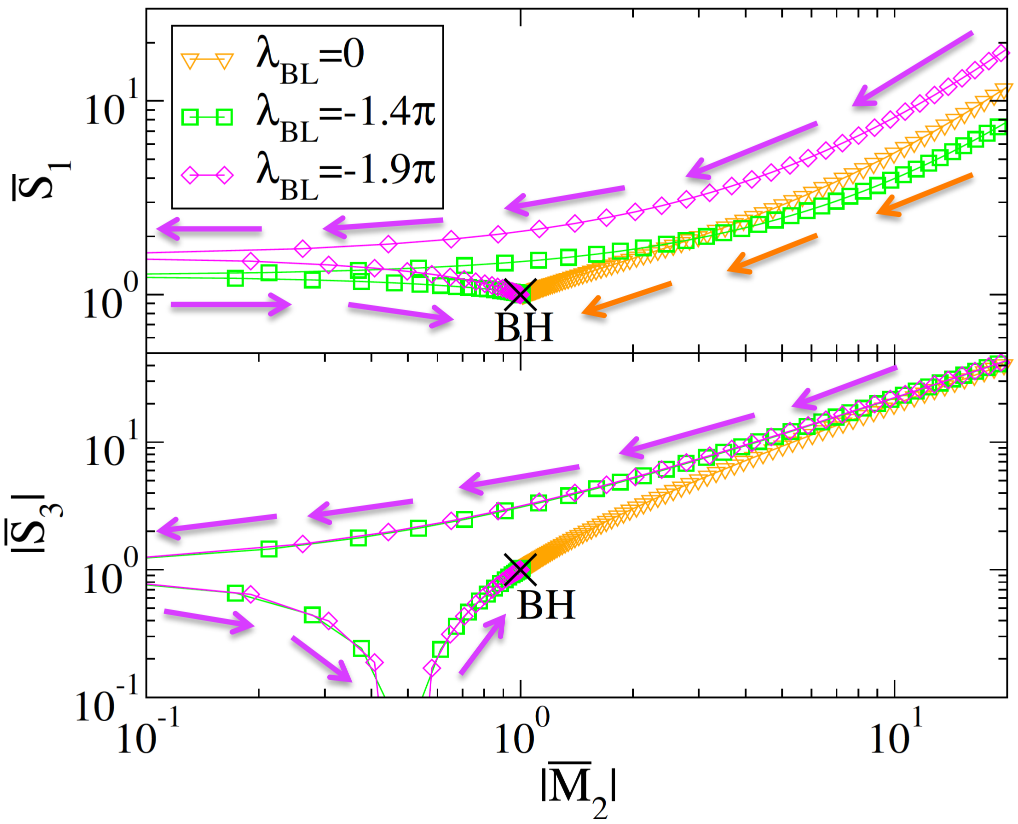


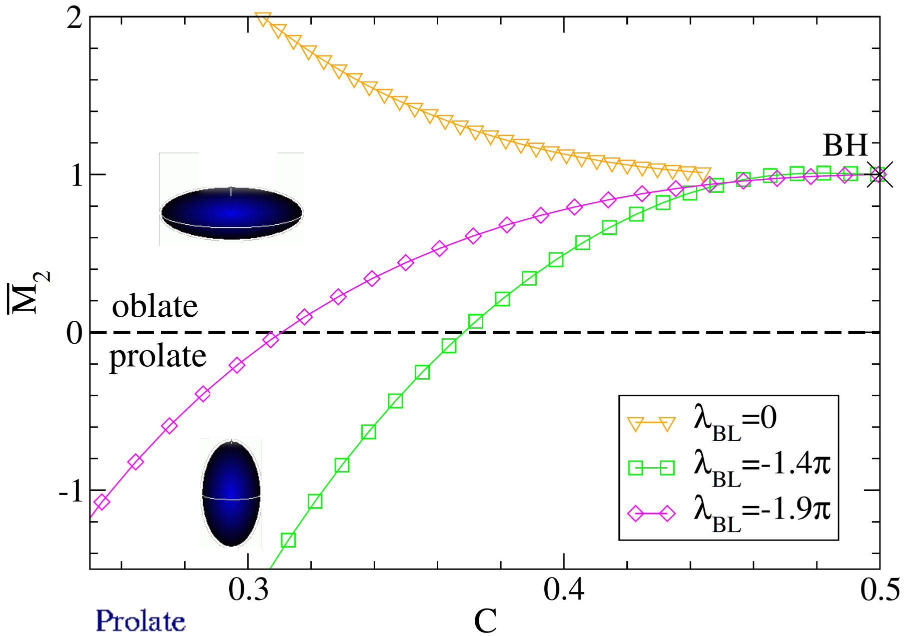
strongly anisotropic

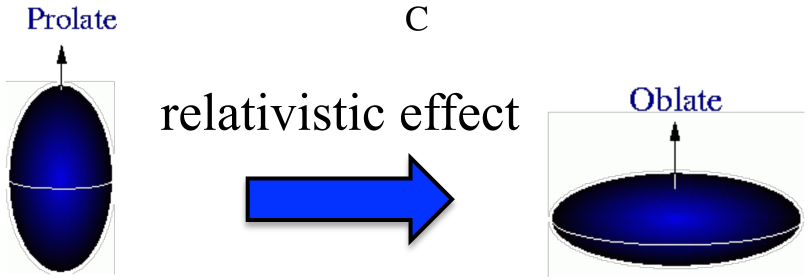
Multipole Relations for Anisotropic Stars

$$\bar{A} \equiv \frac{A}{A_{\rm BH}}$$

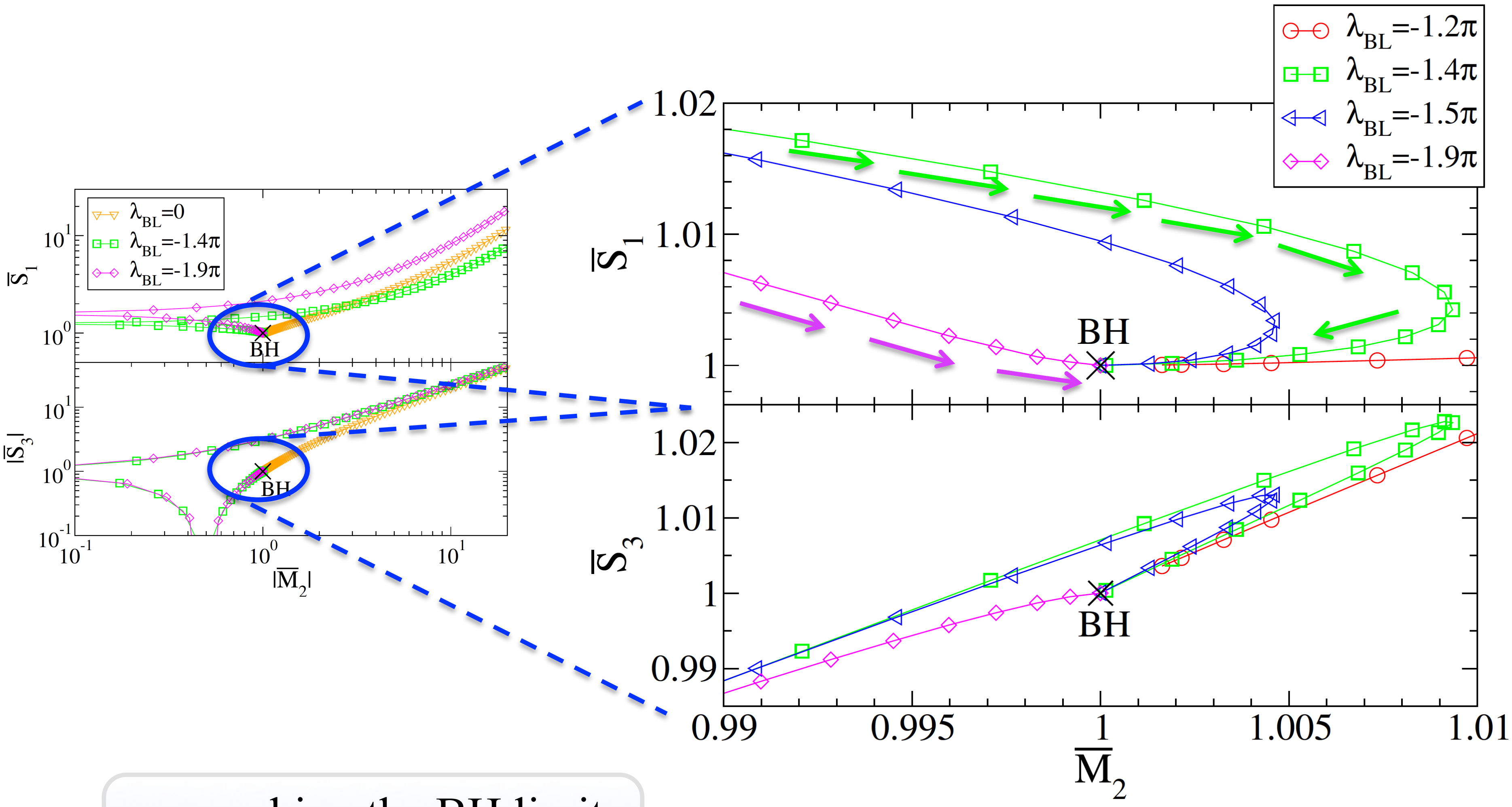
c.f. Newtonian analysis
[Glampedakis et al. (2013)]





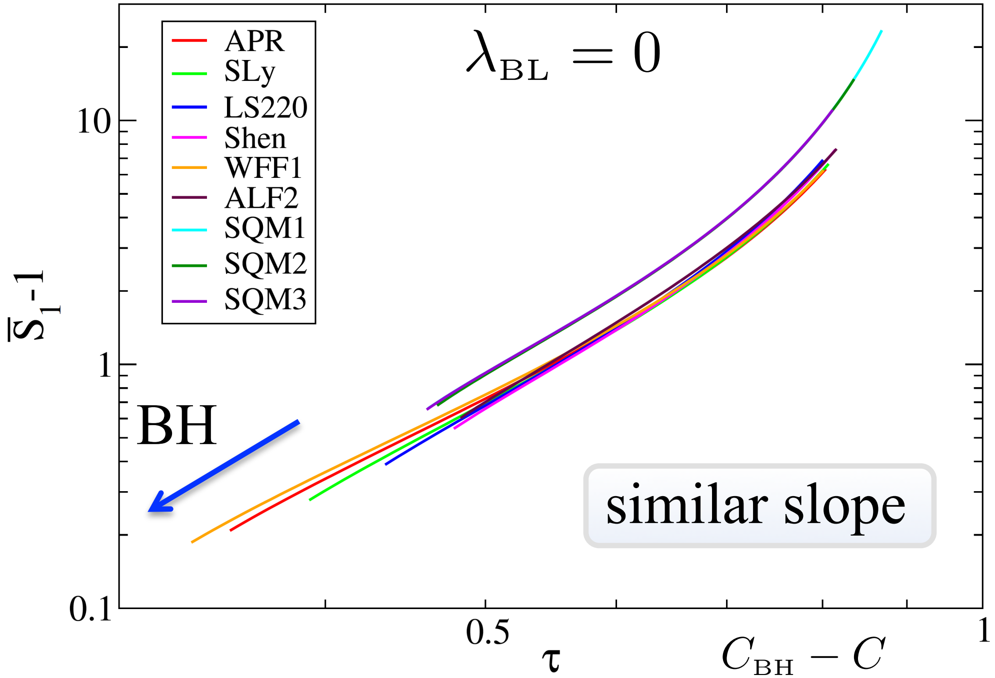


Zoom In!!



approaching the BH limit
in a non-trivial way!!

Phase Transition…?



$$\tau = \frac{C_{\rm BH} - C}{C_{\rm BH}}$$

$$\bar{A}_\ell - 1 \propto \tau^{k_{\bar{A}_\ell}}$$

	Isotropic
$\lambda_{\rm BL}$	0
$k_{\bar{S}_1}$	3.90($\pm$0.49)
$k_{\bar{M}_2}$	4.22($\pm$0.45)
$k_{\bar{S}_3}$	4.19($\pm$0.49)

EoS variation

EoS universality of ~10% in the scaling exponent

Relations to 2nd order phase transition?

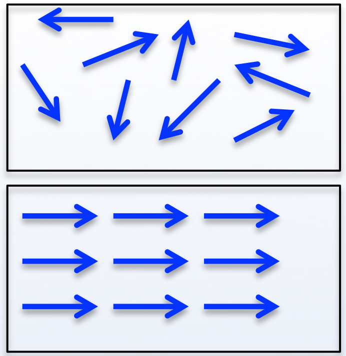
e.g. ferromagnetism

Curie temperature

critical exponent

universality

magnetic susceptibility $\chi \propto |T - T_c|^\gamma$



Conclusions & Future Work

Conclusions & Future Work

-universal relations are useful for probing nuclear and gravitational physics with NS observations

-approximate no-hair (3-hair) relations for NS multipole moments

-multipole relations for anisotropic stars approach the BH limit in a non-trivial way

Relations to
- phase transitions?
- critical behaviors?
- gravity/fluid correspondence?

Gravitational collapse simulations of rotating NSs

⇒ How the multipole relations behave from NSs to BHs in a realistic situation