Phenomenology of dark matter via a bimetric extension of general relativity

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Plan

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Some Relativistic MOND theories

Modified Dark Matter and bimetric gravity

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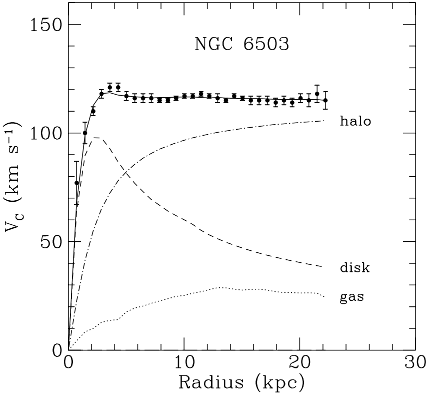
Flat rotation curves of galaxies

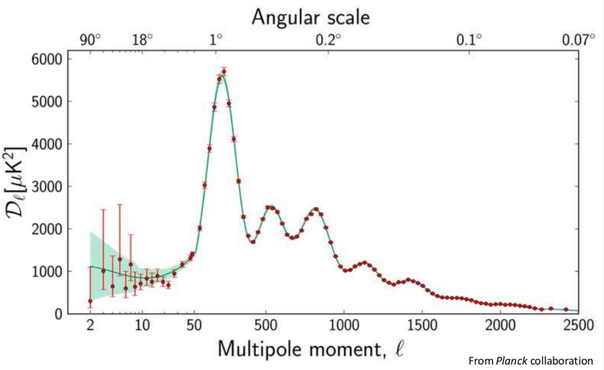


- For a circular orbit we expect

$$\frac{v^2}{r} = g_N = \frac{GM(r)}{r^2} \quad \Longrightarrow \quad v(r) = \sqrt{\frac{GM(r)}{r}},$$

- Instead we find $v(r)$ constant $\Longrightarrow$ Dark Matter (DM) halo : $\rho_{\text{halo}} \sim \frac{1}{r^2}$.

The cosmological concordance model ΛCDM



- Discrepancy between dynamical and visible masses in clusters of galaxies,
- Formation and growth of large scale structures,
- Temperature fluctuations in the cosmic microwave background,
- Content of our universe: dark energy ($\sim 68\%$, unknown), dark matter ($\sim 27\%$, unknown) and baryons ($\sim 5\%$).

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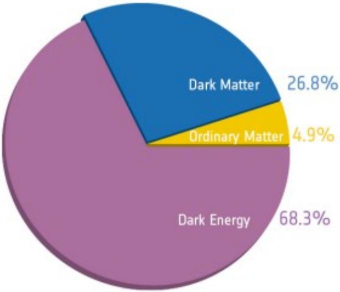

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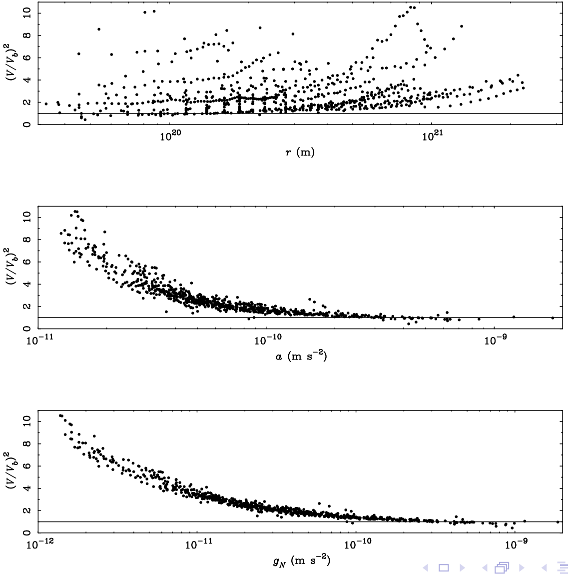
The mass discrepancy - acceleration relation [Famaey & McGaugh, 2012]



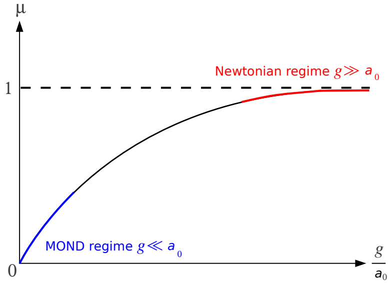
Milgrom's law (1983)

Modification of the Newtonian gravitational acceleration

$$\mu\left(|\mathbf{g}|/a_0\right)\mathbf{g} = \mathbf{g}_N\,,$$

- $a_0 \approx 1.2\times 10^{-10}\ \mathrm{m\,s^{-2}}$ is the MOND acceleration constant,
- μ is the MOND interpolating function :

$$\begin{cases} \mu(x) \underset{x\gg 1}{\longrightarrow} 1 & \text{in the newtonian regime } g \gg a_0\,, \\ \mu(x) \underset{x\ll 1}{\longrightarrow} x & \text{in the MOND regime } g \ll a_0\,. \end{cases}$$



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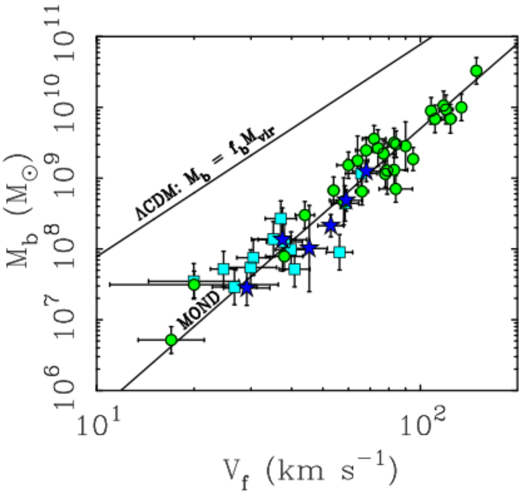
- We recover the flat rotation curves of galaxies,

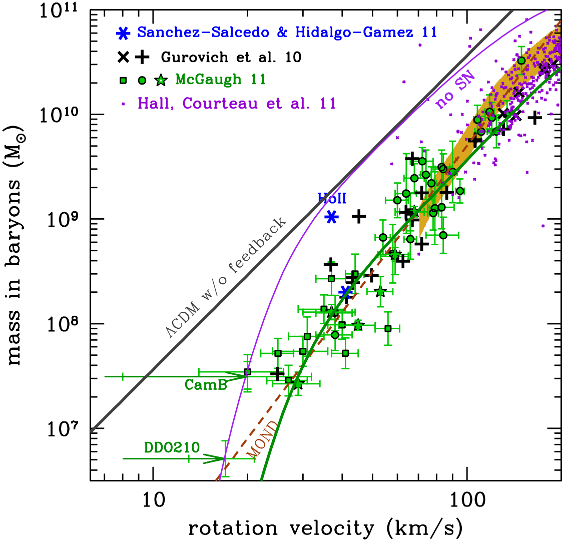
$$\frac{V_c^2}{r} = g = \sqrt{\frac{GMa_0}{r^2}} \implies V_c^4 = GMa_0 \,.$$



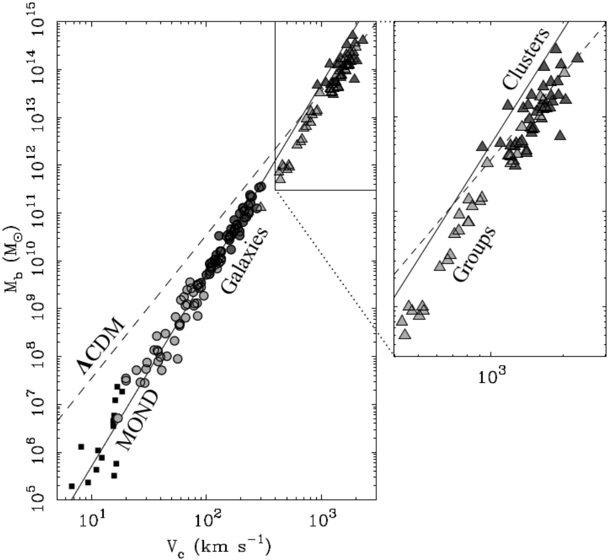
$$V_f \sim (a_0 G M_b)^{1/4} ,$$

$$a_0 \approx 1.2 \times 10^{-10}\ \mathrm{m\,s^{-2}}$$

The Baryonic Tully-Fisher Relation [Silk & Mamon, 2012]



Baryonic mass vs rotation velocity [McGaugh, 2014]



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Modified dark matter theories

- Dipolar Dark Matter [Blanchet & Le Tiec 2008;2009]

The MOND equation and its dielectric analogy

Modified Poisson equation for the gravitational field

[Bekenstein & Milgrom, 1984]

$$\boldsymbol{\nabla} \cdot \left(\mu \left(\frac{g}{a_0} \right) \mathbf{g} \right) = -4\pi G \rho_b \,, \quad \text{with } \mathbf{g} = \boldsymbol{\nabla} U \,.$$

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Analogy with a dielectric medium

Writing $\mu = 1 + \chi$ where χ is the gravitational susceptibility, the analogy with a dielectric medium is apparent,

$$\Delta U = -4\pi G \left(\rho_b + \rho_{\text{pol}} \right) \,,$$

where $\rho_{\text{pol}} = -\boldsymbol{\nabla} \cdot \mathbf{P}$ and $\mathbf{P} = -\frac{\chi}{4\pi G} \mathbf{g}$ is the polarization of some DM medium and $\chi < 0$ (because $\mu < 1$).

Dipolar Dark Matter [Blanchet & Le Tiec 2008;2009]

Dark matter fluid endowed with a dipole moment vector field ξ^μ,

$$S_{\mathrm{DDM}} = \int \mathrm{d}^4x \sqrt{-g}\left[-\rho + J^\mu \dot{\xi}_\mu - V(P_\perp)\right],$$

with $P_\perp = \rho\,\xi_\perp$ the polarization field and

$$V(P_\perp) = \frac{\Lambda}{8\pi} + 2\pi\, P_\perp^2 + \frac{16\pi^2}{3a_0}\, P_\perp^3 + \mathcal{O}(P_\perp^4)\,.$$

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- Indistinguishable from Λ-CDM at first order in cosmological perturbations.

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Drawbacks

- Requires a **weak clustering hypothesis** to recover the MOND equation : the dipolar DM medium should not cluster much in galaxies compared to baryonic matter and stays at rest, $\rho \approx \rho_0 \ll \rho_b$.
- Instability of the evolution of the dipole moment vector $\xi_\perp^\mu$ (with a very long time scale).
- No microscopic description for the dipole moment.

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Microscopic description of a dipolar DM medium

- We describe the dipolar DM medium as made of individual dipole moments $\mathbf{p} = m\boldsymbol{\xi}$, with a polarization field $\mathbf{P} = n\mathbf{p}$.

- The polarization field $\mathbf{P}$ should be aligned with the gravitational field,

$$\mathbf{P} = -\frac{\chi}{4\pi G}\,\mathbf{g} \quad \text{and} \quad \rho_{\text{pol}} = -\boldsymbol{\nabla}\cdot\mathbf{P}\,,$$

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- The dipole moment can be seen as pairs of particles with positive and negative gravitational masses $(m_i, m_g) = (m, \pm m) \longrightarrow$ cannot be coupled to GR.

Non-relativistic description of a dipolar DM medium

- To describe the individual dipole moments correctly, the two species of DM particles couple to two different gravitational potential U and $\underline{U}$,

$$\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \boldsymbol{\nabla}(U + \phi)\,, \qquad \frac{\mathrm{d}\underline{\mathbf{v}}}{\mathrm{d}t} = \boldsymbol{\nabla}(\underline{U} - \phi)\,, \qquad \frac{\mathrm{d}\mathbf{v}_b}{\mathrm{d}t} = \boldsymbol{\nabla}U\,.$$

- **A non-gravitational internal force** ϕ is necessary to stabilize the dipolar medium

$$\Delta\phi = \frac{-4\pi G}{\chi}\left(\rho - \underline{\rho}\right).$$

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- When weakly excited, the dipolar dark matter medium behaves as a polarizable and stable plasma of particles,

$$\frac{\mathrm{d}^2\boldsymbol{\xi}}{\mathrm{d}t^2} + \omega^2\boldsymbol{\xi} = 2\mathbf{g}$$

Going to a relativistic model

- One needs **two metrics $\mathbf{g}_{\mu\nu}$ and $\underline{\mathbf{g}}_{\mu\nu}$** interacting with each other through $f_{\mu\nu}$ algebraically defined by the implicit relation

$$f_{\mu\nu} = f^{\rho\sigma}\, g_{\rho\mu}\, \underline{g}_{\nu\sigma} = f^{\rho\sigma}\, g_{\rho\nu}\, \underline{g}_{\mu\sigma}\,,$$

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- **Two kinds of dark matter** ρ and $\underline{\rho}$, with mass currents $J^{\mu} = \rho\, u^{\mu}$ and $\underline{J}^{\mu} = \underline{\rho}\, \underline{u}^{\mu}$, and respectively coupled to $g_{\mu\nu}$ and $\underline{g}_{\mu\nu}$,

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- **Ordinary baryonic matter** ρ_b living in the sector $g_{\mu\nu}$,
- **A vector field** K_{μ} living in the interacting sector $f_{\mu\nu}$ and with a non-canonical kinetic term.

The action

$$S = \int \mathrm{d}^4x \left\{ \sqrt{-g}\left(\frac{R-2\lambda}{32\pi} - \rho_{\mathrm{b}} - \rho\right) + \sqrt{-\underline{g}}\left(\frac{\underline{R}-2\underline{\lambda}}{32\pi} - \underline{\rho}\right) \right.$$
$$\left. + \sqrt{-f}\left[\frac{\mathcal{R}[f]-2\lambda_f}{16\pi\varepsilon} + (j^\mu - \underline{j}^\mu)K_\mu + \frac{a_0^2}{8\pi}W\left(-\frac{H^{\mu\nu}H_{\mu\nu}}{2a_0^2}\right)\right] \right\}$$

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It is divided in three sectors

- the ordinary sector : $g_{\mu\nu}$, λ, ρ_b and ρ,
- the hidden sector : $\underline{g}_{\mu\nu}$, $\underline{\lambda}$, and $\underline{\rho}$,

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It is divided in three sectors :

- the ordinary sector : $g_{\mu\nu}$, λ, ρ_b and ρ,
- the hidden sector : $\underline{g}_{\mu\nu}$, $\underline{\lambda}$, and $\underline{\rho}$,
- the interacting sector : $f_{\mu\nu}[g,\underline{g}]$, λ_f and K_μ.

The action

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- Three different cosmological constants in the three sectors, will be related to the observed cosmological constant Λ_{obs}.

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- ε measures the **strength of the interaction** between the two sectors. In the (post-)Newtonian limit we will assume $\varepsilon \ll 1$.

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- Three different cosmological constants in the three sectors, will be related to the observed cosmological constant $\Lambda_{\rm obs}$.
- ε measures the **strength of the interaction** between the two sectors. In the (post-)Newtonian limit we will assume $\varepsilon \ll 1$.
- The function W is determined phenomenologically to recover
  - MOND in the weak field limit $X \to 0$,

$$W(X) = X - \frac{2}{3}X^{3/2} + \mathcal{O}(X^2)\,,$$

  - 1PN limit of GR in the strong field limit $X \gg 1$,

$$W(X) = A + \frac{B}{X^\alpha} + \mathcal{O}(X^{-\alpha-1})\,, \quad \alpha > 0\,.$$

Equation of motion

Einstein field equations

$$\sqrt{-g}\,(G^{\mu\nu}+\lambda g^{\mu\nu})+\frac{\sqrt{-f}}{\varepsilon}\,\mathcal{A}^{\mu\nu}_{\rho\sigma}\left(\mathcal{G}^{\rho\sigma}+\lambda_f f^{\rho\sigma}\right)=16\pi\left[\sqrt{-g}\,(T^{\mu\nu}_{\mathrm{b}}+T^{\mu\nu})+\sqrt{-f}\,\mathcal{A}^{\mu\nu}_{\rho\sigma}\,\tau^{\rho\sigma}\right],$$

$$\sqrt{-\underline{g}}\,(\underline{G}^{\mu\nu}+\underline{\lambda}\underline{g}^{\mu\nu})+\frac{\sqrt{-f}}{\epsilon}\,\underline{\mathcal{A}}^{\mu\nu}_{\rho\sigma}\left(\mathcal{G}^{\rho\sigma}+\lambda_f f^{\rho\sigma}\right)=16\pi\left[\sqrt{-\underline{g}}\,\underline{T}^{\mu\nu}+\sqrt{-f}\,\underline{\mathcal{A}}^{\mu\nu}_{\rho\sigma}\,\tau^{\rho\sigma}\right]$$

Equations of motion

$$\begin{aligned} a_b^\mu &= 0\,,\\ a^\mu &= u^\nu\,H_{\mu\nu}\,,\\ \underline{a}^\mu &= -\underline{u}^\nu\,H_{\mu\nu}\,. \end{aligned}$$

$$\mathcal{D}_\nu\left(W'\,H^{\mu\nu}\right)=4\pi\left(j^\mu-\underline{j}^\mu\right)\;.$$

Perturbative solution to the implicit relation for $f_{\mu\nu}$

- Matrix formulation : we define $G_\mu^\nu = f^{\nu\rho} g_{\mu\rho}$ and $\underline{G}_\mu^\nu = f^{\nu\rho} \underline{g}_{\mu\rho}$, the implicit relation becomes

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$$\boxed{G\underline{G} = \underline{G}G = \mathbb{1}}$$

- Defining $H = \frac{1}{2}\big(G - \underline{G}\big)$, we get the perturbative solution

$$\boxed{\begin{aligned} G &= H + \sqrt{\mathbb{1} + H^2} \\ \underline{G} &= -H + \sqrt{\mathbb{1} + H^2} \end{aligned}}$$

with $\sqrt{\mathbb{1} + H^2} = \sum_{p=0}^{+\infty} \gamma_p \, H^{2p}$ with $\gamma_p = \frac{(-)^{p+1}(2p-3)!!}{2^p \, p!}$.

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- Returning to a metric formulation,

$$g_{\mu\nu} = \Big(f_{\mu\nu} + h_{\mu\nu} + x_{\mu\nu}\Big) \,, \quad \text{and} \quad \underline{g}_{\mu\nu} = \Big(f_{\mu\nu} - h_{\mu\nu} + x_{\mu\nu}\Big)$$

with $x_{\mu\nu} = \sum_{p=1}^{+\infty} \gamma_p \, H^{\rho_1}_\mu H^{\rho_2}_{\rho_1} \cdots H^{\rho_{2p-1}}_{\rho_{2p-2}} h_{\nu\rho_{2p-1}}$.

Linearizing matter and gravitational fields

- **Pertubative solution for $\boldsymbol{f_{\mu\nu}}$:**

$$g_{\mu\nu} = f_{\mu\nu} + h_{\mu\nu} + \mathcal{O}(h^2)\,, \qquad \underline{g}_{\mu\nu} = f_{\mu\nu} - h_{\mu\nu} + \mathcal{O}(h^2)\,,$$

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- **Plasma-like hypothesis :**
  - The two fluid of DM particles differ from a common equilibrium configuration by small displacement vectors y^μ and $\underline{y}^\mu$,

$$\begin{aligned} j^\mu &= j_0^\mu + \mathcal{D}_\nu \left(j_0^\nu y_\perp^\mu - j_0^\mu y_\perp^\nu \right) + \mathcal{O}\left(y^2\right)\,, \\ \underline{j}^\mu &= j_0^\mu + \mathcal{D}_\nu \left(j_0^\nu \underline{y}_\perp^\mu - j_0^\mu \underline{y}_\perp^\nu \right) + \mathcal{O}\left(y^2\right)\,, \end{aligned}$$

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$$W' H^{\mu\nu} = \alpha \left(j_0^\nu \xi_\perp^\mu - j_0^\mu \xi_\perp^\nu \right) + \mathcal{O}(2)\,.$$

Linearizing matter and gravitational fields

- **Pertubative solution for $\boldsymbol{f_{\mu\nu}}$:**

$$g_{\mu\nu} = f_{\mu\nu} + h_{\mu\nu} + \mathcal{O}(h^2)\,, \qquad \underline{g}_{\mu\nu} = f_{\mu\nu} - h_{\mu\nu} + \mathcal{O}(h^2)\,,$$

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- All perturbation variables are of the **same order of magnitude**

$$\nabla y \sim \nabla \underline{y} \sim h \sim \mathcal{O}(1)\,.$$

Cosmological perturbations

Background solutions : FLRW metrics

- $$\left.\begin{array}{l} g^{\text{FLRW}}_{\mu\nu}\,[a, \gamma_{ij}] \\ \underline{g}^{\text{FLRW}}_{\mu\nu}\,[\underline{a}, \gamma_{ij}] \end{array}\right\} \quad \Longrightarrow \quad f^{\text{FLRW}}_{\mu\nu}\left[\sqrt{a\underline{a}}, \gamma_{ij}\right]$$
- We recover the standard background equations with the observed cosmological constant being $\Lambda_{\text{obs}} = \lambda = \alpha\,\lambda_f = \alpha^2\,\underline{\lambda}$, with $\alpha = \frac{a}{\underline{a}} = \text{cste}$.

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First order cosmological perturbations

- Cosmological perturbations variables
  - in the g-sector : $\{\Psi,\ \Phi,\ \Phi^i,\ E^{ij}\}$, $\{\delta^F,\ V,\ V^i\}$ and $\{\rho_b,\ u_b^\mu\}$,
  - in the $\underline{g}$-sector : $\{\underline{\Psi},\ \underline{\Phi},\ \underline{\Phi}^i,\ \underline{E}^{ij}\}$ and $\{\underline{\delta}^F,\ \underline{V},\ \underline{V}^i\}$,
  - in the f-sector : $\xi_\perp^\mu = \big(0,\ D^i z + z^i\big)$.
- Then we compare the ordinary sector $g_{\mu\nu}$ on which ordinary matter moves with Λ-CDM scenario $\longrightarrow$ identify the observed dark matter variables in the sector $g_{\mu\nu}$.

First order cosmological perturbations

- Introducing **new effective dark matter variables** in the observable g-sector

$$\mathring{\rho}_{\text{DM}} = \frac{2\varepsilon}{1+\varepsilon}\,\mathring{\rho}\,, \qquad \delta^F_{\text{DM}} = \delta^F - \frac{1}{2\varepsilon}\left(\Delta z - (A - \underline{A})\right)\,,$$

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1. we recover the **standard continuity and Euler equations** for the effective dark matter,

$$\begin{aligned} &{\delta'}^{\rm F}_{\rm DM} + \Delta V_{\rm DM} = 0\,,\\ &V'_{\rm DM} + \mathcal{H} V_{\rm DM} + \Psi = 0\,, \qquad {V'}^i_{\rm DM} + \mathcal{H} V^i_{\rm DM} = 0\,. \end{aligned}$$

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- There are similar equations in the unobservable dark sector ; in particular **the whole set of equations is fully consistent**.

Non-relativistic limit of the model

In the limit where $\boldsymbol{\varepsilon} \ll \mathbf{1}$,

1. $U = -\underline{U}$,
2. At equilibrium the polarization field $\mathbf{P}$ is aligned with the gravitational field,
$$\mathbf{P} = \rho_0^* \boldsymbol{\lambda} = \frac{W'}{4\pi} \boldsymbol{\nabla} U \,,$$
3. We recover the MOND formula in the weak field regime with $\mu = 1 - W' = \frac{|\boldsymbol{\nabla} U|}{a_0}$,
$$\boldsymbol{\nabla} \cdot [\boldsymbol{\nabla} U - 4\pi \mathbf{P}] = -4\pi G \rho_b \,,$$
4. The dipolar dark matter medium should undergoes stable plasma-like oscillations
$$\frac{\mathrm{d}^2 \boldsymbol{\xi}}{\mathrm{d}t^2} + \omega^2 \boldsymbol{\xi} = 2\mathbf{g} \,, \quad \text{with } w = \sqrt{\frac{8\pi \rho_0 *}{W'}} \,.$$

Solar system tests

In the limit where $\varepsilon \ll 1$

1. To recover GR in the strong field regime ($X \to \infty$), we impose

$$W(X) = A + \frac{B}{X^\alpha} + \mathcal{O}\Big(\frac{1}{X^{\alpha+1}}\Big)\,, \qquad \alpha < 0\,,$$

2. And expand both metrics up to **second order in h**

$$g_{\mu\nu} = f_{\mu\nu} + h_{\mu\nu} + \frac{1}{2}h_{\mu\rho}h^\rho{}_\nu \text{ and } \underline{g}_{\mu\nu} = f_{\mu\nu} - h_{\mu\nu} + \frac{1}{2}h_{\mu\rho}h^\rho{}_\nu\,.$$

3. **Post-Newtonian expansion**
   - We expand the metrics to get the standard PN potentials

$$g^{1\text{PN}}_{\mu\nu}[V, V^i] \text{ and } \underline{g}^{1\text{PN}}_{\mu\nu}[V, V^i]\,,$$

   - We obtain the same parametrized PN parameters as in GR

$$\beta^{1\text{PN}} = 1\,, \quad \gamma^{1\text{PN}} = 1\,, \quad \text{all others being zero}\,.$$

Investigating the gravitational sector at linear order

$$S_g = \frac{1}{32\pi} \int \mathrm{d}^4x \left\{ \sqrt{-g}\, R + \sqrt{-\underline{g}}\, \underline{R} + \frac{2}{\varepsilon} \sqrt{-f}\, \mathcal{R} \right\} ,$$

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- Sum of **two massless non-interacting spin-two fields** .

Summary

Results

- The model is indistinguishable from the standard Λ-CDM paradigm at cosmological scales,
- It correctly reproduces the phenomenology of MOND in the non-relativistic limit without any weak clustering hypothesis, and the dipolar DM medium is stable,
- It passes solar system tests (same ppN parameters as GR),
- At linear order the gravitational sector is safe.

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- At linear order the gravitational sector is safe.

Remarks and perspectives

- Check the consistency of the model by counting the propagating degrees of freedom at the non-linear level,
- The arbitrary function W should in principle be derived from a more fundamental theory,
- Test the model by performing N-body simulations, in particular to look at the scale of galaxy clusters.