GreCo Seminar

Post-Newtonian higher-order spin effects in inspiraling compact binaries

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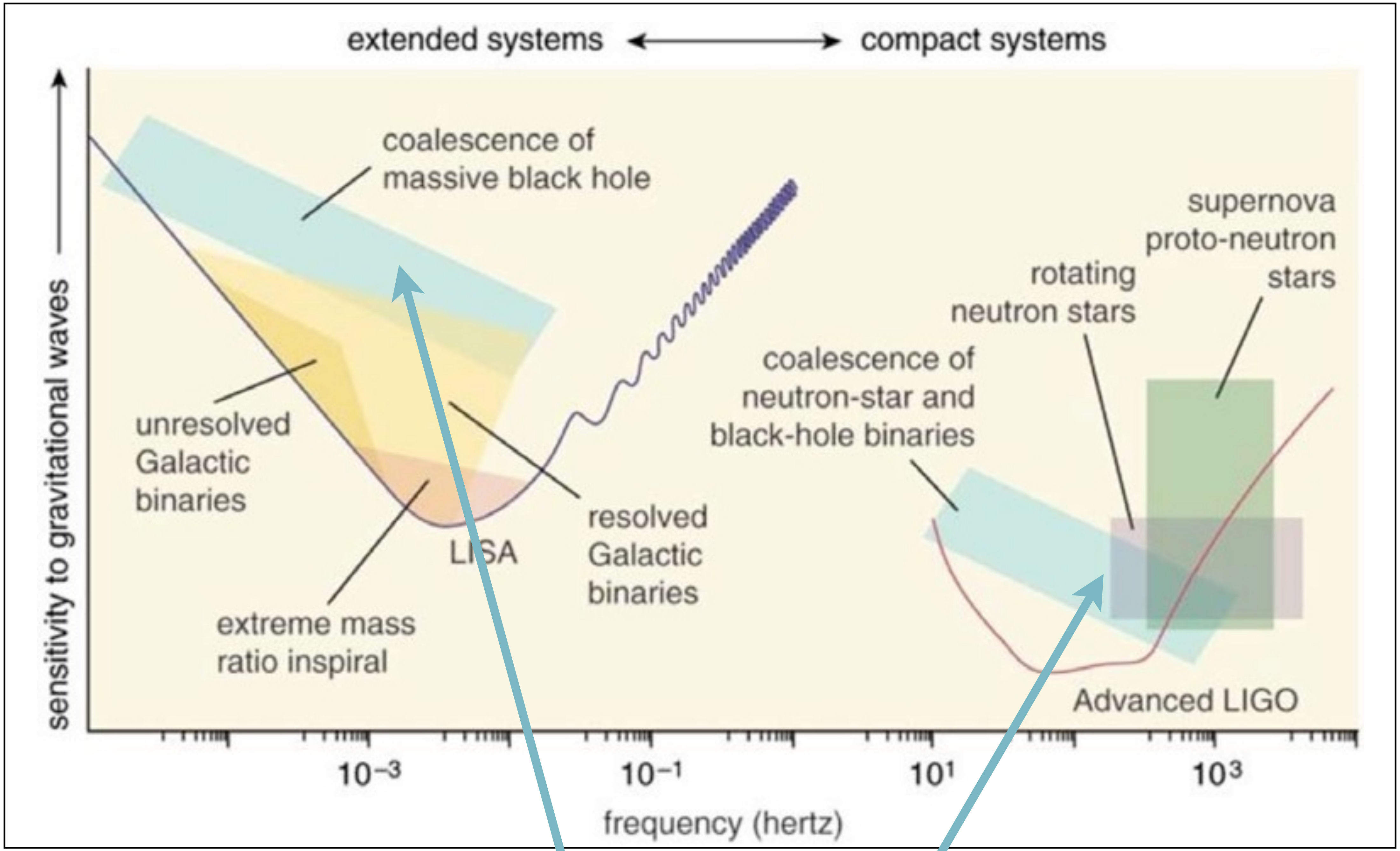
Outline

- Introduction: motivation and spin effects in inspiraling compact binaries
- Post-Newtonian MPN approach: near-zone iteration and wave generation formalism
- Lagrangian formalism for multipolar point particles with spin
- Hereditary effects for precessing orbits
- Results for spin effects in the dynamics and phasing

Introduction

Motivation: PN modeling of spin effects in compact binaries

Part I Astrophysical sources of GW

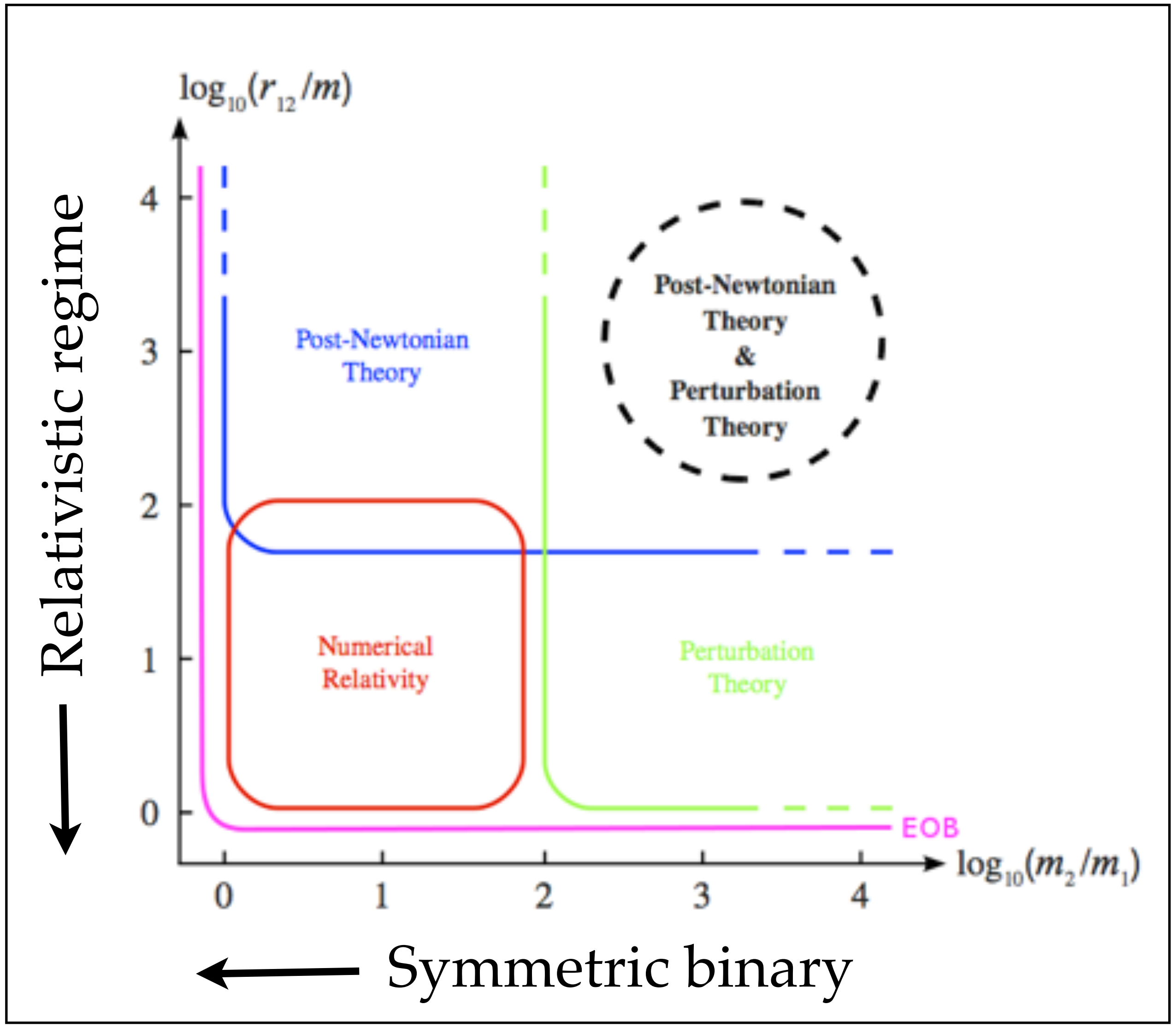


Coalescence of compact objects binaries (black holes/neutron stars)

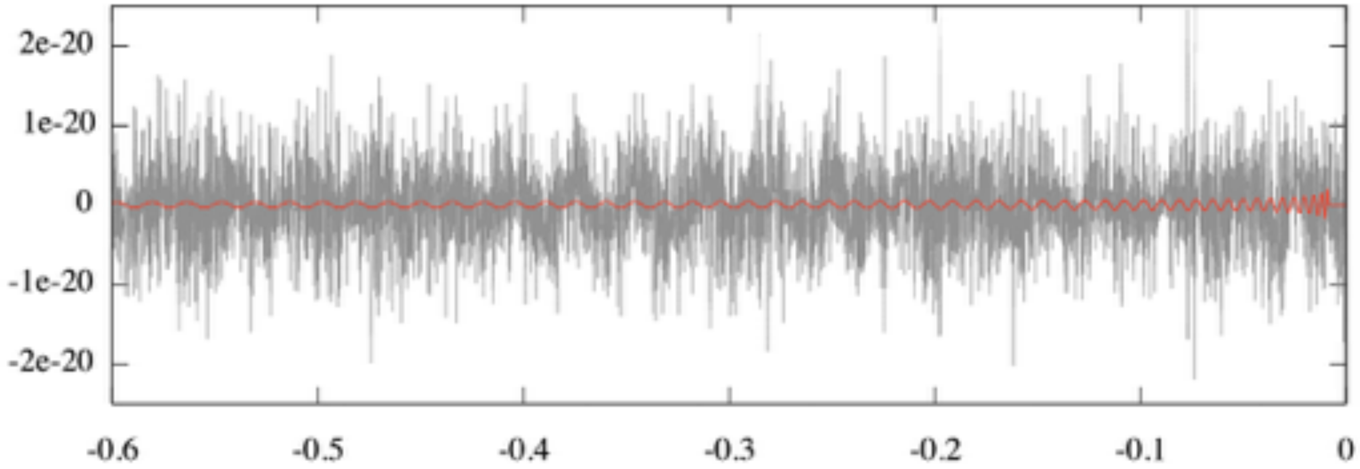
Modeling the GW signal of coalescences

Approximate methods

- **Post-Newtonian theory** (PN)
- Perturbation theory and Self-Force approach
- Numerical Relativity (NR)
- Effective-one-body (EOB)



Motivation for accurate PN templates



Matched filtering

$$(x|y) = 4\mathrm{Re} \int_0^\infty \mathrm{d}f \, \frac{\tilde{x}(f)\tilde{y}^*(f)}{\tilde{S}_n(f)}$$

Advanced LIGO-VIRGO band :

- NS-NS binary: ~10000 cycles
- BH-NS binary: ~3000 cycles
- BH-BH binary: ~600 cycles

High order PN contributions needed for accurate data analysis

Templates ingredients

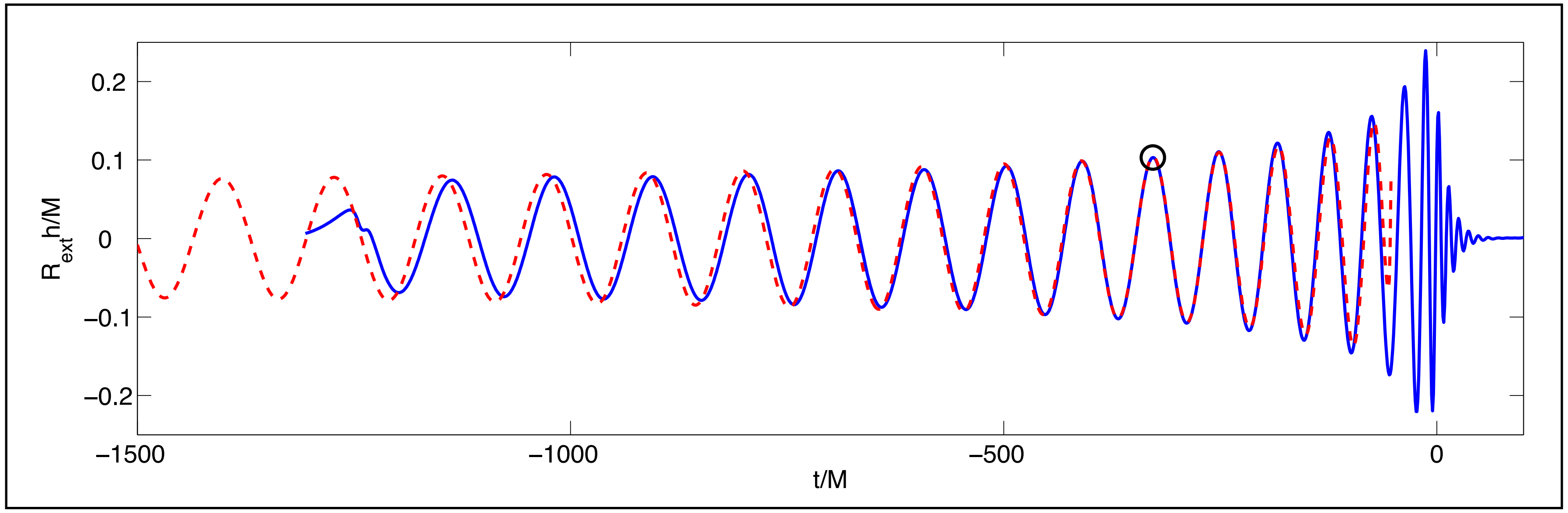
- Phasing for circular orbits: $E(\omega), \mathcal{F}(\omega)$ ⟶ Balance equation $\mathcal{F} = -\mathrm{d}E/\mathrm{d}t$ ⟶ PN approximant ⟶ GW Phase
- GW polarizations (modes): $h_+, h_\times \; (h_{lm})$
- Precessional dynamics: $\dot{\boldsymbol{S}}_1, \dot{\boldsymbol{S}}_2, \dot{\boldsymbol{\ell}}$

Part I

Modeling Inspiral-Merger-Ringdown GW signals

Hybrid waveforms

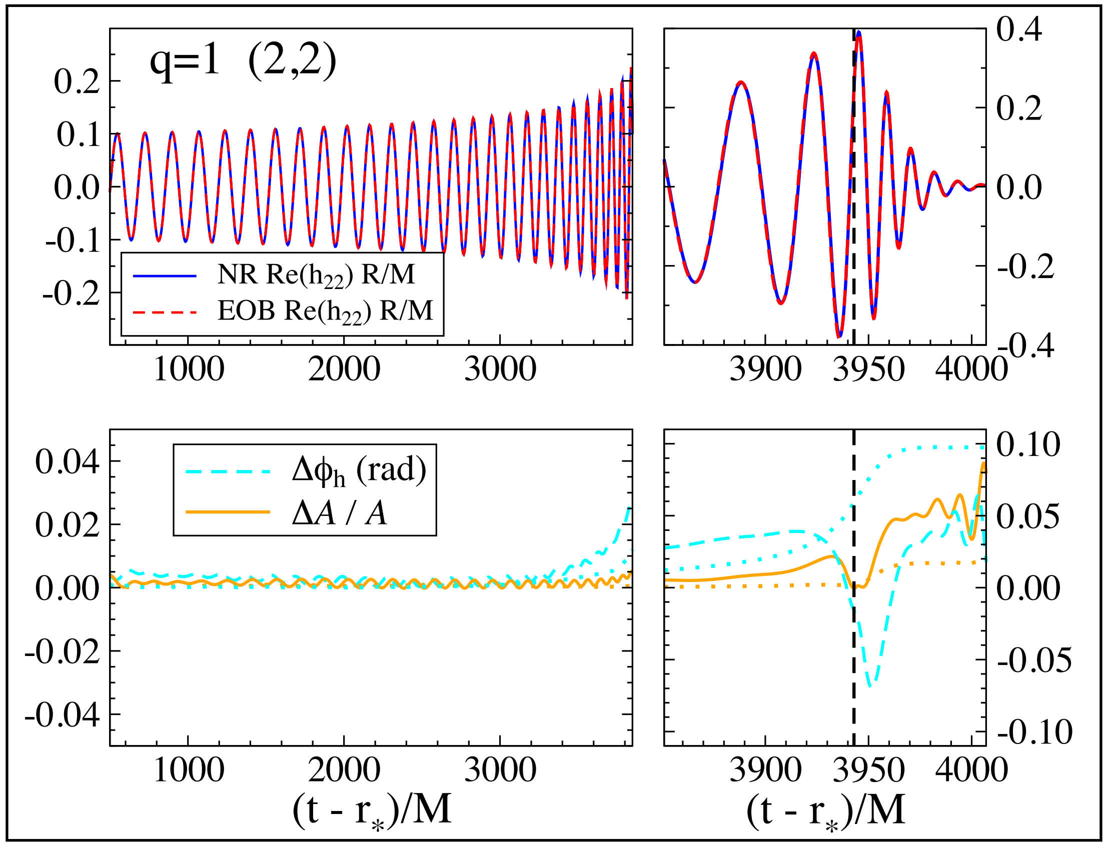
- PN covers inspiral
- Attachment in the late inspiral



Hybrid NR-PN waveform, inspiral-merger-ringdown [Baker&al 07]

Effective-One-Body waveforms

- PN Hamiltonian mapped on a deformed Kerr Hamiltonian and resummed
- PN waveform factorized
- Calibration on NR
- Ringdown attached as a superposition of QNM



EOBNR-NR comparison [Pan&al 11]

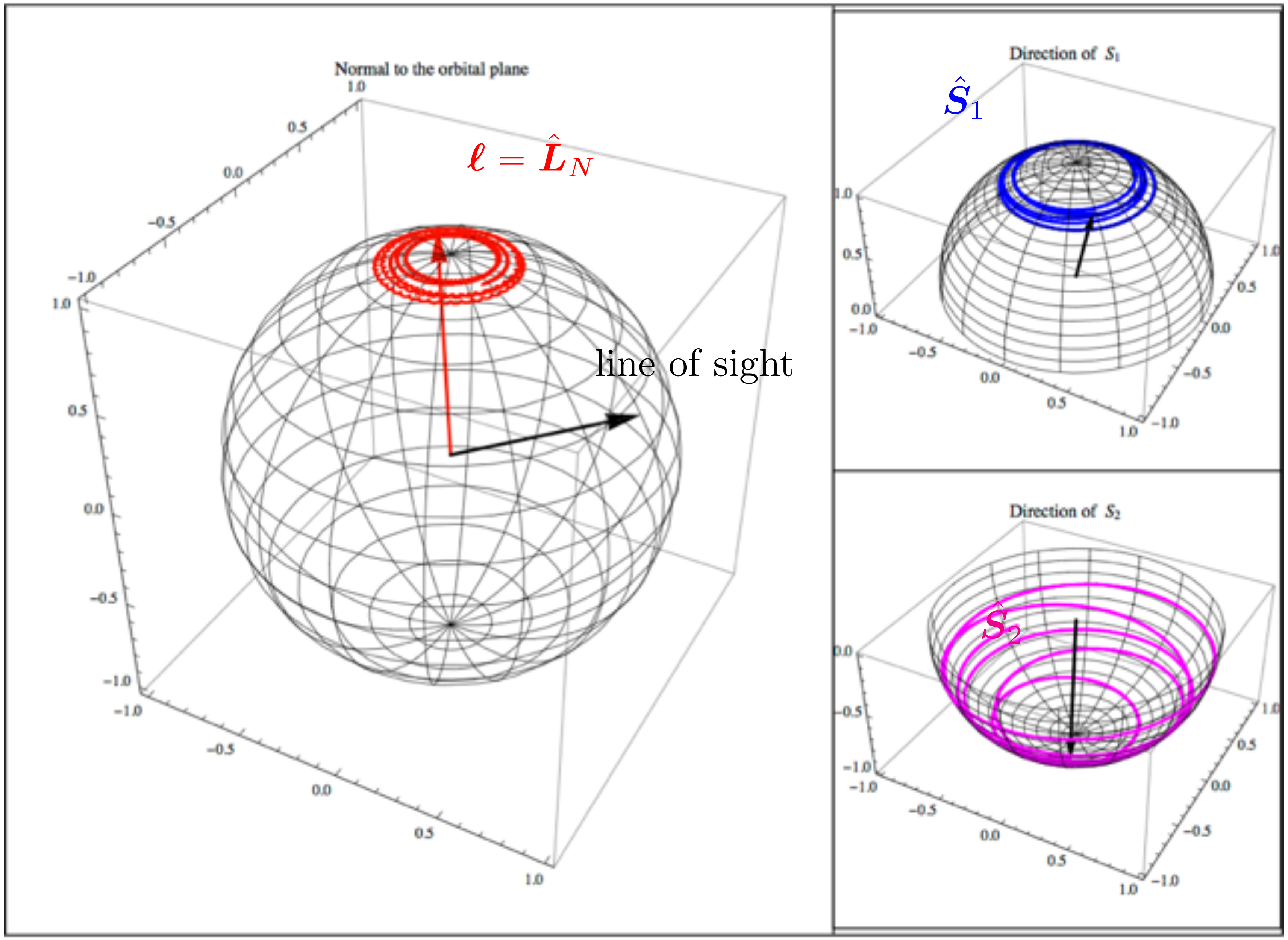
Part I

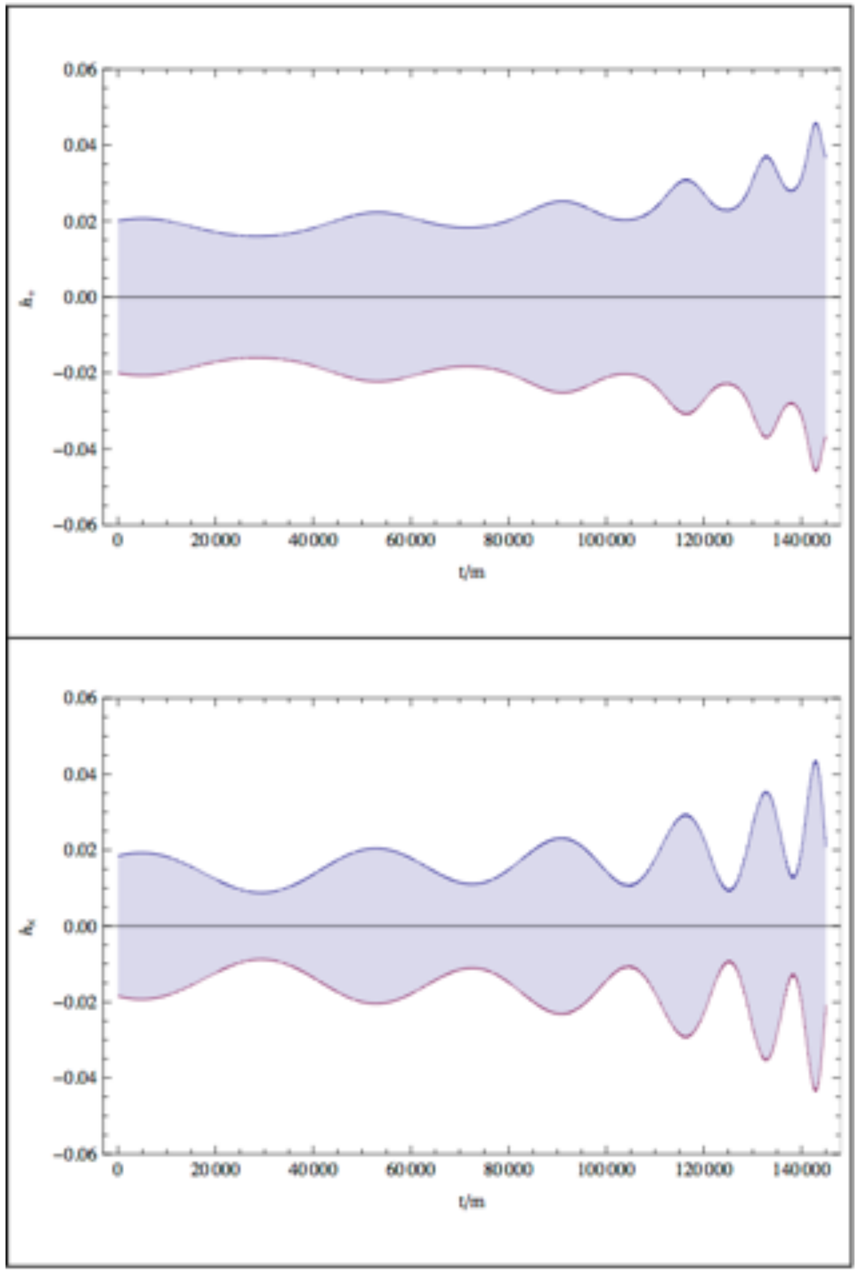
Effects of the spins: phasing and precession

Effects of the spins

- Affect the phasing (aligned)
- Orbital plane precession (misaligned)

Amplitude modulation for $h_+, h_\times$





$$\boldsymbol{J} \simeq \text{cte} = \boldsymbol{L_N} + \boldsymbol{S}/c + \dots$$

$$\dot{\boldsymbol{S}}_A = \boldsymbol{\Omega}_A \times \boldsymbol{S}_A$$

+ precessional phases

Part II

Different post-Newtonian methods

- **DIRE** (Direct Integration of Relaxed Einstein equations): near-zone/far-zone split of retarded integrals [Will, Wiseman, …]
- **Surface-integral** approach [Futamase, Itoh, ...]
- **Effective field theory** [Goldberger, Rothstein, ...]: diagrammatic computation of an effective action
- **ADM Hamiltonian formalism** [Schäfer, Damour, Jaranowski, …]: field degrees of freedom integrated out, obtaining a reduced Hamiltonian
- **Harmonic coordinates, MPM algorithm + matching** [Blanchet, Damour, Iyer, ...]

Validation of results by
different methods welcome !

Post-Newtonian results : where do we stand ?

$1\text{PN} \sim Gm/rc^2 \sim v^2/c^2$

Dynamics

	Leading	Known
NS	N	4PN (ADM)
SO	1.5PN	3.5PN (ADM, H)
SS	2PN	3PN (SS) - 4PN (S1S2) (ADM, EFT, H)
SSS	3.5PN	3.5PN (ADM/EFT, H)
SSSS	4PN	4PN (ADM/EFT)

ADM: reduced Hamiltonian in ADM approach
EFT: effective field theory
H: harmonic coordinates-based method

Energy flux

	Leading	Known
NS	N	3.5PN (H)
SO	1.5PN	3.5PN+4PN (H)
SS	2PN	3PN (SS, S1S2) (partial EFT, H)
SSS	3.5PN	3.5PN (H)

Full waveform

	Leading	Known
NS	N	3PN (H)
SO	1PN	2PN (H)

The harmonic post-Newtonian approach

Near-zone integration and wave generation formalism: overview

Part II

Harmonic coordinates and basics of the method

Einstein equations in harmonic coordinates

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

Harmonic gauge

$$h^{\mu\nu} \equiv \sqrt{-g} g^{\mu\nu} - \eta^{\mu\nu}$$

$$\partial_\nu h^{\mu\nu} = 0$$

→

Einstein equations - can be iterated

$$\Box h^{\mu\nu} = \frac{16\pi G}{c^4} \underbrace{(-g) T^{\mu\nu}}_{\text{matter}} + \underbrace{\Lambda^{\mu\nu}}_{\text{gravitational field}}$$

$\Lambda^{\mu\nu}(h^2, h^3, \dots)$ encodes all non-linearities in h

Retardations expansion and near-zone limitation

- PN retardation expansion of $\Box_{\mathcal{R}}^{-1}$ in the near-zone $r \ll \lambda$ → PN near-zone iteration of field equations
- Iterative multipolar solution in vacuum $r > r_{\text{source}}$ → MPM wave generation formalism in vacuum

Modeling spins for compact objects

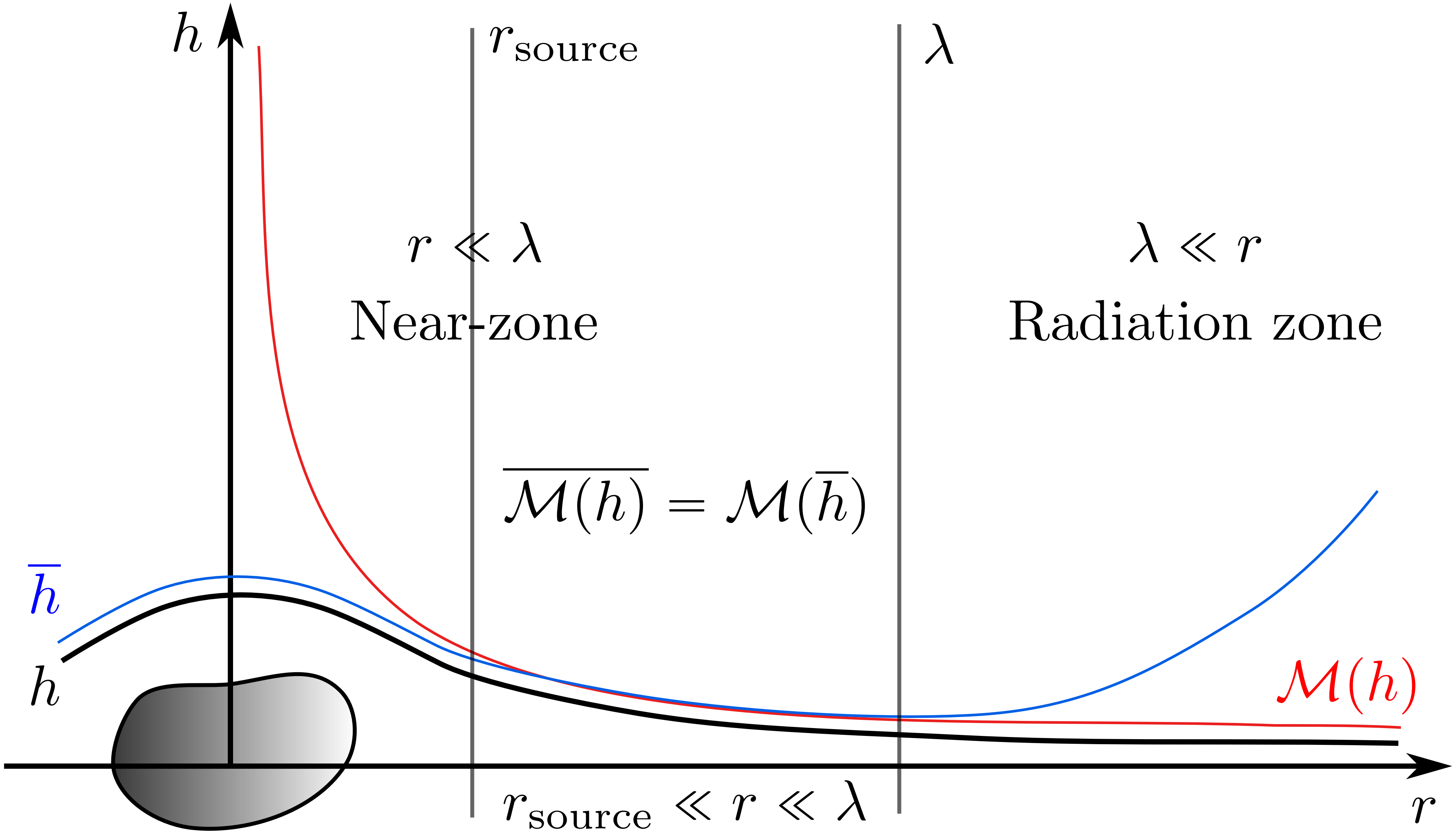
Compact objects as point particles (Dirac deltas)

→ Model of point particles with spin → Higher orders in spin

→ UV regularization scheme

- Hadamard regularization
- Dimensional regularization

Part II Multipolar post-Newtonian wave generation formalism



$\mathcal{M}(h)$: multipolar expansion

$\overline{h}$: post-Newtonian (near-zone) expansion

Part II

Link between source and radiative moments

Outline

- MPM solution parametrized by linear solution source/gauge moments $I_L, J_L, \dots, Z_L$
- Matching: source/gauge moments as spatial integrals $\int \mathrm{d}^3x(\dots)$
- Radiative coordinates and radiatives multipoles U_L, V_L describing waveform at infinity
- Finite part regularization: $\int d^3x \;\to\; FP_{B=0}\int d^3x \left(\frac{|x|}{r_0}\right)^B$

Result of MPM algorithm

Radiative quadrupole: $U_{ij}(u) \;=\; \boxed{I^{(2)}_{ij} + \frac{1}{c^5}\left[I^{(5)}_{ai} I_{ja} + \dots\right]}$ Instantaneous

Hereditary contributions

$+ \boxed{\frac{M}{c^3}\int_0^{+\infty} \mathrm{d}\tau\, I^{(4)}_{ij}(u-\tau)\ln\left(\frac{\tau}{2\tau_0}\right)}$ Tails

$+ \boxed{\frac{1}{c^5}\int \mathrm{d}\tau\, I^{(3)}_{ia} I^{(3)}_{aj} + \dots}$ Memory

$+ \boxed{\frac{M^2}{c^6}\int \mathrm{d}\tau\, I^{(5)}_{ij}\left[\ln^2 + \ln + \mathrm{cte}\right]}$ Tails of tails

Part II

Result of the matching equation and waveform

Results of the matching

[Blanchet 98]

- Source and gauge moments expressed as integrals over the source :

$$I_L = \mathrm{FP}\int d^3\mathbf{x}\,\hat{x}_L\left(\sigma - \frac{1}{c^2}\Delta(V^2) + \frac{1}{c^4}(V\sigma_{ii} + V_i\partial_t\partial_i V) + \ldots\right)$$

...

- The near-zone PN metric from matching (4PN tails) [Blanchet&Poujade 02]

Waveform and energy flux

[Thorne 80]

Wave (Transverse-Traceless):

$$h_{ij}^{\mathrm{TT}} = \frac{1}{c^2 R}\Lambda_{ij}^{\mathrm{TT}}(\boldsymbol{N})\sum_{\ell\geqslant 2}\frac{1}{c^\ell}\left[NU_L + \frac{1}{c}N\varepsilon V_L\right]$$

Emitted energy flux:

$$\mathcal{F} = \sum_{\ell\geqslant 2}\frac{1}{c^{2\ell+1}}\left[\dot{U}_L\dot{U}_L + \frac{1}{c^2}\dot{V}_L\dot{V}_L\right]$$

Part III

PN near-zone metric iteration

Matching for the near-zone metric

$$\Box h^{\mu\nu} = \tau^{\mu\nu}$$

$$\boxed{\overline{h}^{\mu\nu} = \widetilde{\Box_B^{-1}}\, \overline{\tau}^{\mu\nu} + h^{\mu\nu}_{\mathrm{tail}}}$$

- $\widetilde{\Box_B^{-1}}$ PN-expanded inverse d'Alembertian with $FP_{B=0}$ reg.
- $h^{\mu\nu}_{\mathrm{tail}}$ 4PN hereditary contribution (tails in RR)

Metric potentials

$$\sigma \leftrightarrow T^{\mu\nu}$$

$$\begin{aligned} g_{00} &\to V/c^2,\ \hat{X}/c^6\ , \hat{T}/c^8 + \mathcal{O}(10) \\ g_{0i} &\to V_i/c^3\ , \hat{R}_i/c^5\ , \hat{Y}_i/c^7 + \mathcal{O}(9) \\ g_{ij} &\to \delta_{ij} V/c^2\ , \hat{W}_{ij}/c^4\ , \hat{Z}_{ij}/c^6 + \mathcal{O}(8) \end{aligned}$$

Metric parametrized by potentials

$$\begin{aligned} V &= \Box_{\mathcal{R}}^{-1}[-4\pi G\, \sigma] \\ V_i &= \Box_{\mathcal{R}}^{-1}[-4\pi G\, \sigma_i] \\ \hat{W}_{ij} &= \Box_{\mathcal{R}}^{-1}\left[-4\pi G\left(\sigma_{ij} - \delta_{ij}\sigma_{kk}\right) - \partial_i V \partial_j V\right] \\ \hat{X} &= \Box_{\mathcal{R}}^{-1}\left[-4\pi G\, V \sigma_{ii} + \hat{W}_{ij}\partial_{ij} V + \dots\right] \end{aligned}$$

Source equations for potentials
compact and non-compact support

Solution for the potentials

- Relies on explicit solutions
  e.g. $\Delta^{-1}(1/r_1 r_2) = \ln(r_1 + r_2 + r_{12})$
- Regularization & distributional derivatives
- Potentials in all space or regularized

Representing higher-order spin effects

Lagrangian formalism for spin-induced finite-size effects

Part II

Point particles with spin: earlier approaches

Papapetrou approach [Papapetrou 51], generalization [Dixon]

Non-covariant approach : $\mathscr{T}^{\mu\nu} \equiv \sqrt{-g} T^{\mu\nu}$, **pole-dipole** hypothesis :

$$\int \mathrm{d}^3 x \ \mathscr{T}^{\mu\nu} \neq 0, \ \int \mathrm{d}^3 x \ \delta x^\rho \mathscr{T}^{\mu\nu} \neq 0, \ \delta x = x - \bar{x}$$

Composite definitions for spin, linear momentum and mass :

$$S^{\mu\nu} \equiv \int \mathrm{d}^3 x \ 2\delta x^{[\mu} \mathscr{T}^{\nu]0}, \ p^\mu \equiv m u^\mu - u_\nu \frac{\mathrm{D}S^{\mu\nu}}{\mathrm{d}\tau}$$

Evolution equations :

$$\frac{\mathrm{D}p^\mu}{\mathrm{d}\tau} = -\frac{1}{2} R^\mu{}_{\nu\rho\sigma} u^\nu S^{\rho\sigma} \qquad \frac{\mathrm{D}S^{\mu\nu}}{\mathrm{d}\tau} = 2p^{[\mu} u^{\nu]}$$

Gravitational skeleton approach [Mathisson 37], [Tulczyjew 59]

Ansatz on the stress-energy tensor $T^{\mu\nu} = \int \mathrm{d}\tau \ \left[t^{\mu\nu}\delta + \nabla_\rho \left(t^{\mu\nu\rho}\delta \right) + \nabla_\rho \nabla_\sigma \left(t^{\mu\nu\rho\sigma}\delta \right) + \dots \right]$

Method : unicity of the canonical decomposition $\sum_k \int \mathrm{d}\tau \nabla_{\alpha_1 \dots \alpha_k} \left(A^{\alpha_1 \dots \alpha_k \beta_1 \dots \beta_m} \delta \right)$

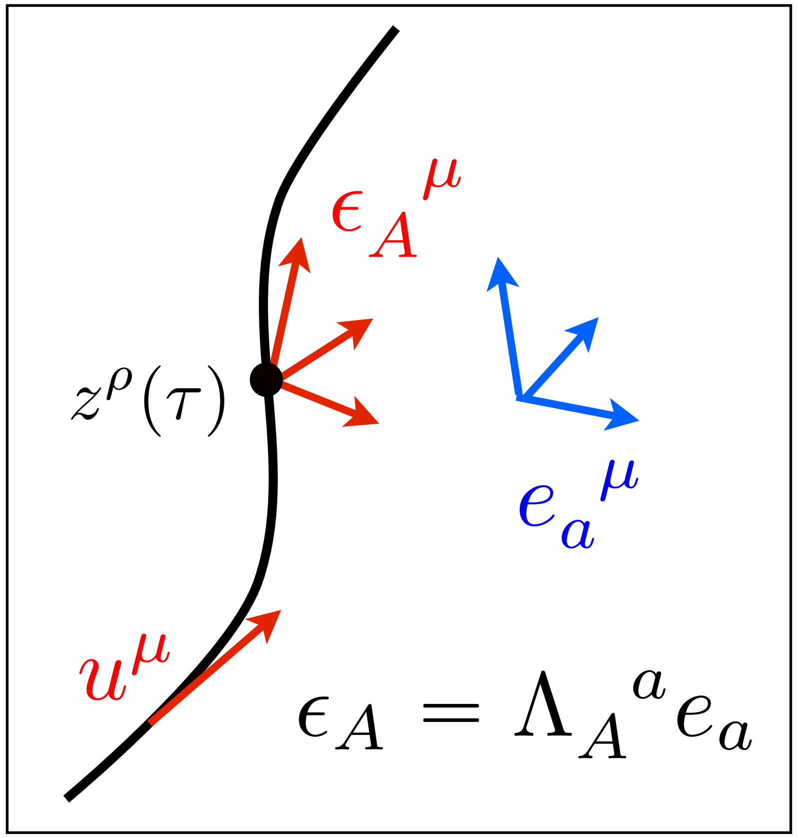
(for the α_i, symmetry and orthogonality to u^μ)

$\nabla_\nu T^{\mu\nu} = 0$ rewritten in canonical form ⟶ equations of evolution

Point particles with spin : Lagrangian formalism

Geometric definitions

[Hanson&Regge 74], [Bailey&Israel 75], [Porto 05]



$e_a{}^\mu$: field tetrad

$\epsilon_A{}^\mu$: tetrad attached to the body

$\Lambda_A{}^a$: Lorentz matrices, 6 internal degrees of freedom

$$u^\mu = \frac{\mathrm{d}z^\mu}{\mathrm{d}\tau}$$: 4-velocity

$$\Omega^{\mu\nu} \equiv \epsilon^{A\mu} \frac{\mathrm{D}\epsilon_A{}^\nu}{\mathrm{d}\tau}$$: rotation coefficients (antisymmetric)

Ansatz for the Lagrangian

$$S = \int \mathrm{d}\tau \, L \left[u^\mu, \Omega^{\mu\nu}, g_{\mu\nu}, \underbrace{R_{\mu\nu\rho\sigma}, \nabla_\lambda R_{\mu\nu\rho\sigma}, \ldots}_{\text{Finite size effects}} \right]$$

Part II Lagrangian formalism: multipoles and identities

Conjugate Moments

Linear momentum: $p_\mu \equiv \frac{\partial L}{\partial u^\mu}$ Spin tensor: $S_{\mu\nu} \equiv 2\frac{\partial L}{\partial \Omega^{\mu\nu}}$

Multipolar Moments

Quadrupolar moment: $J^{\mu\nu\rho\sigma} \equiv -6\frac{\partial L}{\partial R_{\mu\nu\rho\sigma}}$

Octupolar moment: $J^{\lambda\mu\nu\rho\sigma} \equiv -12\frac{\partial L}{\partial \nabla_\lambda R_{\mu\nu\rho\sigma}}$

... and higher orders

Homogeneity condition

invariance by reparametrization of the world line

$L = p_\mu u^\mu + \frac{1}{2} S_{\mu\nu}\Omega^{\mu\nu}$ (regardless of couplings to the Riemann)

Scalar condition

the Lagrangian must be a scalar - eliminates $\partial L/\partial g_{\mu\nu}$

$$2\frac{\partial L}{\partial g_{\mu\nu}} = p^\mu u^\nu + S^{\mu\rho}\Omega^\nu{}_\rho + \frac{2}{3}R^\mu{}_{\lambda\rho\sigma}J^{\nu\lambda\rho\sigma} + \frac{1}{3}J^{\lambda\nu\tau\rho\sigma}\nabla_\lambda R^\mu{}_{\tau\rho\sigma} + \frac{1}{12}J^{\nu\lambda\tau\rho\sigma}\nabla^\mu R_{\lambda\tau\rho\sigma}$$

Lagrangian formalism: equations of motion/precession

Equation of motion

covariantization of the variation of the worldline: $\delta z^\rho \partial_\rho L = \delta z^\rho \nabla_\rho L$

$$\frac{\mathrm{D}p_\mu}{\mathrm{d}\tau} = -\frac{1}{2} R_{\mu\nu\rho\sigma} u^\nu S^{\rho\sigma} - \frac{1}{6} J^{\lambda\nu\rho\sigma} \nabla_\mu R_{\lambda\nu\rho\sigma} - \frac{1}{12} J^{\tau\lambda\nu\rho\sigma} \nabla_\mu \nabla_\tau R_{\lambda\nu\rho\sigma}$$

Immediate generalization to higher orders

Equation of precession

variation of rotational degrees of freedom: $\delta\theta^{ab} \equiv \Lambda^{Aa} \delta\Lambda_A{}^b$

$$\frac{\mathrm{D}S^{\mu\nu}}{\mathrm{d}\tau} = \Omega^\mu{}_\rho S^{\nu\rho} - \Omega^\nu{}_\rho S^{\mu\rho}$$

- Valid at any multipolar order
- Conserved spin norm, independently of the SSC: $s^2 \equiv S_{\mu\nu} S^{\mu\nu}/2 = \mathrm{const}$

With the scalar condition:

$$\frac{\mathrm{D}S^{\mu\nu}}{\mathrm{d}\tau} = 2p^{[\mu}u^{\nu]} + \frac{4}{3} R^{[\mu}{}_{\lambda\rho\sigma} J^{\nu]\lambda\rho\sigma} + \frac{2}{3} \nabla^\lambda R^{[\mu}{}_{\tau\rho\sigma} J_\lambda{}^{\nu]\tau\rho\sigma} + \frac{1}{6} \nabla^{[\mu} R_{\lambda\tau\rho\sigma} J^{\nu]\lambda\tau\rho\sigma}$$

Lagrangian formalism: stress-energy tensor

Variation $\delta g_{\mu\nu}$

Defining the world line density: $w = \int \mathrm{d}\tau \delta^4(x-z)/\sqrt{-g}$

Pole-dipole terms:

$$T^{\mu\nu}_{\text{pole}-\text{dipole}} = p^{(\mu}u^{\nu)}w - \nabla_\rho \left[S^{\rho(\mu}u^{\nu)} w \right]$$

Quadrupole terms:

$$T^{\mu\nu}_{\text{quad}} = \frac{1}{3} R^{(\mu}{}_{\lambda\rho\sigma} J^{\nu)\lambda\rho\sigma} w - \nabla_\rho \nabla_\sigma \left[\frac{2}{3} J^{\rho(\mu\nu)\sigma} w \right]$$

Octupole terms:

No direct generalization

$$\begin{aligned} T^{\mu\nu}_{\text{oct}} = & \left[\frac{1}{6} \nabla^\lambda R^{(\mu}{}_{\xi\rho\sigma} J_\lambda{}^{\nu)\xi\rho\sigma} + \frac{1}{12} \nabla^{(\mu} R_{\xi\tau\rho\sigma} J^{\nu)\xi\tau\rho\sigma} \right] w \\ & + \nabla_\rho \left\{ \left[-\frac{1}{6} R^{(\mu}{}_{\xi\lambda\sigma} J^{|\rho|\nu)\xi\lambda\sigma} - \frac{1}{3} R^{(\mu}{}_{\xi\lambda\sigma} J^{\nu)\rho\xi\lambda\sigma} + \frac{1}{3} R^{\rho}{}_{\xi\lambda\sigma} J^{(\mu\nu)\xi\lambda\sigma} \right] w \right\} \\ & + \nabla_\lambda \nabla_\rho \nabla_\sigma \left[\frac{1}{3} J^{\sigma\rho(\mu\nu)\lambda} w \right] \end{aligned}$$

Part II Lagrangian formalism: SSC and definition of the mass

Spin supplementary condition

$S^{\mu\nu}$ six degrees of freedom $\longrightarrow$ impose 3 conditions $V_\mu S^{\mu\nu} = 0$

Covariant SSC: $p_\mu S^{\mu\nu} = 0$

Impose conservation of the SSC: relation $p^\mu \leftrightarrow u^\mu$

Definition of the mass

$m^2 = -p_\mu p^\mu$ is **not** conserved at order SS 3PN

Alternative definition (not general at all PN orders):

$$\tilde{m} \equiv -p_\mu u^\mu - \frac{1}{6} J^{\lambda\nu\rho\sigma} R_{\lambda\nu\rho\sigma}\,, \quad \frac{\mathrm{d}\tilde{m}}{\mathrm{d}\tau} = \mathcal{O}(S^3/c^9)$$

Part II

Lagrangian formalism: conserved norm spin

The spin covector

Using the SSC to define a spin covector :

$$S_\mu \equiv \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\frac{p^\nu}{m}S^{\rho\sigma} \qquad S_\mu p^\mu = 0$$

Defining Euclidean norm spin vector

Defining a tetrad : $({e_0}^\mu, {e_I}^\mu)$ with ${e_0}^\mu \equiv u^\mu$

Conserved norm vector : $s^2 = (g^{\mu\nu} + u^\mu u^\nu)S_\mu S_\nu = \delta^{IJ} S_I S_J$

$$\boxed{S_I = e_I^\mu S_\mu}$$

Fixing the convention for the spatial part of the tetrad :

$$\gamma_{ij} = g_{ij} + u_i u_j = \delta^{IJ} e_{Ii} e_{Jj} \longrightarrow$$

e_{Ij} chosen as the unique symmetric positive-definite square root of γ_{ij}

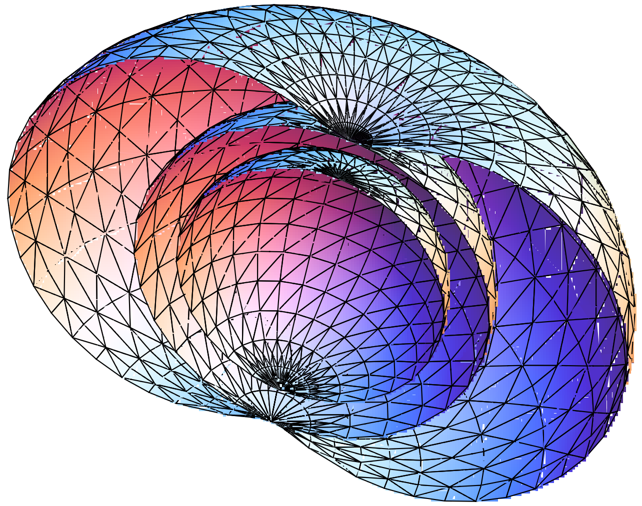
Precession equation

$$\boxed{\frac{\mathrm{d}\boldsymbol{S}}{\mathrm{d}t} = \boldsymbol{\Omega} \times \boldsymbol{S}}$$

- Leading SO terms1PN, leading SS terms 1.5PN
- Simplify the structure of equations (hereditary integrals)
- Important when applying the balance equation

Lagrangian formalism: spin-induced moments

Representing spin-induced structure



- Elimination of $R_{\mu\nu}$ in the Lagrangian: $C_{\mu\nu\rho\sigma}$ write all possible couplings with the Weyl tensor
- Write the couplings directly with the spin tensor and using the SSC

Spin-induced moments

unique solutions

- Quadrupole:

$$J^{\mu\nu\rho\sigma} = -\frac{3\kappa}{m} u^{[\mu}\Theta^{\nu][\rho}u^{\sigma]}$$

$$\Theta^{\mu\nu} \equiv S^{\mu\lambda}S^{\nu}{}_{\lambda}$$

- Octupole:

$$J^{\lambda\mu\nu\rho\sigma} = \frac{\lambda}{4m^2}\Big[\Theta^{\lambda[\mu}u^{\nu]}S^{\rho\sigma} + \Theta^{\lambda[\rho}u^{\sigma]}S^{\mu\nu} - \Theta^{\lambda[\mu}S^{\nu][\rho}u^{\sigma]} - \Theta^{\lambda[\rho}S^{\sigma][\mu}u^{\nu]} - S^{\lambda[\mu}\Theta^{\nu][\rho}u^{\sigma]} - S^{\lambda[\rho}\Theta^{\sigma][\mu}u^{\nu]}\Big]$$

Polarizability constants

$\kappa\,,\lambda$

to be determined

- By matching to a Kerr black hole
- Numerically for neutron stars

Generalization at all orders in spin (leading order in the Weyl tensor) [Levi&Steinhoff 15]

Computation of the SO tail integrals

Hereditary effects at linear order in spin for a precessional dynamics

Part IV

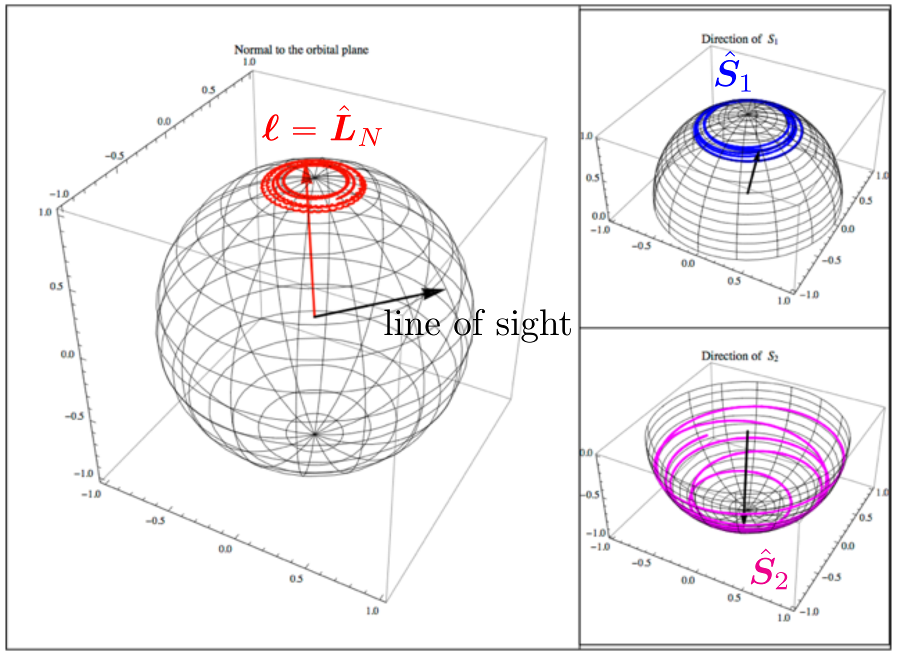
Tail spin-orbit 4PN contributions

Structure of tail contributions

example of the quadrupole

$$U_{ij}^{\text{tail}}(T_R) = \frac{M}{c^3}\int_0^{+\infty} \mathrm{d}\tau\, I_{ij}^{(4)}(T_R-\tau)\left[\ln\left(\frac{\tau}{2\tau_0}\right)+\kappa_2\right]$$

Hereditary integral : requires controlling **precessional** dynamics in the past



- Restriction to quasi-circular orbits
- Conservative dynamics only, neglecting $\mathcal{O}(\ln c/c^5)$ corrections

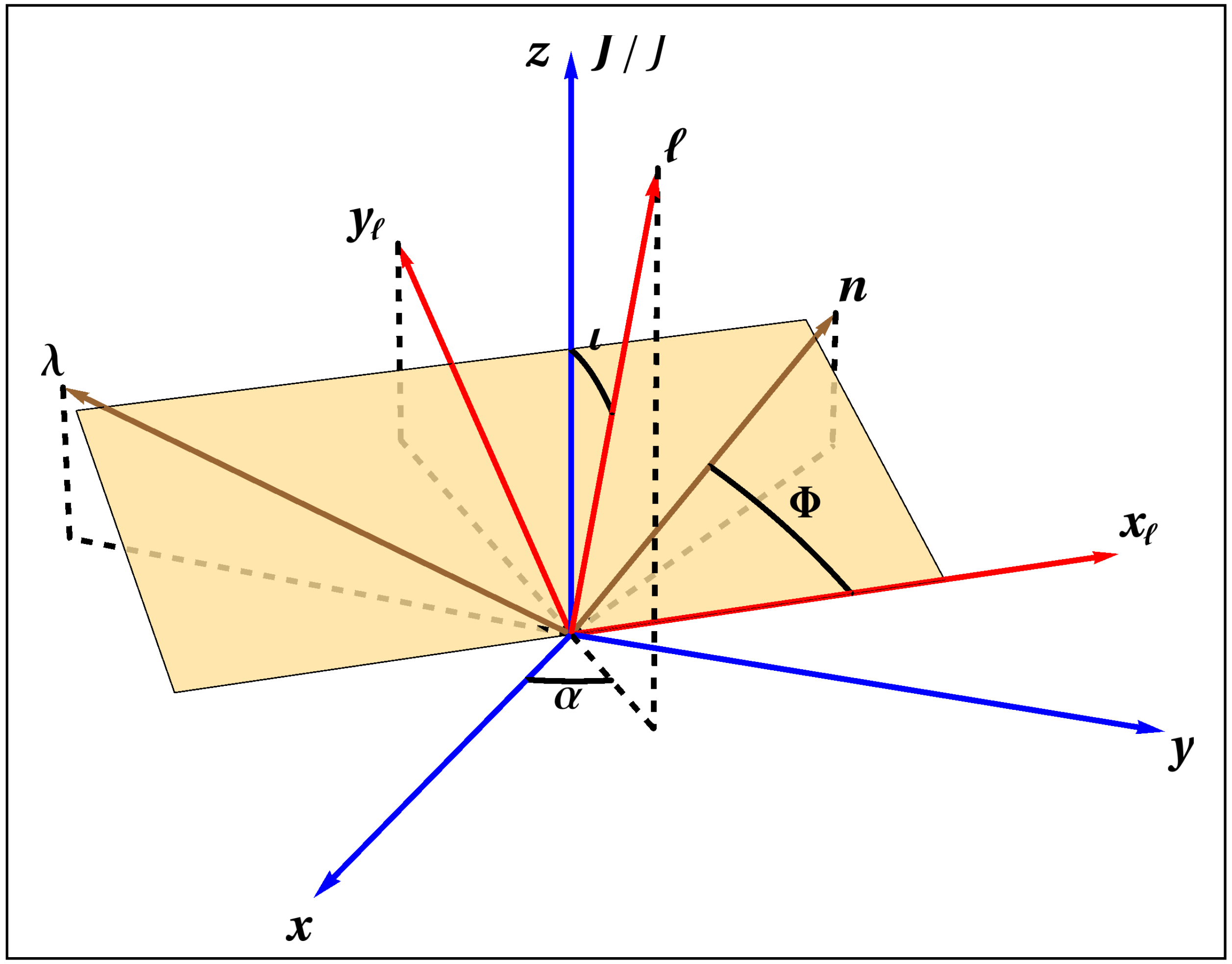
Spin-orbit tail contributions to the energy flux

- At 3PN and 4PN, for dimensional reasons **only** tail contributions
- At linear order in spin, the contribution of the precession of the orbital plane **cancels out**
- Not true for contributions to the waveform h_{ij}^{TT}

Part IV Conservative precessing dynamics : structure

Geometry of the problem

- $\boldsymbol{J}$ constant total angular momentum
- Normal to the orbital plane $\boldsymbol{\ell}$
- Center-of-mass frame - moving triad $(\boldsymbol{n}, \boldsymbol{\lambda}, \boldsymbol{\ell})$
- Euler angles α, ι, Φ
- Orbital phase $\phi = \int \mathrm{d}t\, \omega$



Angular velocities

$$\dot{\boldsymbol{n}} = \omega \boldsymbol{\lambda}$$
$$\dot{\boldsymbol{\lambda}} = -\omega \boldsymbol{n} + \varpi \boldsymbol{\ell}$$
$$\dot{\boldsymbol{\ell}} = -\varpi \boldsymbol{\lambda}$$

Equations of motion

$$\boldsymbol{x} = r\boldsymbol{n}$$
$$\boldsymbol{v} = \dot{r}\boldsymbol{n} + r\omega\boldsymbol{\lambda}$$
$$\boldsymbol{a} = -r\omega^2\boldsymbol{n} + (r\dot{\omega} + 2\dot{r}\omega)\boldsymbol{\lambda} + r\omega\varpi\boldsymbol{\ell}$$

Radiation reaction terms $\mathcal{O}(5)$

Precession due to spins $\mathcal{O}(3)$

Precession equations

$$\dot{\boldsymbol{S}} = \boldsymbol{\Omega} \times \boldsymbol{S}$$
$$\boldsymbol{\Omega} = \Omega\, \boldsymbol{\ell}$$

Angular momentum

$$\boldsymbol{J}_{NS} = J_{NS}\, \boldsymbol{\ell}$$

Scalars (energy, flux)

$$(\boldsymbol{n}, \boldsymbol{v}, \boldsymbol{S}) \propto S_\ell$$
$$\dot{S}_\ell = \mathcal{O}(S^2)$$

Part IV

Tail spin-orbit contributions : calculation

Result for conservative orbital evolution

Extending [Blanchet, Buonanno, Faye 11]

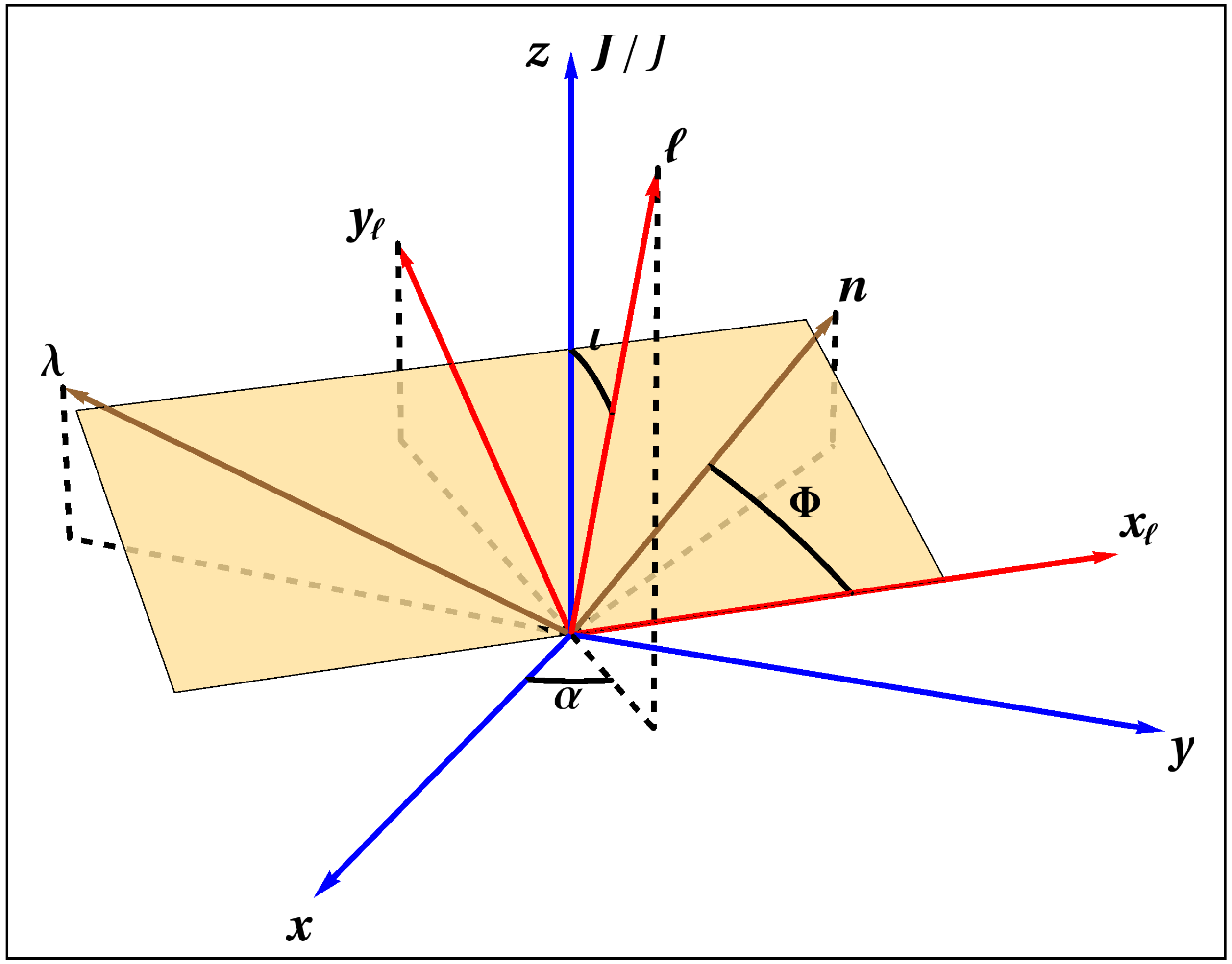
Formally, at linear order in spin, evolution of the moving triad $(\boldsymbol{n}, \boldsymbol{\lambda}, \boldsymbol{\ell})$ entirely expressed with :

$$\boldsymbol{m} \equiv \frac{1}{\sqrt{2}}(\boldsymbol{n} + \mathrm{i}\boldsymbol{\lambda})$$

$$\boldsymbol{m} = e^{-\mathrm{i}(\phi-\phi_0)}\boldsymbol{m}_0 + \frac{\mathrm{i}}{\sqrt{2}}\left(\sin\iota\, e^{\mathrm{i}\alpha} - \sin\iota_0\, e^{\mathrm{i}\alpha_0}\right) e^{-\mathrm{i}\phi}\boldsymbol{\ell}_0 + \mathcal{O}(S^2)\,,$$

$$\boldsymbol{\ell} = \boldsymbol{\ell}_0 + \left[\frac{\mathrm{i}}{\sqrt{2}}\left(\sin\iota\, e^{-\mathrm{i}\alpha} - \sin\iota_0\, e^{-\mathrm{i}\alpha_0}\right) e^{\mathrm{i}\phi_0}\boldsymbol{m}_0 + \mathrm{c.c.}\right] + \mathcal{O}(S^2)$$

$$\sin\iota\, e^{\mathrm{i}\alpha} = -\mathrm{i}\frac{J_S^n + \mathrm{i}J_S^\lambda}{|\boldsymbol{J}_{\mathrm{NS}}|}e^{\mathrm{i}\phi} + \mathcal{O}(S^2)$$



Resulting time dependence

$$e^{\mathrm{i}(m\omega + p\Omega_1 + q\Omega_2)t}$$

$$m \in \mathbb{Z}\ ,\ \ (p,q) \in \{-1,0,1\}$$

ω orbital frequency

Ω_A spin precession frequency

→ Straightforward computation of tail integrals in Fourier domain

Overview of the results

New PN contributions for spin effects

Results and checks

Summary

- Symbolic computation : Mathematica®, xAct [Martin-Garcia], PNComBin [Faye]
- 3.5PN spin-orbit dynamics and flux-phasing (NNLO)
- 4PN spin-orbit tail terms in the flux and phasing (NLO for the tails)
- 3PN spin-spin dynamics and flux-phasing (NLO)
- 3.5PN spin-spin-spin dynamics and flux-phasing (LO)

Tests of the results: dynamics

- Lorentz invariance of the EOM (must hold in harmonic gauge)
- Existence of a set of conserved quantities : energy, angular momentum, linear momentum, center-of-mass integral
- Test-mass limit in agreement with a spinning test particle in a Kerr background
- Equivalence of results with the ADM ones: existence of a contact transformation and spin transformation matching the dynamics

Tests of the results: flux

- Test-mass limit in agreement with the flux emitted by a test particle in a Kerr background
- Source moments for boosted Kerr black holes
- Equivalence with EFT ?

Part IV

Example of results

The energy flux for quasi-circular spin-aligned orbits

$$
\begin{aligned}
\mathcal{F} = \frac{32\nu^2}{5G}c^5x^5 \Bigg(1 &+ \left(-\frac{1247}{336} - \frac{35}{12}\nu\right)x + \dots \\
&+ \left(\left(-\frac{3839}{252} - 43\nu\right)S_\ell^2 + \left(-\frac{1375}{56} - 43\nu\right)\delta S_\ell\Sigma_\ell + \left(-\frac{227}{28} + \frac{3481\nu}{168} + 43\nu^2\right)\Sigma_\ell^2\right)x^6 \\
&+ \left(\left(\frac{476645}{6804} + \frac{6172}{189}\nu - \frac{2810}{27}\nu^2\right)S_\ell + \left(\frac{9535}{336} + \frac{1849}{126}\nu - \frac{1501}{36}\nu^2\right)\frac{\delta m}{m}\Sigma_\ell\right)x^{\frac{7}{2}} \\
&+ \left(-\frac{16}{3}S_\ell^3 + \frac{2}{3}\delta S_\ell^2\Sigma_\ell + \left(\frac{9}{2} - \frac{56\nu}{3}\right)S_\ell\Sigma_\ell^2 + \left(\frac{35}{24} - 6\nu\right)\delta\Sigma_\ell^3\right)x^{7/2} \\
&+ \left(\left(-\frac{3485\pi}{96} + \frac{13879\pi}{72}\nu\right)S_\ell + \left(-\frac{7163\pi}{672} + \frac{130583\pi}{2016}\nu\right)\frac{\delta m}{m}\Sigma_\ell\right)x^4\Bigg)
\end{aligned}
$$

PN parameter: $x \equiv \left(Gm\omega/c^3\right)^{2/3}$ 1PN

Masses: $\nu = m_1m_2/m^2$ $\delta = (m_1 - m_2)/m$

Spins: $S \sim S_1 + S_2$ $\Sigma \sim S_2 - S_1$

Part IV

Result for the number of cycles

Phasing for circular orbits $E(\omega)$ $\mathcal{F}(\omega)$ $\longrightarrow$ **Balance equation** $\mathcal{F} = -\mathrm{d}E/\mathrm{d}t$ $\xrightarrow{\text{PN approximant}}$ GW Phase

Taylor T2 Number of cycles between f~10Hz and $\omega = \omega_{ISCO}$ $(x_{\mathrm{ISCO}} = 1/6)$

- Question of the convergence of the PN series
- Rough estimate of the importance of the new terms
- Approximant-dependent

LIGO/Virgo	$10M_\odot + 1.4M_\odot$	$10M_\odot + 10M_\odot$
Newtonian	3558.9	598.8
1PN	212.4	59.1
1.5PN	$-180.9 + 114.0\chi_1 + 11.7\chi_2$	$-51.2 + 16.0\chi_1 + 16.0\chi_2$
2PN	$9.8 - 10.5\chi_1^2 - 2.9\chi_1\chi_2$	$4.0 - 1.1\chi_1^2 - 2.2\chi_1\chi_2 - 1.1\chi_2^2$
2.5PN	$-20.0 + 33.8\chi_1 + 2.9\chi_2$	$-7.1 + 5.7\chi_1 + 5.7\chi_2$
3PN	$2.3 - 13.2\chi_1 - 1.3\chi_2$ $-1.2\chi_1^2 - 0.2\chi_1\chi_2$	$2.2 - 2.6\chi_1 - 2.6\chi_2$ $-0.1\chi_1^2 - 0.2\chi_1\chi_2 - 0.1\chi_2^2$
3.5PN	$-1.8 + 11.1\chi_1 + 0.8\chi_2 + (\mathrm{SS})$ $-0.7\chi_1^3 - 0.3\chi_1^2\chi_2$	$-0.8 + 1.7\chi_1 + 1.7\chi_2 + (\mathrm{SS})$ $-0.05\chi_1^3 - 0.2\chi_1^2\chi_2 - 0.2\chi_1\chi_2^2 - 0.05\chi_2^3$
4PN	$(\mathrm{NS}) - 8.0\chi_1 - 0.7\chi_2 + (\mathrm{SS})$	$(\mathrm{NS}) - 1.5\chi_1 - 1.5\chi_2 + (\mathrm{SS})$

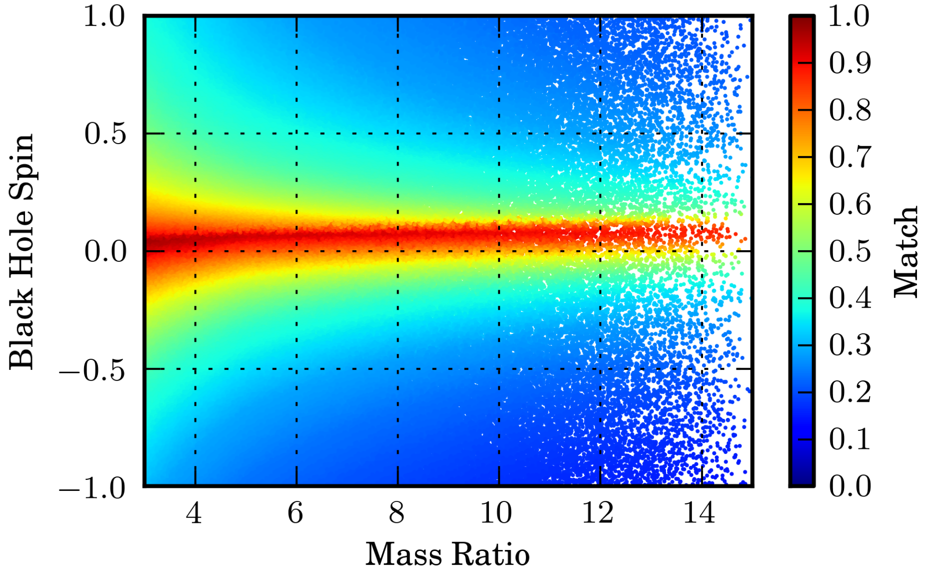
Part IV

Illustration: phasing for the spin-aligned case

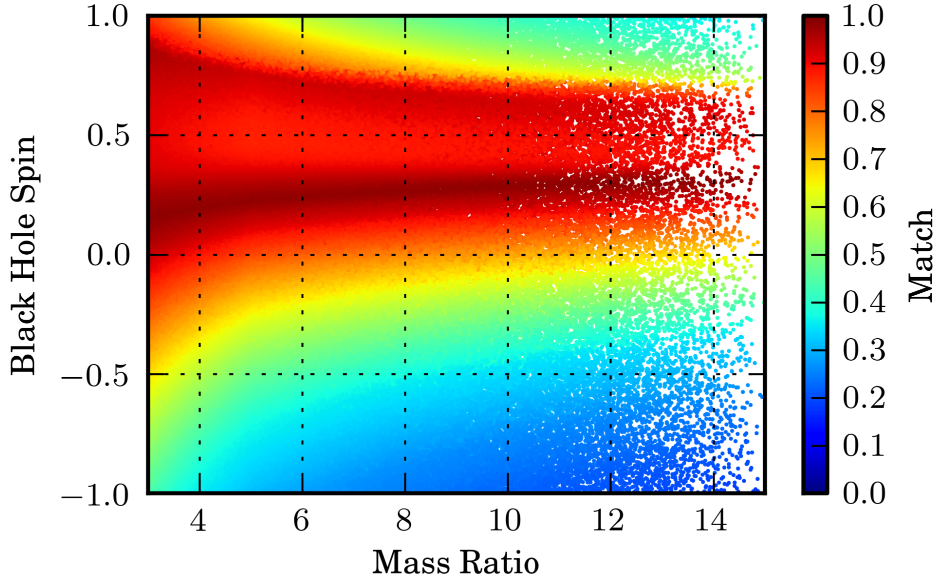
[Nitz&al 13] : matches between templates computed for aligned spins with fixed physical parameters

Agreement between approximants, at a given PN order :

TF2 - T4 S-O 2.5PN

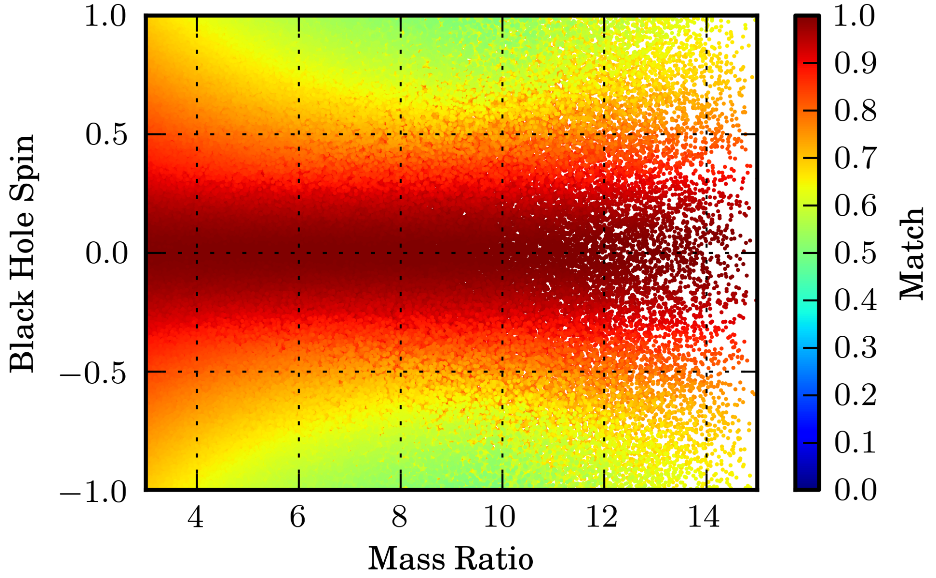


TF2 - T4 S-O 3.5PN

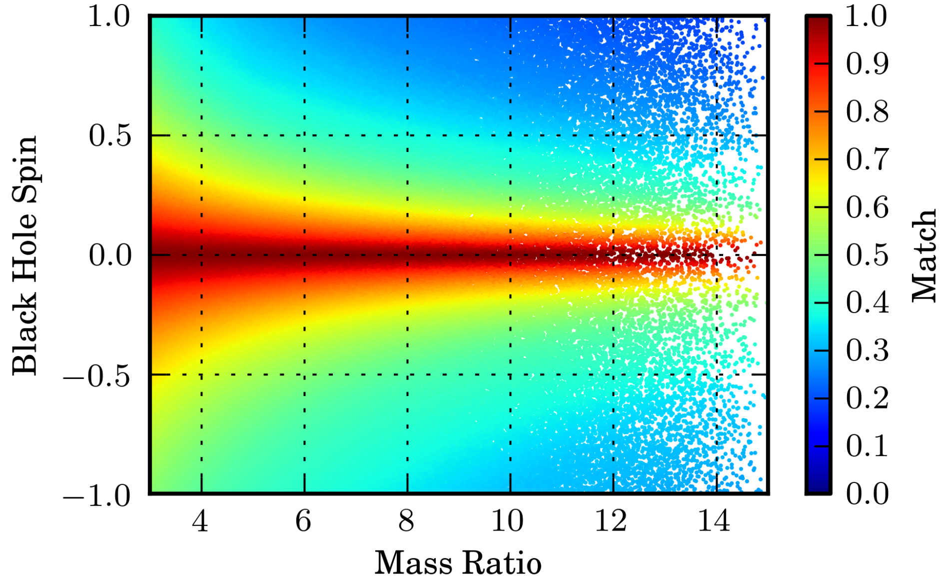


Agreement between successive PN orders for each approximant :

TF2 S-O 2.5PN - TF2 S-O 3.5PN



T4 S-O 2.5PN - T4 S-O 3.5PN



More complete study needed to quantify this in terms of parameter estimation bias.

Illustration: comparison of PN/NR precession

[Ossokine&al 15]: comparison of PN (harmonic) and NR (Spec) precession

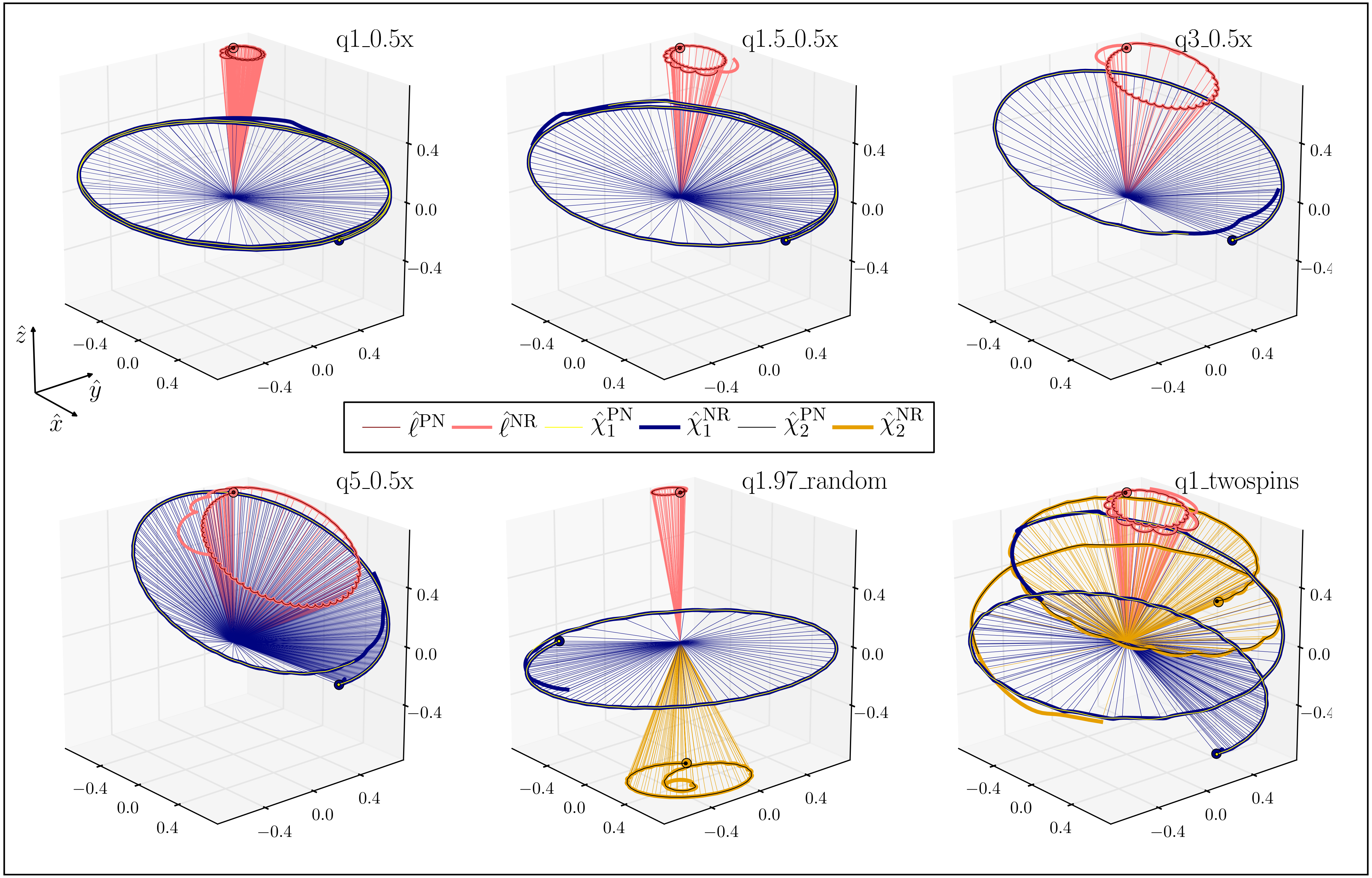
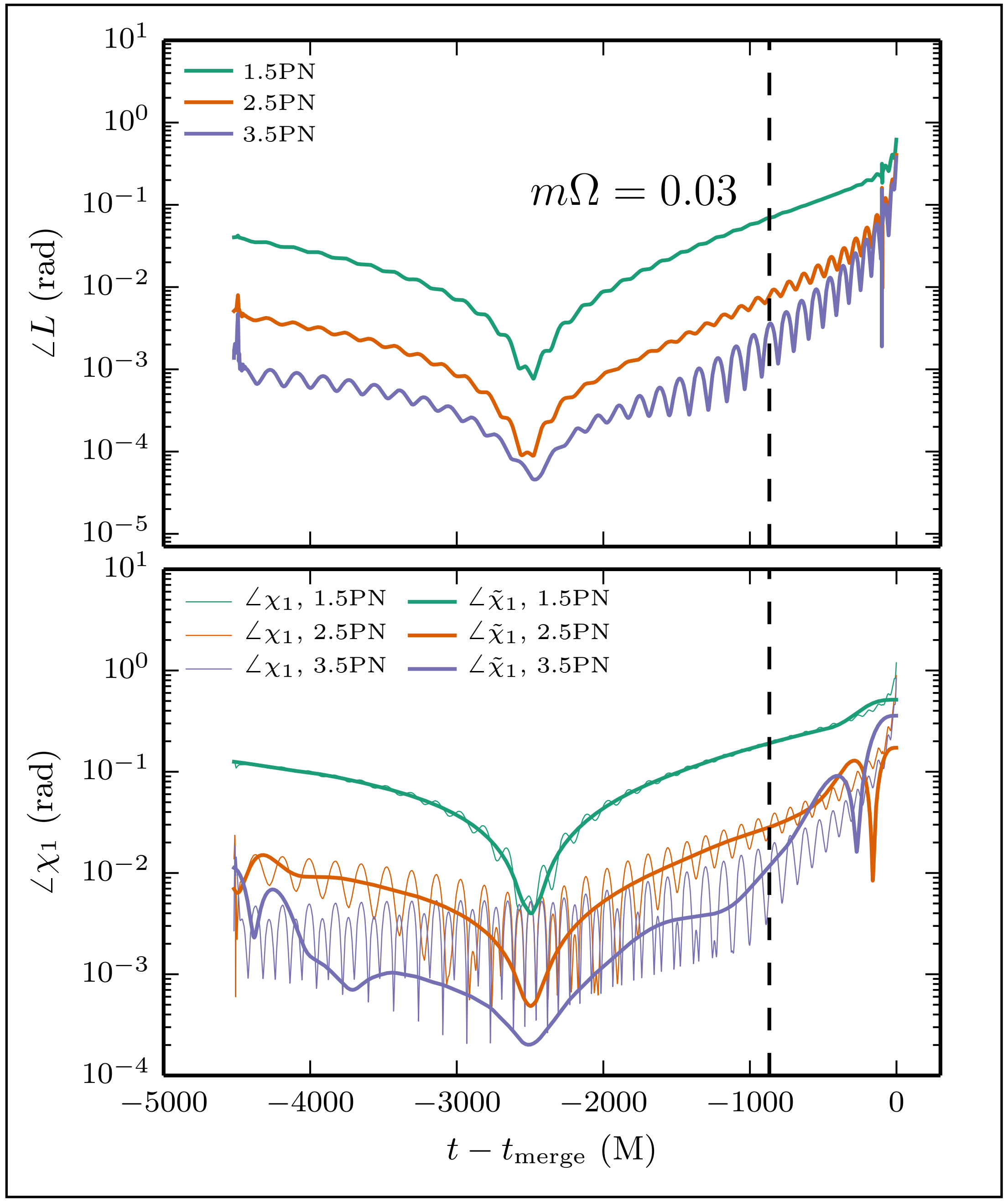
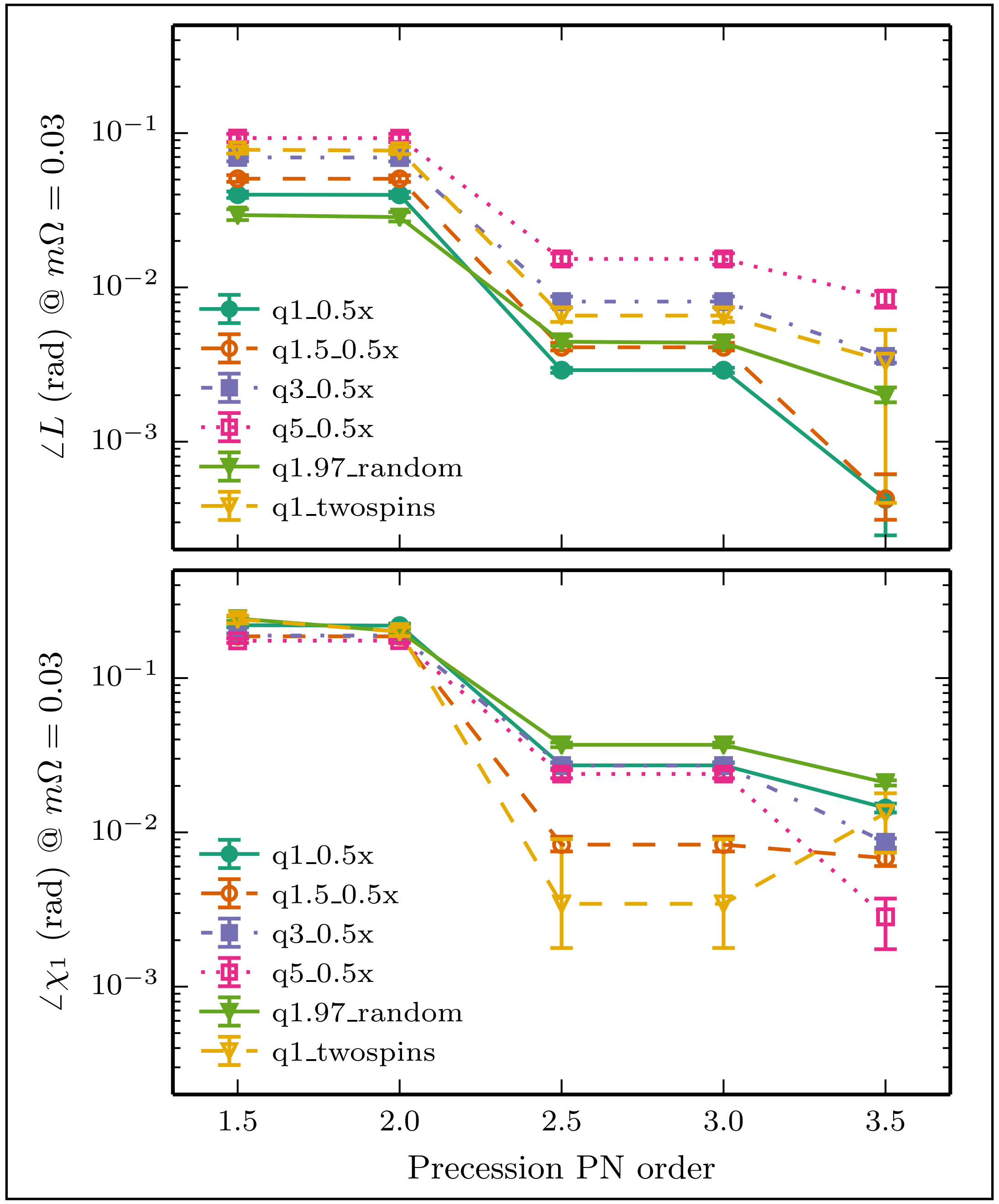


Illustration: comparison of PN/NR precession

[Ossokine&al 15]: comparison of PN (harmonic) and NR (Spec) precession



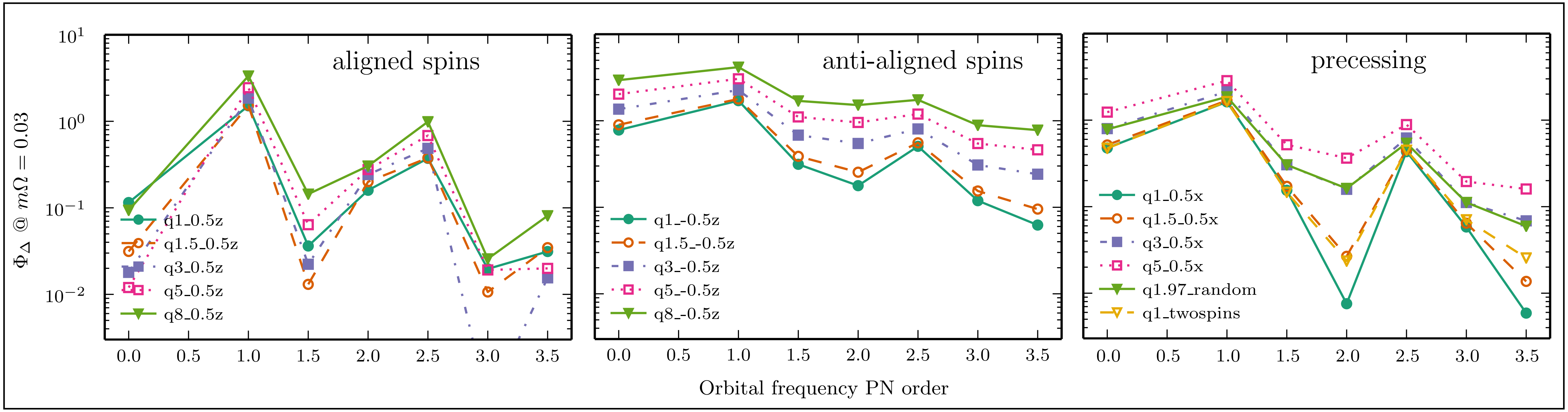
Angles $(\boldsymbol{\ell}_{\rm PN}, \boldsymbol{\ell}_{\rm NR})$ and $(\boldsymbol{S}_{\rm PN}, \boldsymbol{S}_{\rm NR})$
varying PN order



Angles at a specific time
varying PN order

Illustration: comparison of PN/NR phasing

[Ossokine&al 15]: comparison of PN (SpinTaylorT4) and NR (Spec) phasing



Comparison

- Satisfying agreement for the precession (even if gauge-dependent quantities)
- Convergence less clear for the orbital phase…

Summary and Conclusion

Results

- 3.5PN spin-orbit dynamics, 4PN spin-orbit flux/phasing
- 3PN spin-spin dynamics and flux/phasing
- Lagrangian formalism for higher-order spin effects
- 3.5PN spin-cube dynamics and flux/phasing

Comparisons

- PN/PN: still important differences at 3.5PN
- PN/NR: convergence for precession, less clear for orbital phase

Work in progress

- 3.5PN spin-orbit and 3PN spin-spin polarizations (or spherical modes)
- 3.5PN spin-spin tail effects
- 4PN non-spinning dynamics (and flux/phasing later)
- Spin effects at higher order: 4PN spin-spin, 4PN spin^4, 4.5PN spin-orbit ?

Part I

The spin of black holes : observations

The Kerr black hole

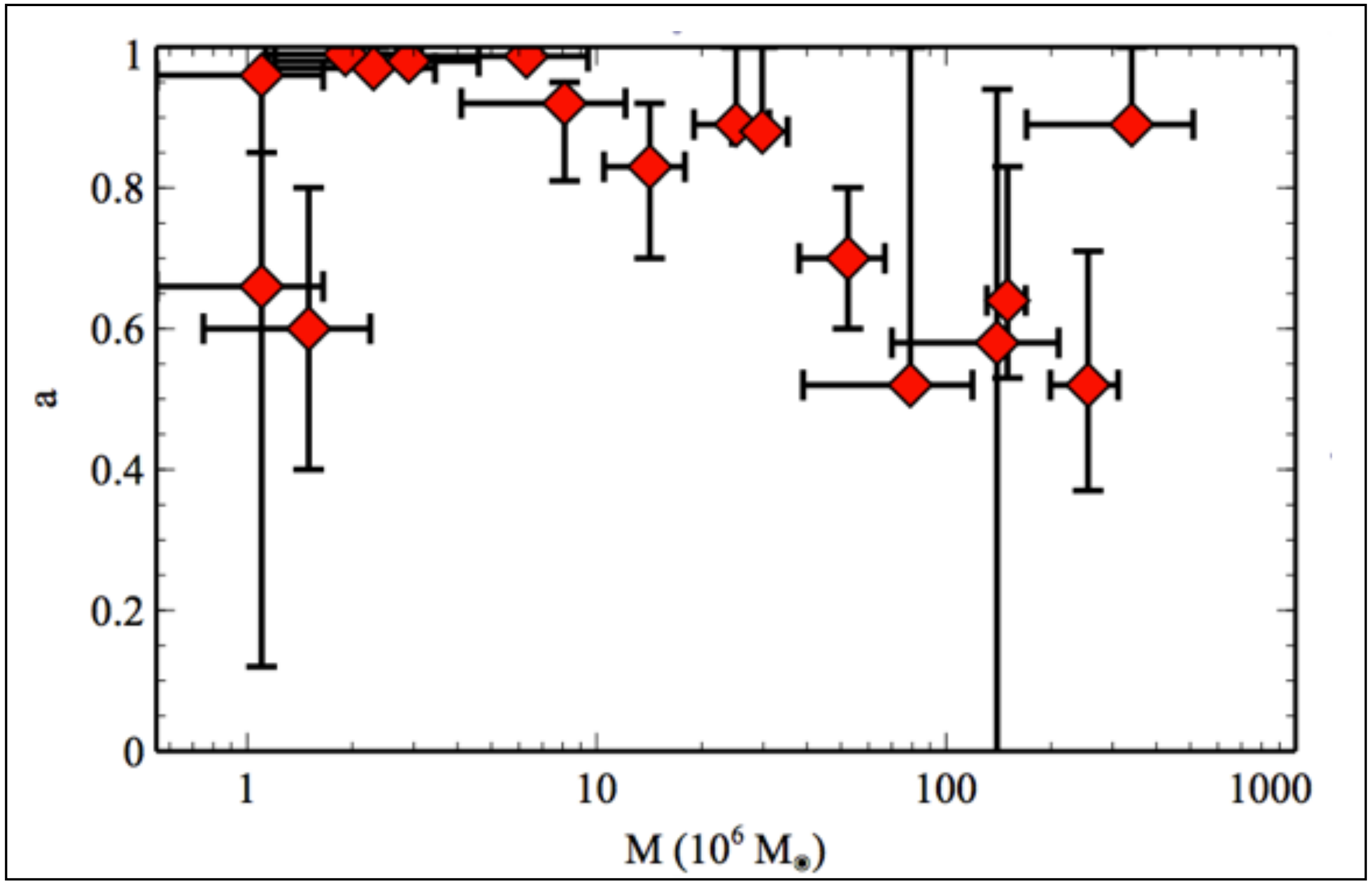
Most general stationary, axisymmetric vacuum solution to Einstein equations : the rotating Kerr black hole

Dimensionless Kerr parameter :
(1 for maximally rotating black hole)

$$a \equiv \frac{cJ}{Gm^2}$$

X-Ray spectroscopy of accretion disks

Example for stellar mass black holes :
[Gou&al 11] a>0.95 for Cygnus X-1



Summary for SMBH [Reynolds 13]

Part I

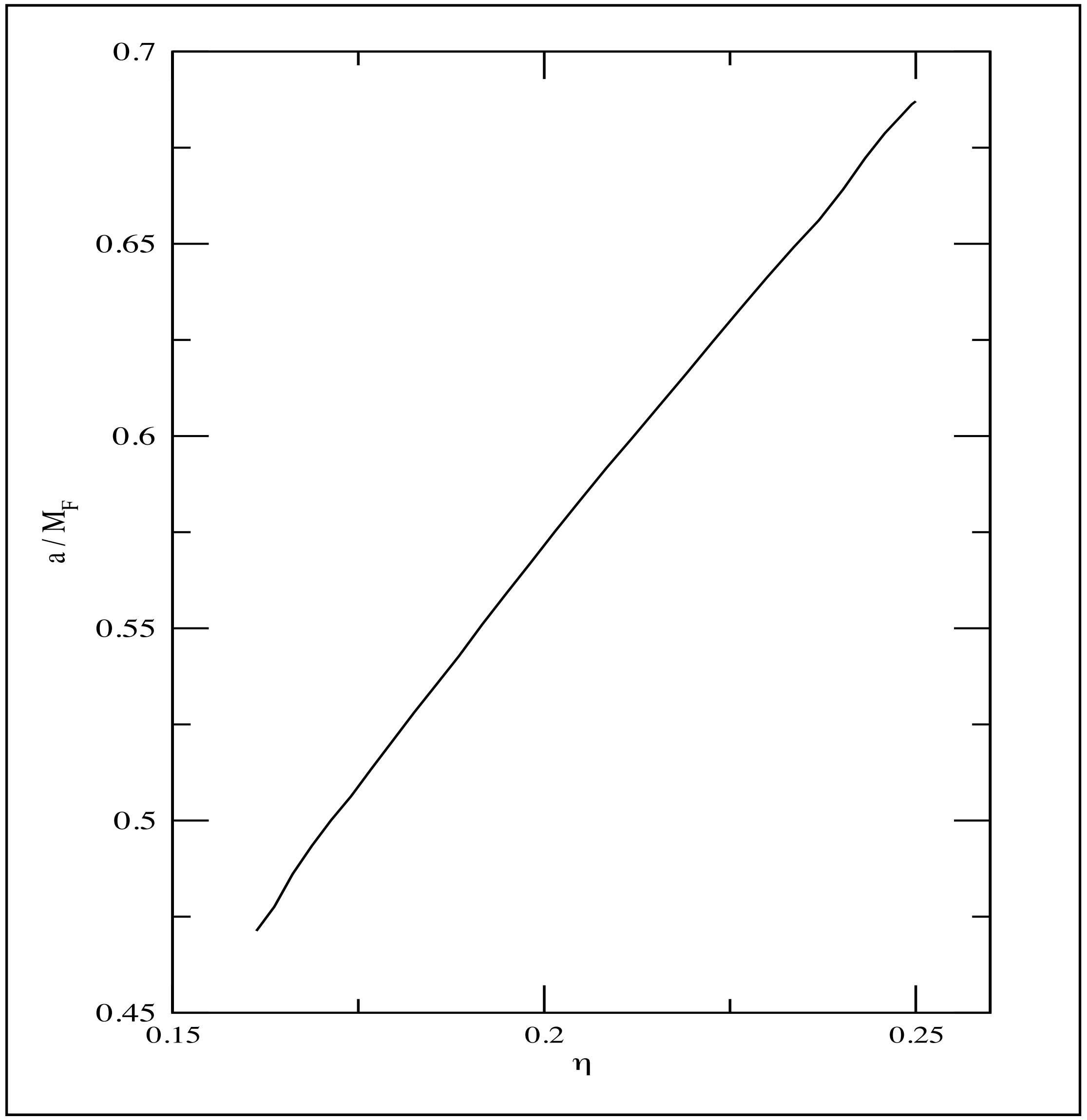
The spin of black holes : theoretical insight

The spin of a merger remnant

Numerical relativity results :

Spin of the remnant for nonspinning black holes [e.g. Gonzalez&al 07] :

Effective formulas for spinning BH binaries [e.g. Rezzolla&al 08]



Link with astrophysics

Inverse problem : what will the measured distribution of spins tell us about their environment, and about the growth history (accretion or merger) of SMBH ?

Post-Newtonian expansion

PN conventions

- Slowly-varying, weakly-gravitating regime : $\mathrm{1PN} \sim Gm/rc^2 \sim v^2/c^2$
- Convention : $S = cJ = Gm^2 a$, of Newtonian order for an extremal BH.

Spin corrections to the equations of motion

ADM Hamiltonian derived by [Hartung-Steinhoff 11]

$$\frac{d\mathbf{v}}{dt} = \mathbf{A}_N + \frac{1}{c^2}\mathbf{A}_{1PN} + \frac{1}{c^4}\mathbf{A}_{2PN} + \frac{1}{c^5}\mathbf{A}^{RR}_{2.5PN} + \frac{1}{c^6}\mathbf{A}_{3PN} + \frac{1}{c^7}\mathbf{A}^{RR}_{3.5PN}$$
$$+ \frac{1}{c^3}\mathbf{A}^{SO}_{1.5PN} + \frac{1}{c^5}\mathbf{A}^{SO}_{2.5PN} + \boxed{\frac{1}{c^7}\mathbf{A}^{SO}_{3.5PN}} + \mathcal{O}\left(\frac{1}{c^8}\right)$$

Spin corrections to the energy flux

(spin-spin terms not shown)

Addressed in this work

$$\mathcal{F} = F_N + \frac{1}{c^2}F_{\mathrm{1PN}} + \frac{1}{c^3}F^{\mathrm{tails}}_{\mathrm{1.5PN}} + \frac{1}{c^4}F_{\mathrm{2PN}} + \frac{1}{c^5}F^{\mathrm{tails}}_{\mathrm{2.5PN}} + \frac{1}{c^6}F_{\mathrm{3PN}} + \frac{1}{c^7}F^{\mathrm{tails}}_{\mathrm{3.5PN}}$$
$$+ \frac{1}{c^3}F^{\mathrm{SO}}_{\mathrm{1.5PN}} + \frac{1}{c^5}F^{\mathrm{SO}}_{\mathrm{2.5PN}} + \frac{1}{c^6}F^{\mathrm{SO-tails}}_{\mathrm{3PN}} + \boxed{\frac{1}{c^7}F^{\mathrm{SO}}_{\mathrm{3.5PN}}} + \boxed{\frac{1}{c^8}F^{\mathrm{SO-tails}}_{\mathrm{4PN}}} + \mathcal{O}\left(\frac{1}{c^9}\right)$$

Part II

Latest post-Newtonian results for spin effects

ADM Hamiltonian results :

- Next-to-leading order Hamiltonian, S-O [Damour, Jaranowski, Schäfer 07]
- Next-to-leading order Hamiltonian, S1-S2 [Steinhoff, Hergt, Schäfer 07]
- Next-to-leading order Hamiltonian, S^2 [Hergt, Steinhoff, Schäfer 10]
- **Next-to-next-to-leading order Hamiltonian,** S-O and S1-S2 [Hartung&Steinhoff 11]

EFT results :

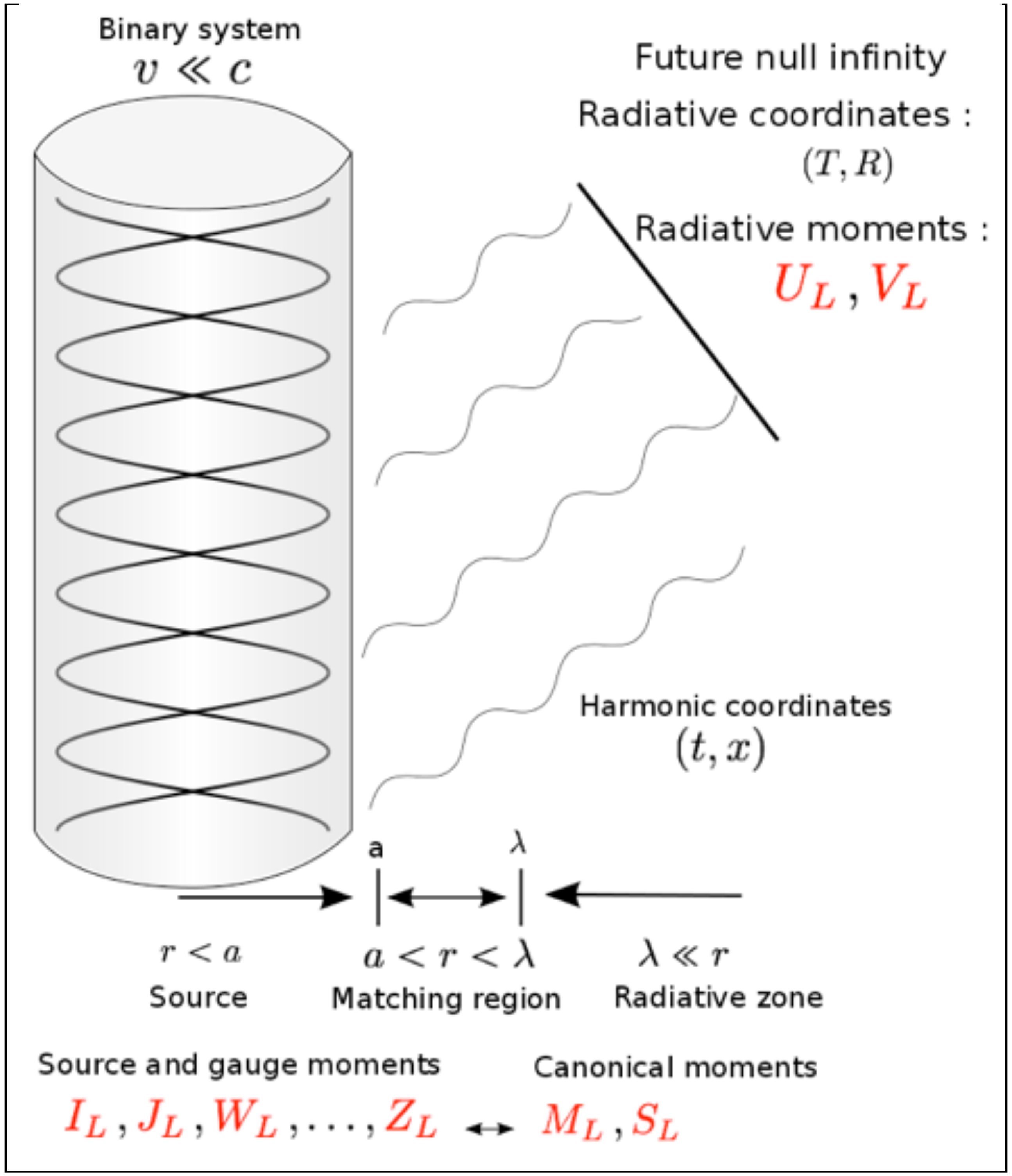
- Next-to-leading order, S-O [Porto 10]
- Next-to-leading order, S1-S2 and S^2 [Porto&Rothstein 10, Levi 08, Levi 10]
- Next-to-next-to-leading order S1-S2 [Porto&Rothstein 11, Levi 11]

(And so far incomplete results for the waveform and flux)

Harmonic coordinates results :

- Next-to-leading order, S-O (EOM and flux) [Faye, Blanchet, Buonanno 06]
- Next-to-leading order, S-O (full waveform) [Arun&al 08]
- Leading order, S1-S2 and S^2 (full waveform) [Buonanno, Faye, Hinderer 12]
- **Next-to-next-to-leading order S-O** (EOM and flux) [this work]

Multipolar post-newtonian wave generation formalism



Outline

- Iteration of $h = \Box^{-1}\Lambda(h)$ outside the source starting with a linear solution parametrized by source and gauge moments $I_L, J_L, \ldots, Z_L$
- Existence of a matching region for a PN source matching of asymptotic expansions
  → $I_L, \ldots, Z_L$ as integrals over the source
  → consistent PN iteration in the near zone
- Radiative coordinates and radiatives multipoles U_L, V_L describing waveform at infinity
- Alternative parametrization in terms of only two sets of canonical moments M_L, S_L
  → relation found by a gauge transformation

Finite part regularization

$$\int d^3x \;\rightarrow\; FP_{B=0}\int d^3x \left(\frac{|x|}{r_0}\right)^B$$

Part II UV regularization

Hadamard regularization

- Regularized value of singular functions :

$$F(\mathbf{x}) = \sum_{p_0 \leqslant p \leqslant N} r_1^p \underset{1}{f}_p(\mathbf{n}_1) + o(r_1^N) \ , \ (F)_1 = \langle \underset{1}{f}_0(\mathbf{n}_1) \rangle$$

- Non-distributive : $F\delta_1 \neq (F)_1\delta_1, \ (FG)_1 \neq (F)_1(G)_1$
- Prescription for distributional derivatives (not unique, no Leibniz rule)
- Regularization of integrals : removal of the diverging part $\mathrm{Pf}_{s_1,s_2} \int \mathrm{d}^3 x F(x)$
- Apparition of ambiguities at the 3PN NS order

Dimensional regularization

- $d \to 3 + \varepsilon$ and analytical continuation in ε
- Structure : $F^{(d)}(\mathbf{x}) = \sum_{\substack{p_0 \leqslant p \leqslant N \\ q_0 \leqslant q \leqslant q_1}} r_1^{p+q\varepsilon} \underset{1}{f}{}^{(\varepsilon)}_{p,q}(\mathbf{n}_1) + o(r_1^N) \, , \quad \underset{1}{f}_p(\mathbf{n}_1) = \sum_{q_0 \leqslant q \leqslant q_1} \underset{1}{f}{}^{(0)}_{p,q}(\mathbf{n}_1)$
- Distributive, well-defined distributional prescription, regular integrals
- In practice : 'pure Hadamard-Schwartz' supplemented by dimreg

→ Determined the 3PN ambiguities

Results for the metric and applications

Metric in the whole near-zone

$$\begin{aligned}(g_{00})_S &\rightarrow \mathcal{O}(7)\\(g_{0i})_S &\rightarrow \mathcal{O}(6)\\(g_{ij})_S &\rightarrow \mathcal{O}(7)\end{aligned}$$

Can be used for :

- Building approximate solutions by asymptotic matching to a perturbed black hole [Gallouin&al 12]
- Simulating a circumbinary MHD disk in a PN-approximated spacetime [Noble&al 09]
- Building realistic initial conditions for NR using PN information [Kelly&al 09]

Regularized metric

$$\begin{aligned}(g^S_{00})_1 &\rightarrow \mathcal{O}(9)\\(g^S_{0i})_1 &\rightarrow \mathcal{O}(8)\\(g^S_{ij})_1 &\rightarrow \mathcal{O}(7)\end{aligned}$$

- Used for the first law of binary black holes [Blanchet&al 12]

With EOM :

Allows computation of the emitted waveform and energy flux

Computation of the potentials I

Compact-support terms

Dirac-delta terms (stress-energy tensor or distributional contributions), treated with pHS :

$$\int \mathrm{d}^3\mathbf{x} F(\mathbf{x})\delta_1 = (F)_1$$

'Easy' non-compact-support terms

Particular solution : $\Delta g = \frac{1}{r_1 r_2}$,

$$g \equiv \ln(r_1 + r_2 + r_{12})$$

Quadratic terms with lowest-order potentials $V,\ V_i$ can be readily integrated :

$$\Delta^{-1}\left[\partial_i\left(\frac{1}{r_1}\right)\partial_{jk}\left(\frac{1}{r_2}\right)\right] = -\partial^1_i\partial^2_{jk} g$$

'Difficult' non-compact-support terms

Only the regularized potential is evaluated, using generic formulas :

$$P(\mathbf{x}) = -\frac{1}{4\pi}\mathrm{Pf}_{s_1,s_2}\int \frac{\mathrm{d}^3\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|}F(\mathbf{x}')$$

$$(P)_1 = -\frac{1}{4\pi}\mathrm{Pf}_{s_1,s_2}\int \frac{\mathrm{d}^3\mathbf{x}}{r_1}F(\mathbf{x}) + \left[\ln\left(\frac{r'_1}{s_1}\right) - 1\right](r_1^2 F)_1$$

s_1, s_2, r'_1, r'_2

Regularization constants

Computation of the potentials II

'Difficult' non-compact-support terms

→ No apparition of reg. constants in the spin part of potentials

$\left(\partial_{jk}\hat{Y}_i^{NS}\right)_1$ → Apparition of gauge constants in one no-spin potential : dimreg computation

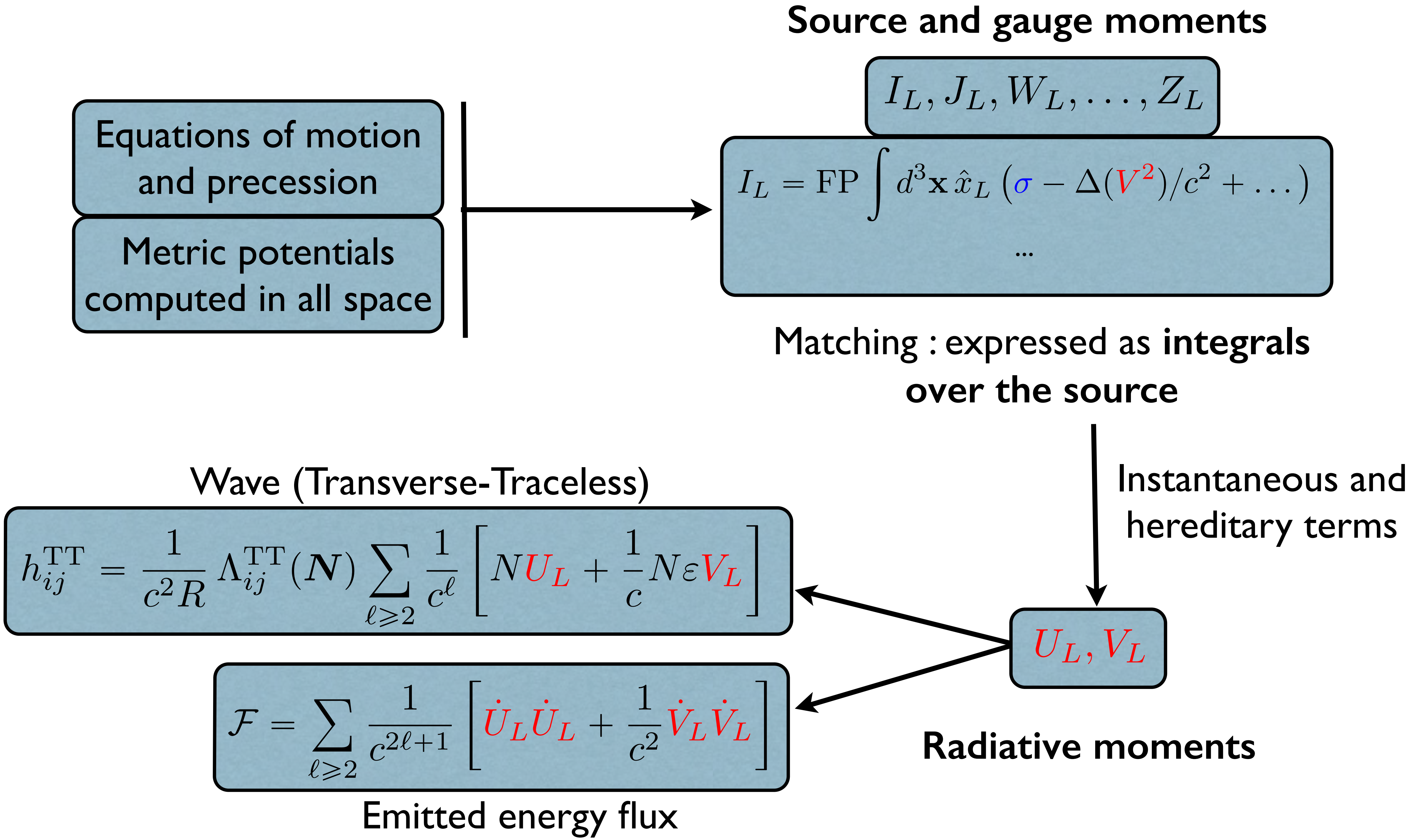
Dimreg contributions

$$\mathcal{D}(\partial_{ij}P)(1) \equiv (\partial_{ij}P^{(d)})(\mathbf{y}_1) - (\partial_{ij}P)_1$$

Result for the pole : $\mathcal{D}(\partial_{jk}\hat{Y}_i)(1) = \frac{1}{\varepsilon}\frac{G^3 m_1^2 m_2}{252} v_{12}^l \partial^1_{ijkl}\left(\frac{1}{r_{12}}\right) + \mathcal{O}(\varepsilon^0)$

$\left(\partial_{j[k}\hat{Y}_{i]}^{NS}\right)_1$ → Cancellation of all dimreg contributions

Overview for the waveform and flux and 3.5PN order



Source and gauge moments

$I_L, J_L, W_L, \dots, Z_L$

Equations of motion and precession

Metric potentials computed in all space

$$I_L = \mathrm{FP} \int d^3\mathbf{x}\, \hat{x}_L \left(\sigma - \Delta(V^2)/c^2 + \dots \right)$$

...

Matching : expressed as **integrals over the source**

Instantaneous and hereditary terms

Wave (Transverse-Traceless)

$$h_{ij}^{\mathrm{TT}} = \frac{1}{c^2 R} \Lambda_{ij}^{\mathrm{TT}}(\boldsymbol{N}) \sum_{\ell \geqslant 2} \frac{1}{c^\ell} \left[N U_L + \frac{1}{c} N \varepsilon V_L \right]$$

U_L, V_L

$$\mathcal{F} = \sum_{\ell \geqslant 2} \frac{1}{c^{2\ell+1}} \left[\dot{U}_L \dot{U}_L + \frac{1}{c^2} \dot{V}_L \dot{V}_L \right]$$

Radiative moments

Emitted energy flux

- At 3.5PN order, only leading order instantaneous contributions intervene (with leading tail terms at 3PN) : $U_L = I_L^{(l)}, V_L = J_L^{(l)}$
- Computation of the source moments and their derivatives using EOM and metric

Part III

Results for quasi-circular orbits

Equations of motion

Corrections in Kepler's law : $x \equiv \left(Gm\omega/c^3\right)^{2/3}$ 1PN

$$\frac{Gm}{rc^2} = x\left\{1 + x\left(1 - \frac{1}{3}\nu\right) + \ldots \right.$$
$$\left. + \frac{x^{7/2}}{Gm^2}\left[\left(5 - \frac{127}{12}\nu - 6\nu^2\right)S_\ell + \frac{\delta m}{m}\left(3 - \frac{61}{6}\nu - \frac{8}{3}\nu^2\right)\Sigma_\ell\right] + \mathcal{O}(8)\right\}.$$

Conserved quantities

Corrections in the orbital energy :

$$E = -\frac{m\nu c^2 x}{2}\left\{1 + x\left(-\frac{3}{4} - \frac{1}{12}\nu\right) + \ldots \right.$$
$$\left. + \frac{x^{7/2}}{Gm^2}\left[\left(\frac{135}{4} - \frac{367}{4}\nu + \frac{29}{12}\nu^2\right)S_\ell + \frac{\delta m}{m}\left(\frac{27}{4} - 39\nu + \frac{5}{4}\nu^2\right)\Sigma_\ell\right] + \mathcal{O}(8)\right\}$$

From the energy and flux to the phase

Spin contributions in the balance equation

$$\mathcal{F} = -\frac{\mathrm{d}E}{\mathrm{d}t} \longrightarrow \dot{x}\frac{\mathrm{d}E}{\mathrm{d}x} + \dot{S}\frac{\mathrm{d}E}{\mathrm{d}S} = -\mathcal{F}$$

Post-Newtonian orders : control of the evolution of the spins ?

$$\mathcal{O}(5)(\mathcal{O}(0) + \cdots + \mathcal{O}(7)) + \dot{S}_\ell(\mathcal{O}(3) + \cdots + \mathcal{O}(7)) = \mathcal{O}(5)(\mathcal{O}(0) + \cdots + \mathcal{O}(7))$$

Secular spin variables at linear order in spin : $\dot{S}_\ell = \mathcal{O}(S^2)$ since $\dot{\boldsymbol{S}} = \boldsymbol{\Omega} \times \boldsymbol{S}$, $\boldsymbol{\Omega} \propto \boldsymbol{\ell}$

Illustration of the computation of the phase

- Taylor T2 : solve analytically after PN-expanding the system

$$\frac{\mathrm{d}\phi}{\mathrm{d}x} = -\frac{c^3}{Gm}x^{3/2}\frac{\mathrm{d}E/\mathrm{d}x}{\mathcal{F}(x)}$$

$$\frac{\mathrm{d}t}{\mathrm{d}x} = -\frac{\mathrm{d}E/\mathrm{d}x}{\mathcal{F}(x)}$$

- Taylor T1 : solve numerically without re-expanding the system

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{\mathcal{F}}{\mathrm{d}E/\mathrm{d}x}$$

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \frac{c^3}{Gm}x^{3/2}$$

Result for the number of cycles

Taylor T2

Number of cycles between f~10Hz and $\omega = \omega_{ISCO} \; (x_{\rm ISCO} = 1/6)$

	$1.4M_\odot + 1.4M_\odot$	$10M_\odot + 1.4M_\odot$	$10M_\odot + 10M_\odot$
N	15952.6	3558.9	598.8
1PN	439.5	212.4	59.1
1.5PN	$-210.3 + 65.6\kappa_1\chi_1 + 65.6\kappa_2\chi_2$	$-180.9 + 114.0\kappa_1\chi_1 + 11.7\kappa_2\chi_2$	$-51.2 + 16.0\kappa_1\chi_1 + 16.0\kappa_2\chi_2$
2PN	9.9	9.8	4.0
2.5PN	$-11.7 + 9.3\kappa_1\chi_1 + 9.3\kappa_2\chi_2$	$-20.0 + 33.8\kappa_1\chi_1 + 2.9\kappa_2\chi_2$	$-7.1 + 5.7\kappa_1\chi_1 + 5.7\kappa_2\chi_2$
3PN	$2.6 - 3.2\kappa_1\chi_1 - 3.2\kappa_2\chi_2$	$2.3 - 13.2\kappa_1\chi_1 - 1.3\kappa_2\chi_2$	$2.2 - 2.6\kappa_1\chi_1 - 2.6\kappa_2\chi_2$
3.5PN	$-0.9 + 1.9\kappa_1\chi_1 + 1.9\kappa_2\chi_2$	$-1.8 + 11.1\kappa_1\chi_1 + 0.8\kappa_2\chi_2$	$-0.8 + 1.7\kappa_1\chi_1 + 1.7\kappa_2\chi_2$
4PN	$(\mathrm{NS}) - 1.5\kappa_1\chi_1 - 1.5\kappa_2\chi_2$	$(\mathrm{NS}) - 8.0\kappa_1\chi_1 - 0.7\kappa_2\chi_2$	$(\mathrm{NS}) - 1.5\kappa_1\chi_1 - 1.5\kappa_2\chi_2$

$\kappa_i, \; \chi_i$ parameters for the orientation and magnitude of the spins

Taylor T1

Aligned spins, 0.1 for neutron stars and 1 for black holes

	$1.4M_\odot + 1.4M_\odot$	$10M_\odot + 1.4M_\odot$	$10M_\odot + 10M_\odot$
N	16028.2	3575.8	601.6
1PN	474.4	248.7	75.8
1.5PN	$-237.1 + (+13.7)_S$	$-214.9 + (122.5)_S$	$-67.2 + (35.0)_S$
2PN	-18.5	$-182.$	-8.0
2.5PN	$20.8 + (0.6)_S$	$33.6 + (16.2)_S$	$16.6 + (3.9)_S$
3PN	$-10 + (0.2)_S$	$-30.3 + (4.6)_S$	$-11.6 + (1.8)_S$
3.5PN	$-0.1 + (-0.01)_S$	$2.7 + (1.3)_S$	$-0.2 + (-0.3)_S$
4PN	$(\mathrm{NS}) + (-0.005)_S$	$(\mathrm{NS}) + (0.4)_S$	$(\mathrm{NS}) + (-0.1)_S$

Part I

The spin of neutron stars

Observations of rotation of neutron stars :

Two main pulsar populations :
- Young, normal pulsars
- Recycled pulsars : P ~ few milliseconds

Dimensionless Kerr parameter :

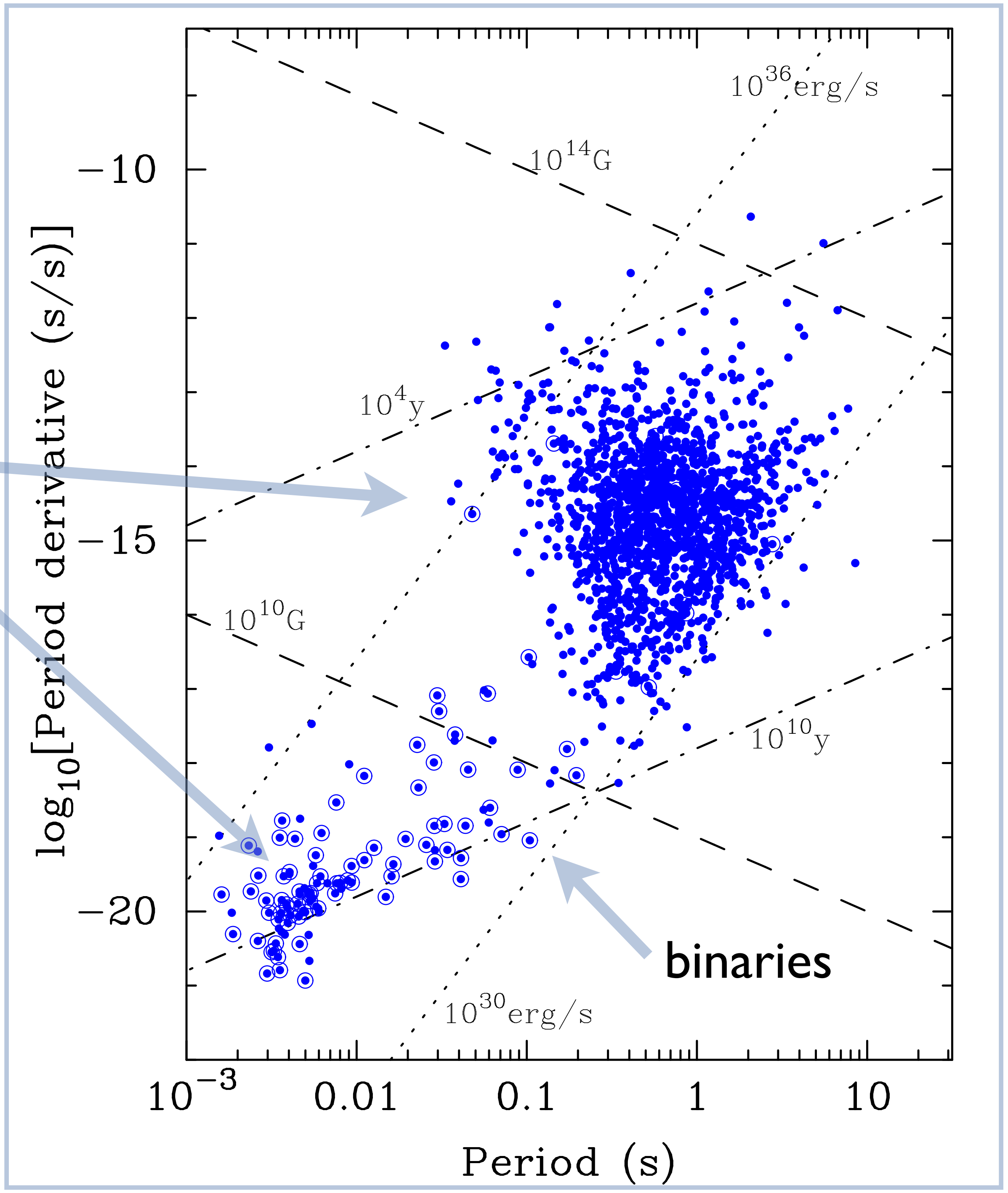
Fastest known pulsar : J1748-2446 , 716 Hz

Order-of-magnitude estimate (I not known) :

$$a \sim 0.4$$

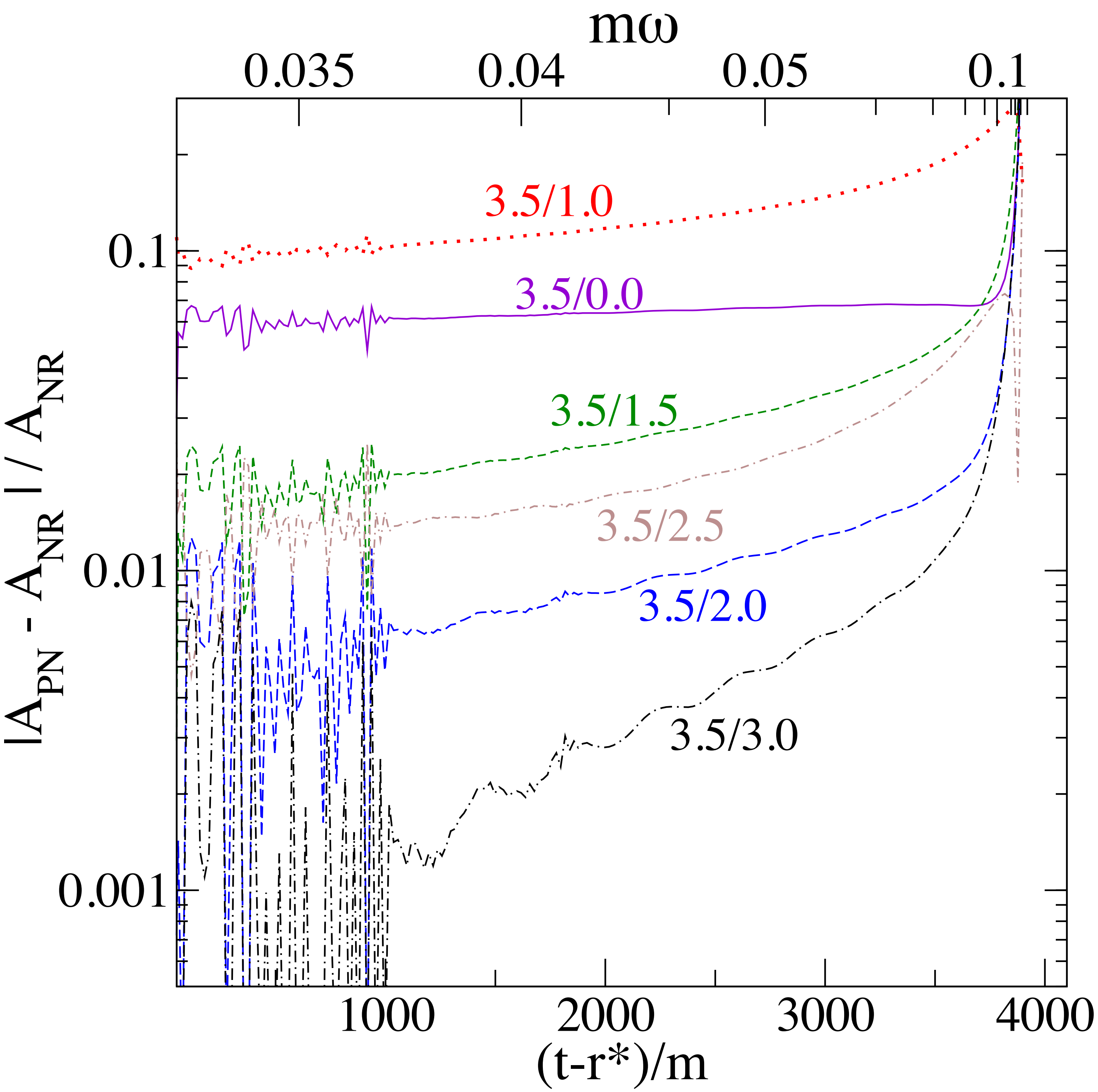
Typical value in binaries :

$$a \sim 0.1$$



$P - \dot{P}$ diagram for NS [Lorimer 08]

Comparison NR/PN for the 22 mode

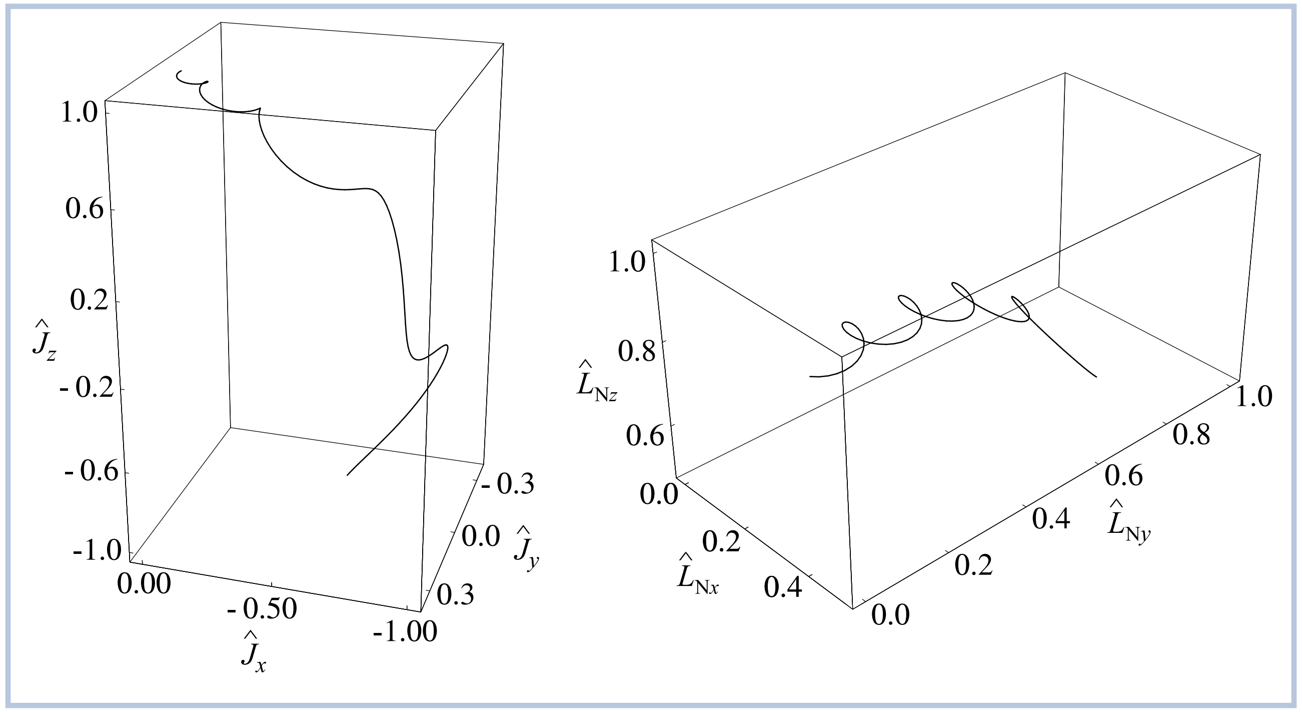


[Boyle&al 07]

Part I

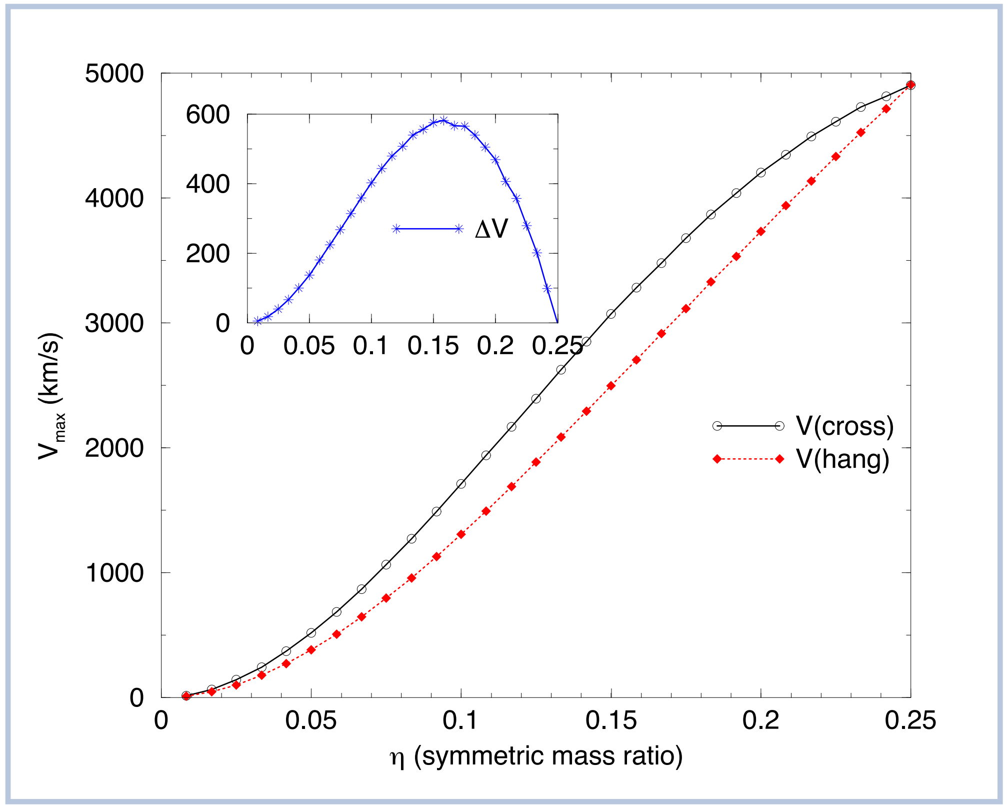
Spin effects : transitional precession and recoil

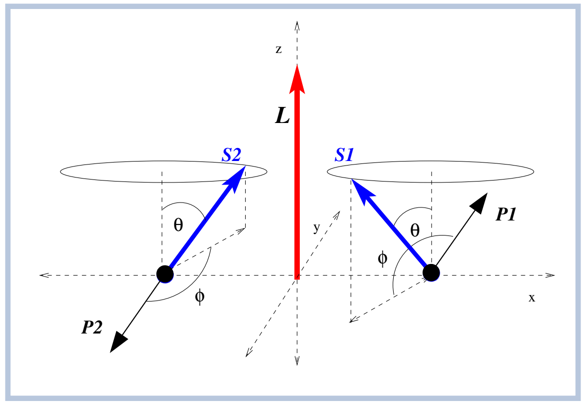
Transitional precession



Transitional precession (20+5)M [BCV 02] : regime where S and L almost cancel, and direction of J changes rapidly

Recoil of the remnant





Maximal kick : "Hangup" configurations [Lousto&Zlochower 12]