GRECO seminar — 7 October 2019 — IAP

Backreacting Quantum Fields on de Sitter Space — lessons from gauge-gravity duality

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with Jewel Kumar Ghosh, Elias Kiritsis and Francesco Nitti (APC)

Motivation

Cosmology is the study of the evolution of the universe according to the laws of gravity subject to the matter content.

Matter in our universe is a manifestation of quantum fields which necessarily exhibit **quantum fluctuations**.

According to the **equivalence principle**, quantum fluctuations are expected to gravitate thus influencing the evolution of the universe.

How do quantum fluctuations **backreact** on a given background spacetime ?

Motivation

Absent a theory of quantum gravity the question of backreaction can be addressed in the framework of **QFT on curved spacetime**.

While quantisation in curved spacetime is textbook material, explicit calculations regarding backreaction are **technically hard**.

More tractable when considering backgrounds with **maximal symmetry**, with **de Sitter (dS) space** of particular phenomenological importance.

How do quantum fluctuations **backreact** on dS space?

Motivation

Long-standing set of hints that **dS space** may be **unstable against fluctuations**:

- Hints for a **thermodynamic** instability [Mottola '85]
- Possible instability due to **graviton** fluctuations and fluctuations of **massless scalars**

  [Antoniadis, Iliopoulos, Tomaras '86; Tsamis, Woodard '96, '97]

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Observed difficulty of obtaining de Sitter vacua from **string theory compactifications**:

- Conjecture that de Sitter vacua are forbidden in quantum gravity. [Obied, Ooguri, Spodyneiko, Vafa '18; Ooguri, Palti, Shiu, Vafa, '18]

I. Thermodynamics

Hints for a **thermodynamic** instability: [Mottola '85]

dS temperature: $T = \frac{H}{2\pi}$ dS horizon entropy: $S_H \sim \frac{1}{T^2}$

$$\Rightarrow \quad \frac{dS_H}{dT} \sim -\frac{1}{T^3} < 0$$

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$\Rightarrow$ **Thermodynamically unstable**

2. Graviton Fluctuations

Hints for instability due to **graviton fluctuations**: [Tsamis, Woodard '96]

Consider the theory: $$S = \frac{1}{\kappa^2}\int d^4x\sqrt{-g}\Big(R-2\lambda\Big)$$

Compute perturbatively: $$\langle 0|g_{\mu\nu}(t,\vec{x})dx^\mu dx^\nu|0\rangle = -dt^2 + a^2(t)d\vec{x}\cdot d\vec{x}$$

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At **two graviton loops** one finds:

$$H_{\text{eff}}(t) = H\left\{1 - \left(\frac{\kappa H}{4\pi}\right)^4 \left[\tfrac{1}{6}(Ht)^2 + \ldots\right] + \mathcal{O}(\kappa^6)\right\} , \qquad 3H^2 = \lambda$$

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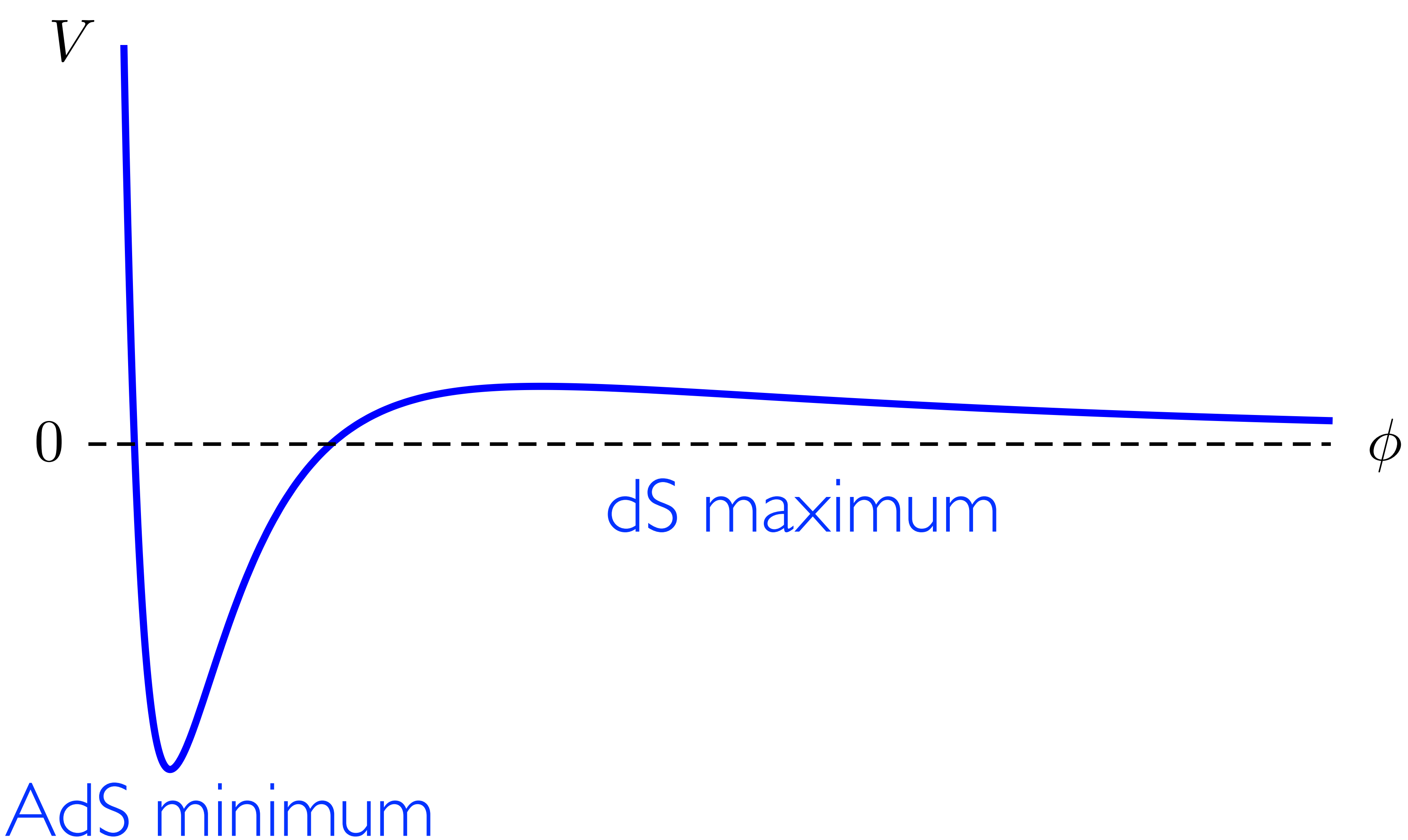
Could this be an artefact of perturbation theory?

3. String Theory

Hints from **string theory compactifications**:

Typical potential for a string compactification including classical + leading quantum contributions:



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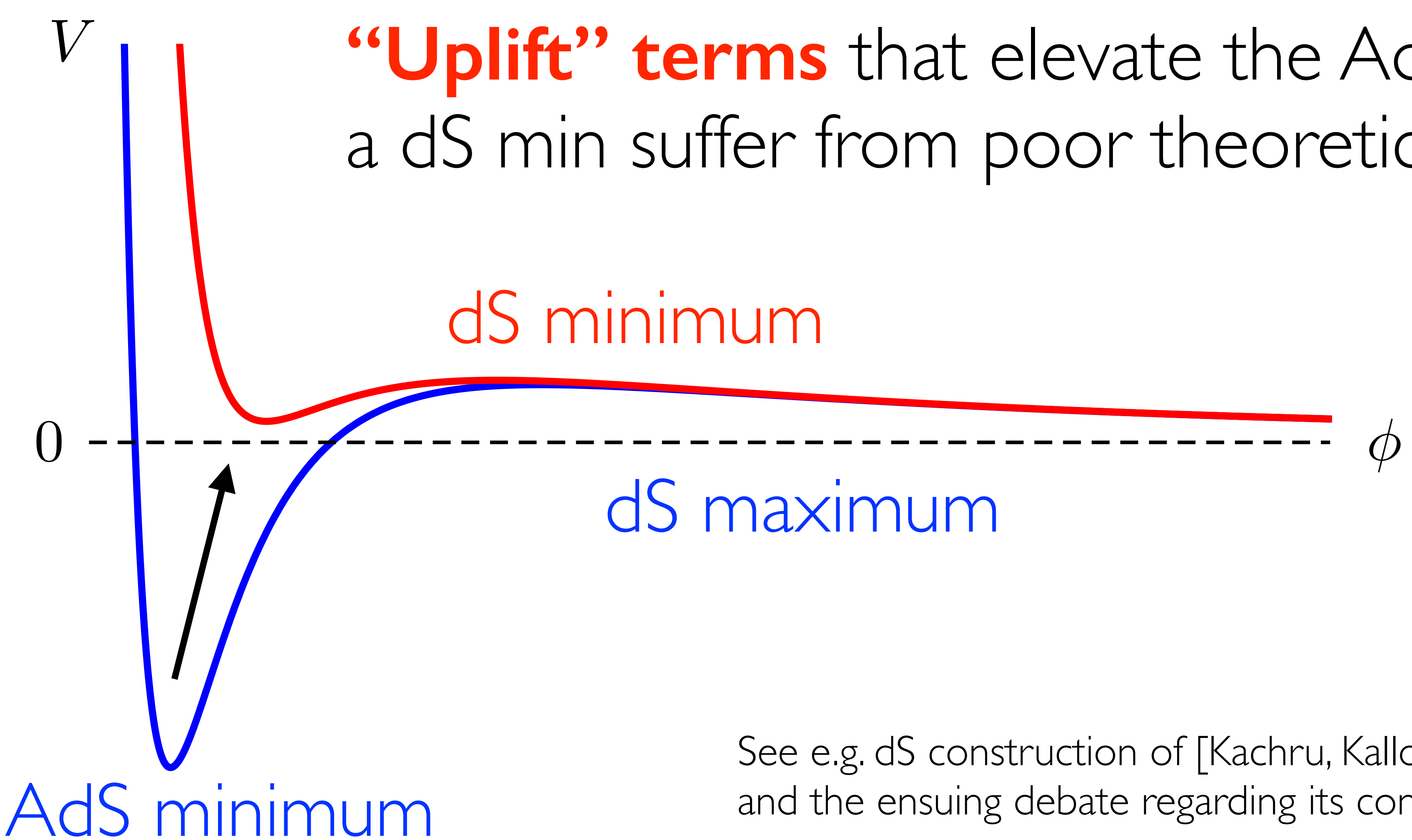

"Uplift" terms that elevate the AdS min. to a dS min suffer from poor theoretical control.

See e.g. dS construction of [Kachru, Kallosh, Linde, Trivedi '03] and the ensuing debate regarding its consistency.

3. String Theory

Hints from **string theory compactifications**:

- AdS minima and dS maxima are generic.
- dS minima difficult to construct & theoretical control is in doubt.

Conjecture that dS minima are forbidden. [Obied, Ooguri, Spodyneiko, Vafa '18; Ooguri, Palti, Shiu, Vafa, '18]

This can be ensured by requiring:

$$V' \geq \frac{c}{M_p} V \,, \qquad\qquad V'' \leq -\frac{c'}{M_p^2} V \,.$$

Note that the conditions in this form do not follow directly from string theory.

Motivation

Long-standing set of hints that **dS space** may be **unstable against fluctuations.**

Observed difficulty of obtaining de Sitter vacua from **string theory compactifications.**

Could the instabilities (due to massless fields) could just be **artefacts of perturbation theory?**

Here: use techniques of **gauge-gravity duality** to study back-reaction of **holographic QFTs** on dS.

Backreacting a QFT

To study backreaction, construct the effective action for the metric $g_{\mu\nu}$ obtained by integrating out the QFT:

$$Z = \int d[g]\, d[\Phi]\, e^{iS_0[g] + iS_{\mathrm{QFT}}[g,\Phi]}$$

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$$\Rightarrow \quad S_{\text{eff}}[g] = S_0[g] + W_{\text{QFT}}[g]$$

Motivation

$$S_{\text{eff}}[g] = S_0[g] + W_{\text{QFT}}[g]$$

- **1st step:**
  study back-reaction on **constant-curvature backgrounds**

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- **Here:** focus on backgrounds with

$$\nabla_\rho R_{\mu\nu} = 0 \quad \Leftrightarrow \quad R_{\mu\nu} = \kappa g_{\mu\nu}\,, \quad R = 4\kappa$$

This includes the phenomenologically interesting cases of max. symmetric space-times (Minkowski, AdS, dS).

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So far, this analysis has been performed for special cases:

- **massive free scalar on de Sitter** [see e.g. Mazur, Mottola 1986]
- ϕ^4**-theory on de Sitter** via non-perturbative RG techniques. [Moreau, Serreau 2018]

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Here, we will employ **holography** to integrate out a QFT on an Einstein manifold and calculate $W_{\text{QFT}}[g]$.

Outline

1.) Setup:

- What types of QFTs are back-reacted?
- Integrating out via holography
- UV divergences & renormalisation

2.) Results for constant-curvature solutions:

- Case I: The physical system is UV-complete
- (Case II: QFT with a UV cutoff)

3.) Stability of the dS solutions

4.) Conclusions & open questions

Setup

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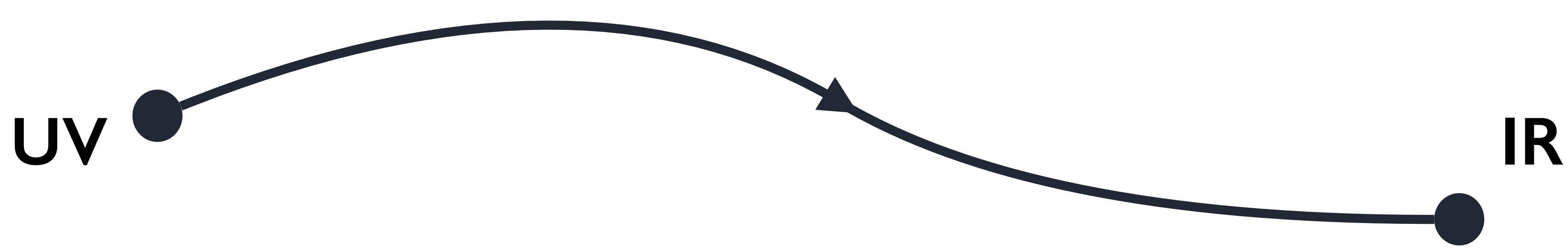
In particular:

1.) CFTs

2.) RG flow QFTs:

RG flows driven by a relevant operator $\mathcal{O}$ of dimension Δ^{UV} from a UV fixed point to a IR fixed point.



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Setup: QFTs

We will consider **large-N_c** theories at **infinite coupling.**
In particular:

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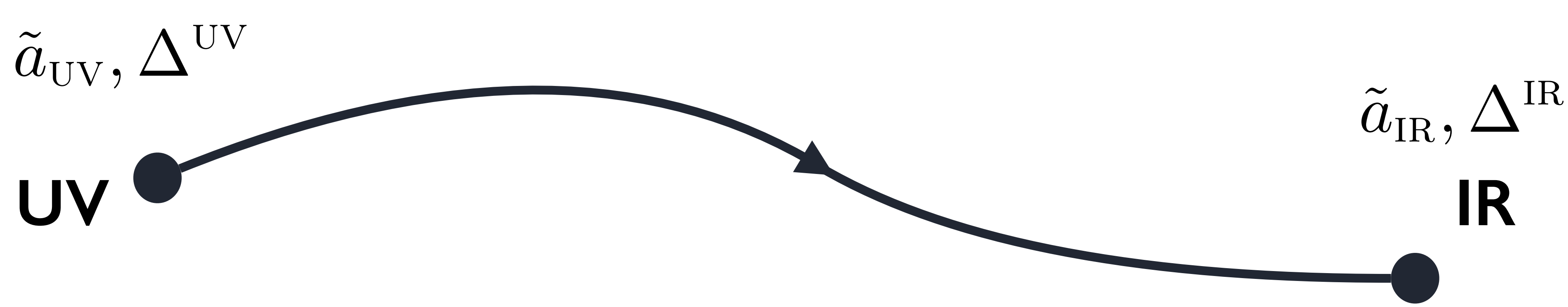

$$S_{\text{QFT}} = S_{\text{UV-CFT}} + \int m^{4-\Delta^{\text{UV}}} \mathcal{O}$$

Setup: Integrating out via holography

Postulate that the **4d QFT** possesses a holographic dual given by a **5d gravitational theory**.

Duality: $$Z_{\mathrm{QFT},4d}[g] = Z_{\mathrm{grav},5d}[g]$$

with $$Z_{\mathrm{QFT},4d}[g] = \int d[\Phi]\, e^{iS_{\mathrm{QFT}}[g,\Phi]} = e^{iW_{\mathrm{QFT}}[g]}$$

and $$Z_{\mathrm{grav},5d}[g] = \int_{G|_{\partial\mathcal{M}}=g} d[G]\, e^{iS_{\mathrm{grav}}[G]}$$

Setup: Integrating out via holography

We take the QFTs to be at **large** N_c and at **infinite coupling**.

The gravity dual is dominated by classical gravity, i.e.

$$Z_{\mathrm{grav},5d}[g] = \int_{G|_{\partial\mathcal{M}}=g} d[G]\, e^{iS_{\mathrm{grav}}[G]} = e^{iS^{\text{on-shell}}_{\mathrm{grav}}[g]}$$

$$\Rightarrow \qquad W_{\mathrm{QFT}}[g] = S^{\text{on-shell}}_{\mathrm{grav}}[g]$$

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The limit $N_c \to \infty$ implies $\tilde{a}, \tilde{a}_{\mathrm{UV}}, \tilde{a}_{\mathrm{IR}} \to \infty$ but $\tilde{a}_{\mathrm{UV}}/\tilde{a}_{\mathrm{IR}}$ can be chosen finite.

Setup: the holographic dual

$$S_{\mathrm{grav},5d} = M^3 \int du\, d^4x \sqrt{|G|} \left(R^{(G)} - (\partial\varphi)^2 - V(\varphi) \right) + S_{\mathrm{GHY}}\,.$$

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Ansatz:

$$ds^2 = du^2 + e^{A(u)} g_{\mu\nu} dx^\mu dx^\nu \,, \qquad \varphi(u, x^\mu) = \varphi(u)$$

Einstein manifold

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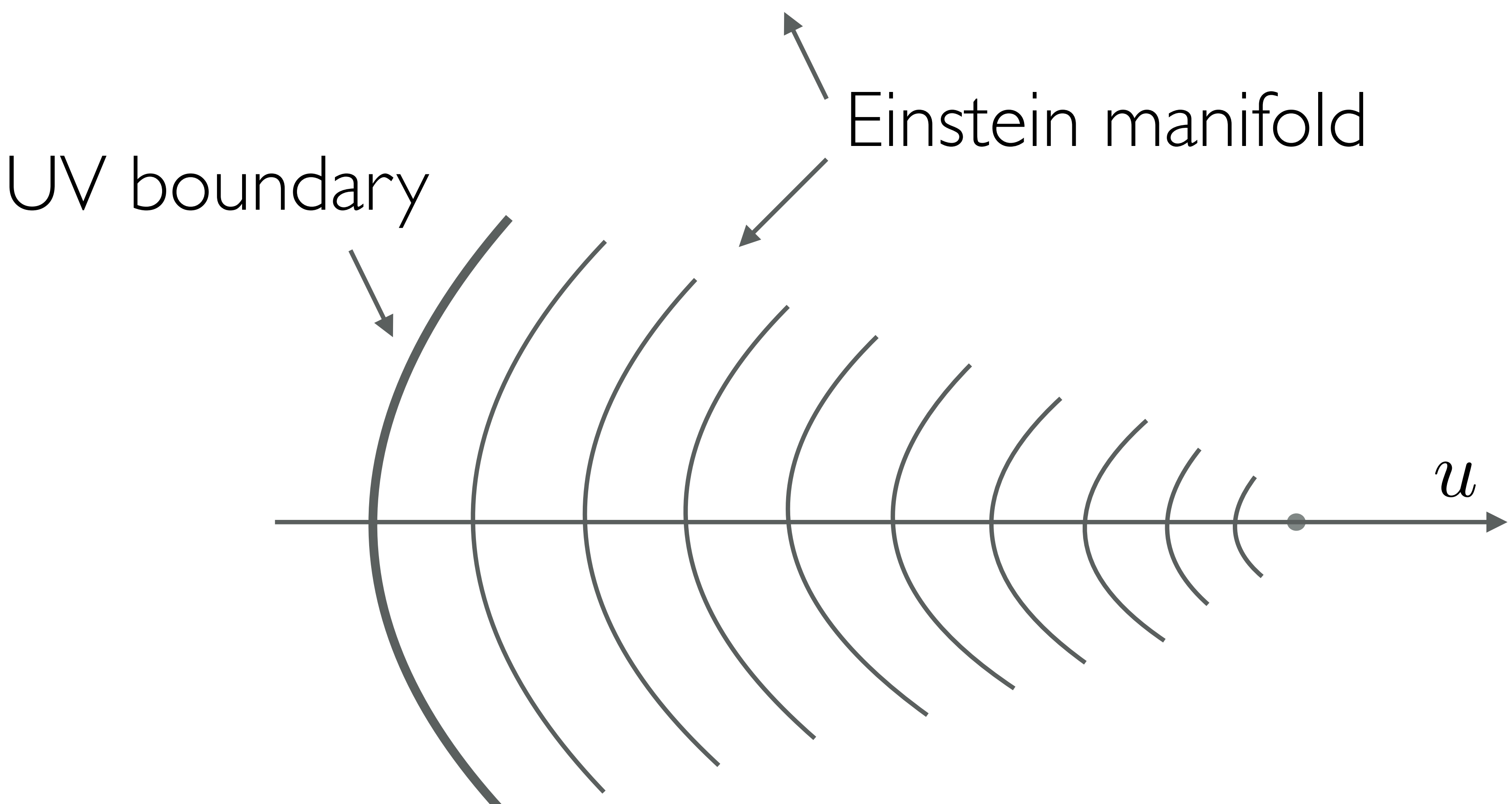

[Ghosh, Kiritsis, Nitti, LW 2017]

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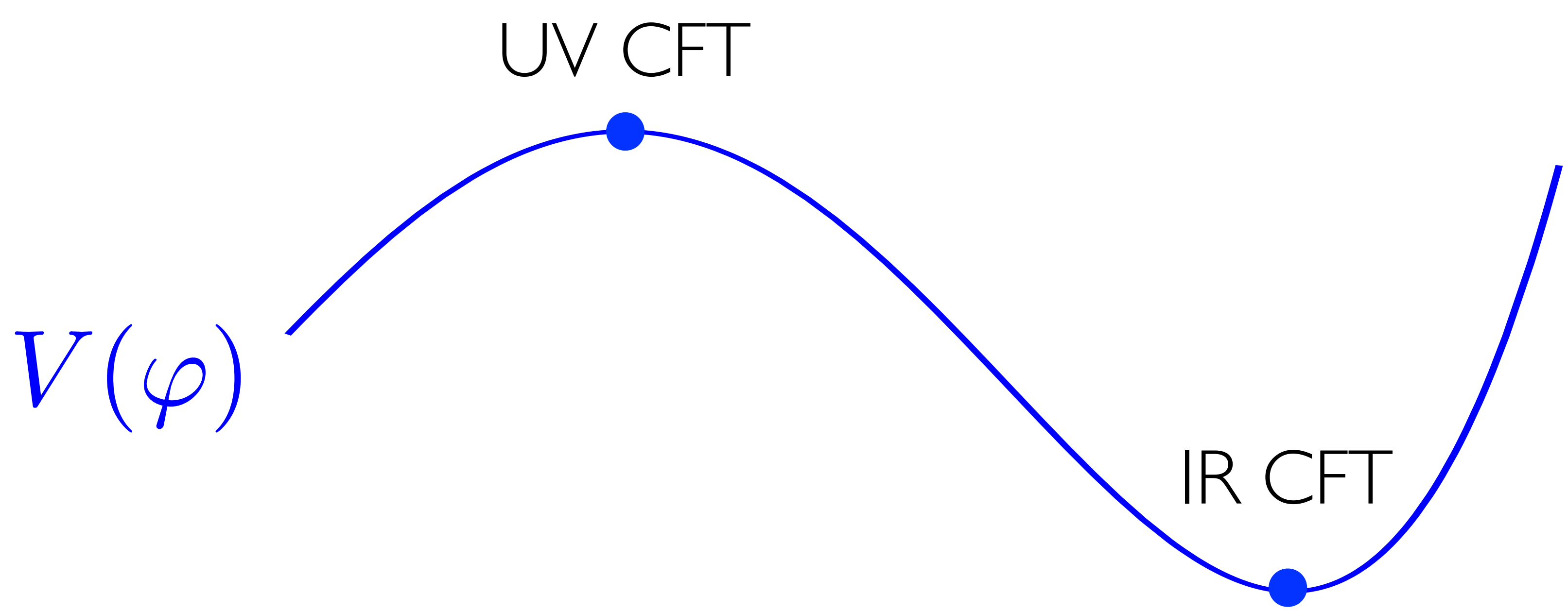
Dilaton potential:



[Ghosh, Kiritsis, Nitti, LW 2017]

Setup: Summary

We will consider **large-N_c** theories at **infinite coupling** of the following type and integrate out via **holography**:

1.) CFTs

2.) RG flow QFTs:

RG flows driven by a relevant operator $\mathcal{O}$ of dimension Δ^{UV} from a UV fixed point to a IR fixed point.

The QFTs are defined on **Einstein manifolds**.

$$S_{\text{eff}}[g] = S_0[g] + S_{\text{grav}}^{\text{on-shell}}[g]$$

Setup: UV divergences

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Regulate UV divergences via a UV cutoff.

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$$S_{\mathrm{grav}}^{\mathrm{on\text{-}shell}}[g] = \int d^4x \sqrt{|g|}\, f_{\mathrm{QFT}}(R)$$

$$\sim \int \left(a_1 \Lambda^4 + a_2 \Lambda^2 R + a_3 R^2 \log R \Lambda^{-2} \right)$$

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$$\sim \int \left(a_1 \Lambda^4 + a_2 \Lambda^2 R + a_3 R^2 \log R\Lambda^{-2} \right)$$

- The term $\sim \Lambda^4$ renormalizes the cosmological constant
- The term $\sim \Lambda^2 R$ renormalizes the Planck scale.
- The term $\sim R^2 \log R\Lambda^{-2}$ renormalizes the R^2-term.

Setup: Case 1

The system of bare grav. theory and QFT is "UV-complete".

$\Lambda \to \infty$ Take cutoff to infinity

$$S_0[g] = \int d^4x \sqrt{|g|} \left[\frac{M_0^2}{2} R - M_0^2 \lambda_0 + a_0 R^2 \right]$$

$$S_{\rm grav}^{\rm on\text{-}shell}[g] = \int d^4x \sqrt{|g|}\, f_{\rm QFT}(R)$$

Absorb the divergent terms in renormalized quantities:

$$M_{\rm ren}^2 \lambda_{\rm ren} = M_0^2 \lambda_0 + f_{\rm QFT}\big|_{R=0,\,\Lambda\to\infty}$$

$$\frac{M_{\rm ren}^2}{2} = \frac{M_0^2}{2} + \frac{d}{dR} f_{\rm QFT}\big|_{R=0,\,\Lambda\to\infty}$$

Setup: Case I

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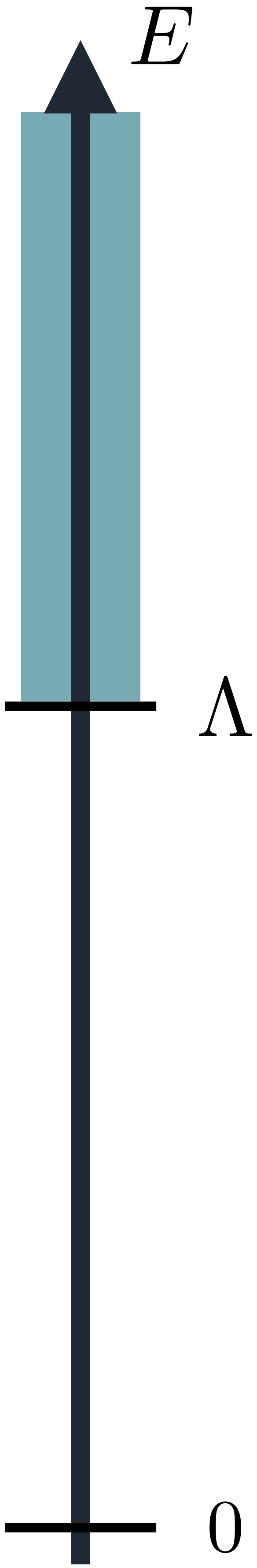
Absorb the divergent terms in renormalized quantities:

$$S_{\text{eff}}[g] = \int d^4x \sqrt{|g|}\, f(R \,|\, M_{\text{ren}}, \lambda_{\text{ren}}, m)$$

Setup: Case 2



In **Case 2** it is assumed that S_0 is an effective theory at some scale Λ.

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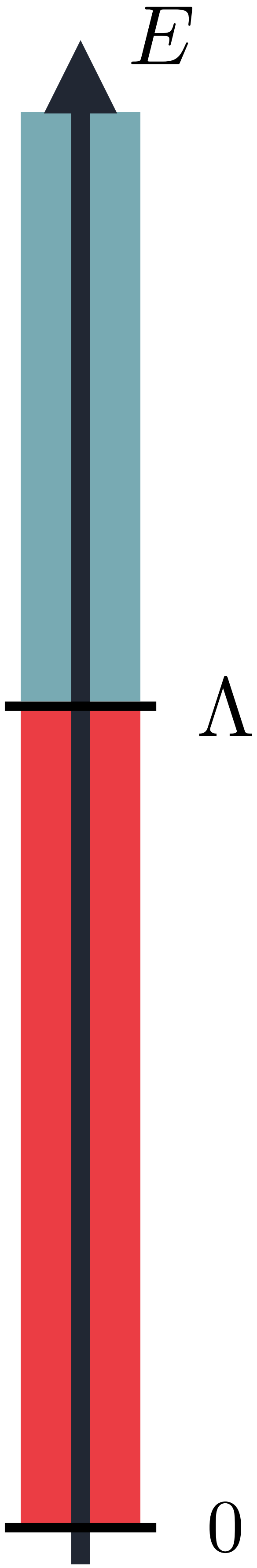

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$$S_0[g] = \int d^4x \sqrt{|g|} \left[\frac{M_0^2}{2} R - M_0^2 \lambda_0 + a_0 R^2 \right]$$

Then couple a QFT with UV cutoff Λ to the background described by $g_{\mu\nu}$.

$$S_{\text{grav}}^{\text{on-shell}}[g] = \int d^4x \sqrt{|g|}\, f_{\text{QFT}}(R \,|\, \Lambda, m)$$

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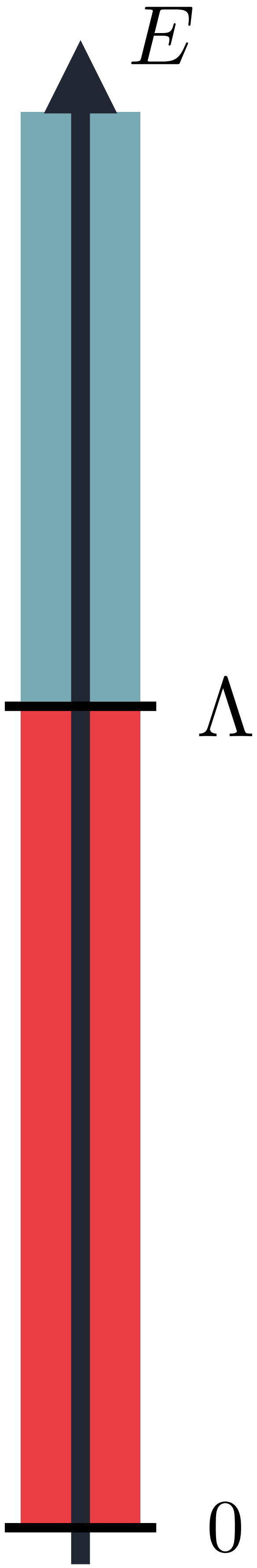

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$$S_{\text{grav}}^{\text{on-shell}}[g] = \int d^4x \sqrt{|g|}\, f_{\text{QFT}}(R \,|\, \Lambda, m)$$

The combined system is described by:

$$S_{\text{eff}}[g] = \int d^4x \sqrt{|g|}\, f(R \,|\, M_0, \lambda_0, \Lambda, m)$$

Results

Results: UV complete case (I)

The system of bare grav. theory and QFT is "UV-complete".

$\Lambda \to \infty$ Absorb divergent terms in renormalized quantities.

Equation for constant-curvature solutions:

$$M_{\rm ren}^2 R - 4M_{\rm ren}^2 \lambda_{\rm ren} + \langle T_\mu^{{\rm ren},\mu} \rangle = 0$$

$$\langle T_\mu^{{\rm ren},\mu} \rangle = -\frac{2}{\sqrt{|g|}} g^{\mu\nu} \frac{\delta}{\delta g^{\mu\nu}} S_{\rm grav}^{\text{on-shell},{\rm ren}}$$

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1.) CFT: $$\langle T_\mu^{{\rm ren},\mu} \rangle = -\frac{\tilde{a}}{48} R^2$$

2.) RG flow QFT: $$\langle T_\mu^{{\rm ren},\mu} \rangle = -\frac{\tilde{a}_{\rm UV}}{48} R^2 + (4-\Delta^{\rm UV}) m^{4-\Delta^{\rm UV}} \langle \mathcal{O} \rangle (R)$$

Results: UV complete case (I)

1.) CFT:

$$R = \frac{24}{\tilde{a}}\, M_{\rm ren}^2 \left(1 \pm \sqrt{1 - \frac{\tilde{a}}{3}\frac{\lambda_{\rm ren}}{M_{\rm ren}^2}} \right)$$

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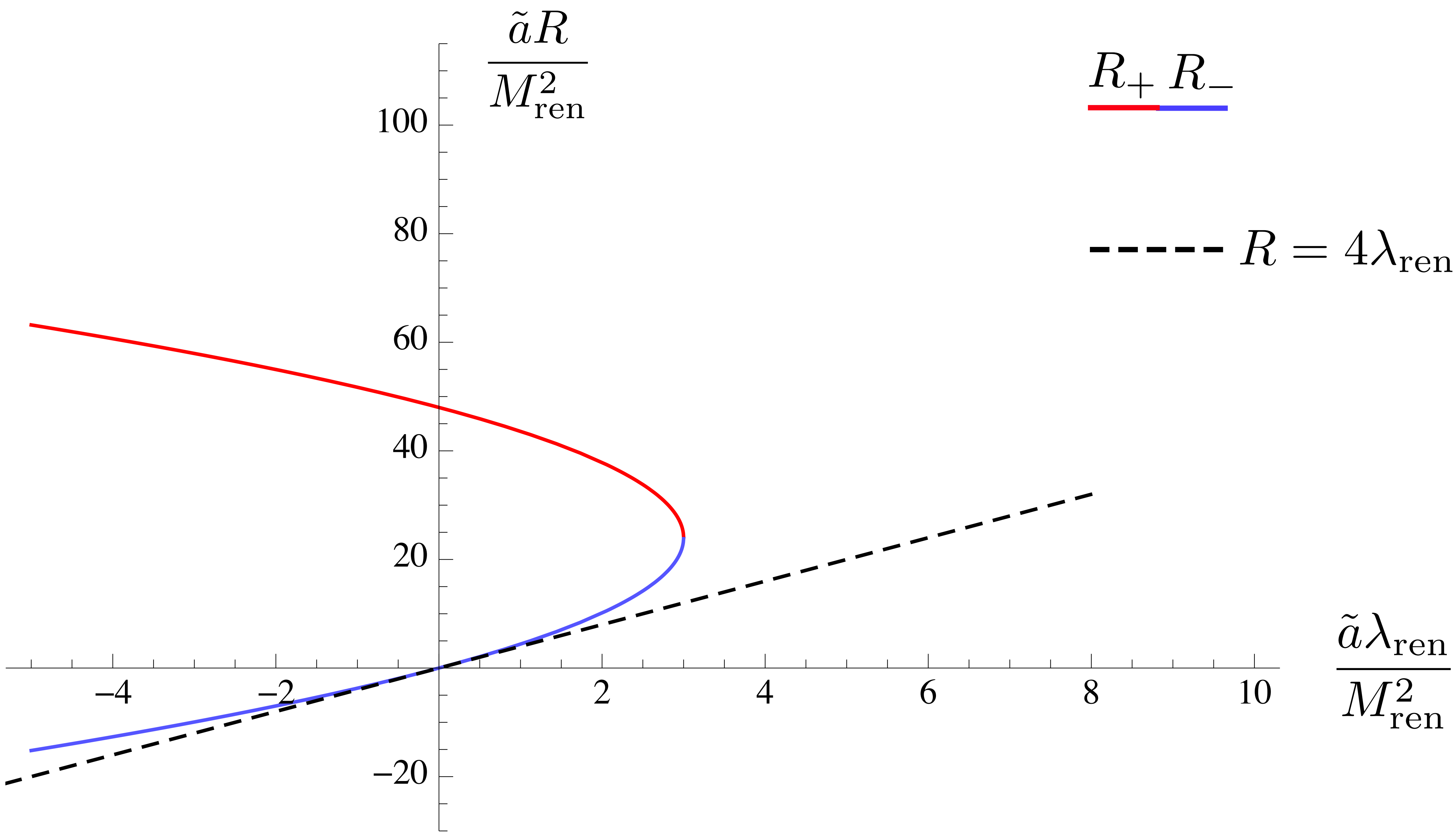

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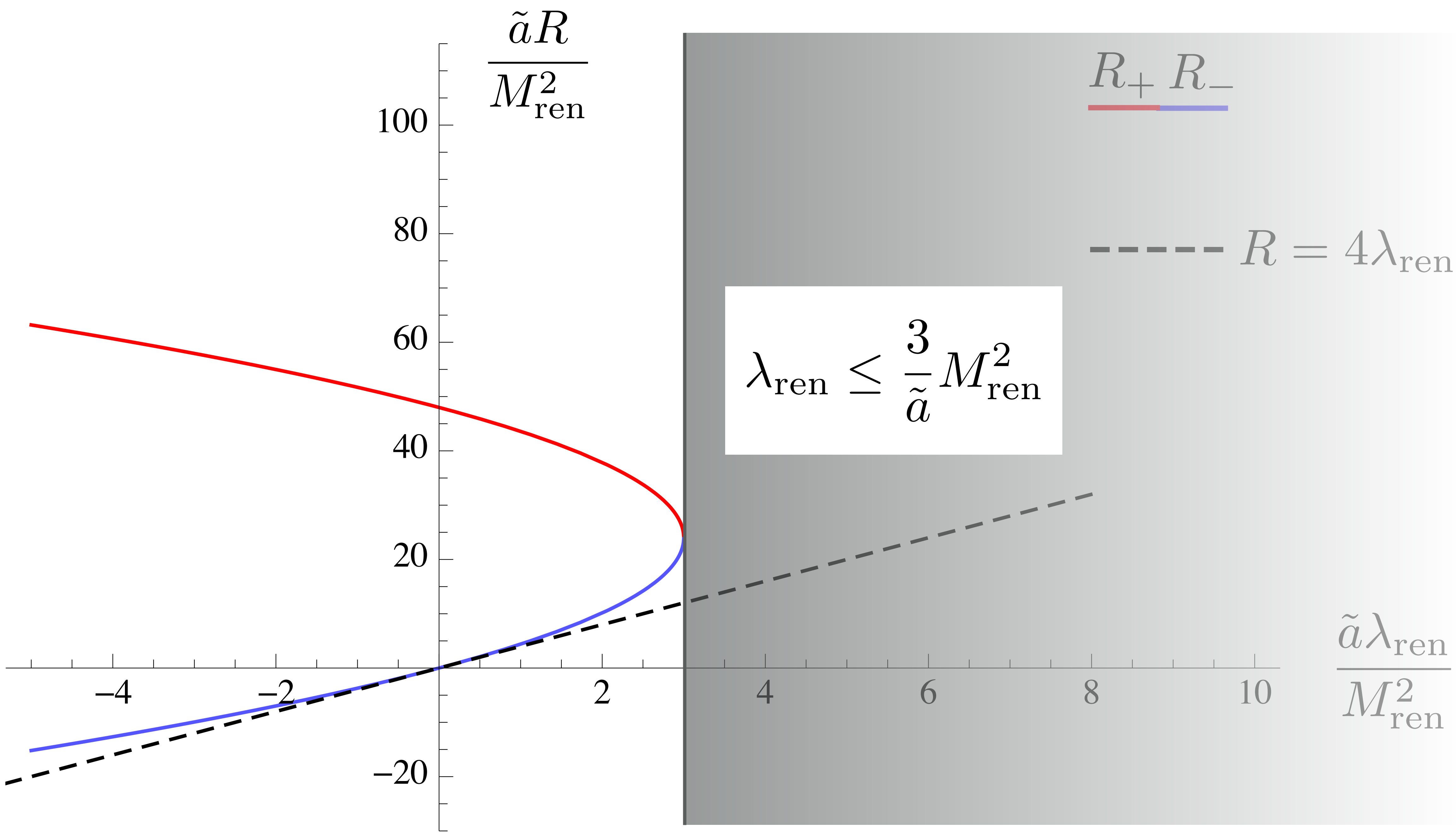

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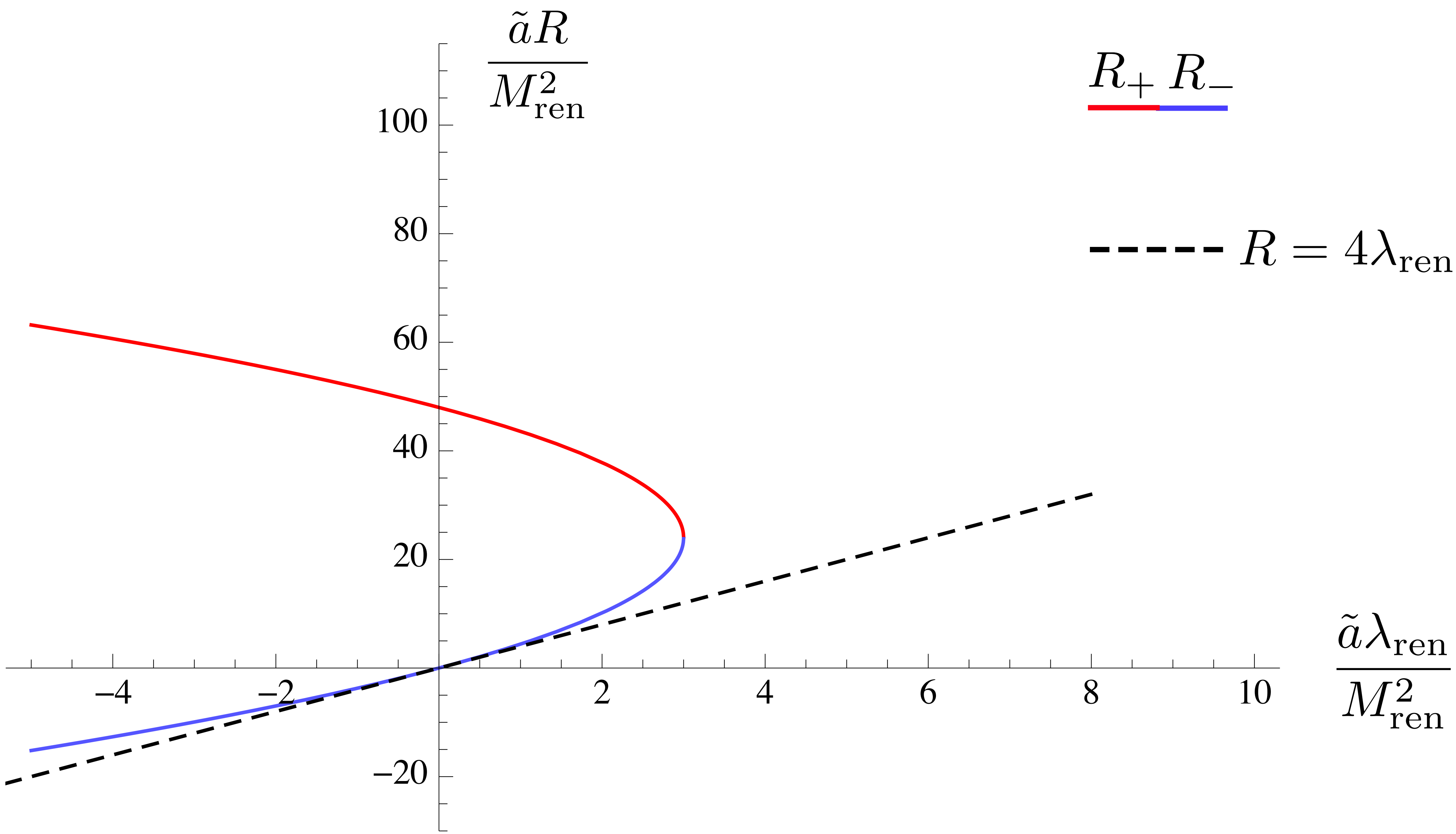

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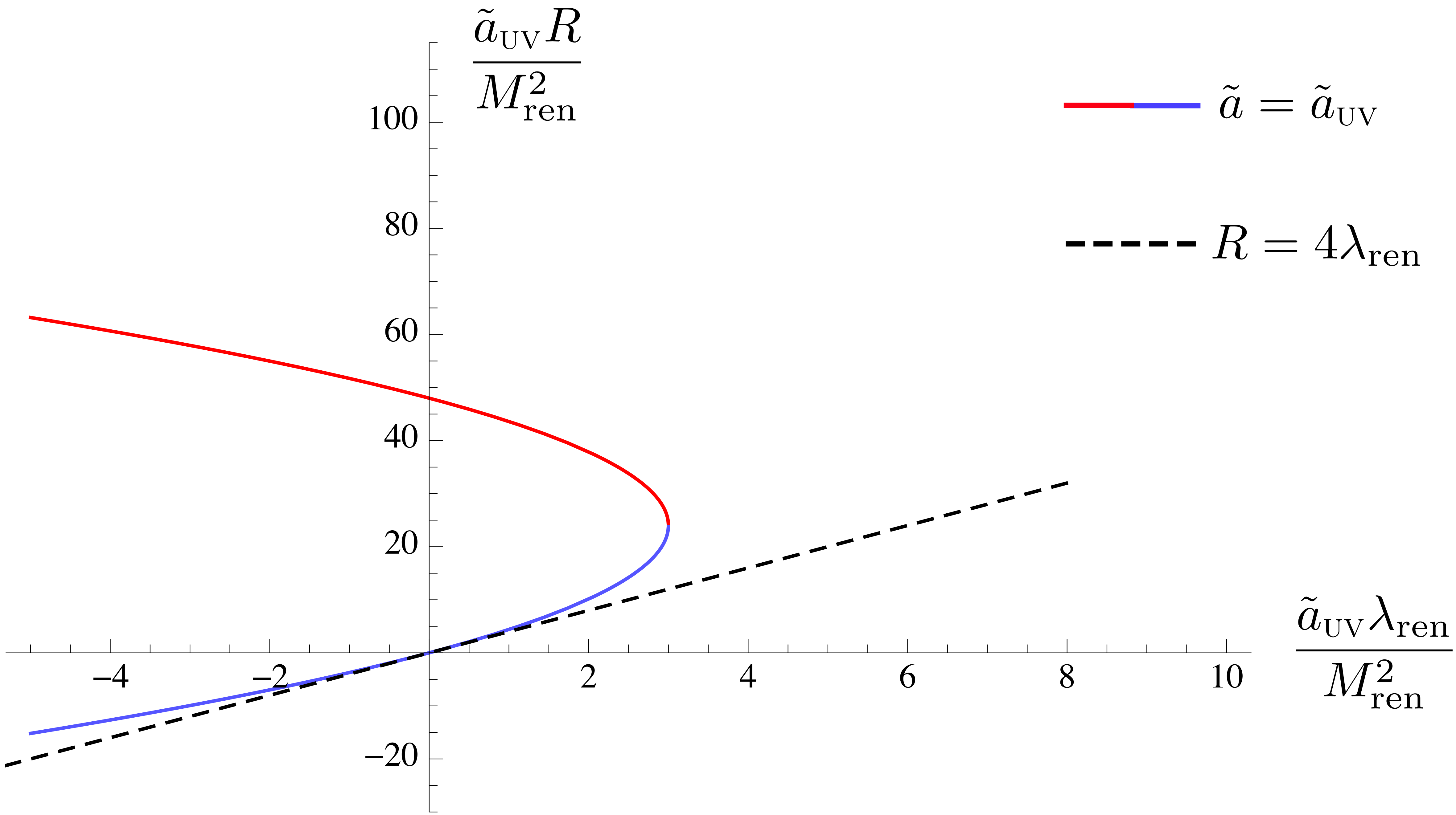

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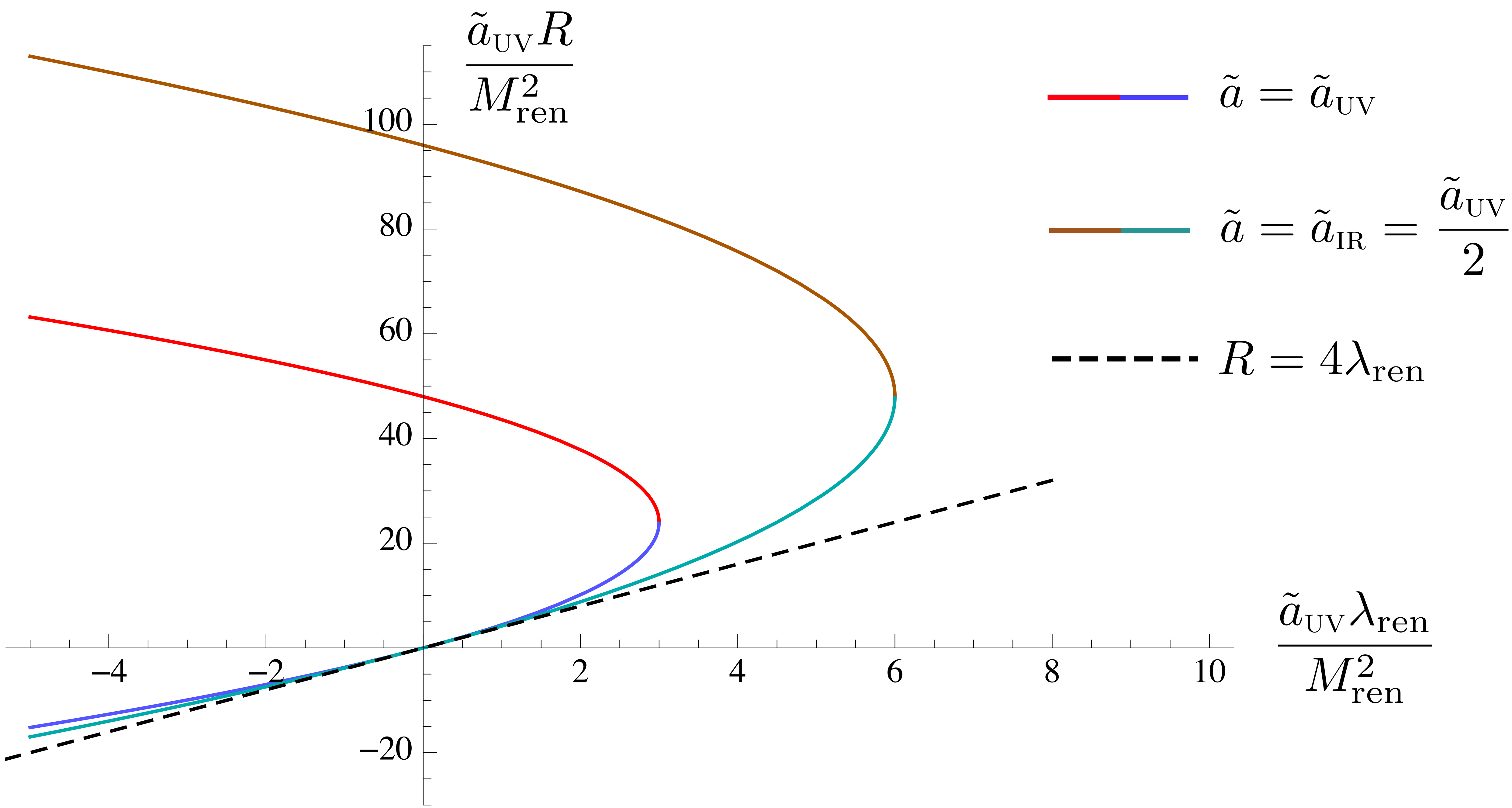
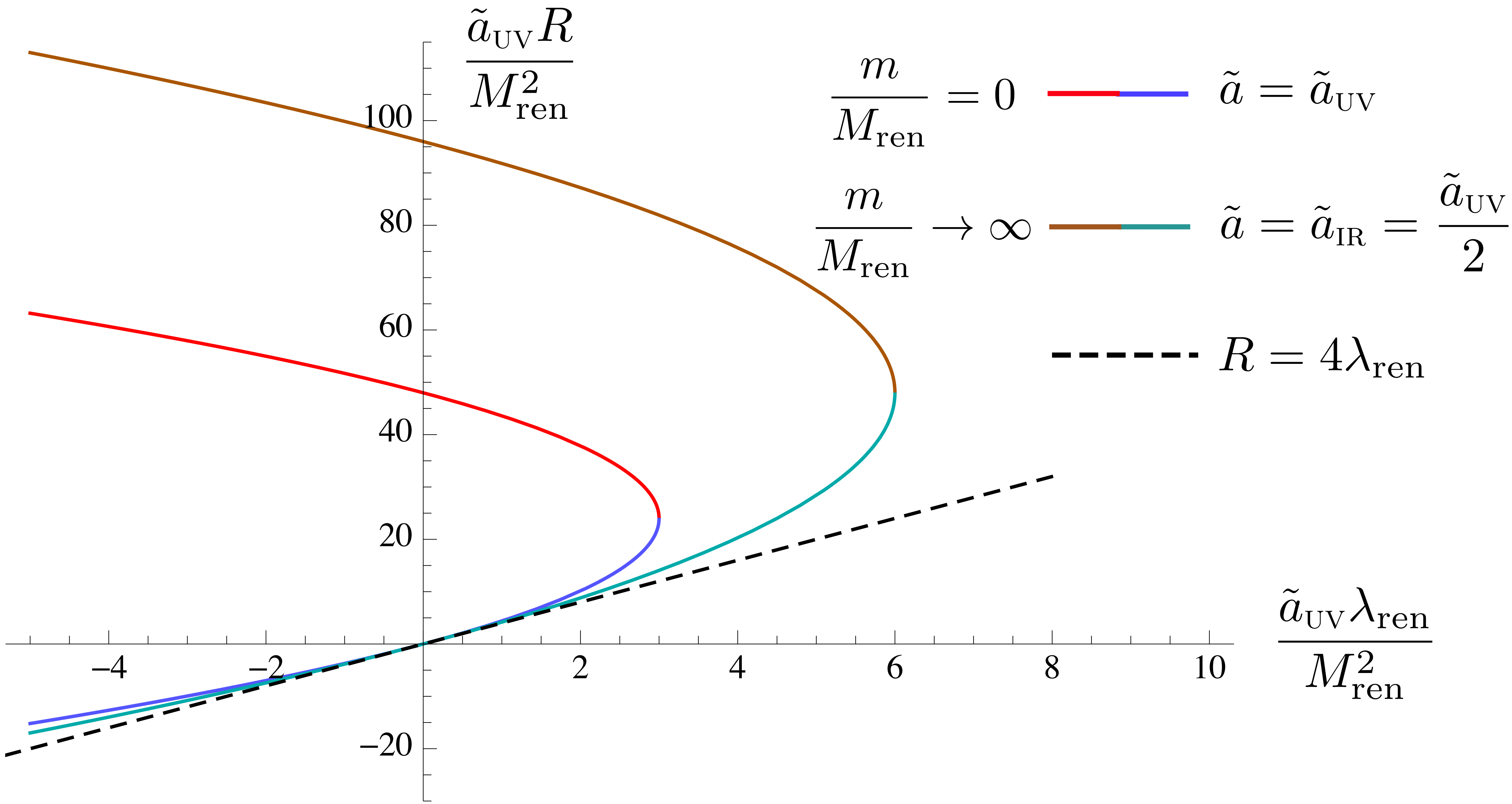


Results: UV complete case (I)

2.) RG flow: $M^2_{\rm ren} R - 4M^2_{\rm ren}\lambda_{\rm ren} - \frac{\tilde{a}_{\rm UV}}{48}R^2 + (4-\Delta^{\rm UV})m^{4-\Delta^{\rm UV}}\langle\mathcal{O}\rangle = 0$



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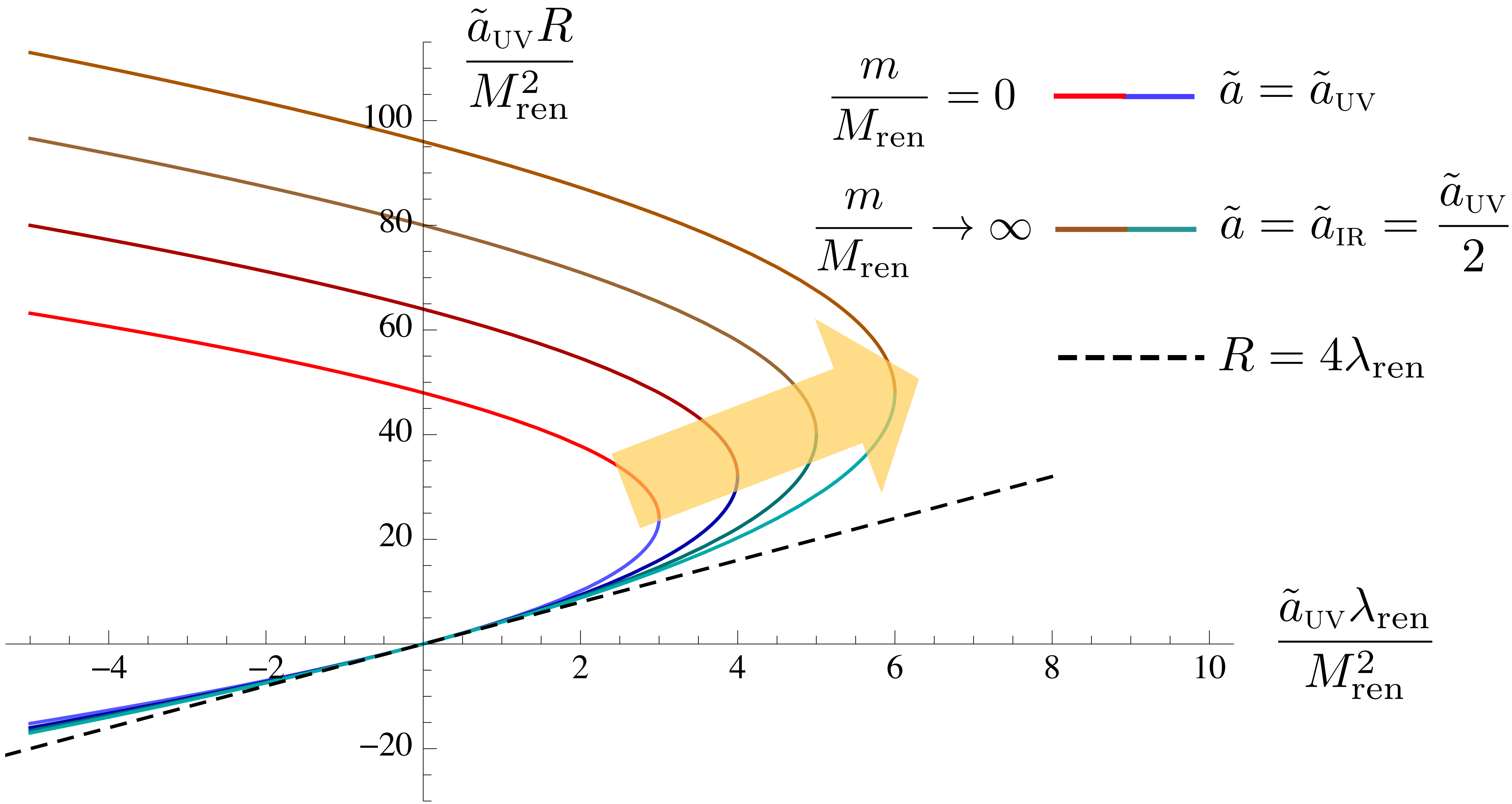
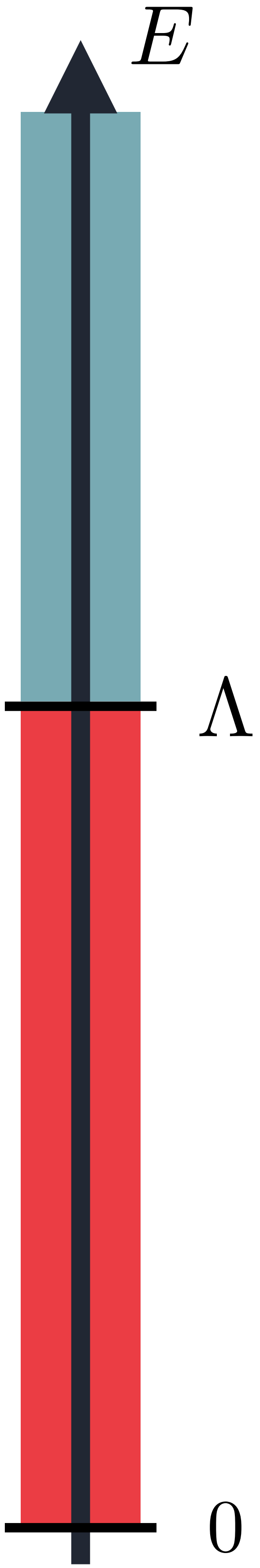


Results: cutoff QFT (2)



In **Case 2** it is assumed that S_0 is an effective theory at some scale Λ.

$$S_0[g] = \int d^4x \sqrt{|g|} \left[\frac{M_0^2}{2} R - M_0^2 \lambda_0 + a_0 R^2 \right]$$

Then couple a QFT with UV cutoff Λ to the background described by $g_{\mu\nu}$.

$$S_{\text{grav}}^{\text{on-shell}}[g] = \int d^4x \sqrt{|g|}\, f_{\text{QFT}}(R \,|\, \Lambda, m)$$

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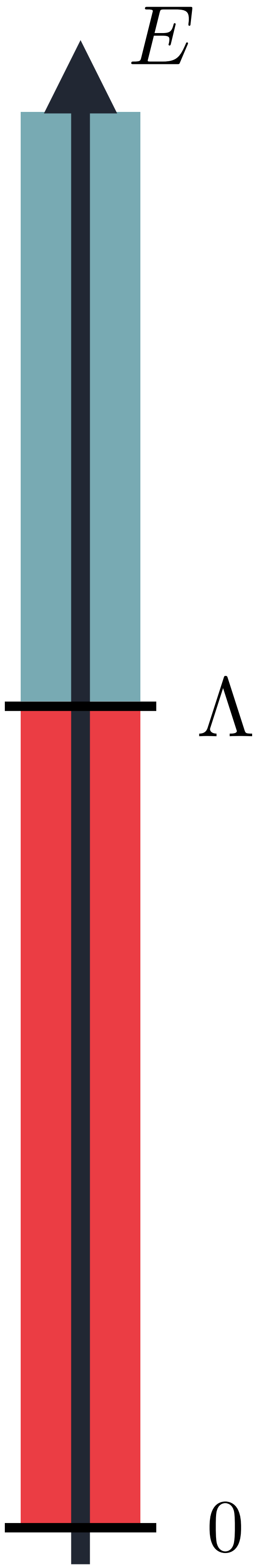

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For a CFT: $$f_{\text{CFT}} = \tilde{a} \left[6\Lambda^4 \sqrt{1 + \frac{R}{12\Lambda^2}} + \frac{R\Lambda^2}{4} \sqrt{1 + \frac{R}{12\Lambda^2}} + \frac{R^2}{48} \log\left(\sqrt{1 + \frac{12\Lambda^2}{R}} - \sqrt{\frac{12\Lambda^2}{R}} \right) \right]$$

Results: cutoff QFT (2)

Eq. for const.-curv. sol.: $M_0^2 R - 4M_0^2\lambda_0 + 24\tilde{a}\Lambda^4\sqrt{1+\frac{R}{12\Lambda^2}} = 0$

Solution: $R = 4\lambda_0 - 24\tilde{a}^2\frac{\Lambda^6}{M_0^4}\left(\sqrt{1+\frac{M_0^4}{\tilde{a}^2\Lambda^4}+\frac{M_0^4\lambda_0}{3\tilde{a}^2\Lambda^6}}-1\right)$

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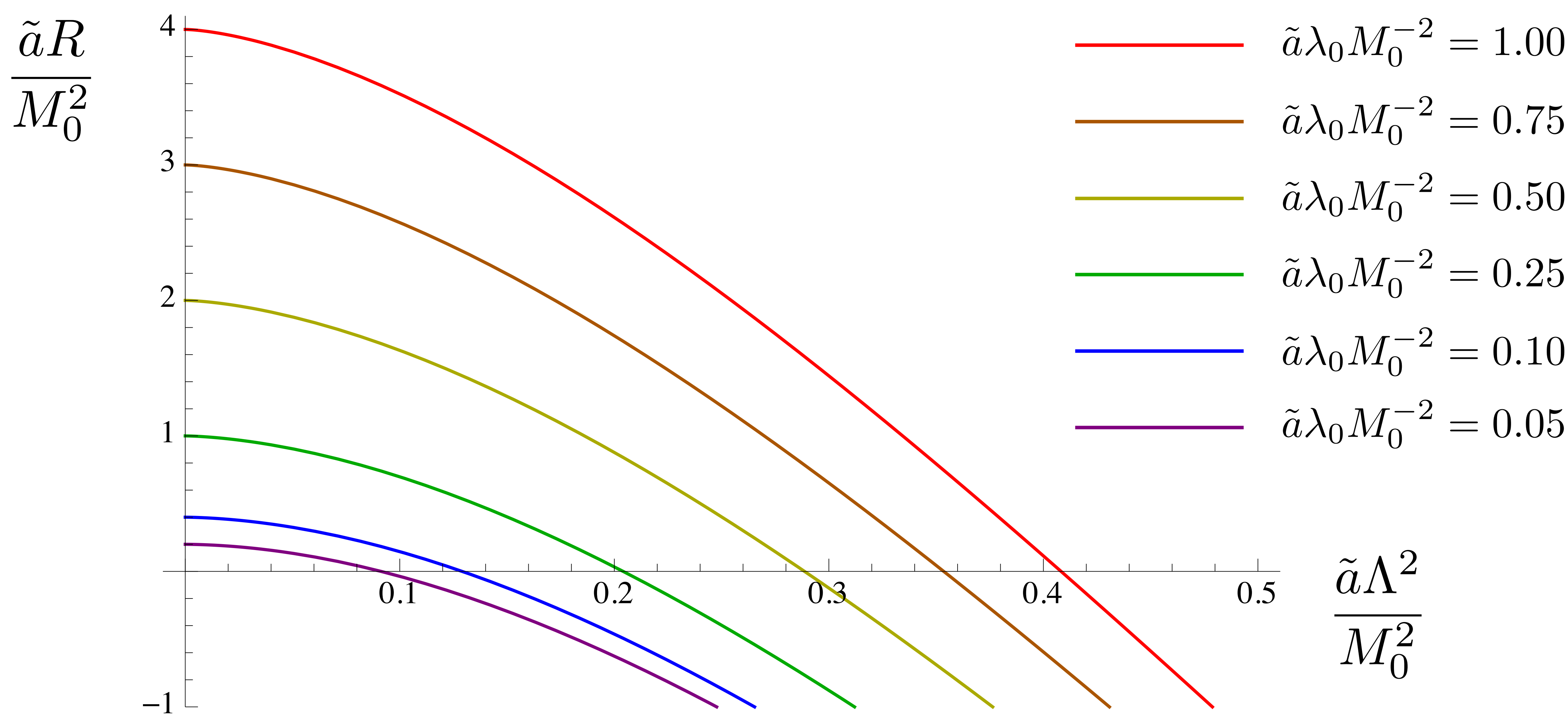

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- For $\Lambda = 0$ have $R(\Lambda = 0) = 4\lambda_0$.
- Increasing Λ always decreases R .
- For sufficiently large Λ the curvature R becomes negative.
- The (thermal) entropy of dS space scales as $S_{\rm th} \sim R^{-1}$. Increasing Λ thus increases $S_{\rm th}$ of dS which may be naively expected. Can this entropic argument be made precise?

Stability

Stability

$$S_{\text{eff}}[g] = \frac{1}{2\kappa}\int d^4x \sqrt{|g|}\, f(R)$$

Condition on **thermodynamic stability**: [Mazur, Mottola '86]

$$\frac{dS_E}{d\beta} < 0 \quad \Leftrightarrow \quad \frac{d^2 S_{\text{eff}}}{d\beta^2} < 0 \qquad \text{with} \quad \beta = \frac{1}{T} = \frac{2\sqrt{12}\pi}{\sqrt{R}}$$

For a $f(R)$-theory stability about a constant-R-solution implies:

$$f_R - R f_{RR} < 0$$

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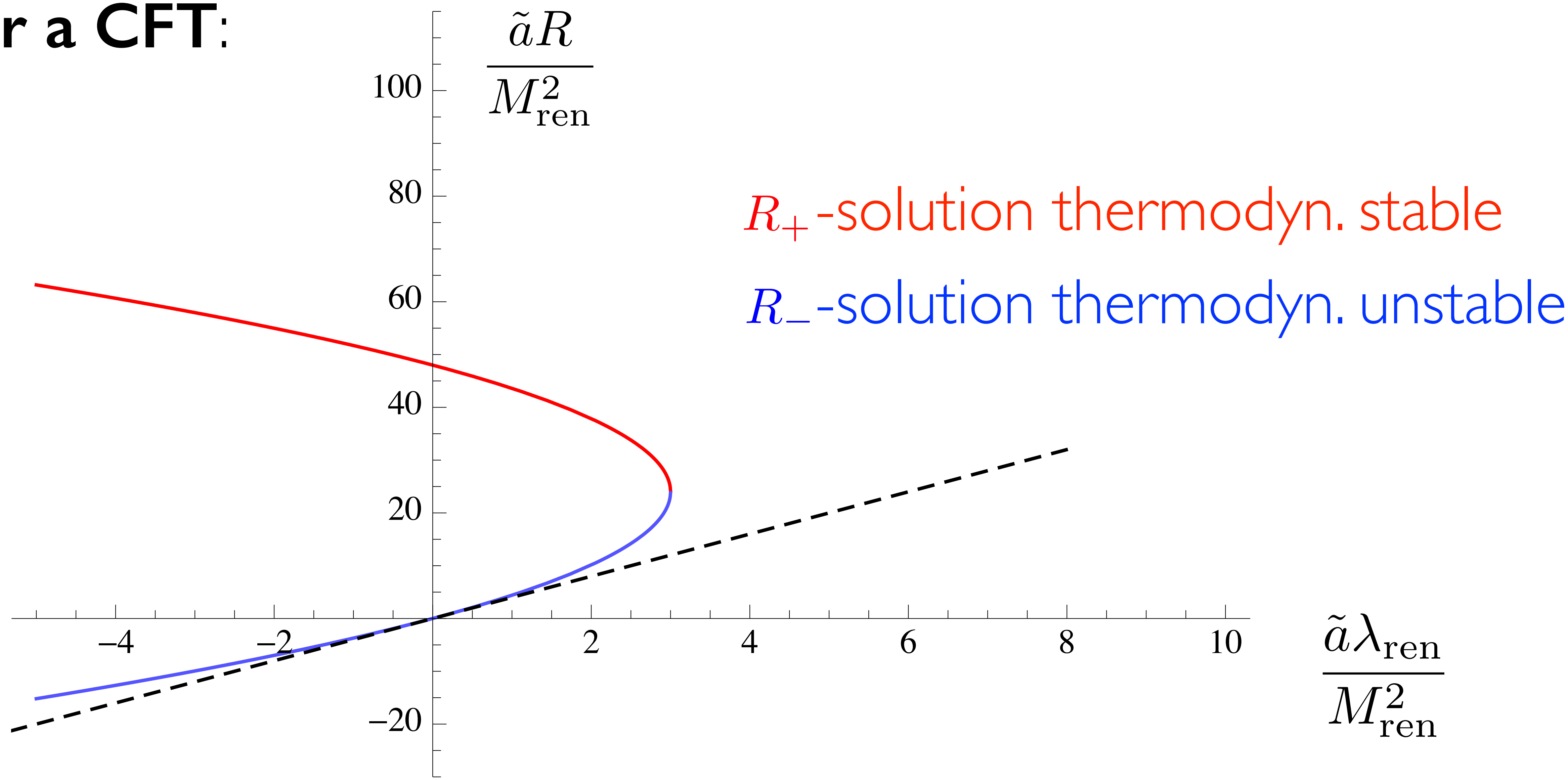

Stability

$$S_{\text{eff}}[g] = \frac{1}{2\kappa}\int d^4x\sqrt{|g|}\,f(R)$$

Note that a $f(R)$ -theory can be written as an **Einstein-dilaton theory with dilaton potential:**

$$S_{\text{eff}}[\tilde{g},\phi] = \int d^dx\,\sqrt{|\tilde{g}|}\,\left[\frac{1}{2\kappa}\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$$

$$V(\phi) = \frac{1}{2\kappa}\frac{Rf_R(R) - f(R)}{f_R^2(R)}$$

$$f_R(R) = \exp\left(\sqrt{\frac{2\kappa}{3}}\phi\right)$$

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Can define stability in terms of stability of the dilaton:

Maxima = unstable **Minima = stable**

Stability

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Note that a $f(R)$ -theory can be written as an **Einstein-dilaton theory with dilaton potential:**

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$$f_R(R) = \exp\left(\sqrt{\frac{2\kappa}{3}} \phi \right)$$

Can define stability in terms of stability of the dilaton:

$$\left. \frac{d^2V}{d\phi^2} \right|_{\text{extremum}} = \frac{1}{6\kappa} \frac{f_R - R f_{RR}}{f_R f_{RR}} > 0$$

for stability

Stability

$$S_{\text{eff}}[g] = \frac{1}{2\kappa}\int d^4x \sqrt{|g|}\, f(R)$$

Thermodynamic stability: $f_R - R f_{RR} < 0$

Dilaton stability: $\dfrac{f_R - R f_{RR}}{f_R f_{RR}} > 0$

For the graviton not to be a ghost require $f_R > 0$.

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The sign of f_{RR} will depend on the quadratic term $aR^2 \subset f(R)$.

Note that this term drops out from $f_R - Rf_{RR}$.

The coefficient a is constrained by the condition $f_R > 0$.

Stability

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For a CFT: for any fixed a find $f_{RR} < 0$ for $R \to 0$.

In the Minkowski limit **thermodyn. stability = dilation stability**

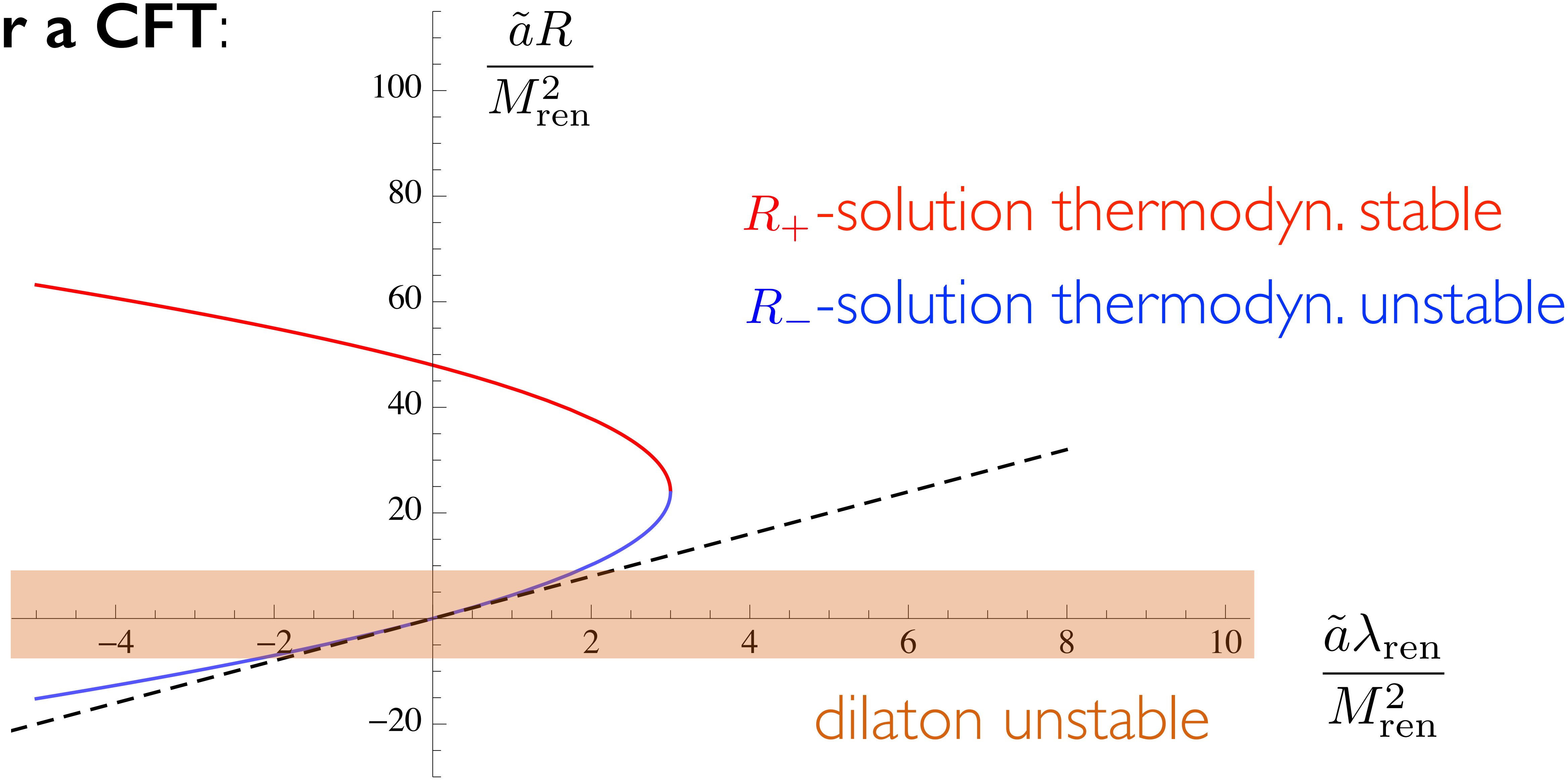
Stability

$$S_{\text{eff}}[g] = \frac{1}{2\kappa}\int d^4x\sqrt{|g|}\, f(R)$$

thermo. stab.: $f_R - R f_{RR} < 0$

dilaton stab.: $\dfrac{f_R - R f_{RR}}{f_R f_{RR}} > 0$

For a CFT:



Summary

0.) Advantages from holography:

- Integrating out a QFT via its gravity-dual is highly tractable.

1.) UV complete setting (case 1)

- **CFTs**: only have solution if $\lambda_{\mathrm{ren}} \leq \frac{3}{\tilde{a}} M_{\mathrm{ren}}^2$.
- **RG flow QFTs**: back-reaction effect interpolates between that of the UV CFT and the IR CFT.

2.) Stability

- Thermodyn. stability and stability in dilaton-formulation coincide for sufficiently small background curvature.
- These solutions are then **unstable** according to both criteria.

Open Questions

- Is it possible to develop a precise and quantitative **entropic** understanding of the back-reaction effect of a cutoff QFT?

- Are the solutions found stable under small **perturbations** that deform the geometry away from dS? To what extent can this question be addressed in the simplified setup considered here with $\nabla_\rho R_{\mu\nu} = 0$?

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Many thanks for your attention!