#### Light Propagation in Massive, Non-Linear, (SuSy) Standard-Model Extension theories

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- Context and motivations
- Non-linear and Massive theories (Born-Infeld, Heisenberg-Euler, de Broglie-Proca, Stueckelberg,...)
  - Results. Non-Linear (magnetars). Massive (photon mass upper limits from solar wind and FRBs).
- Standard-Model Extension and Lorentz(-Poincaré) Symmetry Violation.
  - Results: effective photon mass, dispersion, sub-super luminal velocities, birefringence, non-conservation
- Non-Linear: non-conservation.
- Applications to cosmology: LSV (and nL) Dark energy.

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#### Since 2016 Non-Maxwellian EM (before GR)

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- Physics at the end of the XIX century:
  - Laws of physics are valid anywhere and anytime.
  - 2 Galilei transformations (GT) hold.
  - S Michelson-Morley: light speed constancy, Maxwell equations hold.
- Conclusions: GT are invalid and replaced by Lorentz-Poincaré transformations (LPT), classical mechanics rewritten, æther does not exist, and light has to be reinterpreted.
- Physics at the end of the XX century:
  - Expansion is accelerating (questioned) and rotation curves.
  - I GR holds and works perfectly so far.
  - In the second second
- Two options: search more and better the dark universe or extend GR.
- Third **complementary** option: light has to be reinterpreted again. No pretension of completeness.

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# Motivations: 1/2

- GW detection 2015, but universe understanding based on EM observations.
- As photons are the main messengers, fundamental physics has a concern in testing the foundations of electromagnetism.
- 96% universe dark (unknown), only part of 4% is known: yet precision cosmology.
- Striking contrast: complex and multi-parameterised cosmology linear electromagnetism from the 19<sup>th</sup> century.
- There is no theoretical prejudice against a photon small mass, technically natural, in that all radiative corrections are proportional to mass ('t Hooft).
- Electromagnetic radiation has zero rest mass to propagate at c. Since it carries momentum and energy, it has non-zero inertial mass. Hence, for EP, it has non-zero gravitational mass: → light must be heavy ('t Hooft).
- The Einstein demonstration of the equivalence of mass and energy (wagon at rest on frictionless rails, photon shot *inside* end to end) implies a massive photon.

# Motivations: 2/2

- The photon is the only free massless particle of the Standard Model.
- The SM successful but shortcomings: Higgs is too light, neutrinos are massive, no gravitons...



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# Presenting the nM electromagnetism

- non-linear Born-Infeld (for renormalisation of singularities);
   Heisenberg-Euler (2<sup>nd</sup> order QED as photon splitting, merging, photon-photon interaction, birefringence) or massive (de Broglie-Proca).
- Massive photons evoked for dark matter, inflation, magnetic monopoles, red-shifts, superconductors and "light shining through walls" exp.
- The dBP theory is not gauge invariant, but others are (quantizable Stueckelberg theory presents a scalar compensating field. Boulware showed the renormalizability and unitarity of QED with a dBP photon). If mass rises from the spontaneous symmetry U(1) breaking, gauge invariance is insured also after breaking, possibly determined by the Higgs mechanism (but see Guendelman).
- For charge conservation (dBP Gauss law) the coupling of the photon mass to the scalar potential implies a density of "pseudo-charge" proportional to the squared mass, added to the ordinary charges. The two kinds of charges are conserved separately (but see Nussinov).
- Impact on relativity? Difficult answer: variety of the theories above; removal of ordinary landmarks and rising of interwoven implications (TLP and dBP).

### Non-linear theories: Born-Infeld 1/2

• The Born-Infeld Lagrangian

$$\mathcal{L} = \sqrt{1+F} - 1 + j^{\mu}A_{\mu} \tag{1}$$

The equations are

$$\partial_{\mu}\left(\frac{F^{\mu\nu}\left(1+F\right)^{-\frac{1}{2}}}{2}\right) = j^{\nu}$$
 (2)

- Electromagnetic field gives origin to the mass of the charge.
- Avoidance of infinities out of self-energy  $\phi(0) = 1.8541 \frac{e}{r_0}$ .
- The parameter r<sub>0</sub> is computed out of analytic expressions.

# Non-linear theories: Heisenberg-Euler 2/2

• The Heisenberg-Euler Lagrangian

$$\mathcal{L} = -\frac{F_{\mu\nu}F^{\mu\nu}}{4} + \frac{e^2}{\hbar c} \int_0^\infty d\eta \frac{e^{-\eta}}{\eta^3} \cdot \left\{ i\frac{\eta^2}{2} F^{\mu\nu}F^*_{\mu\nu} \cdot \frac{1}{2} \int_0^\infty d\eta \frac{e^{-\eta}}{\eta^3} \cdot \left\{ i\frac{\eta^2}{2} F^{\mu\nu}F^*_{\mu\nu} \right\} \right\}$$
$$\cdot \frac{\cos\left[\frac{\eta}{\mathfrak{E}_k}\sqrt{\frac{-F_{\mu\nu}F^{\mu\nu}}{2} + iF^{\mu\nu}F^*_{\mu\nu}}\right] + \cos\left[\frac{\eta}{\mathfrak{E}_k}\sqrt{\frac{-F_{\mu\nu}F^{\mu\nu}}{2} - iF^{\mu\nu}F^*_{\mu\nu}}\right]}{\cos\left[\frac{\eta}{\mathfrak{E}_k}\sqrt{\frac{-F_{\mu\nu}F^{\mu\nu}}{2} + iF^{\mu\nu}F^*_{\mu\nu}}\right] - \cos\left[\frac{\eta}{\mathfrak{E}_k}\sqrt{\frac{-F_{\mu\nu}F^{\mu\nu}}{2} - iF^{\mu\nu}F^*_{\mu\nu}}\right]} + |\mathfrak{E}_k|^2 + \frac{\eta^3}{6} \cdot F_{\mu\nu}F^{\mu\nu}\right\}$$
(3)

$$F^*_{\mu\nu} = \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} \tag{4}$$

- Photon-Photon interaction and Photon splitting since HE theory relates to second order QED.
- Vacuum polarisation occurs for  $E_c > 1.3 \times 10^{18}$  V/m or  $B_c > 4.4 \times 10^{13}$  G.

#### Non-linear theories: Magnetar

Heisenberg-Euler on magnetars overcritical magnetic field. Blue or red shift depending on polarisation for a photon emitted up to similar values to the gravitational redshift.



Fig.1. EMS (Electromagnetic shift) of the two photon polarisations versus the ratio of the magnetic/overcritical fields (upper panel), and the azimuthal angle (lower panel). The EMS can reach comparable values to the gravitational Einstein shift. The figure is taken from [Bonetti, Perez Bergliaffa, Spallicci, 2016].

- The concept of a massive photon has been vigorously pursued by Louis de Broglie from 1922 throughout his life. Through dispersions in 1923 he defines the value of the mass to be lower than 10<sup>-53</sup> kg (PDG value 10<sup>-54</sup> after many experiments and observations). In 1936 he writes the modified Maxwells equations in a non-covariant form.
- Insted, the original aim of Alexandru Proca, de Broglie's student, was the description of electrons and positrons. Despite Proca's several assertions on the photons being massless, his work has been used.

#### Massive theories: de Broglie-Proca 2/3

$$\mathcal{L} = -\frac{1}{4\mu} F_{\alpha\beta} F^{\alpha\beta} - \frac{\mathcal{M}^2}{2\mu} A_{\alpha} A^{\alpha} - j^{\alpha} A_{\alpha}$$
<sup>(5)</sup>

 $F_{\mu\nu} = \partial_{\mu}A^{\nu} - \partial_{\nu}A^{\mu}$ . Minimal action (Euler-Lagrange)  $\rightarrow$  inhomogeneous eqs. Ricci Curbastro-Bianchi identity  $\partial^{\lambda}F^{\mu\nu} + \partial^{\nu}F^{\lambda\mu} + \partial^{\mu}F^{\nu\lambda} = 0 \rightarrow$  homogeneous eqs.

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} - \mathcal{M}^2 \phi , \qquad (6)$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} - \mathcal{M}^2 \vec{A} , \qquad (7)$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} , \qquad (8)$$

$$\nabla \cdot \vec{B} = 0 , \qquad (9)$$

 $\epsilon_0$  permittivity,  $\mu_0$  permeability,  $\rho$  charge density,  $\vec{j}$  current,  $\phi$  and  $\vec{A}$  potential.  $\mathcal{M} = m_{\gamma}c/\hbar = 2\pi/\lambda$ ,  $\hbar$  reduced Planck (or Dirac) constant, c speed of light,  $\lambda$ Compton wavelength,  $m_{\gamma}$  photon mass.

Eqs. (6, 7) are Lorentz-Poincaré transformation but not Lorenz gauge invariant, though in static regime they are not coupled through the potential.

# Massive theories: de Broglie/Proca 3/3

From the Lagrangian we get  $\partial_{\alpha}F^{\alpha\beta} + \mathcal{M}^2A^{\beta} = \mu j^{\beta}$ . With the Lorentz subsidiary condition  $\partial_{\gamma}A^{\gamma} = 0$ ,

$$\left[\partial_{\mu}\partial^{\mu} + \mathcal{M}^{2}\right]A^{\nu} = 0 \tag{10}$$

Through Fourier transform, at high frequencies (photon rest energy < the total energy;  $\nu \gg 1$  Hz), the positive difference in velocity for two different frequencies ( $\nu_2 > \nu_1$ ) is

$$\Delta v_{g} = v_{g2} - v_{g1} = \frac{c^{3} \mathcal{M}^{2}}{8\pi^{2}} \left(\frac{1}{\nu_{1}^{2}} - \frac{1}{\nu_{2}^{2}}\right) , \qquad (11)$$

being  $v_g$  the group velocity. For a single source at distance d, the difference in the time of arrival of the two photons is

$$\Delta t = \frac{d}{v_{g1}} - \frac{d}{v_{g2}} \simeq \frac{\Delta v_g d}{c^2} = \frac{dc\mathcal{M}^2}{8\pi^2} \left(\frac{1}{\nu_1^2} - \frac{1}{\nu_2^2}\right)$$
$$\simeq \frac{d}{c} \left(\frac{1}{\nu_1^2} - \frac{1}{\nu_2^2}\right) 10^{100} m_{\gamma}^2 . \tag{12}$$

#### Experimental mass limits: Particle Data Group

Citation: M. Tanalashi et al. (Particle Data Group), Phys. Rev. D 988, 030001 (2012)

$$\gamma$$
 (photon)  $l(J^{PC}) = 0.1(1^{--})$ 

#### 7 MASS

Results prior to 2008 are critiqued in GOLDHABER 10. All experimental results published prior to 2005 are summarized in detail by TU 05.

The following conversions are useful:  $1~eV=1.783\times 10^{-33}~g=1.957\times 10^{-6}~m_{\mu}, X_{C}=(1.973\times 10^{-7}~m)\times (1~eV/m_{\gamma}).$ 

VALUE (VV) Q		DOCUMENT (E)		COMMENT.			
<1 × 10 <sup>-18</sup>	m the full	I RYUTOV twing data for avera	07 grs. 1	MHD of solar wind fits. limits, etc. • • •			
<2.2 × 10 <sup>-14</sup>		<sup>2</sup> BONETTI	17	Fast Radio Barsts, FRB 121102			
<1.8 = 10 <sup>-14</sup>		BONETTI	10	Fast Radio Buesta, FRB 150418			
<1.9 = 10-15	_	<sup>4</sup> RETINO	16	Amuere's Law in solar wind			
$<2.3 \pm 10^{-9}$	- 05	<sup>5</sup> ECOROV	14	Lensed quasar position			
		<sup>6</sup> ACCIOEY	10	Anorralian reage: mom-			
<1 = 10~20		7 ADELBERGER	07A	Proca galactic field			
so limit feasible		ADELBERGER	07A	y as Higgs particle			
<1 × 12 19		# TV	16	Tarque on rotating magnetized toroid			
$<1.4 = 10^{-7}$		ACCIOLY	64	Dispersion of GHz radio waves by			
<2 × 10 <sup>-16</sup>		<sup>9</sup> FULLEKRUG	84	Speed of 5-50 Hz radiation is at-			
<7 = 10-19		10100	03	Tomps on rotating magnetized			
<1 = 10"-17		11 LAKES	98.	Torque on toroid balance			
<6 × 10-17		12 RYUTOV	97	MHD of solar wind			
<8 = 10-16	90	13 FISCHBACH	94	Earth magnetic field			
<5 × 10-13		14 CHERNIKOV	92	Ampere's Law null test			
<1.5 × 10 <sup>-9</sup>	90	15 RYAN	85	Coulomb's Law null test			
<3 = 10-27		16 CHIBISOV	76	Galactic magnetic field			
-10-10	99.7	17 DAVIS	75	Jupiter's magnetic field			
<7.3 = 10-16		HOLLWEG	74	Alfven waves			
<6 = 10-17		18 FEANKEN	71	Low freq. res. circuit			
<2.4 × 10-13		19 KROLL	714	Dispersion in atmosphere			
<1 > 10-14		20 WILLIAMS	71	Tests Coulomb's Law			
<2.3 × 10-15		GOLDHABER	60	Satellite data			

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# Experimental mass limits: warnings

- Quote "Quoted photon-mass limits have at times been overly optimistic in the strengths of their characterisations. This is perhaps due to the temptation to assert too strongly something one knows to be true. A look at the summary of the Particle Data Group (Amsler et al.. 2008) hints at this. In such a spirit, we give here our understanding of both secure and speculative mass limits." Goldhaber and Nieto, Rev. Mod. Phys., 2000
- The lowest theoretical limit on the measurement of any mass is dictated by the Heisenberg's principle  $m \geq \hbar/2\Delta tc^2$ , and gives  $1.35 \times 10^{-69}$  kg, where  $\Delta t$  is the supposed age of the Universe.
- Photon mass reproduces plasma dispersion for the frequency  $f^{-2}$  dependence of the group velocity. There is not the possibility to disentangle the two effects, unless a different z dependence.

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#### Experimental mass limits: Cluster





- Highly elliptical evolving orbits in tetrahedron: perigee 4  $R_{\oplus}$  apogee 19.6  $R_{\oplus}$ , visited a wide set of magnetospheric regions. Inter-spacecraft separation ranging from  $10^2$  to  $10^4$  km.
- Small mass → precise experiment or very large apparatus (Compton wavelength). The largest-scale magnetic field accessible to *in situ* spacecraft measurements, *i.e.* the interplanetary magnetic field carried by the solar wind.

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# Experimental mass limits: Cluster

•  $j_P = 1.86 \cdot 10^{-7} \pm 3 \cdot 10^{-8} \text{ A m}^{-2}$ , while  $j_B = |\nabla \times \vec{B}|/\mu_0$  is  $3.5 \pm 4.7 \cdot 10^{-11} \text{ A m}^{-2}$ .  $A_H$  is an estimate, not a measurement.

$$A_{H}^{\frac{1}{2}}(m_{\gamma} + \Delta m_{\gamma}) = A_{H}^{\frac{1}{2}}\left(m_{\gamma} + \left|\frac{\partial m_{\gamma}}{\partial j_{P}}\right|\Delta j_{P} + \left|\frac{\partial m_{\gamma}}{\partial j_{B}}\right|\Delta j_{B}\right) = k\left[(j_{P} - j_{B})^{\frac{1}{2}} + \frac{\Delta j_{P} + \Delta j_{B}}{2(j_{P} - j_{B})^{\frac{1}{2}}}\right].$$
(13)

Considering  $j_P$  and  $\Delta j_P$  of the same order,  $j_P = 0.62 \ \Delta j_P$ , and both much larger than  $j_B$  and  $\Delta j_B$ , Eq. (13), after squaring, leads to

$$A_{H}^{\frac{1}{2}}(m_{\gamma} + \Delta m_{\gamma}) \sim k (j_{P} + \Delta j_{P})^{1/2}$$
 (14)

Table: The values of  $m_{\gamma}$  (according to the estimate on  $A_H$ ).

$m_{\gamma} \; [ m kg] \;   \; 1.4  imes 10^{-49} \;   \; 1.6  imes 10^{-50} \;   \; 3.4  imes 10^{-51} \; ]$	<i>А<sub>Н</sub></i> [Т m]	0.4	29 (Z)	637	
	$m_\gamma~[{ m kg}]$	$1.4 imes10^{-49}$	$1.6 imes10^{-50}$	$3.4  imes 10^{-51}$	

# Experimental mass limits: Cluster

- The particle current density  $\vec{j} = \vec{j}_P = ne(\vec{v}_i \vec{v}_e)$  from ion and electron currents; *n* is the number density, *e* the electron charge and  $\vec{v}_i$ ,  $\vec{v}_e$  the velocity of the ions and electrons, respectively.
- An accurate assessment of the particle current density in the solar wind is difficult due to inherent instrument limitations.
- $j_P >> j_B$  (up to four orders of magnitude), mostly due to the differences in the i, e velocities, while the estimate of density is reasonable. While we can't exclude that this difference is due to the dBP massive photon, the large uncertainties related to particle measurements hint to instrumental limits.



Alessandro D.A.M. Spallicci 24 September 2020, Institut d'Astrophysique de Paris 18/40

# Non-conservation: difference Maxwell dBP

The dBP equations of motion

$$\partial_{\alpha} F_{\rm T}^{\alpha\beta} + \mathcal{M}^2 A_{\rm T}^{\beta} = \mu_0 j^{\beta} , \qquad (15)$$

Splitting the EM tensor field and the EM 4-potential in the background (capital letters) and photon (small letters) contributions, we have

$$A_{\rm T}^{\beta} = A^{\beta} + a^{\beta} \qquad \qquad F_{\rm T}^{\alpha\beta} = F^{\alpha\beta} + f^{\alpha\beta} , \qquad (16)$$

which replaced in Eq. (15) provide

$$\partial_{\alpha} f^{\alpha\beta} + \mathcal{M}^2 a^{\beta} = \mu_0 j^{\beta} - \partial_{\alpha} F^{\alpha\beta} - \mathcal{M}^2 A^{\beta} .$$
(17)

The dBP photon interacts with the background through the potential even when the background field is constant. Indeed, if a field is constant, its associated potential is not  $F^{\alpha\beta} = \partial^{\alpha}A^{\beta} - \partial^{\beta}A^{\alpha}$ 

Conversely, this is not the case for the Maxwell photon, Eq. (18), that interacts only with a non-constant field

$$\partial_{\alpha} f^{\alpha\beta} = \mu_0 j^{\beta} - \partial_{\alpha} F^{\alpha\beta} . \tag{18}$$

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The energy-momentum density tensor  $\theta^{lpha}_{\ au}$  [Jm<sup>-3</sup>] for the dBP photon is

$$\theta^{\alpha}_{\ \tau} = \frac{1}{\mu_0} \left[ f^{\alpha\beta} f_{\beta\tau} + \mathcal{M}^2 \mathbf{a}^{\alpha} \mathbf{a}_{\tau} + \mathcal{M}^2 \mathbf{A}^{\alpha} \mathbf{a}_{\tau} + \delta^{\alpha}_{\tau} \left( \frac{1}{4} f^2 - \frac{1}{2} \mathcal{M}^2 \mathbf{a}^2 - \mathcal{M}^2 \mathbf{A}^{\beta} \mathbf{a}_{\beta} \right) \right] . \tag{19}$$

The energy-momentum density tensor variation  $\partial_{\alpha} \theta^{\alpha}_{\ \tau}$  [Jm<sup>-4</sup>] is given by

$$\partial_{\alpha}\theta^{\alpha}_{\tau} = \underbrace{j^{\alpha}_{\mu_{0}} f_{\alpha\tau} - \frac{1}{\mu_{0}} (\partial_{\alpha}F^{\alpha\beta}) f_{\beta\tau}}_{\text{Maxwellian terms}} + \underbrace{\frac{1}{\mu_{0}} \mathcal{M}^{2} (\partial_{\tau}A^{\beta}) \mathbf{a}_{\beta}}_{\text{de Broglie-Proca term}} .$$
(20)

In conclusion, the energy-momentum density tensor of the dBP photon is not conserved. On top of the Maxwellian terms, the mass couples with the background potential time-derivative.

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# (SuSy and) LoSy breaking

Planck Scale	10 <sup>19</sup> GeV	CPT-even	CPT-odd
LoSy Violation	10 <sup>17</sup> -10 <sup>19</sup> GeV	Class 3	Class 1
GUT	10 <sup>36</sup> GeV	Class 3	Class 1
SuSy Breaking	(0 <sup>11</sup> -10 <sup>10</sup> GeV	Classe 3 or 4	Class 1 or 2
		k <sub>e</sub> model	CFJ model

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# LoSy breaking

- Four models involving (Super and) Lorentz symmetries breaking. Dispersion relations show a non-Maxwellian behaviour for CPT even and odd sectors. Birefringence.
- An effective mass photon behaviour for both odd and pair CPT. In the odd CPT classes,  $f^{-2}$  in the group velocities emerges.
- A massive and gauge invariant Carroll-Field-Jackiw term in the Lagrangian is extracted and shown to be proportional to the background vector (or tensor).
- Caution in differentiating an effective from a real mass: Higgs for charged leptons and quarks, the W and Z Bosons, while the Chiral Symmetry (Dynamical) Breaking (CSB) for (mostly) composite hadrons (baryons and mesons). Is it epistemologically legitimate to consider such mechanisms as producing an effective mass to massless particles. What is real or effective?
- Frame dependency renders the LSV mass unusual, but acceptable being the dimension indeed that of a mass.
- The effective mass upper value is compatible with experimental data and

The Lagrangian  $L_1$  reads

$$\mathcal{L}_{1} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - \frac{1}{2} \epsilon^{\mu\nu\sigma\rho} k^{\mathrm{AF}}_{\mu} A_{\nu} F_{\sigma\rho} . \qquad (21)$$

where  $F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$  and  $F^{\mu\nu} = \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$  are the covariant and contravariant forms, respectively, of the EM tensor;  $\epsilon^{\mu\nu\sigma\rho}$  is the contravariant form of the Levi-Civita pseudo-tensor, and  $A_{\mu}$  the potential covariant four-vector. We observe the coupling between the EM field and the breaking vector  $k_{\alpha}^{AF}$ .

The Lagrangian  $L_3$  reads

$$L_3 = (k_F)_{\mu\nu\alpha\beta} F^{\mu\nu} F^{\alpha\beta} . \qquad (22)$$

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- The SME-LSV factors:  $k_{\alpha}^{AF}$  [metre <sup>-1</sup>] 4-vector (CPT odd);  $k_{F}^{\alpha\nu\rho\sigma}$  [dimensionless] tensor (CPT even).

-  $k_{\alpha}^{AF}$  vector coming from the Carroll-Field-Jackiw Lagrangian induces an effective mass. The  $k_{F}^{\alpha\nu\rho\sigma}$  tensor induces a mass only in a supersymmetrised context after photino integration.

- The LSV tensor does not violate CPT conversely to the LSV vector (the frequency LSV shift is an observable of CPT violation.

Indicating with the symbol \* the dual field, the photon energy-momentum density tensor  $\theta^{\alpha}_{~\tau}~[\rm Jm^{-3}]$  is

$$\theta^{\alpha}_{\ \tau} = \frac{1}{\mu_0} \left( f^{\alpha\nu} f_{\nu\tau} + \frac{1}{4} \delta^{\alpha}_{\tau} f^2 - \frac{1}{2} k^{\rm AF}_{\tau} {}^* f^{\alpha\nu} a_{\nu} + k^{\alpha\nu\kappa\lambda}_{\rm F} f_{\kappa\lambda} f_{\nu\tau} + \frac{1}{4} \delta^{\alpha}_{\tau} k^{\kappa\lambda\nu\beta}_{\rm F} f_{\kappa\lambda} f_{\nu\beta} \right). \tag{23}$$

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# Non-conservation: SME

The energy-momentum density tensor variation  $\partial_{\alpha}\theta^{\alpha}_{\ \tau}$  [Jm<sup>-4</sup>] is given by

$$\partial_{\alpha}\theta^{\alpha}_{\tau} = \underbrace{j^{\nu}f_{\nu\tau} - \frac{1}{\mu_{0}}\left(\partial_{\alpha}F^{\alpha\nu}\right)f_{\nu\tau}}_{\text{Maxwellian terms}} - \frac{1}{\mu_{0}}\left[\underbrace{\frac{1}{2}\left(\partial_{\alpha}k_{\tau}^{\text{AF}}\right)^{*}f^{\alpha\nu}a_{\nu} - \frac{1}{4}\left(\partial_{\tau}k_{\text{F}}^{\alpha\nu\kappa\lambda}\right)f_{\alpha\nu}f_{\kappa\lambda}}_{\text{EM background independent terms}} + \underbrace{\partial_{\alpha}\left(k_{\text{F}}^{\alpha\nu\kappa\lambda}F_{\kappa\lambda}\right)f_{\nu\tau}}_{\text{non-constant term}} + \underbrace{k_{\alpha}^{\text{AF}*}F^{\alpha\nu}f_{\nu\tau}}_{\text{constant term}}\right].$$
(24)

• Maxwellian, LSV independent, terms.

Three massive contributions (though not all of components are mass dependent).

- EM background independent terms for which non-conservation if the LSV fields are space-time dependent. This is really a distinctive term of the SME.
- LSV and EM space-time dependent terms.
- A constant term (constant EM background and a constant k<sub>AF</sub>) coming solely from the CPT-odd handedness. Its action entails a non-constant potential. Indeed, there is an explicit x<sup>α</sup> coordinate dependence at the level of the Lagrangian, exactly as in the dBP theory.

**Table**: Upper limits of the LSV parameters (the last value is in SI units): <sup>a</sup>Energy shifts in the spectrum of the hydrogen atom; <sup>b</sup>Rotation of the polarisation of light in resonant cavities; <sup>c,e</sup>Astrophysical observations. Such estimates are close to the Heisenberg limit on the smallest measurable energy or mass or length for a given time *t*, set equal to the age of the universe; <sup>d</sup>Rotation in the polarisation of light in resonant cavities. <sup>f</sup>Typical value.

$ ec{k}^{ m AF} $	a	$<10^{-10}~\text{eV}=1.6\times10^{-29}$ J; $5.1\times10^{-4}~\text{m}^{-1}$
$ ec{k}^{ m AF} $	b	$< 8 \times 10^{-14} \mbox{ eV} = 1.3 \times 10^{-32} \mbox{ J};  4.1 \times 10^{-7} \mbox{ m}^{-1}$
$ ec{k}^{ m AF} $	с	$<10^{-34}~\text{eV}=1.6\times10^{-53}~\text{J};~5.1\times10^{-28}~\text{m}^{-1}$
$k_0^{ m AF}$	d	$<10^{-16}~\text{eV}=1.6\times10^{-35}~\text{J};~5.1\times10^{-10}~\text{m}^{-1}$
$k_0^{ m AF}$	е	$<10^{-34}~\text{eV}=1.6\times10^{-53}~\text{J};~5.1\times10^{-28}~\text{m}^{-1}$
$k_{ m F}$	f	$\simeq 10^{-17}$

The leading term is proportional to  $k_0^{\text{AF}} * F^{0i} f_{i0}$ .

- The  $k_0^{\rm AF}$  component of the LSV vector is supposed large scale. We need to integrate over the light travel time. For a source at z = 0.5, the look-back time is  $t_{LB} = 1.57 \times 10^{17}$  s (Lemaître-Hubble-Humason constant = 70 km/s per m,  $\Omega_m = 0.3$ ,  $\Omega_{\Lambda} = 0.7$ ).
- A safe margin ρ for the many magnetic fields, B = 5 × 10<sup>-10</sup> 5 × 10<sup>-9</sup> T each, differently oriented, crossed by light (Not considered a possible presence of a strong magnetic field at the source)..

The wave energy density variation  $\Delta E$ 

$$|\Delta \mathsf{E}|_{z=0.5} = \frac{c}{\mu_0} |k_0^{\rm AF}| |Bf_{i0}| \varrho \ t_{LB} \approx 1.02 \times 10^{23} |k_0^{\rm AF}| \varrho |f_{i0}| \ . \tag{25}$$

For  $h=6.626 imes 10^{-34}$  Js, the frequency variation  $\Delta 
u$  is

$$|\Delta\nu|_{z=0.5} = \frac{1.023 \times 10^{23}}{h} |k_0^{\rm AF}|\varrho|f_{i0}| \approx 1.55 \times 10^{56} k_0^{\rm AF}\varrho|f_{i0}| .$$
 (26)

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We now need to compute  $|f_{i0}| = |\mathcal{E}|/c$ , the electric field of the photons. We consider the Maxwellian - in first approximation - classic intensity  $I = \epsilon_0 c \mathcal{E}^2 = \epsilon_0 c^3 |f_{i0}|^2 (c\mathcal{B} = \mathcal{E})$ . The frequency  $\nu = 4.86 \times 10^{14}$  Hz corresponds to the Silicon absorption line at 6150

The frequency  $\nu = 4.86 \times 10^{47}$  Hz corresponds to the Silicon absorption line at 6150 Å, of SN 1A Supernova type. The monochromatic AB magnitude is defined as the logarithm of a spectral flux density *SFD* 

$$m_{\rm AB} = -2.5 \log_{10} SFD - 48.6 , \qquad (27)$$

in cgs units. For  $m_{\rm AB} = -19$ , we get  $SFD = 10^{-15} \text{ Js}^{-1} \text{ Hz}^{-1} \text{ m}^{-2}$  having converted to SI units. We integrate over the frequency width of a bin, that is 30 Å or 2.37 THz and get  $I = 2.37 \times 10^{-3} \text{ Js}^{-1} \text{ m}^{-2}$ . For  $\epsilon_0 = 8.85 \times 10^{-12} \text{ Fm}^{-1}$ , we have

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From astrophysical data

$$f_{i0} = \sqrt{\frac{l}{\epsilon_0 c^3}} \approx 3.79 \times 10^{-9} \mathrm{V sm}^{-2}$$
 (28)

Finally, from Eq. (26), we get

$$|\Delta\nu|_{z=0.5}^{\nu=486\,\mathrm{THz}} = 3.6 \times 10^{47} \ k_0^{\mathrm{AF}} \varrho \ . \tag{29}$$

The parameter  $k_0^{\rm AF}$  has a laboratory upper limit of  $10^{-10} {\rm m}^{-1}$  but a more stringent, and less favourable for our study, astrophysical upper limit of  $5.1 \times 10^{-28} {\rm m}^{-1}$ . In this worst case, it is sufficient that  $\varrho \geq 1.6 \times 10^{-7}$ , to get  $z_{\rm LSV}$  in the order of 10% of z.

# Impact on cosmology: dark energy

The LSV as vacuum energy.

- The LoSy breaking four-vector,  $k_{\rm AF}$ , and the rank-four tensor,  $k_{\rm F}$ , correspond to the vacuum condensation of a vector and a tensor field in string models.
- They describe part of the vacuum structure, in the form of space-time anisotropies.
- Their presence reveals that vacuum effects are responsible for the energy variation of light waves and thus photon frequency shift.

Superposing the shifts.

- $z = \Delta \nu / \nu_o$  where  $\Delta \nu = \nu_e \nu_o$  is the difference between the observed  $\nu_o$  and emitted  $\nu_e$  frequencies, or else  $z = \Delta \lambda / \lambda_e$  for the wavelengths.
- Expansion causes  $\lambda_e$  to stretch to  $\lambda_c$  that is  $\lambda_c = (1 + z_C)\lambda_e$ . The wavelength  $\lambda_c$  could be further stretched or shrunk for the LSV shift to  $\lambda_o = (1 + z_{\rm LSV})\lambda_c = (1 + z_{\rm LSV})(1 + z_C)\lambda_e$ . But since  $\lambda_o = (1 + z)\lambda_e$ , we have  $1 + z = (1 + z_C)(1 + z_{\rm LSV})$ .

$$z = z_{\rm C} + z_{\rm LSV} + z_{\rm C} z_{\rm LSV} . \tag{30}$$

(日)

The second order is not negligible for larger  $z_{\rm C}$ .

# Impact on cosmology: dark energy

Behaviour of the LSV shift with distance.

- $z_{\rm C}$  takes into consideration the universe expansion, while  $z_{\rm LSV}$  is based on the comoving distance. The frequency variation is proportional
- Type 1 to the instantaneous frequency and to the distance.
- Type 2 to the emitted frequency and the distance.
- Type 3 to the distance.
- (Type 4 to the observed frequency and the distance.)

Table: LSV shift types.  $k_{1,2}$  have the dimensions of Mpc<sup>-1</sup>,  $k_3$  of Mpc<sup>-1</sup>s<sup>-1</sup>. The positiveness of the distance r constraints  $z_{\text{LSV}/1} > -1$  for  $k_1 < 0$ , and  $-1 < z_{\text{LSV}/1} < 0$  for  $k_1 > 0$ .

Туре	1	2	3
$\Delta \nu$	k <sub>1</sub> νdr	$k_2  u_{ m e} dr$	k <sub>3</sub> dr
$\nu_{\rm o}$	$ u_{ m e} \exp^{k_1 r} $	$\nu_{\mathrm{e}}(1+k_{2}r)$	$\nu_{ m e} + k_3 r$
$z_{ m LSV}$	$\exp^{-k_1r}-1$	$-rac{k_2r}{1+k_2r}$	$-\frac{k_3r}{\nu_e+k_3r}$

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TABLE III: We hold to the observed  $z_m$ , Eq. (23), and show the values that the cosmological shift  $z_C$  should assume for a fixed luminosity distance  $d_L$  in the first column, but different h and  $\Omega$  densities; in the second column for matter density  $\Omega_m = 0.28$ , energy density  $\Omega_\Lambda = 0.72$  and h = 0.7, we pose  $z_{1,SV} = 0$  and thereby  $z = z_C$ ; in the third column for  $\Omega_m = 0.28$  but  $\Omega_\Lambda = 0$  and h = 0.7, the values of  $z_C$  which determine the same  $d_L$ ; in the fourth, eighth and twelfth columns, the rate  $\frac{z_{1,SV}}{z_C}$ ; in the fifth, ninth and thirteenth columns, from Eq. (23),  $z_{1,SV} = \frac{z - z_C}{1 + z_C}$ ; in the sixth, tenth and fourteenth columns, the rate  $\frac{z_{1,SV}}{z_C}$ ; in the seventh column for  $\Omega_m = 0.28$  but  $\Omega_\Lambda = 0$  and h = 0.67, the values of  $z_C$  which determine the same  $d_L$ . The curvature and radiation densities are set to zero,  $\Omega_h = \Omega_{2,S}$  but  $\Omega_\Lambda = 0$  and h = 0.74, the values of  $z_C$  which determine the same  $d_L$ . The curvature and radiation densities are set to zero,  $\Omega_h = \Omega_{2,S}$  but distant superluminous SN is at z = 3.8893 [73], and the most distant galaxy is at z = 11.09 [74]. The numerical values are derived from a Cosmology Simulator [70, 71].

1	П	Ш	IV	v	VI	VII	VIII	IX	Х	XI	$X\Pi =$	XIII	XIV
$d_{\rm L}$ [Gpc]	$\begin{array}{c} h=0.7\\ \Omega_{\rm in}=0.28\\ \Omega_{\Lambda}=0.72\\ z_{\rm LSV}=0\\ z=z_{\rm C}\\ \end{array}$	$\begin{array}{c} h=0.7\\ \Omega_{\rm in}=0.28\\ \Omega_{\Lambda}=0\\ z_{\rm LSV}\neq 0,\\ z\neq z_{\rm C}\\ \end{array}$	$\frac{z_{\rm C}-z}{[\tilde\%]}$	≎LSV	$\frac{z_{\rm LSV}}{[\%]}$	$ \begin{array}{c} h=0.67\\ \Omega_{\rm in}=0.28\\ \Omega_{\Lambda}=0\\ z_{\rm LSV}\neq 0,\\ z\neq z_{\rm C} \end{array} $	$\frac{z_{\rm C}-z}{[\%]}$	\$LSV	$\frac{z_{\rm LSV}}{[\%]}$	$ \begin{array}{c} h=0.74\\ \Omega_{\rm in}=0.28\\ \Omega_{\Lambda}=0\\ z_{\rm LSV}\neq 0,\\ z\neq z_{C} \end{array} $	$\frac{z_{\rm C}-z}{[\%]}$	*LSV	$\frac{z_{LSV}}{[\tilde{\tilde{\aleph}}]}$
0.0007	0.05000	2C	1.00	0.00070	1.10	.2C	0.50	0.00100	0.47	2C	7.90	0.00010	0.00
0.2225	0.05000	0,05063	1.20	-0.00059	-1.19	0.04812	- 2.50	0.00122	.2.44	0.05308	1.30	-0.00349	-0.98
0.4610	0.10000	0.10314	3.14	-0.00285	-2.85	0.09888	-1.12	0.00107	1.07	0.10877	8.77	-0.00791	-7.91
2.8528	0.50000	0.54649	9.30	-0.04425	-8.88	0.52645	5.29	-0.01733	-3.46	0.57322	14.64	-0.04654	-9.31
6.6874	1.00000	1.10489	10.49	-0.04983	-4.98	1.06682	6.68	-0.03233	-3.23	1.15473	15.47	-0.07181	-7.81
11.0776	1.50000	1.63897	9.26	-0.05266	-3.51	1.58473	5.65	-0.03278	-2.18	1.71022	14.01	-0.07756	-5.17
15.8128	2.00000	2.14731	7.36	-0.04681	-2.34	2.07771	3.88	-0.02525	-1.26	2.23873	11.93	-0.07371	-3.68
36.6276	4.00000	3.99729	-0.07	0.00054	0.01	3,87115	-3.22	0.02645	0.66	4.16317	4.08	-0.03160	-0.79
118.5408	11.00000	9.47515	-13.86	0.14557	1.32	9.17182	-16.62	0.17973	1.63	9,87518	-10.22	0.10343	0.94

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#### Impact on cosmology: dark energy



Figure: TOP: for type 1 of LSV shift,  $\Omega_{rad} = \Omega_k = \Omega_{\Lambda} = 0$  and  $\Omega_m = 0.28$ ,  $\mu(z)$ ; BOTTOM: possible values of  $z_{LSV}$  for a given z (only 73 SNeIa for z > 0.8).

# Impact on cosmology: dark energy

- **1** The total red-shift z is the combination of the expansion red-shift  $z_{\rm C}$  and of a static, red or blue shift  $z_{\rm LSV}(r)$ , due to the energy non-conservation of a photon propagating through EM fields (host galaxy, intergalactic and Milky Way). Such propagation may be described by the dBP or others. In the latter case, the non-conservation stems from the vacuum expectation value of the vector and tensor LSV fields.
- 2 Then,  $z_{\text{LSV}}$  is a manifestation of an effective dark energy caused by the expectation values of the vacuum under LSV. If so, dark energy, *i.e.* LSV vacuum energy, is not causing an accelerated expansion but a frequency shift.
- **3** The single  $z_{\rm LSV}$  shift from a single SNIa may be small or large, red or blue, depending on the orientations of the LSV (vector or tensor) and of the EM fields (host galaxy, intergalactic medium, Milky Way), as well as the distance of the source. Anyway, the colour of  $z_{\rm LSV}$  is the final output of a series of shifts, both red and blue, encountered along the path.
- If the  $z_{\rm LSV}$  shift is blue, the photon gains energy; it implies that the real z, traditionally the red-shift, is larger than the measured z, as  $z_{\rm LSV}$  is subtracted from  $z_{\rm C}$ , the expansion red-shift. If red,  $z_{\rm LSV}$  corresponds to dissipation along the photon path; it implies that the real z is smaller than the measured z, as  $z_{\rm LSV}$  is added to  $z_{\rm C}$ .

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- Recasting z, as average, we observe a blue static shift for z ≤ 2, but red in our local Universe for smaller values of the Hubble(-Humason)-Lemâitre parameter (67 74 km/s per Mpc), and always red for z > 4.
- 2 A single mechanism could explain all the positions of the SNeIa in the  $(\mu, z)$  plan,  $\mu$  being the distance modulus, including the outliers. The experimental and observational limits on LSV and magnetic fields are fully compatible with our findings.

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# A general non-linear Lagrangian. Summary. I.

- A non-linear and general Lagrangian (including BI and EH, depending upon powers of the EM field tensor and its dual), in flat spacetime.  $\mathcal{L} = \mathcal{L} (\mathcal{F}, \mathcal{G})$ . - Field = background + light-wave.  $F = F_{\rm B} + f$ ,  $\tilde{F} = G = \tilde{F}_{\rm B} + \tilde{f} = G_{\rm B} + g$ .

$$F_{\sigma\tau} = \partial_{\sigma} A_{\tau} - \partial_{\tau} A_{\sigma} , \qquad \qquad \tilde{F}_{\sigma\tau} = G_{\sigma\tau} = \frac{1}{2} \epsilon_{\sigma\tau\kappa\lambda} F^{\kappa\lambda} \qquad (31)$$

4-potential  $A^{\sigma} = \left(\frac{\phi}{c}, \vec{A}\right)$ ,  $\phi$  and  $\vec{A}$ , time (scalar) and space (vector) components.

$$\mathcal{F} = -\frac{1}{4\mu_0} F^2 = -\frac{1}{4\mu_0} F_{\sigma\tau} F^{\sigma\tau} = \frac{1}{2\mu_0} \left( \frac{\vec{E}^2}{c^2} - \vec{B}^2 \right) , \qquad (32)$$

and

$$\mathcal{G} = -\frac{1}{4\mu_0} F_{\sigma\tau} \tilde{F}^{\sigma\tau} = -\frac{1}{4\mu_0} F_{\sigma\tau} G^{\sigma\tau} = \frac{1}{\mu_0} \frac{\vec{E}}{c} \cdot \vec{B} , \qquad (33)$$

where  $\mu_0 = 4\pi \times 10^{-7} \approx 1.256$  H m<sup>-1</sup> or V s A<sup>-1</sup> m<sup>-1</sup> is the vacuum permeability.

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# A general non-linear Lagrangian. Summary. II.

Lagrangian meaning (1<sup>st</sup> order no interaction; 2<sup>nd</sup> order interaction photon-background; 3<sup>rd</sup> order photon splitting or merging (three photons), with background; 4<sup>th</sup> order photon-photon (four photons), with background.) Non-conservation of the photon energy-momentum tensor  $\Theta_{\rm ff}$  (here 2<sup>nd</sup> order) when the EM external field is not constant in space-time.

$$(\Theta_{\rm ff})^{\mu}_{\alpha} = C_1 f^{\mu\nu} f_{\nu\alpha} - \frac{1}{2} k^{\mu\nu\kappa\lambda} f_{\kappa\lambda} f_{\nu\alpha} - \frac{1}{2} t^{\mu\nu\kappa\lambda} \tilde{f}_{\kappa\lambda} f_{\nu\alpha} - \frac{1}{4} \epsilon^{\mu\nu\kappa\lambda} t_{\kappa\lambda\rho\sigma} f^{\rho\sigma} f_{\nu\alpha} + \delta^{\mu}_{\alpha} \left( \frac{1}{4} C_1 f^2 - \frac{1}{8} \kappa^{\nu\mu\kappa\lambda} f_{\nu\mu} f_{\kappa\lambda} - \frac{1}{4} t^{\nu\rho\kappa\lambda} f_{\nu\rho} \tilde{f}_{\kappa\lambda} \right) .$$

$$(34)$$

$$\mathcal{T}_{\alpha} = -\partial_{\mu} \left( C_{1} F_{\mathrm{B}}^{\mu\nu} + C_{2} \tilde{F}_{\mathrm{B}}^{\mu\nu} \right) f_{\nu\alpha} + \frac{1}{4} \left( \partial_{\alpha} C_{1} \right) f^{2} + \frac{1}{4} \left( \partial_{\alpha} C_{2} \right) \tilde{f} f \\ - \frac{1}{8} \left( \partial_{\alpha} k^{\nu\mu\kappa\lambda} \right) f_{\nu\mu} f_{\kappa\lambda} - \frac{1}{4} \left( \partial_{\alpha} t^{\nu\mu\rho\sigma} \right) f_{\nu\mu} f_{\rho\sigma} .$$
(35)

The continuity equation

$$\partial_{\mu} \left(\Theta_{\rm ff}\right)^{\mu}_{\alpha} = \mathcal{T}_{\alpha} \ .$$
 (36)

# Source apparent time dilation??



$$\Delta t \simeq \frac{d}{c} \left( \frac{1}{\nu_1^2} - \frac{1}{\nu_2^2} \right) 10^{100} m_\gamma^2 H_\gamma(z) , \qquad (37)$$

where

$$H_{\gamma}(z) \equiv \int_{0}^{z} \frac{dz'}{(1+z')^{2}\sqrt{\Omega_{\Lambda}+(1+z')^{3}\Omega_{m}}}.$$
 (38)

If SN spectrum shifts towards lower frequencies, massive photon may mimic time dilation, even if the source is not moving. Relevant corrections? It seems not.

Introducing non-Maxwellian electromagnetism in astrophysics and cosmology allow new interpretations of data.

# Grazie per la vostra attenzione



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