On infrared aspects of gravity and flat space holography

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GReCO seminar @ IAP Paris, 30 March 2020

The spacetime of the universe

Maximally symmetric spacetimes relevant for our universe:

- $\Lambda > 0$ **de Sitter**
- $\Lambda = 0$ **Minkowski**
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- de Sitter: Identification of symmetries of dS with proposed dual conformal structure at the boundary in the future ⇒ dS/CFT with time emergent.

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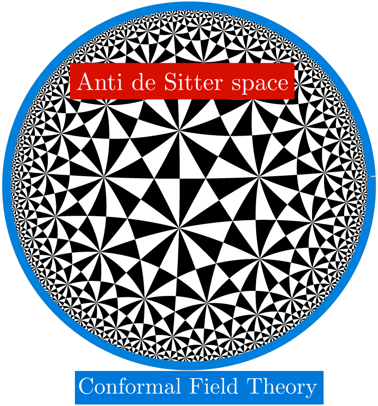
- de Sitter: Identification of symmetries of dS with proposed dual conformal structure at the boundary in the future ⇒ dS/CFT with time emergent. Positive cosmological constant: no box.

- Minkowski: *What are the symmetries? Is there a dual CFT? Where does it live? What are its properties? ...*

Holographic screen of AdS spacetime

- Ground-breaking tool: **AdS/CFT correspondence** (holography)

[Maldacena'97]

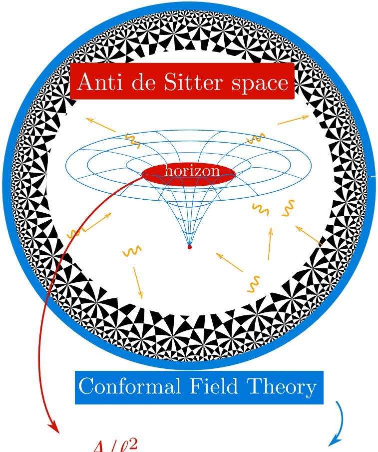


Quantum gravity in Anti de Sitter space
=
Conformal Field Theory on boundary

Black hole information in AdS

- Ground-breaking tool: **AdS/CFT correspondence** (holography)

[Strominger,Vafa'96; Strominger'97]



$$S = \frac{A/\ell_P^2}{4} \quad = \quad S = \log N$$

perfect match!

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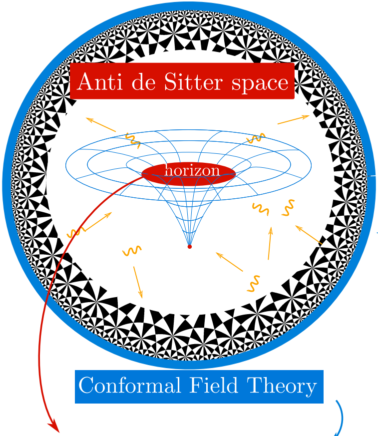

Information encoded at boundary of Anti de Sitter space!

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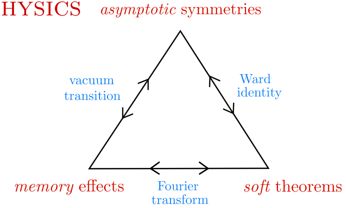
Holographic screen of Minkowski spacetime

- Novel development: **flat space holography?**

4D spacetime encoded on 2D celestial sphere?

Outline

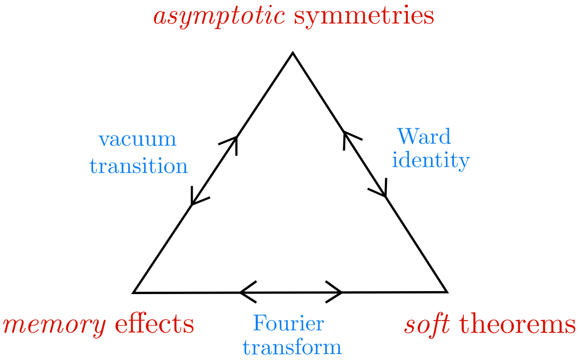
I. INFRARED PHYSICS



II. CELESTIAL SPHERE: FLAT SPACE HOLOGRAPHY?

- 4D S-matrix $\leftrightarrow$ 2D correlator
- 4D spacetime symmetries $\leftrightarrow$ 2D conformal soft primaries
- 4D memory effects $\leftrightarrow$ 2D conformal memory primaries
- 4D soft theorems $\leftrightarrow$ 2D conformal soft theorems

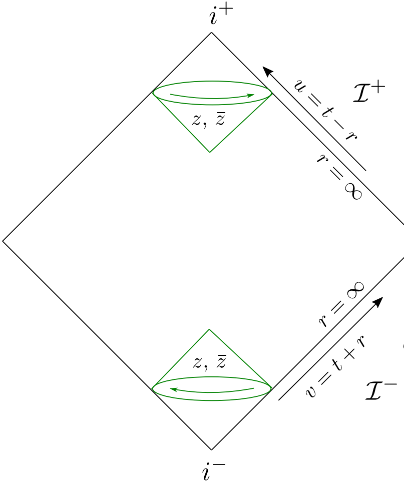
I. Infrared physics



Causal structure of Minkowski spacetime

Natural holographic screen: conformal boundary of Minkowski space

null infinity $\mathcal{I} = \mathcal{C}S^2 \times \mathbb{R}$
"celestial sphere"



$$ds^2 = -du^2 - 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z}$$

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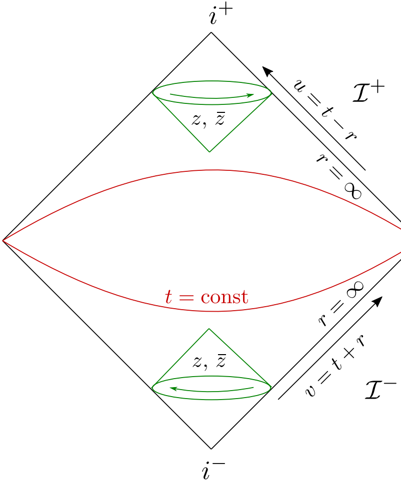

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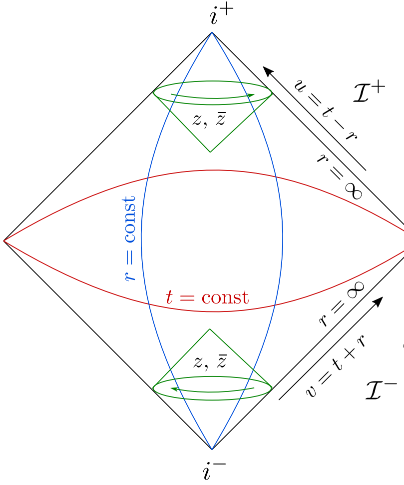

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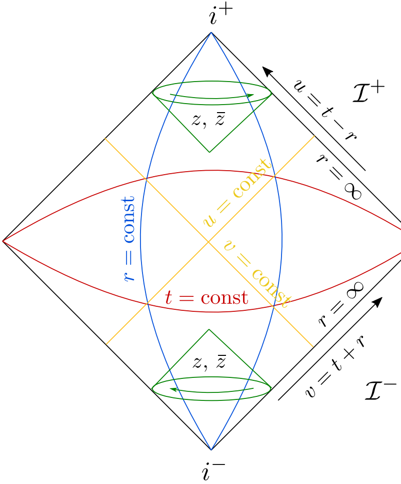

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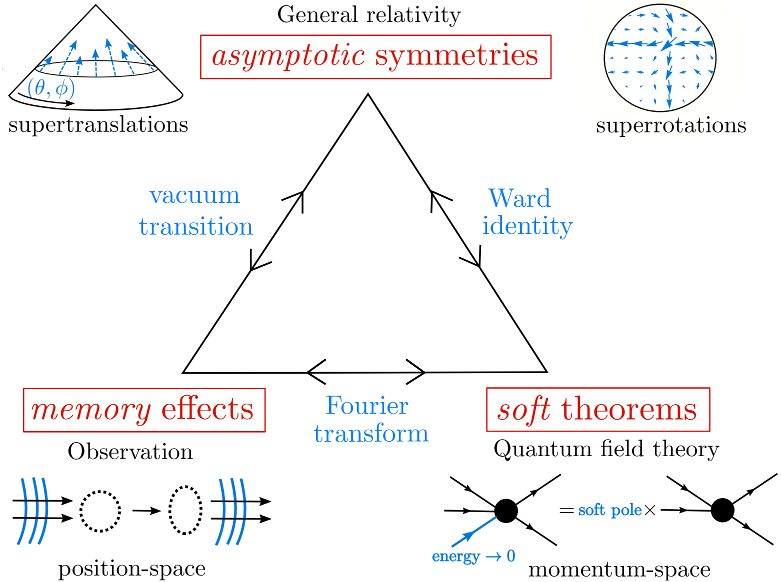
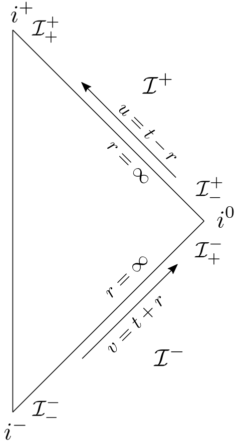
The infrared triangle



Asymptotic symmetries

$$\text{Asymptotic Symmetry Group} = \frac{\text{allowed gauge symmetry}}{\text{trivial gauge symmetry}}$$



- choose boundary conditions
  strong enough to avoid pathologies, weak enough to allow all relevant configurations
- find residual gauge/diffeos respecting boundary conditions

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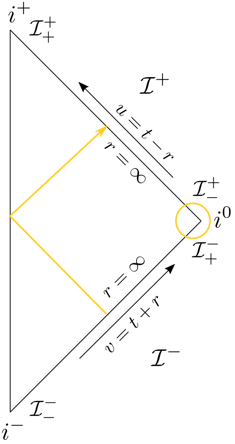

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antipodal matching as $\to i^0$
$\Rightarrow \infty$ of charge conservation laws

[Strominger'13]

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$Q_\varepsilon^+ = \frac{1}{e^2} \int_{\mathcal{I}_-^+} \varepsilon * F = \frac{1}{e^2} \int_{\mathcal{I}_+^-} \varepsilon * F = Q_\varepsilon^-$

→ Electric charge conservation

Asymptotically flat space

$$ds^2 = -\,du^2 + 2\,du dr + 2r^2\gamma_{z\bar{z}}dz d\bar{z} \qquad \text{... flat space}$$

Asymptotically flat space

[Bondi,van der Burg,Metzner,Sachs'62]

$$
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ds^2 = & - du^2 + 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} \qquad \text{... flat space} \\
& + \frac{2m_B}{r}du^2 + rC_{zz}dz^2 + D^z C_{zz}dudz + c.c + \\
& + \frac{1}{r}\left(\frac{4}{3}\left(N_z + u\partial_z m_B\right) - \frac{1}{4}\partial_z(C_{zz}C^{zz})\right)dudz + c.c + \dots
\end{aligned}
$$

- m_B...Bondi mass aspect
- C_{zz}...free gravitational data
- N_z...angular momentum aspect

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→ Energy conservation

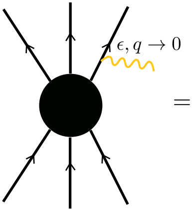
[deBoer,Solodukhin'03][Banks'03][Barnich,Troessaert'09'11]

Superrotations: $\delta_Y^{shift} C_{zz} = -u D_z^3 Y^z$ with $Y^z = Y^z(z, \bar{z})$

$Q_Y^+ = \frac{1}{8\pi G} \int_{\mathcal{I}_-^+} d^2 z (Y_z N_{\bar{z}} + Y_{\bar{z}} N_z) = \frac{1}{8\pi G} \int_{\mathcal{I}_+^-} d^2 z (Y_z N_{\bar{z}} + Y_{\bar{z}} N_z) = Q_Y^-$

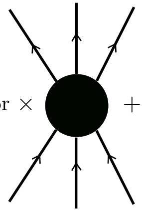
→ Angular momentum conservation

Soft theorems



$\langle out|a_{\pm}(\vec{q})\mathcal{S}|in\rangle$

$=$ soft factor $\times$



$+ \mathcal{O}(q^0)$

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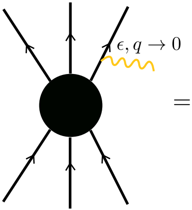

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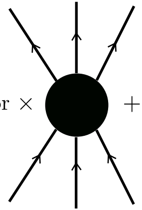

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$$S_0^\pm = e\sum_k \frac{\epsilon_\mu^\pm p_k^\mu Q_k}{p_k \cdot q}$$

[Bloch,Nordsiek'37][Low'54] ...

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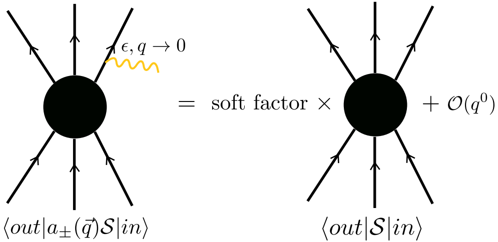

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[Weinberg'65]

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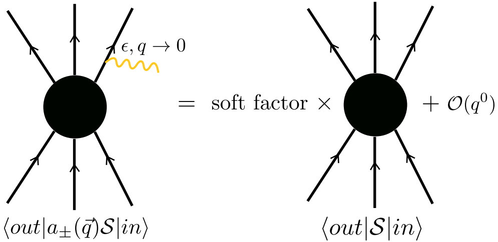

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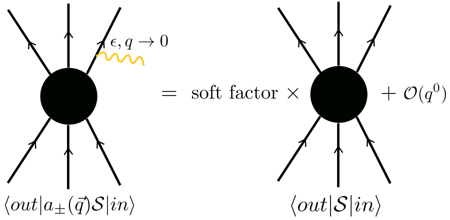
[Cachazo,Strominger'15]

Soft theorems ⇔ Asymptotic symmetries

soft theorems



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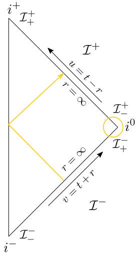

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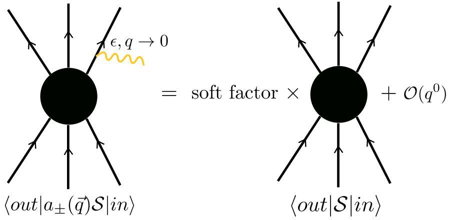
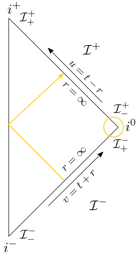
soft theorems $\xleftarrow{\textbf{Ward ID}}$ asymptotic symmetries





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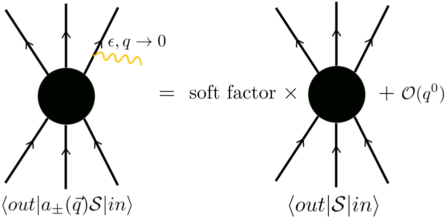

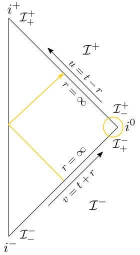

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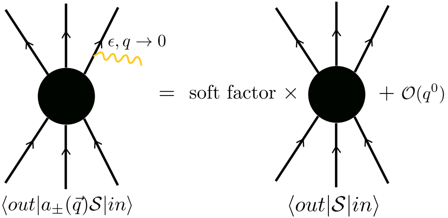

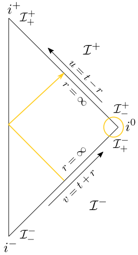

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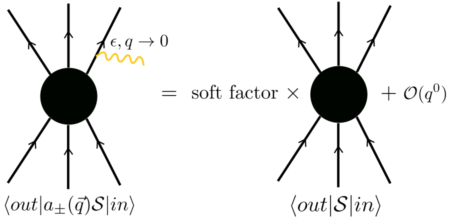

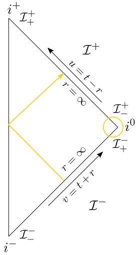

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Memory effects

Asymptotically flat space: [Bondi,van der Burg,Metzner,Sachs'62]

$$
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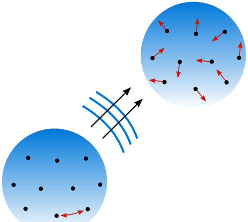
& + \frac{1}{r}\left(\frac{4}{3}\left(N_z + u\partial_z m_B\right) - \frac{1}{4}\partial_z(C_{zz}C^{zz})\right)dudz + c.c + \dots
\end{aligned}
$$

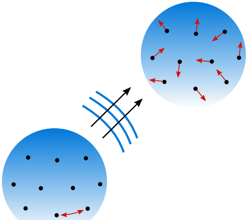


Memory effects

Asymptotically flat space: [Bondi,van der Burg,Metzner,Sachs'62]

$$
\begin{aligned}
ds^2 =& - du^2 + 2dudr + 2r^2\gamma_{z\bar{z}}dzd\bar{z} \\
&+ \frac{2m_B}{r}du^2 + rC_{zz}dz^2 + D^z C_{zz}dudz + c.c+ \\
&+ \frac{1}{r}\left(\frac{4}{3}\left(N_z + u\partial_z m_B\right) - \frac{1}{4}\partial_z(C_{zz}C^{zz})\right)dudz + c.c + \dots
\end{aligned}
$$

geodesic deviation



relative angular separation
$s(z,\bar{z})$ of detectors

$$
\begin{aligned}
r^2\gamma_{z\bar{z}}\partial_u^2 s^{\bar{z}} =& - R_{uzuz}s^z \\
=& \frac{r}{2}\partial_u^2 C_{zz}s^z
\end{aligned}
$$

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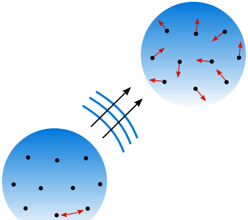
$$
\begin{aligned}
ds^2 = & - du^2 + 2 du dr + 2 r^2 \gamma_{z\bar{z}} dz d\bar{z} \\
& + \frac{2m_B}{r} du^2 + r C_{zz} dz^2 + D^z C_{zz} du dz + c.c + \\
& + \frac{1}{r} \left(\frac{4}{3} (N_z + u \partial_z m_B) - \frac{1}{4} \partial_z (C_{zz} C^{zz}) \right) du dz + c.c + \ldots
\end{aligned}
$$

geodesic deviation



$$
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= & \frac{r}{2} \partial_u^2 C_{zz} s^z
\end{aligned}
$$

gravitational memory effect

$$
\Delta s^{\bar{z}} = \frac{\gamma^{z\bar{z}}}{2r} \Delta C_{zz} s^z
$$

[Zel'dovich,Polnarev'74][Braginski,Thorne'87][Christodoulou'91] [Blanchet,Damour'92] ...

Soft theorems ⇔ Memory effects

soft theorems

elementary particle collisions

[Weinberg'65]

The dominance of the $1/(p\cdot q)$ pole in (2.5) implies that the effect of attaching one soft-graviton line to an arbitrary diagram is to supply a factor equal to the sum of (2.5) over all external lines in the diagram

$$(8\pi G)^{1/2}\sum_n \eta_n p_n{}^\mu p_n{}^\nu/[p_n\cdot q - i\eta_n\epsilon]. \qquad (2.7)$$

memory effects

black hole/neutron star collisions

[Braginsky, Thorne'87]

permanent change in the gravitational-wave field (the burst's memory) $\delta h_{ij}^{\mathrm{TT}}$ is equal to the 'transverse, traceless (TT) part'[36] of the time-independent, Coulomb-type, $1/r$ field of the final system minus that of the initial system. If $\mathbf{P}^A$ is the 4-momentum of mass A of the system and P_i^A is a spatial component of that 4-momentum in the rest frame of the distant observer, and if $\mathbf{k}$ is the past-directed null 4-vector from observer to source, then $\delta h_{ij}^{\mathrm{TT}}$ has the following form:

$$\delta h_{ij}^{\mathrm{TT}} = \delta\left(\sum_A \frac{4P_i^A P_j^A}{\mathbf{k}\cdot\mathbf{P}^A}\right)^{\mathrm{TT}} \qquad (1)$$

Here we use units with $G = c = 1$. In the observer's local Car-

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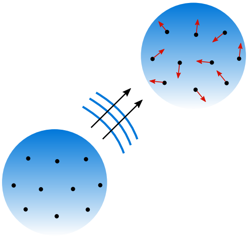
Equivalence:

- $P_i^A \leftrightarrow p_n^\mu$
- different conventions Newton's constant G and normalization
- momentum space soft theorem $\xrightarrow{\int dt e^{i\omega t}}$ position-space memory effect

Gravitational memory in the sky?

- *displacement* memory: Pulsar Timing Array [van Haasteren+'10; Wang et al'15], LIGO [Lasky+'16]
  gravitational wave → relative angular displacement of detectors
- *spin* memory: LISA, Einstein Telescope [Nichols'17]
  gravitational wave → relative time delays of counterorbiting object



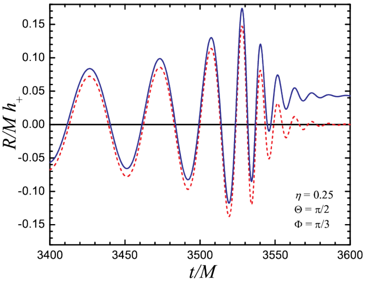
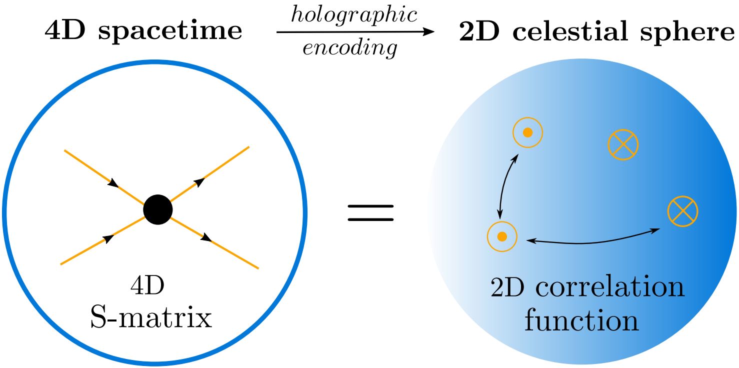


Figure: *Angular displacement of evenly spaced array of detectors caused by gravitational wave (left). h_+ polarization for an equal-mass binary black hole coalescence with and without memory (right) [Favata'10].*
(M total mass, η reduced mass, R distance and (Θ, Φ) angles of source to observer)

II. Celestial sphere: flat space holography?

Evidence for flat space holography



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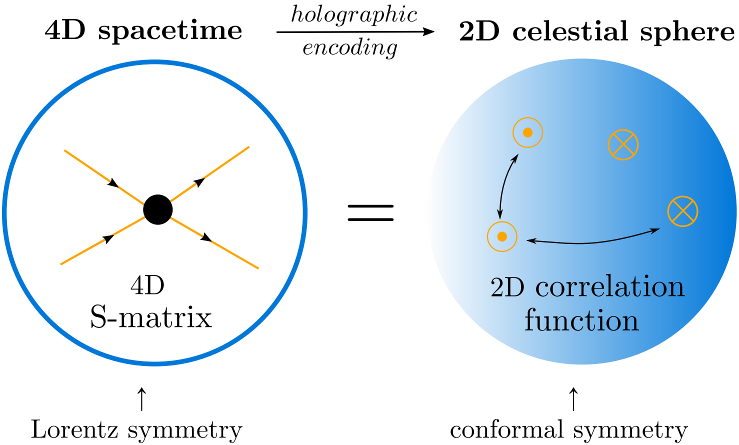

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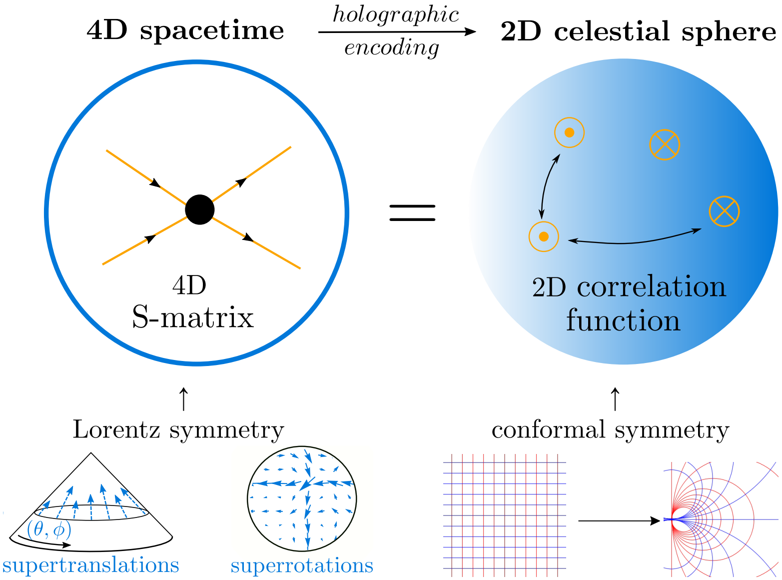


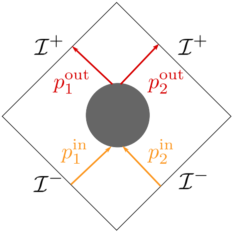
Basic idea of flat space holography

Bulk Mink[1,3]

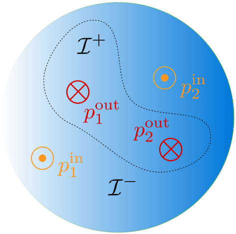
4D $SL(2,\mathbb{C})$ Lorentz
4D superrotations
4D supertranslations

Boundary $\mathcal{CS}^2$

2D global conformal
2D local conformal
2D Kac-Moody



$=$



4D $\mathcal{S}$-matrix
$\langle out|\mathcal{S}|in\rangle$

$\omega, \vec{p}, \ell$

2D conformal correlator
$\langle \mathcal{O}_1(z_1, \bar{z}_1)\ldots\mathcal{O}_n(z_n, \bar{z}_n)\rangle$

$(z, \bar{z})$, Δ, J

Basic elements of flat space holography

standard formulation

energy-momentum basis

$p_k^\mu = (\omega_k, \vec{p}_k)\,,\ \ell_k$

holographic formulation

conformal basis

$z_k = \dfrac{p_k^1 + i p_k^2}{p_k^3 + p_k^0}\,,\ J_k = \ell_k\,,\ \Delta_k$

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plane waves

$$e^{ip\cdot X} = e^{i\omega\, q\cdot X}$$

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holographic formulation

conformal basis

Mellin transform of plane waves*

$$\int_0^\infty d\omega \omega^{\Delta-1} e^{i\omega\, q\cdot X} = \frac{\mathcal{N}(\Delta)}{(-q\cdot X)^\Delta}$$

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Solution to Maxwell ($\partial^\mu A_\mu = 0$ and $X^\mu A_\mu = 0$):

$$A_\mu = \frac{\Delta - 1}{\Delta}\frac{\epsilon_\mu}{(-q\cdot X)^\Delta} + \partial_\mu \alpha$$

Solution to linearized Einstein ($\partial^\mu h_{\mu\nu} = 0$ and $X^\mu h_{\mu\nu} = 0$):

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A holographic formulation of 4D QFT

standard formulation

energy-momentum basis

→ plane waves

$\mathcal{A}(\omega_1, \vec{p}_1, \ldots, \omega_n, \vec{p}_n)$

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holographic formulation

conformal basis

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[de Boer,Solodukhin'03] [Pasterski,Shao,Strominger'17]
[Cheung,de la Fuente,Sundrum'17]...

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translational symmetry manifest
conformal properties obscured

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"soft" particle: $\omega \to 0$

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"conformal soft" particle: $\lambda \to 0$**

Large gauge symmetry & EM memory

Goldstone mode of spontaneously broken large gauge symmetry

$$A^{\mathrm{G}}_{\mu;a} \equiv \lim_{\Delta\to 1} A^{\Delta,\pm}_{\mu;a} = \partial_\mu \alpha^{\Delta=1}_a \quad \rightarrow \quad A^1_z = D_z \varepsilon(z,\bar{z})$$

Its canonical partner in energy-momentum basis is soft photon = EM memory.

Not captured by Mellin transforms of plane waves → need new solution!

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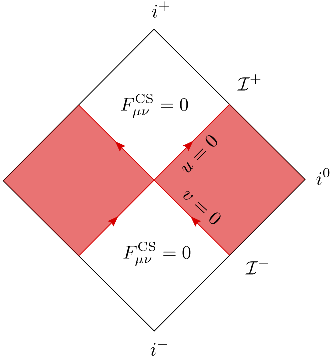
Conformal soft photon [Donnay,AP,Strominger'18]

$$A^{\rm CS}_{\mu;a} = [\log[X^2](q\cdot X)\delta(q\cdot X) + \Theta\left(X^2\right)]A^{\rm G}_{\mu;a}$$

is a new $\Delta = 1$ conformal primary with $(h,\bar{h}) = (1,0)$ (for $a = w$).

Conformal soft photon



On $\mathcal{I}^+$:

$$F^{\rm CS}_{uz;w} = \frac{\delta(u)}{(z-w)^2}\,, \quad F^{\rm CS}_{u\bar{z};w} = -2\pi\delta(u)\delta^{(2)}(z-w)$$

$$r^2 F^{\rm CS}_{ur;w} = 4\pi\gamma^{z\bar{z}}\partial_z\delta^{(2)}(z-w)\Theta(-u)\,, \quad F^{\rm CS}_{z\bar{z};w} = 0$$

On $\mathcal{I}^-$:

$$F^{\rm CS}_{vz;w} = \frac{\delta(v)}{(z-w)^2}\,, \quad F^{\rm CS}_{v\bar{z};w} = -2\pi\delta(v)\delta^{(2)}(z-w)$$

$$r^2 F^{\rm CS}_{vr;w} = 4\pi\gamma^{z\bar{z}}\partial_z\delta^{(2)}(z-w)\Theta(v)\,, \quad F^{\rm CS}_{z\bar{z};w} = 0$$

Radiative shockwave: initial data at $v = 0$ emerges from $\mathcal{I}^-$, impinges on origin, reemerges at $u = 0$ at $\mathcal{I}^+$

Coulombic fields: produced by and confined to future of incoming shock, annihilated by and confined to past of outgoing shock

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Not captured by Mellin transforms of plane waves → need new solution!

Conformal soft photon [Donnay,AP,Strominger'18]

$$A^{\mathrm{CS}}_{\mu;a} = [\log[X^2](q\cdot X)\delta(q\cdot X) + \Theta\left(X^2\right)] A^{\mathrm{G}}_{\mu;a}$$

is a new $\Delta = 1$ conformal primary with $(h,\bar{h}) = (1,0)$ (for $a = w$).

Pair of zero-modes $\{A^{\mathrm{G}}, A^{\mathrm{CS}}\}$ completes the conformal primary basis

$$i(A^{\mathrm{CS}}_w(w), A^{\mathrm{G}}_{w'}(w'))_{\mathcal{I}^+} = 8\pi^2\delta^{(2)}(w-w') \quad \checkmark$$

Large gauge symmetry & EM memory

Goldstone mode of spontaneously broken large gauge symmetry

$$A^{\rm G}_{\mu;a} \equiv \lim_{\Delta\to 1} A^{\Delta,\pm}_{\mu;a} = \partial_\mu \alpha_a^{\Delta=1} \quad \rightarrow \quad A^1_z = D_z \varepsilon(z,\bar z)$$

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$$i(A^{\rm CS}_w(w), A^{\rm G}_{w'}(w'))_{\mathcal{I}^+} = 8\pi^2\delta^{(2)}(w-w') \quad \checkmark$$

$J_w = i(\widehat{A}, A^G_w)$ is the $(h,\bar h) = (1,0)$ *conformal soft photon current*.

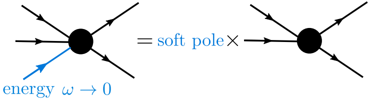
[Donnay,AP,Strominger'18]

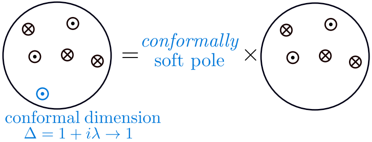
It generates on the celestial sphere a $U(1)$ Kac-Moody symmetry.

[Strominger'13][He,Mitra,Porfyriadis,Strominger'14][Nande,Pate,Strominger'15]

Conformally soft theorem in gauge theory

Insertions of the $(\Delta, J) = (1, \pm 1)$ soft photon/gluon current:





Soft gluon theorem

[Yennie,Frautschi,Suura'61]

Conformal soft gluon theorem

[Fan,Fotopoulos,Taylor'19]

[Pate,Raclariu,Strominger'19]

[Nandan,Schreiber,Volovich,Zlotnikov'19]

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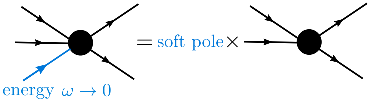

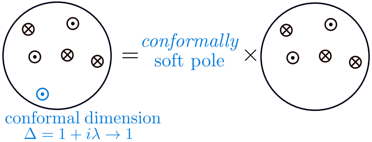

Soft gluon theorem

[Yennie,Frautschi,Suura'61]

$$\lim_{\omega_n \to 0} \omega_n \mathcal{A}_n(\omega_i, z_i, \bar{z}_i) = S^{(0)} \mathcal{A}_{n-1}(\omega_i, z_i, \bar{z}_i)$$

with

$$S^{(0)} = -\frac{1}{2} \frac{z_{n-1\, n+1}}{z_{n-1\, n}\, z_{n\, n+1}}$$

where $z_{ij} = z_i - z_j$

Conformal soft gluon theorem

[Fan,Fotopoulos,Taylor'19]

[Pate,Raclariu,Strominger'19]

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$$\lim_{\lambda_n \to 0} i\lambda_n \widetilde{\mathcal{A}}_n(\lambda_i, z_i, \bar{z}_i) = S^{(0)} \widetilde{\mathcal{A}}_{n-1}(\lambda_i, z_i, \bar{z}_i)$$

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where $z_{ij} = z_i - z_j$

Both expected and surprising: amplitude factorization for zero energy in energy basis but Mellin transform = superposition of all energies.

Supertranslations & displacement memory

Goldstone mode of spontaneously broken supertranslation symmetry

$$h^{\mathrm{G}}_{\mu\nu;a} \equiv \lim_{\Delta\to 1} h^{\Delta,\pm}_{\mu\nu;a} = \partial_{(\mu}\xi^{\Delta=1}_{\nu);a} \quad \to \quad C^1_{zz} = -2D_z^2 f(z,\bar{z})$$

Its canonical partner in energy-momentum basis is soft graviton = displacement memory. Not captured by Mellin transforms of plane waves → need new solution!

Conformal soft graviton [Donnay,AP,Strominger'18]

$$h^{\mathrm{ST}}_{\mu\nu;a} = [\log[X^2](q\cdot X)\delta(q\cdot X) + \Theta\left(X^2\right)] h^{\mathrm{G}}_{\mu\nu;a}$$

is a new $\Delta = 1$ conformal primary with $(h,\bar{h}) = (\frac{3}{2}, -\frac{1}{2})$ for $a = ww$.

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Pair of zero-modes $\{h^{\mathrm{G}}, h^{\mathrm{ST}}\}$ completes the conformal primary basis

$$(h^{\mathrm{ST}}_{ww}(w), h^{\mathrm{G}}_{ww}(w'))_{\mathcal{I}^+} = \frac{i\pi^2}{2}\gamma_{w\bar{w}}\delta^{(2)}(w-w') \quad \checkmark$$

Supertranslations & displacement memory

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$P_w = 4iD^w(\hat{h}, h^{\mathrm{G}}_{ww})$ is the $(h,\bar{h}) = (\frac{3}{2},\frac{1}{2})$ supertranslation current.

[Donnay,AP,Strominger'18]

It generates on the celestial sphere a BMS supertranslation symmetry.

[Strominger'13] [Barnich,Troessaert'09'11]

Conformally soft theorem in gravity

Insertions of the $(\Delta, J) = (1, \pm 2)$ BMS supertranslation current:

[Donnay,AP,Strominger'18]

$$P_w \mathcal{O}_\omega(z) \sim \frac{\omega}{w-z}\mathcal{O}_\omega(z) \quad \to \quad P_w \mathcal{O}_{(\Delta,J)}(z) \sim \frac{1}{w-z}\mathcal{O}_{(\Delta+1,J)}(z)$$

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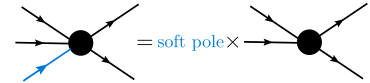

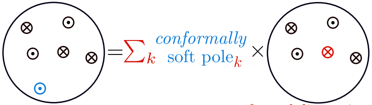

Weinberg's soft graviton theorem

[Weinberg'65]

$$\lim_{\omega_n \to 0} \omega_n \mathcal{H}_n(\omega_1, \ldots, \omega_k, \ldots, \omega_n)$$
$$= S^{(0)} \mathcal{H}_{n-1}(\omega_1, \ldots, \omega_k, \ldots, \omega_{n-1})$$

with

$$S^{(0)} = \sum_{k=1}^{n-1} \omega_k \frac{\epsilon_k}{\epsilon_n} \frac{\bar{z}_{nk}}{z_{nk}} \frac{z_{xk} z_{yk}}{z_{xn} z_{yn}}$$

where x, y is a choice of reference spinors

Conformal soft graviton theorem

[AP'19][Adamo,Mason,Sharma'19][Guevara'19]

$$\lim_{\lambda_n \to 0} i\lambda_n \widetilde{\mathcal{H}}_n(\lambda_1, \ldots, \lambda_k, \ldots, \lambda_n)$$
$$= \sum_{k=1}^{n-1} \widetilde{S}_k^{(0)} \widetilde{\mathcal{H}}_{n-1}(\lambda_1, \ldots, \lambda_k - i, \ldots, \lambda_{n-1})$$

with

$$\widetilde{S}_k^{(0)} = \frac{\epsilon_k}{\epsilon_n} \frac{\bar{z}_{nk}}{z_{nk}} \frac{z_{xk} z_{yk}}{z_{xn} z_{yn}}$$

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Celestial conformal symmetry

Goldstone mode of spontaneously broken Diff(S^2) symmetry

$$\lim_{\Delta\to 0} h^{\Delta,\pm}_{\mu\nu;a} = \partial_{(\mu}\zeta^{\Delta=0}_{\nu);a} \quad \to \quad C^{0}_{zz} = -uD^3_z\, Y(z,\bar{z})$$

[Pasterski,Shao'17] [Donnay,Pasterski,AP - to appear]

Goldstone mode of spontaneously broken superrotation symmetry

$$\lim_{\Delta\to 2} \tilde{h}^{\Delta,\pm}_{\mu\nu;a} = \partial_{(\mu}\zeta^{\Delta=2}_{\nu);a} \quad \to \quad \tilde{C}^{2}_{zz} = -uD^3_z\, \tilde{Y}(z,\bar{z})$$

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[Pasterski,Shao'17] [Donnay,AP,Strominger'18] [Donnay,Pasterski,AP - to appear]

$T_{ww} \sim i(\hat{h}, \tilde{h}^2_{ww})$ is the $(h,\bar{h}) = (2,0)$ 2D stress tensor for 4D gravity.

$\Uparrow$

up to subtleties

[Donnay,AP,Strominger'18]

It generates on the celestial sphere a celestial conformal symmetry.

[Kapec,Mitra,Raclariu,Strominger'16]

Summary

4D scattering amplitudes ⟶ 2D celestial correlators

energy, momentum, helicity — point on sphere, conformal dimension, spin

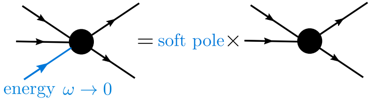
4D spacetime symmetries ⟶ 2D conserved currents

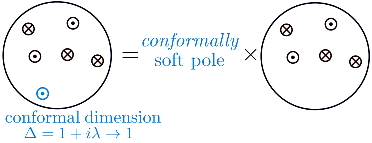
large gauge symmetry — conformal soft photon/gluon current

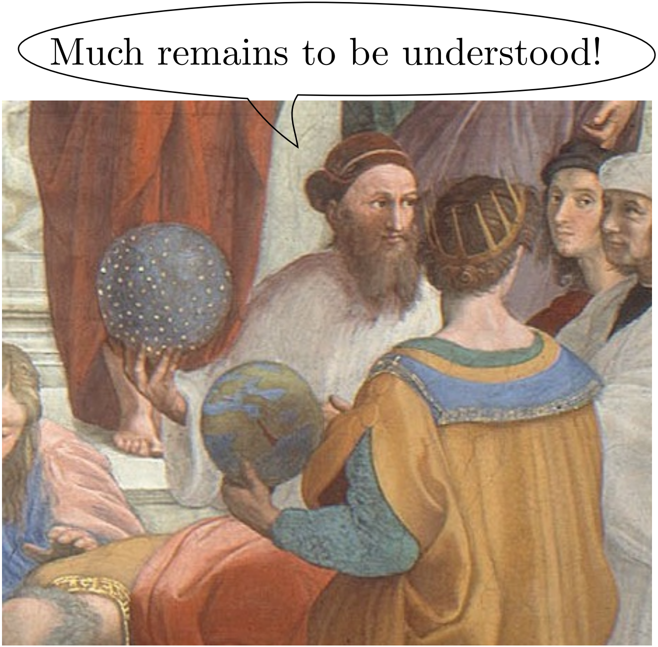
supertranslations — supertranslation current

superrotations — stress tensor

4D *"soft"* theorems ⟶ 2D *"conformally soft"* theorems






Much remains to be understood!