The rôle of neutrinos in supernovaes and neutron star formation

GreCo seminar

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LUTh (Laboratoire Univers et Théories), Observatoire de Paris, PSL

Plan

1. The core-collapse mechanism

2. Electron captures on nuclei during the infall

3. The formation of neutron stars

4. PNS modelling within the quasi-stationnary approximation

5. Impact of different approximation for charged current reactions on nucleons

6. Convection in proto-neutron stars

The core-collapse mechanism

The core-collapse mechanism : infall

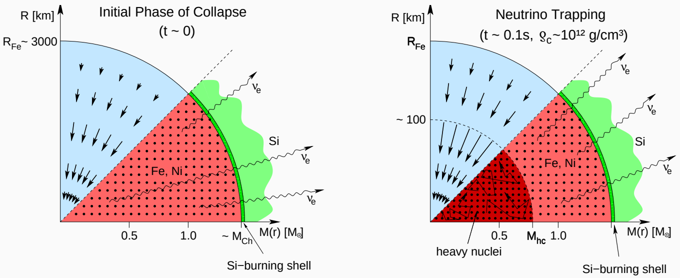

Figure 1: Core-collapse mechanism, figure extracted from Janka et al. (2007)

Iron core beyond the Chandrasekhar mass $M_{\mathrm{Ch}} \approx 1.2\,\mathrm{M}_\odot \Rightarrow$ collapse

Electron captures during the infall :

$$^{A}_{Z}\mathrm{X} + e^- \rightarrow {}_{Z-1}^{\;\;A}\mathrm{Y} + \nu_e$$

The limit of the zone at high densities and temperatures in which neutrinos are *trapped* because of their low mean free path is called the *neutrinosphere*

The core-collapse mechanism : bounce and shock

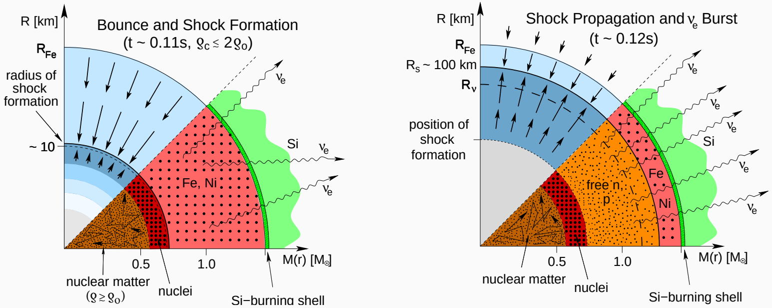

Figure 2: Core-collapse mechanism, figure extracted from Janka et al. (2007)

density of roughly nuclear saturation : $n_0 = 0.16\,\mathrm{fm}^{-3}$

$\Rightarrow$ nuclei dissociation, core bounce and shock generation

shock propagation ν-burst when the shock reaches the neutrinosphere

exhaustion of the shock by dissociation of infalling material

The core-collapse mechanism : shock stalling and revival

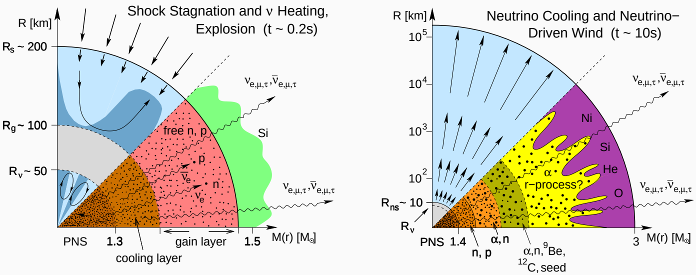

Figure 3: Core-collapse mechanism, figure extracted from Janka et al. (2007)

shock stalling and accretion

ν-heating (coupled with SASI and strong asymmetries) $\Rightarrow$ possible revival of the shock and final explosion

Relevant weak processes occuring during core-collapse

Neutrinos absorption/emission via charge exchange

$$p + e^- \leftrightarrows n + \nu_e \qquad p \leftrightarrows n + e^+ + \nu_e$$

$$n + e^+ \leftrightarrows p + \bar{\nu}_e \qquad n \leftrightarrows p + e^- + \bar{\nu}_e$$

$${}^{A}_{Z}\mathrm{X} + e^- \leftrightarrows {}_{Z-1}^{\;\;A}\mathrm{Y} + \nu_e$$

Thermal pair production of neutrinos

$$e^- + e^+ \leftrightarrows \nu + \bar{\nu}$$

$$N + N \leftrightarrows N + N + \nu + \bar{\nu} \quad \text{(nucleon bremsstrahlung)}$$

Neutrino scattering

$$N + \nu \leftrightarrows N + \nu$$

$${}^{A}_{Z}\mathrm{X} + \nu \leftrightarrows {}^{A}_{Z}\mathrm{X} + \nu$$

$$e^{\pm} + \nu \leftrightarrows e^{\pm} + \nu$$

Electron captures on nuclei during the infall

Electron capture rate

$$^{A}_{Z}\mathrm{X} + e^- \leftrightarrows {}^{\;\;A}_{Z-1}\mathrm{Y} + \nu_e$$

$$j_{\nu\text{-prod}} = n_A \times \Gamma_{\mathrm{EC}}$$

where

$j_{\nu\text{-prod}}$ is the neutrino production rate per volume unit

n_A is the density of nuclei $^{A}_{Z}\mathrm{X}$

Γ_{EC} is the electron capture rate on the nuclei $^{A}_{Z}\mathrm{X}$

$\Rightarrow$ we need individual cross sections AND nuclear abundances

Composition of the medium

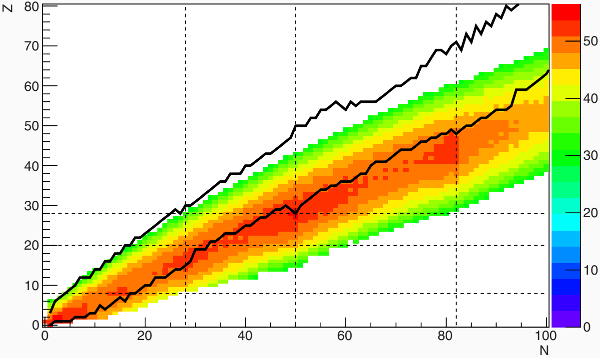

Figure 4: Typical nuclear abundance near the end of the collapse[1] (arbitrary unit), solid lines mark boundaries of experimental mass measurements, dashed lines mark magic numbers

[1] Raduta, Gulminelli, and Oertel 2016.

Most relevant nuclei for electron captures

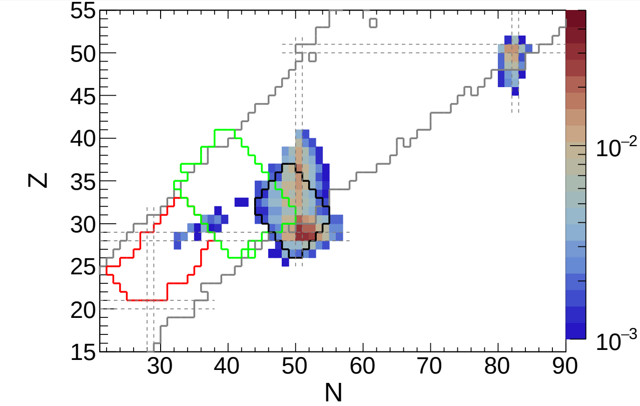

Figure 5: Time integrated relative deleptonization rate (color scale) associated to the different nuclear species identified by their proton Z and neutron N number.[2]

[2]Pascal et al. 2020.

3 models for electrons captures on nuclei

$$ {}^{A}_{Z}\mathrm{X} + e^- \rightarrow {}^{A}_{Z-1}\mathrm{Y} + \nu_e $$

Bruenn : approx. of independant particles[3]

LMP (Langanke and Martínez-Pinedo) : fit on shell model results[4]

ISO : same fit but done with more parameters[5]

[3] Bruenn 1985.
[4] Langanke et al. 2003.
[5] Raduta, Gulminelli, and Oertel 2017.

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Bruenn model

Computation : weak interaction + lowest order shell model
+ approximation of independant particles

$\Rightarrow$ Predicts no captures on nuclei with $N \geq 40$
(fewer captures at the end of the collapse, where neutron rich nuclei dominates the composition)

[3] Bruenn 1985.
[4] Langanke et al. 2003.
[5] Raduta, Gulminelli, and Oertel 2017.

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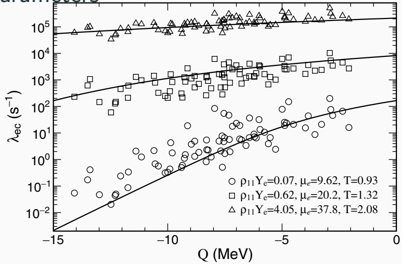
LMP model

Fit on results of nuclear shell models using the $Q - value$ dependance of the rate

⇒ all nuclei contribute to captures



[3] Bruenn 1985.
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3 models for electrons captures on nuclei

$$^A_Z\mathrm{X} + e^- \to {}^{\ \ A}_{Z-1}\mathrm{Y} + \nu_e$$

- Bruenn : approx. of independant particles[3]
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- ISO : same fit but done with more parameters[5]

Raduta's improvement of LMP fit

Improvement of the previous fit, done with more parameters :

- the *Q-value* ($Q = M(A, Z-1) - M(A, Z)$)
- thermodynamic conditions : T, $n_e = Y_e n_b$
- nuclear parameters : $I = (N - Z)/A$ and pairing

[3] Bruenn 1985.

[4] Langanke et al. 2003.

[5] Raduta, Gulminelli, and Oertel 2017.

Evolution of capture rates

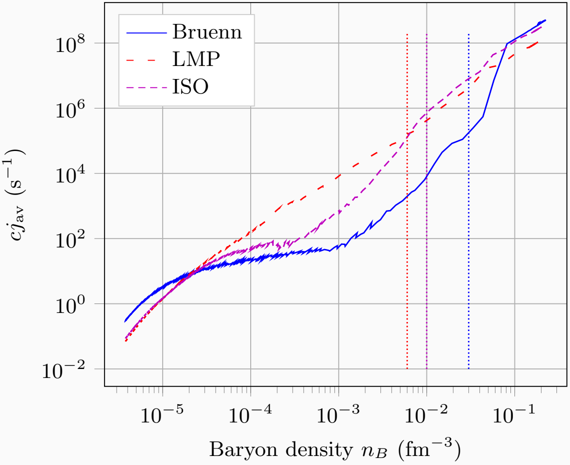


Figure 6: Evolution of the electron capture rates (on nuclei and free protons), in the central element, during the collapse. The vertical dashed lines show when β-equilibrium sets in, see Pascal et al. (2020)

Evolution of electron fraction

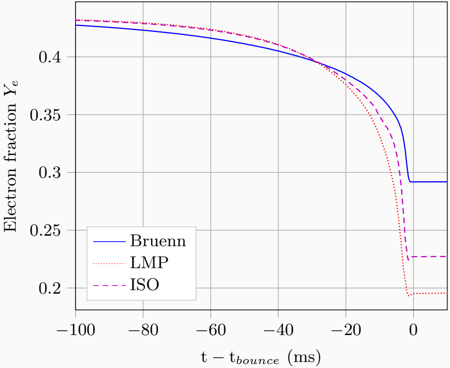


Figure 7: Evolution of the electron fraction in the central element, see Pascal et al. (2020)

Model	Bruenn	LMP	ISO
BC mass	0.45	0.31	0.4

Table 1: Mass of inner bouncing core (units of $M_\odot$)

$$Y_e \nearrow \quad \Rightarrow \quad P_{nuc} \searrow \quad \Rightarrow \quad M_{BC} \nearrow$$

(weaker nuclear pressure because nuclear matter is more symmetric)

Dynamic of the shock

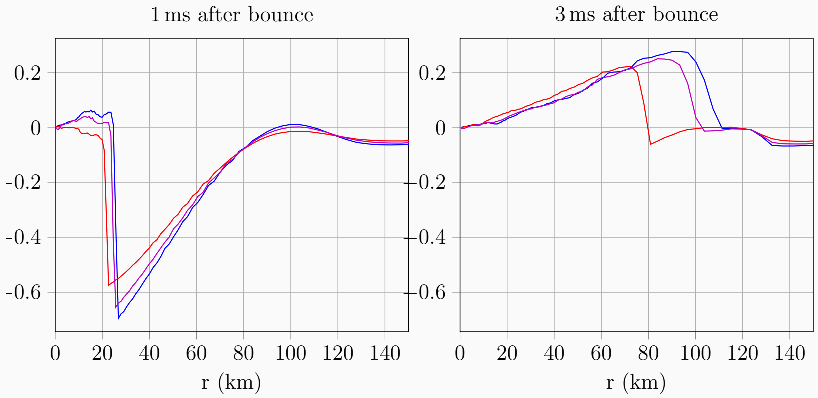
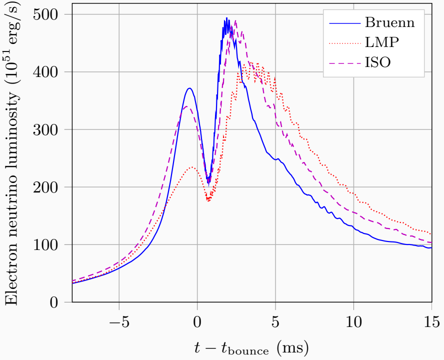
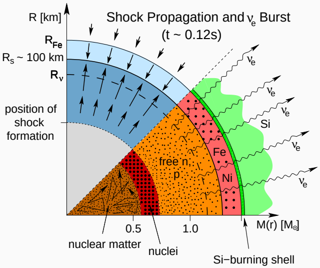


Figure 8: Radial velocity profiles in the early post-bounce phase, see Pascal et al. (2020)

Neutrino luminosity





(Janka et al. (2007))

Figure 9: Electron neutrino luminosity, as a function of time after bounce, see Pascal et al. (2020)

The formation of neutron stars

Proto Neutron Star and ν-emission

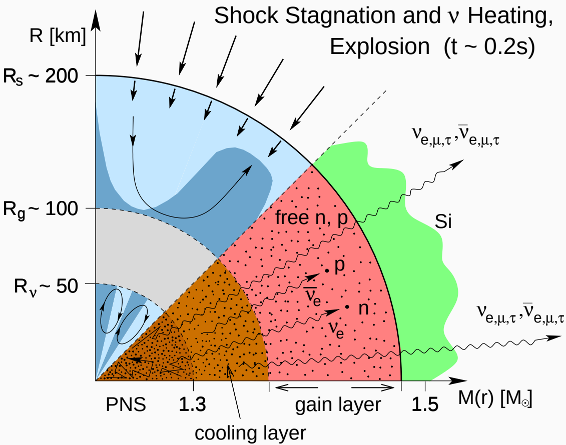


Figure 10: Core-collapse mechanism, figure extracted from Janka et al. (2007)

Proto-neutron star structure

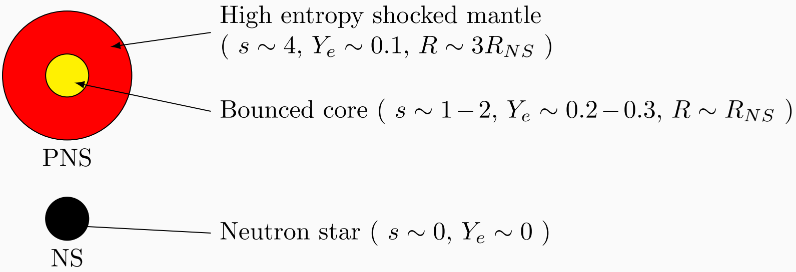


Figure 11: Schematic representation of a proto neutron star structure (PNS), compared to the corresponding cold catalysed neutron star (NS)

PNS cooling : $T_{PNS} \sim 10\,\text{MeV}\ (10^{11}\,\text{K}) \Rightarrow T_{NS} \sim 10\,\text{keV}\ (10^{8}\,\text{K})$

main mechanism : energy loss and deleptonization via emission of ν_e, ν_μ, ν_τ
$\Rightarrow$ mantle contraction with Kelvin-Helmoltz mechanism :
cooling via radiation $\rightarrow$ heating via contraction $\rightarrow$ cooling...

Relevant timescales

Acoustic timescale :

$$t_{\rm ac} = \frac{R}{c_{\rm sound}} = \left(\frac{R}{10\,\rm km}\right)\left(\frac{c_{\rm sound}}{10^8\,\rm m\,s^{-1}}\right)^{-1} \times 10^{-1}\,\rm ms$$

Deleptonization timescale :

$$t_{\rm delep} = \frac{Y_e N_B}{L_{\nu,n}} \approx \left(\frac{Y_e}{0.2}\right)\left(\frac{M}{1.6\,\rm M_\odot}\right)\left(\frac{L_{\nu,n}}{10^{55}\,\rm s^{-1}}\right)^{-1} \times 30\,\rm s$$

Where N_B is the total baryon number, M is the total mass and $L_{\nu,n}$ the total neutrino number-luminosity.

Kelvin-Helmholtz (star contraction) timescale :

$$t_{\rm KH} = \frac{GM^2}{RL_{\nu,e}} \approx \left(\frac{M}{1.6\,\rm M_\odot}\right)^2\left(\frac{R}{10\,\rm km}\right)^{-1}\left(\frac{L_{\nu,e}}{10^{52}\,\rm erg\,s^{-1}}\right)^{-1} \times 30\,\rm s$$

Where $L_{\nu,e}$ is the total luminosity.

Timescales

$$t_{\text{ac}} = 10^{-1}\,\text{ms}$$
$$t_{\text{delep}} = 30\,\text{s}$$
$$t_{\text{Kelvin-Helmoltz}} = 30\,\text{s}$$

We want to simulate $\sim 60\,\text{s}$ but the acoustic timescale limits timesteps to $\delta t \sim 10\,\mu\text{s}$

$\Rightarrow$ we use a quasi-stationary approximation to average acoustic effects and evolve the PNS over KH-time

Neutrino trapping/transparency

Neutrinos are trapped in dense, hot matter. Therefore they diffuse, as do photons in main sequences stars, and ν-radiation is close to a black body.

ν-matter cross sections are extremely temperature-dependant (because of Pauli-blocking effects) :

$$\sigma \propto T^5 \quad \text{(charge exchange with Nucleons)}$$
$$\sigma \propto T^8 \quad \text{(NN bremsstrahlung pair production)}$$

$\Rightarrow$ as the star cool down, it will progressively become ν-transparent

We need a neutrino radiation-transfer scheme

Open questions on PNS evolution

- how do uncertainties on microphysics (EoS and weak cross sections) influence the cooling ?
- how and when the NS does the crust form ? and what influence does it have on cooling ?
- what is the influence of the neutrino transport scheme
- to which extent convection effects contributes to the cooling ?
- what are the effects of rotation (meridional circulation, horizontal turbulence, magneto-dynamo...)
- what is the GW emission of a PNS ?

PNS modelling within the quasi-stationnary approximation

Hydrostatic approximation

We assume the star contracts slowly :
$\frac{\partial n_B}{\partial t} \sim 0$, $\frac{\partial p}{\partial t} \sim 0$, $\frac{\partial g_{\mu\nu}}{\partial t} \sim 0$

(but we still have $\frac{\partial s}{\partial t} \neq 0$ and $\frac{\partial Y_e}{\partial t} \neq 0$!)

$\Rightarrow p$ is computed via the TOV equations

Closure is obtained with a hot equation of state for dense matter[6] :
$(p, s, Y_e) \mapsto$ density, temperature, composition, chemical potentials, ...

[6] Oertel et al. 2017

Hydrostatic equilibrium - TOV equation

Metric in spherical symmetry :

$$ds^2 = -\alpha^2 c^2 dt^2 + \psi^2 dr^2 + r^2 (d\theta^2 + \sin^2\theta d\varphi^2)$$

Einstein equations :

$$\frac{1}{\psi} = \sqrt{1 - \frac{2Gm}{rc^2}}$$

$$\frac{dm}{dr} = 4\pi r^2 \frac{E}{c^2}$$

$$\frac{d\ln\alpha}{dr} = \psi^2 \frac{G}{c^2}\left(\frac{m}{r^2} + 4\pi r \frac{p}{c^2}\right)$$

Hydrostatic equilibrium equation :

$$\frac{dp}{dr} = -(E+p)\frac{d\ln\alpha}{dr}$$

Evolution equations

Despite the quasi-stationary approximation, we still have $\frac{\partial Y_e}{\partial t} \neq 0$ and $\frac{\partial s}{\partial t} \neq 0$ and we use evolution equations for Y_e and s to compute the next quasi-stationary state

The time evolution of Y_e and s comes from the source of electrons s_n and the source of energy s_e :

$$\nabla_\mu (n_B Y_e u^\mu) = s_n$$
$$u_\nu \nabla_\mu (T^{\mu\nu}) = s_e$$

which can be recasted as

$$\frac{1}{\alpha c} \frac{DY_e}{Dt} = \frac{s_n}{n_B}$$
$$\frac{1}{\alpha c} \frac{Ds}{Dt} = \frac{\alpha s_e - \mu_e s_n}{n_B T}$$

s_n and s_e have to be computed with a neutrino radiation-transfert scheme

Neutrino radiation-transfert scheme

we need the source terms for evolution :

$$s_n = -\frac{1}{c}\left(\Gamma_{\nu_e} - \Gamma_{\bar{\nu}_e}\right)$$
$$s_e = -\frac{1}{c}\left(Q_{\nu_e} + Q_{\bar{\nu}_e} + 4Q_{\nu_x}\right)$$

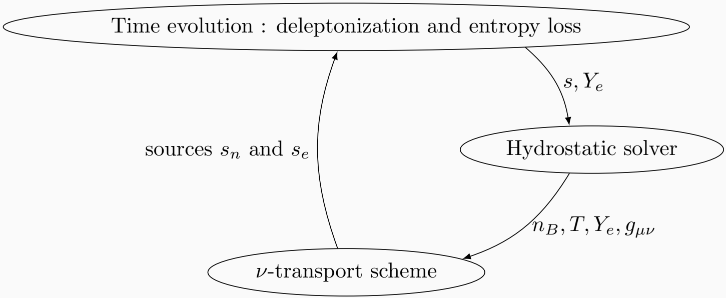
we use the Fast Multigroup Transport scheme[7] a *stationnary* approximation of the transport equation :

$$p^i\frac{\partial f}{\partial x^i} - \Gamma^i{}_{\mu\nu}p^\mu p^\nu\frac{\partial f}{\partial p^i} = u_\mu p^\mu \mathcal{B}[f]$$

at high optical depth we use the *two-stream approximation*
at low optical depth we use a *two-moment closure*

[7] Müller and Janka 2015.

The algorithm



Initial data

We simulate the collapse of the iron core of a $15\,M_\odot$ star[8]

For this we used the CoCoNuT[9]-Lorene[10] core-collapse code, with a full general relativistic hydrodynamic treatment.
Neutrinos transport scheme : fast multigroup transport (FMT)[11]

Simulation is stopped 500 ms after bounce, we discard matter beyond the stalled shock
$\Rightarrow$ extracted baryon mass : $M_B = 1.6\,M_\odot$

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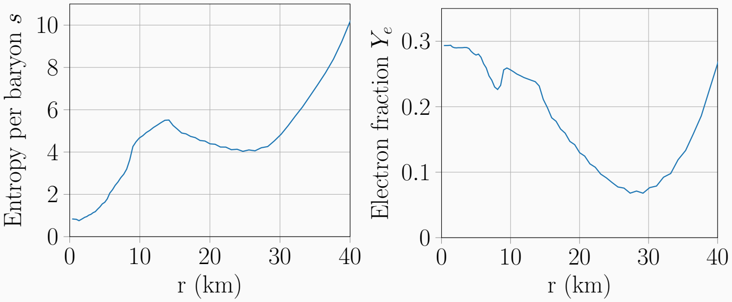
[8] Woosley, Heger, and Weaver 2002.
[9] Dimmerlmeier, Novak, and Cerdá-Durán 2001-2007.
[10] Gourgoulhon et al. 1997-2012.
[11] Müller and Janka 2015.

Initial data



(a) Entropy per baryon **(b)** Electron fraction

Figure 12: Initial profiles

Some exemples of results

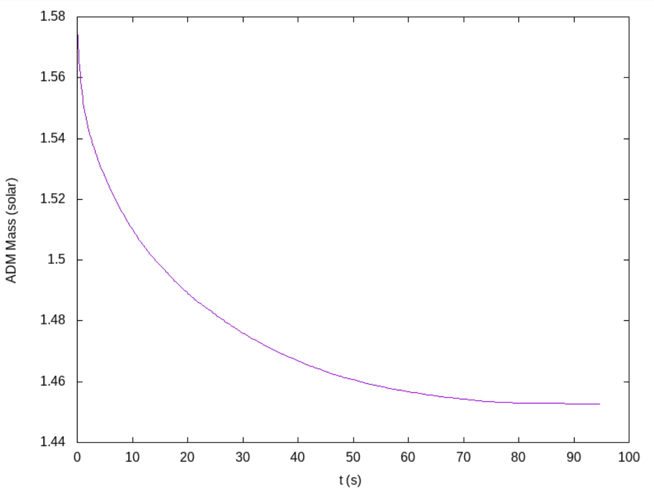


Figure 13: Evolution of the mass of the PNS

Some exemples of results

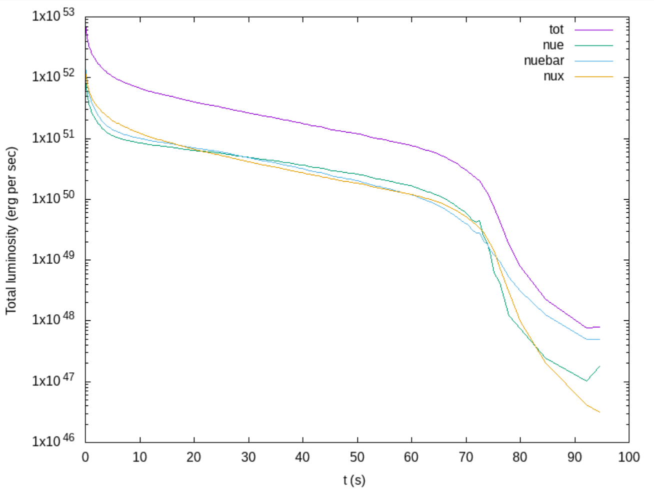


Figure 14: Evolution of the ν-luminosity of the PNS

Impact of different approximation for charged current reactions on nucleons

Models for charged current reactions on nucleons

$$p + e^- \leftrightarrows n + \nu_e \qquad p \leftrightarrows n + e^+ + \nu_e$$

$$n + e^+ \leftrightarrows p + \bar{\nu}_e \qquad n \leftrightarrows p + e^- + \bar{\nu}_e$$

- Elastic[12] : zero momentum transfert, independant particles
- Mean Field (MF)[13] : full kinematics, mean field corrections
- Random Phase Approximation (RPA)[14] : full kinematics, some correlation effects added to mean field theory[15]

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[12] Bruenn 1985.
[13] Oertel et al. 2020.
[14] Oertel et al. 2020.
[15] in extremely dense mediums mean field theory might not be good enough

Influence of various models for nucleons charged currents

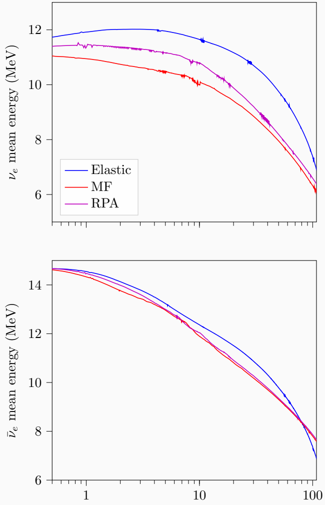

Figure 15: Mean energy (in MeV) of neutrinos emission as a function of time (in s), for various models of nucleons charged currents

Influence of various models for nucleons charged currents

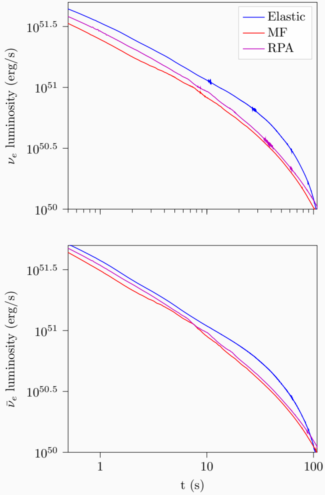

Figure 16: Luminosity (in erg/s) of neutrinos emission as a function of time (in s), for various models of nucleons charged currents

Convection in proto-neutron stars

Origin of convection

In Proto-Neutron Stars the stability of the stratification depend on s and Ye profiles, and on how the pressure varies according to these two quantities $p = p(n_B, s, Ye)$

Ledoux stability criterion[16] :

$$\left(\frac{\partial \log p}{\partial \log s}\right)_{n_B, Y_e} \frac{\partial \log s}{\partial r} + \left(\frac{\partial \log p}{\partial \log Y_e}\right)_{n_B, s} \frac{\partial \log Y_e}{\partial r} > 0$$

The loss of energy in the mantle layers makes the stratification unstable.

$\Rightarrow$ mixing of layers and uniformisation of s and Y_e profiles in unstable regions

[16] Roberts et al. 2012.

Influence of convection

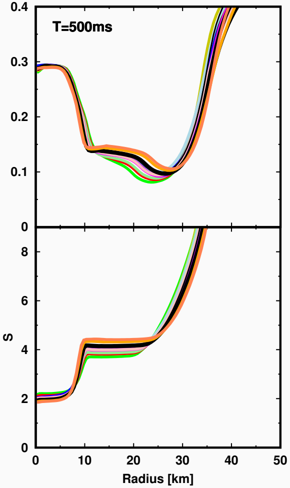

Figure 17: Angle averaged Y_e and s profiles at $T = 500\,\text{ms}$ in the case of a 3D simulation (Nagakura et al. (2020))

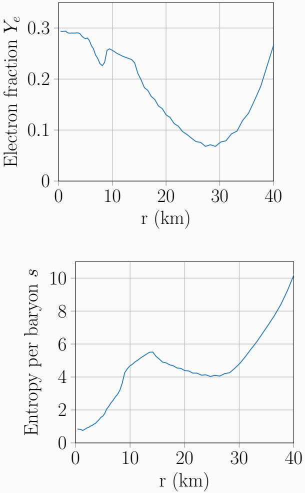

Figure 18: Y_e and s profiles from a 1D simulation

Approachs to convection modelling over long timescales

From the less accurate/expensive to the most accurate/expensive

- Mixing Length Theory : use a diffusion equation for s and Y_e in unstable areas
- Two columns formalism Stökl 2008
- Anelastic 2D models
- full hydrodynamics

Mixing length theory

This is a 1D model of convection. We introduce the buoyancy frequency :

$$\omega_b^2 = \frac{g}{\left(\frac{\partial \log p}{\partial \log n_B}\right)_{Y_e,s}} \left[\left(\frac{\partial \log p}{\partial \log s}\right)_{n_B,Y_e} \frac{\partial \log s}{\partial r} + \left(\frac{\partial \log p}{\partial \log Y_e}\right)_{n_B,s} \frac{\partial \log Y_e}{\partial r} \right]$$

with $g = c^2 \frac{d \ln \alpha}{dr}$ is the local gravitational acceleration.
In areas where $\omega_b^2 \leq 0$, we model the mixing of Y_e and s by a diffusion equation

The diffusion coefficient D_{MLT} is given by

$$D_{\mathrm{MLT}} = n_B v_c \lambda_c$$

The mixing length is coupled to the pressure scale height :
$\lambda_c = \xi p \left(\frac{dp}{dr}\right)^{-1}$

And the convection velocity is given by $v_c = \lambda_c \sqrt{-2\omega_b^2}$,

Exemple of results

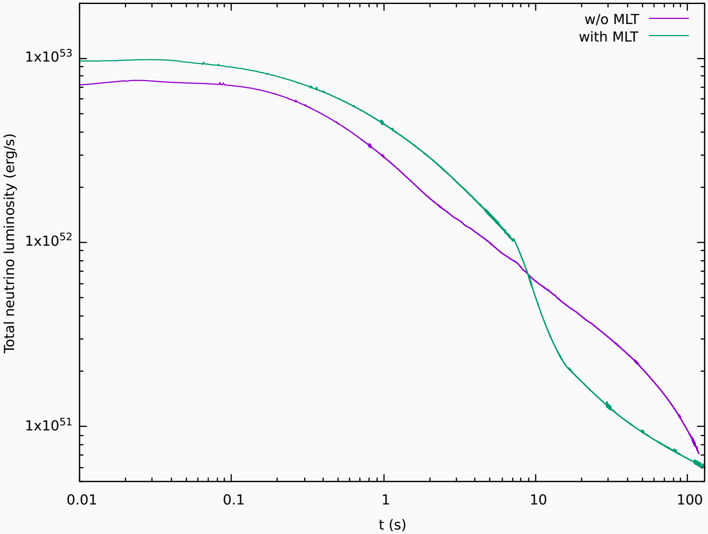

Figure 19: Neutrino luminosity as a function of time with and without MLT

Conclusion

- the model of electron capture rates is of great influence on the results of a core-collapse simulation
- a new code for modelling proto-neutron star cooling has been developped
- we are currently investigating the effect of charged currents in proto-neutron stars modelisation using this new cooling code
- convection effects on the neutrino emission and the cooling are important and are investigated using the new cooling code