Tidal effects up to second post-Newtonian order in inspiralling binary neutron-star systems

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This presentation is based on three papers:

- "Tidal effects in the equations of motion of compact binary systems to next-to-next-to-leading post-Newtonian order"
  Phys. Rev. D 101, 064047 [HFB20a]

- "Tidal effects in the gravitational-wave phase evolution of compact binary systems to next-to-next-to-leading post-Newtonian order"
  Phys. Rev. D 102, 044033 [HFB20b]

- "Hamiltonian for tidal interactions in compact binary systems to next-to-next-to-leading post-Newtonian order"
  Phys. Rev. D 102, 124074 [HFB20c]

Outline

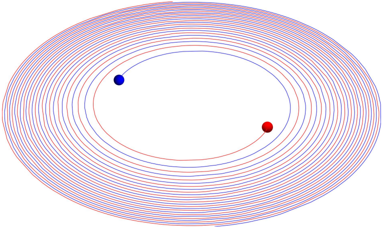
1) Context and motivations

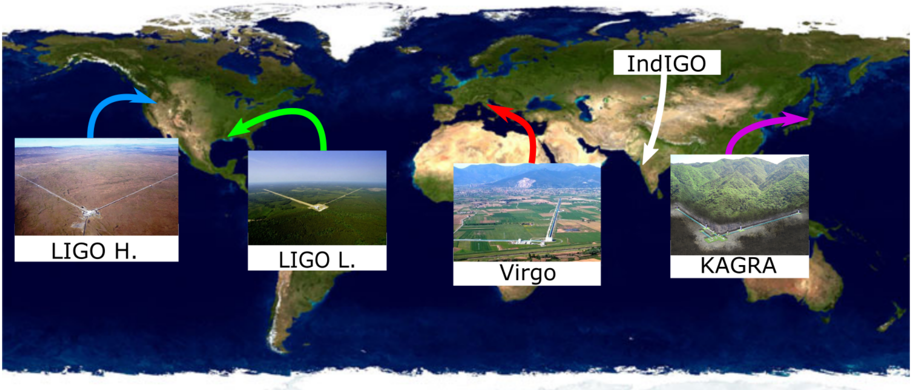
2) Effective action at 2PN

3) Conservative sector

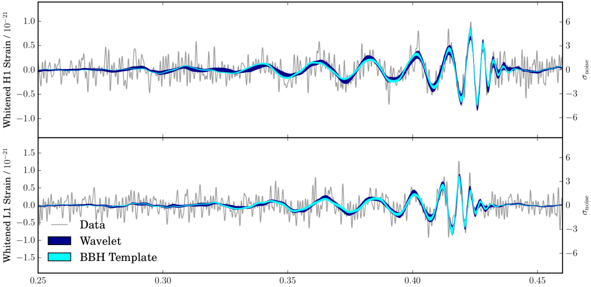
4) Radiative sector

5) Conclusion and perspectives




IndIGO
LIGO H.
LIGO L.
Virgo
KAGRA

GW150914:


Whitened H1 Strain / 10^{-21}
Whitened L1 Strain / 10^{-21}
σ_{noise}
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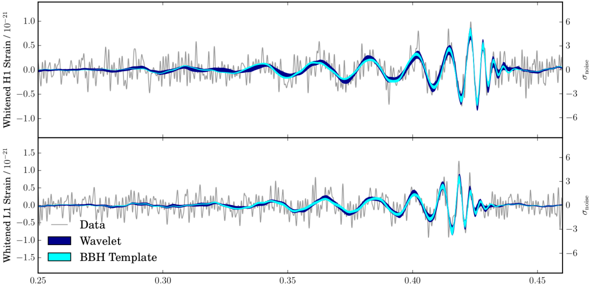
GW150914:



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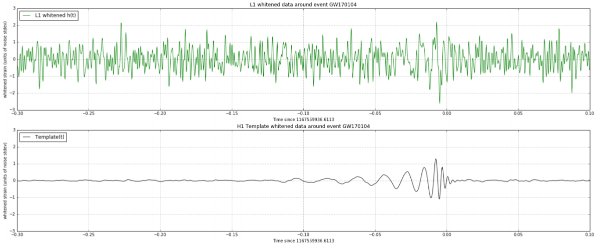
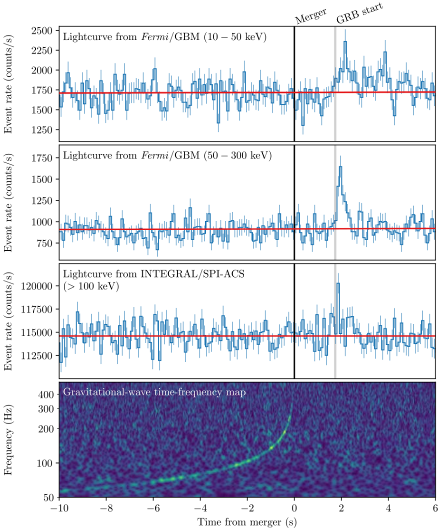


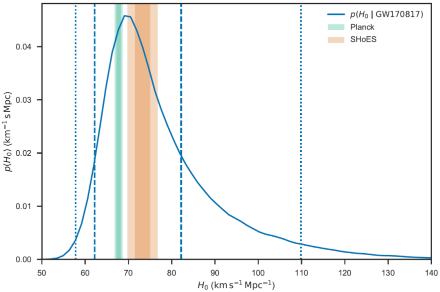
GW170817



$$D \sim 40Mpc \quad \Rightarrow \quad |c_{\rm g} - c_{\rm em}| \lesssim 10^{-15} c_{\rm em}$$

BNS mergers: what can we learn ?

- Discarded alternative theories of gravity through the constrain on c
- Bounding the mass of the graviton $m_g \lesssim 10^{-22}$eV
- Allows an independant measurement of the Hubble-Lemaitre constant.



- Constrain the EOS of neutron stars.

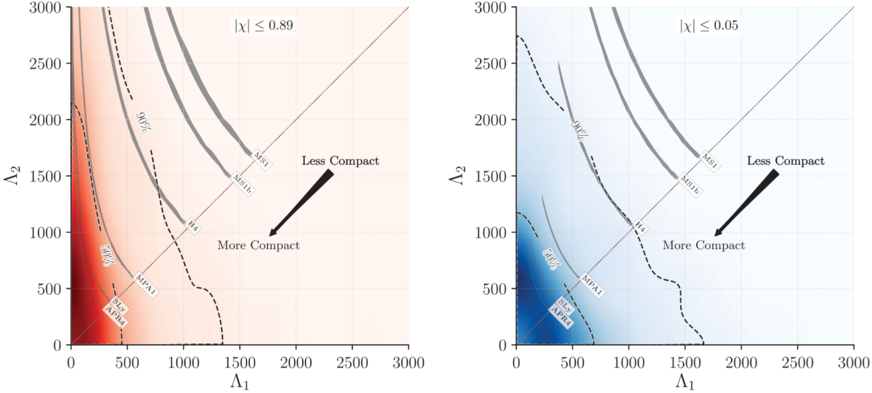
Link between tidal effects and internal structure

We parametrize the deformability of an body in a static tidal field through a set of parameters called Love numbers.

The constrain on EOS from GW170817

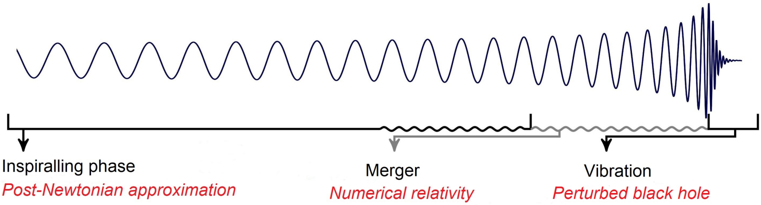
$$\Lambda = \frac{2k^{(2)}}{3}\left(\frac{c^2 R}{Gm}\right)^5$$

$k^{(2)}$: second Love number, parametrizes the quadrupolar deformation.



[LIGO, Virgo PRL 116, 061102 (2016)]

The post-Newtonian formalism



$\hookrightarrow$ Tidal effects negligible in the inspiralling phase but measurable in the pre-merger.

PN formalism :

- Perturbative expansion of the equations of GR.
- Weak field, small velocities : $(v/c)^2 \sim Gm/rc^2 \ll 1$.
- 2^{nd} relative PN order $\rightarrow O(1/c^4)$.
- Decompose the problem in conservative and radiative sectors

$$\mathscr{F} = -\frac{\mathrm{d}E}{\mathrm{d}t}$$

Motivations on the 2PN tidal computation

- All GW detections from LIGO/Virgo came from compact binaries.

	GW150914	GW170817
Chirp mass ($M_\odot$)	$30.4^{+2.1}_{-1.9}$	$1.188^{+0.004}_{-0.002}$
Cycles	8	~ 3000

[LIGO, Virgo PRL 116, 061102 (2016); PRL 119, 161101 (2017)]

- NR inadequate for high number of cycles.
- Necessity to use analytical models for data analysis.
- Build more precise templates.
- Extend overlap between PN and NR.
- Compare with different analytical methods.
- Allows to constrain the equation of state of neutron stars through Love numbers $\{k^{(\ell)}, j^{(\ell)}\}$.

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2) Effective action at 2PN

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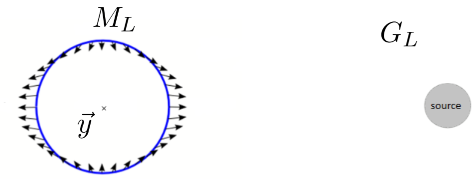
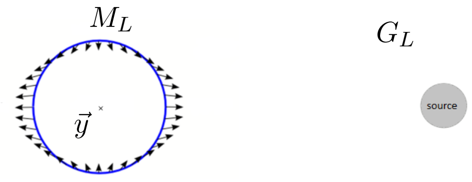
Tidal effects in Newtonian gravity



Extended body in a non homogeneous gravitational field
$\rightarrow$ Taylor expansion around $\vec{y}$.

$$F^i_{\text{ext}} = m\,\partial_i U_{\text{ext}} + \sum_{\ell\geq 2} \frac{1}{\ell!} M_L G_{iL} \quad \leftarrow L = i_1 \ldots i_\ell$$

Tidal effects in Newtonian gravity



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In the adiabatic approximation : $\boxed{M_L = \mu^{(\ell)} G_L}$

GR effective action for static finite-size effects without spins

Most general non minimal worldline action satisfying:

- depending only on the metric and 4-velocity
- invariance under reparametrisation
- parity invariance

$$S_{\rm nm} = \sum_A \sum_{\ell\geq 2} \left[\frac{1}{2\ell!}\mu_A^{(\ell)} \int \mathrm{d}\tau_A\, G^A_{\alpha_1\dots\alpha_\ell}(\tau_A)\, G_A^{\alpha_1\dots\alpha_\ell}(\tau_A) \right.$$
$$+ \frac{\ell}{2(\ell+1)\ell!\, c^2}\sigma_A^{(\ell)} \int \mathrm{d}\tau_A\, H^A_{\alpha_1\dots\alpha_\ell}(\tau_A)\, H_A^{\alpha_1\dots\alpha_\ell}(\tau_A)$$
$$+ \frac{1}{2\ell!\, c^2}\mu_A^{'(\ell)} \int \mathrm{d}\tau_A\, \dot{G}^A_{\alpha_1\dots\alpha_\ell}(\tau_A)\, \dot{G}_A^{\alpha_1\dots\alpha_\ell}(\tau_A)$$
$$\left. + \frac{\ell}{2(\ell+1)\ell!\, c^4}\sigma_A^{'(\ell)} \int \mathrm{d}\tau_A\, \dot{H}^A_{\alpha_1\dots\alpha_\ell}(\tau_A)\, \dot{H}_A^{\alpha_1\dots\alpha_\ell}(\tau_A) + \dots \right]$$

$\hookrightarrow$ G and H are the electric and magnetic parts of the Weyl tensor.
$\hookrightarrow$ Contains also cubic terms (higher order).

Effective action at 2PN

$$S = S_{\rm EH} + S_{\rm pp} + S_{\rm T}$$

$$S_{\rm T} = \sum_{\rm A=1,2} \int {\rm d}\tau_A \left[\frac{\mu_A^{(2)}}{4} G^A_{\alpha\beta} G^{\alpha\beta}_A + \frac{\sigma_A^{(2)}}{6c^2} H^A_{\alpha\beta} H^{\alpha\beta}_A + \frac{\mu_A^{(3)}}{12} G^A_{\alpha\beta\gamma} G^{\alpha\beta\gamma}_A \right]$$

- $G_{\alpha\beta} \equiv -R_{\alpha\mu\beta\nu} u^\mu u^\nu$: tidal quadrupole moment (mass type)
- $H_{\alpha\beta} \equiv 2R^*_{\alpha\mu\beta\nu} u^\mu u^\nu$: tidal quadrupole moment (current type)
- $G_{\alpha\beta\gamma}$: tidal octupole moment (mass type)

$\hookrightarrow$ We regularise the tensors on the location of body A.

[Bini, Damour, Faye 2012]

Effective action at 2PN

$$S_{\mathrm{T}} = \sum_{\mathrm{A}=1,2} \int \mathrm{d}\tau_A \left[\frac{\mu_A^{(2)}}{4} G^A_{\alpha\beta} G_A^{\alpha\beta} + \frac{\sigma_A^{(2)}}{6c^2} H^A_{\alpha\beta} H_A^{\alpha\beta} + \frac{\mu_A^{(3)}}{12} G^A_{\alpha\beta\gamma} G_A^{\alpha\beta\gamma} \right]$$

- $\mu^{(\ell)}$ and $\sigma^{(\ell)}$ linked to Love numbers : $\mu_A^{(\ell)} \propto k_A^{(\ell)} R_A^{2\ell+1}$.
- $\mathscr{C} = \dfrac{Gm}{Rc^2} \sim 1$ for compact objects.
- $\mu^{(2)} \sim O\left(\dfrac{1}{c^{10}}\right) \sim \sigma^{(2)} \to$ Newtonian (leading) order (5PN)
- $\mu^{(3)} \sim O\left(\dfrac{1}{c^{14}}\right) \quad \to$ 2PN relative (7PN)

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The Fokker Lagrangian, reduced Lagrangian

$$\boxed{S[q, g_{\mu\nu}] = S_{\rm EH} + S_{\rm pp} + S_{\rm T}} \qquad \text{with} \qquad S_{\rm T} \ll S_{\rm EH} + S_{\rm pp}$$

$$\frac{\delta S}{\delta g_{\mu\nu}} = 0 \Rightarrow g_{\mu\nu}^{(\rm sol)}[q] \qquad (q = \{\vec{y}_A, \vec{v}_A, \vec{a}_A, \ldots, \vec{a}_A^{(n)}\})$$

Fokker action : $S_{\rm Fokker}[q] \equiv S[q, g_{\mu\nu}^{(\rm sol)}[q]]$

→ Same EoM
→ No need of tidal effects in $g_{\mu\nu}^{(\rm sol)}$

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Reduced Lagrangian :

- $S_{\rm Fokker} \to L_{\rm Fokker}(\vec{y}_A, \vec{v}_A, \vec{a}_A, \dots, \vec{a}^{(n)}_A)$
- $L_{\rm Fokker} \to$ Reduction method $\to L(\vec{y}_A, \vec{v}_A, \vec{a}_A)$

The Fokker Lagrangian, reduced Lagrangian

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- $L_{\rm Fokker} \rightarrow$ Reduction method $\rightarrow L(\vec{y}_A, \vec{v}_A, \vec{a}_A)$

10 conserved quantities : $\{P^i, J^i, G^i, E\}$

Computations in the considered problem

The matter Lagrangian:

$$L_{\mathrm{m}} = \sum_A \left(-m_A c^2 + \frac{\mu_A^{(2)}}{4} G^A_{\alpha\beta} G_A^{\alpha\beta} + \frac{\sigma_A^{(2)}}{6c^2} H^A_{\alpha\beta} H_A^{\alpha\beta} + \frac{\mu_A^{(3)}}{12} G^A_{\alpha\beta\gamma} G_A^{\alpha\beta\gamma} \right)$$

The 2PN metric: parametrized by a set of potentials

$$g_{00} = -1 + \frac{2V}{c^2} - \frac{2V^2}{c^4} + \frac{8}{c^6}\left(\hat{X} + V_i V_i + \frac{V^3}{6}\right) + O\left(\frac{1}{c^8}\right),$$

$$g_{0i} = -\frac{4V_i}{c^3} - \frac{8\hat{R}_i}{c^5} + O\left(\frac{1}{c^7}\right),$$

$$g_{ij} = \delta_{ij}\left(1 + \frac{2V}{c^2} + \frac{2V^2}{c^4}\right) + \frac{4\hat{W}_{ij}}{c^4} + O\left(\frac{1}{c^6}\right).$$

↪Insert the metric in the 3+1 decomposition of $L = L[V, V_i, \hat{W}_{ij}, \hat{R}_i, \hat{X}]$

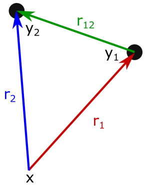
↪Insert the values of the regularised point-particle potentials.

Regularisation

We use the **pure Hadamard** regularisation:

↪ Laurent expansion of a function

$$F(\vec{x}) = \sum_{k_0 \leq k \leq 0} r_1^k f_k(\vec{n}_1) + o(r_1)$$



↪ The value of a function at 1

$$(F)_1 = \int \frac{\mathrm{d}\Omega}{4\pi} f_0(\vec{n}_1)$$

↪ Equivalent to dimensional regularisation up to 2PN order.

$$V = \frac{Gm_1}{r_1} + \frac{Gm_2}{r_2} + O\left(\frac{1}{c^2}\right) \quad \Rightarrow \quad (V)_1 = \frac{Gm_2}{r_{12}} + O\left(\frac{1}{c^2}\right)$$

Reduction method for the Lagrangian

At this stage, we have $L_{\text{Fokker}}(\vec{y}_A, \vec{v}_A, \vec{a}_A, \ldots, \vec{a}_A^{(n)})$

$\hookrightarrow$ Replace $\vec{a}_A^{(k)} = \vec{A}_A^{(k)} + \delta\vec{a}_A^{(k)}$ where $\vec{A}_A^{(k)}$ is the onshell value, $\delta\vec{a}^{(k)}|_{\text{os}} = 0$

$\hookrightarrow$ Three types of terms:

- No $\delta\vec{a}_A^{(k)} \Rightarrow$ depends only on $(\vec{y}_A, \vec{v}_A)$
- Linear in $\delta\vec{a}_A^{(k)}$
- At least quadratic in $\delta\vec{a}_A^{(k)} \Rightarrow 0$ after variation of the Lagrangian

$\hookrightarrow$ Replace back $\delta\vec{a}_A^{(k)} = \vec{a}_A^{(k)} - \vec{A}_A^{(k)}$

$$\vec{C}(\vec{y}_A, \vec{v}_A).\vec{a}_A^{(n)} \to -\dot{\vec{C}}(\vec{y}_A, \vec{v}_A).\vec{a}_A^{(n-1)}$$

$\hookrightarrow$ Reiteration of the procedure until the obtention of $L(\vec{y}_A, \vec{v}_A, \vec{a}_A)$

Straightforward to obtain the conserved quantities $\{P^i, J^i, G^i, E\}$.

2PN CoM energy for circular orbits

$$x \equiv \left(\frac{Gm\omega}{c^3}\right)^{2/3}, \qquad \nu \equiv \frac{m_1 m_2}{m^2}, \qquad \Delta \equiv \frac{m_1 - m_2}{m}$$

$$\begin{aligned}
E = & -\frac{\mu c^2 x}{2}\Bigg\{1 + \left(-\frac{3}{4} - \frac{\nu}{12}\right)x + \left(-\frac{27}{8} + \frac{19}{8}\nu - \frac{\nu^2}{24}\right)x^2 \\
& -18\,\widetilde{\mu}_+^{(2)} x^5 + \left[\left(-\frac{121}{2} + 33\nu\right)\widetilde{\mu}_+^{(2)} - \frac{55}{2}\Delta\,\widetilde{\mu}_-^{(2)} - 176\widetilde{\sigma}_+^{(2)}\right]x^6 \\
& + \left[\left(-\frac{20865}{56} + \frac{5434}{21}\nu - \frac{91}{4}\nu^2\right)\widetilde{\mu}_+^{(2)} + \Delta\left(-\frac{11583}{56} + \frac{715}{12}\nu\right)\widetilde{\mu}_-^{(2)}\right. \\
& \left. + \left(-\frac{2444}{3} + \frac{1768}{3}\nu\right)\widetilde{\sigma}_+^{(2)} - \frac{884}{3}\Delta\,\widetilde{\sigma}_-^{(2)} - 130\widetilde{\mu}_+^{(3)}\right]x^7\Bigg\}
\end{aligned}$$

[HFB20a]

Hamiltonian for tidal effects

$\hookrightarrow$ Hamiltonian computed by other groups with different methods (EFT, scattering amplitudes) in isotropic coordinates in the CoM.

- Transform $L(\vec{y}_A, \vec{v}_A, \vec{a}_A)$ into $L'(\vec{y}_A^{\,\prime}, \vec{v}_A^{\,\prime})$ using a coordinate shift.
- Get a Hamiltonian in the CoM.
- Canonical transformation to get a Hamiltonian in isotropic coordinates.
- Comparison: overlap in full agreement with the literature [Solon et al. 2020, Porto et al. 2020].

[HFB20c]

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(Very) brief overview of the PN-MPM formalism

$$\varphi(\omega)$$

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$$\mathscr{F} = \frac{G}{c^5}\left[\frac{1}{5}\mathrm{U}^{(1)}_{ij}\mathrm{U}^{(1)}_{ij} + \frac{1}{c^2}\left(\frac{1}{189}\mathrm{U}^{(2)}_{ijk}\mathrm{U}^{(2)}_{ijk} + \frac{16}{45}\mathrm{V}^{(1)}_{ij}\mathrm{V}^{(1)}_{ij}\right) + \dots\right]$$

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$$\mathrm{I}_{ij} = \underset{B=0}{\mathrm{FP}} \int \mathrm{d}^3x\, r^B \hat{x}^{ij} \left[\sigma - \frac{1}{\pi G c^2}\partial_k V \partial_k V + \dots\right]$$

(Very) brief overview of the PN-MPM formalism

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$$\sigma = \rho + \dots, \qquad V = -4\pi G \Box^{-1}\sigma = \frac{G m_1}{r_1} + \dots$$

Form of the stress energy tensor composed of multipoles

$$\boxed{T^{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g_{\mu\nu}}}$$

$$T^{\mu\nu}_{\text{pole}} = n p^{(\mu} u^{\nu)}, \quad p^{\mu} = \frac{\partial L}{\partial u^{\mu}}$$

$$T^{\mu\nu}_{\text{quad}} = n\frac{1}{3}R^{(\mu}{}_{\lambda\rho\sigma} J^{\nu)\lambda\rho\sigma} - \nabla_{\rho}\nabla_{\sigma}\left[n\frac{2}{3}J^{\rho(\mu\nu)\sigma}\right], [1] \quad J^{\mu\nu\rho\sigma} = -6\frac{\partial L}{\partial R_{\mu\nu\rho\sigma}}$$

↪ Similar expression for the octupolar part.

↪ $J^{\mu\nu\rho\sigma}$ contains the quadrupolar tidal information.

[Bailey, Israel 1975 ; Marsat 2015]

↪ This method and direct derivations have been done.

↪ Expand the covariant into partial derivatives + Christoffels.

↪ Decompose the expressions in 3+1 (spatial and temporal indices).

[1] 3d covariant Dirac: $n = \delta^{(3)}(x - y(\tau))/(u^0\sqrt{-g})$

3+1 decomposition of the stress-energy tensor

$$
\begin{aligned}
\sigma = {}& \frac{1}{\sqrt{-g}}\partial_t\left[\delta_1\frac{\mu^{(2)}(G_{1a}^a)(v_1^a\partial_a V)-\mu^{(2)}(G_{1ab}v_1^a\ \partial^b V)-2\mu^{(2)}(G_{1ab}\partial^b V^a)}{c^4}\right]+\frac{1}{\sqrt{-g}}\partial_t^2\left[\delta_1\big(\frac{\mu^{(2)}(G_{1a}^a)}{2c^2}-\frac{\mu^{(2)}(G_{1ab}v_1^a v_1^b)+2\mu^{(2)}(G_{1a}^a)V}{c^4}\big)\right]\\
&+\frac{1}{\sqrt{-g}}\partial_t\partial_k\left[\delta_1\big(-\frac{\mu^{(2)}G_{1a}^k v_1^a-\mu^{(2)}(G_{1a}^a)v_1^k}{c^2}+\frac{\mu^{(2)}G_{1a}^k(v_{1a}v_1^a)v_1^a+4\mu^{(2)}G_{1a}^k V v_1^a-\mu^{(2)}(G_{1ab}v_1^a v_1^b)v_1^k-4\mu^{(2)}(G_{1a}^a)Vv_1^k}{c^4}\big)\right]\\
&+\frac{1}{\sqrt{-g}}\partial_k\Big[\delta_1\big(\frac{1}{c^2}(\mu^{(2)}G_{1a}^k\partial^a V-\mu^{(2)}(G_{1a}^a)\partial^k V)-\frac{1}{c^4}(4\mu^{(2)}G_{1a}^k(v_1^a\partial_a V)v_1^a-3\mu^{(2)}(G_{1a}^a)(v_1^a\partial_a V)v_1^k+4\mu^{(2)}(G_{1ab}v_1^a\partial^b V)v_1^k+4\mu^{(2)}(G_{1ab}\partial^b V^a)v_1^k\\
&+\mu^{(2)}G_{1a}^k v_1^a\partial_t V-\mu^{(2)}(G_{1a}^a)v_1^k\partial_t V-4\mu^{(2)}G_{1a}^k\partial_t V^a+2\mu^{(2)}(G_{1a}^a)\partial_t V^k-8\mu^{(2)}G_{1b}^k v_1^a\partial_a V^b+2\mu^{(2)}(G_{1a}^a)v_1^a\partial_a V^k-3\mu^{(2)}G_{1a}^k(v_{1a}v_1^a)\partial^a V\\
&+8\mu^{(2)}G_{1a}^k V\partial^a V-2\mu^{(2)}G_1^{ab}\partial_b\hat{W}^k{}_a-\frac{8}{3}\sigma^{(2)}\epsilon^k{}_{ai}H_{1b}{}^i v_1^a\partial^b V+\frac{8}{3}\sigma^{(2)}\epsilon_{abi}H_1^{ki}v_1^a\partial^b V+4\mu^{(2)}G_{1b}^k v_1^a\partial^b V_a-\frac{8}{3}\sigma^{(2)}\epsilon^k{}_{bi}H_{1a}{}^i\partial^b V^a-\frac{8}{3}\sigma^{(2)}\epsilon_{abi}H_1^{ki}\partial^b V^a\\
&-2\mu^{(2)}G_{1ab}v_1^a\partial^b V^k+\mu^{(2)}(G_{1a}^a)(v_{1a}v_1^a)\partial^k V-3\mu^{(2)}(G_{1ab}v_1^a v_1^b)\partial^k V-6\mu^{(2)}(G_{1a}^a)V\partial^k V-2\mu^{(2)}(G_{1a}^a)v_1^a\partial^k V_a+2\mu^{(2)}G_{1ab}v_1^a\partial^k V^b+\mu^{(2)}G_1^{ab}\partial^k\hat{W}_{ab})\big)\Big]\\
&+\frac{1}{\sqrt{-g}}\partial_l\partial_k\Big[\delta_1\big(\frac{1}{2}\mu^{(2)}G_1^{kl}+\frac{1}{c^2}(\frac{1}{2}\mu^{(2)}G_1^{kl}(v_{1a}v_1^a)-2\mu^{(2)}G_1^{kl}V+\frac{2}{3}\sigma^{(2)}\epsilon^l{}_{ab}H_1^{kb}v_1^a+\frac{2}{3}\sigma^{(2)}\epsilon^k{}_{ab}H_1^{lb}v_1^a-\mu^{(2)}G_{1a}^l v_1^a v_1^k-\mu^{(2)}G_{1a}^k v_1^a v_1^l+\frac{1}{2}\mu^{(2)}(G_{1a}^a)v_1^k v_1^l)\\
&-\frac{1}{c^4}(2\mu^{(2)}G_1^{kl}(v_{1a}v_1^a)V-4\mu^{(2)}G_1^{kl}V^2+\frac{8}{3}\sigma^{(2)}\epsilon^l{}_{ab}H_1^{kb}Vv_1^a+\frac{8}{3}\sigma^{(2)}\epsilon^k{}_{ab}H_1^{lb}Vv_1^a-2\mu^{(2)}G_{1a}^l Vv_1^a v_1^k+\frac{2}{3}\sigma^{(2)}\epsilon^l{}_{bi}H_{1a}{}^i v_1^a v_1^b v_1^k-2\mu^{(2)}G_{1a}^k Vv_1^a v_1^l\\
&+\frac{2}{3}\sigma^{(2)}\epsilon^k{}_{bi}H_{1a}{}^i v_1^a v_1^b v_1^l-\frac{1}{2}\mu^{(2)}(G_{1ab}v_1^a v_1^b)v_1^k v_1^l+2\mu^{(2)}(G_{1a}^a)Vv_1^k v_1^l+\frac{4}{3}\sigma^{(2)}\epsilon^l{}_{ab}H_1^{kb}V^a+\frac{4}{3}\sigma^{(2)}\epsilon^k{}_{ab}H_1^{lb}V^a-2\mu^{(2)}G_{1a}^l v_1^k V^a\\
&-2\mu^{(2)}G_{1a}^k v_1^l V^a-2\mu^{(2)}G_{1a}^l v_1^a V^k-2\mu^{(2)}G_{1a}^k v_1^a V^l+2\mu^{(2)}G_1^{la}\hat{W}^k{}_a+2\mu^{(2)}G_1^{ka}\hat{W}^l{}_a)\big)\Big]\\
&+\delta_1\Big[\frac{1}{c^2}\big(-\frac{1}{4}\mu^{(2)}(G_{1ab}G_1^{ab})-\frac{2}{3}\sigma^{(2)}(G_{1ab}G_1^{ab})\big)+\frac{1}{c^4}\big(-\frac{1}{4}\mu^{(2)}(G_{1ab}G_1^{ab})\ (v_{1a}v_1^a)-\frac{2}{3}\sigma^{(2)}(G_{1ab}G_1^{ab})\ (v_{1a}v_1^a)+\frac{1}{2}\mu^{(2)}(G_{1b}{}^a G_{1ia}v_1^b v_1^i)\\
&+\frac{4}{3}\sigma^{(2)}(G_{1b}{}^a G_{1ia}v_1^b v_1^i)+\mu^{(2)}(\epsilon_{jbi}G_1^{ab}H_{1a}{}^i v_1^j)+4\sigma^{(2)}(\epsilon_{jbi}G_1^{ab}H_{1a}{}^i v_1^j)-2\mu^{(2)}(G_{1a}^a)(v_1^a\partial_t\partial_a V)+2\mu^{(2)}(G_{1ab}v_1^a\partial_t\partial^b V)-\mu^{(2)}(G_{1a}^a)(\partial_a V\partial^a V)\\
&-2\mu^{(2)}(v_{1a}v_1^a)(G_1^{ab}\partial_b\partial_a V)-2\mu^{(2)}(G_{1a}^a)(v_1^a v_1^b\partial_b\partial_a V)+4\mu^{(2)}(G_1^{ab}v_1^i\partial_b\partial_a V_i)+3\mu^{(2)}(G_{1ab}\partial^a V\partial^b V)+4\mu^{(2)}(G_{1b}{}^a v_1^b v_1^i\partial_i\partial_a V)-4\mu^{(2)}(G_1^{ab}v_1^i\partial_i\partial_b V_a)\\
&-\frac{8}{3}\sigma^{(2)}\ (\epsilon_{ibj}H_1^{ab}v_1^i\partial^j\partial_a V)-\ \frac{8}{3}\sigma^{(2)}(\epsilon_{bij}H_1^{ab}\ \partial^j\partial_a V^i)+\frac{3}{2}\mu^{(2)}(G_{1ab}\ G_1^{ab})V+4\sigma^{(2)}(G_{1ab}G_1^{ab})V+4\mu^{(2)}\ (G_1^{ab}\partial_b\partial_a V)V\big)\Big]\\
&+\frac{1}{\sqrt{-g}}\partial_m\partial_l\partial_k\Big(-\frac{1}{6}\mu^{(3)}\delta_1 G_1^{klm}\Big)+1\leftrightarrow 2+O\left(\frac{1}{c^5}\right)
\end{aligned}
$$

3+1 decomposition of the stress-energy tensor

$$
\begin{aligned}
\sigma = {} & \frac{1}{\sqrt{-g}}\partial_t \left[\delta_1 \frac{\mu^{(2)}(G^a_{1a})(v^a_1\partial_a V) - \mu^{(2)}(G_{1ab}v^a_1\ \partial^b V) - 2\mu^{(2)}(G_{1ab}\partial^b V^a)}{c^4}\right] + \frac{1}{\sqrt{-g}}\partial_t^2 \left[\delta_1 \big(\frac{\mu^{(2)}(G^a_{1a})}{2c^2} - \frac{\mu^{(2)}(G_{1ab}v^a_1 v^b_1) + 2\mu^{(2)}(G^a_{1a})V}{c^4}\big)\right] \\
& + \frac{1}{\sqrt{-g}}\partial_t\partial_k \left[\delta_1\big(-\frac{\mu^{(2)}G^k_{1a}v^a_1 - \mu^{(2)}(G^a_{1a})v^k_1}{c^2} + \frac{\mu^{(2)}G^k_{1a}(v_{1a}v^a_1)v^a_1 + 4\mu^{(2)}G^k_{1a}Vv^a_1 - \mu^{(2)}(G_{1ab}v^a_1 v^b_1)v^k_1 - 4\mu^{(2)}(G^a_{1a})Vv^k_1}{c^4}\big)\right] \\
& + \frac{1}{\sqrt{-g}}\partial_k \Big[\delta_1\big(\frac{1}{c^2}(\mu^{(2)}G^k_{1a}\partial^a V - \mu^{(2)}(G^a_{1a})\partial^k V) - \frac{1}{c^4}(4\mu^{(2)}G^k_{1a}(v^a_1\partial_a V)v^a_1 - 3\mu^{(2)}(G^a_{1a})(v^a_1\partial_a V)v^k_1 + 4\mu^{(2)}(G_{1ab}v^a_1\partial^b V)v^k_1 + 4\mu^{(2)}(G_{1ab}\partial^b V^a)v^k_1 \\
& + \mu^{(2)}G^k_{1a}v^a_1\partial_t V - \mu^{(2)}(G^a_{1a})v^k_1\partial_t V - 4\mu^{(2)}G^k_{1a}\partial_t V^a + 2\mu^{(2)}(G^a_{1a})\partial_t V^k - 8\mu^{(2)}G^k_{1b}v^a_1\partial_a V^b + 2\mu^{(2)}(G^a_{1a})v^a_1\partial_a V^k - 3\mu^{(2)}G^k_{1a}(v_{1a}v^a_1)\partial^a V \\
& + 8\mu^{(2)}G^k_{1a}V\partial^a V - 2\mu^{(2)}G^{ab}_1\partial_b\hat{W}^k{}_a - \frac{8}{3}\sigma^{(2)}\epsilon^k{}_{ai}H_{1b}{}^i v^a_1\partial^b V + \frac{8}{3}\sigma^{(2)}\epsilon_{abi}H^{ki}_1 v^a_1\partial^b V + 4\mu^{(2)}G^k_{1b}v^a_1\partial^b V_a - \frac{8}{3}\sigma^{(2)}\epsilon^k{}_{bi}H_{1a}{}^i\partial^b V^a - \frac{8}{3}\sigma^{(2)}\epsilon_{abi}H^{ki}_1\partial^b V^a \\
& - 2\mu^{(2)}G_{1ab}v^a_1\partial^b V^k + \mu^{(2)}(G^a_{1a})(v_{1a}v^a_1)\partial^k V - 3\mu^{(2)}(G_{1ab}v^a_1 v^b_1)\partial^k V - 6\mu^{(2)}(G^a_{1a})V\partial^k V - 2\mu^{(2)}(G^a_{1a})v^a_1\partial^k V_a + 2\mu^{(2)}G_{1ab}v^a_1\partial^k V^b + \mu^{(2)}G^{ab}_1\partial^k\hat{W}_{ab})\big)\Big] \\
& + \frac{1}{\sqrt{-g}}\partial_l\partial_k \Big[\delta_1\big({\color{red}\frac{1}{2}\mu^{(2)}G^{kl}_1} + \frac{1}{c^2}(\frac{1}{2}\mu^{(2)}G^{kl}_1(v_{1a}v^a_1) - 2\mu^{(2)}G^{kl}_1 V + \frac{2}{3}\sigma^{(2)}\epsilon^l{}_{ab}H^{kb}_1 v^a_1 + \frac{2}{3}\sigma^{(2)}\epsilon^k{}_{ab}H^{lb}_1 v^a_1 - \mu^{(2)}G^l_{1a}v^a_1 v^k_1 - \mu^{(2)}G^k_{1a}v^a_1 v^l_1 + \frac{1}{2}\mu^{(2)}(G^a_{1a})v^k_1 v^l_1) \\
& - \frac{1}{c^4}(2\mu^{(2)}G^{kl}_1(v_{1a}v^a_1)V - 4\mu^{(2)}G^{kl}_1V^2 + \frac{8}{3}\sigma^{(2)}\epsilon^l{}_{ab}H^{kb}_1Vv^a_1 + \frac{8}{3}\sigma^{(2)}\epsilon^k{}_{ab}H^{lb}_1Vv^a_1 - 2\mu^{(2)}G^l_{1a}Vv^a_1v^k_1 + \frac{2}{3}\sigma^{(2)}\epsilon^l{}_{bi}H_{1a}{}^i v^a_1 v^b_1 v^k_1 - 2\mu^{(2)}G^k_{1a}Vv^a_1 v^l_1 \\
& + \frac{2}{3}\sigma^{(2)}\epsilon^k{}_{bi}H_{1a}{}^i v^a_1 v^b_1 v^l_1 - \frac{1}{2}\mu^{(2)}(G_{1ab}v^a_1 v^b_1)v^k_1 v^l_1 + 2\mu^{(2)}(G^a_{1a})Vv^k_1 v^l_1 + \frac{4}{3}\sigma^{(2)}\epsilon^l{}_{ab}H^{kb}_1V^a + \frac{4}{3}\sigma^{(2)}\epsilon^k{}_{ab}H^{lb}_1V^a - 2\mu^{(2)}G^l_{1a}v^k_1V^a \\
& - 2\mu^{(2)}G^k_{1a}v^l_1V^a - 2\mu^{(2)}G^l_{1a}v^a_1V^k - 2\mu^{(2)}G^k_{1a}v^a_1V^l + 2\mu^{(2)}G^{la}_1\hat{W}^k{}_a + 2\mu^{(2)}G^{ka}_1\hat{W}^l{}_a)\big)\Big] \\
& + \delta_1 \Big[\frac{1}{c^2}\big(-\frac{1}{4}\mu^{(2)}(G_{1ab}G^{ab}_1) - \frac{2}{3}\sigma^{(2)}(G_{1ab}G^{ab}_1)\big) + \frac{1}{c^4}\big(-\frac{1}{4}\mu^{(2)}(G_{1ab}G^{ab}_1)\ (v_{1a}v^a_1) - \frac{2}{3}\sigma^{(2)}(G_{1ab}G^{ab}_1)\ (v_{1a}v^a_1) + \frac{1}{2}\mu^{(2)}(G_{1b}{}^a G_{1ia}v^b_1 v^i_1) \\
& + \frac{4}{3}\sigma^{(2)}(G_{1b}{}^a G_{1ia}v^b_1 v^i_1) + \mu^{(2)}(\epsilon_{jbi}G^{ab}_1 H_{1a}{}^i v^j_1) + 4\sigma^{(2)}(\epsilon_{jbi}G^{ab}_1 H_{1a}{}^i v^j_1) - 2\mu^{(2)}(G^a_{1a})(v^a_1\partial_t\partial_a V) + 2\mu^{(2)}(G_{1ab}v^a_1\partial_t\partial^b V) - \mu^{(2)}(G^a_{1a})(\partial_a V\partial^a V) \\
& - 2\mu^{(2)}(v_{1a}v^a_1)(G^{ab}_1\partial_b\partial_a V) - 2\mu^{(2)}(G^a_{1a})(v^a_1 v^b_1\partial_b\partial_a V) + 4\mu^{(2)}(G^{ab}_1 v^i_1\partial_b\partial_a V_i) + 3\mu^{(2)}(G_{1ab}\partial^a V\partial^b V) + 4\mu^{(2)}(G_{1b}{}^a v^b_1 v^i_1\partial_i\partial_a V) - 4\mu^{(2)}(G^{ab}_1 v^i_1\partial_i\partial_b V_a) \\
& - \frac{8}{3}\sigma^{(2)}\ (\epsilon_{ibj}H^{ab}_1 v^i_1\partial^j\partial_a V) - \frac{8}{3}\sigma^{(2)}(\epsilon_{bij}H^{ab}_1\ \partial^j\partial_a V^i) + \frac{3}{2}\mu^{(2)}(G_{1ab}\ G^{ab}_1)V + 4\sigma^{(2)}(G_{1ab}G^{ab}_1)V + 4\mu^{(2)}\ (G^{ab}_1\partial_b\partial_a V)V\big)\Big] \\
& + \frac{1}{\sqrt{-g}}\partial_m\partial_l\partial_k\Big(-\frac{1}{6}\mu^{(3)}\delta_1 G^{klm}_1\Big) + 1 \leftrightarrow 2 + O\left(\frac{1}{c^5}\right)
\end{aligned}
$$

Computation of the potentials

Sources:

$$\sigma = \frac{T^{00} + T^{ii}}{c^2}, \qquad \sigma_i = \frac{T^{0i}}{c}, \qquad \sigma_{ij} = T^{ij}$$

In the effective approach, $\sigma \propto \partial \ldots \partial \delta^{(3)}(\vec{x} - \vec{y}_A)$.

Potentials:

We require $\{V, V_i, \hat{W}_{ij}\}$ for the computation of the multipoles.

$$\begin{aligned} \Box V &= -4\pi G \sigma \\ \Box V_i &= -4\pi G \sigma_i \\ \Box \hat{W}_{ij} &= -4\pi G (\sigma_{ij} - \delta_{ij}\sigma_{kk}) - \partial_i V \partial_j V \end{aligned}$$

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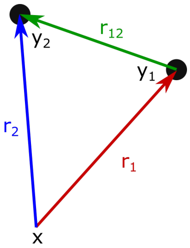


Defining $\partial_{1i} \equiv \partial / \partial y_1^i$,

$$\partial_i V \partial_j V = G^2 m_1\, m_2\, \partial_{1i} \partial_{2j} \frac{1}{r_1 r_2} + \ldots$$

$\hookrightarrow \Delta^{-1} \frac{1}{r_1 r_2} = \ln(r_1 + r_2 + r_{12})$

Source multipole moments

$$\mathrm{I}_L = \underset{B=0}{\mathrm{FP}} \int \mathrm{d}^3\mathbf{x} \left(\frac{r}{r_0}\right)^B \int_{-1}^{1} \mathrm{d}z \left[\delta_\ell \hat{x}_L \Sigma + \frac{\alpha_\ell}{c^2} \hat{x}_{iL} \dot{\Sigma}_i + \frac{\beta_\ell}{c^4} \hat{x}_{ijL} \ddot{\Sigma}_{ij}\right] \left(\mathbf{x}, u + \frac{zr}{c}\right)$$

$$\Sigma = \frac{\overline{\tau}^{00} + \overline{\tau}^{ii}}{c^2} \qquad \Sigma_i = \frac{\overline{\tau}^{0i}}{c} \qquad \Sigma_{ij} = \overline{\tau}^{ij}$$

Σ, Σ_i and Σ_{ij} contain the σ, σ_i, σ_{ij} and the potentials $\{V, V_i, \hat{W}_{ij}\}$[2]

Regularisation: Hadamard *Partie finie*

$$F(\vec{x}) = \sum_{k_0 \leq k \leq 0} r_1^k f_k(\vec{n}_1) + o(r_1)$$

↪ Choose B such that the integral converges.
↪ Only keep the coefficient B^0.

---

[2] $\tau^{\mu\nu} = |g| T^{\mu\nu} + \frac{c^4}{16\pi G} \Lambda^{\mu\nu}(h, \partial h, \partial^2 h)$

Computation of the multipoles

3 types of terms:

- compact support: $(\sigma \propto \delta^{(3)}(\vec{x} - \vec{y}_A))$

$$\underset{B=0}{\mathrm{FP}} \int \mathrm{d}^3x \, r^B \hat{x}^{ij} \sigma V$$

- non-compact support:

$$\underset{B=0}{\mathrm{FP}} \int \mathrm{d}^3x \, r^B \hat{x}^{ij} \, \hat{W}_{ab} \partial_{ab} V$$

- surface:

$$\underset{B=0}{\mathrm{FP}} \int \mathrm{d}^3x \, r^B \partial_k \left[r^2 \hat{x}^{ij} \partial_k (V^2) \right]$$

Computation of the multipoles

3 types of terms:

- compact support: $(\sigma \propto \delta^{(3)}(\vec{x} - \vec{y}_A))$

$$\underset{B=0}{\mathrm{FP}} \int \mathrm{d}^3x\, r^B \hat{x}^{ij} \sigma V$$

- non-compact support:

$$\underset{B=0}{\mathrm{FP}} \int \mathrm{d}^3x\, r^B \hat{x}^{ij}\, \hat{W}_{ab} \partial_{ab} V$$

- surface:

$$\underset{B=0}{\mathrm{FP}} \int \mathrm{d}^3x\, r^B \partial_k \left[r^2 \hat{x}^{ij} \partial_k (V^2) \right]$$

$\hookrightarrow$ Had to take into account distributional parts.

$$\partial_{ab} \left(\frac{1}{r_1} \right) \Bigg|_{\mathrm{Distr}} = -\frac{4\pi}{3} \delta_{ab}\, \delta^{(3)}(\vec{x} - \vec{y}_1)$$

Radiative multipole moments and flux

Flux :

$$\mathscr{F} = \sum_{\ell=2}^{+\infty} \frac{G}{c^{2\ell+1}} \left[a_\ell \left(\mathrm{U}_L^{(1)} \right)^2 + \frac{b_\ell}{c^2} \left(\mathrm{V}_L^{(1)} \right)^2 \right]$$

Link between source and radiative moments:

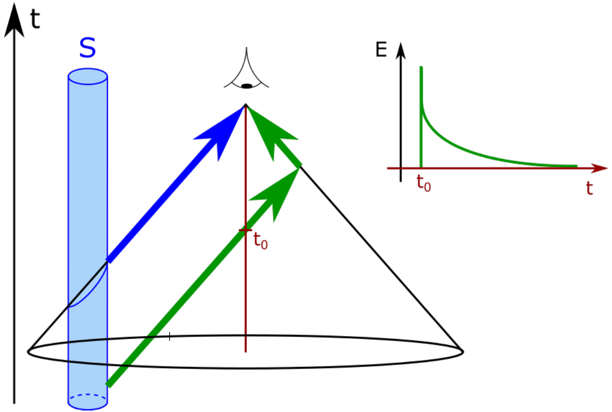
$$\mathrm{U}_L(t) = \mathrm{I}_L^{(\ell)}(t) + \frac{2GM}{c^3} \int_0^{+\infty} \mathrm{d}\tau \, \mathrm{I}_L^{(\ell+2)}(t-\tau) \ln\left(\frac{\tau}{\tau_\ell}\right) + O\left(\frac{1}{c^5}\right)$$

$\hookrightarrow$ Similar expression for V_L.

$\hookrightarrow$ M is the ADM mass $\Rightarrow$ take into account PN corrections.

$\hookrightarrow$ We perform the quasi-circular orbits approximation.

Tails



- Direct propagation
- GW scatters on the background

2.5PN CoM flux for circular orbits

$$
\begin{aligned}
\mathscr{F}_{\text{tidal}} = \frac{192 c^5 \nu\, x^{10}}{5G} \Bigg\{ & (1+4\nu)\widetilde{\mu}_+^{(2)} + \Delta \widetilde{\mu}_-^{(2)} + \Bigg[\left(-\frac{22}{21} - \frac{1217}{168}\nu - \frac{155}{6}\nu^2\right)\widetilde{\mu}_+^{(2)} \\
& + \Delta\left(-\frac{22}{21} - \frac{23}{24}\nu\right)\widetilde{\mu}_-^{(2)} + \left(-\frac{1}{9} + \frac{76}{3}\nu\right)\widetilde{\sigma}_+^{(2)} - \frac{1}{9}\Delta\widetilde{\sigma}_-^{(2)}\Bigg] x \\
& + 4\pi\left[(1+4\nu)\widetilde{\mu}_+^{(2)} + \Delta\widetilde{\mu}_-^{(2)}\right] x^{3/2} \\
& + \Bigg[\left(\frac{167}{54} - \frac{722429}{18144}\nu + \frac{15923}{336}\nu^2 + \frac{965}{12}\nu^3\right)\widetilde{\mu}_+^{(2)} + \Delta\left(\frac{167}{54} + \frac{66719}{2016}\nu\right. \\
& \left. - \frac{2779}{144}\nu^2\right)\widetilde{\mu}_-^{(2)} + \left(-\frac{173}{756} + \frac{145}{3}\nu - 208\nu^2\right)\widetilde{\sigma}_+^{(2)} + \Delta\left(-\frac{173}{756}\right. \\
& \left. + \frac{1022}{27}\nu\right)\widetilde{\sigma}_-^{(2)} + \frac{80}{3}\nu\widetilde{\mu}_+^{(3)}\Bigg] x^2 + 4\pi\Bigg[\left(-\frac{22}{21} - \frac{5053}{1344}\nu - \frac{2029}{48}\nu^2\right)\widetilde{\mu}_+^{(2)} \\
& + \Delta\left(-\frac{22}{21} - \frac{351}{64}\nu\right)\widetilde{\mu}_-^{(2)} + \left(-\frac{1}{18} + \frac{226}{9}\nu\right)\widetilde{\sigma}_+^{(2)} - \frac{\Delta}{18}\widetilde{\sigma}_-^{(2)}\Bigg] x^{5/2} \Bigg\}
\end{aligned}
$$

[HFB20b]

Balance equation

$$\mathscr{F} = -\frac{\mathrm{d}E}{\mathrm{d}t} \qquad \Rightarrow \qquad \varphi = -\int \omega(x) \frac{\mathrm{d}E/\mathrm{d}x}{\mathscr{F}(x)} \mathrm{d}x.$$

What is new in the phase?

ψ_{tidal}	Mass quadrupole	Current quadrupole	Mass octupole
5PN (L)	✓	×	×
6PN (NL)	✓	✓	×
7PN (NNL)	new	new	✓
6.5PN (tail)	✓	×	×
7.5PN (tail)	disagreement	new	×

2.5PN SPA phase for circular orbits

$v = \left(\frac{\pi G m f}{c^3}\right)^{1/3}$ where f is the orbital frequency.

$$\psi_{\rm pp} = \frac{3}{128\nu v^5}\Bigg\{1 + \left(\frac{3715}{756} + \frac{55}{9}\nu\right)v^2 - 16\pi v^3 + \left(\frac{15293365}{508032} + \frac{27145}{504}\nu + \frac{3085}{72}\nu^2\right)v^4 + \left(\frac{38645}{252} - \frac{65}{3}\nu\right)\pi v^5 \ln\left(\frac{v}{v_0}\right)\Bigg\}$$

$$\begin{aligned}\psi_{\rm tidal} = -\frac{9v^5}{16\nu^2}\Bigg\{&(1+22\nu)\widetilde{\mu}_+^{(2)} + \Delta\widetilde{\mu}_-^{(2)} + \Bigg[\left(\frac{195}{112} + \frac{1595}{28}\nu + \frac{325}{84}\nu^2\right)\widetilde{\mu}_+^{(2)} + \Delta\left(\frac{195}{112} + \frac{4415}{336}\nu\right)\widetilde{\mu}_-^{(2)} \\ &+ \left(-\frac{5}{126} + \frac{1730}{21}\nu\right)\widetilde{\sigma}_+^{(2)} - \frac{5}{126}\Delta\widetilde{\sigma}_-^{(2)}\Bigg]v^2 - \pi\left[(1+22\nu)\widetilde{\mu}_+^{(2)} + \Delta\widetilde{\mu}_-^{(2)}\right]v^3 \\ &+ \Bigg[\left(\frac{136190135}{27433728} + \frac{975167945}{4572288}\nu - \frac{281935}{6048}\nu^2 + \frac{5}{3}\nu^3\right)\widetilde{\mu}_+^{(2)} + \Delta\bigg(\frac{136190135}{27433728} + \frac{211985}{2592}\nu \\ &\quad + \frac{1585}{1296}\nu^2\bigg)\widetilde{\mu}_-^{(2)} + \left(-\frac{745}{4536} + \frac{1933490}{5103}\nu - \frac{3770}{81}\nu^2\right)\widetilde{\sigma}_+^{(2)} + \Delta\left(-\frac{745}{4536} + \frac{19355}{243}\nu\right)\widetilde{\sigma}_-^{(2)} \\ &\quad + \frac{1000}{27}\nu\widetilde{\mu}_+^{(3)}\Bigg]v^4 + \pi\Bigg[\left(-\frac{397}{112} - \frac{5343}{56}\nu + \frac{1315}{42}\nu^2\right)\widetilde{\mu}_+^{(2)} + \Delta\left(-\frac{397}{112} - \frac{6721}{336}\nu\right)\widetilde{\mu}_-^{(2)} \\ &+ \left(\frac{2}{21} - \frac{8312}{63}\nu\right)\widetilde{\sigma}_+^{(2)} + \frac{2}{21}\Delta\widetilde{\sigma}_-^{(2)}\Bigg]v^5\Bigg\}\end{aligned}$$

[HFB20b]

SPA phase for identical objects

EOB tidal phase of the GW without spin (in the SPA) :

$$\Psi^{\mathrm{T,EOB}}_{\mathrm{2PN}}(v) = -\frac{117}{2}\widetilde{\mu}^{(2)} v^5 \left[1 + \frac{3115}{1248}v^2 - \pi v^3 + \left(\frac{28024205}{3302208} + \frac{20}{351}\beta_2^{22}\right) v^4 - \frac{4283}{1092}\pi v^5\right]$$

[Damour, Nagar, Villain 2012]

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$$\psi_{\mathrm{tidal}} = -\frac{117}{2}\widetilde{\mu}^{(2)}v^5\left[1+\frac{3115}{1248}v^2-\pi v^3+\frac{379931975}{44579808}v^4-\pi\frac{2137}{546}v^5\right]$$

$\hookrightarrow$ Our work fixes $\beta_2^{22} = \dfrac{642083}{1016064} \simeq 0.632$

$\hookrightarrow$ Slight disagreement on the tail term ($\propto v^5$) under investigation.

Outline

1) Context and motivations

2) Effective action at 2PN

3) Conservative sector

4) Radiative sector

5) Conclusion and perspectives

Conclusion

Summary:

- Computation of the Lagrangian, EOM and conserved quantities.
  - $\hookrightarrow$ Equations of motion
  - $\hookrightarrow$ Energy in agreement with literature
  - $\hookrightarrow$ Isotropic Hamiltonian in agreement with literature
- Computation of the flux and phase up to relative 2.5PN.
  - $\hookrightarrow$ Flux
  - $\hookrightarrow$ Phase in time domain
  - $\hookrightarrow$ SPA phase

Perspectives:

- Coupling of tidal effects to spins
  - $\hookrightarrow$ Already known up to 1.5PN
- Relax the adiabatic approximation
  - $\hookrightarrow$ Shown that dynamical tides are not negligible at leading order
- Relax the quasi-ciruclar orbits approximation
  - $\hookrightarrow$ Include eccentricities in order to derive the phase

Thank you for your attention !