



Primordial black holes in an early matter era and stochastic inflation

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This talk is based on [1912.01638], [2001.08220] and [2006.14597]

Introduction

- Primordial Black Holes are relevant dark matter candidates. They are interesting because they do not require [physics beyond inflation](#).
- A large window of masses remains [viable](#)¹

$$10^{-16} M_{\odot} \lesssim M_{\text{PBH}} \lesssim 10^{-11} M_{\odot}.$$

- Their [astrophysical signatures](#) (gravitational waves, lensing, etc.) could be probed within the next decade.²
- We wish to determine the effects on the PBH abundance of
 - 1 The [equation of state](#) of the Universe at the time of their formation.
 - 2 The [stochastic inflation](#) formalism.
- We explore these aspects in the context of three different inflationary models.

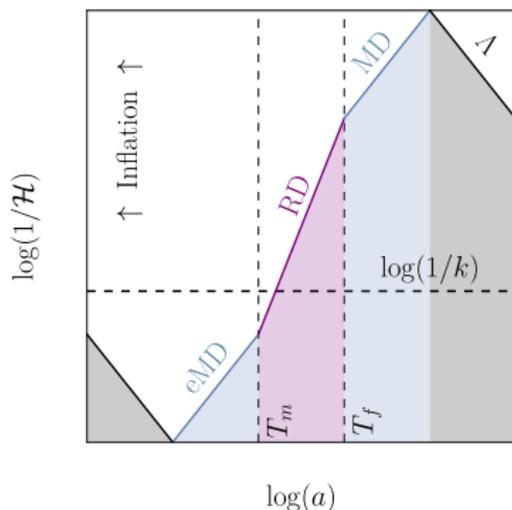
¹B. Carr, et al. [0912.5297], A. Arbey et al. [1906.04750], H. Niikura et al. [1701.02151], A. Katz et al. [1807.11495]

²M. Sasaki et al. [1801.05235]

Primordial Black Holes

Primordial Black Holes are...

Black holes formed in the early universe by mechanisms different to the usual stellar collapse. There are many possibilities, from the collision of vacuum bubbles and RD and collapse of topological defects to [inflation](#).



For PBHs to form, we need [large density fluctuations](#) $\delta = \delta\rho/\rho$. These are produced during [inflation](#). We assume transitions are [instantaneous](#).

Fluctuations leave the horizon and induce collapse upon re-entry.

Collapse in the Radiation Era

The mass of the PBHs that form is proportional to the total energy in a Hubble patch, and thus depends on the **scale of the fluctuation**,³

$$M_{\text{PBH}} = \gamma \frac{4}{3} \pi \rho H^{-3},$$

with $\gamma \lesssim 1$ because of **causality**. After a short calculation,

$$M_{\text{PBH}} = 10^{18} \text{g} \left(\frac{\gamma}{0.2} \right) \left(\frac{g(T_f)}{106.75} \right)^{-1/6} \left(\frac{k}{7 \times 10^{13} \text{Mpc}^{-1}} \right)^{-2}.$$

The fraction of DM in the form of PBHs is ($M_{\text{PBH}} = M_{\text{PBH}}(k)$)

$$f_{\text{PBH}} = \left(\frac{\beta(M_{\text{PBH}})}{8 \times 10^{-16}} \right) \left(\frac{\gamma}{0.2} \right)^{3/2} \left(\frac{g(T_f)}{106.75} \right)^{-1/4} \left(\frac{M_{\text{PBH}}}{10^{18} \text{g}} \right)^{-1/2},$$

with $\beta(M_{\text{PBH}}) \sim \exp[-\delta_c^2/2\mathcal{P}_{\mathcal{R}}(k)]$ for **Gaussian** fluctuations.

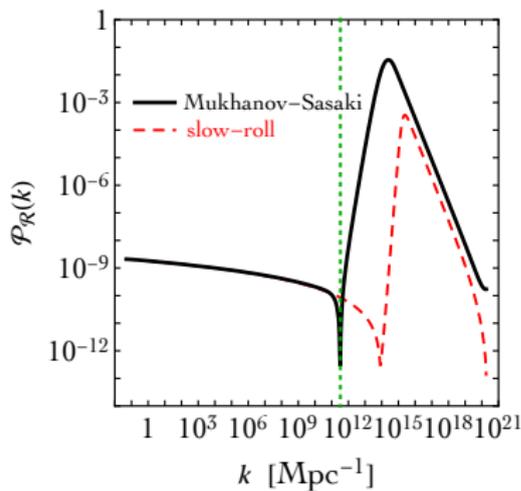
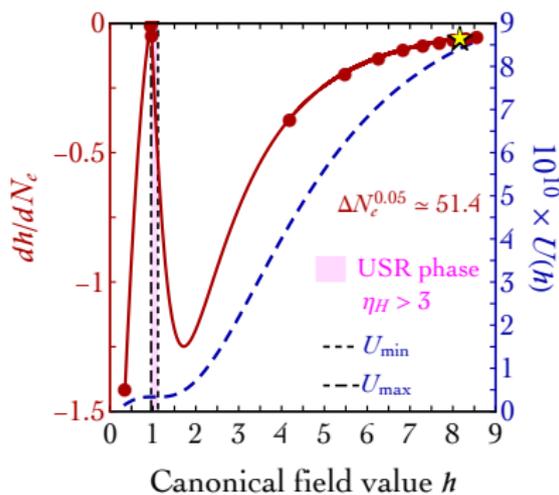
³B. Carr [10.1086/153853]

The **power spectrum** $\mathcal{P}_{\mathcal{R}}(\mathbf{k})$ tells us how these fluctuations are distributed, can be computed from inflation, and is **what CMB experiments measure**.

How do we get PBHs from inflation?

Fluctuations at CMB scales do **not produce enough PBHs to explain all DM**. We need to enhance the power spectrum **at small scales**.

Roughly speaking, $\mathcal{P}_{\delta}(\mathbf{k}) \sim \mathcal{P}_{\mathcal{R}}(\mathbf{k}) \sim H^4/\dot{\phi}^2$,



The Simplest Model

Consider a scalar field coupled to gravity in the [Jordan frame](#)⁴

$$S = \int d^4x \sqrt{-g} \left[-\frac{1}{2} (M_p^2 + \xi \phi^2) R + \frac{1}{2} g_{\mu\nu} \partial^\mu \phi \partial^\nu \phi - V(\phi) \right].$$

We can get rid of the coupling to R by redefining the fields,

$$\Omega^2 \equiv 1 + \xi \phi^2 / M_p^2,$$

$$g_{\mu\nu} \rightarrow \Omega^2[\phi] g_{\mu\nu},$$

$$\Omega^2 \frac{dh}{d\phi} = \left[\Omega^2 + \frac{3}{2} M_p^2 \left(\frac{d\Omega^2}{d\phi} \right)^2 \right]^{1/2},$$

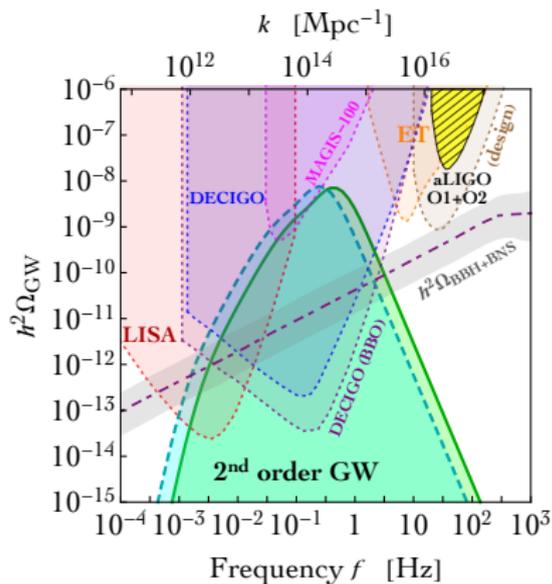
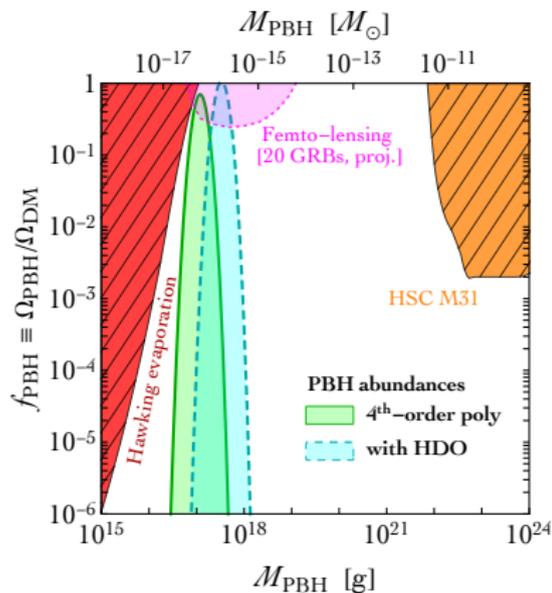
where h is such that the kinetic term is [canonically normalized](#). The resulting potential is (denominator helps fit CMB)

$$U(h) \equiv \frac{V}{\Omega^4} = \frac{a_2 \phi^2 + a_3 \phi^3 + a_4 \phi^4}{(1 + \xi \phi^2 / M_p^2)^2} \Big|_{\phi=\phi(h)}$$

⁴G. Ballesteros and M. Taoso [1709.05565], G. Ballesteros et al. [2001.08220]

By adjusting the tilt of $\mathcal{P}_{\mathcal{R}}$ at CMB scales we run into problems with evaporation bounds, $n_s^{\text{poly}} \simeq 0.949$ but $n_s^{\Lambda\text{CDM}} = 0.9649 \pm 0.0042$.

- 1 Extend ΛCDM , since $n_s^{\Lambda\text{CDM}+N_{\text{eff}}+dn_s/d\log(k)} = 0.950 \pm 0.011$ ⁵
- 2 Add higher-dimensional operators $c_n \phi^n / \Lambda^{n-4}$ (expected anyway)



⁵Y. Akrami et al. [1807.06211]

Subtleties

Do non-Gaussianities matter?

In this case, no. There are no constraints on the potential parameters, so a large change in the abundance

$$\beta(M_{\text{PBH}}) \sim \exp[-\delta_c^2/2\mathcal{P}_{\mathcal{R}}(\mathbf{k})]$$

can be compensated with a small change in the [power spectrum](#)⁷ (beware of [stochastic effects](#), however ⁸). The n_s problem occurs at large field values, whereas the peak in the spectrum is at low field values.

The gauge problem for gravitational waves

Induced GWs are [gauge-dependent](#)⁹. There are two relevant observables, Ω_{GW} and $\Delta\tau$, neither of which is properly understood [beyond leading order](#).

⁷V. De Luca et al. [1904.00970], M. Taoso and A. Urbano [2102.03610]

⁸J. M. Ezquiaga et al. [1912.05399]

⁹V. De Luca et al. [1911.09689], Y. Lu et al. [2006.03450]

The Stochastic Formalism

What is Stochastic Inflation?

In [stochastic inflation](#), quantum fluctuations backreact on the classical trajectory of the inflaton, modifying its background evolution¹⁰,

$$\frac{d\bar{\phi}}{dN} = -\frac{\partial_{\phi}V}{3H^2} + \frac{H}{2\pi}\xi_{\phi} \quad \rightarrow \quad \mathcal{P}_{\mathcal{R}} \ll 1 \quad (\text{slow roll})$$

The field is split into a [coarse-grained](#) part and a [perturbation](#),

$$\phi(t, \mathbf{x}) = \bar{\phi}(t) + \underbrace{\int \frac{d^3\mathbf{k}}{(2\pi)^{3/2}} W[\mathbf{k} - \mathbf{k}_{\sigma}(t)] \left[a_{\mathbf{k}}\phi_{\mathbf{k}}(t)e^{i\mathbf{k}\cdot\mathbf{x}} + \text{hc.} \right]}_{\hat{\phi}_{\mathcal{Q}}(t, \mathbf{x})},$$

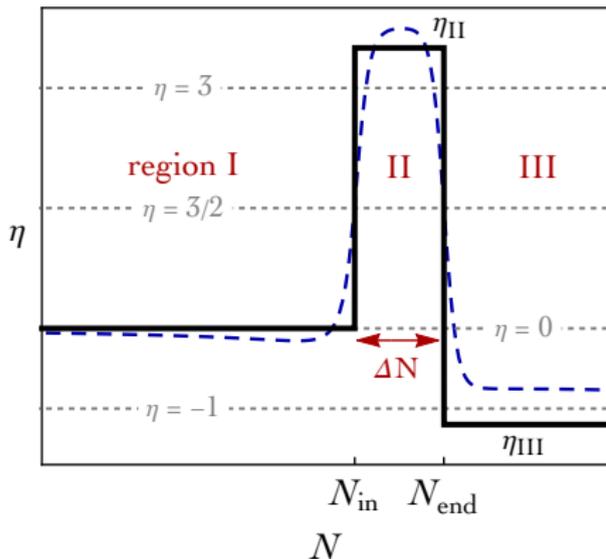
where $\mathbf{k}_{\sigma} = \sigma aH$ is a cutoff that separates classical, superhorizon modes from quantum, subhorizon modes. Fields satisfy the [Langevin equations](#),

$$\frac{d\bar{\phi}}{dt} = \frac{\bar{\pi}}{a^3} + \xi_{\phi}, \quad \text{and} \quad \frac{d\bar{\pi}}{dt} = -a^3 \frac{dV}{d\phi} \Big|_{\bar{\phi}} + \xi_{\pi}.$$

¹⁰G. Ballesteros, and M. Taoso [1709.05565], A. Starobinsky [10.1007/3-540-16452-9-6], M. Biagetti et al. [1804.07124], J. M. Ezquiaga and J. García-Bellido [1805.06731]

Analytical Model

The enhancement of the power spectrum due to a near-inflection point in the potential can be understood by considering a **three-region model**,



$$\eta \sim \frac{\ddot{\phi}}{H\dot{\phi}}$$

The potential, trajectory, etc. can all be reconstructed.

This behaviour is **generic** for all models of this type.

The evolution of the perturbations can then be obtained by solving the Mukhanov-Sasaki equation in each region and **matching the solutions**.

The noise and field perturbations are **classical stochastic variables** described by their **statistical moments**. The **power spectrum** is

$$\mathcal{P}_{\mathcal{R}} = \frac{1}{2\epsilon_{\text{cl}}} \left[D_{\phi\phi} + 2\langle\delta\phi_{\text{st}}\delta\pi_{\text{st}}\rangle - 2(\epsilon_{\text{cl}} - \eta_{\text{cl}})\langle\delta\phi_{\text{st}}^2\rangle \right].$$

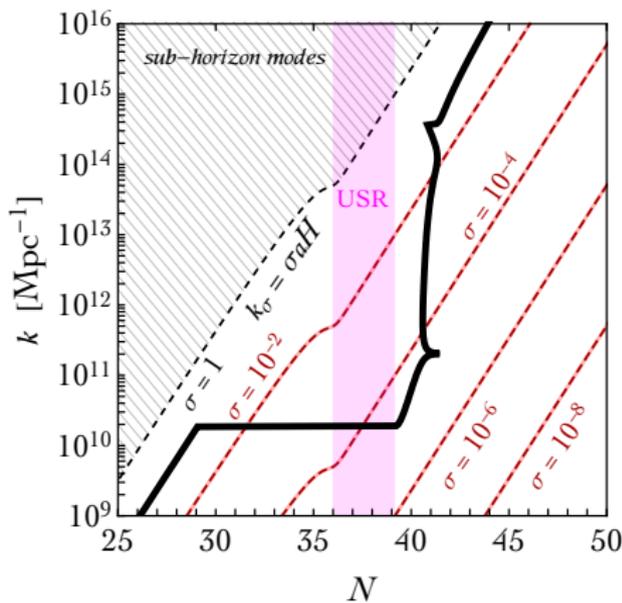
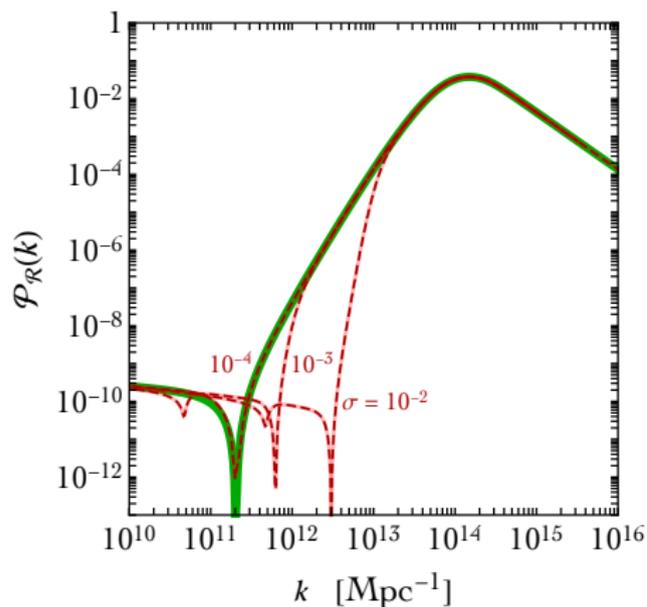
The biggest advantage of this approach is that we can find explicit **analytical expressions** for the **noise matrix** $\Theta_{ab} \sim \langle\xi_a\xi_b\rangle$

| $\sigma \rightarrow 0$ | region I | region II | region III |
|------------------------|----------------------|---|--|
| $\Theta_{\phi\phi}(N)$ | $\frac{H^2}{4\pi^2}$ | $\frac{H^2}{4\pi^2} e^{-2\eta_{\text{II}}(N-N_{\text{in}})}$ | $\frac{H^2}{4\pi^2} e^{-2\eta_{\text{III}}(N-N_{\text{end}})-2\eta_{\text{II}}\Delta N}$ |
| $\Theta_{\pi\phi}(N)$ | 0 | $-\frac{H^2}{4\pi^2} \eta_{\text{II}} e^{-2\eta_{\text{II}}(N-N_{\text{in}})}$ | $-\frac{H^2}{4\pi^2} \eta_{\text{III}} e^{-2\eta_{\text{III}}(N-N_{\text{end}})-2\eta_{\text{II}}\Delta N}$ |
| $\Theta_{\pi\pi}(N)$ | 0 | $\frac{H^2}{4\pi^2} \eta_{\text{II}}^2 e^{-2\eta_{\text{II}}(N-N_{\text{in}})}$ | $\frac{H^2}{4\pi^2} \eta_{\text{III}}^2 e^{-2\eta_{\text{III}}(N-N_{\text{end}})-2\eta_{\text{II}}\Delta N}$ |

The above can be used to show $2\langle\delta\phi_{\text{st}}\delta\pi_{\text{st}}\rangle - 2(\epsilon_{\text{cl}} - \eta_{\text{cl}})\langle\delta\phi_{\text{st}}^2\rangle = 0$.

We have shown that the power spectrum coincides, at the linear level, with the perturbative result in the $\sigma \rightarrow 0$ limit. Numerical results below.

$$\mathcal{P}_{\mathcal{R}} = \frac{1}{2\epsilon_{\text{cl}}} \left[D_{\phi\phi} + \underbrace{2\langle\delta\phi_{\text{st}}\delta\pi_{\text{st}}\rangle - 2(\epsilon_{\text{cl}} - \eta_{\text{cl}})\langle\delta\phi_{\text{st}}^2\rangle}_0 \right].$$



More subtleties

This result is valid at leading order

We have identified the coarse-grained field with the background field

$$\bar{\phi} + \hat{\phi}_Q \simeq \phi_{\text{bg}} + \delta\phi_{\text{st}}.$$

A proper numerical computation involves **re-calculating the coarse-grained field at each time step** ¹¹.

Stochastic inflation affects non-Gaussianities

We focused on the power spectrum, but in stochastic inflation it is possible to compute the **full PDF**. Analyses in **perfect USR** suggest an exponential tail ¹²,

- 1 Can the change in abundance still be reabsorbed in the spectrum?
- 2 What happens outside of perfect USR? Is there a tail? Is it model-dependent?

¹¹D. Figueroa et al. [2012.06551]

¹²J. M. Ezquiaga et al. [1912.05399], C. Pattison et al. [2101.05741], M. Biagetti et al. [2105.07810]

Collapse in Matter Domination

If collapse occurs during an **early matter-dominated era**, the abundance is

$$f_{\text{RD}} \propto \beta_{\text{RD}} M_{\text{RD}}^{-1/2} \quad f_{\text{MD}} \propto \beta_{\text{MD}} T_m$$

The β function represents the **fraction of energy density that collapses**. This function has very different forms in MD and RD ¹³,

$$\beta_{\text{RD}}(\mathbf{k}) \propto \frac{1}{\sqrt{\mathcal{P}_\delta}} \int_{\delta_c}^{\infty} \exp\left(-\frac{\delta^2}{2\mathcal{P}_\delta}\right) d\delta,$$

$$\beta_{\text{MD}}(\mathbf{k}) \propto \mathcal{I}^6 \mathcal{P}_\delta \exp\left[-\alpha \left(\frac{\mathcal{I}^4}{\mathcal{P}_\delta}\right)^{1/3}\right].$$

The latter takes into account the **non-sphericity** and **angular momentum** (related to \mathcal{I}) of the collapsing cloud.

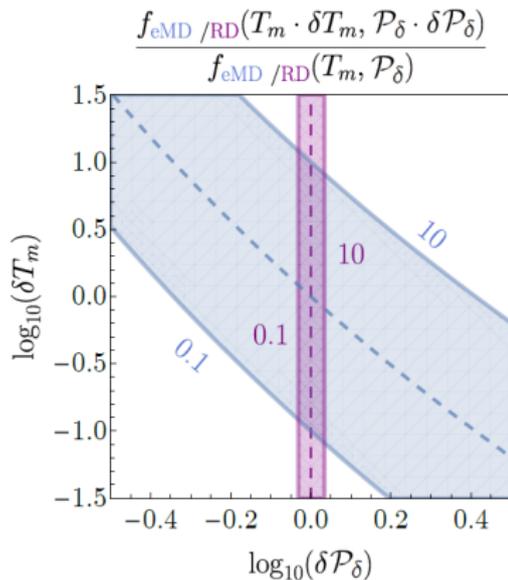
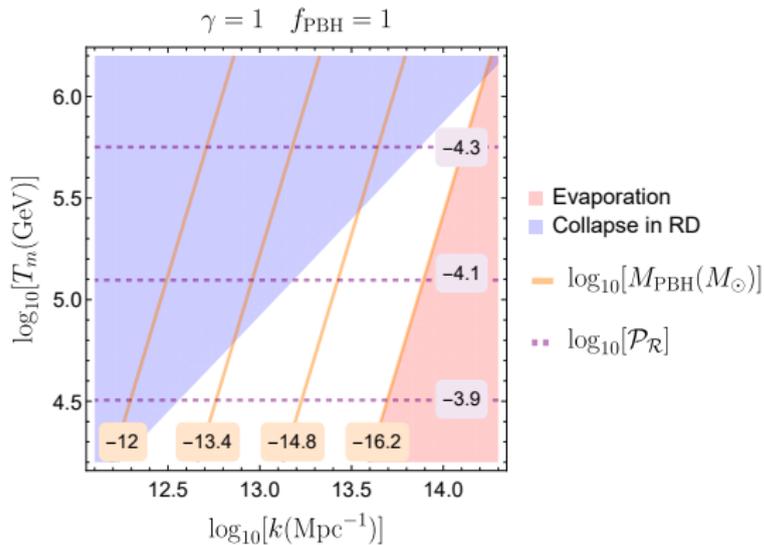
Intuitively...

Collapse is easier during an eMD era because of the **lack of radiation pressure** (roughly speaking).

¹³T. Harada et al. [1609.01588], T. Harada et al. [1707.03595]

Collapse during matter-domination has two big advantages,

- 1 The **power spectrum** required to get a significant PBH abundance is much smaller than in RD ($\mathcal{P}_{\text{RD}} \sim 10^{-2}$ vs $\mathcal{P}_{\text{MD}} \sim 10^{-4}$).
- 2 The abundance is **much** less sensitive to small changes in $\mathcal{P}_{\mathcal{R}}$, since β is different.

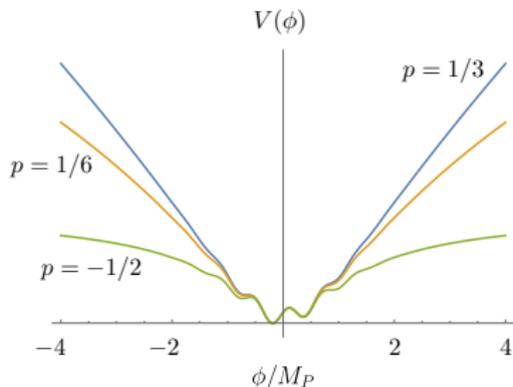


Numerical Examples

Consider the following axion monodromy-inspired potential,

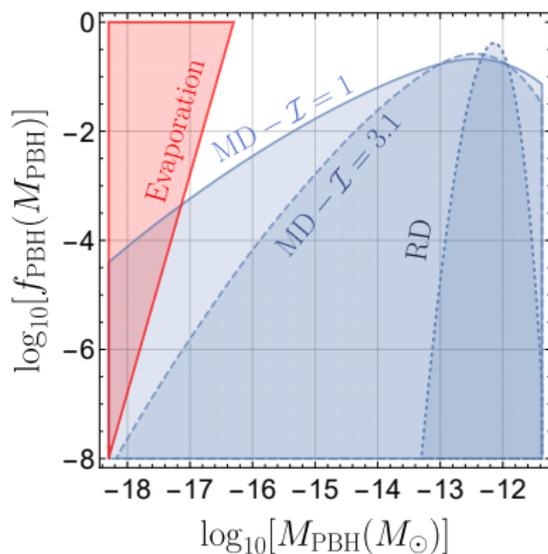
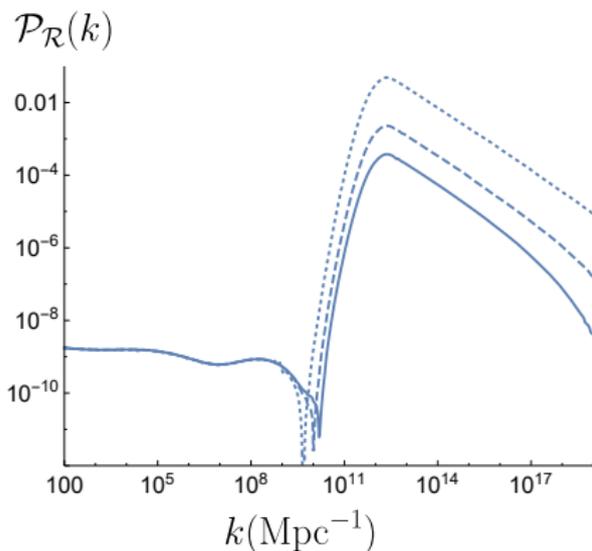
$$V(\phi) = m^2 f^2 \left\{ \frac{1}{2p} \frac{F^2}{f^2} \left[-1 + \left(1 + \frac{\phi^2}{F^2} \right)^p \right] + \kappa e^{-\left(\frac{\phi}{\phi_\Lambda}\right)^{p\Lambda}} \cos\left(\frac{\phi}{f} + \delta\right) \right\} + V_0,$$

This potential features oscillations and is quadratic near the minimum, ensuring a **long epoch of matter domination** if reheating is **perturbative**.



The motivation is to reduce tuning by **having several minima**.

The mechanism is illustrated in these examples. All parameters are identical except for κ (controls the depth of minima in the potential), and \mathcal{I} (from the β function).



Thus, the main advantage of MD is also its biggest drawback: lack of suppression makes it more difficult to evade evaporation bounds.

Even more subtleties

There are many uncertainties

- 1 The efficiency factor γ .
- 2 The angular momentum of the collapsing cloud \mathcal{I} .
- 3 The effect of non-Gaussianities.
- 4 Inhomogeneities.
- 5 ...

These are the best formulas available at the moment (for MD), but there is still a long way to go. The focus is usually on RD.

No numerical simulations yet

The first step towards determining how good these estimates really are is to perform numerical simulations, which only exist in RD at the moment.¹⁴

¹⁴See, however, E. de Jong, et al. [2109.04896]

Conclusions

- The **simplest potential** that can produce PBHs is viable, provided Λ CDM is extended, or **higher-dimensional operators** are considered.
- If dark matter is in the form of PBHs, the corresponding **GW signal should be observable** by **LISA and DECIGO** if they form during RD.
- We have shown, both analytically and numerically that, at leading order, stochastic inflation **does not affect the power spectrum**, even in the presence of a USR phase.
- PBH formation in an early matter-dominated era has **significant advantages**, namely, that a **smaller enhancement** of the power spectrum is required, and the potential parameters are **less tuned**.
- However, the fact that the PBH abundance function is less suppressed in this scenario makes it **more difficult** to evade **evaporation bounds**.

Paths for future work

- We are currently trying to determine how the GW signal is affected if PBHs form in an eMD era.
- The gauge problem of induced gravitational waves remains open. Solving it involves understanding two different observables (Ω_{GW} and $\Delta\tau$) beyond leading order.
- The effect of the stochastic formalism on the probability distribution and differences between this and the classical calculations are still to be studied.
- There are many sources of uncertainty in the MD formulas. A more thorough analytical description is necessary, together with numerical simulations.

Thank you! Questions?

Light: *exists*

Black holes:

