

# The effect of dark matter discreteness on light propagation

Sofie-Marie Koksang and SR  
arXiv:2108.06163

*Syksy Räsänen*

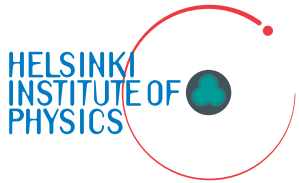
University of Helsinki

Department of Physics and Helsinki Institute of Physics



# Beyond null geodesics

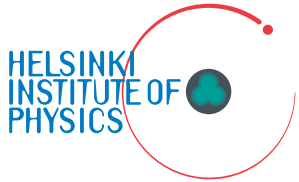
---



- In cosmology, light is usually taken to move on null geodesics.
- This follows from geometrical optics, which assumes that photon wavenumber is larger than any other scale, including spacetime curvature.
- Dark matter particles give density spikes, so the curvature is not smooth.



# Bed of nails

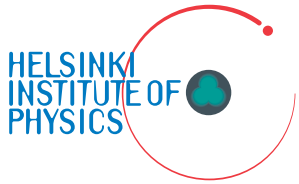


- If dark matter particle of mass  $m$  is localised within a Compton wavelength, it has
  - energy density  $\sim 10^{-3} m^4$
  - curvature  $\sim 10^{-3} m^4/M_{Pl}^2$ .
- If  $m \gtrsim 10^4$  GeV, the curvature scale is larger than CMB photon energy.
- Instead of using null geodesics, we have to go back to the equation of motion.



# Action and equation of motion

---



- Light propagation is governed by the action

$$S = - \int d^4x \sqrt{-g} \frac{1}{4} F_{\alpha\beta} F^{\alpha\beta} .$$

- Variation gives the equation of motion

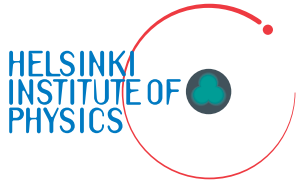
$$\square A^\alpha - \nabla^\alpha \nabla_\beta A^\beta - R^\alpha{}_\beta A^\beta = 0 .$$

- Lorenz gauge:  $\nabla_\alpha A^\alpha = 0$  .



# Post-geometrical approximation

$$\square A^\alpha - R^\alpha{}_\beta A^\beta = 0$$



- Local plane wave form:

$$A^\alpha(x) = \sum_{n=0}^{\infty} \text{Re} \left[ A_n^\alpha(x) e^{iS(x)/\epsilon} \right] \epsilon^n .$$

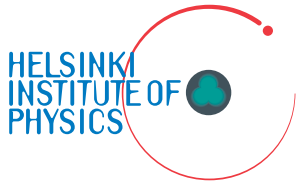
- In geometrical optics only the phase is assigned the factor  $1/\epsilon$ .
- We also assign  $1/\epsilon^2$  to the curvature.
- As in geometrical optics, we keep two leading terms in  $1/\epsilon^2$ .



# Curvature-induced mass

$$M^2 = \frac{\rho - p}{2M_{\text{Pl}}^2}$$

$$\square A^\alpha - R^\alpha{}_\beta A^\beta = 0$$



- Assuming ideal fluid matter, we get ( $k_\alpha = \partial_\alpha S$ )

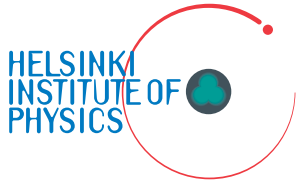
$$k^2 = -\frac{\rho - p}{2M_{\text{Pl}}^2} \iff E^2 = \vec{k}^2 + M^2$$

$$k^\beta \nabla_\beta k_\alpha = -\frac{1}{4M_{\text{Pl}}^2} \partial_\alpha (\rho - p) .$$

- Light gets curvature scale mass.
- Light is pushed off-geodesic by density gradient.



# Clumpy matter



- Model dark matter particles as isolated Gaussian clumps:

$$\rho = m \sum_n \frac{1}{(2\pi)^{3/2} \lambda_c^3} e^{-\frac{r_n^2}{2\lambda_c^2}}$$

- Space inside the particles does not expand.
- Light with  $E < M$  cannot enter dark matter particles, and our approximation is not valid.



# Redshift



- Redshift is given by photon energy:  $1 + z = \frac{E_s}{E_o}$ .
- Along the light ray, we have

$$\frac{dE}{ds} = -k^\beta \nabla_\beta (u^\alpha k_\alpha) = -k^\alpha k^\beta \nabla_\beta u_\alpha - \frac{1}{2} u^\alpha \nabla_\alpha k^2,$$

so integrating gives

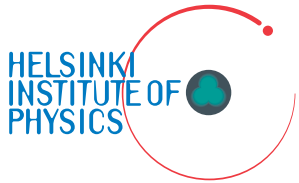
$$\ln(1 + z) = \int_{s_s}^{s_o} ds E^{-1} \left[ v^2 \left( \frac{1}{3} \theta + \sigma_{\alpha\beta} e^\alpha e^\beta \right) - \frac{v}{\rho + p} e^\alpha \partial_\alpha p - \frac{\dot{\rho} - \dot{p}}{4M_{Pl}^2 E^2} \right]$$

- Here  $v$  is photon velocity.





# Negligible effect on redshift



- Redshift is given by:

$$\ln(1+z) = \int_{s_s}^{s_o} ds E^{-1} \left[ v^2 \left( \frac{1}{3} \theta + \sigma_{\alpha\beta} e^\alpha e^\beta \right) - \frac{v}{\rho + p} e^\alpha \partial_\alpha p - \frac{\dot{\rho} - \dot{p}}{4M_{\text{Pl}}^2 E^2} \right]$$

- The effect of curvature spikes is negligible, because particles occupy tiny fraction of volume.
- Distance involves (derivative of the density)<sup>2</sup>.



# Angular diameter distance



- Angular diameter distance  $D_A$  is determined by the area expansion rate  $\tilde{\theta}$  as

$$D_A \propto \exp\left(\frac{1}{2} \int ds \tilde{\theta}\right) .$$

- We get  $\tilde{\theta}$  from  $\nabla_\beta k_\alpha = \frac{1}{2} \tilde{\theta} \tilde{h}_{\alpha\beta} + \tilde{\sigma}_{\alpha\beta} + A_{\alpha\beta}$  .

- Recall that  $k^\beta \nabla_\beta k_\alpha = -\frac{1}{4M_{\text{Pl}}^2} \partial_\alpha (\rho - p)$  .

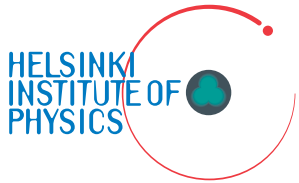
- As with the redshift, we integrate  $\frac{d\tilde{\theta}}{ds}$  .



# Large effect on distance

$$M^2 = \frac{\rho}{2M_{\text{Pl}}^2}$$

$$D_A \propto \exp\left(\frac{1}{2} \int ds \tilde{\theta}\right)$$



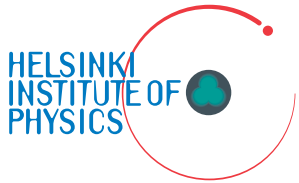
- The leading correction to the angular diameter distance is given by

$$\int ds \tilde{\theta} = \int ds \int ds' \left\{ -\frac{1}{2} \tilde{\theta}^2 - 2\tilde{\sigma}^2 - E^2 \frac{\langle \rho \rangle}{M_{\text{Pl}}^2} \left[ 1 + \frac{5}{128\sqrt{2}\pi^2} \frac{m^2}{E^2} \frac{M^2}{E^2} - \frac{13}{24\sqrt{2}} \frac{M^2}{E^2} + \frac{5}{72\sqrt{3}} \left( \frac{M^2}{E^2} \right)^2 \right] \right\}$$

- Terms on the second line are at most 0.4 and 0.04, but last term on the first line can be large.



# Upper limit on dark matter mass



- The corrected distance equation is

$$\frac{d^2 D_A}{ds^2} = -4\pi G_N \langle \rho \rangle [1 + \alpha (E/E_o)^{-4}] D_A ,$$

$$\text{where } \alpha \equiv \frac{5}{8192\pi^{13/2}} \frac{m^6}{M_{\text{Pl}}^2 E_o^4} .$$

- This could give an alternative to dark energy and/or explain the  $H_0$  tension if  $\alpha$  were negative.
- As it is, it gives the constraint  $m < 100$  MeV.



# Two caveats

---



- Is the post-geometrical approximation valid?
  - Intermediate region, local plane wave form.
- Is the particle size correct?
  - Due to spreading of wavefunction, particle size grows linearly in time, but decoherence localises it.
  - Need a realistic quantum mechanical treatment.



# Spiky and thorny

---



- Dark matter particles lead to spikes in curvature.
  - Photons get a gravitational mass and are pushed off geodesics.
- No change in redshift, possibly big change in distance.
  - Constraint on dark matter mass:  $m < 100 \text{ MeV}$ .
- Need to understand treatment of light propagation and spread of dark matter particle wavefunction.