

Addressing the dark matter problem using extensions of GR



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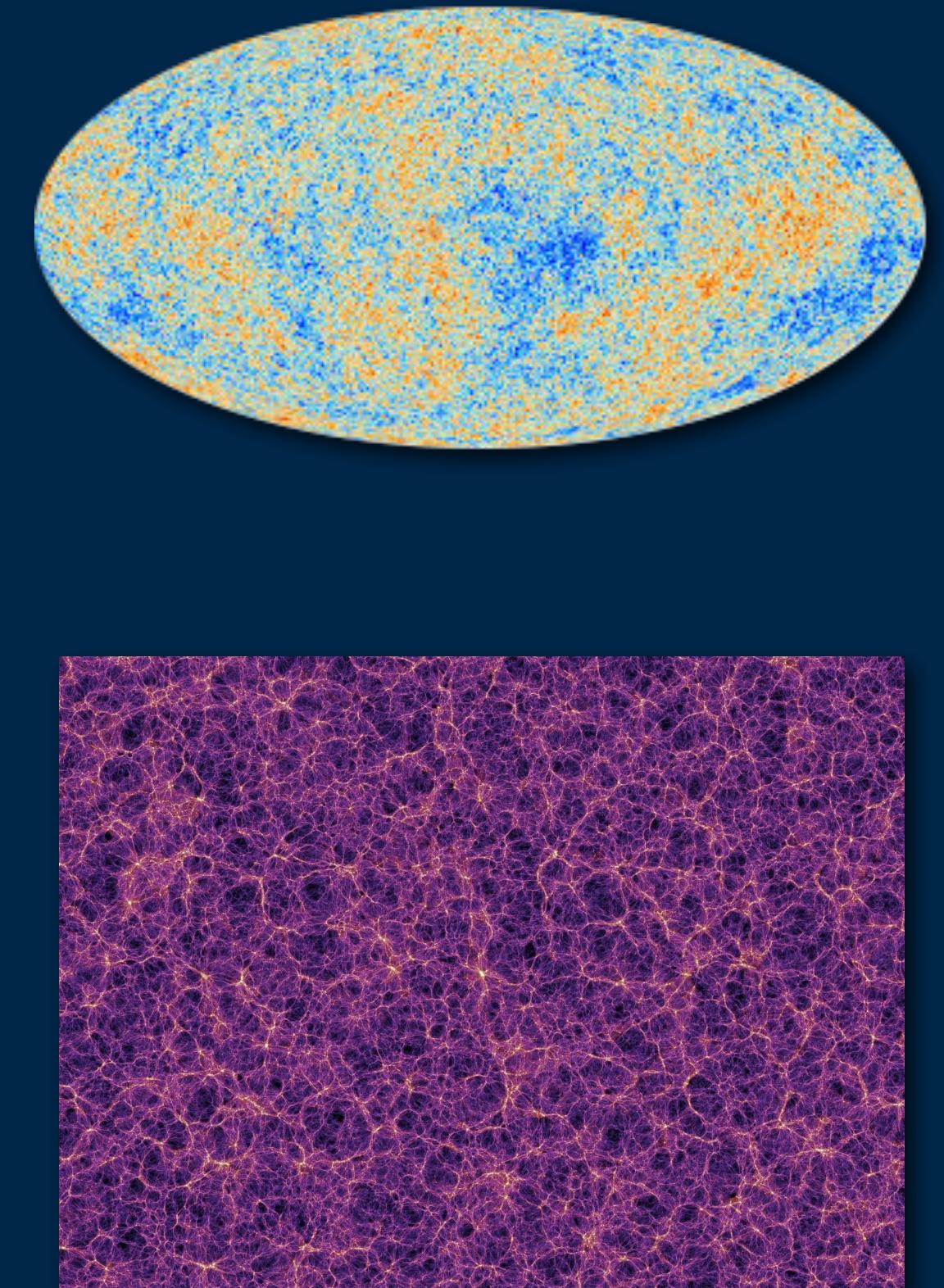
ceico

CENTRAL EUROPEAN INSTITUTE FOR
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The logo for the Institute of Physics of the Czech Academy of Sciences, featuring a blue stylized "F" and "Z" intertwined with a "U".
FZU

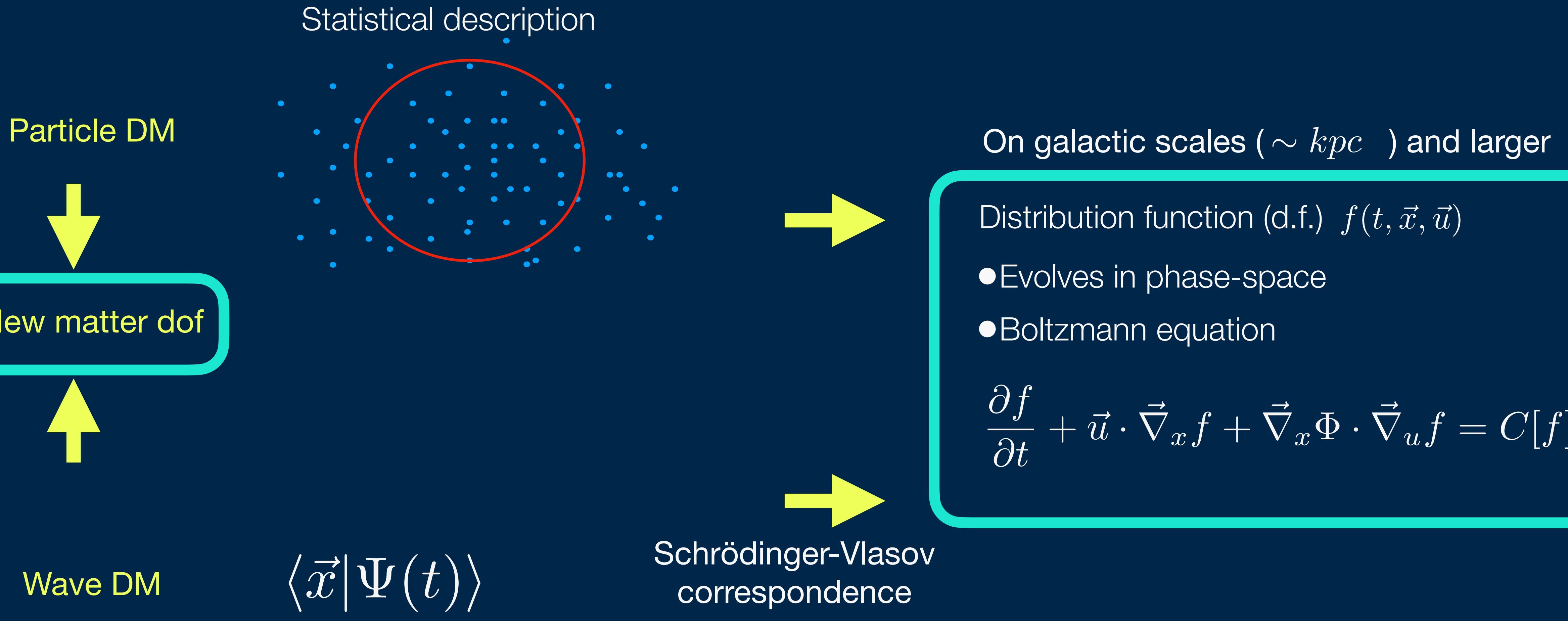
Institute of Physics of the
Czech Academy of Sciences

Visit supported by Barrande mobility programme, grant
no. MSMT-7781/2020-32 8J21FR028



Mismatch between
observed dynamics of visible matter its gravitational influence

Dark matter



Widrow & Kaiser ApJ Lett 416, L71 (1993)
Kopp, Vattis & C.S, PRD (2017) (and ref. therein)

Λ

Cosmological constant

Primordial Black holes

.....

New particle not in standard model
— e.g. neutralino



Cold Dark Matter
CDM

 Λ CDM

Cosmological scales

FLRW

$$\bar{\rho}_{CDM} \propto \frac{1}{a^3} \quad \bar{\rho}_\Lambda \rightarrow \text{constant}$$

Fluctuations

$$\delta \equiv \frac{\delta\rho}{\bar{\rho}}$$

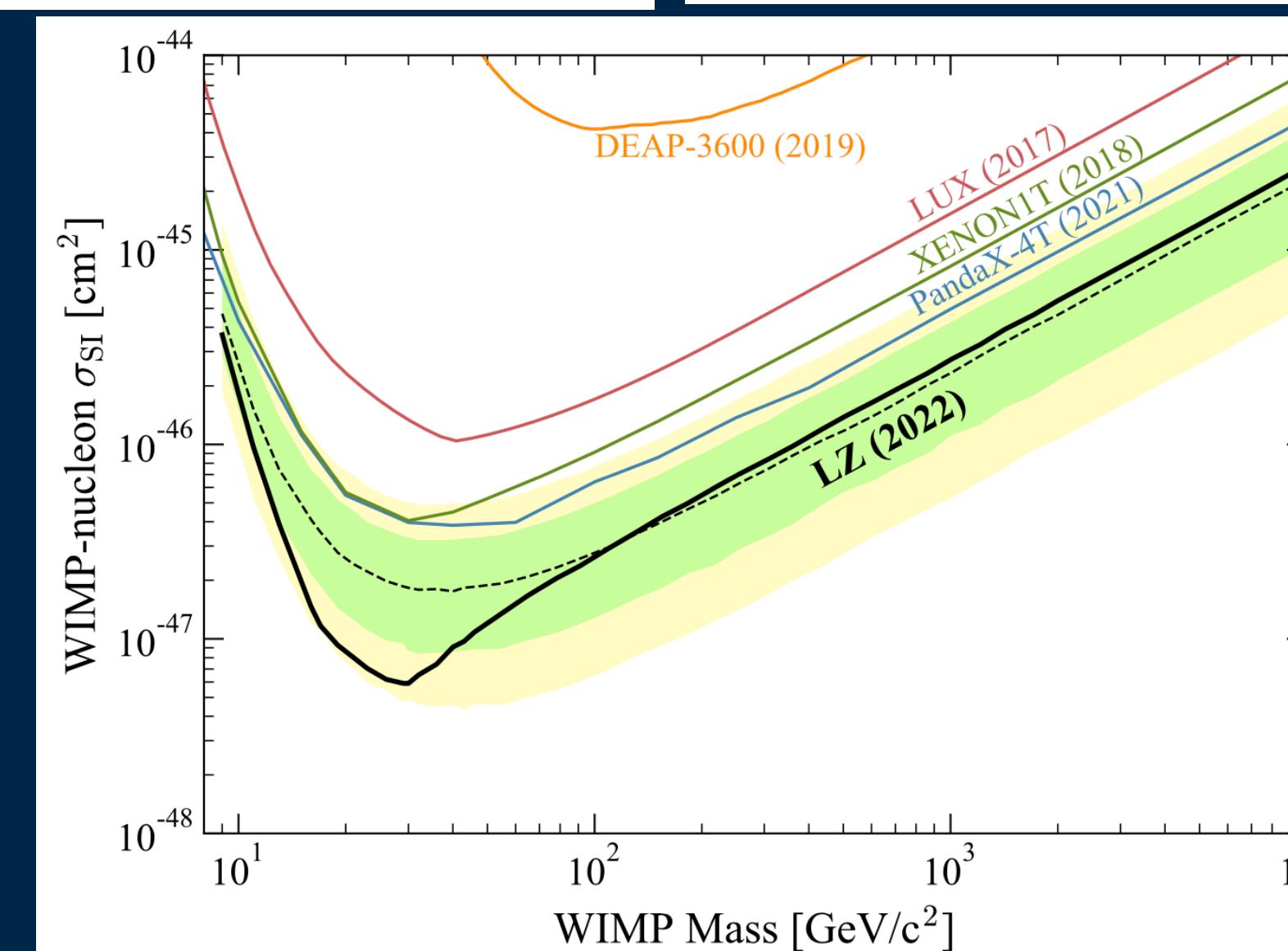
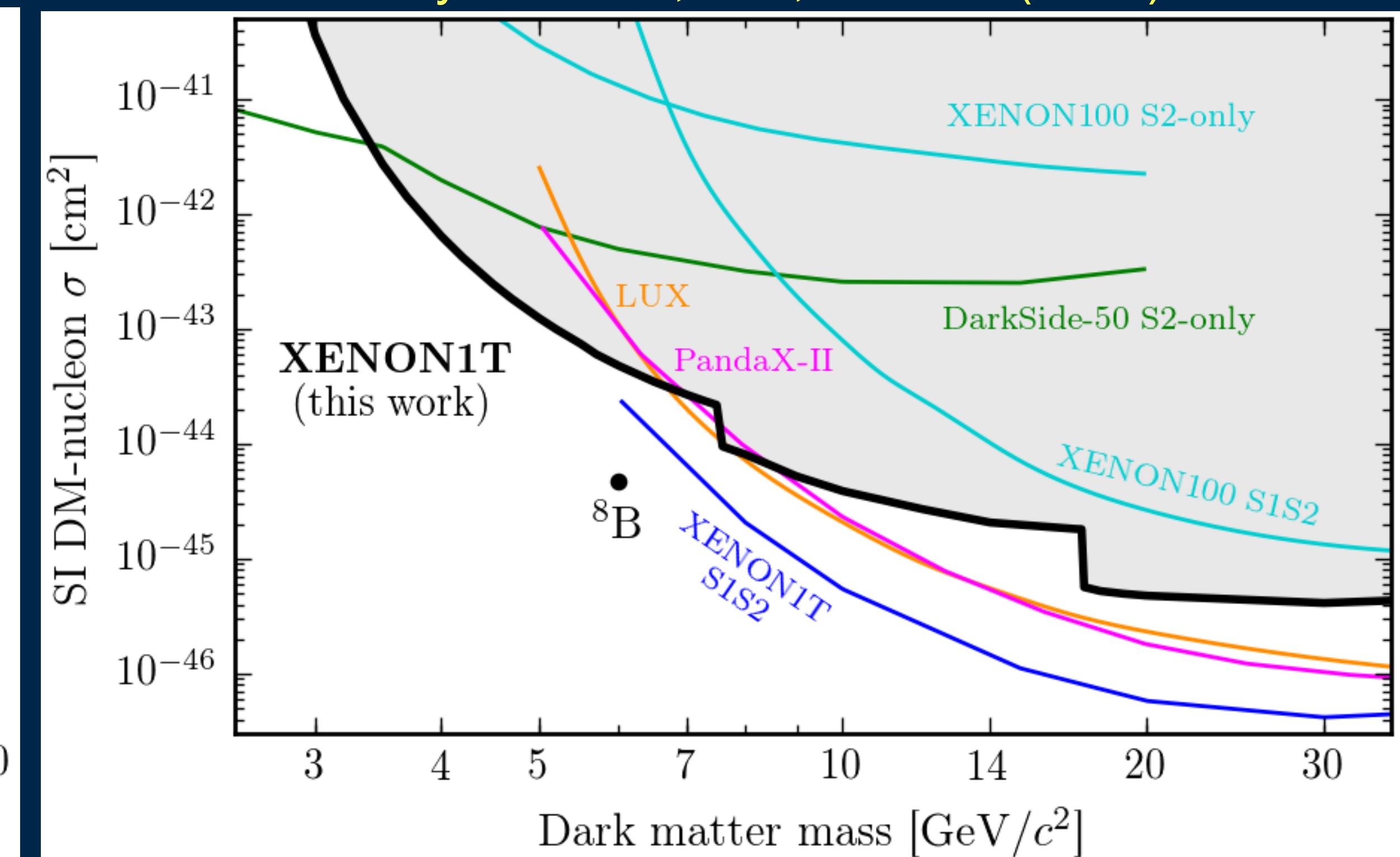
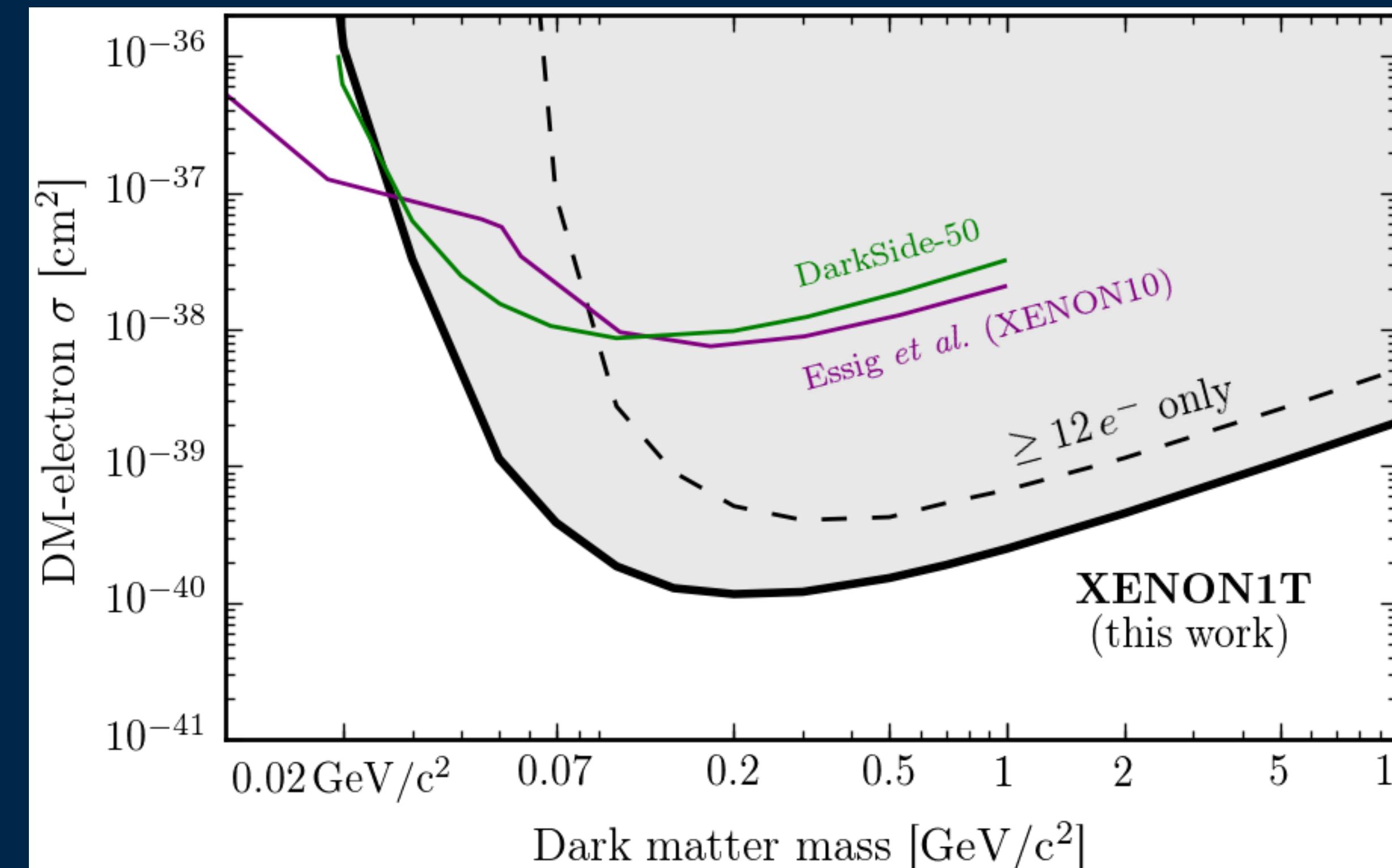
$$\dot{\delta}_{CDM} = 3\dot{\Phi} - \frac{k^2}{a^2}\theta_{CDM}$$

Density contrast

Velocity divergence

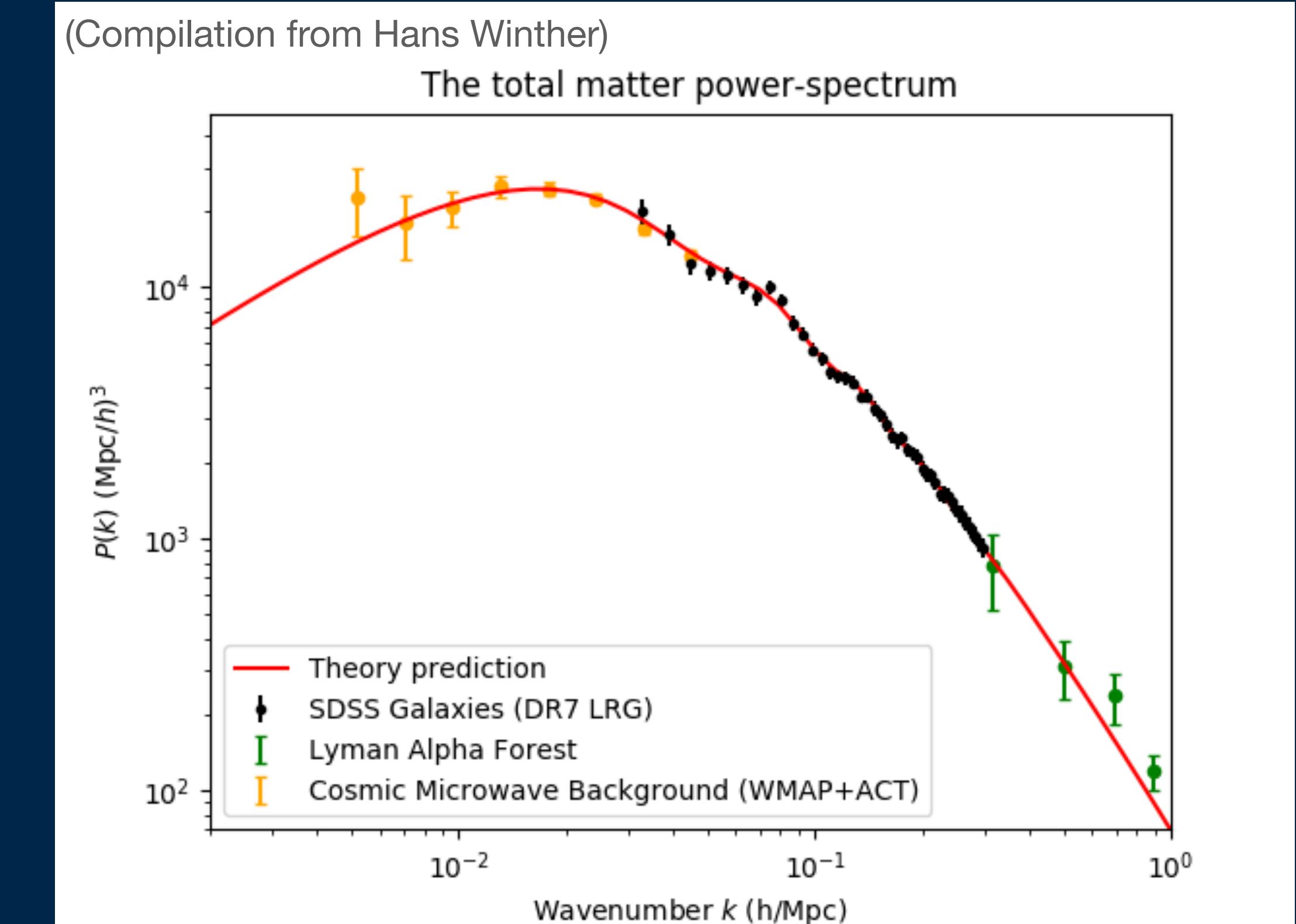
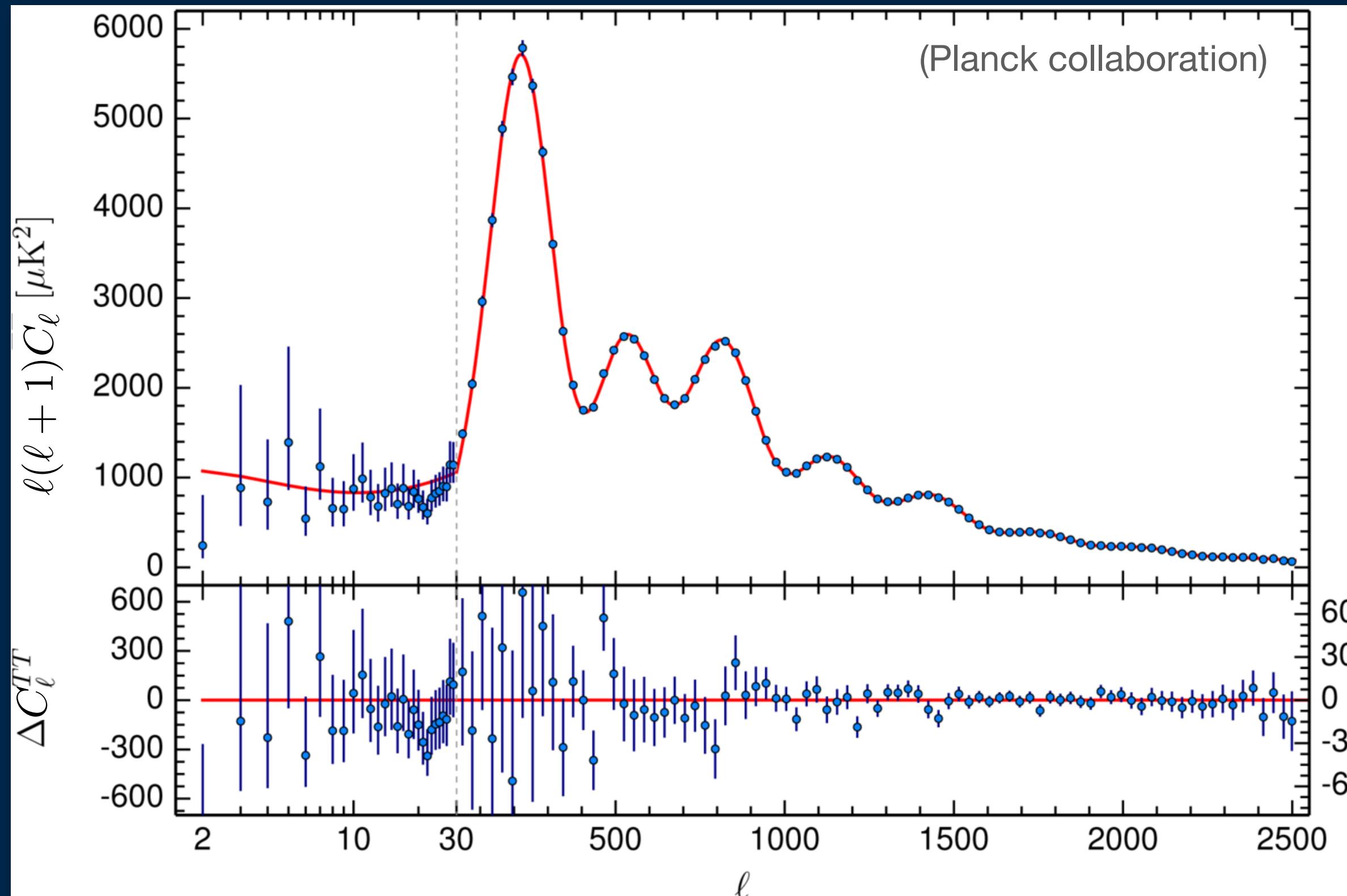
$$\theta \equiv -\frac{i\vec{k} \cdot \vec{v}}{k^2}$$

$$\dot{\theta}_{CDM} = \Psi$$



LZ collaboration
arXiv:2207.03764

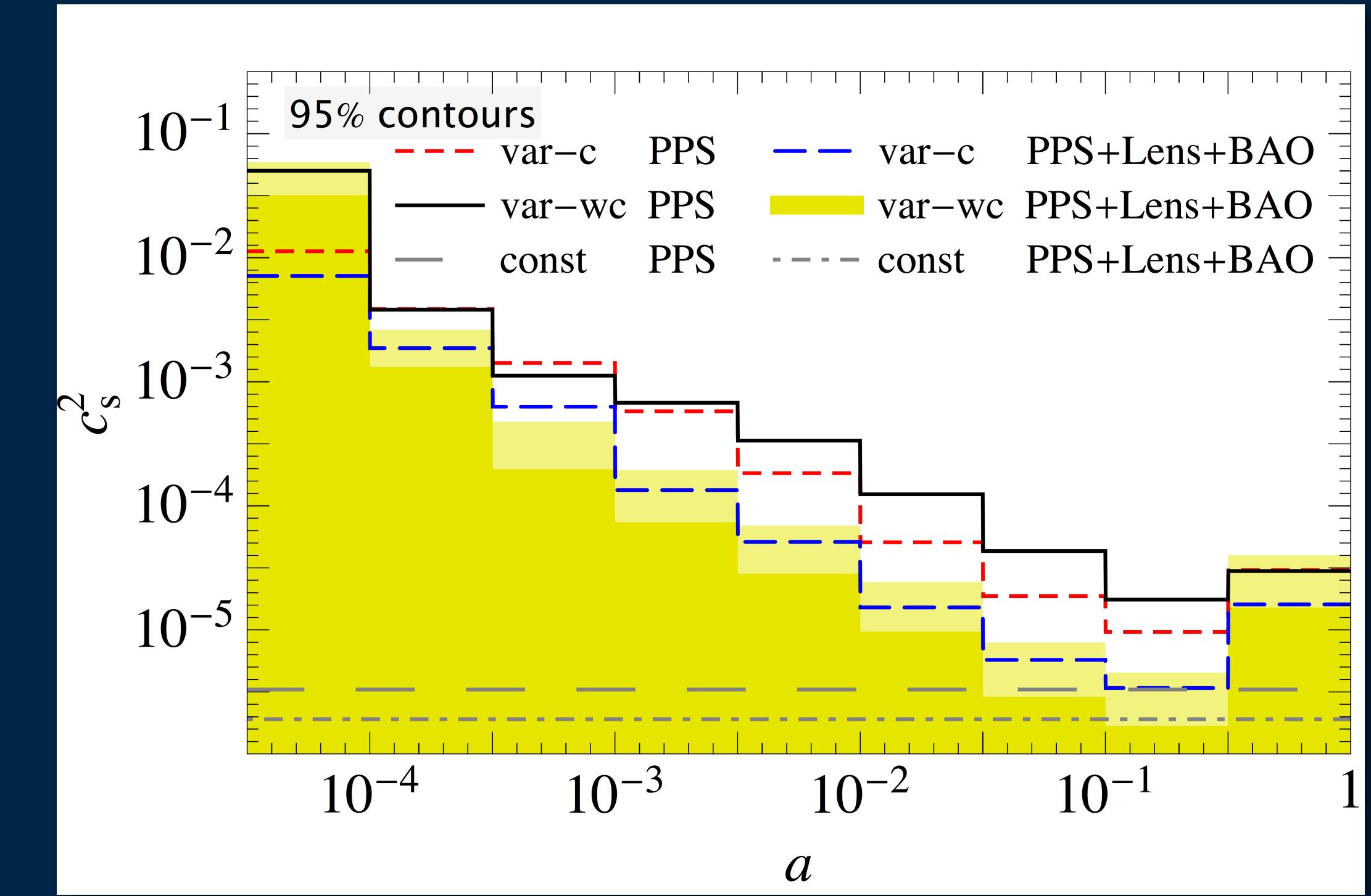
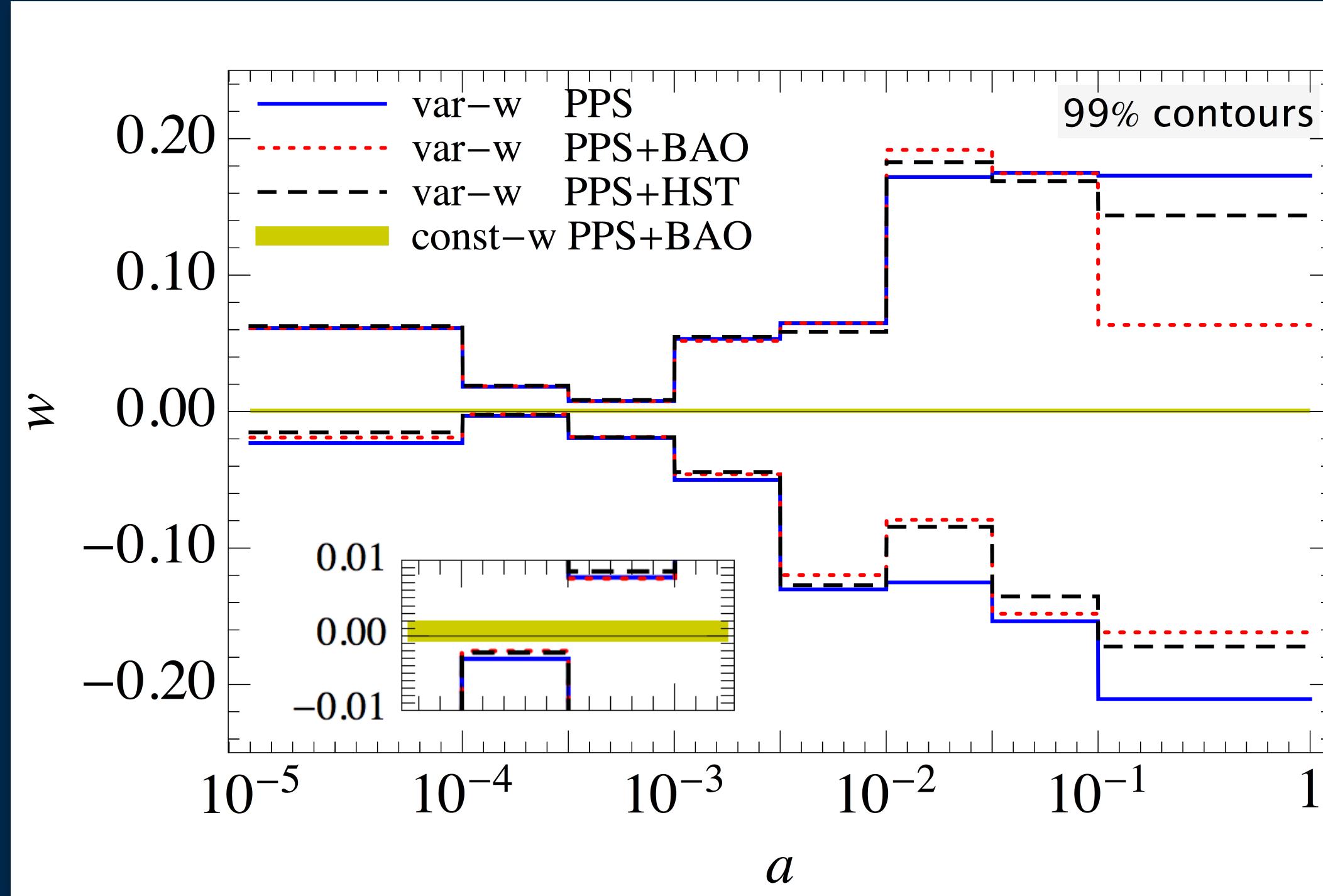
Λ CDM: Success on large scales



$$P = w\rho$$

$$\delta P \sim c_s^2 \delta \rho$$

W. Hu, ApJ 506, 485 (1998)



GR + dust: large scale description of Universe is Λ CDM

$$0 < k \lesssim 0.1(hMpc)^{-1}$$

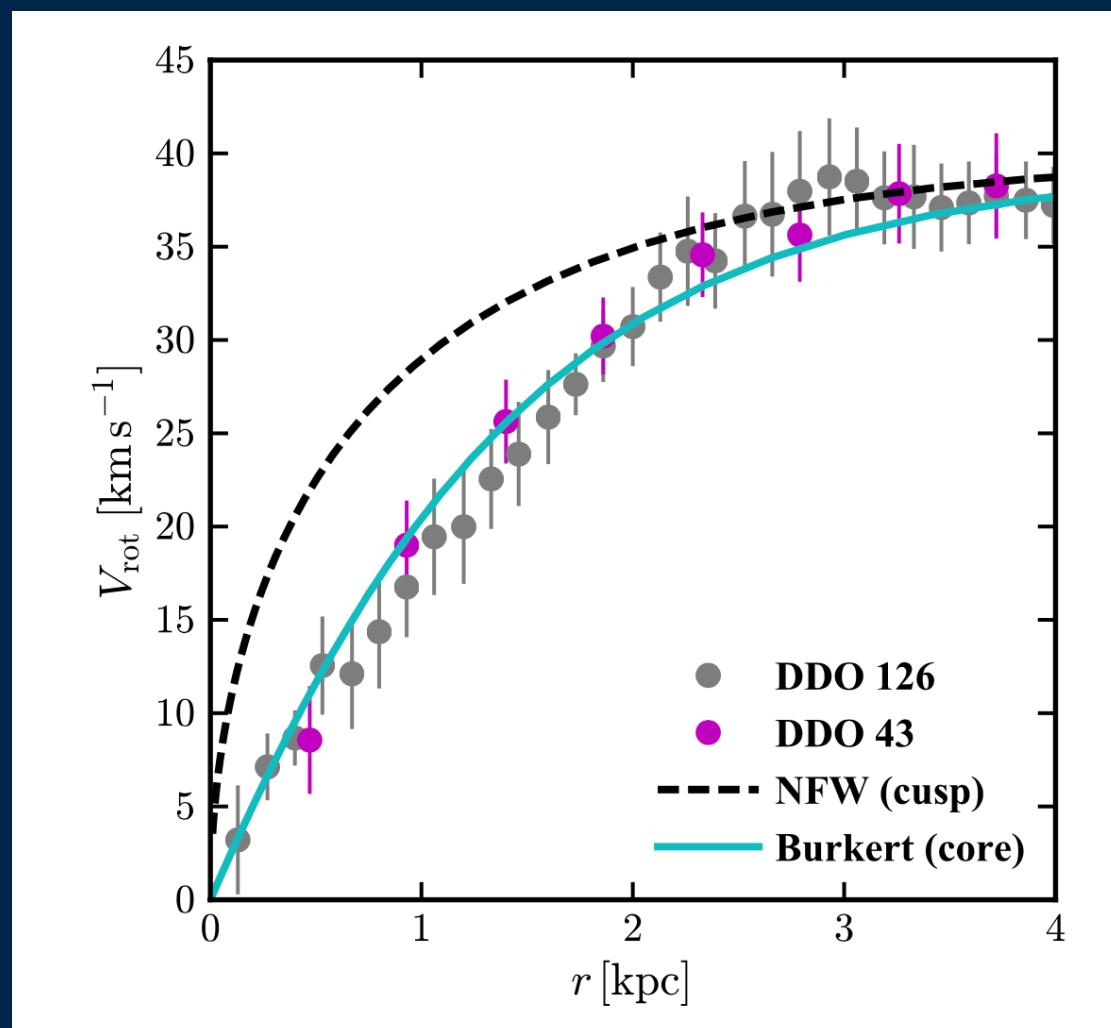
D. Thomas, M. Kopp, CS, S. Ilic, PRL120, 221102 (2018)

S. Ilic, M. Kopp, CS, D. Thomas, PRD 104, 043520 (2021)

Λ CDM: small scales

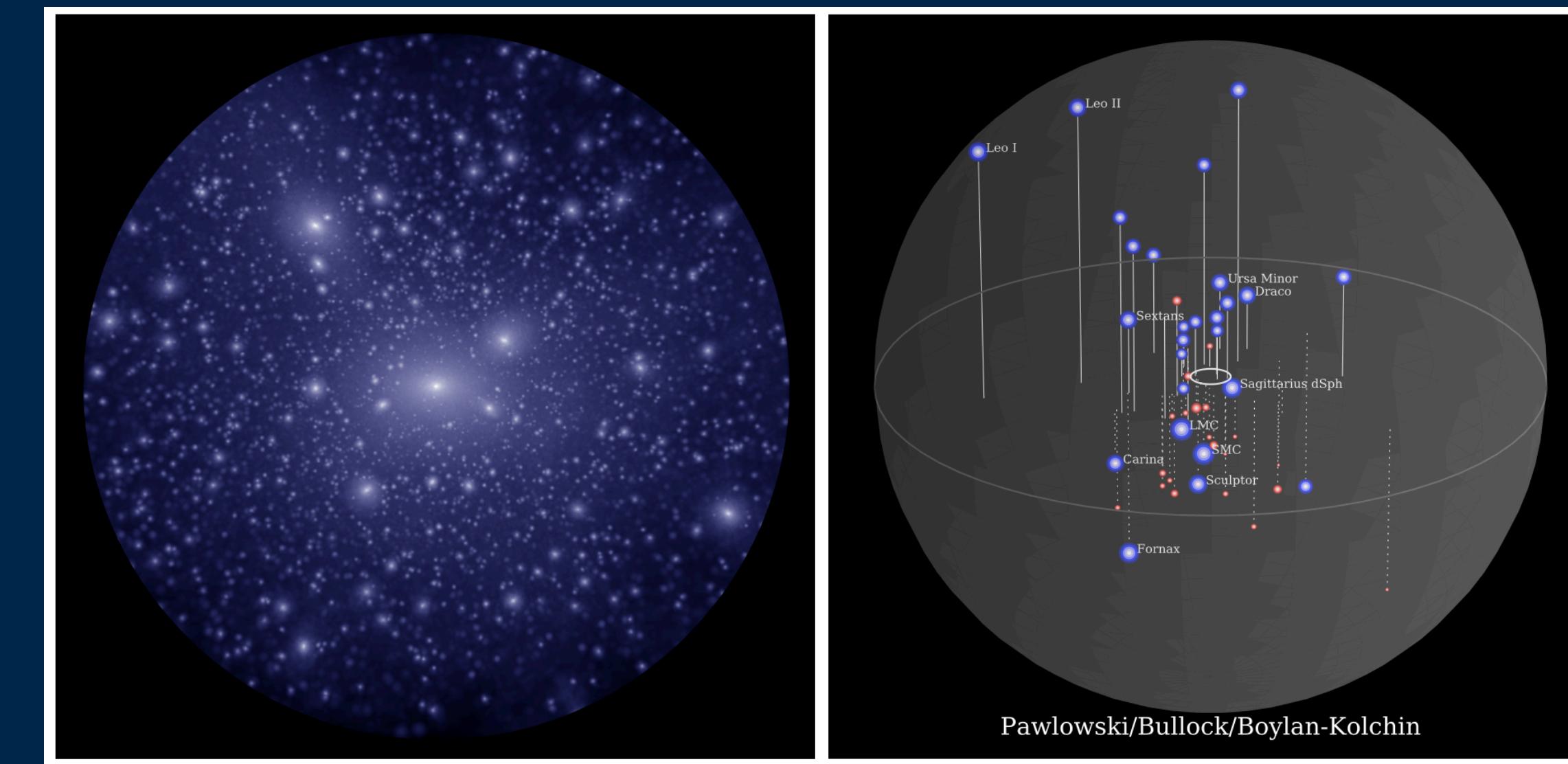
See e.g. Bullock & Boylan-Kolchin (2017)

Core-Cusp — a.k.a diversity
of rotation curves



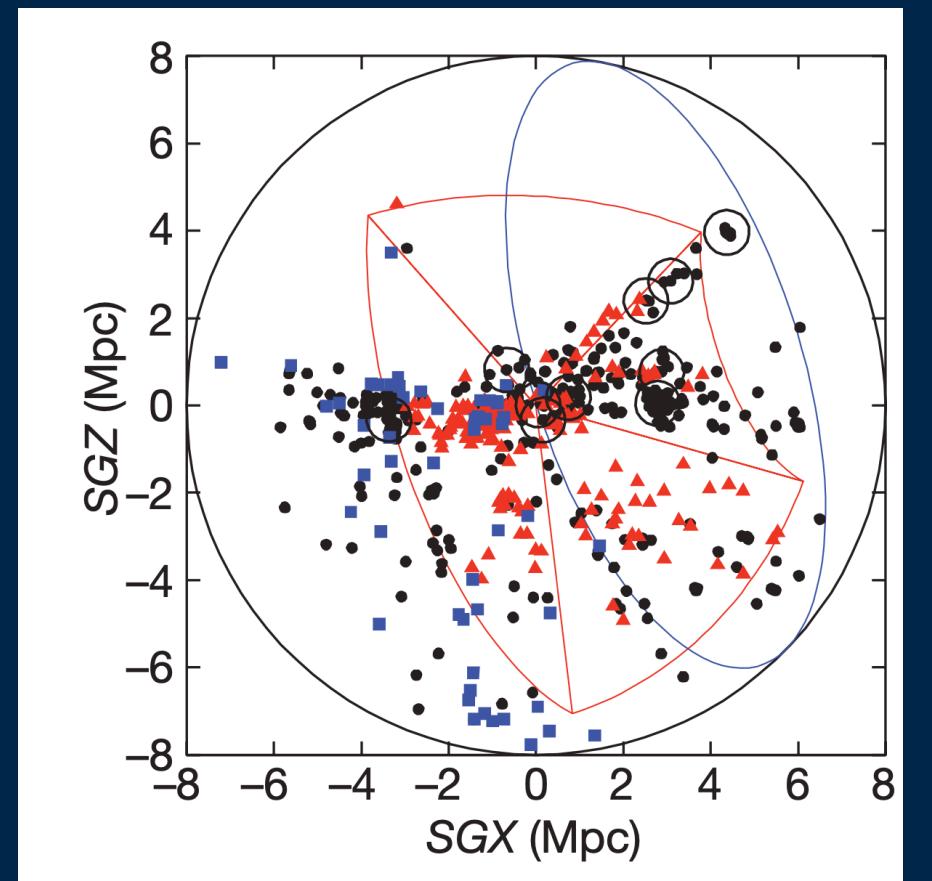
Challenges

Missing satellites

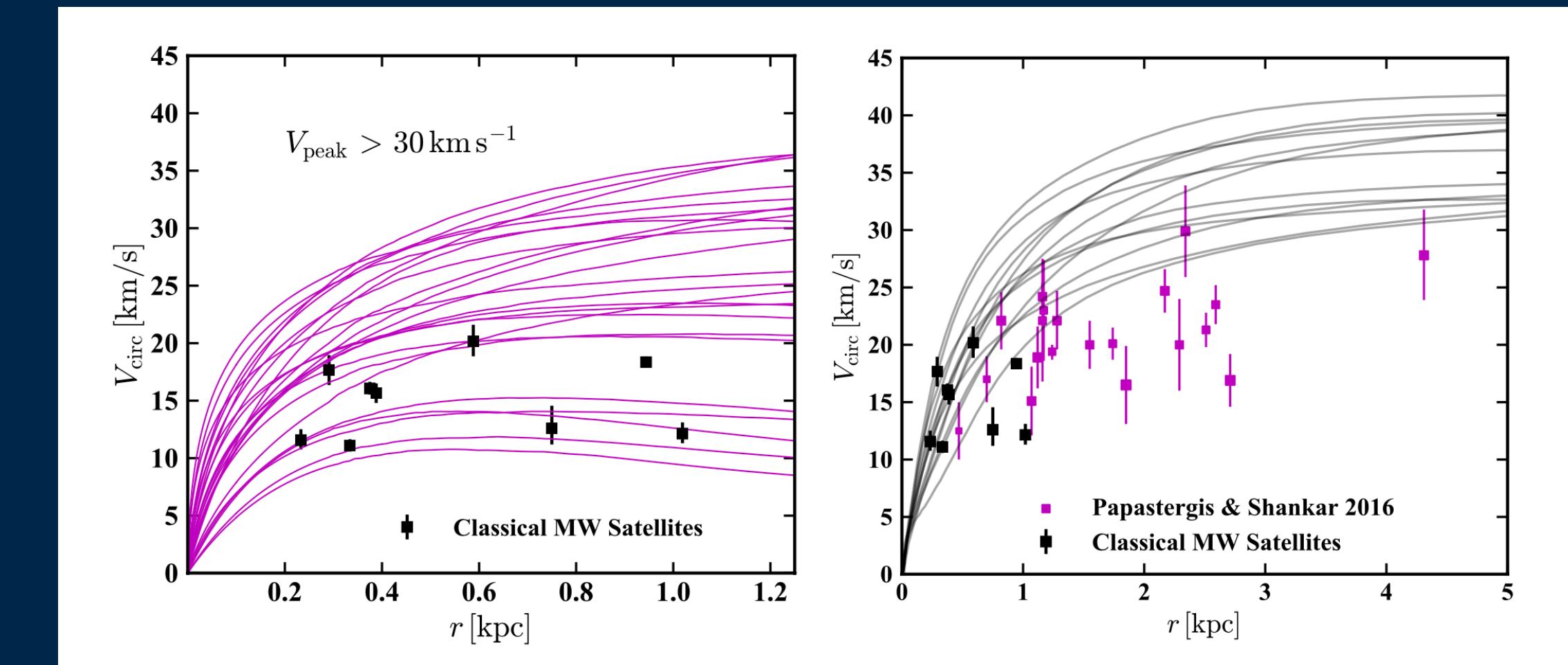


Local void too empty

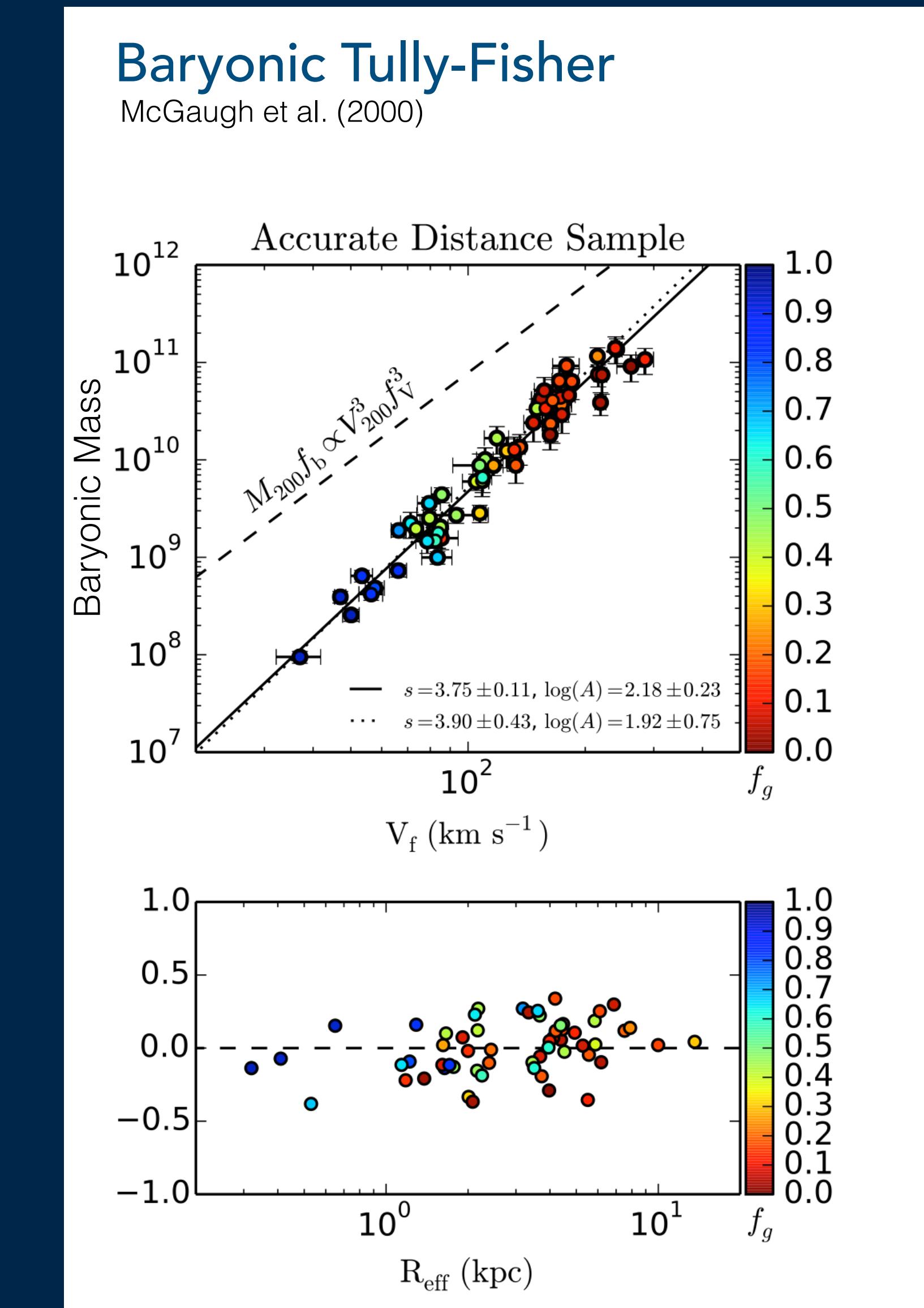
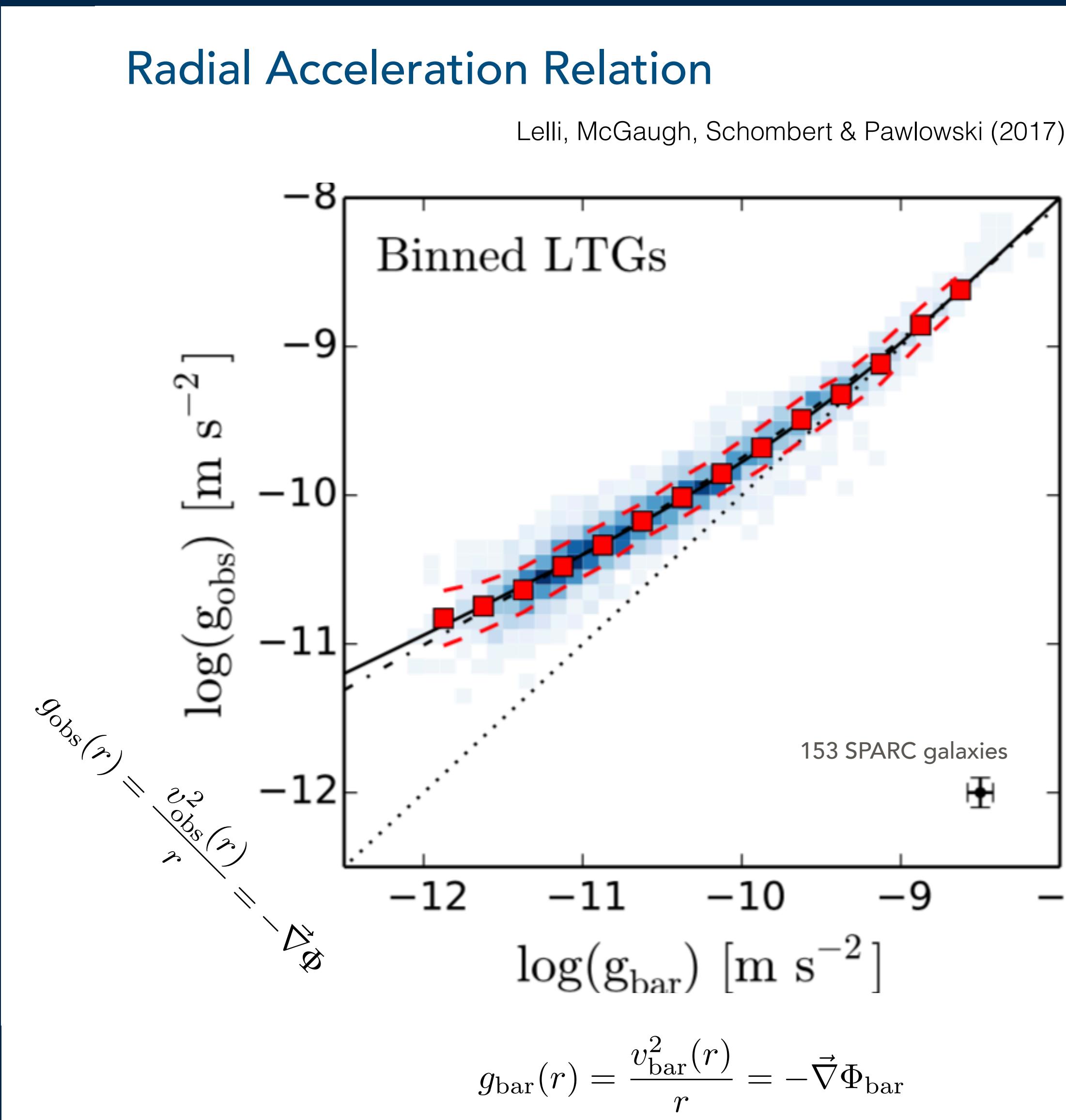
(Peebles & Nusser, Nature 2010)



Too-big-to-fail

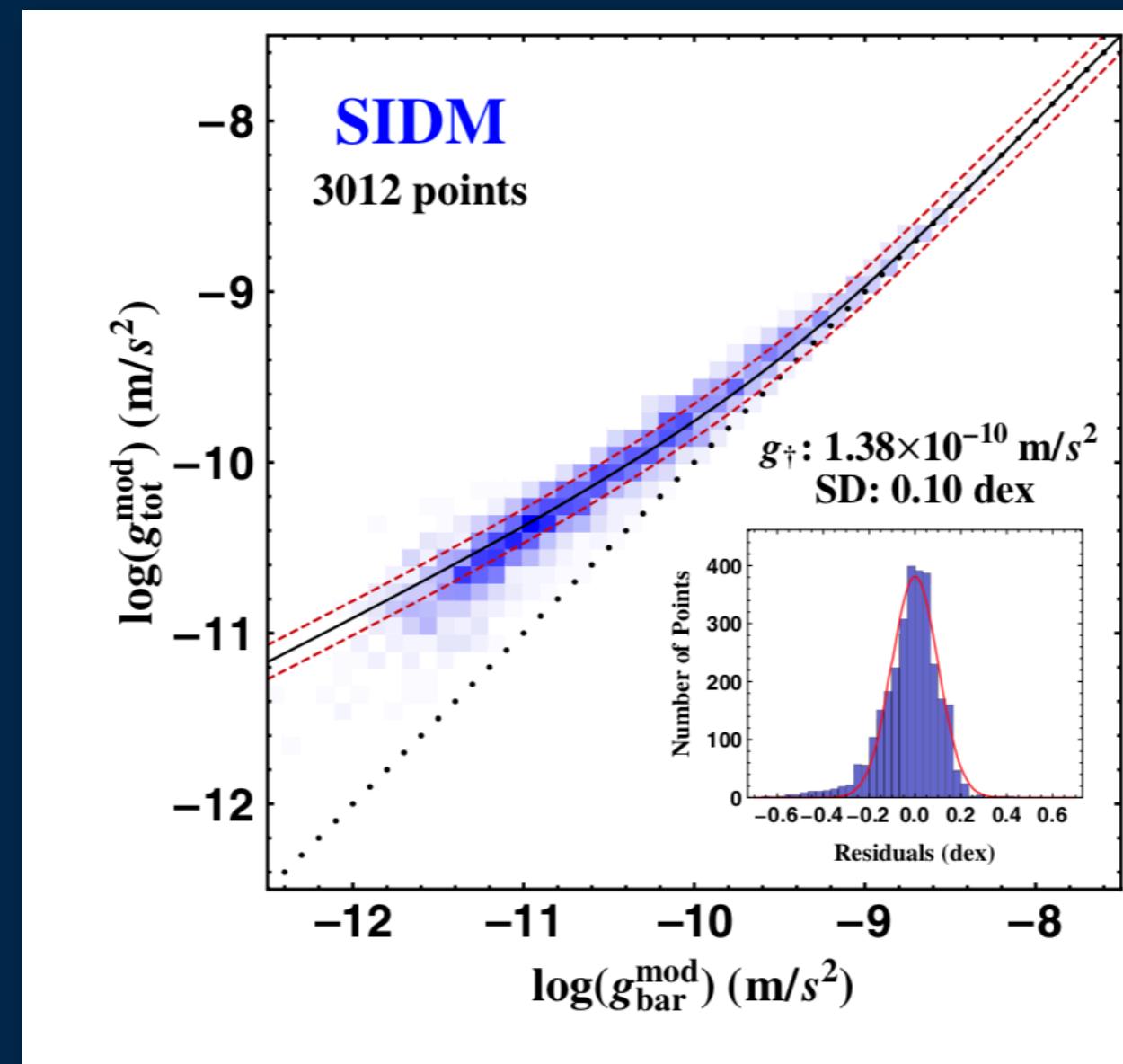


Regularity in galaxies



DARK MATTER ++

Self-interacting Dark Matter



Spergel & Steinhardt, PRL 84, 3760 (2000)

Ren et al, PRX 9, 031020 (2019)

Vogelsberger, Zavala, et al

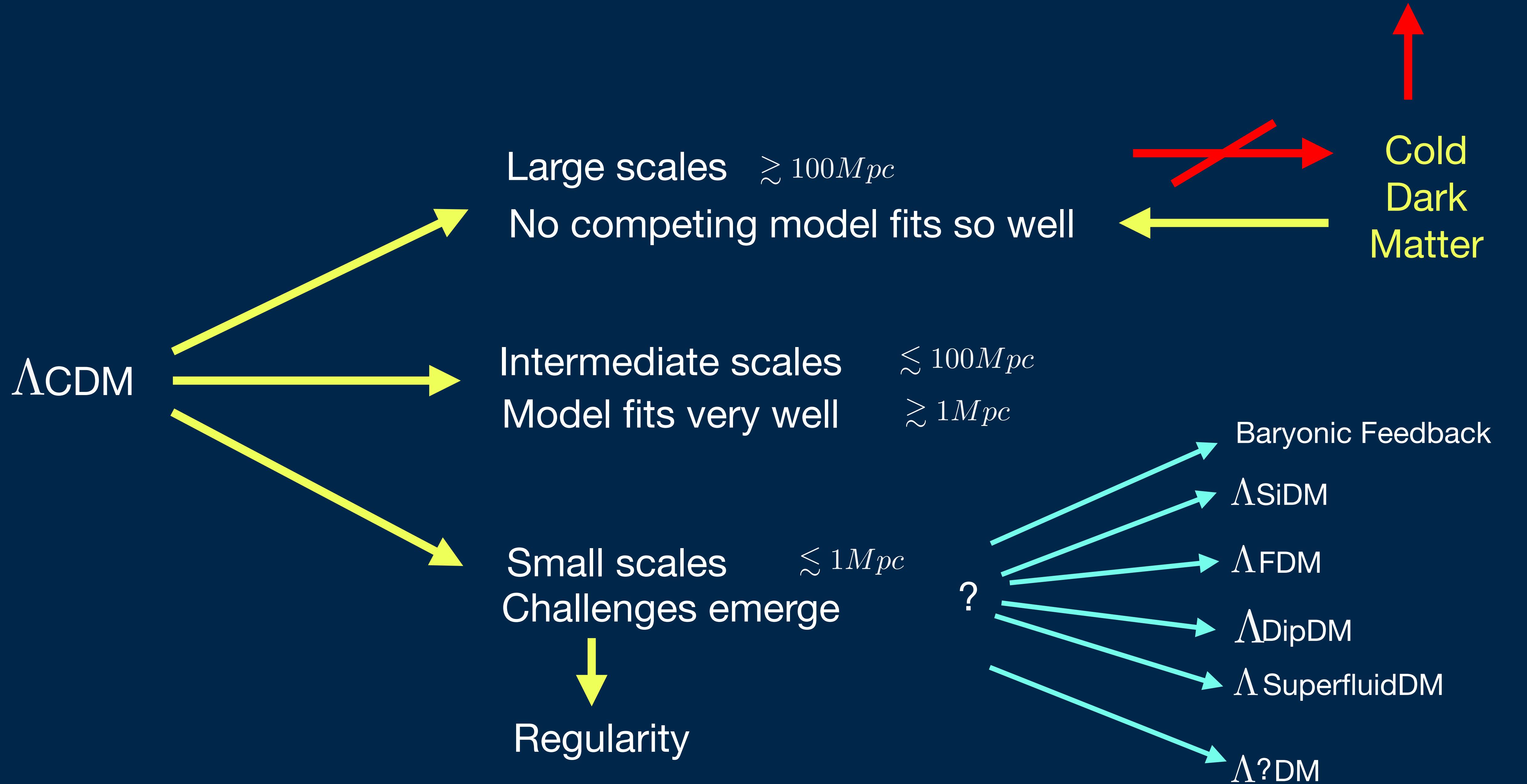
Dipolar dark matter (Blanchet 2007)

“gravitational polarization” → dipole moment → Emergence of universality (MOND)

Superfluid dark matter (Khoury & Berezhiani, 2015)

- Axion-like particles with mass of order eV and strong self-interactions.
- Aptly described as collective excitations: phonons
- Superfluid phonons: Goldstone bosons of a spontaneously broken global U(1) symmetry.
- Lagrangian put in by hand (no fundamental theory)

To summarize



Extending GR

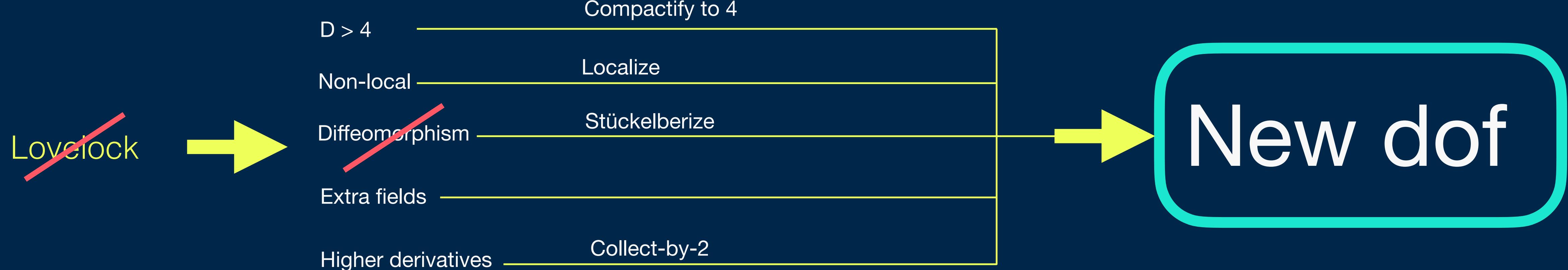
Lovelock's Theorem (1967)

The only

- local,
- diffeomorphism invariant action,
- which leads to 2nd order field equations
- and which depends only on a metric

in 4D

is a linear combination of the Einstein-Hilbert action with a cosmological constant up to a total derivative: GR



New dof

$$\varphi^{(I)} \quad \alpha_\mu^{(I)}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Minkowski

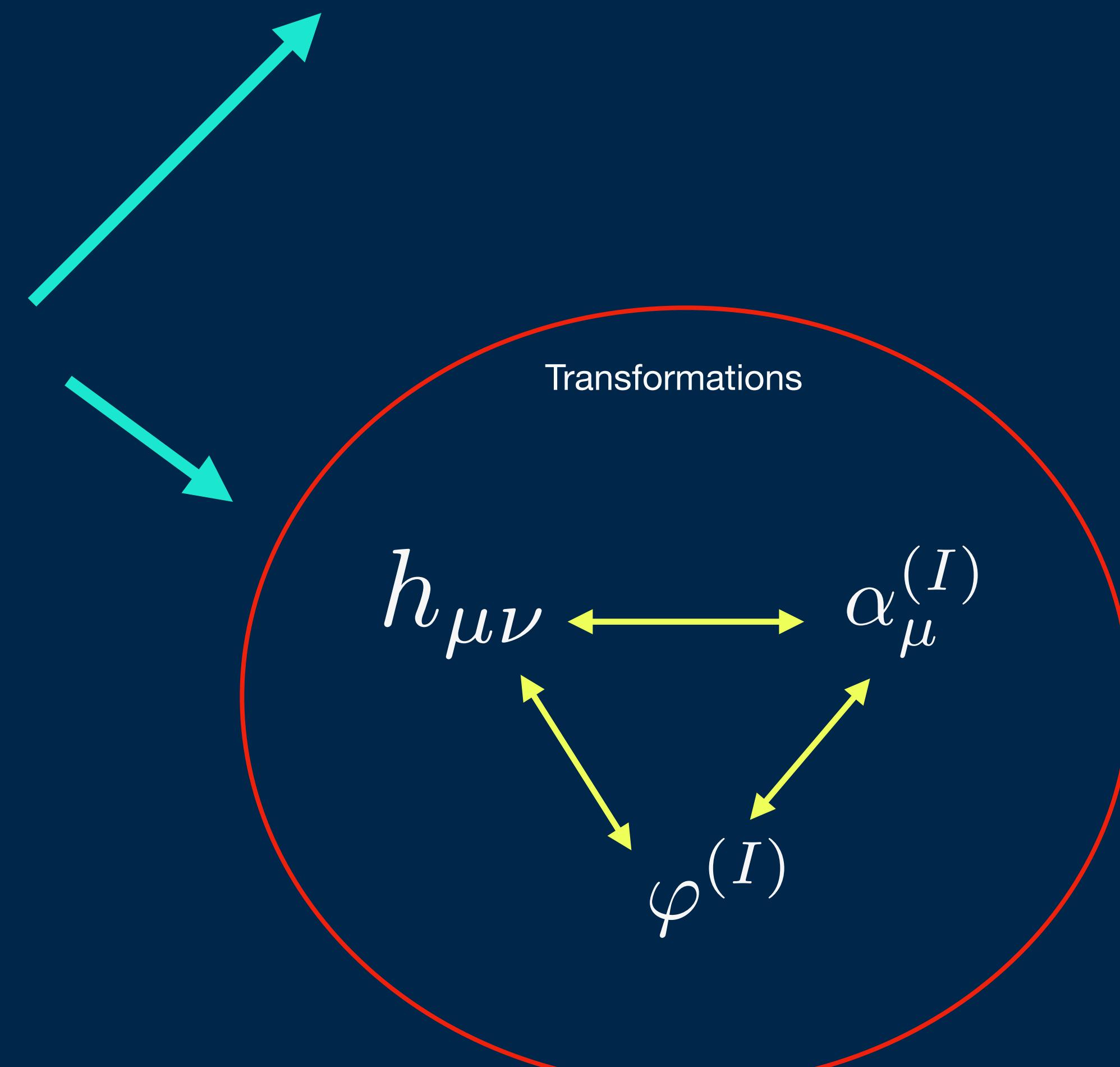
$$h_{\mu\nu}$$

$$\varphi^{(I)}$$

$$\alpha_\mu^{(I)}$$

Matter

e.g. $\mathcal{L} \sim (\partial h)^2 + (\partial \varphi)^2$

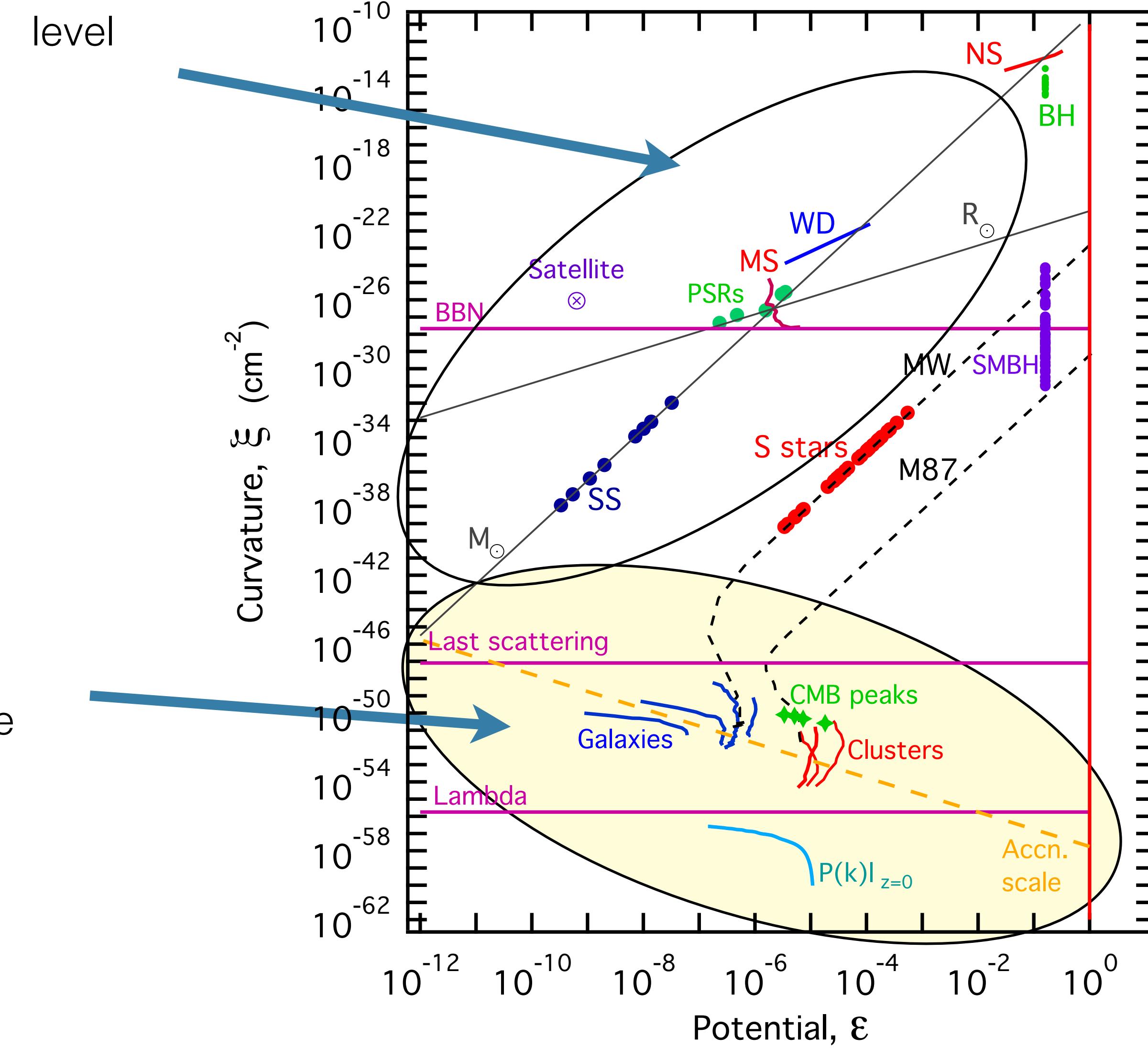


Gravity

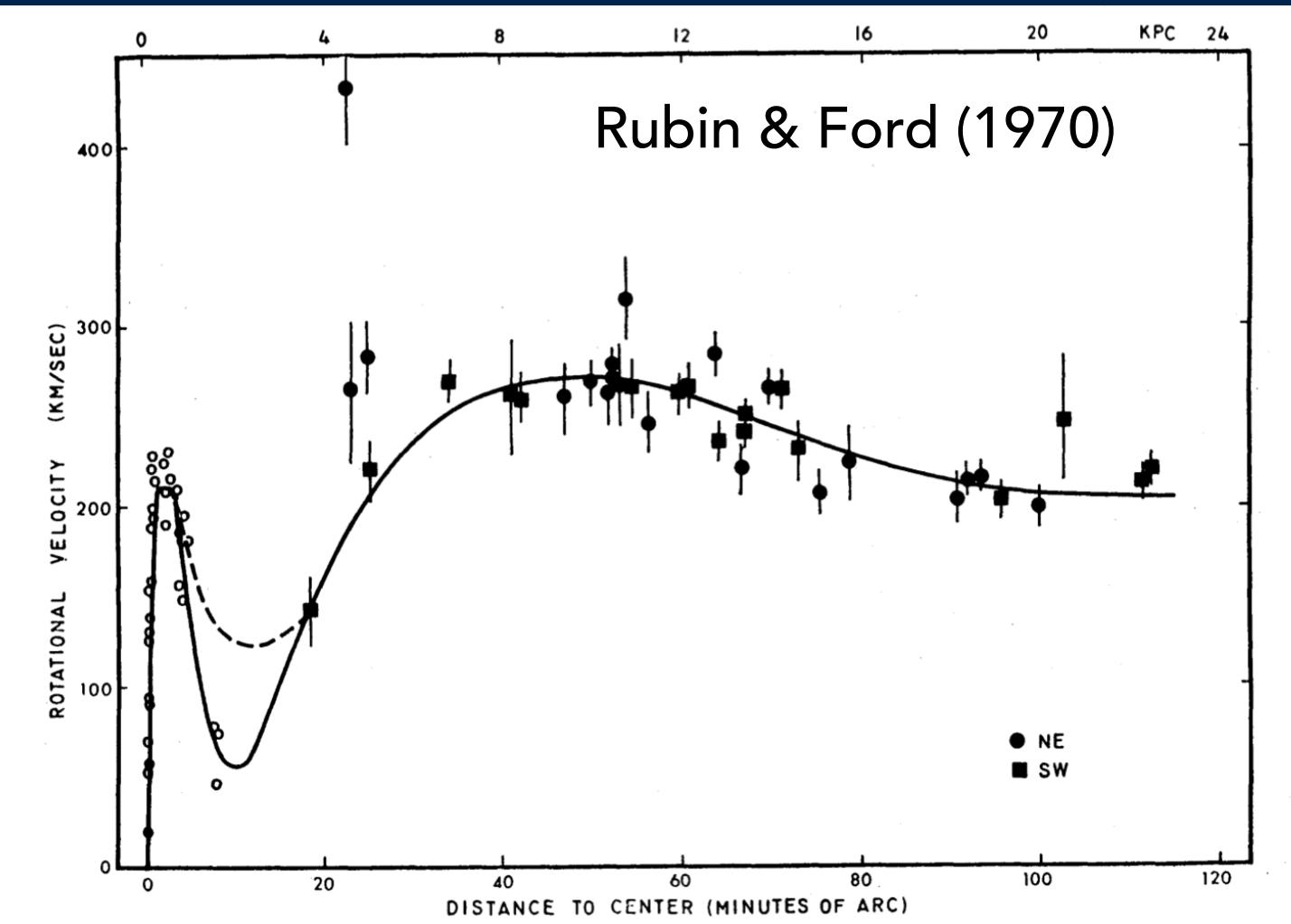
e.g. $\mathcal{L} \sim (\partial h)^2 + (\partial \varphi)^2 + (\partial h)(\partial \varphi)$

- GR experimentally tested
- Deviations at $10^{-4} - 10^{-8}$ level
- New dof suppressed

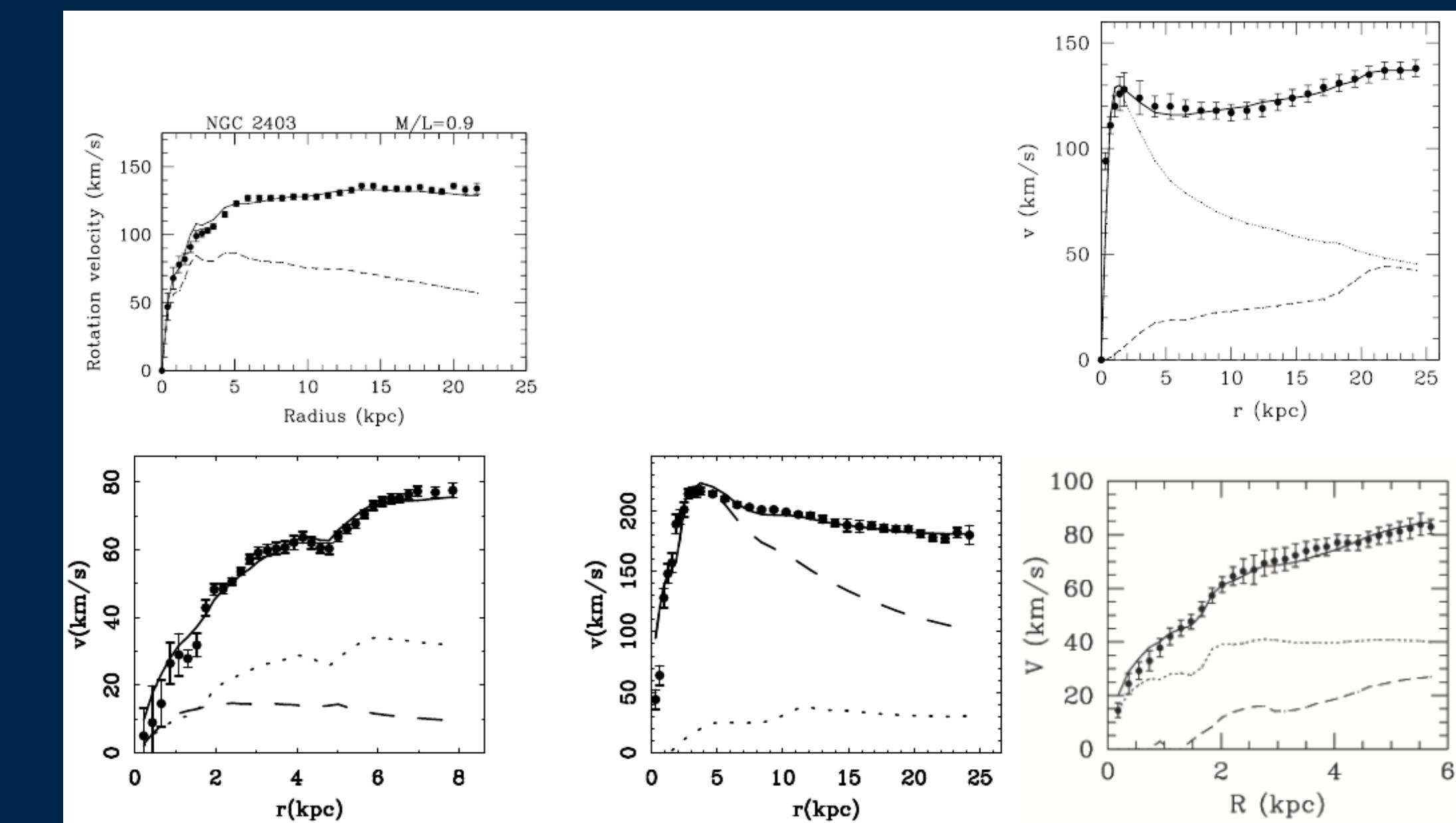
- Need DM
- GR not tested
- New dof active



Phenomenology: galaxies



$$v \sim \text{const} \Rightarrow a \sim \frac{v^2}{r} \sim \frac{1}{r}$$



Modified Newtonian Dynamics

Milgrom (1983), Bekenstein & Milgrom (1984)

Deviation from Newton when

$$a < a_0 \sim 1.2 \times 10^{-10} m/s^2$$



Universal constant

$$\vec{\nabla} \cdot \left(\frac{|\vec{\nabla} \Phi|}{a_0} \vec{\nabla} \Phi \right) = 4\pi G_N \rho$$

Gravitational Lensing

Not valid for

CMB

LSS



Relativistic description

EXTENSION OF GR



Non-relativistic, static: MOND

Bekenstein & Milgrom 1984

$$\Phi$$



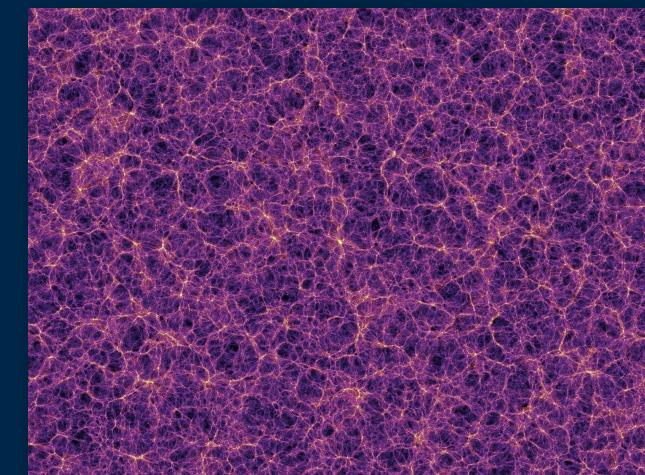
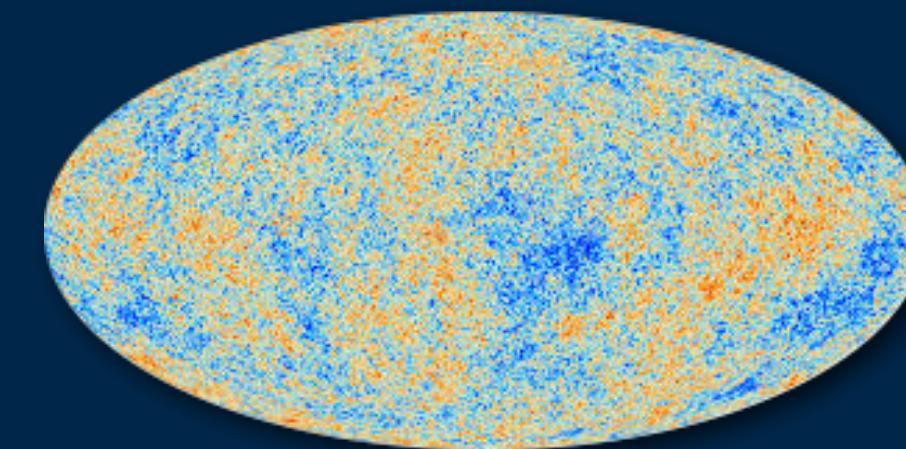
Lensing

$$\Phi = \Psi$$

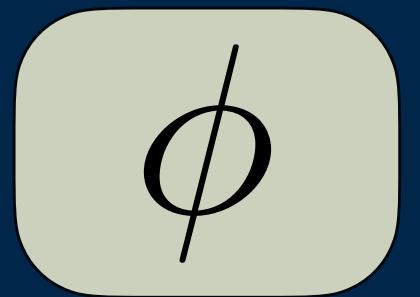
aLIGO/Virgo + EM (2017)
Tensor speed = 1



FRW + linear fluctuations
Effective description: Λ CDM



New scalar dof.



Bekenstein & Milgrom (1984)

(Scalar-tensor theory)

$$g_{\mu\nu} = e^{-2\phi} \tilde{g}_{\mu\nu}$$

AQUAL

$$\vec{\nabla} \cdot \left(\frac{d\mathcal{J}}{d\mathcal{Y}} \vec{\nabla} \phi \right) = 4\pi G_N \rho$$

$$\vec{\nabla} \phi \sim \frac{G_N M}{r^2} + \vec{\nabla} \varphi$$

Parameter ↑

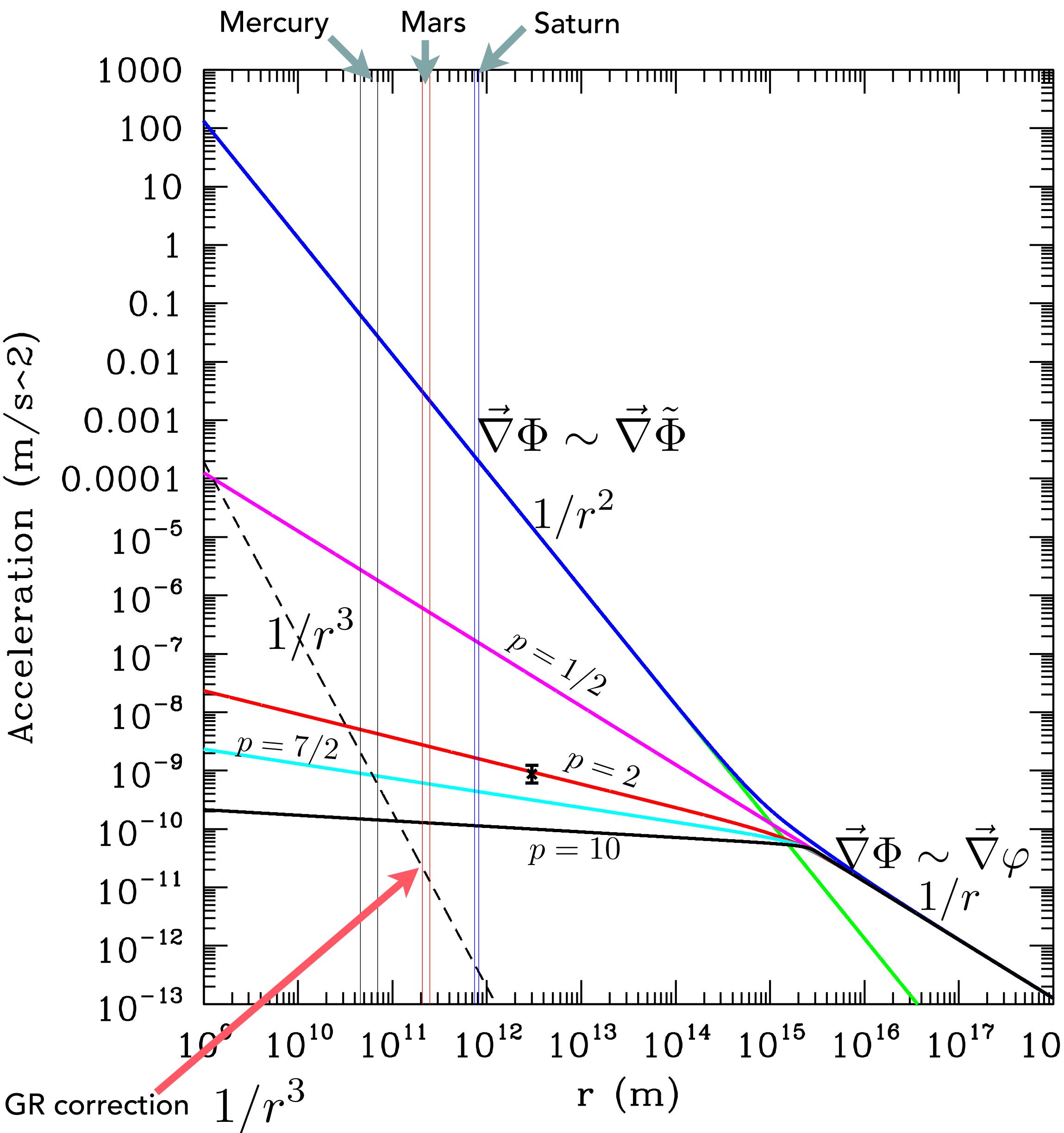
$$\mathcal{J}(\mathcal{Y}) \rightarrow \lambda_s \mathcal{Y} = \lambda_s |\vec{\nabla} \phi|^2 \quad |\vec{\nabla} \phi| \gg a_0$$

Parameter ↓

$$\mathcal{J}(\mathcal{Y}) \rightarrow \frac{\mathcal{Y}^{3/2}}{a_0} = \frac{|\vec{\nabla} \varphi|^3}{a_0} \quad |\vec{\nabla} \phi| \ll a_0$$

Relativistic version gives
wrong lensing formula

Quasistatic weak-field limit



Screening

$$\mathcal{L} \sim \mathcal{J}(\mathcal{Y}) \sim \frac{|\vec{\nabla}\varphi|^3}{a_0} + \beta_p \frac{|\vec{\nabla}\varphi|^{2(p+1)}}{a_0^{2p}}$$

$$p \rightarrow \infty \Rightarrow \vec{\nabla}\phi \rightarrow \text{const}$$



DBion — Burrage & Khouri (2014)

$$f \equiv \frac{d\mathcal{J}}{d\mathcal{Y}}$$

Tracking

$$\vec{\nabla} \cdot \left[f \left(\frac{|\vec{\nabla}\varphi|}{a_0} \right) \vec{\nabla}\varphi \right] = 4\pi G\rho$$

Interpolation function:

$$f \left(\frac{|\vec{\nabla}\varphi|}{a_0} \right)$$

$$\frac{|\vec{\nabla}\varphi|}{a_0} \gg 1$$

Const.

$$\frac{|\vec{\nabla}\varphi|}{a_0} \ll 1$$

?

$$\vec{\nabla} \cdot \left(\frac{d\mathcal{J}}{d\mathcal{Y}} \vec{\nabla} \phi \right) = 4\pi G_N \rho$$

$$\vec{\nabla} \phi \sim \frac{G_N M}{r^2} + \vec{\nabla} \varphi$$

Parameter
↑

$$\mathcal{J}(\mathcal{Y}) \rightarrow \lambda_s \mathcal{Y} = \lambda_s |\vec{\nabla} \phi|^2 \quad |\vec{\nabla} \phi| \gg a_0$$

$$\mathcal{J}(\mathcal{Y}) \rightarrow \frac{\mathcal{Y}^{3/2}}{a_0} = \frac{|\vec{\nabla} \varphi|^3}{a_0} \quad |\vec{\nabla} \phi| \ll a_0$$

↓ Parameter

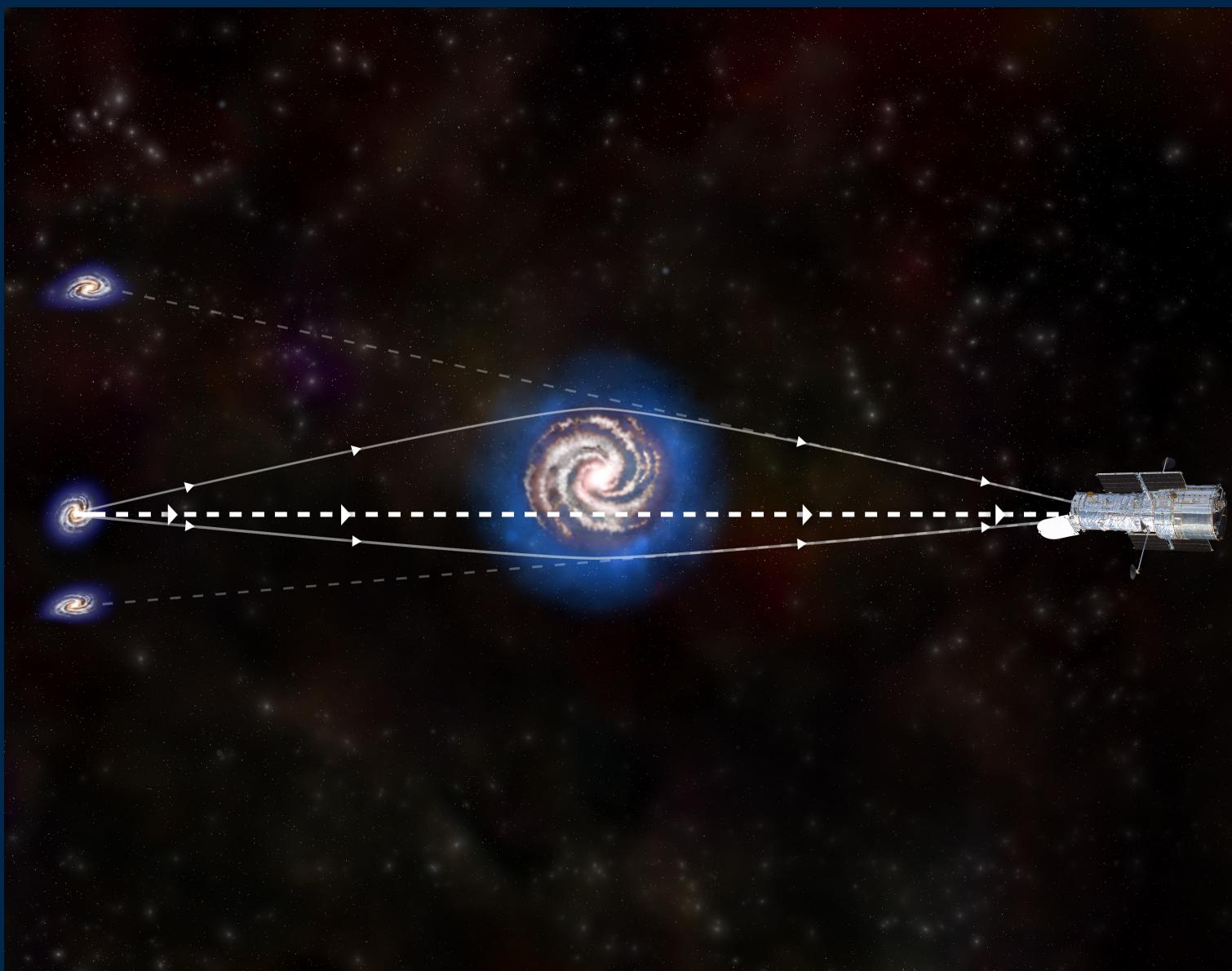
Screening: $\lambda_s \rightarrow \infty$

Tracking: requires $\mathcal{J}(\mathcal{Y})$ To be non-analytic

Example: $\mathcal{J} = \lambda_s \left\{ \mathcal{Y} - 2a_0(1 + \lambda_s)\sqrt{\mathcal{Y}} + 2(1 + \lambda_s)^2 a_0^2 \ln \left[1 + \frac{\sqrt{\mathcal{Y}}}{(1 + \lambda_s)a_0} \right] \right\}$

New time-like vector dof

A_μ



$$ds^2 = -(1 + 2\Phi)dt^2 + (1 - 2\Phi)d\vec{x}^2$$

$$\Phi = \tilde{\Phi} + \varphi$$

$$\rightarrow ds^2 \neq e^{\pm 2\varphi} [-e^{2\tilde{\Phi}}dt^2 + e^{-2\tilde{\Phi}}d\vec{x}^2]$$

But:

$$g_{\mu\nu} = e^{-2\phi}\tilde{g}_{\mu\nu} - (e^{2\phi} - e^{-2\phi}) A_\mu A_\nu$$

$$\tilde{g}^{\mu\nu} A_\mu A_\nu = -1$$

Sanders, ApJ 480, 492 (1997)



Tensor-Vector-Scalar theory: Bekenstein, PRD 70, 083509 (2004)

Disagreement with CMB

Skordis, Mota, Ferreira, Boehm, PRL 96, 011301 (2006)

Dodelson & Liguori, PRL 97, 231301 (2006)

Agreement with matter power spectrum

Tensor mode speed $\neq 1$

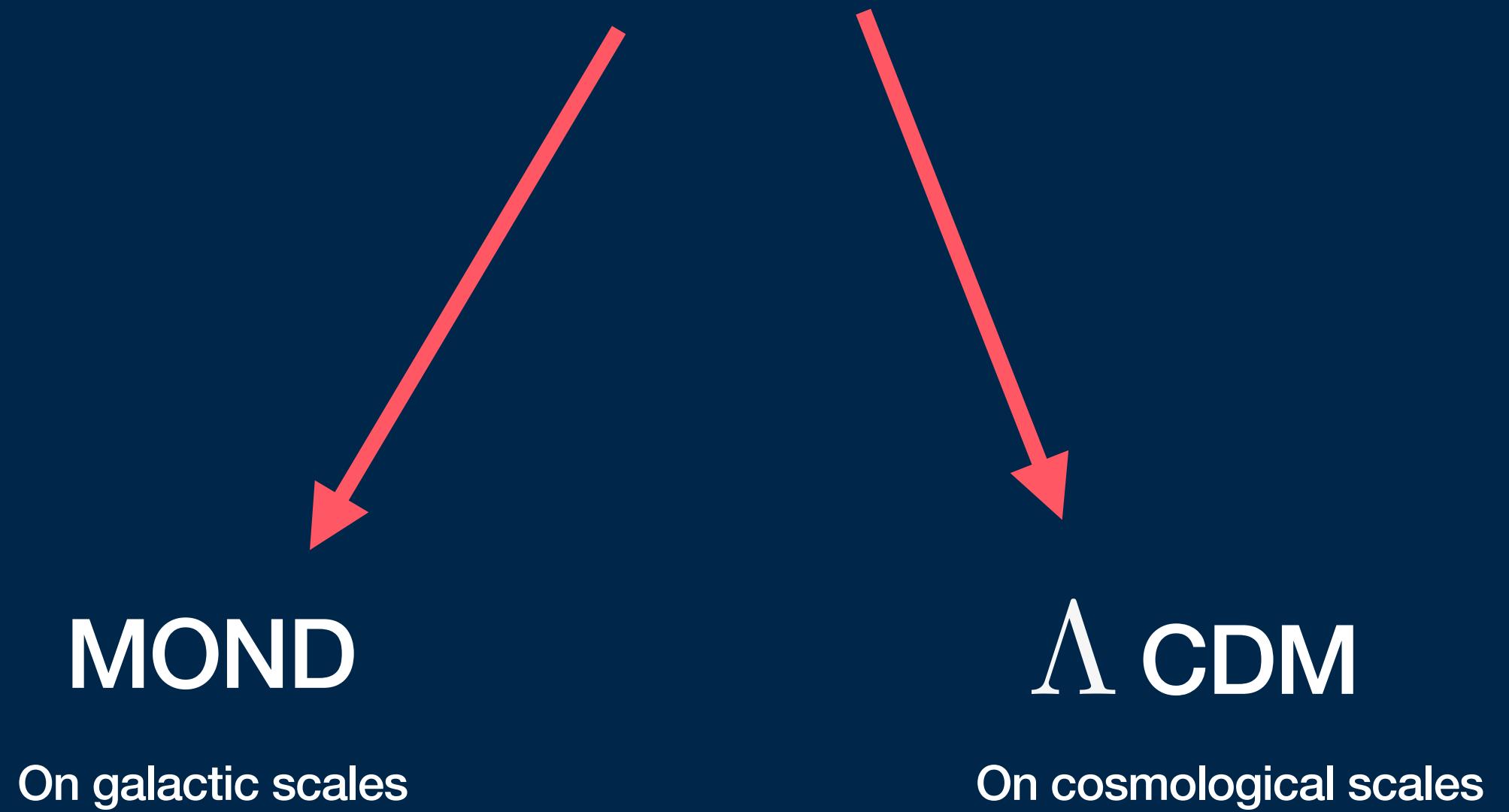
Boran et al., PRD 97, 041501 (2018)

C.S & Zlosnik, PRD 100, 104013 (2019)

One metric:

$$g_{\mu\nu}$$

•Aether Scalar Tensor (AeST)



New scalar dof.

Aether:
New time-like vector dof.

Dirac's new theory of electrons (1963)

Einstein-Aether theory
Jacobson. & Mattingly (2002)

Bumblebee field
Kostelecky & Samuel, PRD 40, 1886 (1989)

$$\phi$$

$$A_\mu$$

Λ CDM

MOND

Sanders (1997), Bekenstein (2004)
TeVeS theory

Gauge ghost condensate
Cheng et al (2006)

Lorentz violation

Scherrer (2004)
Arkani-Hamed et al (2004)

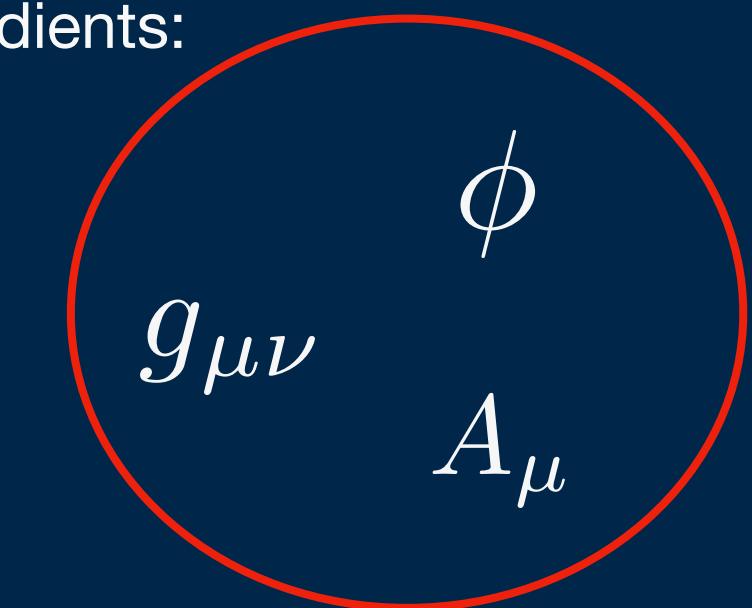
Bekenstein & Milgrom (1984)

Tensor speed = 1

Skordis & Zlosnik (2019)

$$S = \int d^4x \frac{\sqrt{-g}}{16\pi\tilde{G}} \left[R - \frac{K_B}{2} F^{\mu\nu} F_{\mu\nu} + 2(2 - K_B) J^\mu \nabla_\mu \phi - (2 - K_B) \mathcal{Y} - \mathcal{F}(\mathcal{Y}, \mathcal{Q}) - \lambda (A^\mu A_\mu + 1) \right] + S_m[g]$$

Ingredients:



$$A^\mu A_\mu = -1$$

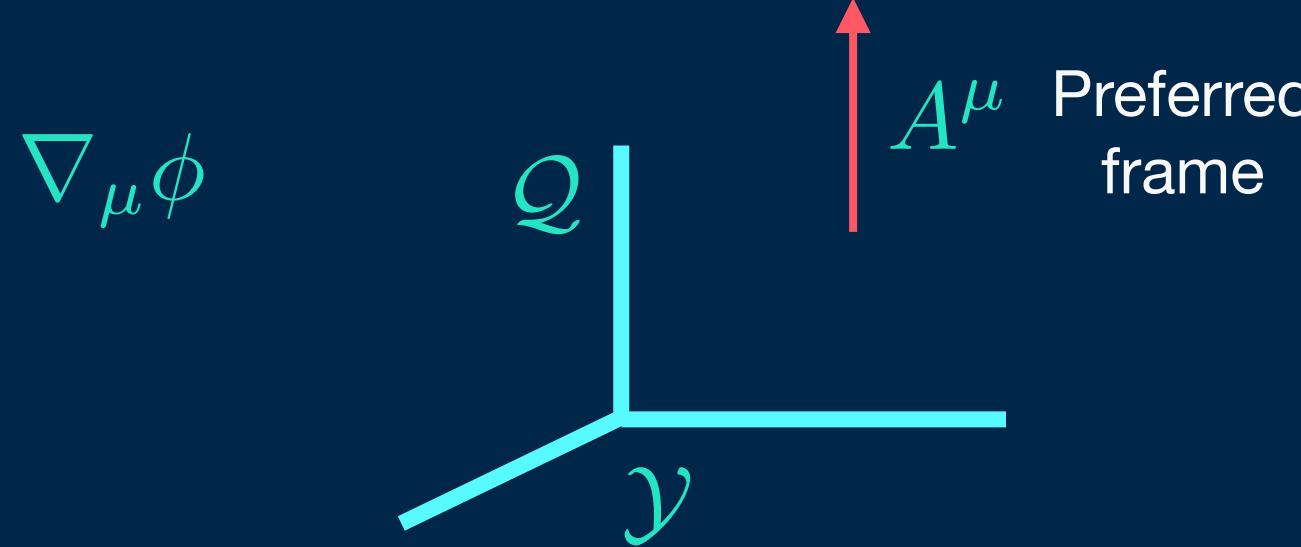
Unit-timelike

“Magic function”

Parameter

$$F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$$

$$J_\mu = A^\nu \nabla_\nu A_\mu$$



Spatial gradients

$$\mathcal{Y} = (g^{\mu\nu} + A^\mu A^\nu) \nabla_\mu \phi \nabla_\nu \phi \rightarrow |\vec{\nabla} \phi|^2$$

$$\mathcal{Q} = A^\mu \nabla_\mu \phi \rightarrow \dot{\phi}$$

Time evolution

Quasistatic weak-field limit

$$\phi = Q_0 t + \varphi(\vec{x})$$

$$ds^2 = -(1 + 2\Psi) dt^2 + (1 - 2\Phi) d\vec{x}^2$$

$A^0 = 1 - \Psi$

Symmetry: $A_i \rightarrow A_i - \vec{\nabla}_i \xi_T$

$\dot{\xi}_T = 0 \quad \varphi \rightarrow \varphi + Q_0 \xi_T$

Ignoring curl, set $A_i = 0$

Field equations



Constraint:
 $\Psi = \Phi$

Lensing ok

$$\vec{\nabla}\phi = \vec{\nabla}\varphi$$

$$\gamma \rightarrow |\vec{\nabla}\varphi|^2$$

$$Q \rightarrow Q_0$$

$$\mathcal{F}(\gamma, Q) \rightarrow \mathcal{J}(\gamma)$$

MOND:

$$\mathcal{J} \sim \frac{\gamma^{3/2}}{a_0} \sim \frac{|\vec{\nabla}\varphi|^3}{a_0}$$

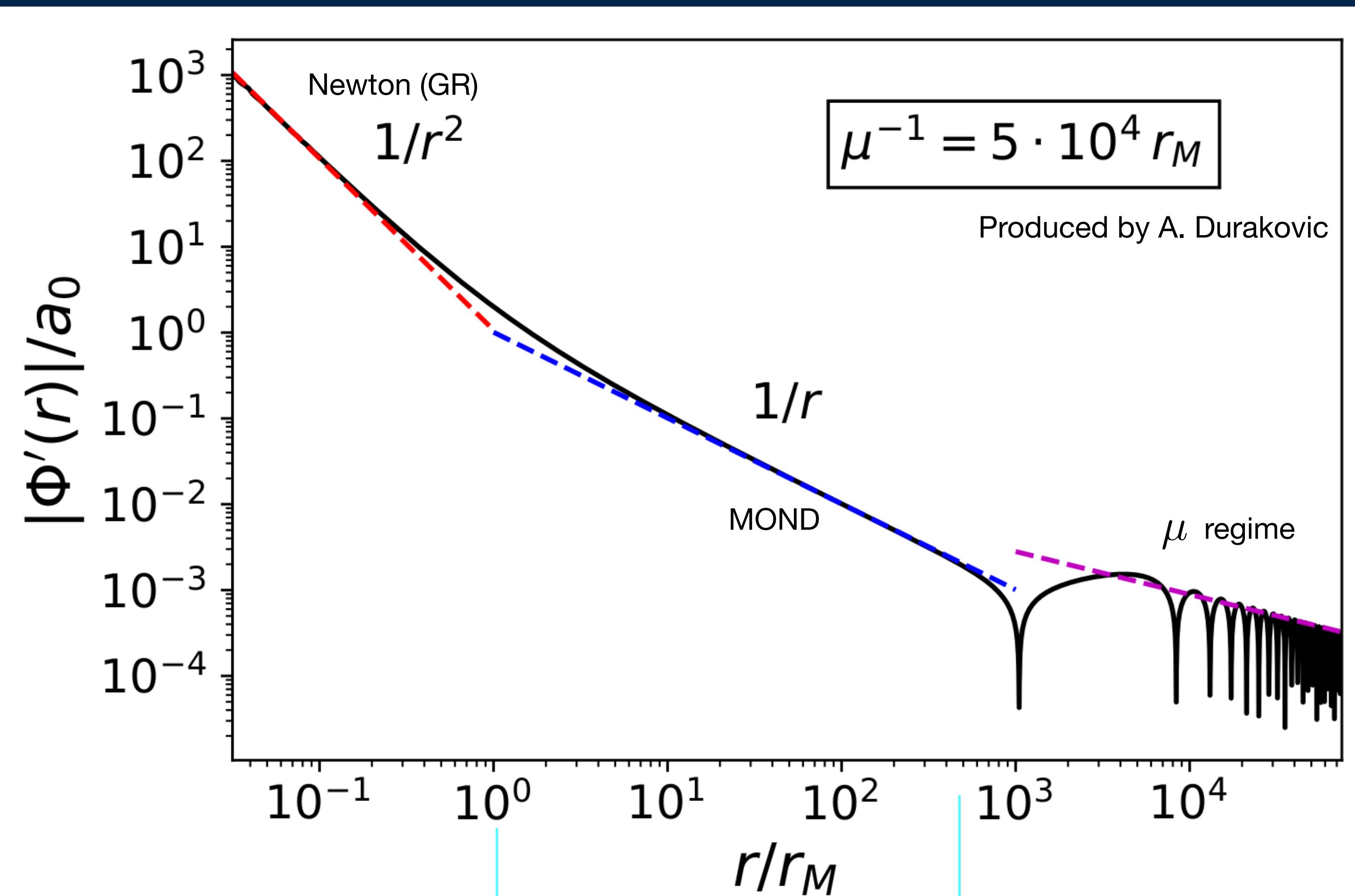
If Ψ correctly given by baryons alone
then
 Ψ also gives correct lensing potential

Quasistatic weak-field limit

$$\vec{\nabla}^2 (\Phi - \varphi) = \vec{\nabla} \cdot \left(\frac{d\mathcal{J}}{d\gamma} \vec{\nabla} \varphi \right)$$

$$\vec{\nabla}^2 (\Phi - \varphi) + \mu^2 \Phi = \frac{8\pi \tilde{G}}{2 - K_B} \rho$$

$$\mu^2 = \frac{2\mathcal{K}_2}{2 - K_B} Q_0^2$$



$$r_M \sim \sqrt{\frac{GM}{a_0}}$$

$$r_C \sim \left(\frac{r_M}{\mu^2} \right)^{1/3}$$

$$\mu^{-1} \gtrsim Mpc$$

$$(\mu \lesssim 6 \times 10^{-30} eV)$$

$$\mathcal{J}(\gamma) \rightarrow (2 - K_B) \lambda_s \gamma$$

$$\mathcal{J}(\gamma) \rightarrow \frac{\gamma^{3/2}}{a_0} = \frac{|\vec{\nabla} \varphi|^3}{a_0}$$

$$\phi = \bar{\phi}(t)$$

$$A^0 \rightarrow 1$$

$$ds^2 = -dt^2 + a^2 d\vec{x}^2$$

Λ CDM



$$\mathcal{Y} = 0$$

$$\mathcal{Q} = \mathcal{Q}(t)$$

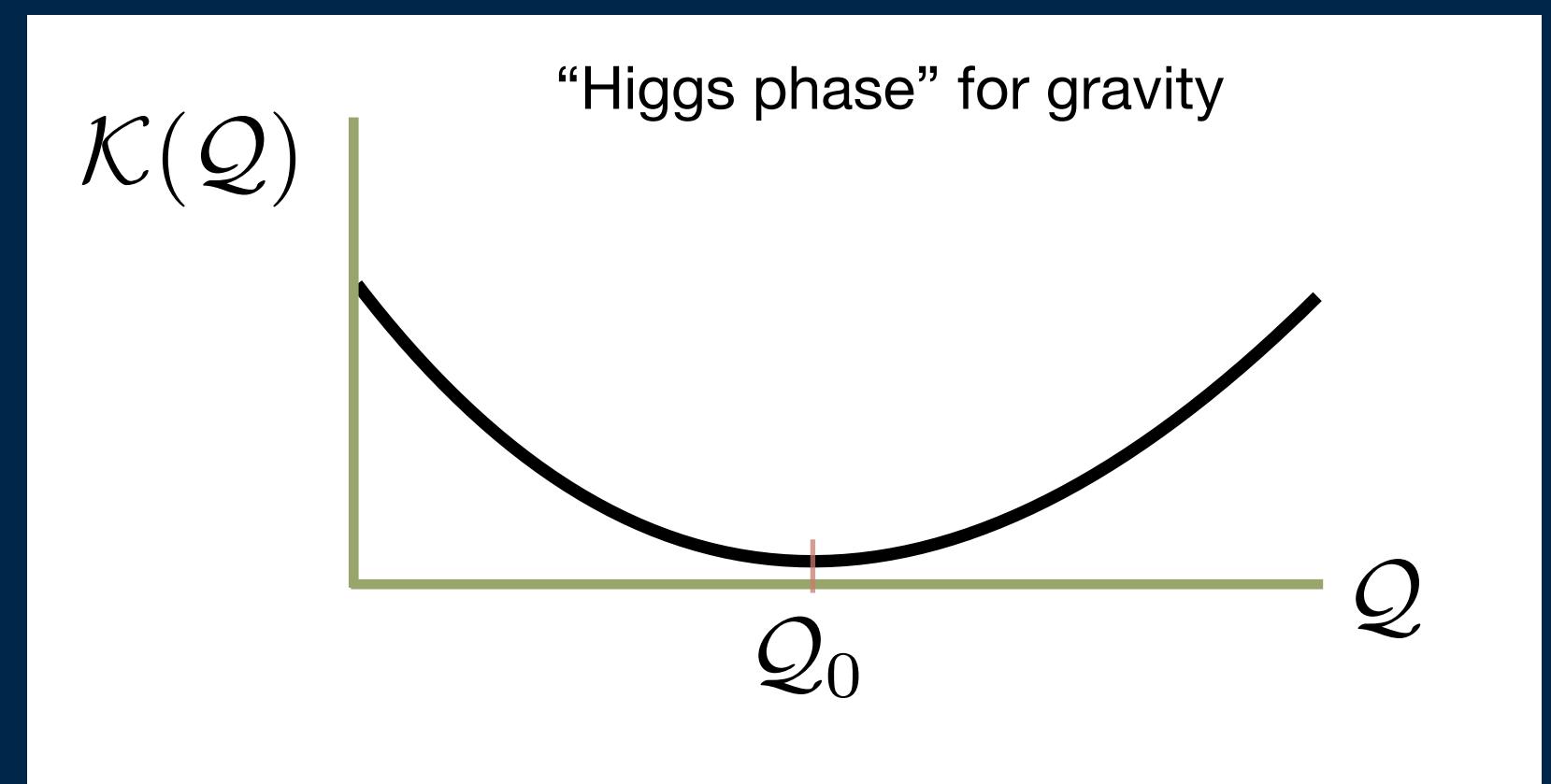


$$\mathcal{F}(\mathcal{Y}, \mathcal{Q}) \rightarrow -2\mathcal{K}(\mathcal{Q})$$

(K-essence)

$$\mathcal{K}(\mathcal{Q}) = \mathcal{K}_2 (\mathcal{Q} - \mathcal{Q}_0)^2 + \dots$$

Parameter



Spontaneous breaking of Lorentz symmetry: Massive fields generated

Shift-symmetric k-essence:

Scherrer, Phys.Rev.Lett. 93, 011301 (2004)

FLRW Limit of Ghost condensate

Arkani-Hamed et al., JHEP 05, 074 (2004)

New scalar dof mixing with metric:

$$\phi = \mathcal{Q}_0 t + \varphi$$

$$h_{00} \rightarrow h_{00} - 2\dot{\xi}$$

$$\varphi \rightarrow \varphi + \mathcal{Q}_0 \xi$$

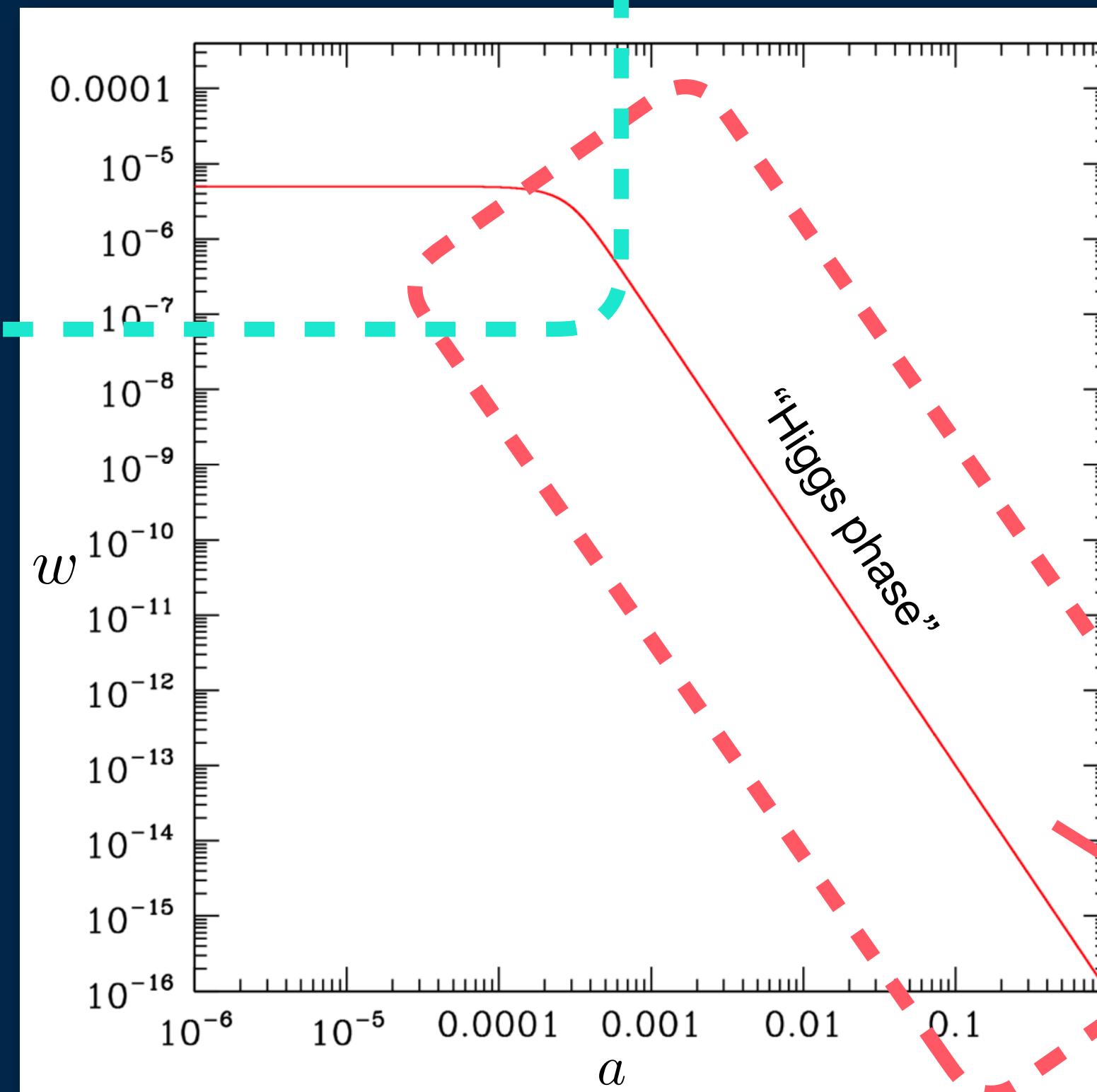
FLRW cosmology

$$\text{EOM: } \frac{d}{dt} \left(a^3 \frac{d\mathcal{K}}{dQ} \right) = 0 \Rightarrow \dot{\phi} = Q_0 + \frac{I_0/2\mathcal{K}_2}{a^3}$$

Initial condition

Early region: depends on form of $\mathcal{K}(\bar{Q})$

Equation of state $w(t)$



Higgs phase: "effective dust"

$$\rho = \frac{Q_0 I_0}{a^3} + \dots$$

Density

$$w = \frac{w_0}{a^3} + \dots$$

Equation of state

$$c_{\text{ad}}^2 = \frac{2w_0}{a^3} + \dots$$

Adiabatic sound speed

Late region: Universal

$$\mathcal{K} = -2\Lambda + \mathcal{K}_2(\bar{Q} - Q_0)^2 + \dots$$

MOND compatibility

$$\mu^2 = \frac{2\mathcal{K}_2}{2 - K_B} Q_0^2$$

$$\mu^{-1} \gtrsim Mpc$$

Higgs phase:

$$w \approx \frac{w_0}{a^3} + \dots$$

$$w_0 = \frac{3H_0^2\Omega_{\mathcal{Q}}}{4Q_0^2\mathcal{K}_2} = \frac{3H_0^2\Omega_{\mathcal{Q}}}{2(2 - K_B)\mu^2}$$

$$w_0 \gtrsim 10^{-8}$$

$$w_{rec} \sim O(1)$$

Data: $w_{rec} \lesssim 10^{-4}$



Higgs-like

$$\mathcal{K} \sim (Q^2 - Q_0^2)^2$$



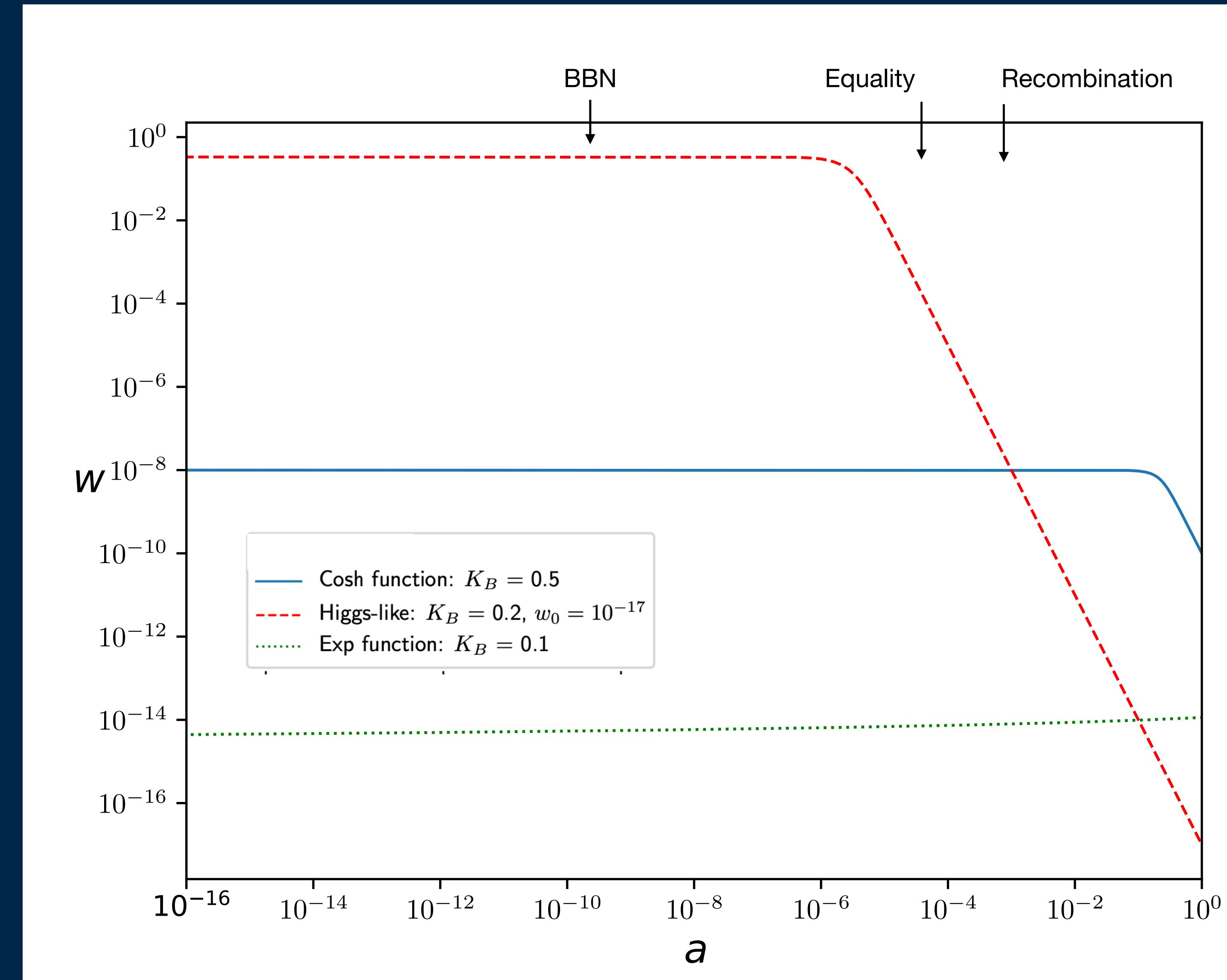
Cosh

$$\mathcal{K} \sim \cosh\left(\frac{Q - Q_0}{z_0}\right)$$



Exp

$$\mathcal{K} \sim e^{\left(\frac{Q - Q_0}{z_0}\right)^2}$$



FLRW + Perturbations

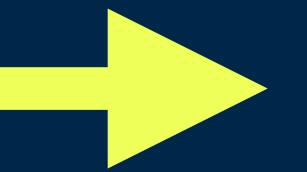
$$\phi = \bar{\phi} + \varphi$$

$$A_i = \vec{\nabla}_i \alpha$$

$$E = \dot{\alpha} + \Psi$$

$$\chi = \varphi + \dot{\bar{\phi}}\alpha$$

$$\gamma = \dot{\varphi} - \dot{\bar{\phi}}\Psi$$



Density contrast

$$\delta = \frac{1+w}{\dot{\bar{\phi}}c_{\text{ad}}^2}\gamma + \frac{1}{8\pi G a^2 \bar{\rho}} \vec{\nabla}^2 [K_B E + (2-K_B)\chi]$$

Velocity divergence

$$\theta = \frac{\varphi}{\dot{\bar{\phi}}}$$

Pressure contrast

$$\Pi = c_{\text{ad}}^2 \delta - \frac{c_{\text{ad}}^2}{8\pi G a^2 \bar{\rho}} \vec{\nabla}^2 [K_B E + (2-K_B)\chi]$$

Fluid-like

$$\dot{\delta} = 3H(w\delta - \Pi) + (1+w) \left(3\dot{\Phi} - \frac{k^2}{a^2}\theta \right)$$

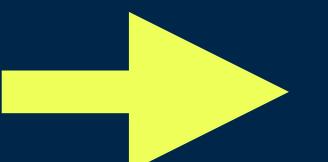
$$\dot{\theta} = 3c_{\text{ad}}^2 H\theta + \frac{\Pi}{1+w} + \Psi$$

Field

$$K_B (\dot{E} + HE) = \frac{d\mathcal{K}}{d\mathcal{Q}}\chi - (2-K_B) \left[\frac{\dot{\bar{\phi}}}{1+w}\Pi + (H + \dot{\bar{\phi}})\chi - 3c_{\text{ad}}^2 H\dot{\bar{\phi}}\alpha \right]$$

$$w \rightarrow 0$$

$$c_{\text{ad}} \rightarrow 0$$



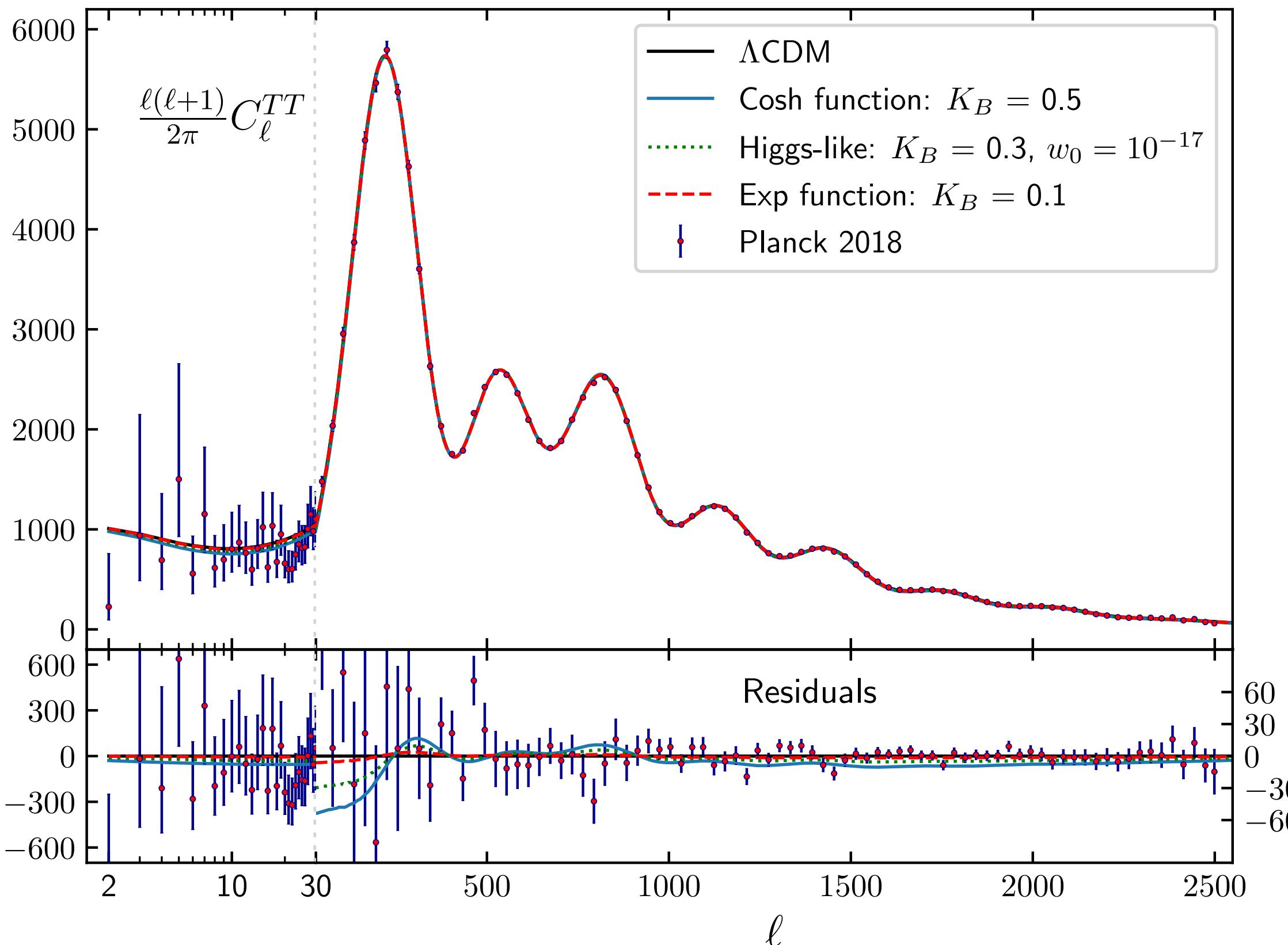
CDM-like

$$\dot{\delta} \approx 3\dot{\Phi} - \frac{k^2}{a^2}\theta$$

$$\dot{\theta} \approx \Psi$$

Field (decoupled)

$$K_B (\dot{E} + HE) \approx \left[\frac{3H_0^2 \Omega_0 \mathcal{Q}}{a^3} - (2-K_B)H\mathcal{Q}_0 \right] (\theta + \alpha)$$



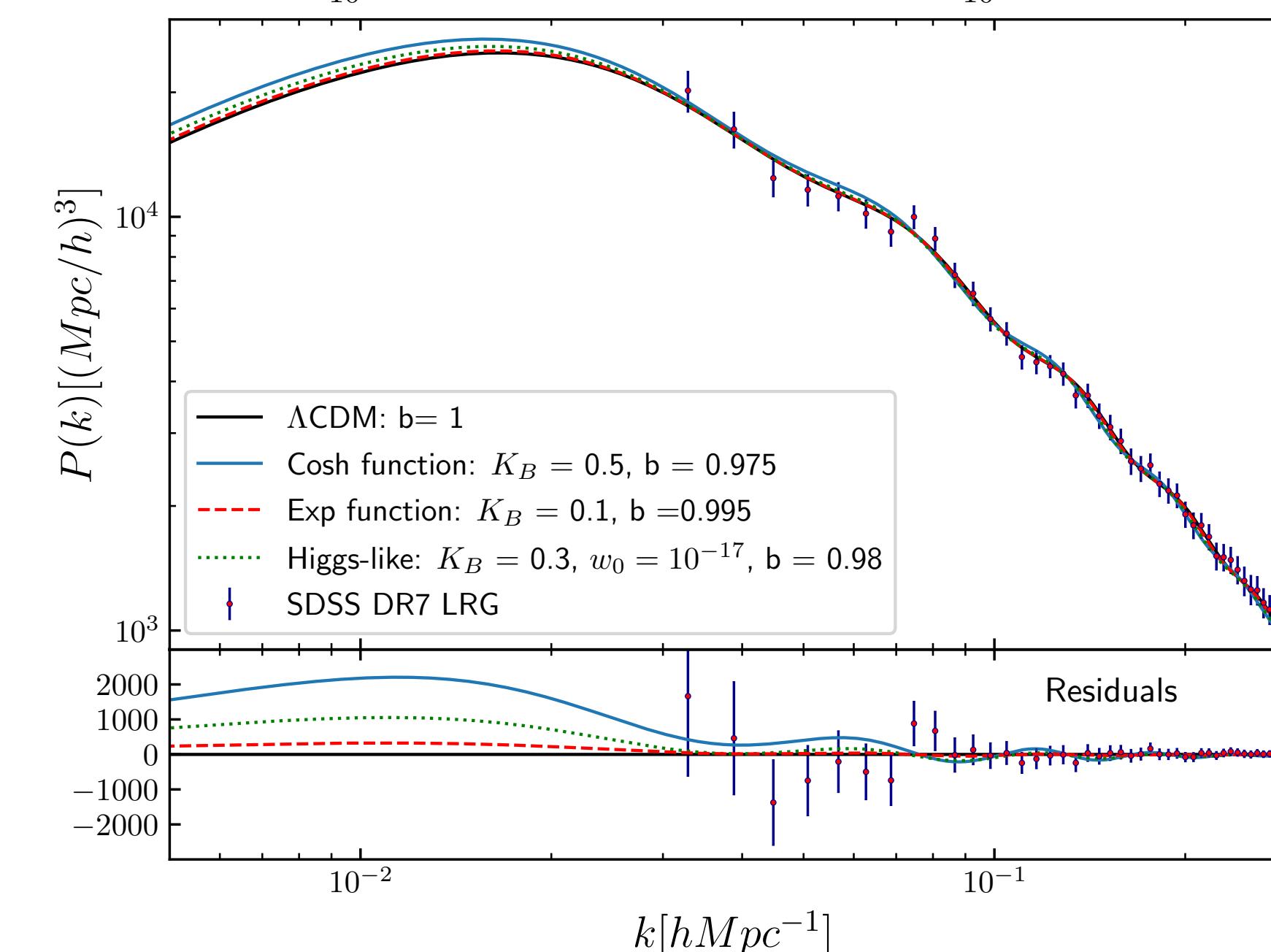
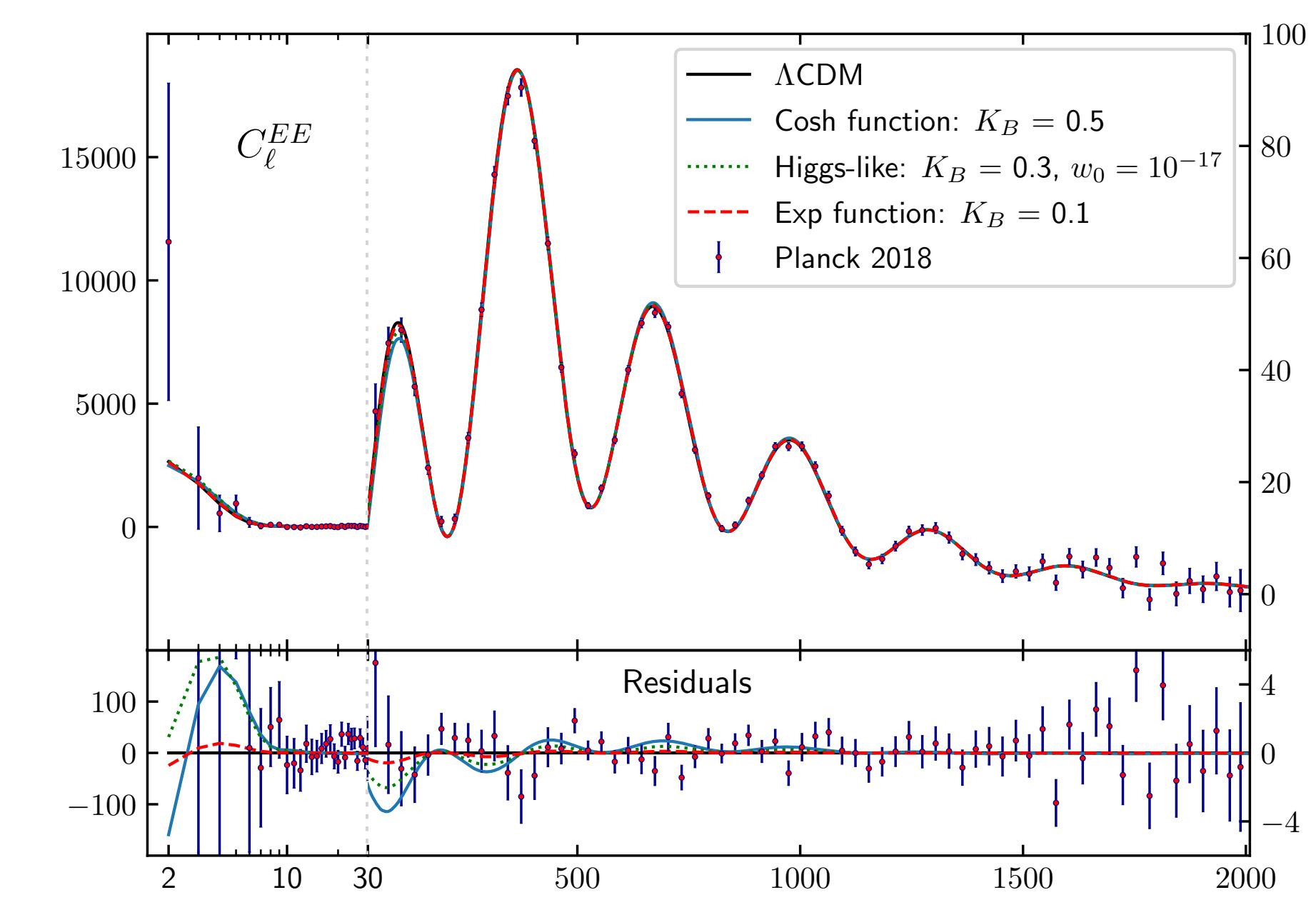
C.S. & Zlosnik, PRL 127, 161302 (2021)

AeST parameters:

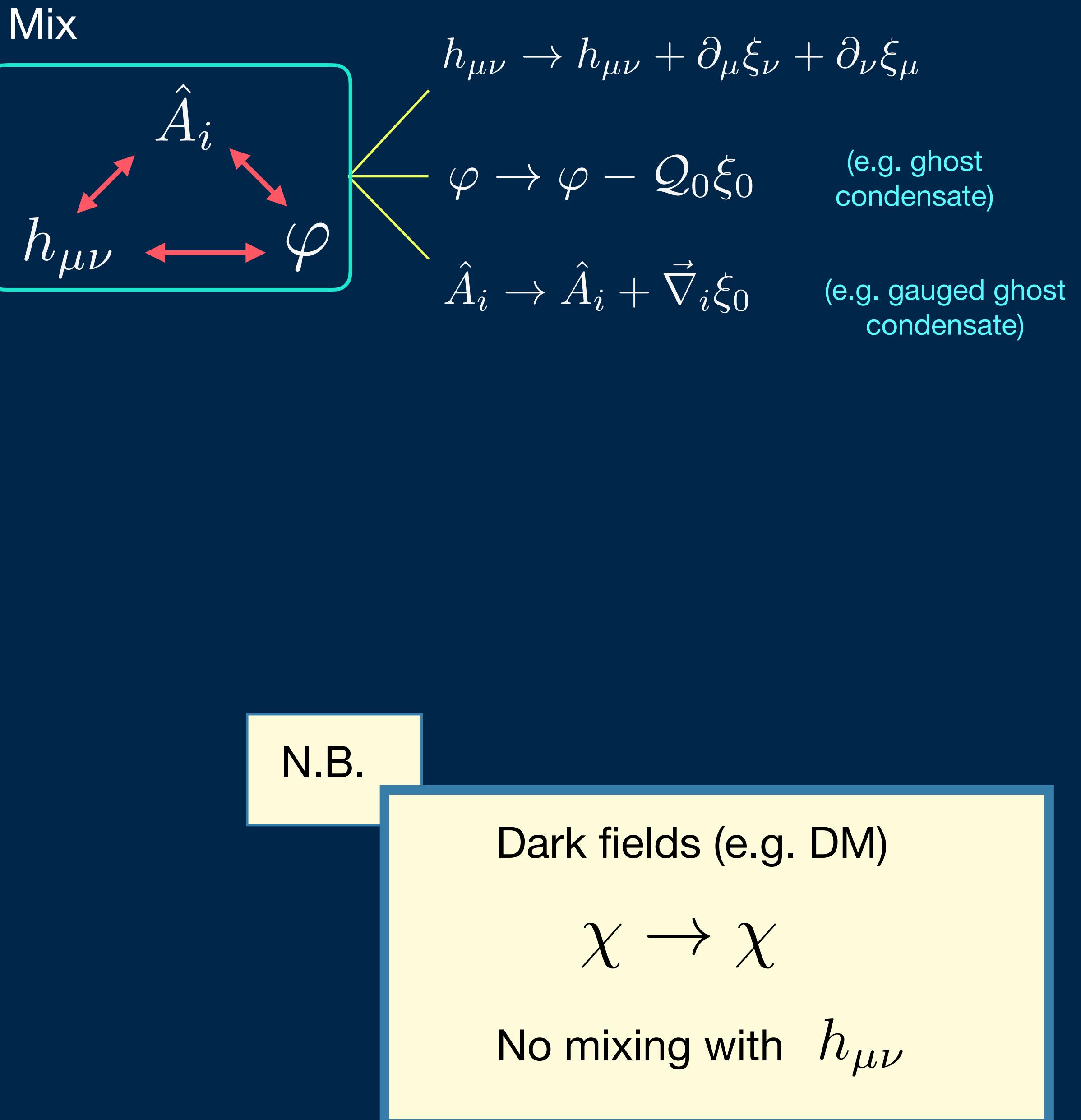
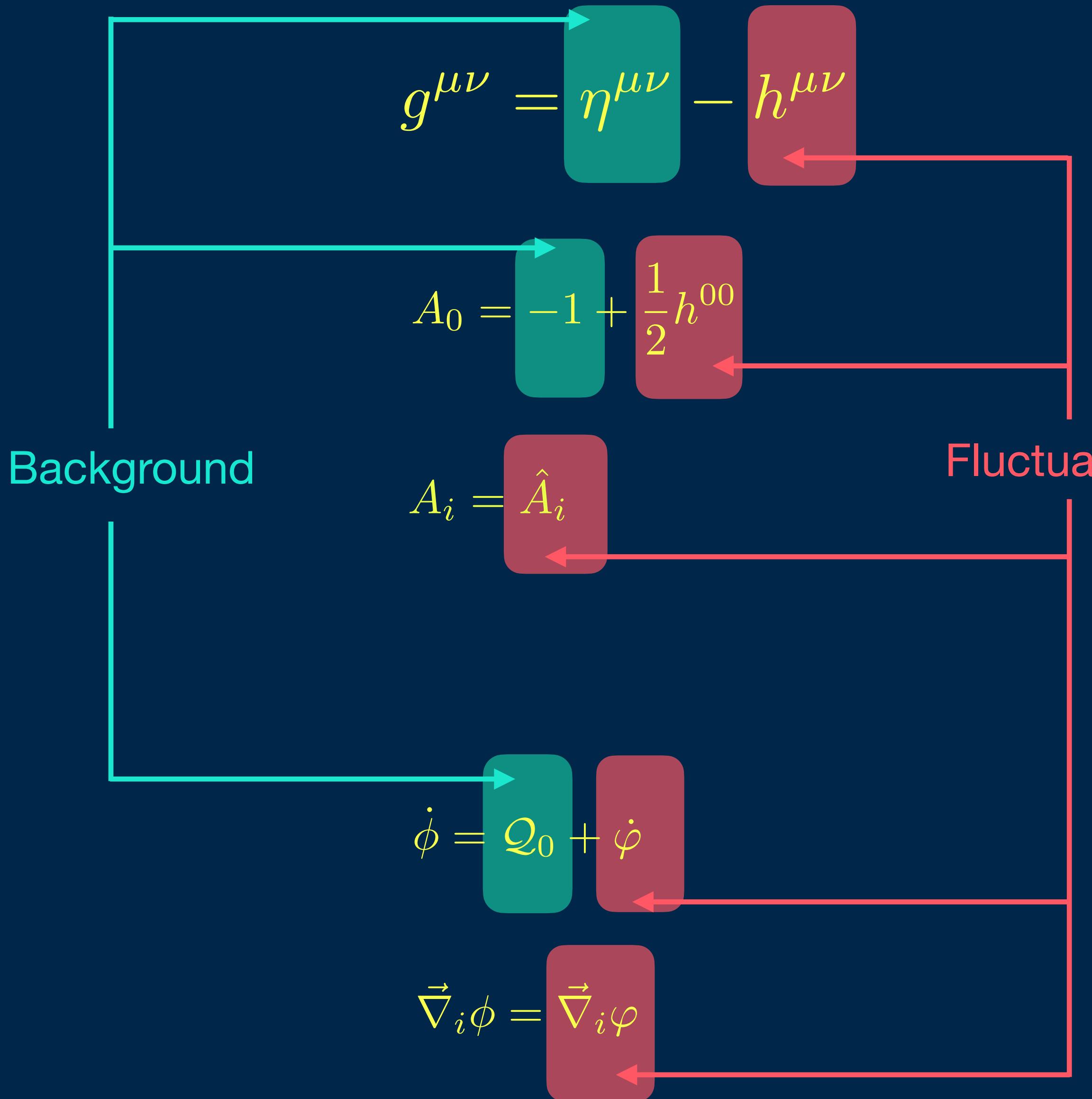
K_B Q_0 \mathcal{K}_2

Initial condition: $I_0 \rightarrow \rho_{0c}$

(MCMC pending)



Gravity vs. Matter fields: on Minkowski



Tensor mode graviton

$c_T = 1$

$$h_{00} \rightarrow h_{00} - 2\dot{\xi}_T$$

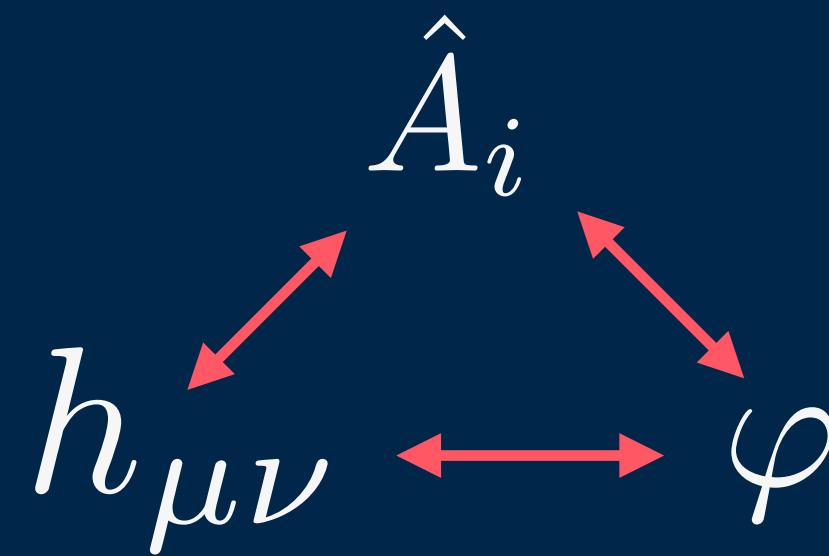
$$h_{0i} \rightarrow h_{0i} + \dot{\xi}_i - \vec{\nabla}_i \xi_T$$

$$h_{ij} \rightarrow h_{ij} + \vec{\nabla}_i \xi_j + \vec{\nabla}_j \xi_i$$

$$S = \int d^4x \left\{ -\frac{1}{2}\partial_\mu h \partial_\nu h^{\mu\nu} + \frac{1}{4}\partial_\rho h \partial^\rho h + \frac{1}{2}\partial_\mu h^{\mu\rho} \partial_\nu h^\nu{}_\rho - \frac{1}{4}\partial^\rho h^{\mu\nu} \partial_\rho h_{\mu\nu} + K_B \left| \dot{\vec{A}} - \frac{1}{2}\vec{\nabla} h^{00} \right|^2 - 2K_B \vec{\nabla}_{[i} \hat{A}_{j]} \vec{\nabla}^{[i} \hat{A}^{j]} \right.$$

$$\left. + (2 - K_B) \left[2(\dot{\vec{A}} - \frac{1}{2}\vec{\nabla} h^{00}) \cdot (\vec{\nabla} \varphi + Q_0 \vec{A}) - (1 + \lambda_s) |\vec{\nabla} \varphi + Q_0 \vec{A}|^2 \right] + 2\mathcal{K}_2 \left| \dot{\varphi} + \frac{1}{2}Q_0 h^{00} \right|^2 + \frac{1}{\tilde{M}_p^2} T_{\mu\nu} h^{\mu\nu} \right\}$$

Gauge Invariant terms



Mixing: genuine modification of gravity

$$h_{00} \rightarrow h_{00} - 2\dot{\xi}_T$$

$$A_i \rightarrow A_i - \vec{\nabla}_i \xi_T$$

$$\varphi \rightarrow \varphi + Q_0 \xi_T$$

Emergent symmetry for static fields:

Only $\chi \equiv \varphi + Q_0 \alpha$ Relevant

$$\vec{A} = \vec{\nabla} \alpha + \vec{\nabla} \times \vec{\beta}$$

Tensor modes $\cdots \omega^2 = k^2$
 (as in GR)

Vector modes: $\cdots \omega^2 = k^2 + \mathcal{M}^2$ $\xrightarrow{\quad}$ $\mathcal{M}^2 = \frac{2 - K_B}{K_B} (1 + \lambda_s) \mathcal{Q}_0^2$

Scalar modes: $\cdots \omega^2 = c_s^2 k^2 + \mathcal{M}^2$

$$c_s^2 = \frac{2 - K_B}{\mathcal{K}_2 K_B} \left(1 + \frac{K_B}{2} \lambda_s \right)$$

Positive Hamiltonian

$\mathcal{K}_2 > 0$
 $0 < K_B < 2$
 $\lambda_s > 0$

$\cdots \omega^2 = 0$ (Non-propagating)

Positive Hamiltonian $k > \mu (\sim Mpc^{-1} \text{ or smaller})$

Negative Hamiltonian $k < \mu$ \rightarrow Linear instability
 (Cosmology?)

Black Holes

C. Skordis, in preparation.

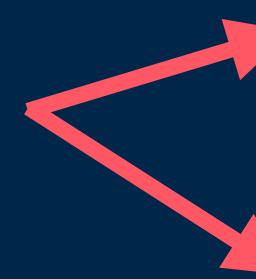
Bernardo & Chen, arxiv:2202.08460 (consider a disconnected sector where) $\phi = \phi(r)$

→ Schwarzchild BH, ϕ Singular at horizon

Better assumption is:

Continuity with cosmology demands that $\nabla_\mu \phi$ be timelike → Global time coordinate: $\phi = Q_0 t$

- Assume static, spherically symmetric



$$ds^2 = -e^{2\Phi(r)} dt^2 + e^{2\Psi(r)} dr^2 + r^2 d\Omega^2$$

$$A_\mu = \{-e^\Phi \chi(r), A(r), 0, 0\}$$

$$\chi = \sqrt{1 + e^{-2\Psi} A^2} \quad (\text{From unit timelike constraint})$$

- Take strong-field limit: $\mathcal{F} = (2 - K_B) \lambda_s \mathcal{Y}$

(Assumes relevant scales are smaller than

$$r_M \sim \sqrt{\frac{GM}{a_0}} \ll \mu^{-1}$$

- Three functions to be determined: $\Phi(r)$ $\Psi(r)$ $A(r)$, depend on two additional parameters: K_B, λ_s

Unique solution: Reissner-Nordstrom:
(Schwarzschild not a solution)

$$ds^2 = - \left(1 + \frac{C_0}{r} + \frac{q^2}{r^2} \right) dt^2 + \frac{dr^2}{1 + \frac{C_0}{r} + \frac{q^2}{r^2}} + r^2 d\Omega^2$$

$$q^2 = \left(1 + \frac{K_B}{2} \lambda_s \right) (1 + \lambda_s) Q_0^2 \alpha_0^2$$

With: $A = \frac{\alpha_0}{r^2}$

Connect with
linearised solution

$$\begin{aligned} C_0 &= -2G_N M \\ \alpha_0 &= \frac{GM}{\lambda_s Q_0} \end{aligned} \rightarrow ds^2 = - \left(1 - \frac{2G_N M}{r} + \frac{2\delta_\beta (G_N M)^2}{r^2} \right) dt^2 + \frac{dr^2}{1 - \frac{2G_N M}{r} + \frac{2\delta_\beta (G_N M)^2}{r^2}} + r^2 d\Omega^2$$

$$\delta_\beta = \beta_{PPN} - 1 = \frac{1}{2} \left(\frac{K_B}{2} + \frac{1}{\lambda_s} \right) \left(1 + \frac{1}{\lambda_s} \right) < 8 \times 10^{-5} \quad (\text{BH always sub-extremal})$$

$$K_B < 3.2 \times 10^{-4} \quad \frac{1}{\lambda_s} < 1.6 \times 10^{-4}$$

A possibility EXTENSION OF GR

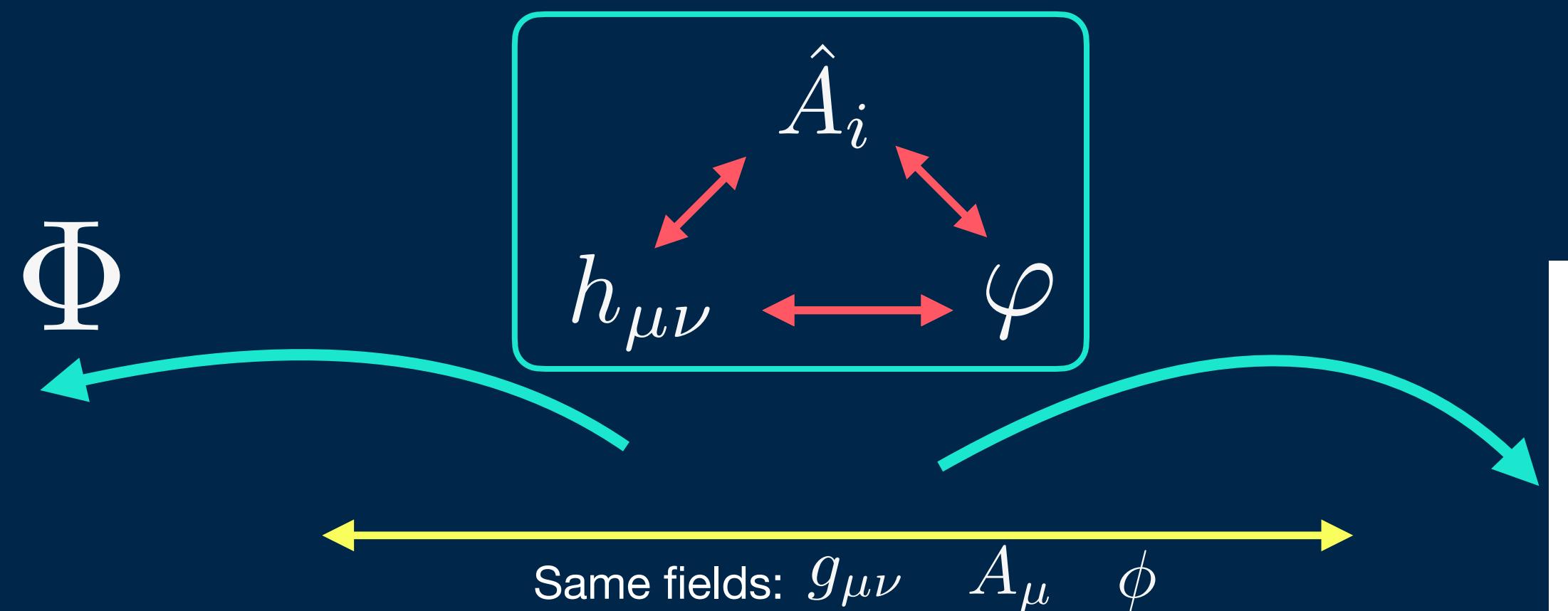


Non-relativistic, static: MOND

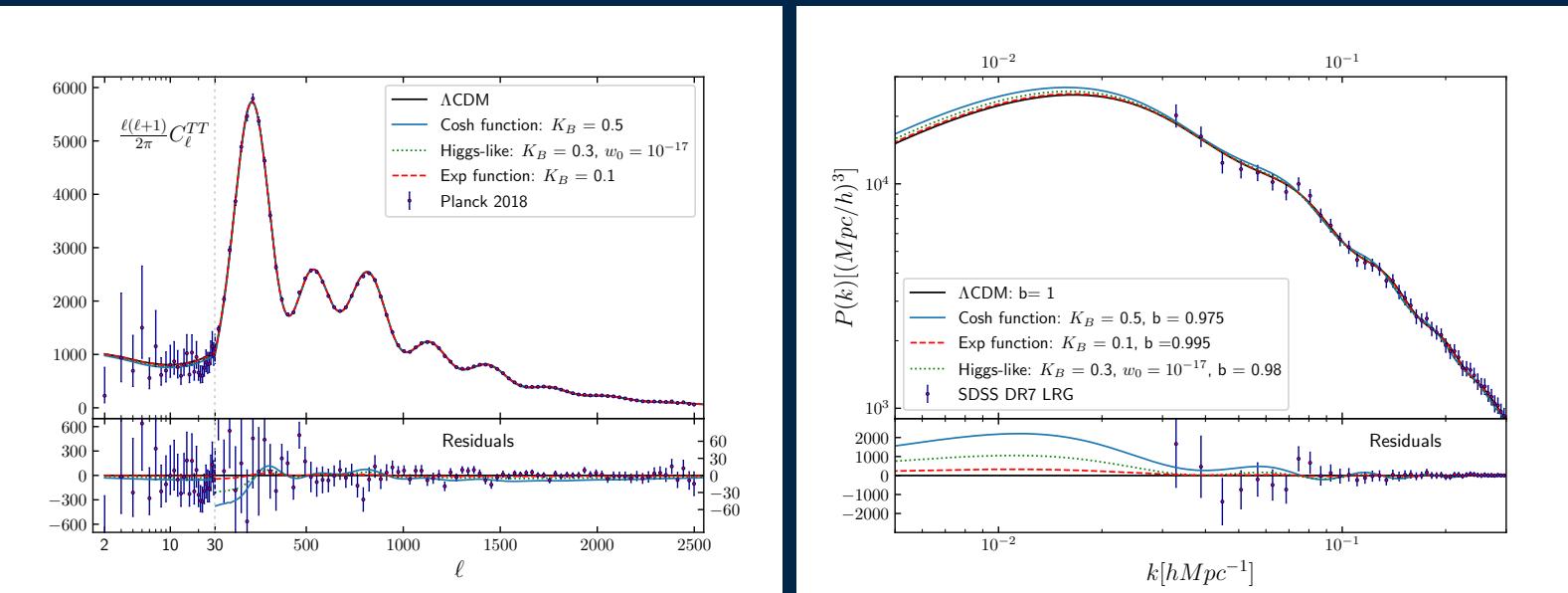
Bekenstein & Milgrom 1984



Lensing



FRW + linear fluctuations
Effective description: Λ CDM



aLIGO/Virgo + EM (2017)
Tensor speed = 1

• Aether Scalar Tensor (AeST)

- New dof mixing with metric perturbation: 1 scalar and 1 unit-timeline vector (Aether)

$$\begin{array}{c} \hat{A}_i \\ h_{\mu\nu} \\ \varphi \end{array}$$



- BH solutions: Reisner-Nordstrom with charge related to (baryonic) mass
- Static solutions: only exist if baryonic mass is present (no non-trivial scalar/vector profiles)

- Non-zero PPN parameters expected

$$\gamma = 1$$

$$\beta \approx 1 + \frac{K_B}{4} + \frac{1}{2\lambda_s}$$

$$\alpha_1 \neq 0$$

$$\alpha_2 \neq 0$$

$$\begin{aligned} K_B &< 3.2 \times 10^{-4} \\ \frac{1}{\lambda_s} &< 1.6 \times 10^{-4} \end{aligned}$$

What now?

Upcoming work:

- Hamiltonian formulation (with M. Bataki (PhD student) & T. Zlosnik)
- Weak-field spherically symmetric solutions (w. A. Durakovic, P. Verwayen (PhD st.), C. Boehm, D. Mota, C. Llinares,)
- PPN parameters
- Black Holes
- MCMC (Cosmological parameters) (w. S. Ilic & T. Zlosnik)

• Non-linear cosmology



• N-body simulations

• EFTofLSS (done in the case of CDM — Senatore, Zaldarriaga, Baumann, et al.)

• Theory needs improvements:

• Magic function

• Term $|\vec{\nabla}\varphi|^3$ Is non-local in Fourier space — not nice

• Scalar gravitational waves



Cosmological background (as wave-like isocurvature modes)

Stellar pulsations

Stochastic background

Current collaborators: T. Zlosnik (CEICO), S. Ilic (former CEICO, current APC), M. Bataki (PhD student, U. Cyprus & CEICO)

A. Durakovic (CEICO), D. Mota (Oslo), C. Boehm (Sydney), P. Verwayen (Sydney), C. Llinares (Lisbon)