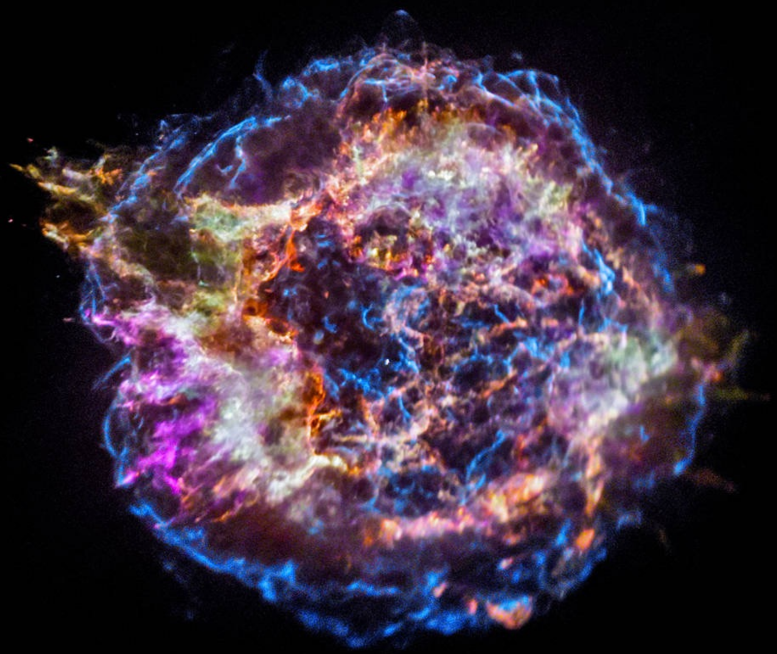
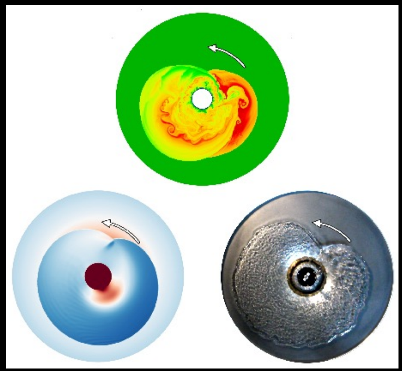
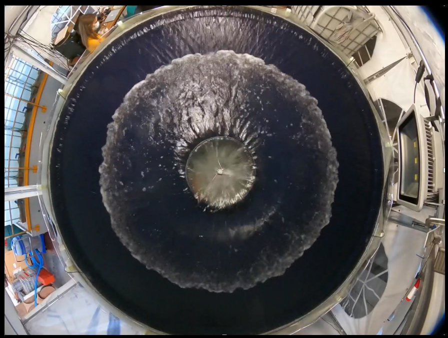


The impact of rotation and turbulence on core-collapse supernovae

Thierry Foglizzo

CEA Saclay





G. Durand, J. Guilet, M. Bugli, M. González,
A-C Buellet, F. Masset, L. Walk, I. Tamborra,

Outline

I. State of the art of core-collapse supernovae

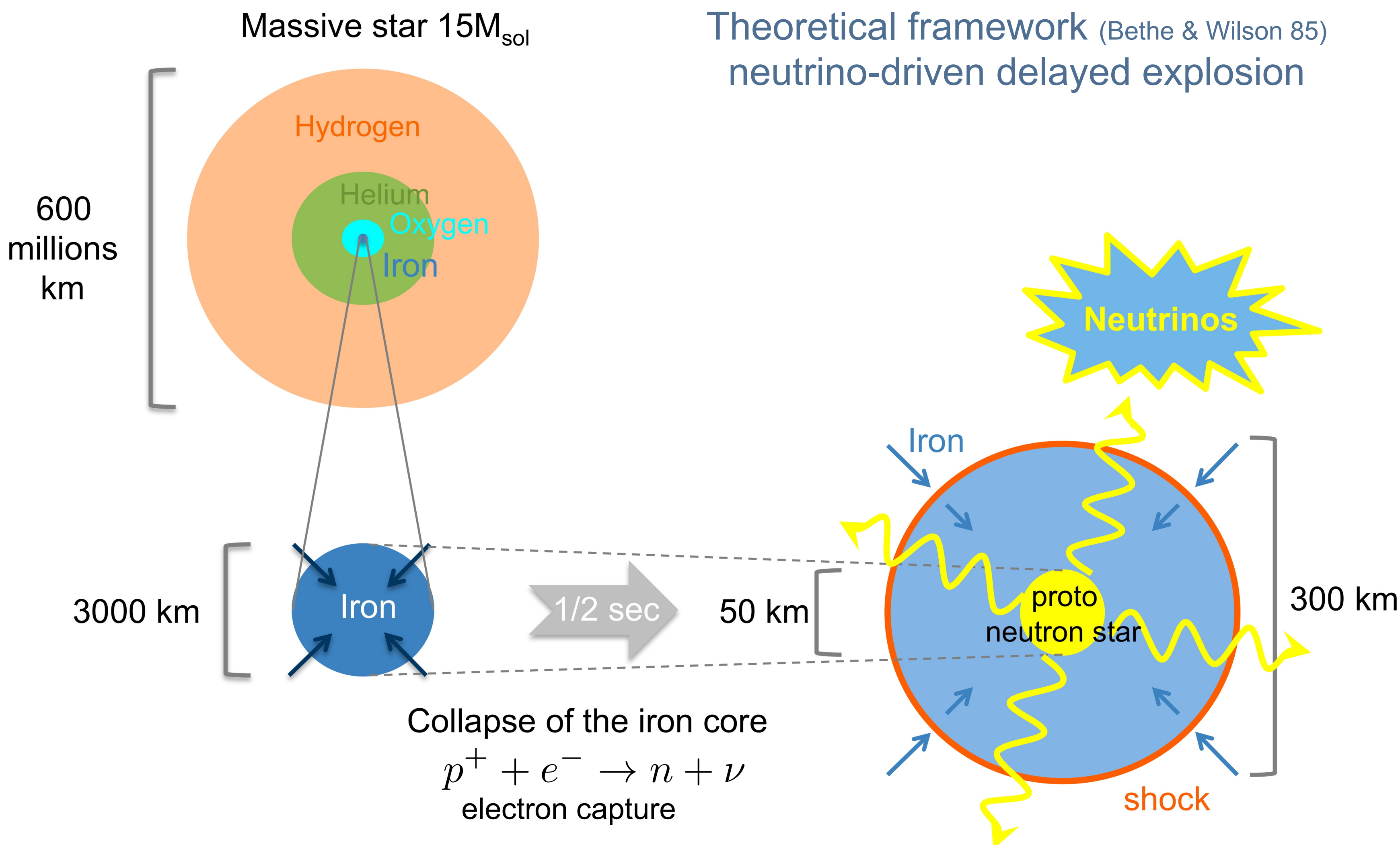
II. Adiabatic model of forced oscillator

III. Rotation effects on SASI clarified

IV. Viscous/turbulent stabilisation of SASI

Astrophysics background in 47s Youtube/CEA "How does a supernova explode" (6mn)

Theoretical framework (Bethe & Wilson 85)
neutrino-driven delayed explosion



$$\frac{GM_{ns}^2}{R_{ns}} \sim 2 \times 10^{53}\text{erg} \left(\frac{30\text{km}}{R_{ns}}\right) \left(\frac{M_{ns}}{1.5M_{sol}}\right)^2$$

modest energy in differential rotation: $E_{diff} < E_{rot} \sim 2.4 \times 10^{50}\text{erg} \left(\frac{M_{ns}}{1.5M_{sol}}\right) \left(\frac{R_{ns}}{10\text{km}}\right)^2 \left(\frac{10\text{ms}}{P_{ns}}\right)^2$

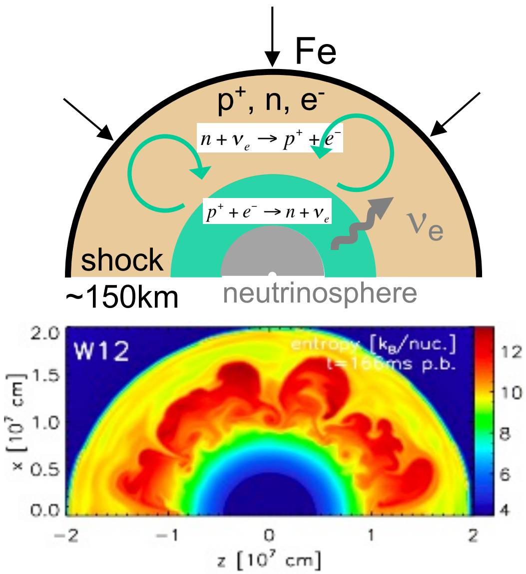
$27M_{sol}$ stellar core collapse in 3D

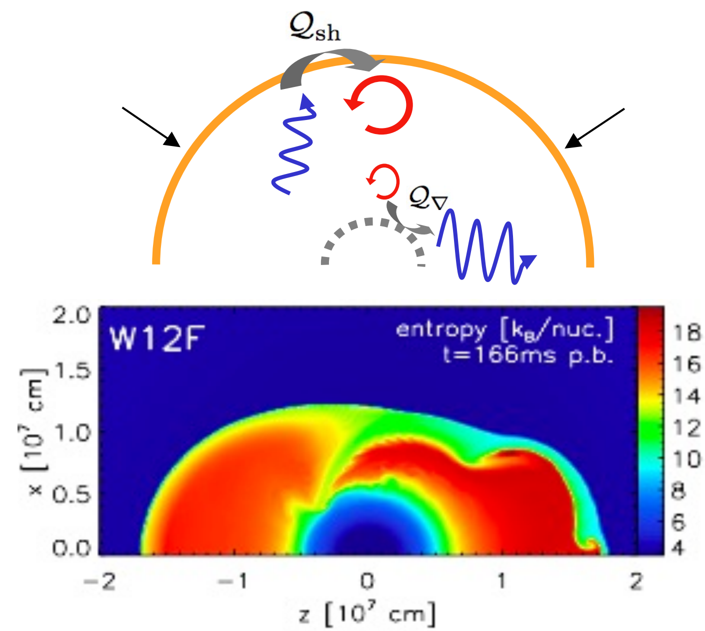
Hanke et al. 13

time evolution: 500ms
diameter: 300km

PRACE
150 million hours
16.000 processors
4.5 months/model

Instabilities during the phase of stalled accretion shock





Neutrino-driven convection

(Herant+92)

- entropy gradient
- angular scale l=5,6

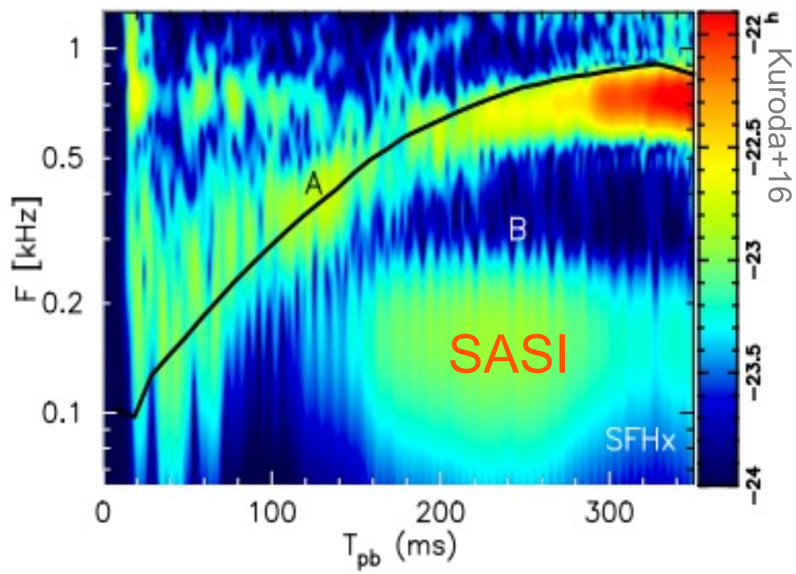
SASI: Standing Accretion Shock Instability

(Blondin+03)

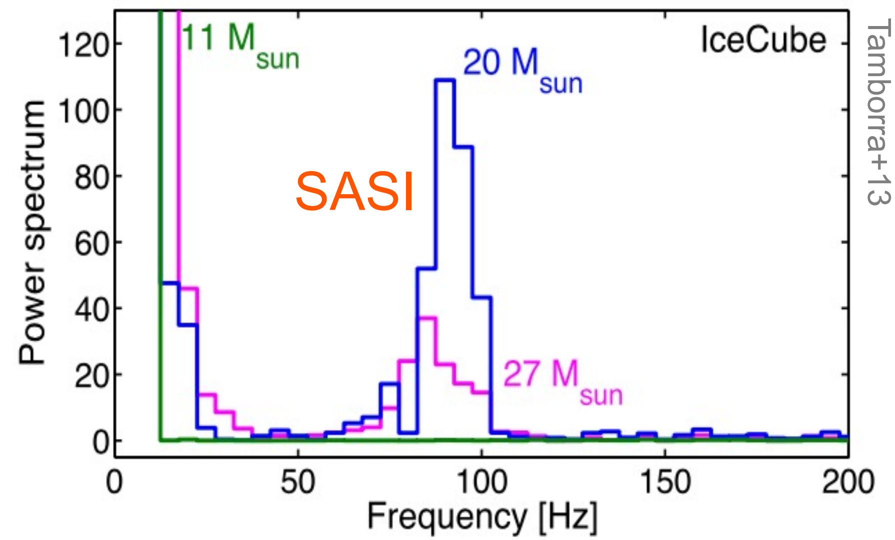
- advective-acoustic cycle
- oscillatory, large angular scale l=1,2

SASI oscillations can leave a direct imprint on the gravitational wave and neutrino signals: reverse engineering?

Gravitational Waves



Oscillations of the neutrino flux



stellar parameters: progenitor mass, compactness, angular momentum, inhomogeneities

\+ **puzzling dynamics:** SASI, ν-driven convection, low T/|W|, PNS dynamo

\+ **uncertain physics:** reaction rates, EOS, neutrino interactions, magnetic fields

\+ **numerical approximations:** neutrino transport, 2D vs 3D, turbulence

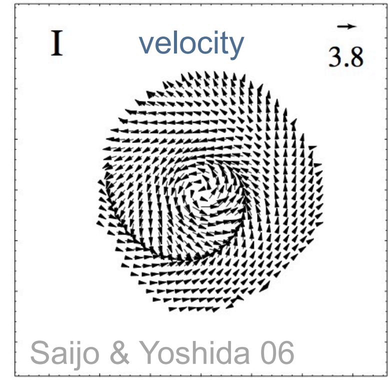
Can gravitational wave and neutrino signatures disentangle so many processes ?

Additional instabilities induced by moderate rotation: uncertain mechanism(s)

-low T/|W| instability?

(Shibata+02, Watts+05, Passamonti & Andersson 15, Takiwaki+21, Bugli+23)

- corotation radius
- vorticity gradient? mid-latitude Rossby waves?

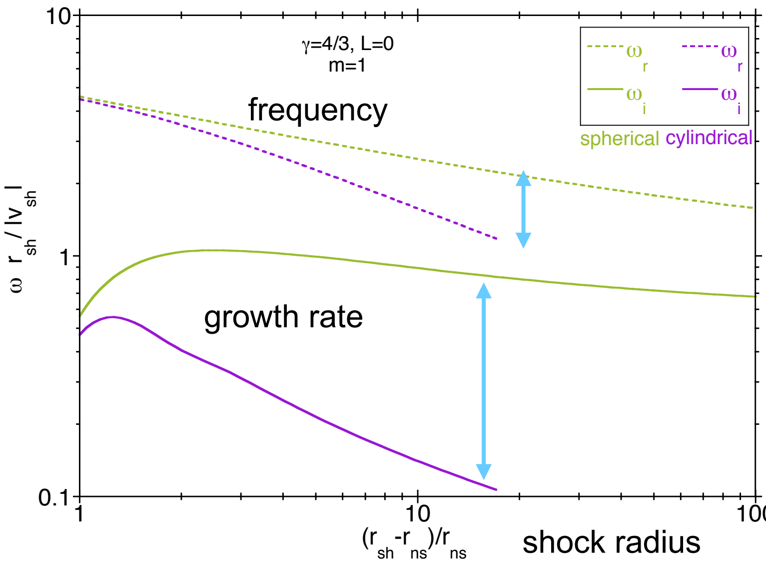


-spiral mode of SASI?

(Blondin & Mezzacappa 07, Yamasaki & Foglizzo 08, Blondin+17, Walk+23)

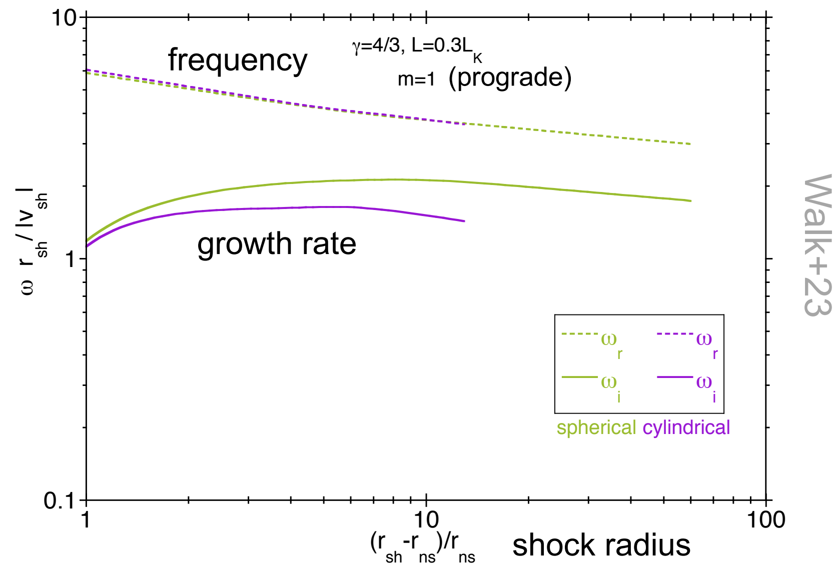
- rotation-enhanced advective-acoustic cycle?
- why such as strong impact of rotation on the prograde SASI mode?

no rotation



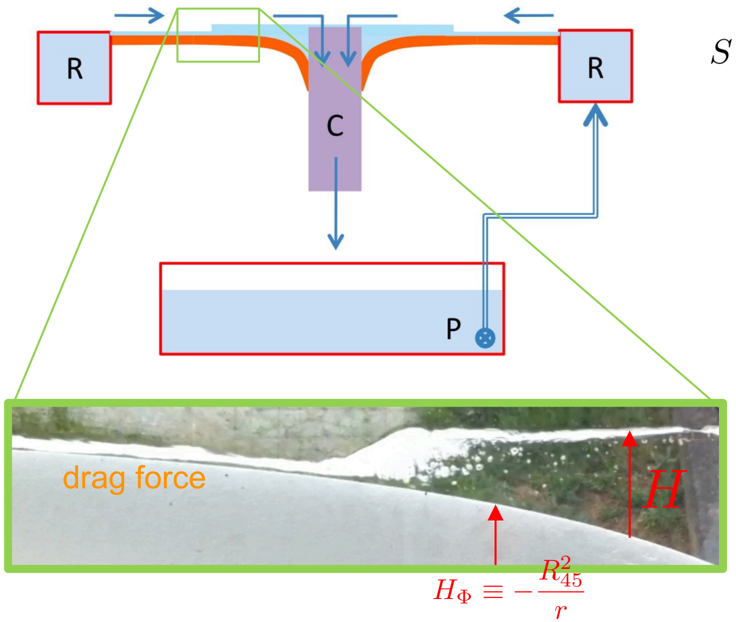
as expected from an advective-acoustic mechanism

with rotation 30% Keplerian



unexpected for an advective-acoustic mechanism??

Shallow water analogy



adiabatic gas

$$S \equiv \frac{1}{\gamma - 1} \log \frac{p}{\rho^\gamma}$$

$$c_s^2 \equiv \frac{\gamma P}{\rho}$$

$$\Phi \equiv -\frac{GM_{ns}}{r}$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0$$

$$\frac{\partial v}{\partial t} + (\nabla \times v) \times v + \nabla \left(\frac{v^2}{2} + \frac{c_s^2}{\gamma - 1} + \Phi \right) = \frac{c_s^2}{\gamma} \nabla S$$

Inviscid shallow water is analogue to a homentropic gas $\gamma=2$

St Venant

$$c_{sw}^2 \equiv gH$$

$$\Phi \equiv gH_\Phi$$

$$\frac{\partial H}{\partial t} + \nabla \cdot (Hv) = 0$$

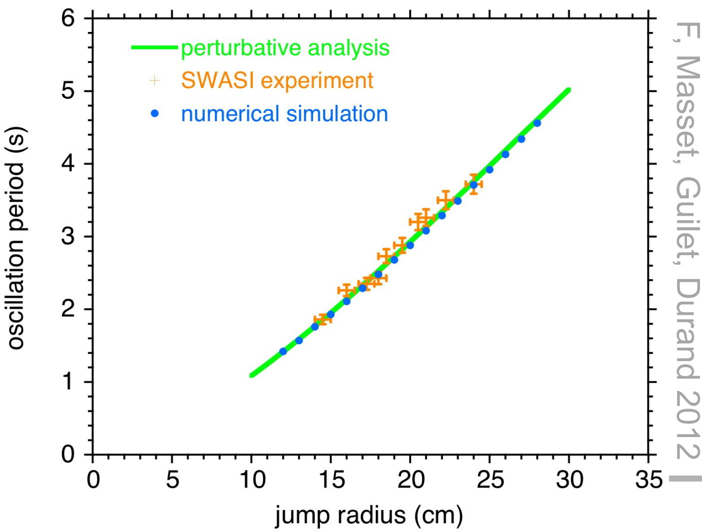
$$\frac{\partial v}{\partial t} + (\nabla \times v) \times v + \nabla \left(\frac{v^2}{2} + c_{sw}^2 + \Phi \right) = 0$$

(+drag force) $-3\nu \frac{v}{H^2}$

acoustic waves shock wave density ρ	⟷	surface waves hydraulic jump depth H

expected scaling

$$\frac{t_{ff}^{sh}}{t_{ff}^{jp}} \equiv \left(\frac{r_{sh}}{r_{jp}} \right) \left(\frac{r_{sh} g H_\Phi^{jp}}{GM_{ns}} \right)^{\frac{1}{2}} \sim 10^{-2}$$



shock radius $\times 10^{-6}$ 200 km → 20 cm

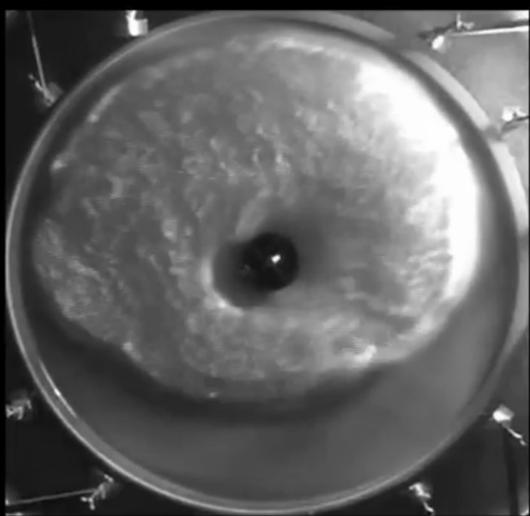
oscillation period $\times 10^2$ 30 ms → 3 s

Dynamics of water
in the fountain

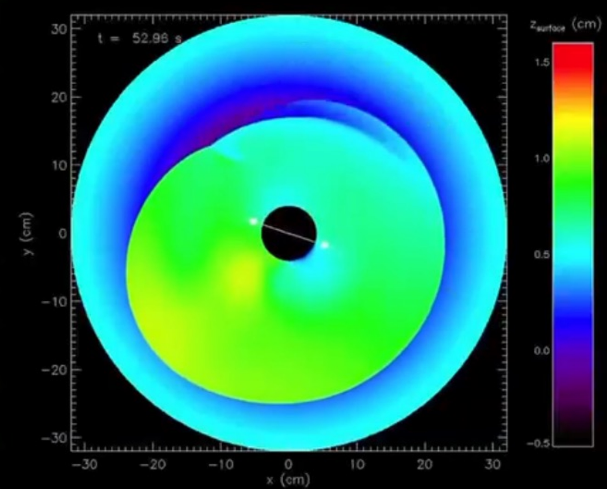
Dynamics of the gas
in the supernova core

diameter 40cm ← 1 000 000 x bigger → diameter 400km

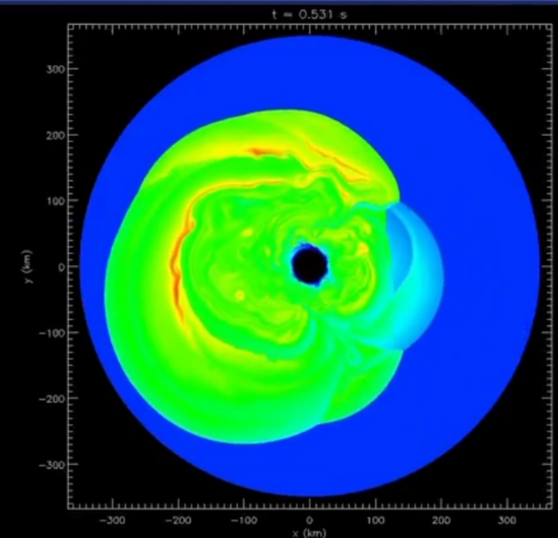
3s/oscillation ← 100 x faster → 0.03s/oscillation



Expérience hydraulique



Simulation numérique de l'expérience hydraulique



Simulation numérique de l'onde de choc dans le coeur de la supernova

SASI dynamics seems to be adiabatic

Adiabatic approximation
inspired by the shallow water experiment

Stellar SASI:

- non adiabatic cooling/heating (ν-processes)

$$\mathcal{L} = A_{\rm cool}\rho^{\beta-\alpha}p^{\alpha}$$

- 4th order differential system

$$\delta w_\perp \equiv r(\nabla \times \delta w)_r$$

$$\delta K \equiv r v \delta w_\perp + l(l+1)\frac{c^2}{\gamma}\delta S$$

$$\frac{\partial \delta f}{\partial r} = \frac{i\omega v}{1-\mathcal{M}^2}\left[\delta h - \frac{\delta f}{c^2} + \left(\gamma - 1 + \frac{1}{\mathcal{M}^2}\right)\frac{\delta S}{\gamma}\right] + \delta\left(\frac{\mathcal{L}}{\rho v}\right), \quad \text{(B1)}$$

$$\frac{\partial \delta h}{\partial r} = \frac{i\omega}{v(1-\mathcal{M}^2)}\left(\frac{\mu^2}{c^2}\delta f - \mathcal{M}^2\delta h - \delta S\right) + \frac{i\delta K}{\omega r^2 v}, \quad \text{(B2)}$$

$$\frac{\partial \delta S}{\partial r} = \frac{i\omega}{v}\delta S + \delta\left(\frac{\mathcal{L}}{p v}\right), \quad \text{(B3)}$$

$$\frac{\partial \delta K}{\partial r} = \frac{i\omega}{v}\delta K + l(l+1)\delta\left(\frac{\mathcal{L}}{\rho v}\right). \quad \text{(B4)}$$

Foglizzo+07

$$\mu^2 \equiv 1 - \frac{l(l+1)}{\omega^2 r^2}(c^2 - v^2)$$

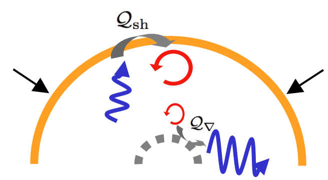
Adiabatic approximation:

- linear conservation of entropy δS and baroclinic vorticity δK

- 2nd order differential system

$$\mathrm{dX} \equiv \frac{v}{1-\mathcal{M}^2}\mathrm{d}r$$

$$\left[\left(\frac{\partial}{\partial X} + \frac{i\omega}{c^2}\right)^2 + \frac{\omega^2\mu^2}{v^2c^2}\right]\delta\mathbf{L} = \frac{\partial}{\partial X}\frac{r\delta\mathbf{w}}{v}$$



perturbed specific angular momentum

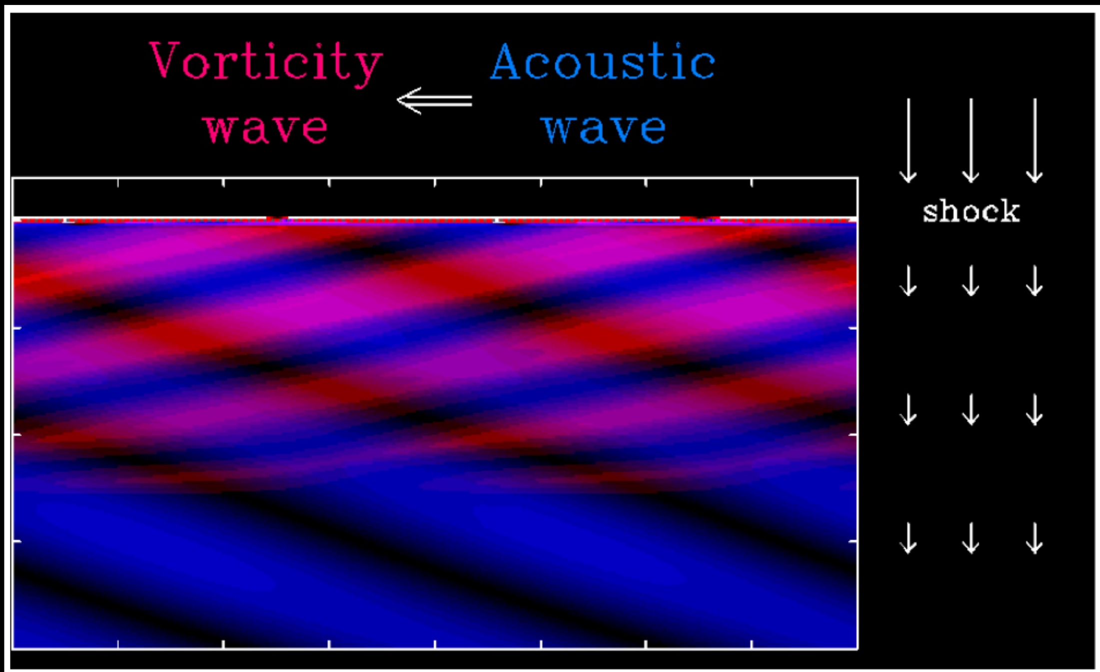
$$\delta\mathbf{L} \equiv \mathbf{r}\times\delta\mathbf{v}$$

perturbed vorticity

$$\delta\mathbf{w} \equiv \nabla\times\delta\mathbf{v}$$

- acoustic oscillator forced by the advection of vorticity

Interaction of advected and acoustic perturbations



Sato+09

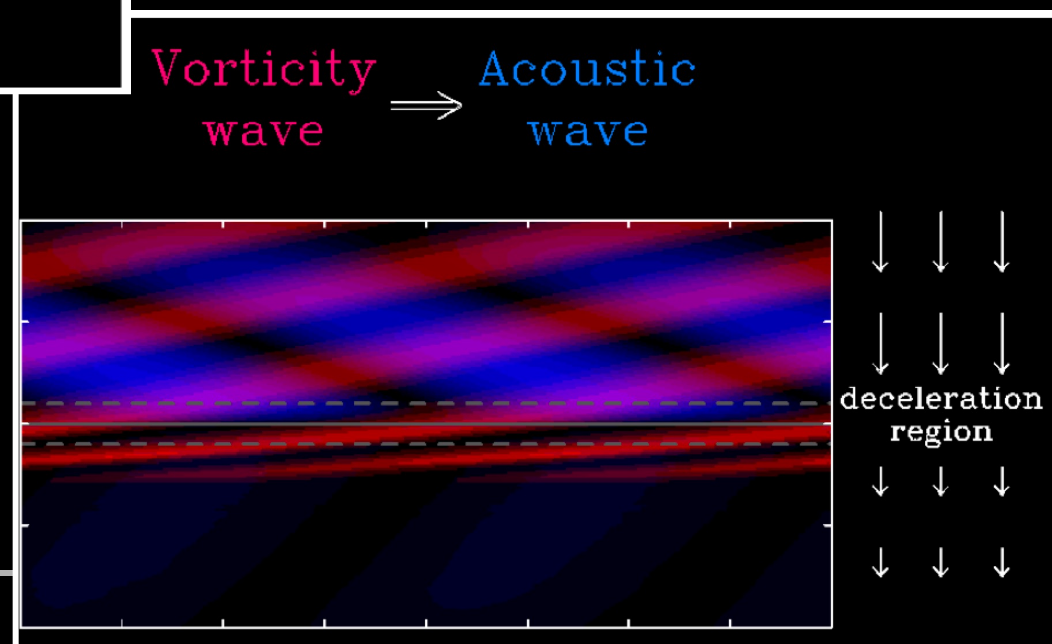
In a uniform stationary flow, advected and acoustic perturbations ignore each other linearly.

If the stationary flow involves gradients, these perturbations are linearly coupled

The advected perturbations are source terms in the acoustic equation

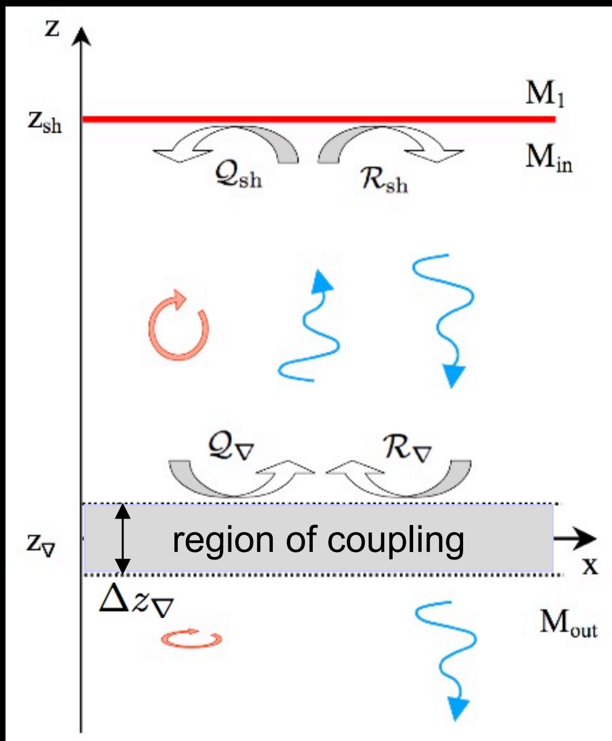
$$\frac{\partial^2 Y}{\partial X^2} + \left[\frac{\omega'^2}{c^2} - \frac{m^2}{r^2}(1-\mathcal{M}^2)\right]\frac{Y}{v_r^2} = \mathcal{S},$$

$$\mathcal{S} \equiv -(r v_r \delta w)_{\rm sh} e^{\int_{\rm sh} \frac{i\omega'}{c^2} {\rm d}X} \frac{\partial}{\partial X}\left(\frac{e^{\int_{\rm sh}\frac{i\omega'}{v_r}{\rm d}r}}{v^2}\right)$$



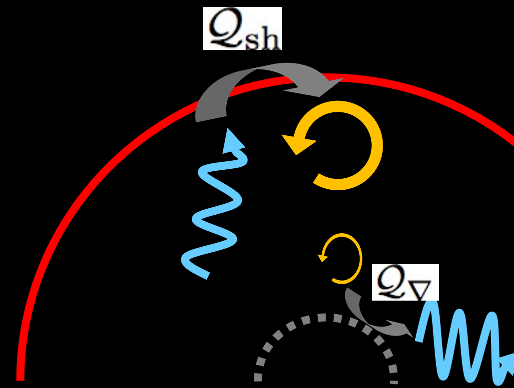
A planar toy model for the advective-acoustic coupling (F2009)

The planar geometry and uniform flow between the shock and the compact deceleration region allows for a fully analytic calculation



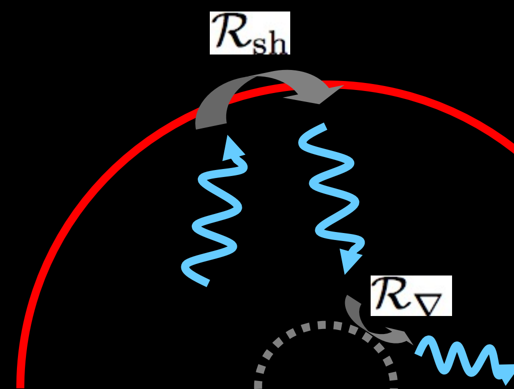
advective-acoustic cycle efficiency $\mathcal{Q} \equiv \mathcal{Q}_{sh}\mathcal{Q}_\nabla$

timescale $\tau_\mathcal{Q}$



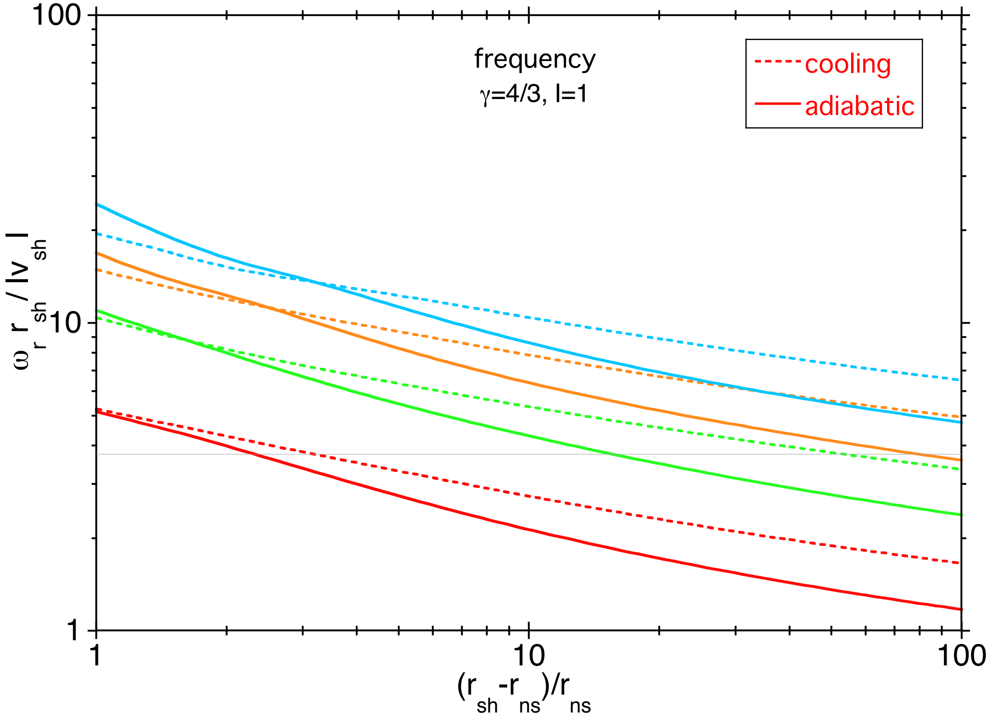
purely acoustic cycle efficiency $\mathcal{R} \equiv \mathcal{R}_{sh}\mathcal{R}_\nabla$

timescale $\tau_\mathcal{R}$



$$\mathcal{Q}e^{i\omega\tau_\mathcal{Q}} + \mathcal{R}e^{i\omega\tau_\mathcal{R}} = 1$$

Comparison of SASI eigenfrequencies with/without a cooling function

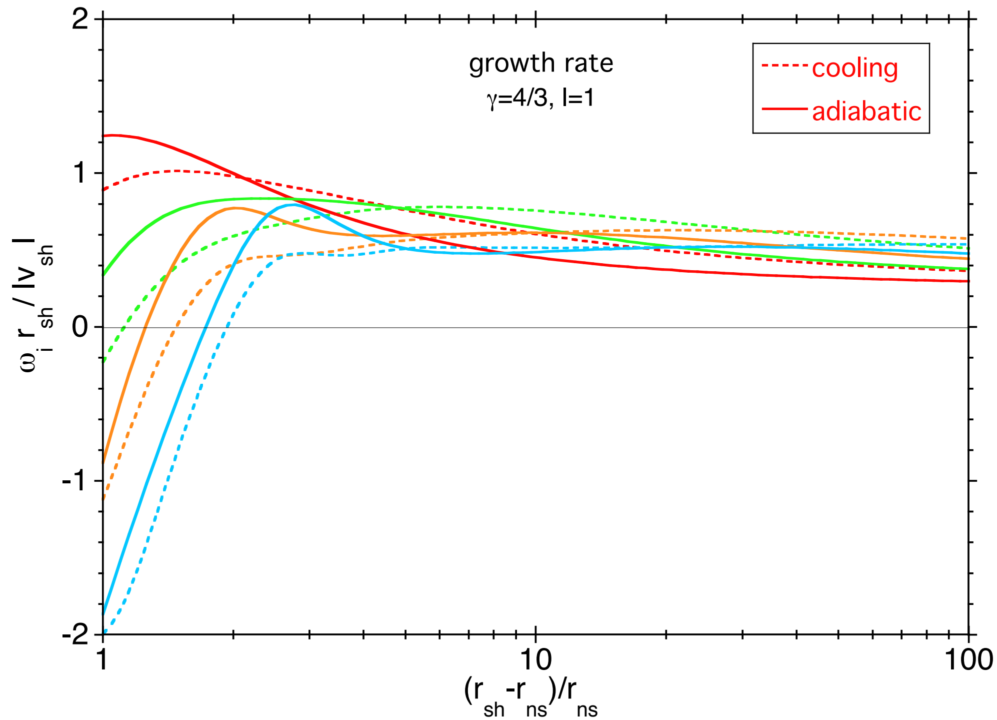


$$\left[\left(\frac{\partial}{\partial X}+\frac{i\omega}{c^2}\right)^2+\frac{\omega^2\mu^2}{v^2c^2}\right]\delta\mathbf{L} \quad = \quad \frac{\partial}{\partial X}\frac{r\delta\mathbf{w}}{v}$$

fundamental mode
1st, 2nd, 3rd harmonics

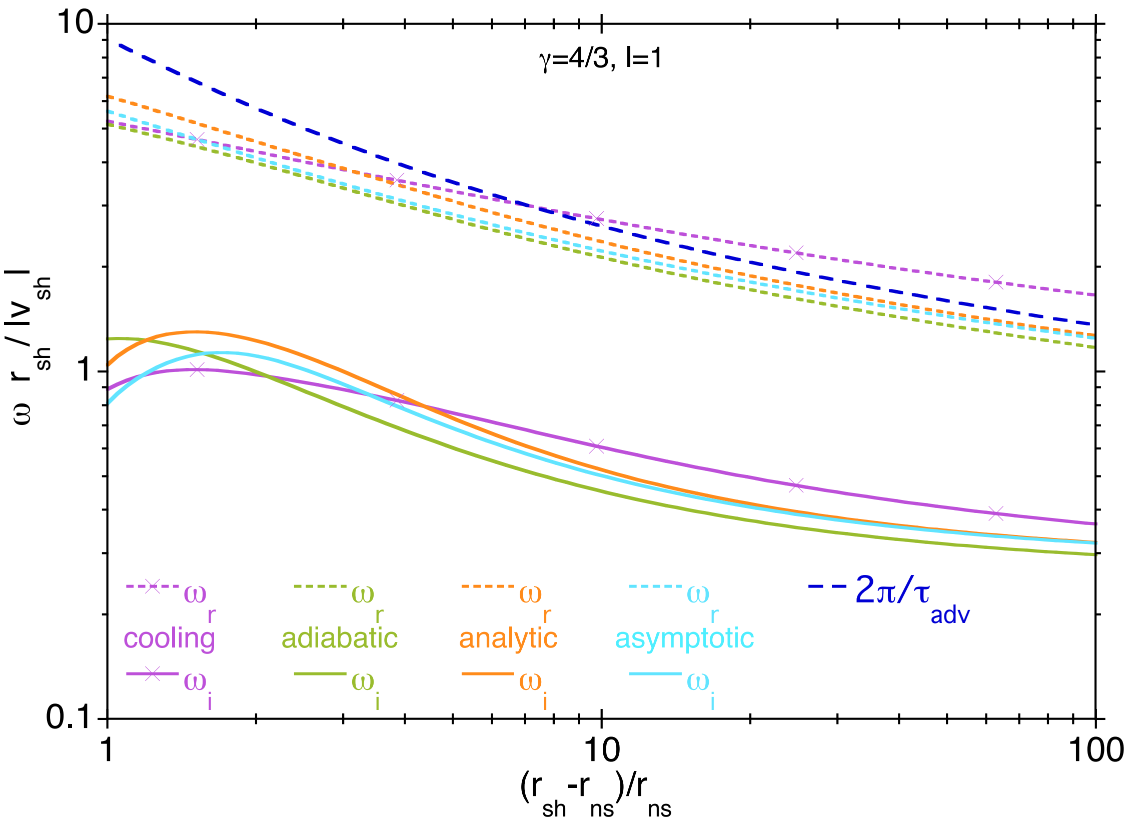
→general trends are captured by the adiabatic approximation

→the physical mechanism of SASI is approximately adiabatic



Analytical estimate of the SASI growth rate and frequency

state of the art = plane parallel model (Foglizzo 2009)



$$\left[\left(\frac{\partial}{\partial X}+\frac{i\omega}{c^2}\right)^2+\frac{\omega^2\mu^2}{v^2c^2}\right]\delta\mathbf{L} = \frac{\partial}{\partial X}\frac{r\delta\mathbf{w}}{v}$$

Forced oscillator + shock & pns boundary conditions

$$\left\{\frac{\partial^2}{\partial X^2}+\frac{\omega^2-\omega_{\rm Lamb}^2}{v^2c^2}\right\}Y_0=0$$ acoustic solution

→integral equation defining the eigenfrequencies

$$a_1'Y_0^{\rm sh}+a_2'r_{\rm sh}\left(\frac{\partial Y_0}{\partial r}\right)_{\rm sh} = -\mathcal{M}_{\rm sh}^2 e^{\int_{\rm sh}^{\rm ns}\frac{i\omega}{v}\frac{dr}{1-\mathcal{M}^2}}Y_0^{\rm ns} - \int_{\rm ns}^{\rm sh}\frac{\partial}{\partial r}\left(Y_0 e^{\int_{\rm sh}\frac{i\omega\mathcal{M}^2}{1-\mathcal{M}^2}\frac{dr}{v}}\right)\frac{\mathcal{M}_{\rm sh}^2}{\mathcal{M}^2}e^{\int_{\rm sh}\frac{i\omega}{v}dr}dr,$$

with a_1', a_2' defined by:

$$a_1' \equiv (\gamma-1)\mathcal{M}_{\rm sh}^2+\frac{\frac{i\omega r_{\rm sh}}{v_{\rm sh}}\frac{v_{\rm sh}}{v_1}}{\frac{v_1}{v_{\rm sh}}\frac{1}{2\eta^2}-2-\left(1-\frac{v_{\rm sh}}{v_1}\right)\frac{i\omega r_{\rm sh}}{v_{\rm sh}}},$$

$$a_2' \equiv \frac{1-\mathcal{M}_{\rm sh}^2}{\frac{v_1}{v_{\rm sh}}\frac{1}{2\eta^2}-2-\left(1-\frac{v_{\rm sh}}{v_1}\right)\frac{i\omega r_{\rm sh}}{v_{\rm sh}}}.$$

→asymptotic approximation $r_{sh} >> r_{ns}$

$$\frac{i\omega r_{\rm sh}}{|v_{\rm sh}|} = b-2+\frac{2ni\pi}{\zeta-d_1}+\mathcal{O}\left(\frac{1}{\zeta^3}\right),$$

$$\mathcal{Q}\left(\frac{i\omega r_{\rm sh}}{|v_{\rm sh}|}\right) = \frac{\left(\frac{r_{\rm sh}}{r_{\rm ns}}\right)^{2-b}}{1+\frac{2ni\pi d_1}{\zeta-d_1}-\frac{4n^2\pi^2 d_2}{b(\zeta-d_1)^2}+\mathcal{O}\left(\frac{1}{\zeta^3}\right)},$$

$$|\mathcal{Q}| = \left(\frac{r_{\rm sh}}{r_{\rm ns}}\right)^{2-[1+l(l+1)]^{\frac{1}{2}}}+\mathcal{O}\left(\frac{1}{\zeta^2}\right),$$

$$\zeta \equiv \log\frac{r_{\rm sh}}{r_{\rm ns}},$$

$$\omega_i^{(0)} = (2-b)\frac{|v_{\rm sh}|}{r_{\rm sh}},$$

$$\omega_i^{(k)} = \frac{1}{\tau_{\rm adv}^{\rm ns}}\log\left|Q\left(\frac{2n\pi}{\zeta-d_1}+\frac{i\omega_i^{(k-1)}r_{\rm sh}}{|v_{\rm sh}|}\right)\right|$$

→analytic approximation

$$\mathcal{Q}(Z) \equiv \frac{2b\left(\frac{r_{\rm sh}}{r_{\rm ns}}\right)^{2-b}\left\{1+\left[(Z+2)^2-b^2\right]\frac{\mathcal{M}_{\rm sh}^2}{l(l+1)x_{\rm sh}^3}\right\}}{\left[1-(Z+2-b)N\right](Z+2+b)-\frac{Z+2-b}{x_{\rm sh}^{2b}}},$$

$$\mathcal{Q}\left(\frac{i\omega r_{\rm sh}}{|v_{\rm sh}|}\right)e^{i\omega\tau_{\rm adv}^{\rm ns}} = 1,$$

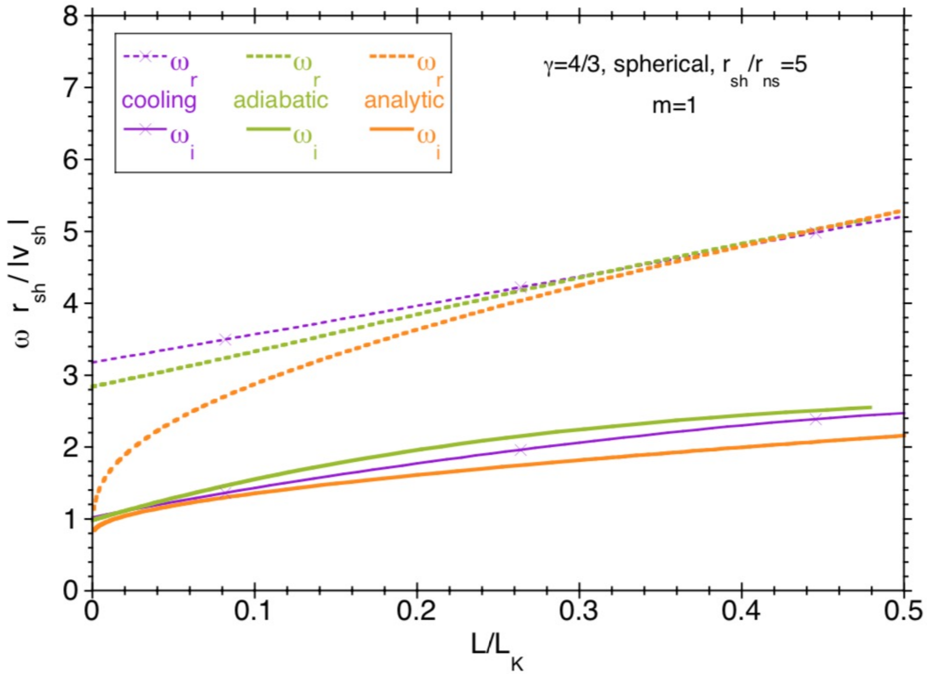
→ practical use for multi-messenger analysis

Physical insight on the impact of rotation on SASI

→adiabatic approximation

$$\left\{\left(\frac{\partial}{\partial X}+\frac{i\omega'}{c^2}\right)^2+\frac{\omega'^2\mu'^2}{v_r^2c^2}\right\}(r\delta v_\varphi)=-\frac{\partial}{\partial X}\left(\frac{r\delta w_\theta}{v_r}\right)$$

$$\omega'\equiv\omega-\frac{mL}{r^2}$$



Modest rotation: differential rotation $\Omega \sim L/r^2$ at small radius increases the radial wavelength $\lambda_r \sim 2\pi v/(\omega - mL/r^2)$ of advected perturbations

→increases the match between the acoustic oscillator and the advected forcing = "un-mixing" of the phase

Strong rotation: corotation radius r_{co} where $\omega'=0$

stationary phase approximation

$$\int_{ns}^{sh}\frac{\partial Y_0}{\partial r}\frac{1}{\mathcal{M}^2}e^{\int_{sh}\frac{i\omega'}{v_r}dr}\frac{dr}{r_{sh}} \sim e^{i\Psi_{co}}\int_{ns}^{sh}\frac{\partial Y_0}{\partial r}\frac{e^{-\omega_i\tau_{adv}(r)}}{\mathcal{M}^2}e^{-i\left(\frac{r-r_{co}}{\Delta r}\right)^2}\frac{dr}{r_{sh}}$$
$$\sim e^{i\Psi_{co}}\pi^{\frac{1}{2}}e^{-i\frac{\pi}{4}}\left(\frac{\partial Y_0}{\partial r}\right)_{co}\frac{e^{-\omega_i\tau_{adv}^{co}}}{\mathcal{M}_{co}^2}\frac{\Delta r}{r_{sh}}$$

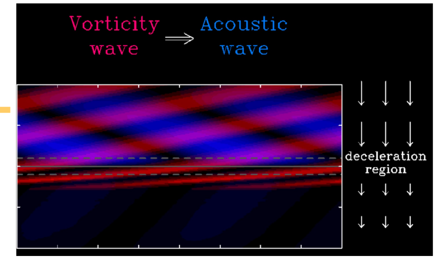
→spiral SASI is produced by an advective-acoustic cycle with an extended coupling in the corotation region

→analytic approximation

$$\mathcal{Q}e^{-\omega_i\tau_{adv}^{co}}=1$$

$$\mathcal{Q}\equiv\frac{\pi^{\frac{1}{2}}\left(\frac{r_{sh}}{r_{co}}\right)^{2a-b}e^{i\left(\Psi_{co}-\frac{5\pi}{4}\right)}}{\left(\frac{\omega_r r_{sh}}{|v_{sh}|}\right)^{\frac{1}{2}}\left[N\left(\frac{i\omega'_{sh}r_{sh}}{|v_{sh}|}\right)+\frac{2b}{m_l^2}\frac{\mathcal{M}_{sh}^2}{x_{sh}^{a+b}}e^{i\omega\tau_{adv}^{sh}}\right]}$$

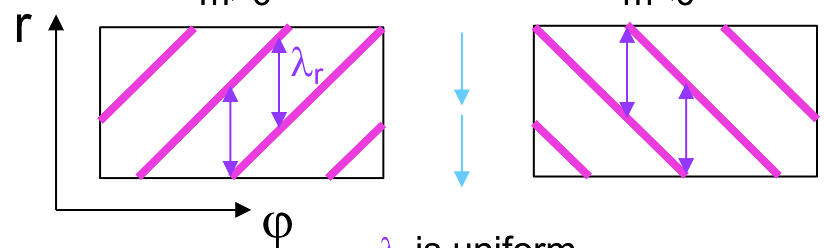
Why is the prograde mode destabilized by rotation ?



$\lambda_r = 2\pi/k_r$: radial wavelength of advected perturbations $\quad e^{-i\omega t + i(k_r r + m\varphi)}$

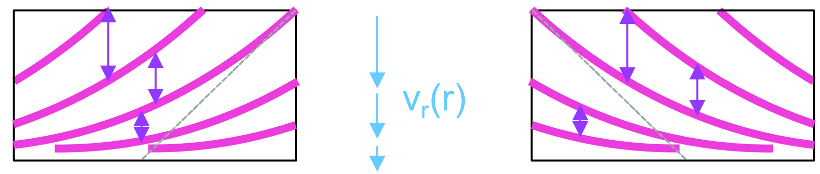
At low frequency, the radial scale of the pressure field is large ~ $r_{sh}-r_{ns}$
Its forcing by advected perturbations is inefficient where $\lambda_r \ll r_{sh}-r_{ns}$

uniform radial velocity v_r



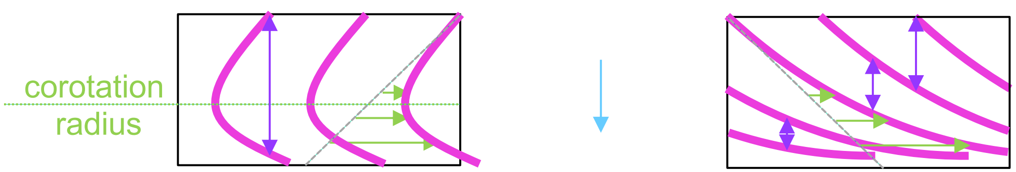
λ_r is uniform

decelerated radial velocity $v_r(r)$



λ_r decreases downward
→phase mixing at small radius

uniform radial velocity v_r
+ horizontal shear $\Omega = L/r^2$



prograde mode
m>0

retrograde mode
m<0

$$\omega' \equiv \omega - \frac{mL}{r^2}$$

$$\lambda_r = \frac{2\pi v}{\omega - m\Omega}$$

$$k_r = \frac{\omega'}{v_r}$$

$$r_{\rm corot} \equiv \left(\frac{mL}{\omega}\right)^{\frac{1}{2}}$$

→best phase match at corotation

→phase mixing at small radius

Turbulent stabilization and rotational destabilization ?

small experiment
laminar regime

$$H_\Phi \equiv -\frac{R_{45}^2}{r}$$

$$\mathrm{Re} \equiv \frac{hv}{\nu} = \mathrm{Fr}\frac{g^{\frac{1}{2}}h^{\frac{3}{2}}}{\nu}$$

$$Q = 2\pi r v h = 2\pi\frac{r}{h}\mathrm{Fr}g^{\frac{1}{2}}h^{\frac{5}{2}}$$

$$\lambda \equiv \frac{R'_{45}}{R_{45}} = 6.25$$

$$6.25^{\frac{3}{2}} \sim 15.6$$

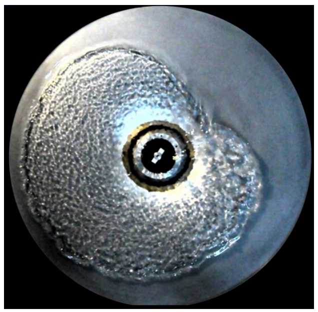
$$6.25^{\frac{5}{2}} \sim 98$$

large experiment
turbulent regime

$R'_{45} = 35cm$

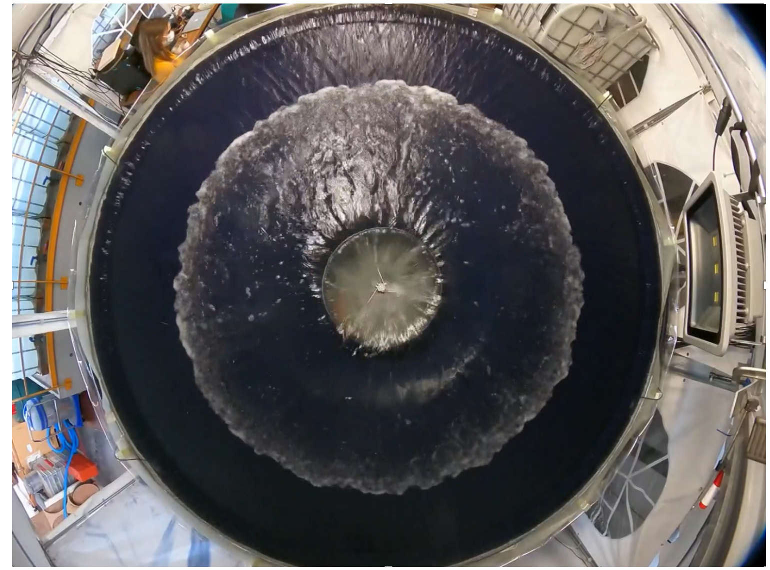
$R_{45} = 5.6cm$

66cm



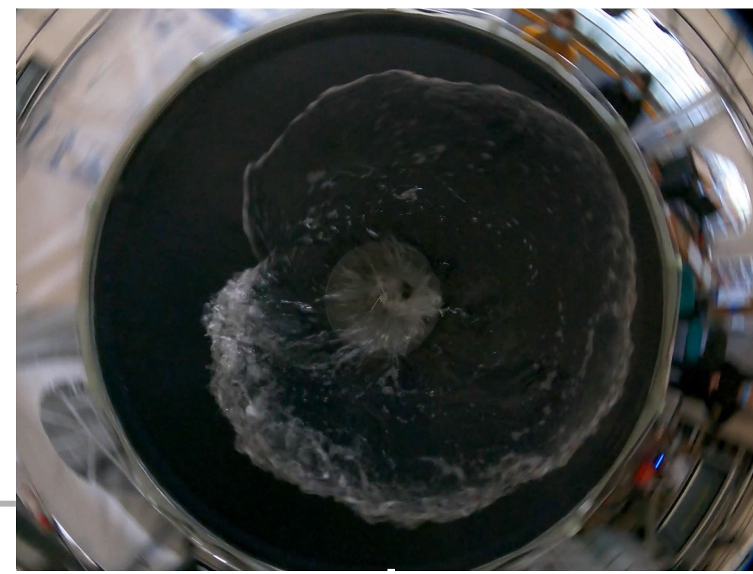
without rotation,
turbulent SASI @ 100L/s
is more stable than
laminar SASI @1L/s

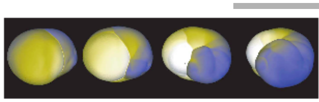
350cm



$R_{45} = 5.6cm$

A small amount
of rotation
is sufficient
to destabilize
the prograde mode



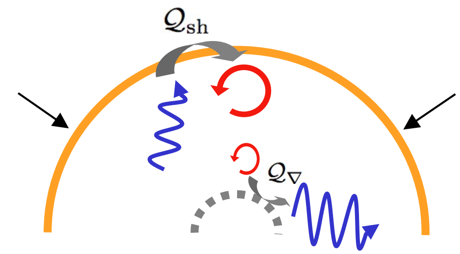


as observed in 3D numerical models
by Blondin & Mezzacappa (2006)

Impact of viscosity ν and thermal diffusivity κ on SASI?

$$\tau : \nabla v = 2\nu\rho \left[\frac{1}{2}(\partial_j v_i + \partial_i v_j) - \frac{1}{3}(\nabla \cdot v)\delta_{ij}\right]^2$$

$$\begin{aligned}
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) &= 0, \\
\frac{\partial v}{\partial t} + (v \cdot \nabla) v + \nabla \Phi &= -\frac{\nabla p}{\rho} + \nu \left[\nabla^2 v + \frac{1}{3}\nabla(\nabla \cdot v)\right] \\
\frac{\partial S}{\partial t} + (v \cdot \nabla) S &= \frac{\gamma \kappa}{\gamma - 1}\frac{\nabla^2 c^2}{c^2} + \frac{1}{p}\tau : \nabla v.
\end{aligned}$$



$$\omega_r^{\rm SASI} \sim \frac{2\pi |v_{\rm sh}|}{r_{\rm sh}} \qquad w_i^{\rm SASI} \sim \frac{|v_{\rm sh}|}{r_{\rm sh}}$$

in a plane parallel uniform flow:

$$\omega_i^{\rm visc} = -\nu k_{\rm adv}^2$$

vorticity perturbations can be damped by viscosity → $\frac{\partial w_i^{\rm adv}}{\partial \nu} \sim -\frac{4\pi^2}{r_{\rm sh}^2}$?

with $k_{\rm adv}^r \sim \frac{\omega_r^{\rm SASI}}{|v_{\rm sh}|} \sim \frac{2\pi}{r_{\rm sh}}$

$$\omega_i^{\rm diff} = -\kappa k_{\rm adv}^2$$

entropy perturbations can be damped by thermal diffusivity → $\frac{\partial w_i^{\rm adv}}{\partial \kappa} \sim -\frac{4\pi^2}{r_{\rm sh}^2}$?

$$\omega_i^{\rm ac} = -\left(\frac{2}{3}\nu + \frac{\gamma - 1}{2}\kappa\right) k_{\rm ac}^2$$

acoustic perturbations can be damped by both →

$\frac{\partial w_i^{\rm ac}}{\partial \nu} \sim -\frac{2}{3}\frac{4\pi^2 \mathcal{M}_{\rm sh}^2}{r_{\rm sh}^2}$?

$\frac{\partial w_i^{\rm ac}}{\partial \kappa} \sim -\frac{\gamma - 1}{2}\frac{4\pi^2 \mathcal{M}_{\rm sh}^2}{r_{\rm sh}^2}$?

with $k_{\rm ac} \sim \frac{\omega_r^{\rm SASI}}{c_{\rm sh}} \sim \frac{2\pi \mathcal{M}_{\rm sh}}{r_{\rm sh}}$

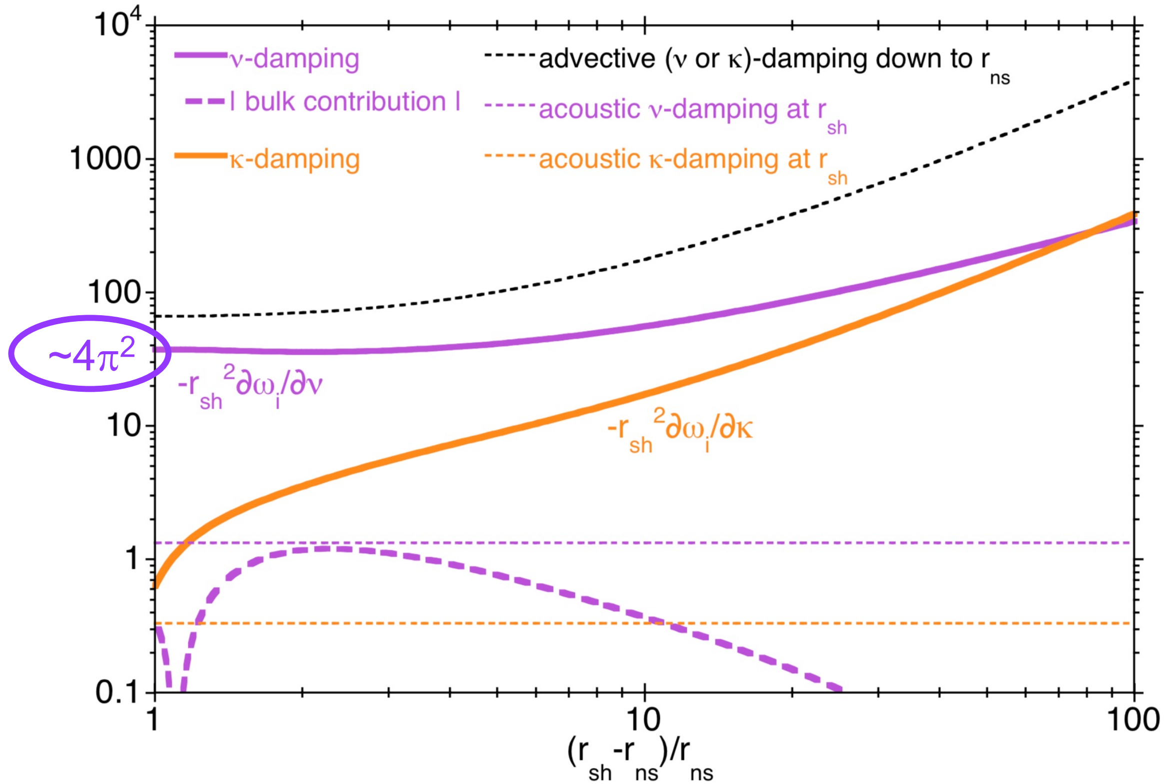
Impact of viscosity ν and thermal diffusivity κ on SASI?

perturbative calculation

→ $$\frac{\partial w_i}{\partial \nu} \sim -\frac{4\pi^2}{r_{sh}^2}$$

as expected if SASI mechanism is governed by the advection of vorticity perturbations

$$\frac{\delta w_i}{\omega_i} \sim -4\pi^2 \frac{\nu}{r_{sh}|v_{sh}|}$$



stabilization by numerical viscosity

$$\nu_{num} \sim \left(\frac{1-C_{CFL}}{2}\right) v\Delta r$$

$$\frac{\delta w_i}{\omega_i} \sim -\frac{4\pi^2}{N_r}\left(\frac{1-C_{CFL}}{2}\right)\frac{r_{sh}-r_{ns}}{r_{sh}}$$

$$\frac{\delta w_i}{\omega_i} \sim -26\% \left(\frac{30}{N_{pns}^{sh}}\right)$$ (C_{CFL}~0.4)

→ 30 grid points from r_{pns}=50km to r_{sh}=150km are insufficient in 3D simulations

stabilization by turbulence

$$\frac{v_{turb}}{v_{sh}} \sim \frac{\nu_{stab}}{r_{sh}|v_{sh}|} \sim \frac{1}{4\pi^2} \sim 3\%$$

→ the "turbulence" invoked for SN explosions
(>30% in Müller & Janka 15, Müller+17)
would stabilize SASI without rotation
(see also Nagakura+19)

Conclusion

The shallow water experiment unexpectedly drew our attention to

-the adiabatic analytical framework to study SASI
-the stabilizing effect of turbulence on SASI

Rotation effects on SASI are clarified using the adiabatic approximation

-forced oscillator sensitive to phase mixing
-prograde vorticity waves sheared by differential rotation

First analytical estimates of SASI growth rate and frequency

Unexpectedly large stabilization of SASI by viscosity (without rotation)

-turbulent velocities ≥ 3% $|v_{sh}|$ can stabilize SASI
-warning on the damping effect of numerical viscosity

What's next? → reverse engineering of multi-messenger signatures

- include the low-T/W instability + SASI + convection
- complementarity of neutrino and GW signatures for each instability

Outreach movie on YouTube 6mn