The Cosmological Flow

A Systematic Approach to Inflationary Correlators

Greco Seminar 24/04/2023

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Based on: ArXiv:2302.00655 (short paper)
ArXiv:2304.xxxxx (long paper)
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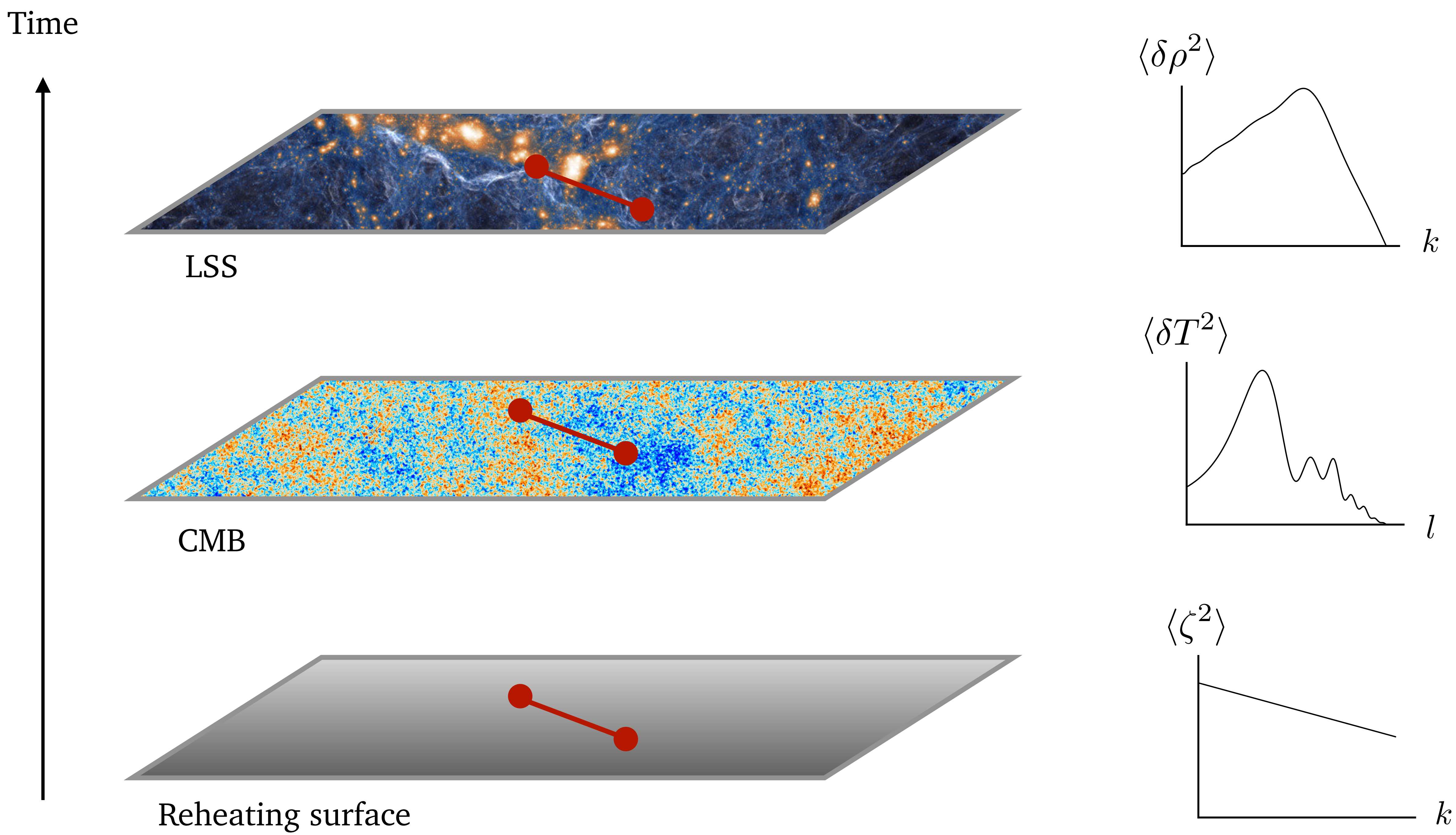
Cosmology: Observing Correlated Fluctuations

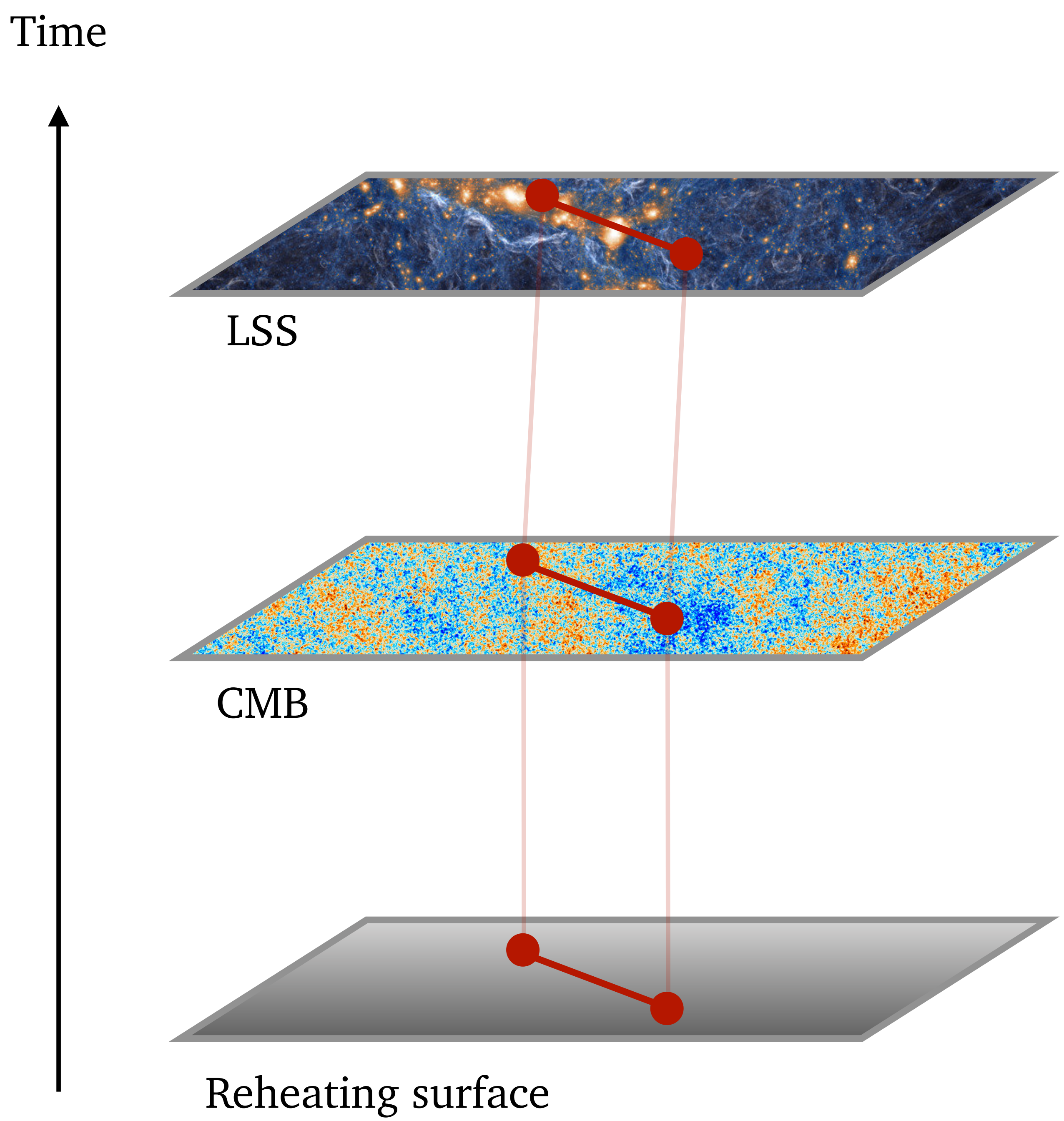
Cosmological fluctuations are **correlated** on large scales



Courtesy of Daniel Baumann

Cosmology: A History of Time

The physics is encoded in the **time evolution** of these fluctuations



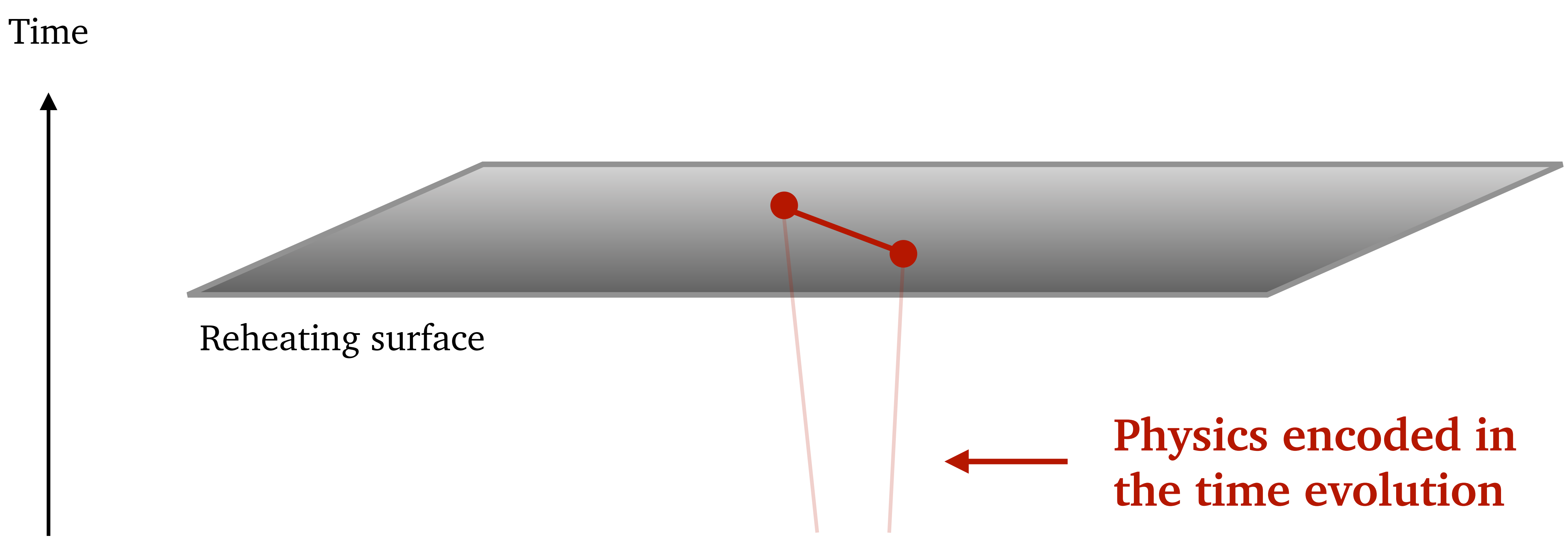
Transfer function

$$\langle \delta X^2(\boldsymbol{k}, t)\rangle = \mathbf{T}^2(\boldsymbol{k}; t, t_0)\langle \delta X^2(\boldsymbol{k}, t_0)\rangle$$

- Linear clustering
- Dark matter, photons, baryons
- …

The Cosmological Flow Philosophy

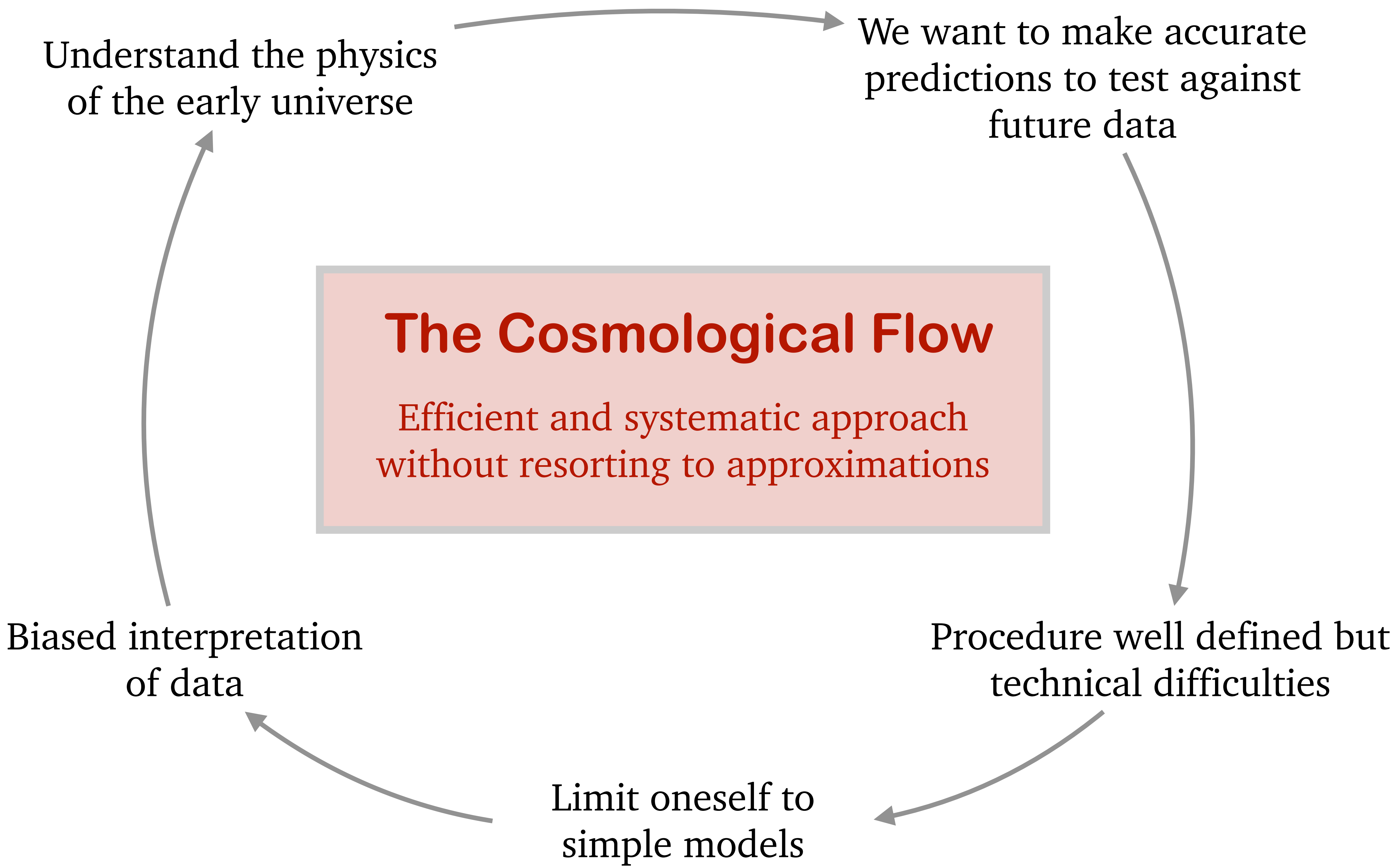
Follow the **time evolution** of primordial fluctuations from their origin as **quantum vacuum fluctuations** to the reheating surface



Theory → **Cosmological Flow** → **Observables**

By studying inflationary fluctuations, we learn about the **origin of structures**

Why the Cosmological Flow: Break the Vicious Circle



Outline

I. The Physics of Inflation

II. The Cosmological Flow

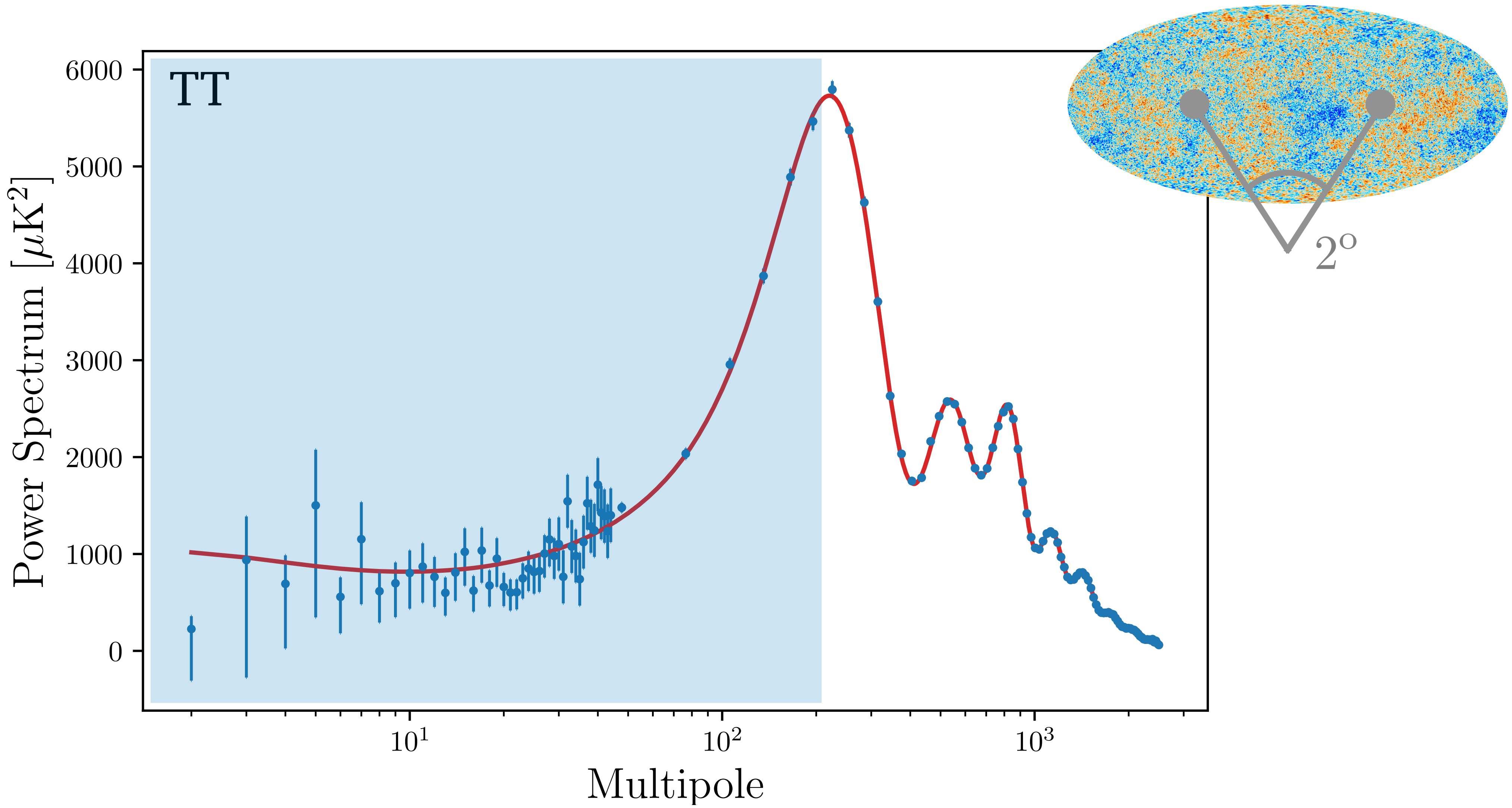
III. Applications

I. The Physics of Inflation

- Basics of Inflation from Observations
- Primordial Non-Gaussianities

Superhorizon Fluctuations

Fluctuations from *a priori* **causally disconnected** patches are correlated



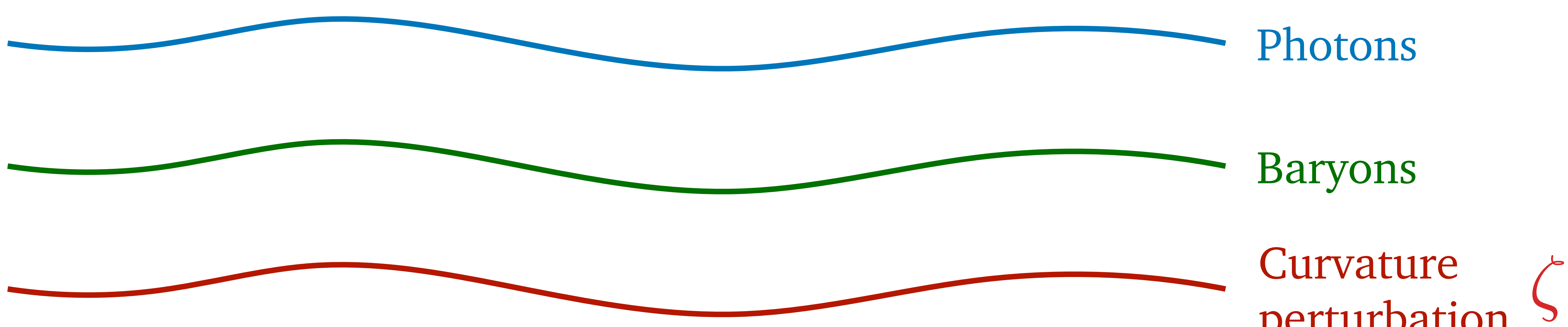
Inflation

These fluctuations were generated during a **period of accelerated expansion**, before the conventional ΛCDM cosmology

Adiabatic Fluctuations

The CMB power spectrum is evidence that the dominant contribution to the primordial perturbations is adiabatic

$$\delta_X(\boldsymbol{x}) \propto \delta_Y(\boldsymbol{x})$$



$$g_{ij} = a^2 e^{2\zeta} \delta_{ij}$$

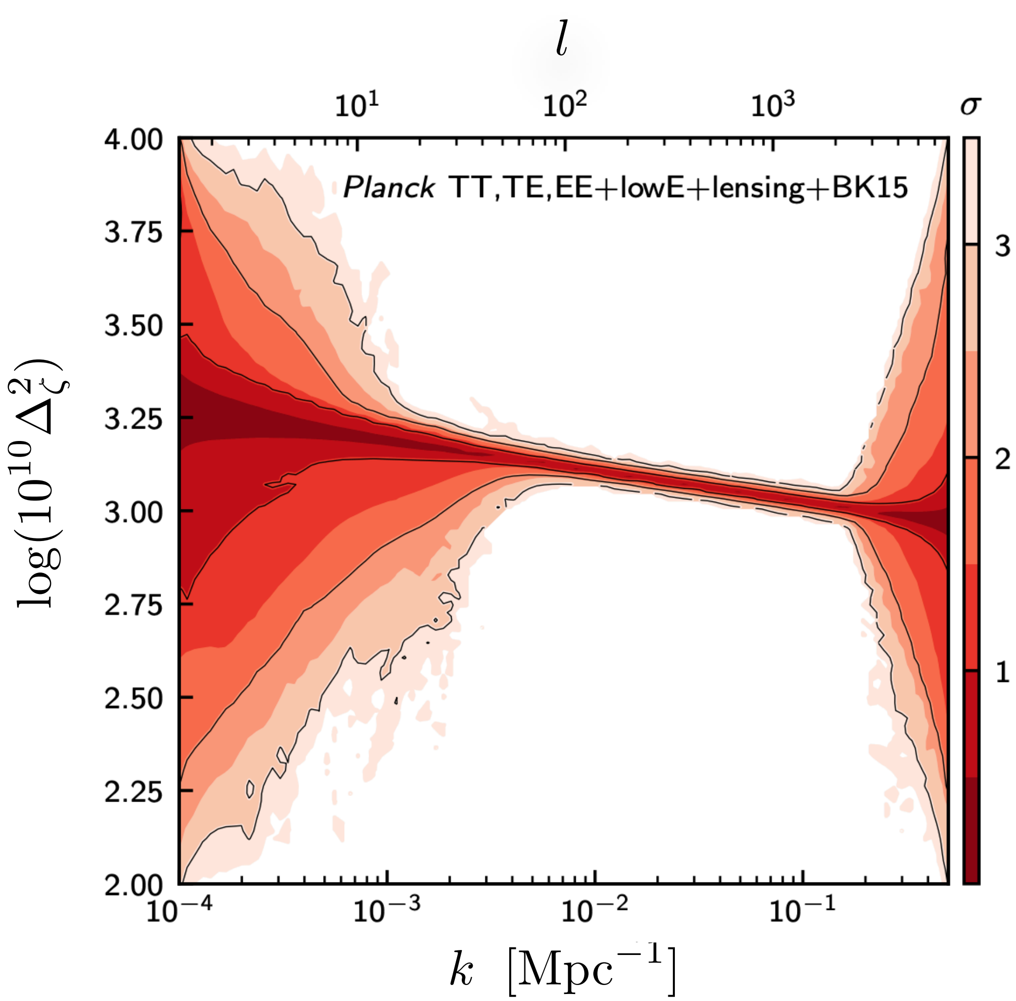
Primordial fluctuations can be described by a **single fluctuating scalar degree of freedom**

Near Scale-Invariant Fluctuations

Primordial fluctuations are **approximately scale-invariant**

$$\Delta_\zeta^2 = \frac{k^3}{2\pi^2}\langle \zeta_{\boldsymbol{k}} \zeta_{-\boldsymbol{k}} \rangle' = A_s \left(\frac{k}{k_\star}\right)^{n_s - 1} \quad \text{with} \quad n_s = 0.9652 \pm 0.0042$$

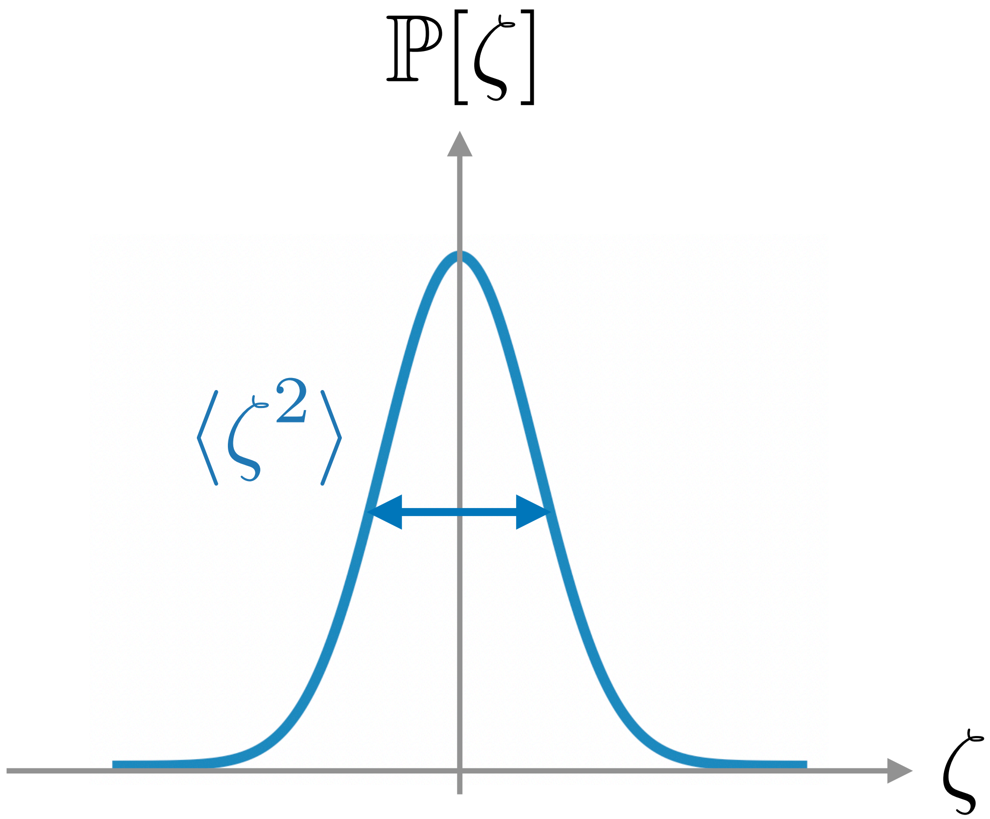
Planck [2018]



On CMB scales, inflation can be described by an **approximate de Sitter spacetime**

Almost Gaussian Fluctuations

Primordial fluctuations are very close to **Gaussian**



$$\frac{\langle \zeta\zeta\zeta \rangle}{\langle \zeta\zeta \rangle^{3/2}} < 10^{-3}$$

The physics of inflation (=**interactions**) is encoded in **deviations from Gaussianity**

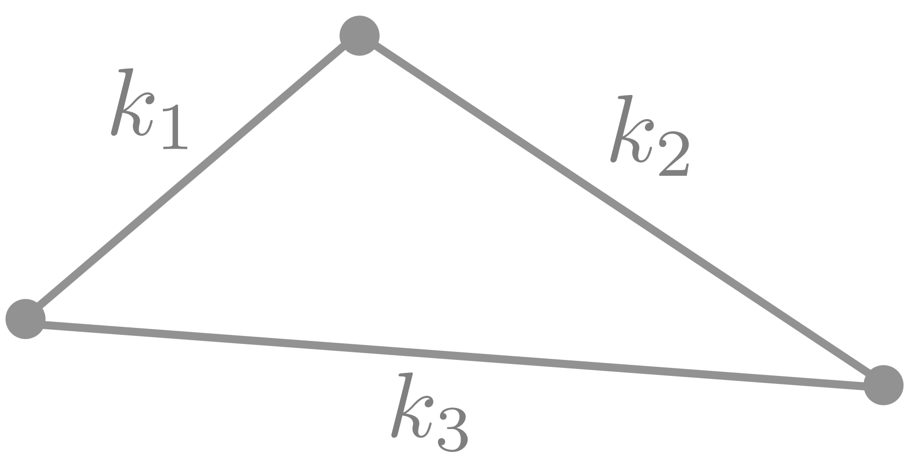
Primordial Non-Gaussianities

For weakly coupled fluctuations, the leading non-Gaussian signature is the three-point correlation function of ζ (=**bispectrum**)

$$\langle \zeta_{\boldsymbol{k}_1} \zeta_{\boldsymbol{k}_2} \zeta_{\boldsymbol{k}_3} \rangle = (2\pi)^3 \delta^{(3)}(\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3) B_\zeta(k_1, k_2, k_3)$$

Homogeneity

Isotropy



The sum of the momentum 3-vectors must form a closed triangle

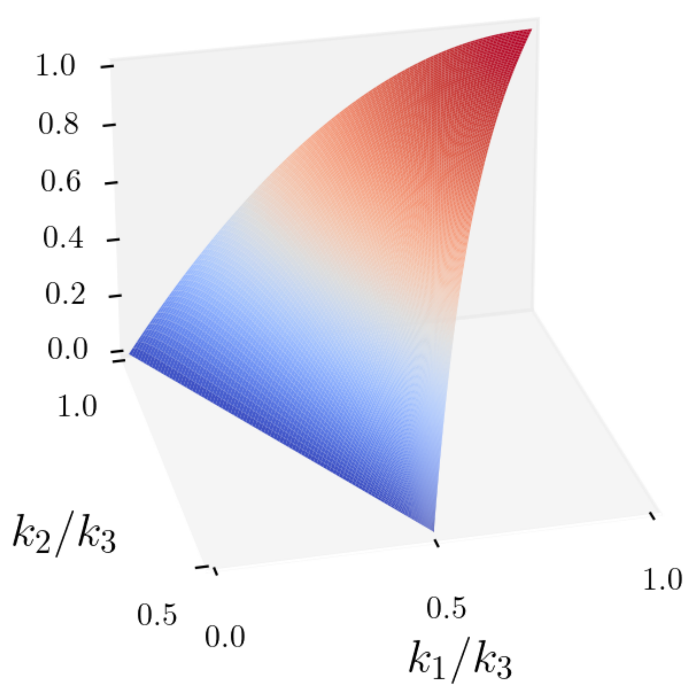
The bispectrum is a "**function of triangle**"

Shapes of Non-Gaussianities

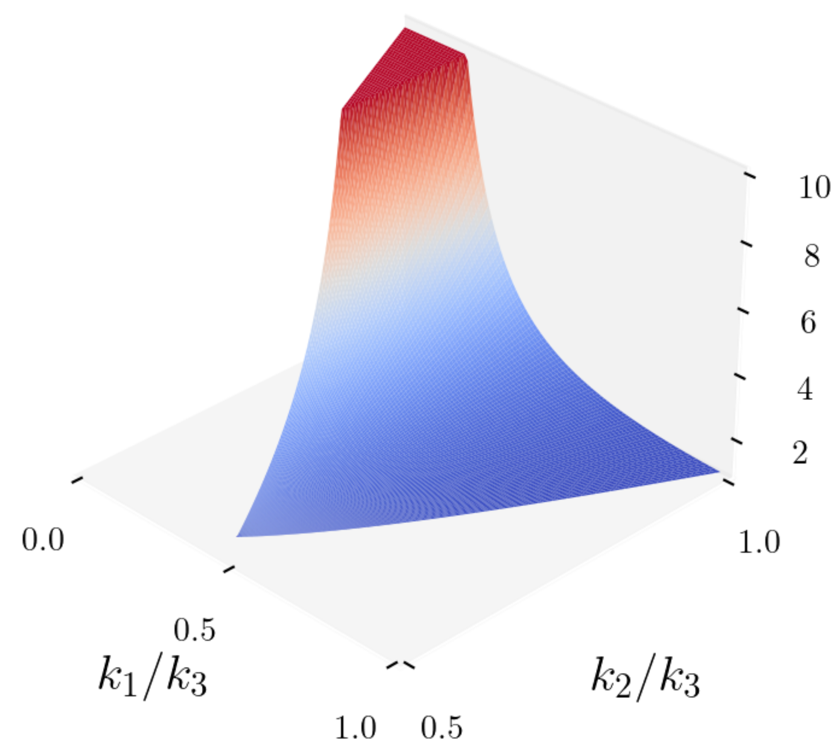
The amplitude of the bispectrum is given by $f_{\rm NL}$ and the physics of inflation is encoded in the **shape function**

$$\langle \zeta_{\boldsymbol{k}_1} \zeta_{\boldsymbol{k}_2} \zeta_{\boldsymbol{k}_3} \rangle' \equiv (2\pi)^4 \frac{S(k_1,k_2,k_3)}{(k_1 k_2 k_3)^2} \Delta_\zeta^4$$

$$f_{\rm NL} \equiv \frac{10}{9} S(k,k,k)$$



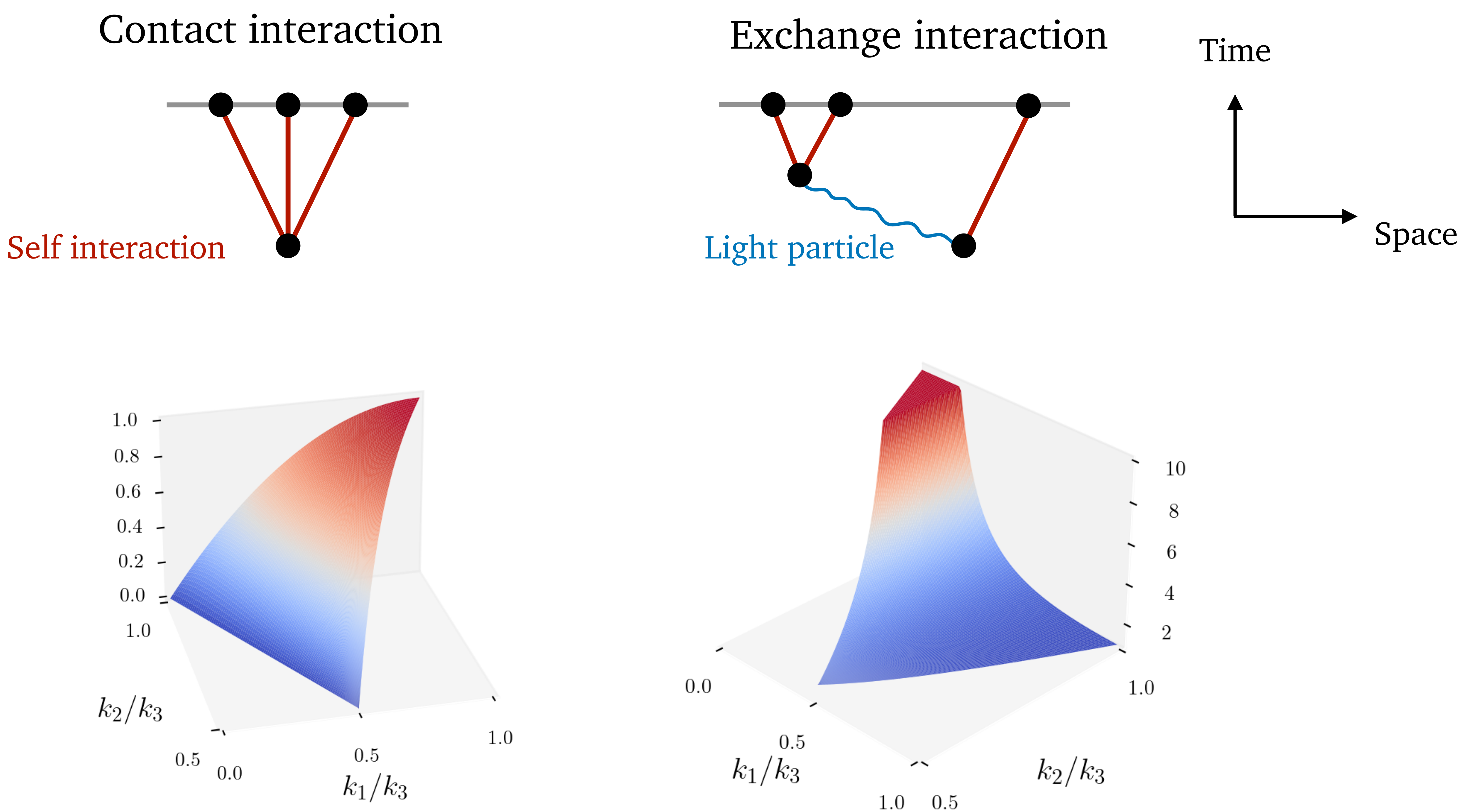
$f_{\rm NL}^{\rm eq} = -26 \pm 47$



$f_{\rm NL}^{\rm loc} = -0.9 \pm 5.1$

Planck [2018]

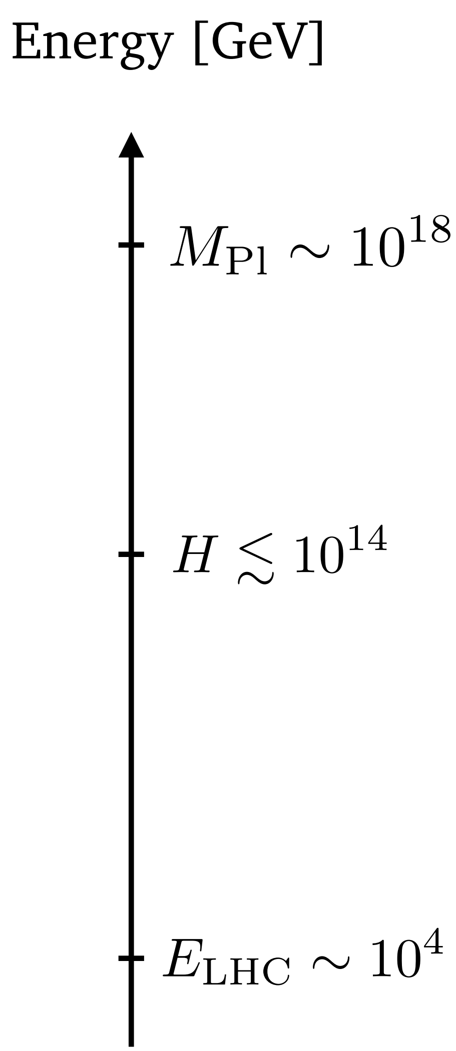
The Physics of Non-Gaussianities

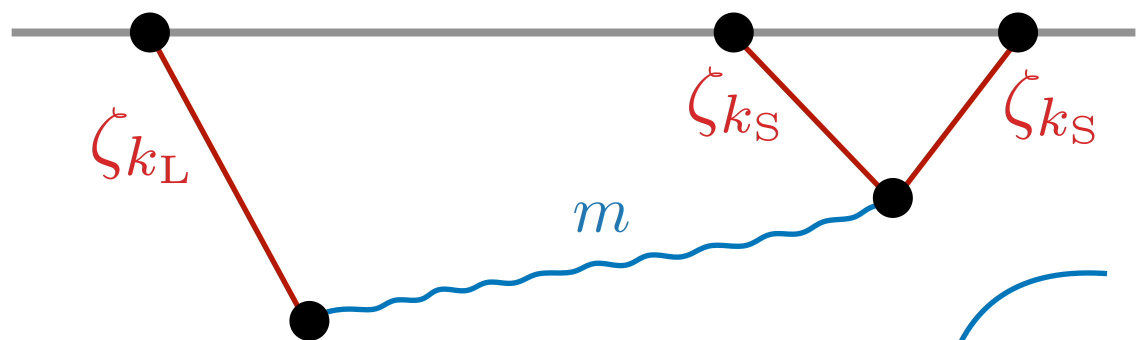


Primordial non-Gaussianities are a probe of the physics of inflation
(=**particle content, interactions, masses, spins, sound speeds, etc**)

Cosmological Collider Physics

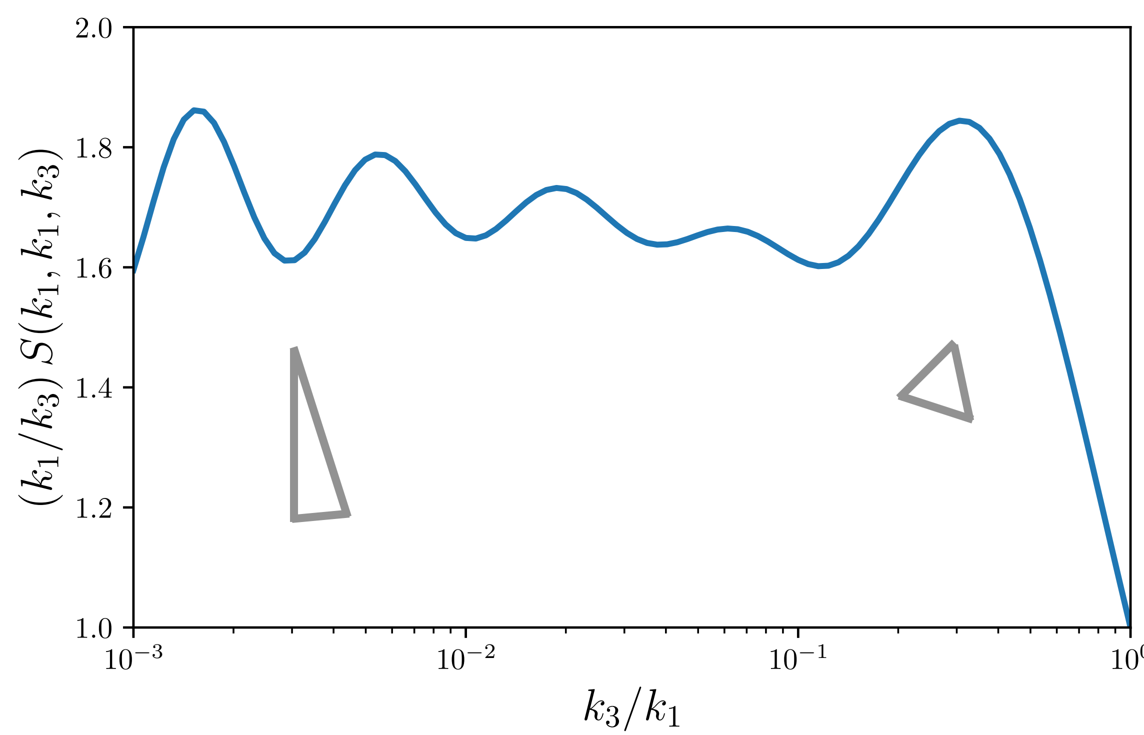
During inflation, very **massive particles** ($\sim 10^{14}$ GeV) can be produced whose decays lead to observable correlations





$$\mu^2 = m^2/H^2 - 9/4$$

$$\lim_{k_3/k_1 \to 0} S(k_1, k_1, k_3) \sim \left(\frac{k_3}{k_1}\right)^{1/2} \cos\left[\mu \log\left(\frac{k_3}{k_1}\right) + \delta\right]$$



Chen, Wang, Maldacena, Arkani-Hamed [2010, 2015]

Primordial non-Gaussianities in a Nutshell

Observations
- CMB-S4
- 3D LSS surveys
- 21cm

Phenomenology
- New physics at the highest reachable energies
- Cosmological Collider physics

Primordial non-Gaussianities

Theory
- QFT in dS
- Derive analytical solutions for these correlators
- Formal aspects and analytical structure
- Similar to flat-space scattering amplitudes or boundary correlators in AdS

Achucarro, Adshead, Baumann, Benincasa, Bonifacio, Chen, Creminelli, Easther, Komatsu, Langlois, Lim, Maldacena, McAllister, Nicolis, Pajer, Pimentel, Renaux-Petel, Seery, Senatore, Sleight, Stefanyszyn, Taronna, Tolley, Vernizzi, Wang, Weinberg…
and many others

II. The Cosmological Flow

- In-in Formula
- Inflationary Correlators
- The Cosmological Flow Approach

In-in Formula

- From first principles, we want to compute **equal-time correlators**

Vacuum

Operator

$$\langle \Omega | \mathcal{O}(t) | \Omega \rangle$$

- The theory is described by a **Hamiltonian**

$$\boldsymbol{X}^a \equiv (\boldsymbol{\varphi}^\alpha, \boldsymbol{p}^\beta)$$

$$H(\boldsymbol{X}^a) = H_0(\boldsymbol{X}^a) + H_I(\boldsymbol{X}^a)$$

Free

Interacting

- We go from the Heisenberg picture to the **interaction picture**

$$X^a \equiv \mathcal{U}^\dagger \boldsymbol{X}^a \,\mathcal{U} \qquad \langle \Omega | \mathcal{O}(\boldsymbol{\varphi}^\alpha, \boldsymbol{p}^\beta) | \Omega \rangle = \langle \Omega | \mathcal{U}\, \mathcal{O}(\varphi^\alpha, p^\beta)\, \mathcal{U}^\dagger | \Omega \rangle$$

Heisenberg-picture fields

Interaction-picture fields

In-in Formula

- The interaction-picture fields evolve with the free Hamiltonian

$$\frac{\mathrm{d}X^a}{\mathrm{d}t} = i\,[H_0(X^b), X^a]$$

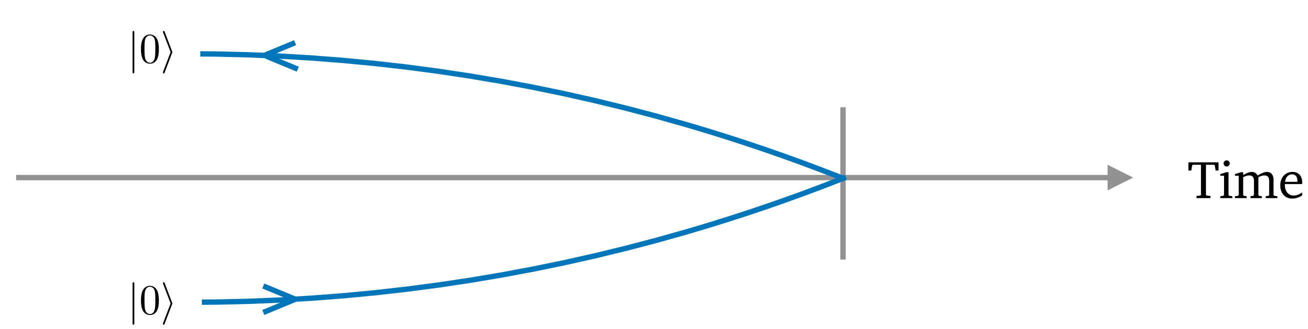
- We choose the unitary operator to evolve with the interacting Hamiltonian

$$\frac{\mathrm{d}\mathcal{U}}{\mathrm{d}t} = i\mathcal{U}H_I(X^a) \longrightarrow \mathcal{U} = \bar{\mathrm{T}}\exp\left(i\int_{-\infty^+}^{t} H_I(t')\mathrm{d}t'\right)$$

Dyson's formula

$i\epsilon$ prescription

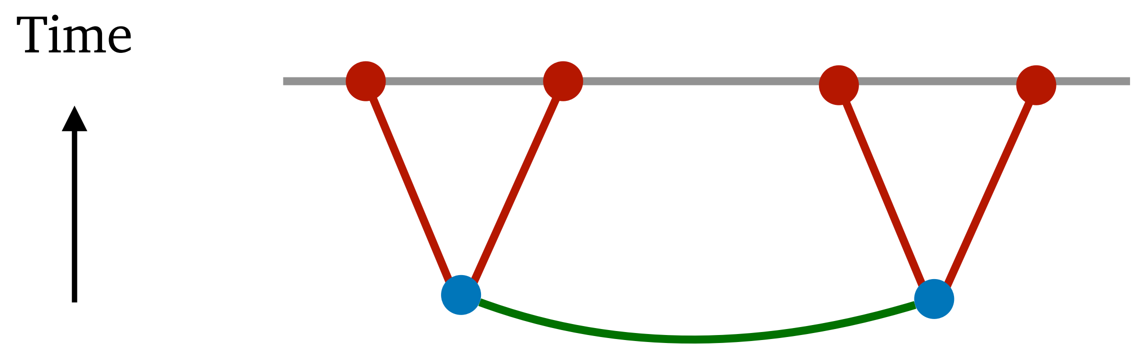
$$\langle\Omega|\,\mathcal{O}(\boldsymbol{X}^a)\,|\Omega\rangle = \langle 0|\left[\bar{\mathrm{T}}\,e^{i\int_{-\infty^+}^{t} H_{\mathrm{I}}(t')\mathrm{d}t'}\right]\mathcal{O}(X^a)\left[\mathrm{T}\,e^{-i\int_{-\infty^-}^{t} H_{\mathrm{I}}(t')\mathrm{d}t'}\right]|0\rangle$$



Weinberg [2005]

Technical Difficulties of Perturbation Theory

In practice, we compute Feynman-Witten **diagrams** involving complicated time integrals



$$\langle \boldsymbol{X}^4 \rangle = \int \mathrm{d}t \int \mathrm{d}t' V(t) V(t') \mathcal{G}(k_{12}, t, t') K(k_1, t) K(k_2, t) K(k_3, t') K(k_4, t')$$

- Background is time-dependent
- Algebraic complexity
- Late-time correlators receive contributions from all times
- We cannot use standard techniques from particle physics
- …

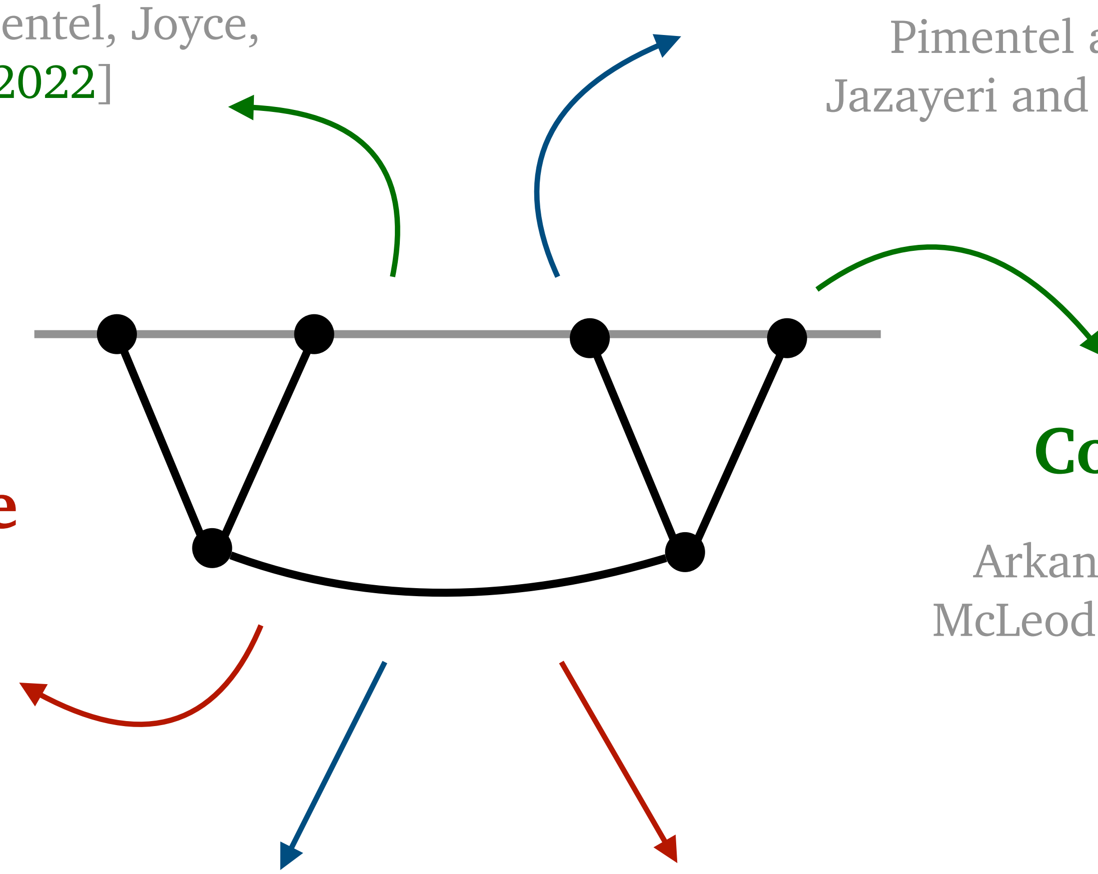
Recent Analytical Developments

Cosmological Bootstrap Program

Arkani-Hamed, Baumann, lee, Pimentel, Joyce, Duaso Pueyo [2019, 2020, 2022]

Bootstrap Equations for Boost-breaking Interactions

Pimentel and Wang [2022], Jazayeri and Renaux-Petel [2022]



AdS-inspired Mellin Space

Sleight and Taronna [2019, 2021]

Cosmological Potytopes

Arkani-Hamed, Benincasa, Postnikov, McLeod [2017, 2018, 2019, 2020, 2022]

Fundamental Principles (Symmetries & Causality & Locality)

Pajer, Stefanyszyn, Supeł, Goodhew, Jazayeri, Melville, Gordon Lee, Bonifacio, Wang [2020, 2021]

Partial Mellin-Barnes Representation

Qin and Xianyu [2022]

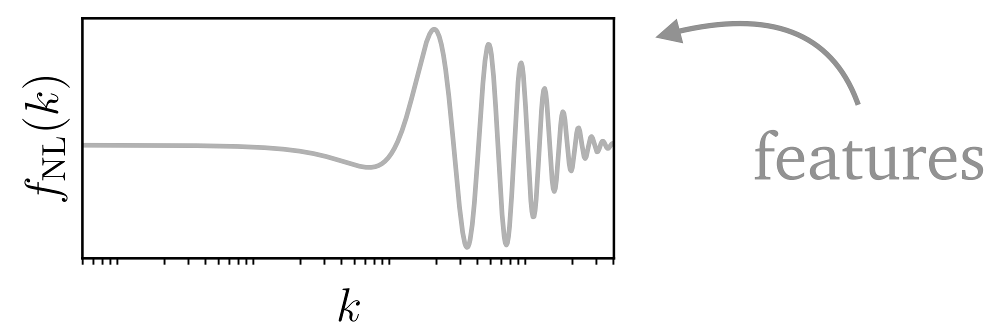
Limitations of Analytical Methods

Weak Quadratic Mixing

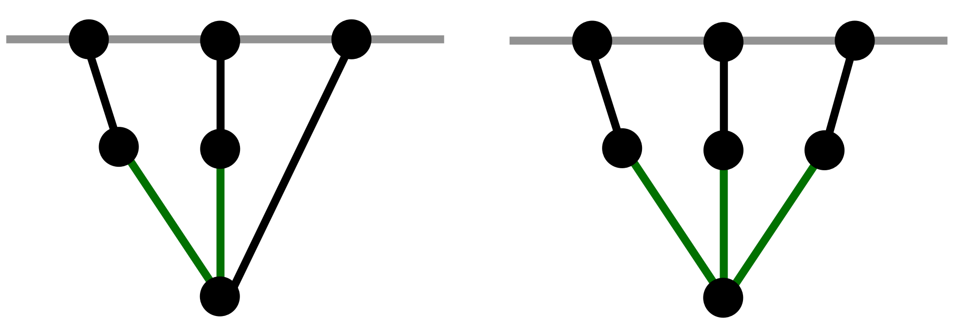
$$\mathcal{L}^{(2)} \supset \rho\dot{\phi}\sigma$$

treated perturbatively

(Near) Scale-Invariance

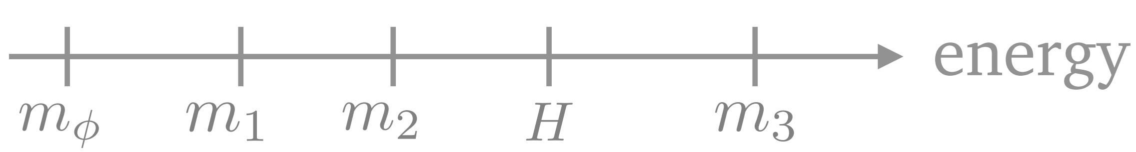


Only Single-Exchange Diagram

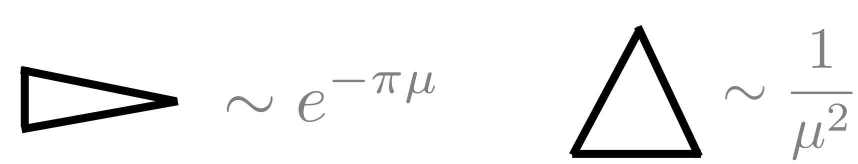


Large hierarchy of masses/couplings but not the intermediate regimes

Often only 1 or 2 Fields



Treatment of Equilateral and Squeezed Configurations Separately



Aside from isolated examples…

Equations of Motion

- Without loss of generality, the Hamiltonian can be written

Any time/momentum dependence

$$H = \frac{1}{2!} H_{\mathsf{ab}} \boldsymbol{X}^{\mathsf{a}} \boldsymbol{X}^{\mathsf{b}} + \frac{1}{3!} H_{\mathsf{abc}} \boldsymbol{X}^{\mathsf{a}} \boldsymbol{X}^{\mathsf{b}} \boldsymbol{X}^{\mathsf{c}} + \frac{1}{4!} H_{\mathsf{abcd}} \boldsymbol{X}^{\mathsf{a}} \boldsymbol{X}^{\mathsf{b}} \boldsymbol{X}^{\mathsf{c}} \boldsymbol{X}^{\mathsf{d}} + \dots$$

- The fully non-linear equations of motion are

Commutator

$$[\boldsymbol{X}^{\mathsf{a}}, \boldsymbol{X}^{\mathsf{b}}] = i\epsilon^{\mathsf{ab}}$$

$$\begin{aligned} \frac{\mathrm{d}\boldsymbol{X}^{\mathsf{a}}}{\mathrm{d}t} &= i\,[H, \boldsymbol{X}^{\mathsf{a}}] \\ &= \epsilon^{\mathsf{ac}} H_{\mathsf{cb}} \boldsymbol{X}^{\mathsf{b}} + \frac{1}{2!} \epsilon^{\mathsf{ad}} H_{\mathsf{dbc}} \boldsymbol{X}^{\mathsf{b}} \boldsymbol{X}^{\mathsf{c}} + \frac{1}{3!} \epsilon^{\mathsf{ae}} H_{\mathsf{ebcd}} \boldsymbol{X}^{\mathsf{b}} \boldsymbol{X}^{\mathsf{c}} \boldsymbol{X}^{\mathsf{d}} + \dots \\ &= u^{\mathsf{a}}{}_{\mathsf{b}} \boldsymbol{X}^{\mathsf{b}} + \frac{1}{2!} u^{\mathsf{a}}{}_{\mathsf{bc}} \boldsymbol{X}^{\mathsf{b}} \boldsymbol{X}^{\mathsf{c}} + \frac{1}{3!} u^{\mathsf{a}}{}_{\mathsf{bcd}} \boldsymbol{X}^{\mathsf{b}} \boldsymbol{X}^{\mathsf{c}} \boldsymbol{X}^{\mathsf{d}} + \dots \end{aligned}$$

Theory dependence

- Interaction-picture operators are solution to the linear equations of motion

$$\frac{\mathrm{d}X^{\mathsf{a}}}{\mathrm{d}t} = u^{\mathsf{a}}{}_{\mathsf{b}} X^{\mathsf{b}}$$

Fourier notation

$$A_{\mathsf{a}} B^{\mathsf{a}} = \sum_a \int \frac{\mathrm{d}^3 k_a}{(2\pi)^3} A_a(\boldsymbol{k}_a) B^a(\boldsymbol{k}_a)$$

Tree-level Two-point Inflationary Correlators

Dias, Fazer, Mulryne, Seery, Ronayne [2010, 2011, 2012, 2013, 2015, 2016, 2018]

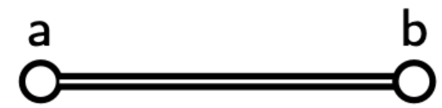
- Two-point correlators read

$$\langle\Omega|\,\boldsymbol{X}^{\mathsf{a}}\boldsymbol{X}^{\mathsf{b}}\,|\Omega\rangle = \langle 0|\,X^{\mathsf{a}}X^{\mathsf{b}}\,|0\rangle$$

- Adopt a **diagrammatic representation** of correlators

$$= \boldsymbol{X}^{\mathsf{a}}(t)$$

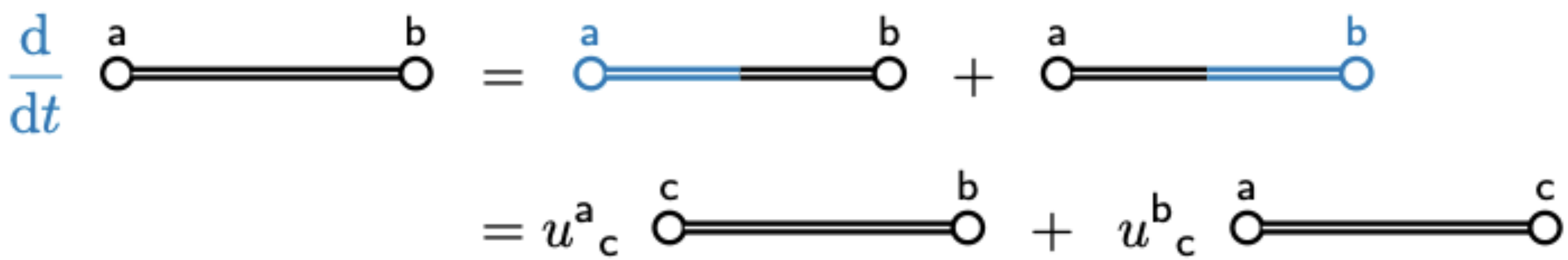
$$\langle\boldsymbol{X}^{\mathsf{a}}\boldsymbol{X}^{\mathsf{b}}\rangle =$$



External field insertion at t

- Using the equation of motion, we obtain

$$\frac{\mathrm{d}X^{\mathsf{a}}}{\mathrm{d}t} = u^{\mathsf{a}}{}_{\mathsf{b}}X^{\mathsf{b}}$$



$$\boxed{\frac{\mathrm{d}}{\mathrm{d}t}\langle\boldsymbol{X}^{\mathsf{a}}\boldsymbol{X}^{\mathsf{b}}\rangle = u^{\mathsf{a}}{}_{\mathsf{c}}\langle\boldsymbol{X}^{\mathsf{c}}\boldsymbol{X}^{\mathsf{b}}\rangle + u^{\mathsf{b}}{}_{\mathsf{c}}\langle\boldsymbol{X}^{\mathsf{a}}\boldsymbol{X}^{\mathsf{c}}\rangle}$$

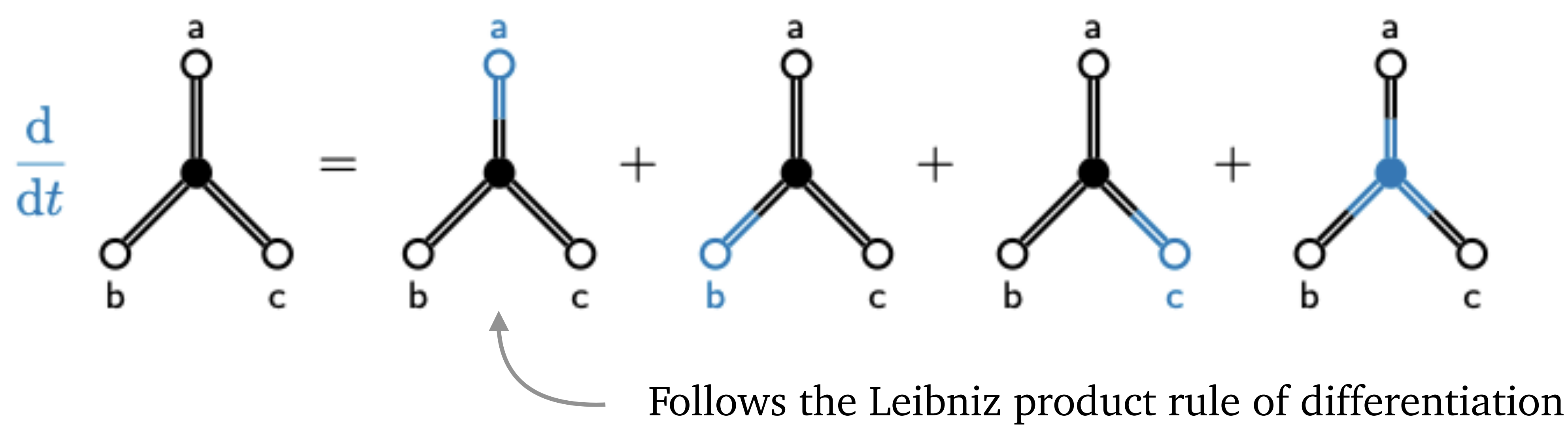
Equivalent to linear equation of motion + quantisation condition

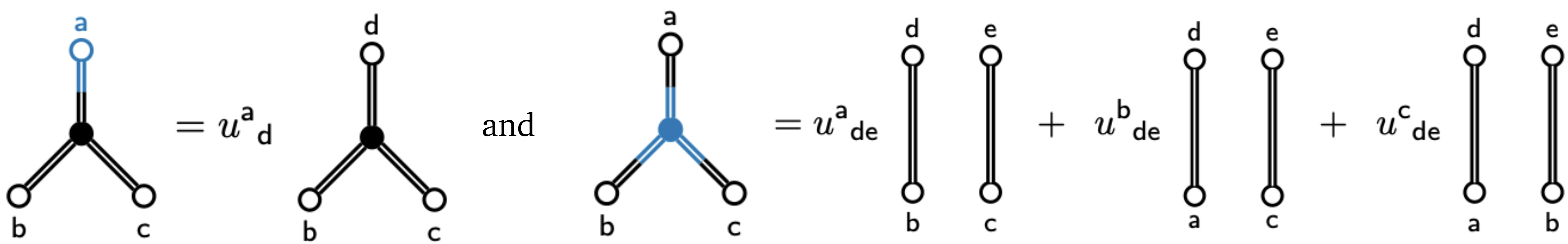
Tree-level Three-point Inflationary Correlators

- Three-point correlators read

$$\langle\Omega|\, \boldsymbol{X}^{\mathsf{a}}\boldsymbol{X}^{\mathsf{b}}\boldsymbol{X}^{\mathsf{c}}\,|\Omega\rangle = \langle 0|\, \frac{i}{3!}\int_{-\infty}^{t} \mathrm{d}t'\, H_{\mathsf{def}}\left[X^{\mathsf{d}}X^{\mathsf{e}}X^{\mathsf{f}}, X^{\mathsf{a}}X^{\mathsf{b}}X^{\mathsf{c}}\right]|0\rangle$$

- Differentiating with respect to time gives





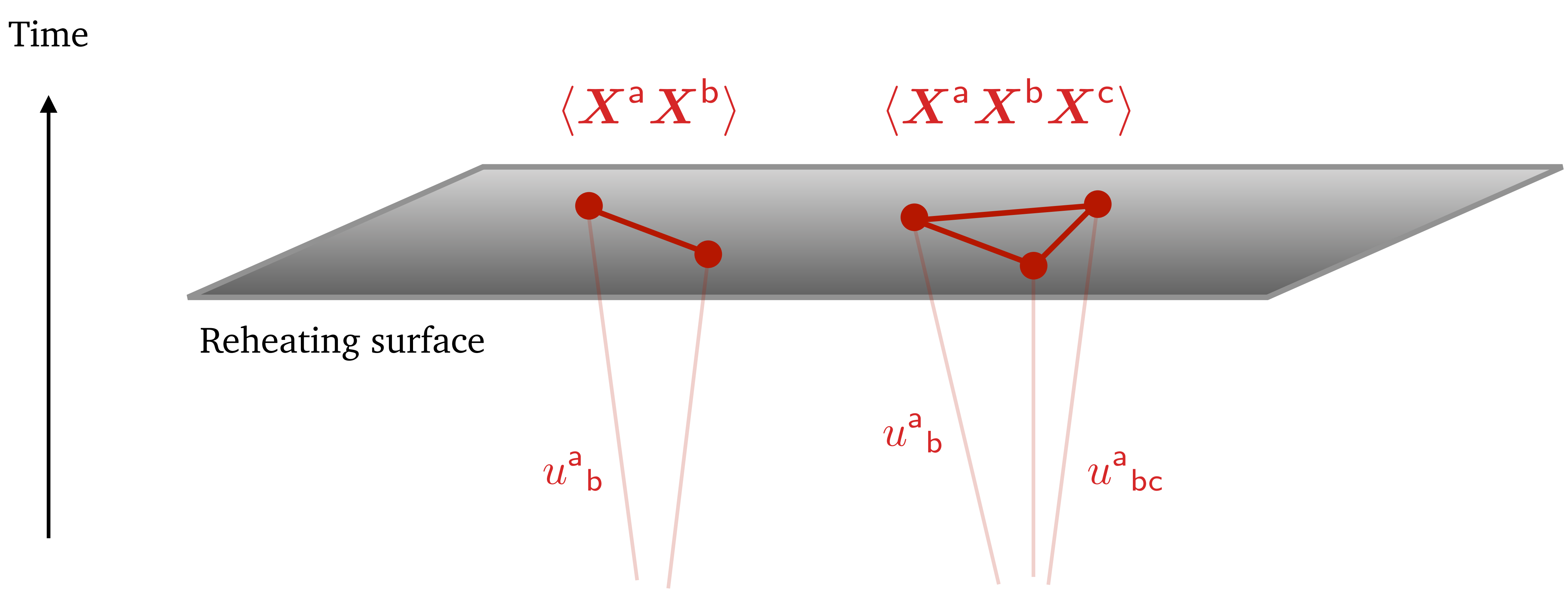
Tree-level Three-point Inflationary Correlators

- The **flow equations** for the three-point correlators are

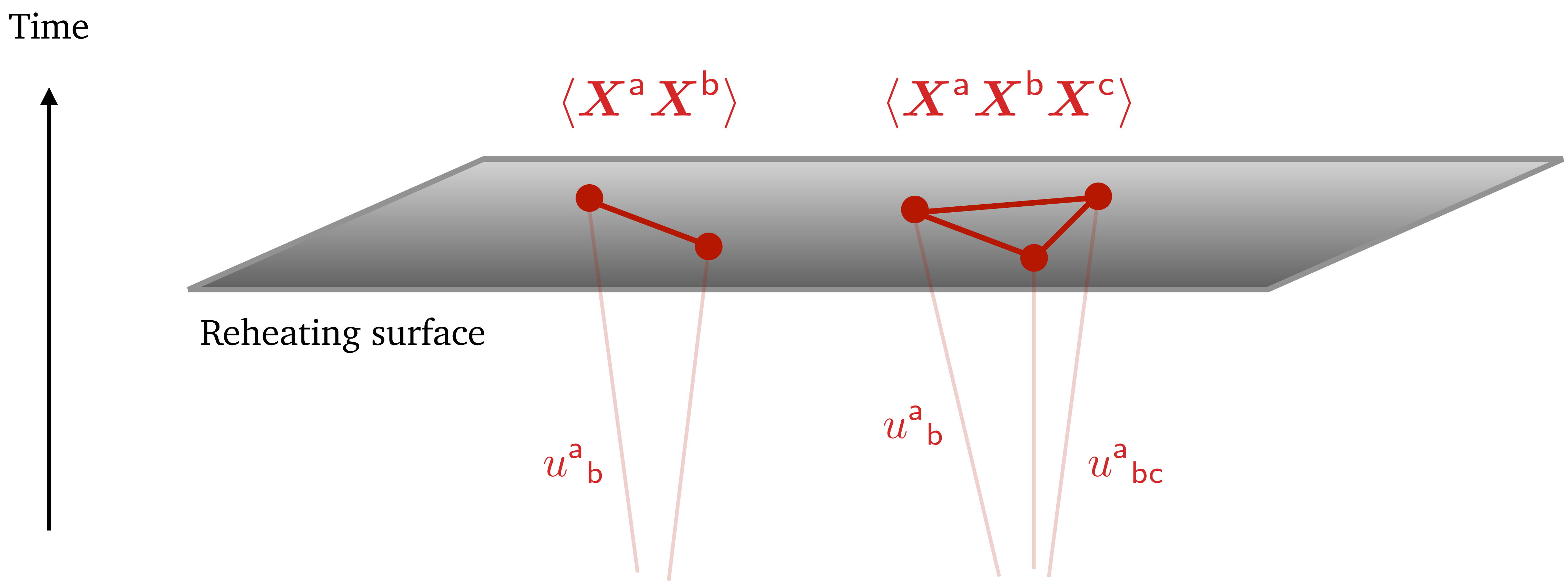
$$\frac{\mathrm{d}}{\mathrm{d}t}\langle \boldsymbol{X}^{\mathsf{a}} \boldsymbol{X}^{\mathsf{b}} \boldsymbol{X}^{\mathsf{c}} \rangle = u^{\mathsf{a}}{}_{\mathsf{d}} \langle \boldsymbol{X}^{\mathsf{d}} \boldsymbol{X}^{\mathsf{b}} \boldsymbol{X}^{\mathsf{c}} \rangle + u^{\mathsf{a}}{}_{\mathsf{de}} \langle \boldsymbol{X}^{\mathsf{b}} \boldsymbol{X}^{\mathsf{d}} \rangle \langle \boldsymbol{X}^{\mathsf{c}} \boldsymbol{X}^{\mathsf{e}} \rangle + (2 \text{ perms})$$

Theory dependence

Non-linear = quantum



Initial Conditions



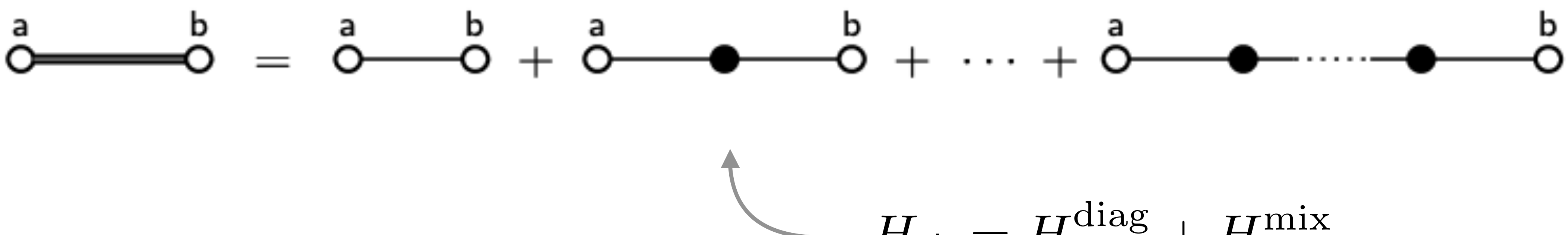
- In the far past, modes do not feel the effect of spacetime curvature
- Set of **uncoupled degrees of freedom**
- Asymptotically reaching the vacuum state and analytical calculations become **tractable**

Resumming Quadratic Mixings

- The flow equations encode an **exact** treatment of quadratic interactions in $H_{\mathsf{ab}}\boldsymbol{X}^{\mathsf{a}}\boldsymbol{X}^{\mathsf{b}}$

$$\langle \boldsymbol{X}^{\mathsf{a}}\boldsymbol{X}^{\mathsf{b}}\rangle = \sum_{n=0}^{\infty} \frac{i^n}{(2!)^n} \int_{-\infty}^{t} \mathrm{d}t' \int_{-\infty}^{t'} \mathrm{d}t'' ... \int_{-\infty}^{t^{(n-1)}} \mathrm{d}t^{(n)}$$

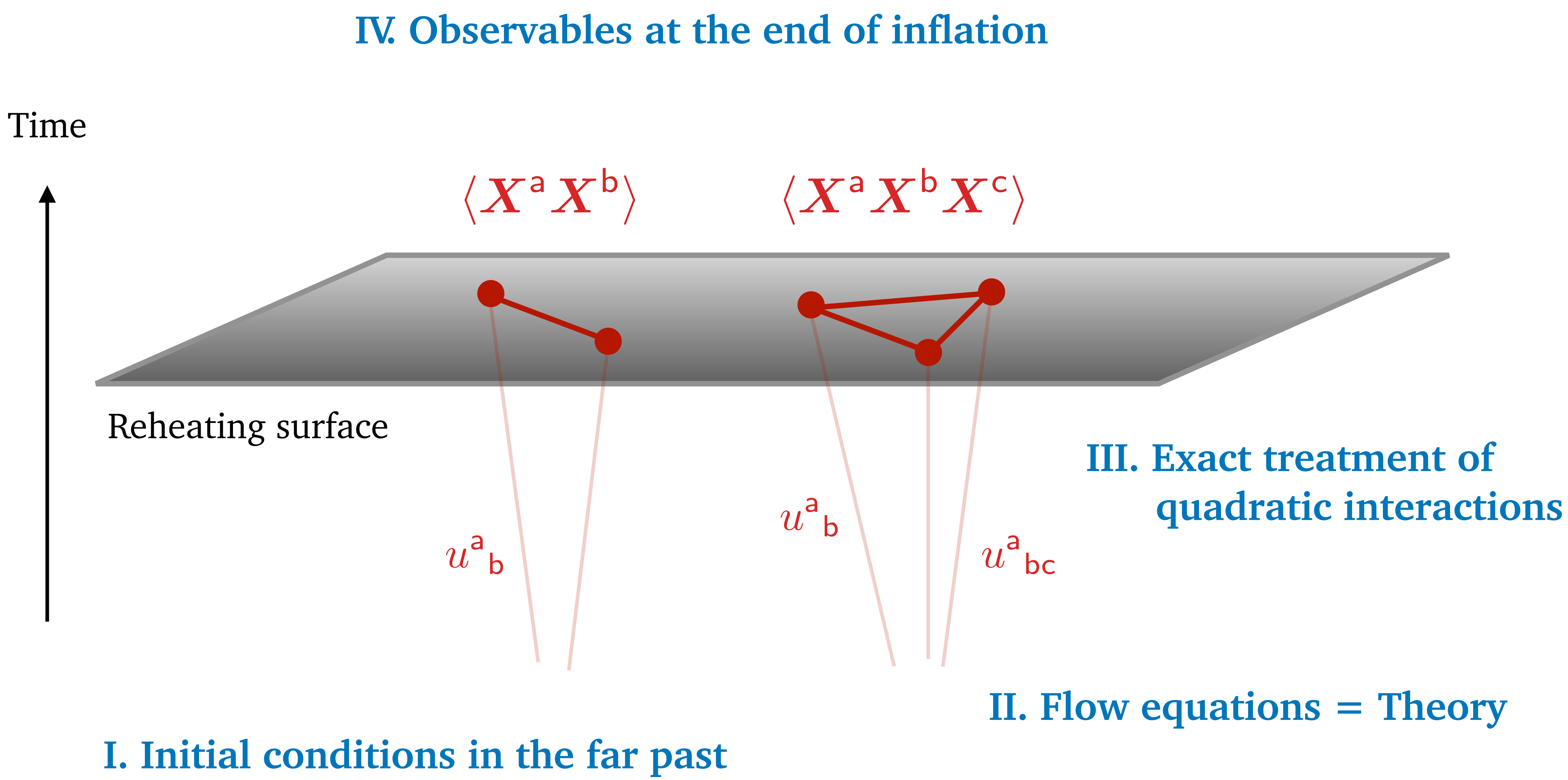
$$\left\langle \left[H^{\text{mix}}_{\mathsf{cd}} \bar{X}^{\mathsf{c}}\bar{X}^{\mathsf{d}}, \left[H^{\text{mix}}_{\mathsf{ef}} \bar{X}^{\mathsf{e}}\bar{X}^{\mathsf{f}}, ..., \left[H^{\text{mix}}_{\mathsf{gh}} \bar{X}^{\mathsf{g}}\bar{X}^{\mathsf{h}}, \bar{X}^{\mathsf{a}}\bar{X}^{\mathsf{b}} \right] ... \right] \right] \right\rangle$$



$$H_{\mathsf{ab}} = H^{\text{diag}}_{\mathsf{ab}} + H^{\text{mix}}_{\mathsf{ab}}$$

- We have converted the problem of computing nested time integrals to **solving a set of coupled differential equations**

Key Ideas of the Cosmological Flow



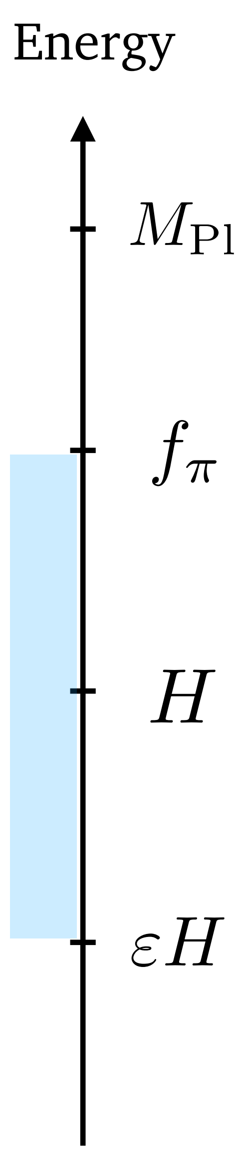
III. Applications

- Goldstone Description of Inflationay Fluctuations
- Cosmological Collider at Strong Mixing
- Cosmological Collider Flow
- Cosmological Collider with Primordial Features

Goldstone Description of Inflationary Fluctuations

Inflation can be described by a process of **spontaneous symmetry breaking**

Preferred space-like foliation spontaneously breaks **time reparametrization invariance**



In the unitary gauge

$$S_\pi = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{1}{2}M_{\rm pl}^2 R + M_{\rm pl}^2\dot{H}g^{00} - M_{\rm pl}^2(3H^2+\dot{H}) + \sum_{n=2}^{\infty}\frac{M_n^4(t)}{n!}\left(\delta g^{00}\right)^n + \dots\right]$$

Action constructed out of all operators invariant under time-dependent spatial diffeomorphisms

$$x^i \to x^i + \xi^i(\boldsymbol{x}, t)$$

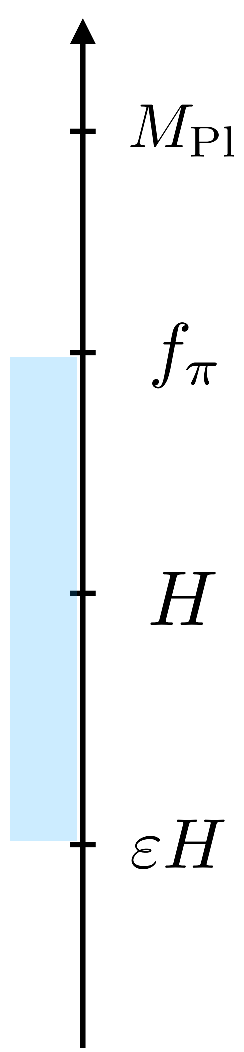
Higher-order derivatives

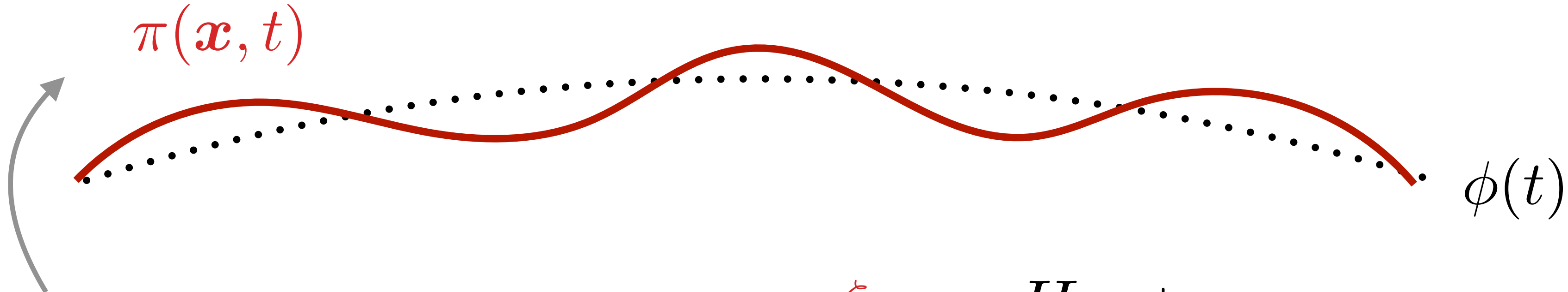
- Stuckelberg trick $t \to t + \pi(\boldsymbol{x}, t)$ $\longrightarrow$ $\delta g^{00} \to -2\dot{\pi} - \dot{\pi}^2 + (\partial_i\pi)^2/a^2$
- Decoupling limit $M_{\rm Pl} \to \infty$ and $\dot{H} \to 0$ while keeping $M_{\rm Pl}^2\dot{H}$ fixed

Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore [2008]

Goldstone Description of Inflationary Fluctuations

The relevant degree of freedom is the **Goldstone boson** associated with the spontaneous breaking of time-translation invariance





Goldstone boson

$$\zeta = -H\pi + \ldots$$

$$\mathcal{L}_\pi / a^3 = \frac{M_{\rm pl}^2 |\dot{H}|}{c_s^2}\left[\dot{\pi}^2 - c_s^2 \frac{(\partial_i \pi)^2}{a^2} + (1 - c_s^2)\left(\dot{\pi}^3 - \dot{\pi}\frac{(\partial_i \pi)^2}{a^2}\right) - \frac{4}{3} M_3^4 \frac{c_s^2}{M_{\rm pl}^2 |\dot{H}|}\dot{\pi}^3\right]$$

Non-linearly realised symmetry

Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore [2008]

Goldstone Boson coupled to an Additional Field

We couple the **Goldstone boson** to an additional **massive scalar field**

π quadratic sector

Canonically normalised

σ quadratic sector

quadratic mixing

$$\mathcal{L}/a^3 = -\frac{1}{2}\left[-\dot{\pi}_c{}^2 + c_s^2\frac{(\partial_i\pi_c)^2}{a^2}\right] - \frac{1}{2}\left[(\partial_\mu\sigma)^2 + m^2\sigma^2\right] + \rho\dot{\pi}_c\sigma$$

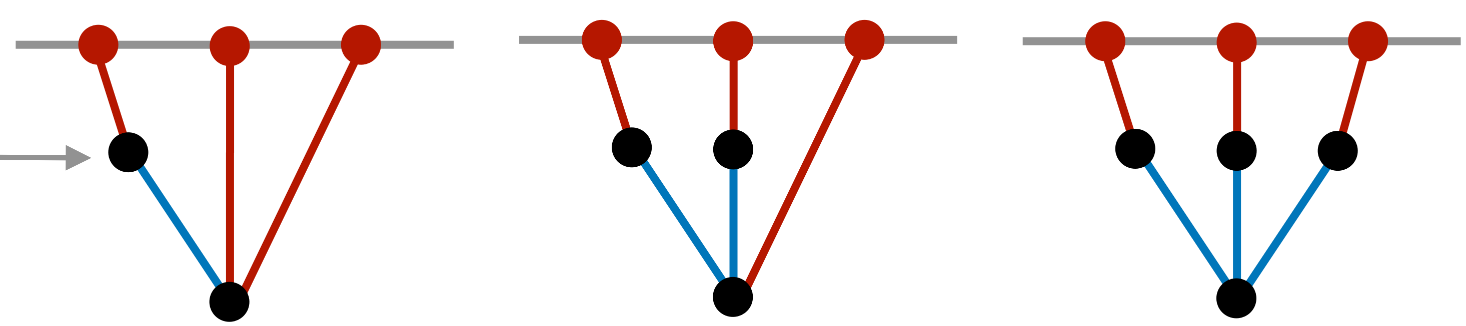
$$-\lambda_1\dot{\pi}_c\frac{(\partial_i\pi_c)^2}{a^2} - \lambda_2\dot{\pi}_c{}^3 - \mu\sigma^3 - \frac{1}{2}\alpha\dot{\pi}_c\sigma^2 - \frac{1}{2\Lambda_1}\frac{(\partial_i\pi_c)^2}{a^2}\sigma - \frac{1}{2\Lambda_2}\dot{\pi}_c{}^2\sigma$$

Self-interactions

Non-linearly realised symmetry

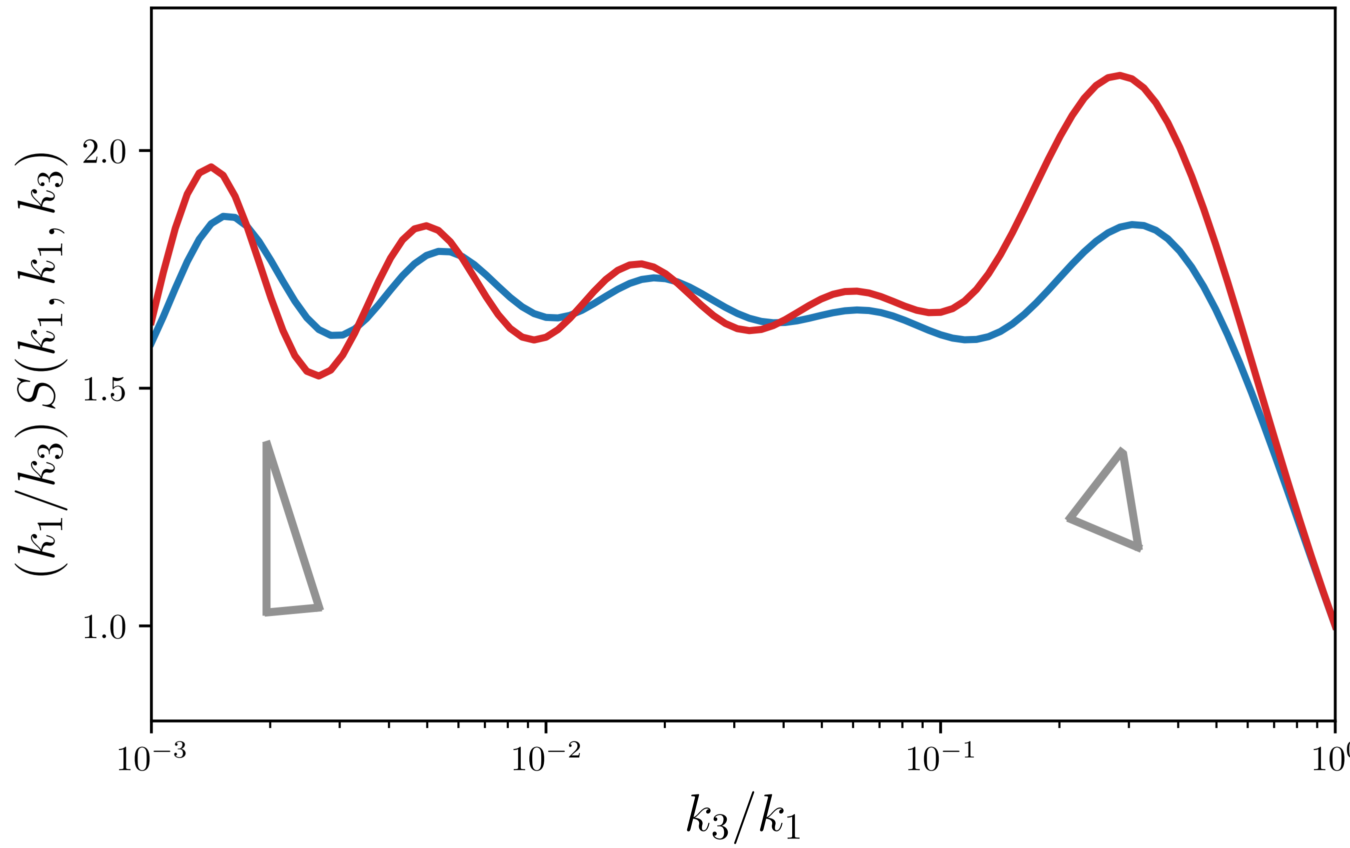
$H/\Lambda_1 \propto \rho/H$

Assuming weak mixing



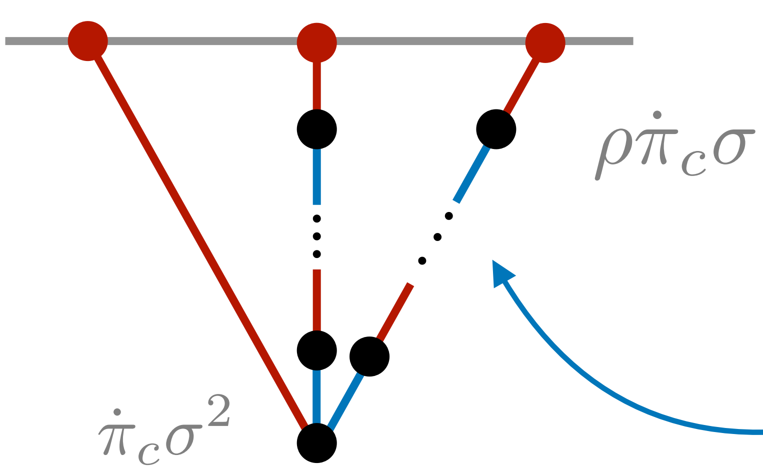
Cosmological Collider Signal at Strong Mixing

The cosmological collider signal of **heavy but weakly mixed** particle oscillates at the same frequency than that of a **light but strongly mixed** particle



Frequency

$$\mu_{\rm eff}^2 = m_{\rm eff}^2/H^2 - 9/4$$

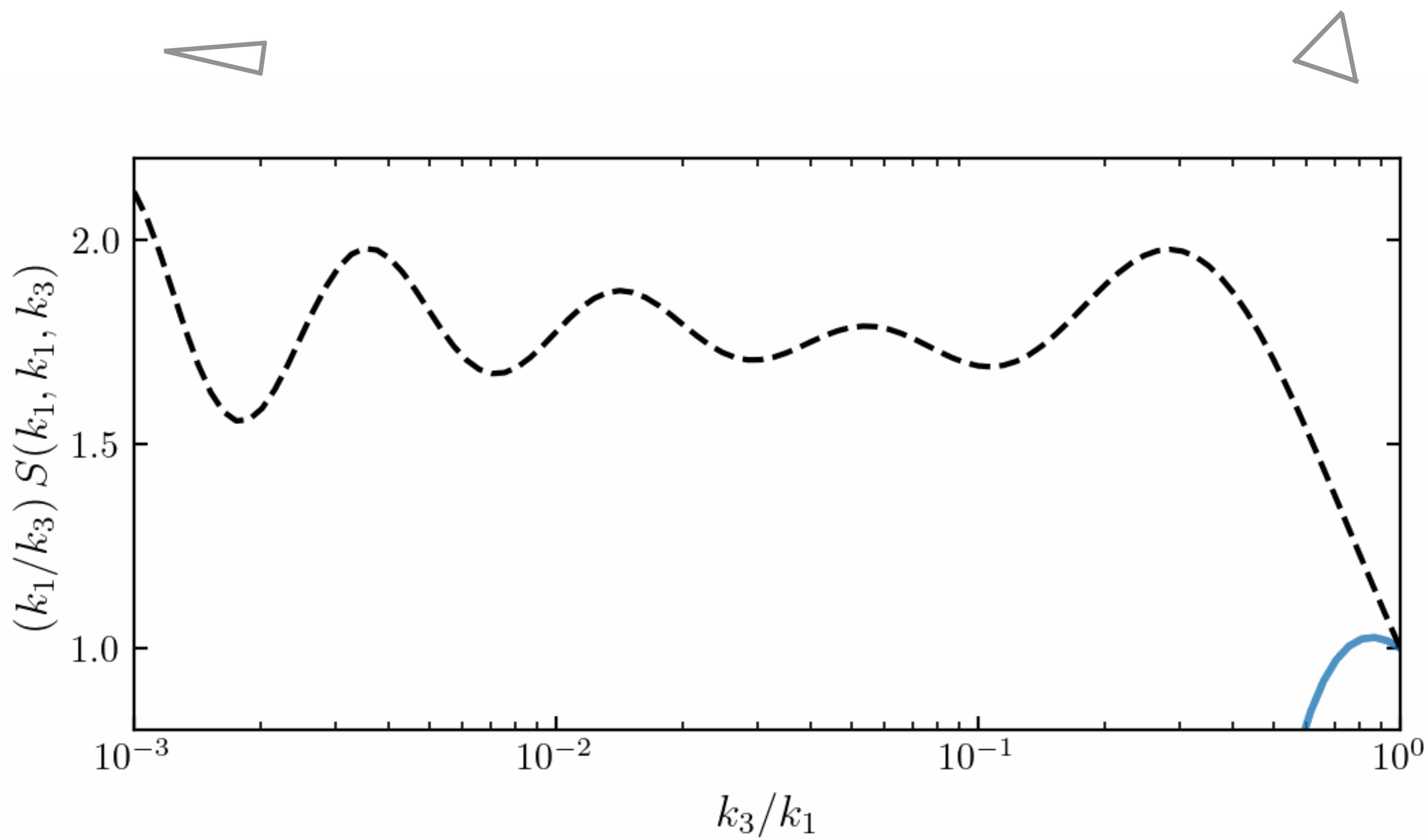


Effective mass for the heavy field

$$m^2 \to m_{\rm eff}^2 = m^2 + \rho^2$$

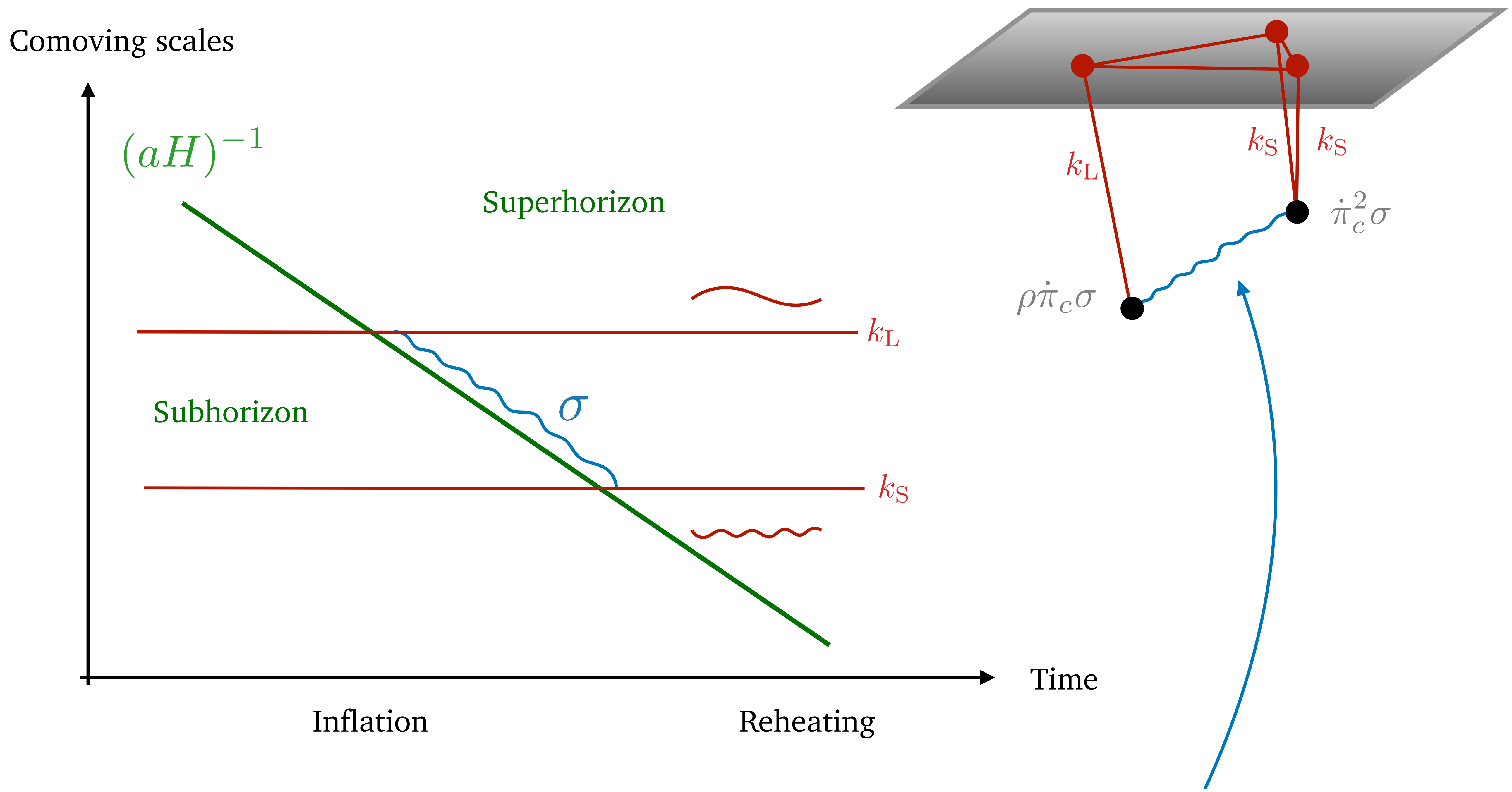
Resummation of quadratic mixings

Cosmological Collider Flow



Cosmological Collider Flow

The Cosmological Flow enables us to shed light on **characteristic time scales** at play



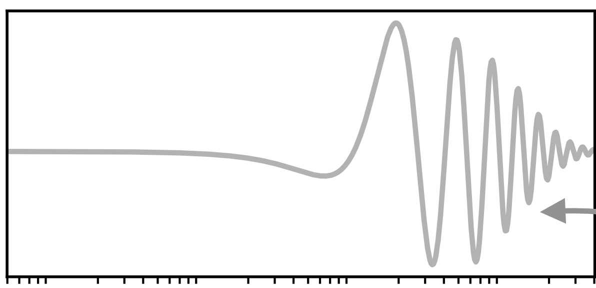
The cosmological collider signal probes the **superhorizon time evolution** of σ

Primordial Features

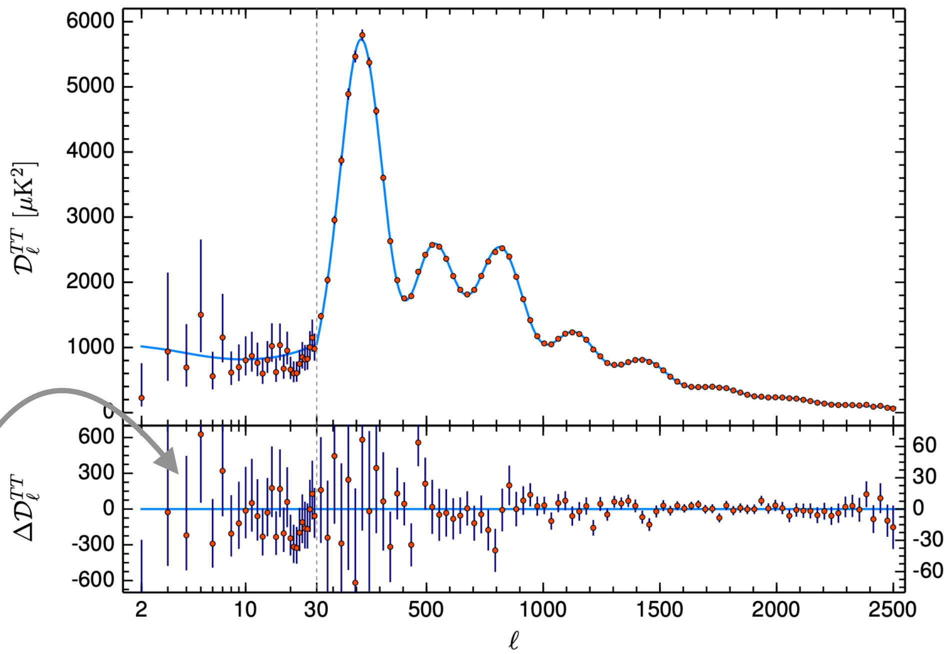
Features arise when the couplings are **time-dependent**

Planck [2018]

$\Delta_\zeta^2(k)$ or $f_{\rm NL}(k)$



Features = breaking of scale-invariance



Observationally motivated

Time-dependent quadratic mixing oscillating at the frequency μ_c

$$\rho(t) \to \rho(t+\pi) \approx \rho(t) + \pi\dot{\rho}(t)$$

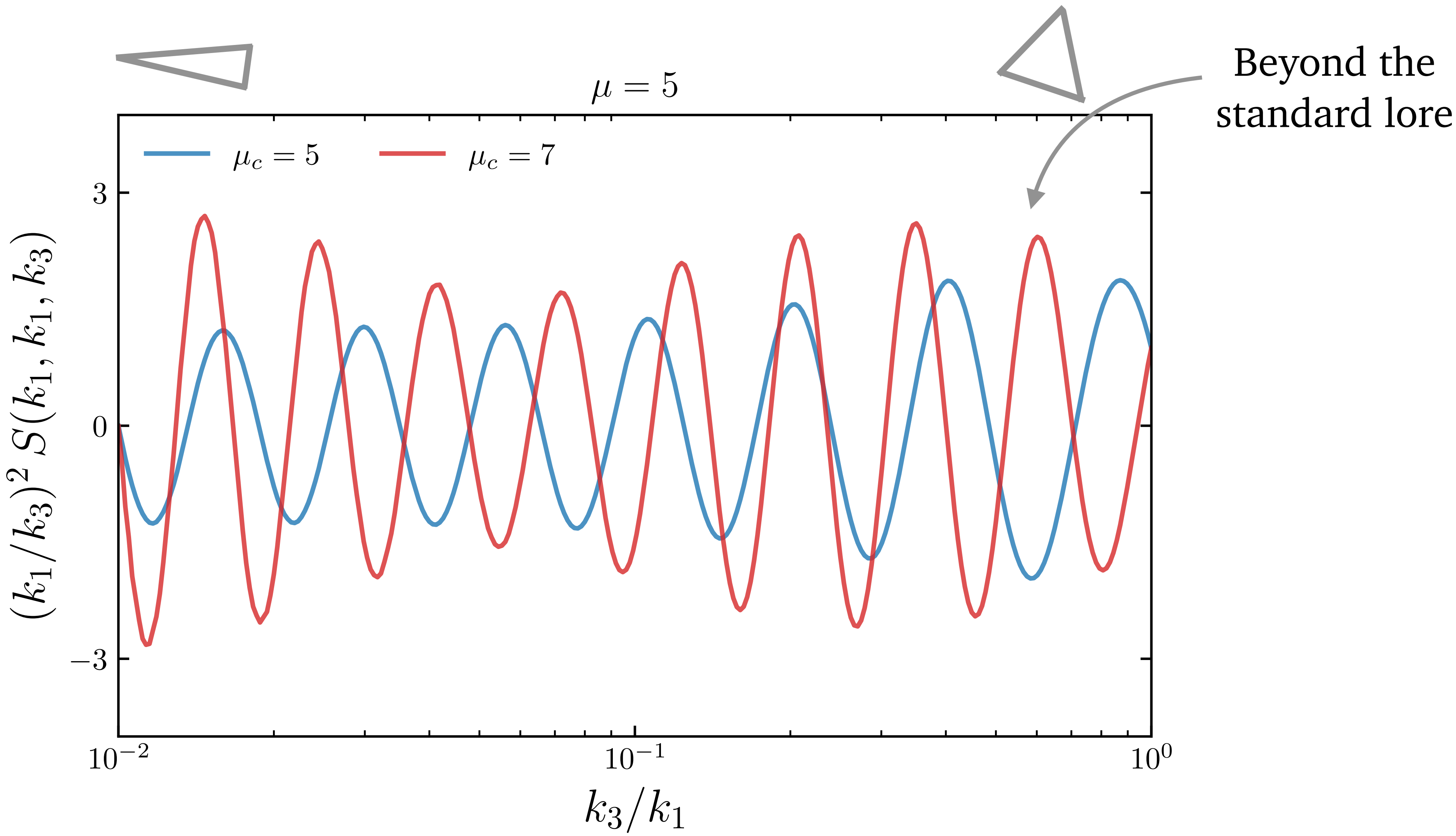
Non-linearly realised symmetry

Mass parameter μ

$$\mathcal{L}_{\pi-\sigma}/a^3 = \rho(t)\dot{\pi}_c\sigma + \frac{\rho(t)}{2f_\pi^2}(\partial_\mu\pi_c)^2\sigma + \frac{\dot{\rho}(t)}{f_\pi^2}\pi_c\dot{\pi}_c\sigma$$

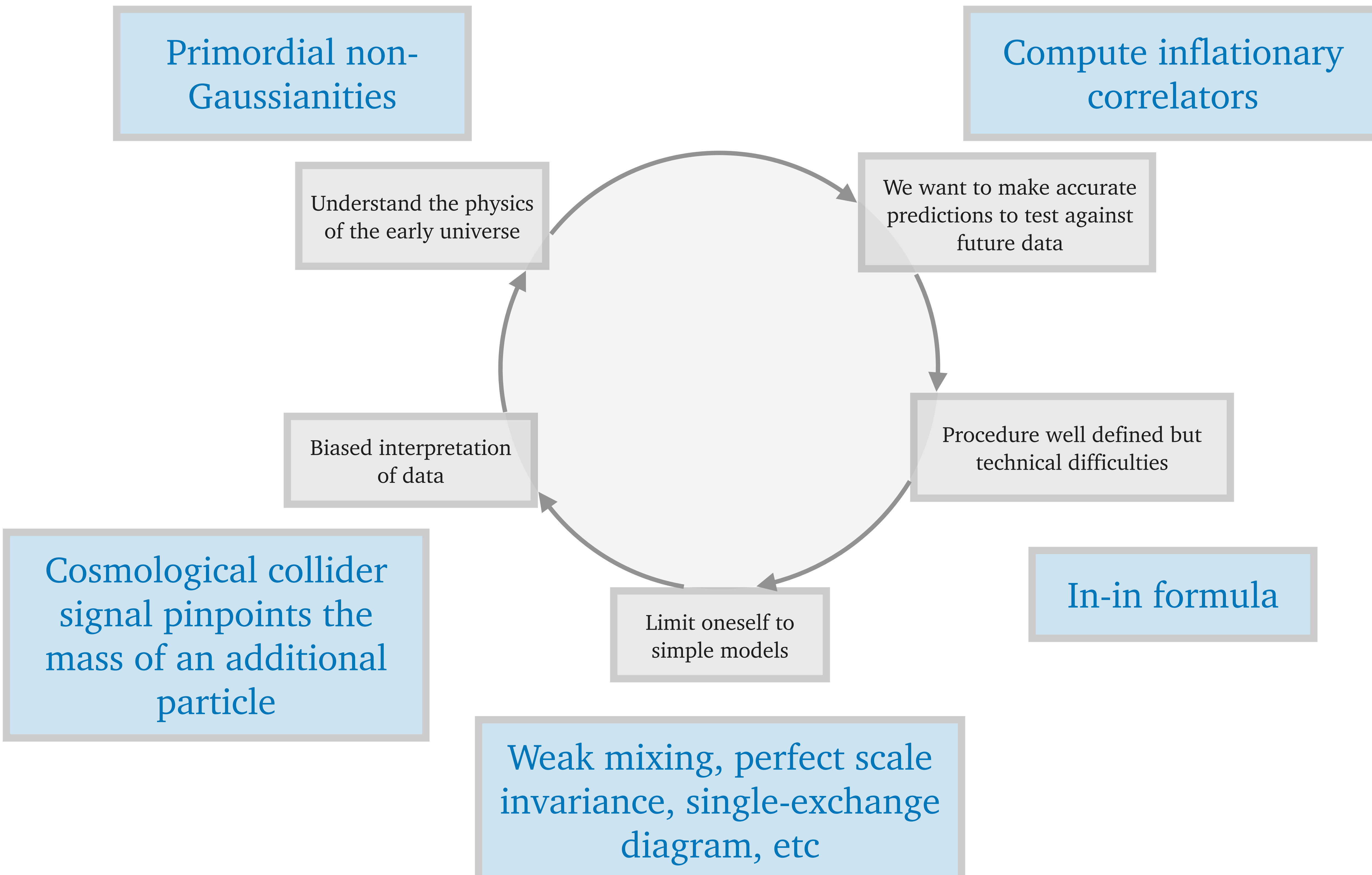
Cosmological Collider Signals with Primordial Features

The presence of features breaks the link between the **mass of the exchanged particle** and the **frequency** of the cosmological collider signal



$$\lim_{k_3/k_1 \to 0} S(k_1, k_1, k_3) \sim \left(\frac{k_3}{k_1}\right)^2 \left[\mathcal{A}_+ \cos\left((\mu + \mu_c)\log\left(\frac{k_3}{k_1}\right) + \delta_+\right) + \mathcal{A}_- \cos\left((\mu - \mu_c)\log\left(\frac{k_3}{k_1}\right) + \delta_-\right)\right]$$

Standard Lore before the Cosmological Flow



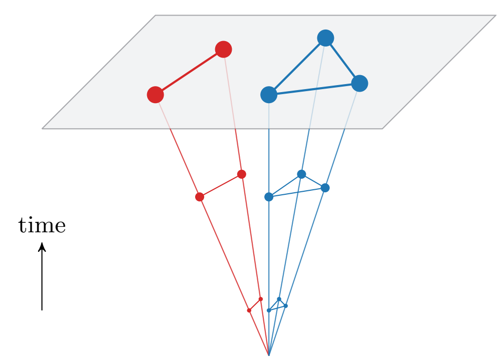
The Cosmological Flow Era

Primordial non-Gaussianities

Compute inflationary correlators

Understand the physics of the early universe

We want to make accurate predictions to test against future data



Procedure well defined but technical difficulties

Limit oneself to simple models

Biased interpretation of data

Classify signals without being limited by technical difficulties

The Cosmological Flow

Any theory: # dofs, couplings, time-dependence, sound speeds, masses, etc

Conclusions

Primordial **non-Gaussianities** to understand the **physics of inflation**, primary target for future missions

Cosmological Collider: probe the laws of physics at the highest reachable energies

The Cosmological Flow

Concentrating on **exploring** and **understanding** the physics in motivated scenarios **in full generality**

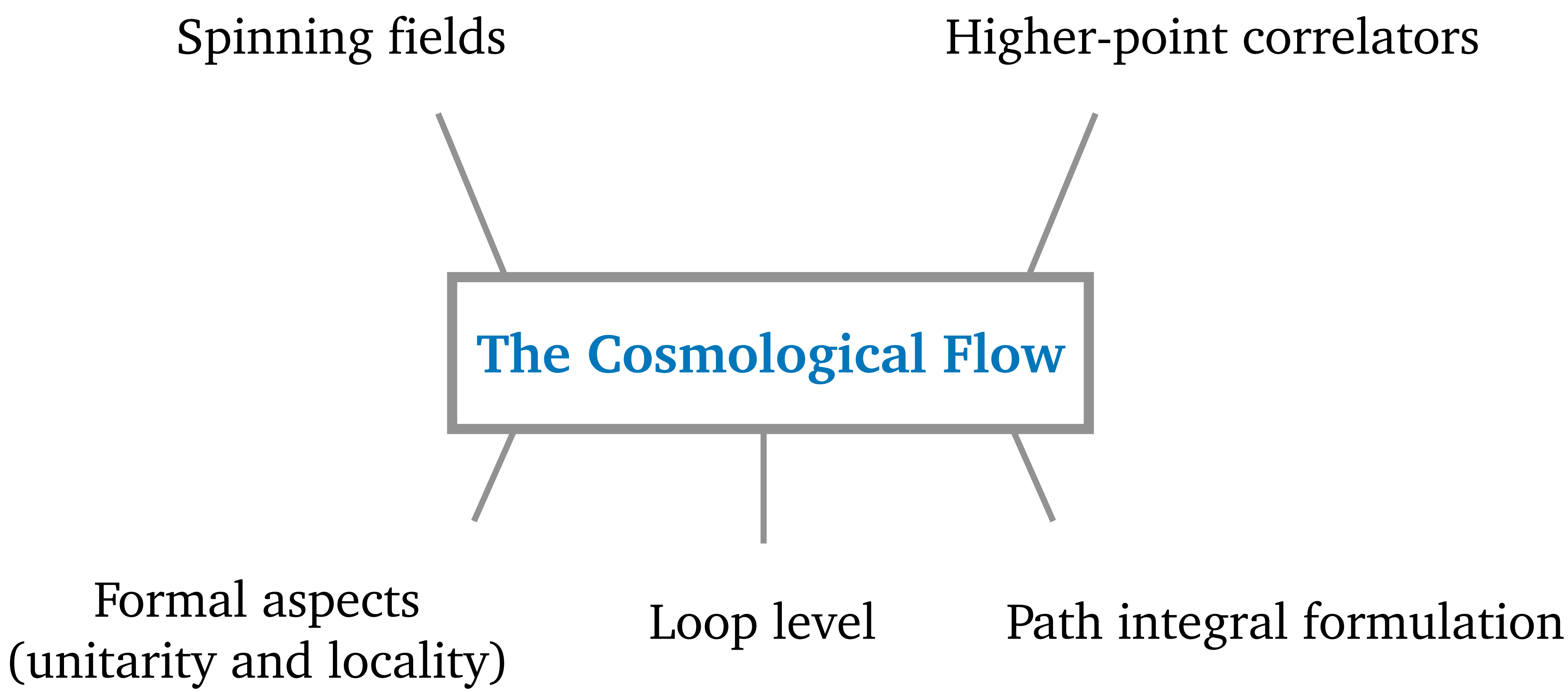
Efficient and **systematic** approach to compute inflationary correlators, avoiding technical difficulties

We have only scratched the tip of the iceberg …

Thank you

Outlook

The cosmological flow offers straight **extensions**



We have paved the way to a systematic investigation of the rich and fascinating subject of inflationary correlators

Effective Field Theory of Inflationary Fluctuations

The relevant degree of freedom is the **Goldstone boson** associated with the spontaneous breaking of time-translation invariance

In the unitary gauge

$$S_\pi = \int d^4x \sqrt{-g} \left[\frac{1}{2} M_{\rm pl}^2 R + M_{\rm pl}^2 \dot{H} g^{00} - M_{\rm pl}^2 (3H^2 + \dot{H}) + \sum_{n=2}^{\infty} \frac{M_n^4(t)}{n!} \left(\delta g^{00}\right)^n + \dots \right]$$

Action constructed out of all operators invariant under time-dependent spatial diffeomorphisms

$$x^i \to x^i + \xi^i(\boldsymbol{x}, t)$$

Higher-order derivatives

- Stuckelberg trick

$$t \to t + \pi(\boldsymbol{x}, t) \quad \longrightarrow \quad \delta g^{00} \to -2\dot{\pi} - \dot{\pi}^2 + (\partial_i \pi)^2 / a^2$$

- Decoupling limit

$M_{\rm Pl} \to \infty$ and $\dot{H} \to 0$ while keeping $M_{\rm Pl}^2 \dot{H}$ fixed

Primordial Features: Effective Single-Field Description

When the field σ is sufficiently massive, we can **integrate it out** and obtain an effective single-field theory for π

- Solve for the linear equation of motion

$$(-\Box + m^2)\sigma = \rho\dot{\pi}_c \longrightarrow \sigma = \frac{\rho}{-\Box + m^2}\dot{\pi}_c \approx \frac{\rho}{m^2}\left(1 - \frac{\Box}{m^2} + \dots\right)\dot{\pi}_c$$

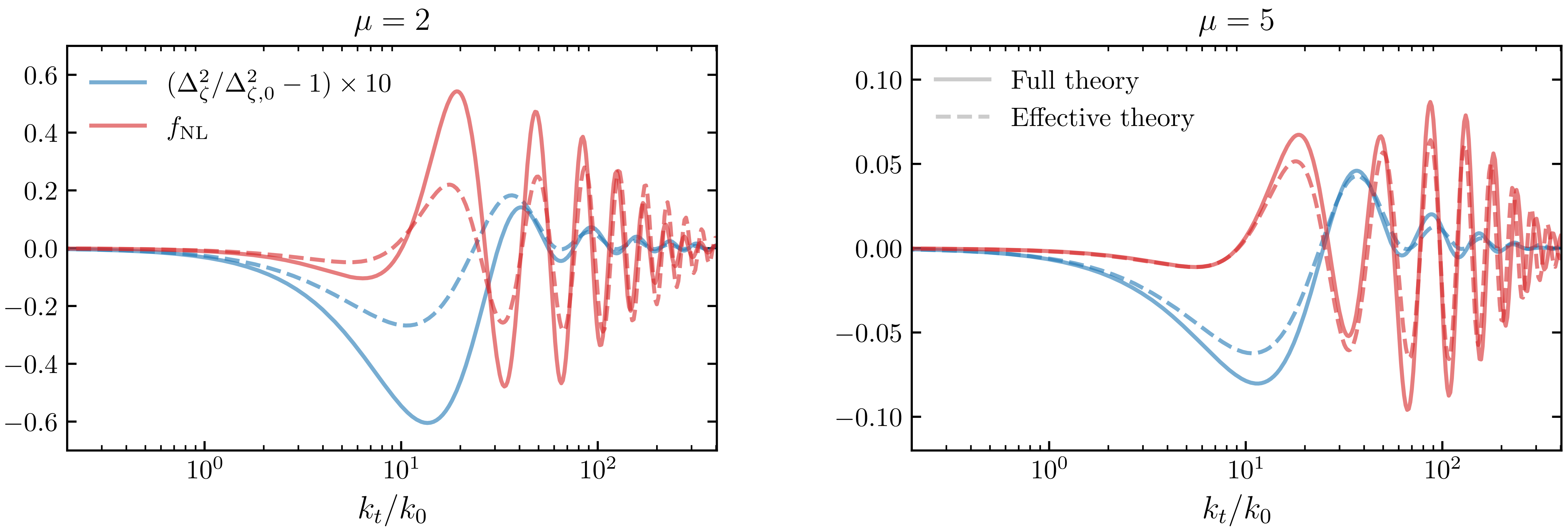
- Reduced speed of sound

$$\mathcal{L}/a^3 = \frac{1}{2\tilde{c}_s^2(t)}\dot{\pi}_c^2 - \frac{1}{2}\frac{(\partial_i\pi_c)^2}{a^2} + \frac{1}{2f_\pi^2}\left(\frac{1}{\tilde{c}_s^2(t)} - 1\right)\dot{\pi}_c(\partial_\mu\pi_c)^2 - \frac{\dot{\tilde{c}}_s(t)}{f_\pi^2\tilde{c}_s(t)}\pi_c\dot{\pi}_c^2$$

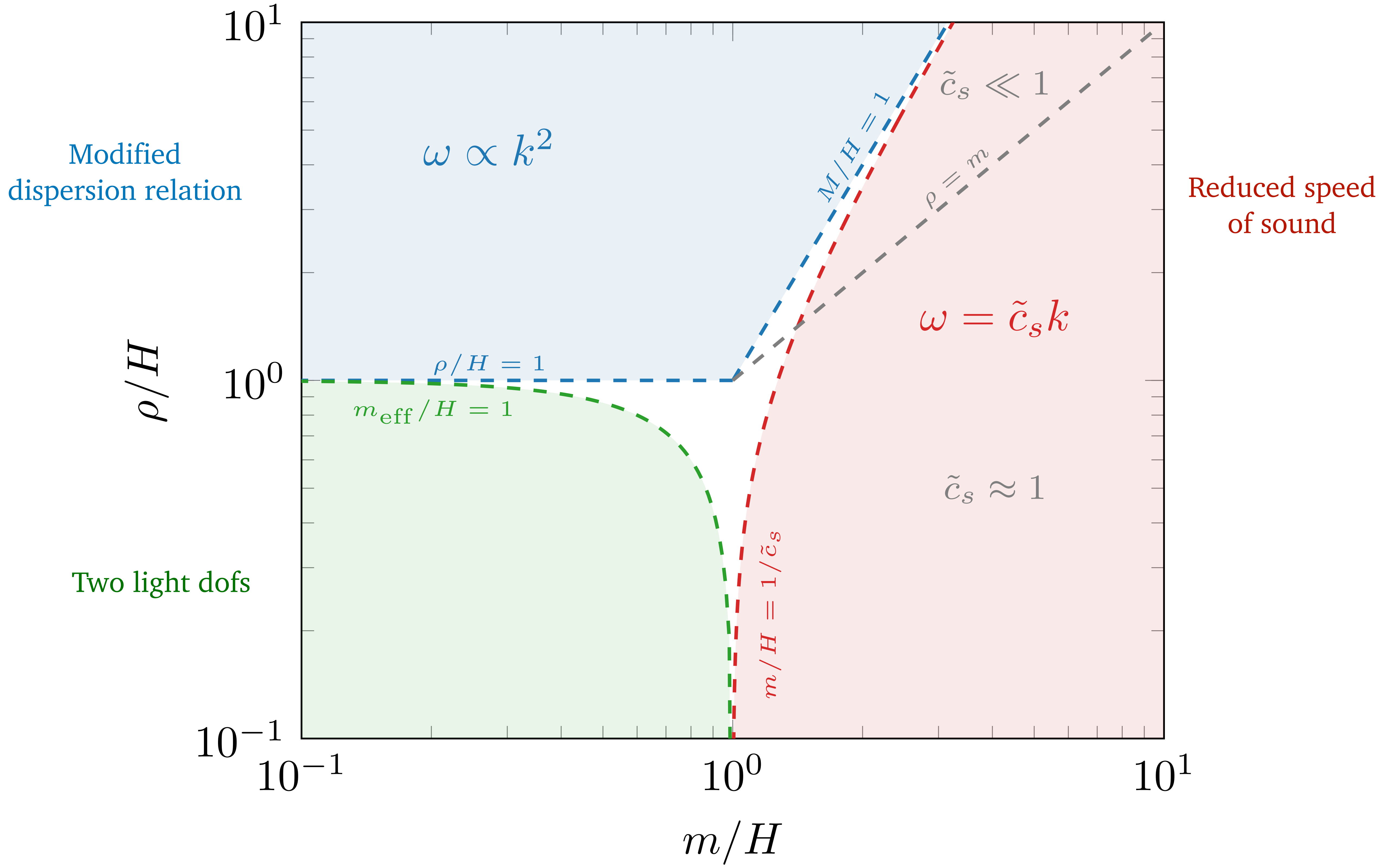
$$\text{with} \quad \tilde{c}_s^{-2}(t) = 1 + \frac{\rho^2}{m^2}$$

Primordial Features: Effective Single-Field Description

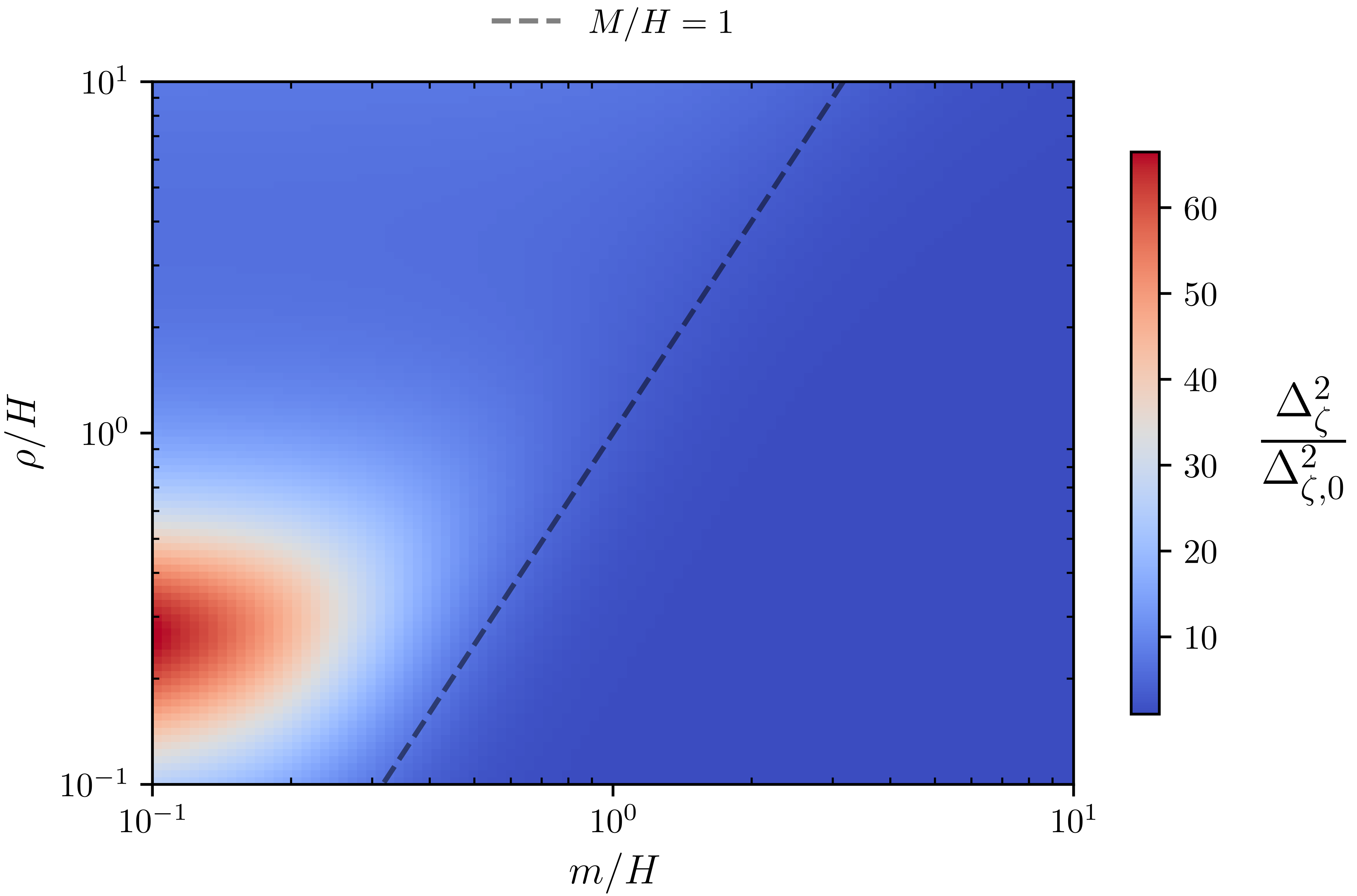
The effective single-field theory gives **wrong predictions**



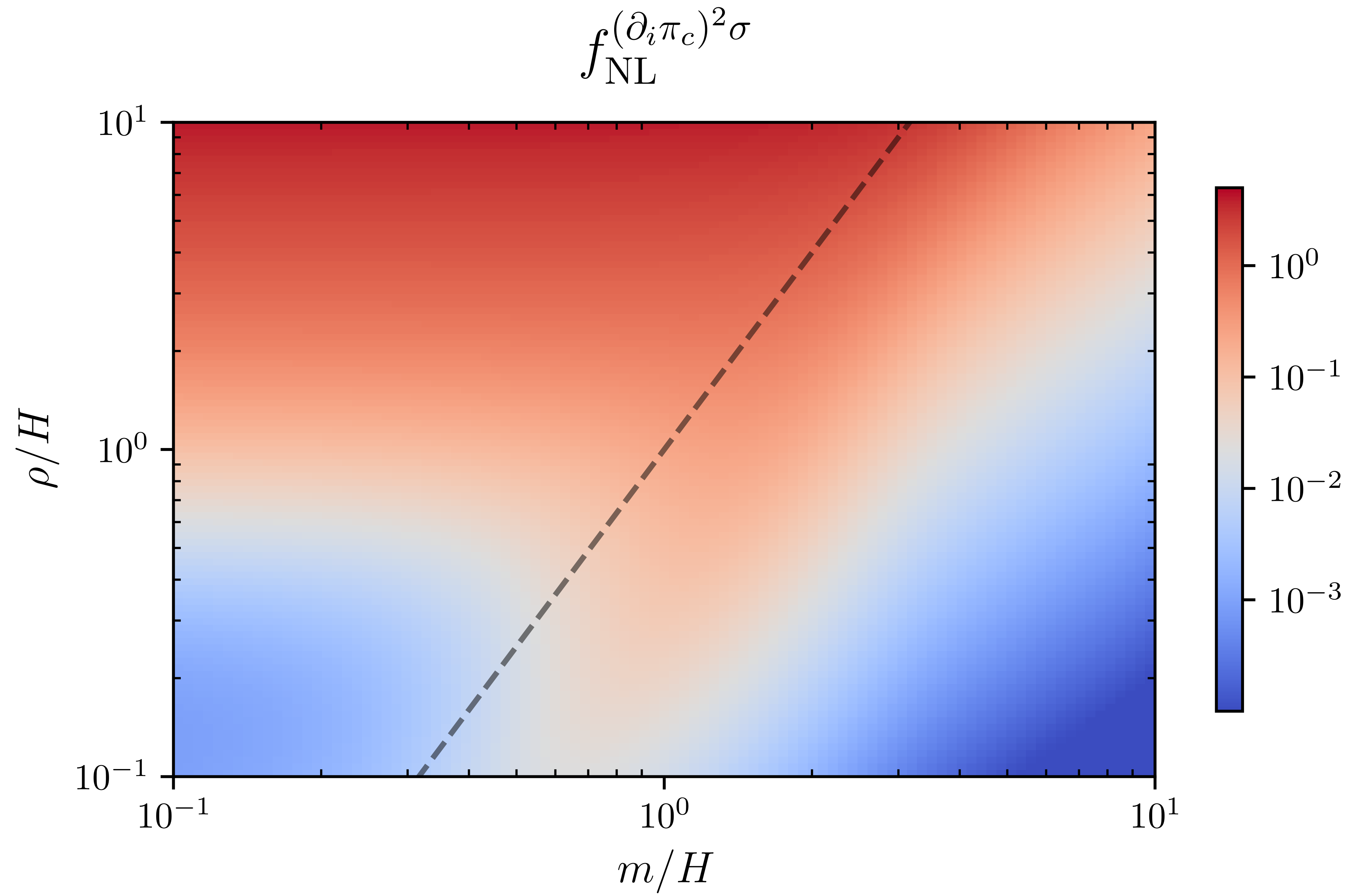
Phase Diagram of the $\pi - \sigma$ Model



Quadratic Theory Phase Diagram



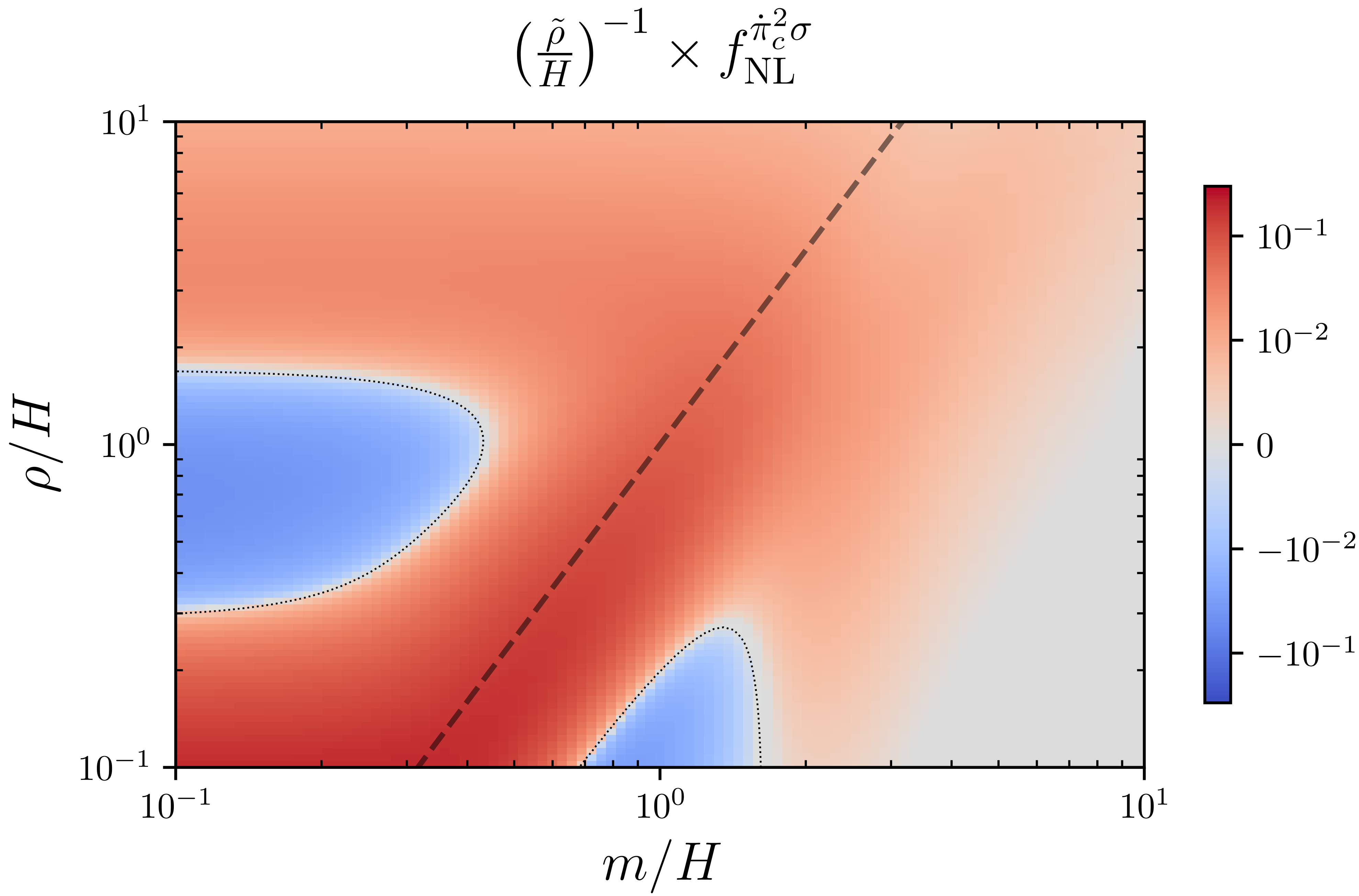
Single-Exchange Diagram Phase Diagram



Weak mixing : $\rho/H \lesssim c_s^{-1/2}$

Strong mixing : $\rho/H \lesssim c_s^{3/4}\dfrac{\kappa^{1/2}}{\Delta_\zeta}$

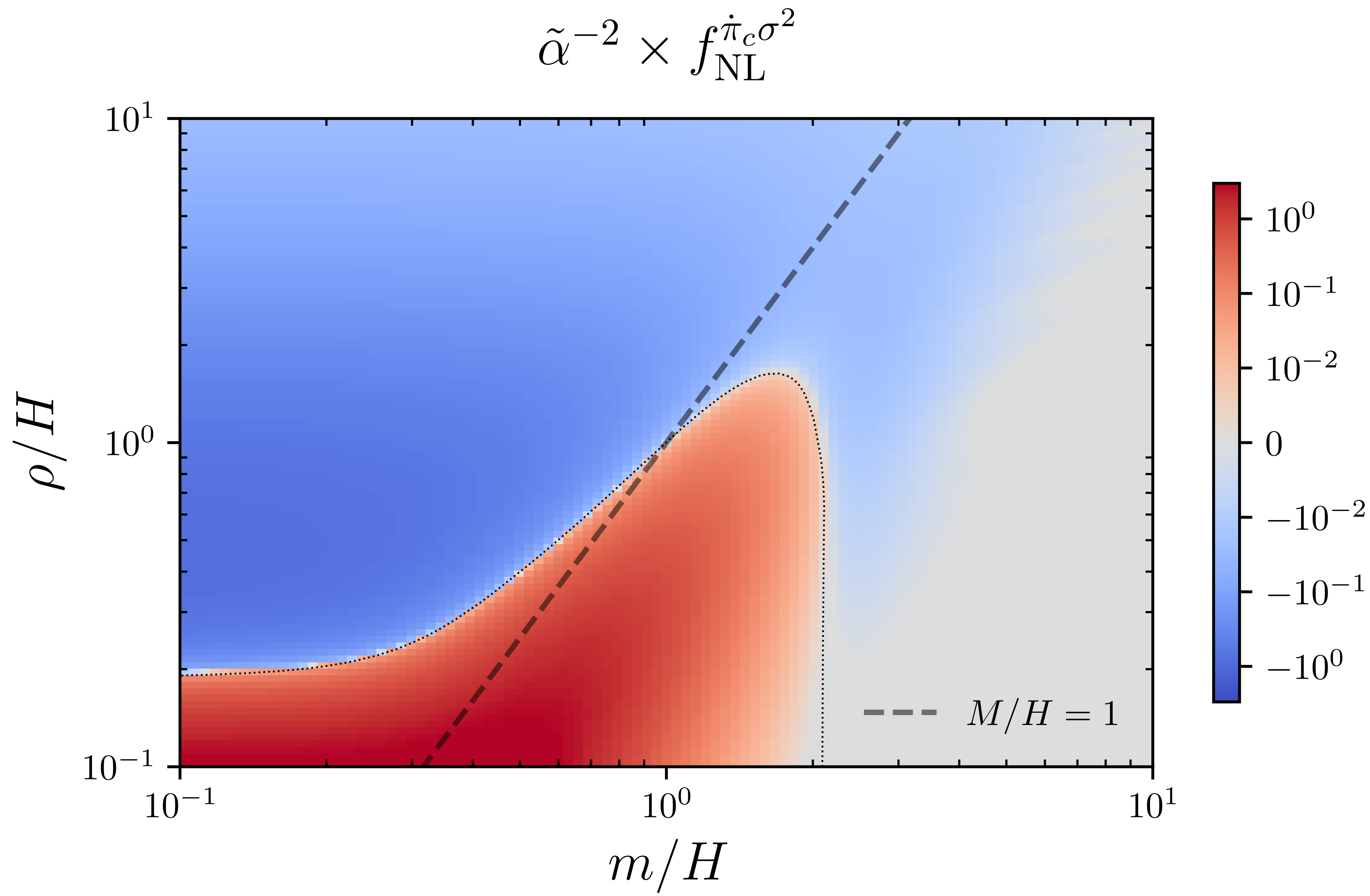
Single-Exchange Diagram Phase Diagram



Weak mixing : $\tilde{\rho}/H \lesssim \dfrac{c_s^{-1/2}}{2\pi\Delta_\zeta}$

Strong mixing : $\tilde{\rho}/H \lesssim \dfrac{\rho}{H}\dfrac{\kappa^{1/2}}{c_s^{1/4}\Delta_\zeta}$

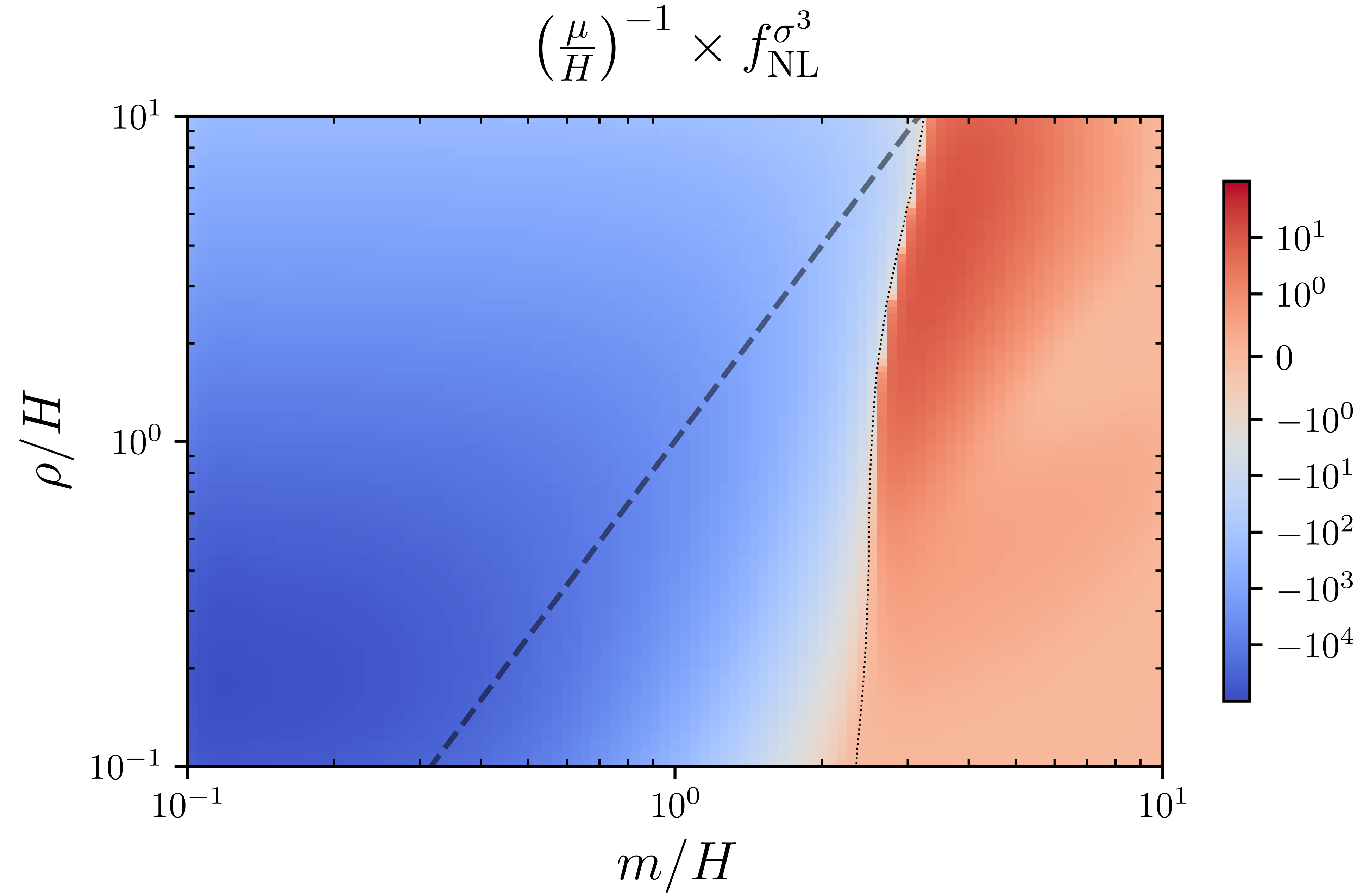
Double-Exchange Diagram Phase Diagram



Weak mixing : $\tilde{\alpha} \lesssim \frac{c_s^{-1/2}}{2} \frac{1}{(2\pi\Delta_\zeta)^{1/2}}$

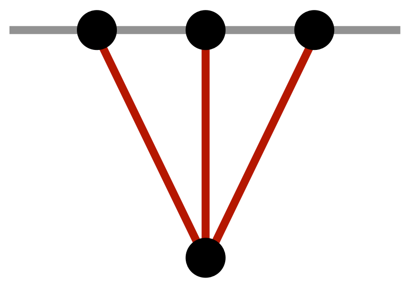
Strong mixing : $\tilde{\alpha} \lesssim \left(\frac{\rho \Delta_\zeta}{16 c_s^{5/2} H \kappa}\right)^{1/4}$

Triple-Exchange Diagram Phase Diagram



Weak mixing : $\mu/H \lesssim 1$ Strong mixing : $\mu/H \lesssim c_s^{-3/4} (\rho/H)^{3/4}$

Numerical Challenges and Developments



$$\langle \phi^3 \rangle(\tau) \sim \int_{-\infty}^{\tau} \mathrm{d}\tau' \, \tau'^n g(\tau') e^{iK(\tau' - \tau)}$$

Direct Calculations (not systematic)

- Wick rotation [Chen and Wang 2010]
- Numerical mode functions [Assassi et al. 2013]
- Holder summation [Junaid et al. 2015]
- Cesaro/Riesz summation [Tran et al. 2022]
- ...

Indirect Calculations

$$\frac{\mathrm{d}}{\mathrm{d}\tau} \langle \phi^3 \rangle = g - iK \langle \phi^3 \rangle$$

- Translate the problem of computing Feynman-type integrals to **solving differential equations in time**

Systematic framework to study inflationary correlators :
the **transport approach**

[Dias, Fazer, Mulryne and Seery 2016]

Codes Available for Inflationary Calculations

Two-point function solvers:

- FieldInf
- ModeCode & MultiModeCode
- PyFlation

Our code:

- Decouple from a specific background
- EFT at the level of the fluctuations

Three-point function solvers:

- BINGO (single-field inflation)

Transport approach:

- CppTransport
- PyTransport

Ringeval, Brax, van de Bruck, Davis, Martin [2006]
Price, Frazer, Xu, Peiris, Easther [2015]
Huston, Malik [2009, 2011]
Hazra, Sriramkumar, Martin [2013]
Dias, Fazer, Seery [2015]
Mulryne [2016]