Induced Gravitational Waves and Matter-Dominated Era

Takahiro Terada (KMI, Nagoya University), seminar at IAP, Nov. 4, 2024.

Outline

- Introduction
- Aspects of Induced Gravitational Waves (Review)
- Induced Gravitational Waves and Matter-Dominated Era
- Conclusions

Introduction:

Gravitational Waves to Probe the Early Universe

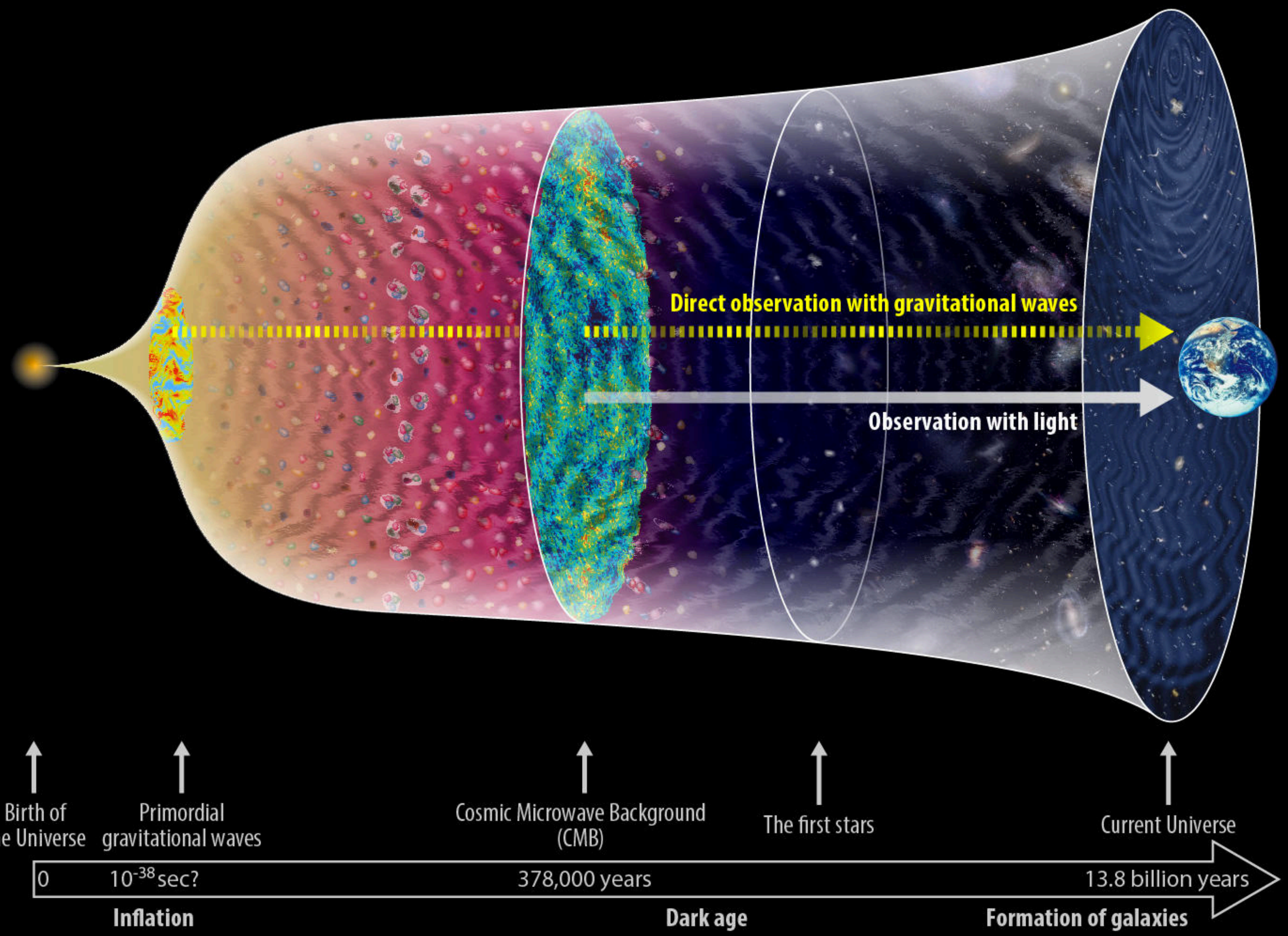
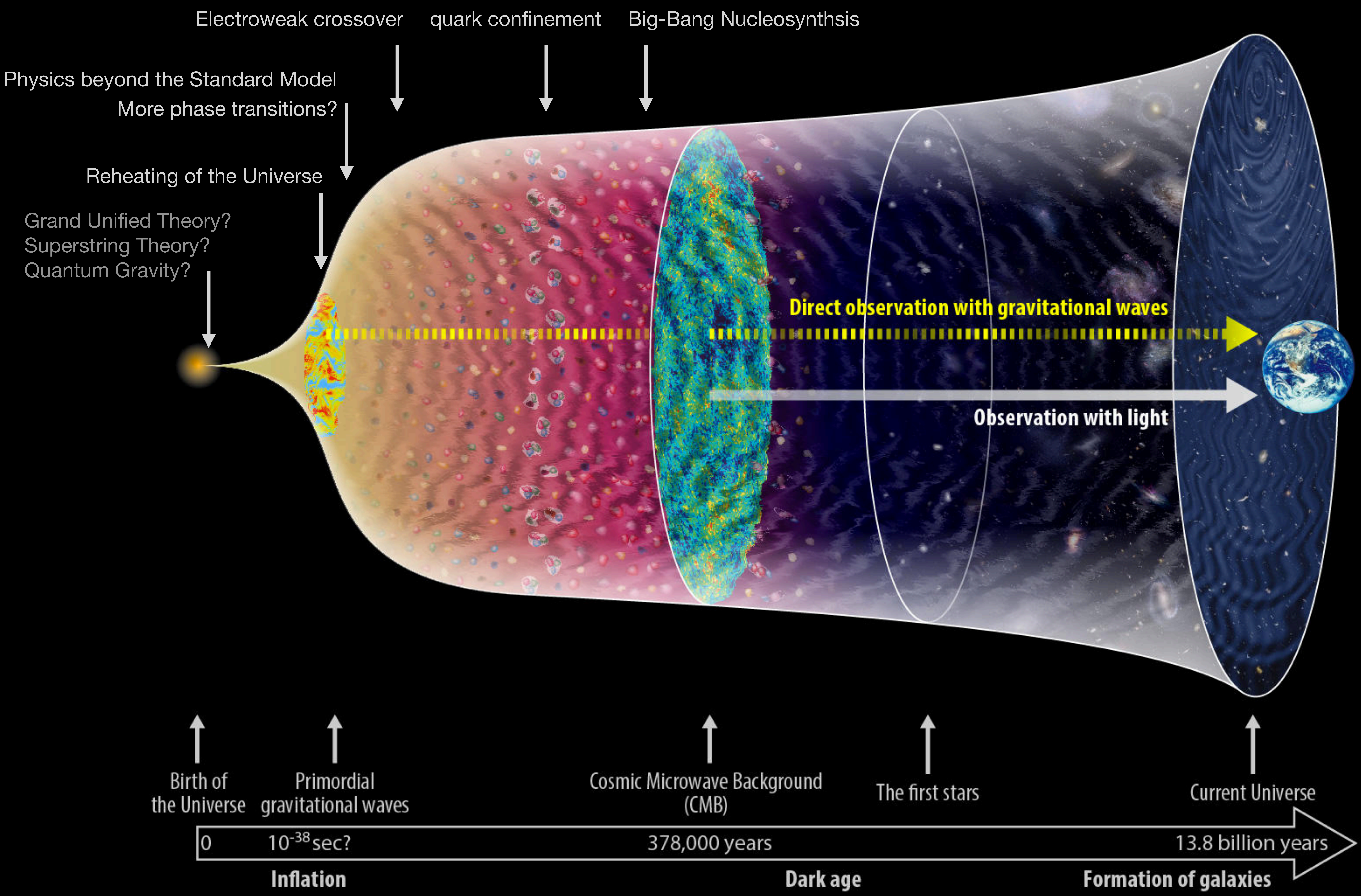


Image credit: NAOJ



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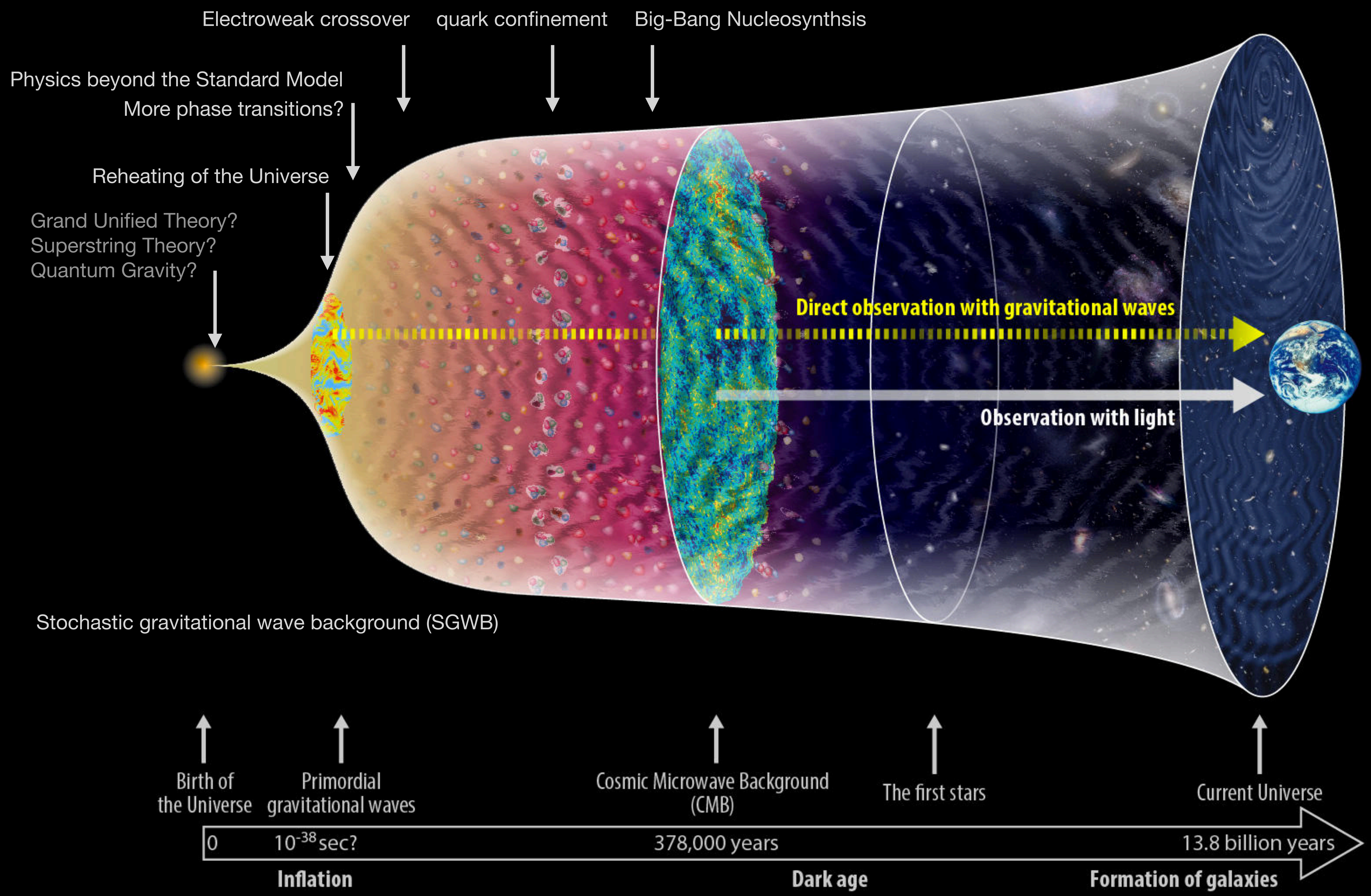

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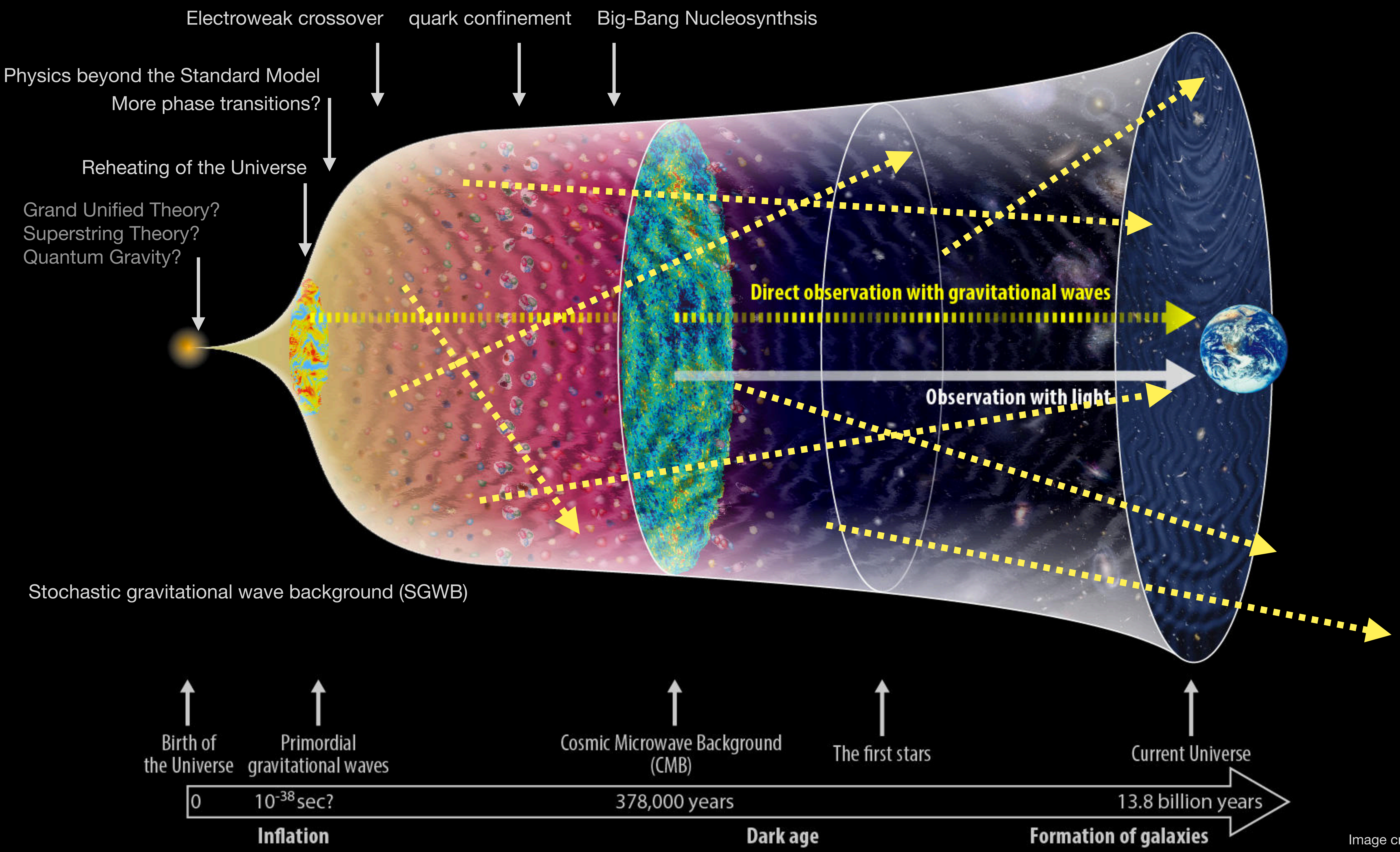

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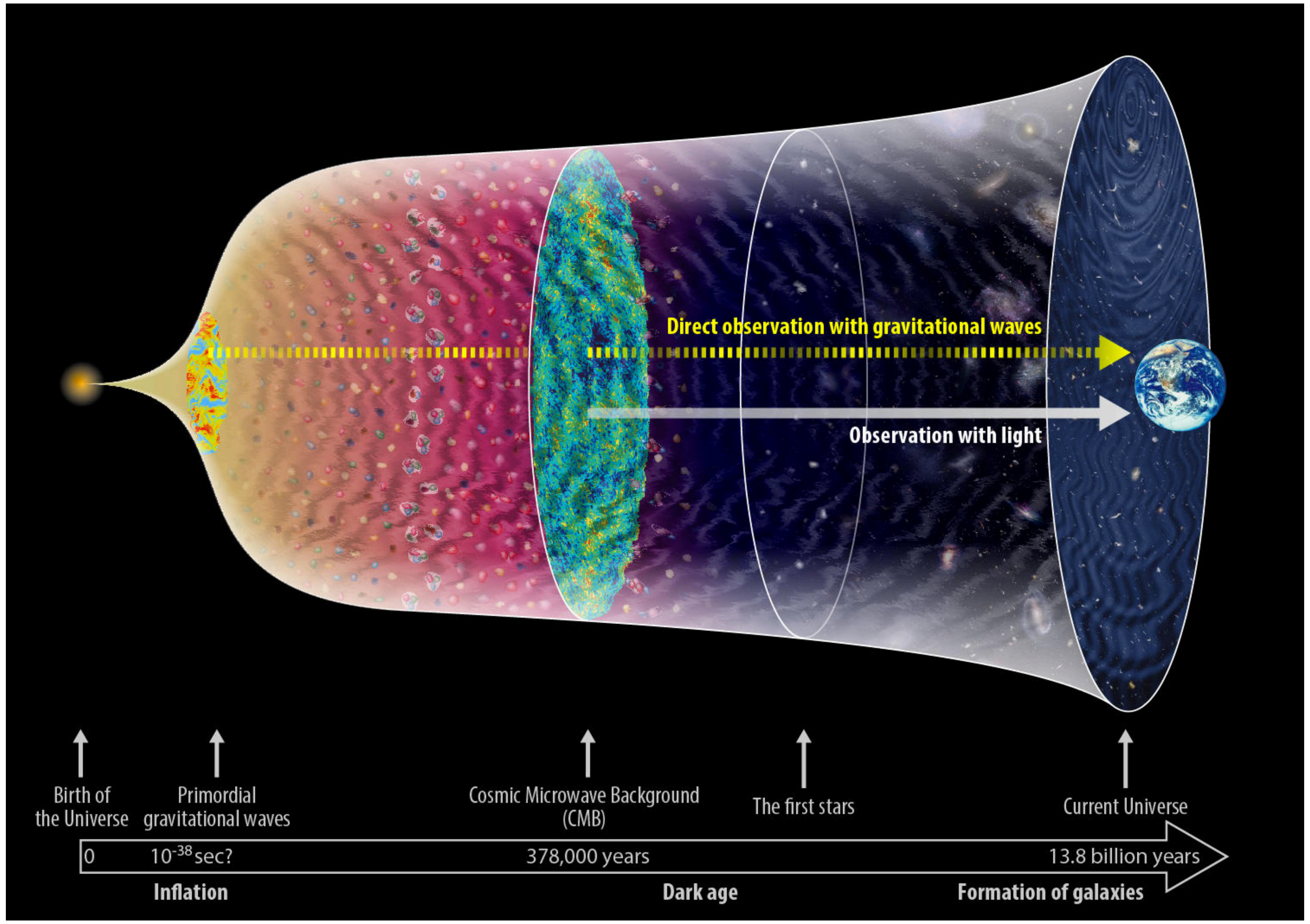


NAOJ

GW sources in the late Universe

- Mergers of compact stars (e.g., black holes)
- Mergers of galaxies

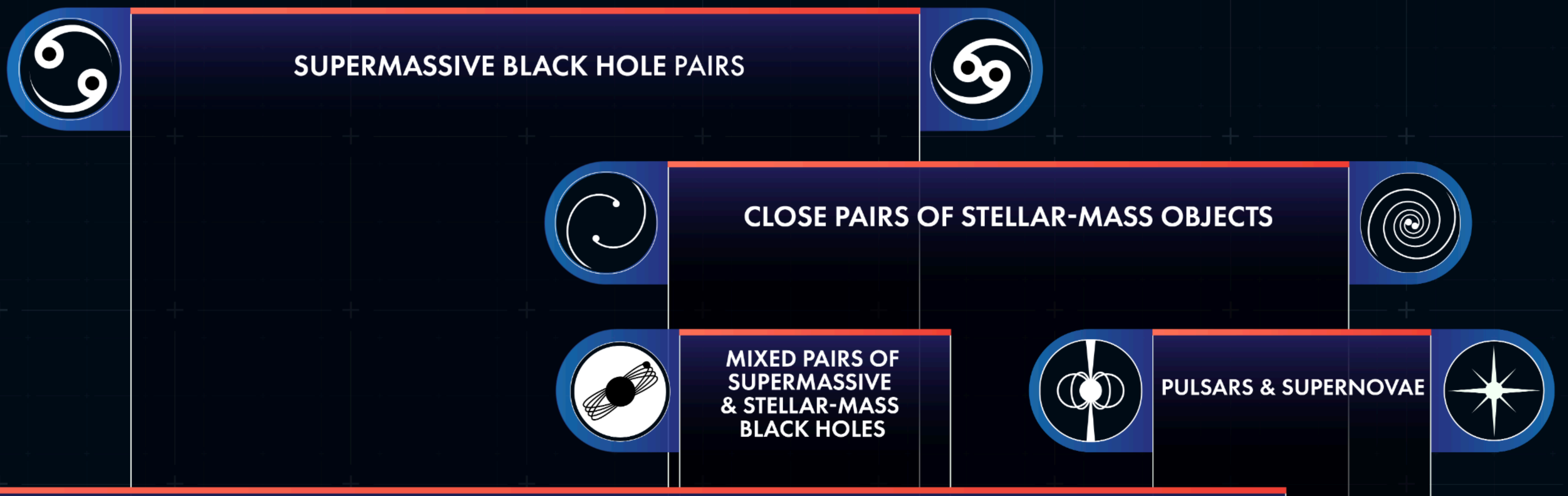
GW sources in the early Universe

- Inflation
- Large curvature perturbations
- Phase transitions
- Cosmic strings & domain walls
- Hawking radiation
- …

THE GRAVITATIONAL WAVE SPECTRUM

Gravitational waves are ripples in space-time traveling at light speed. They're created when massive objects accelerate. Different phenomena produce ripples with wavelengths ranging from a few miles to larger than the observable universe. The general range of waves from some sources are shown here. Merging objects emit ever shorter wavelengths as they spiral inward. Pairs of stellar-mass objects include combinations of black holes, neutron stars, and white dwarfs.

Scientists need different detectors to explore these wavelengths, from human-made facilities on the ground and in space to galaxy-sized pulsar timing arrays – sets of rapidly rotating neutron stars monitored for changes. Details in the cosmic microwave background (CMB), the oldest light in the universe, can reveal gravitational waves generated less than a trillionth trillionth of a second after the big bang.



Credit: NASA's Goddard Space Flight Center Conceptual Image Lab

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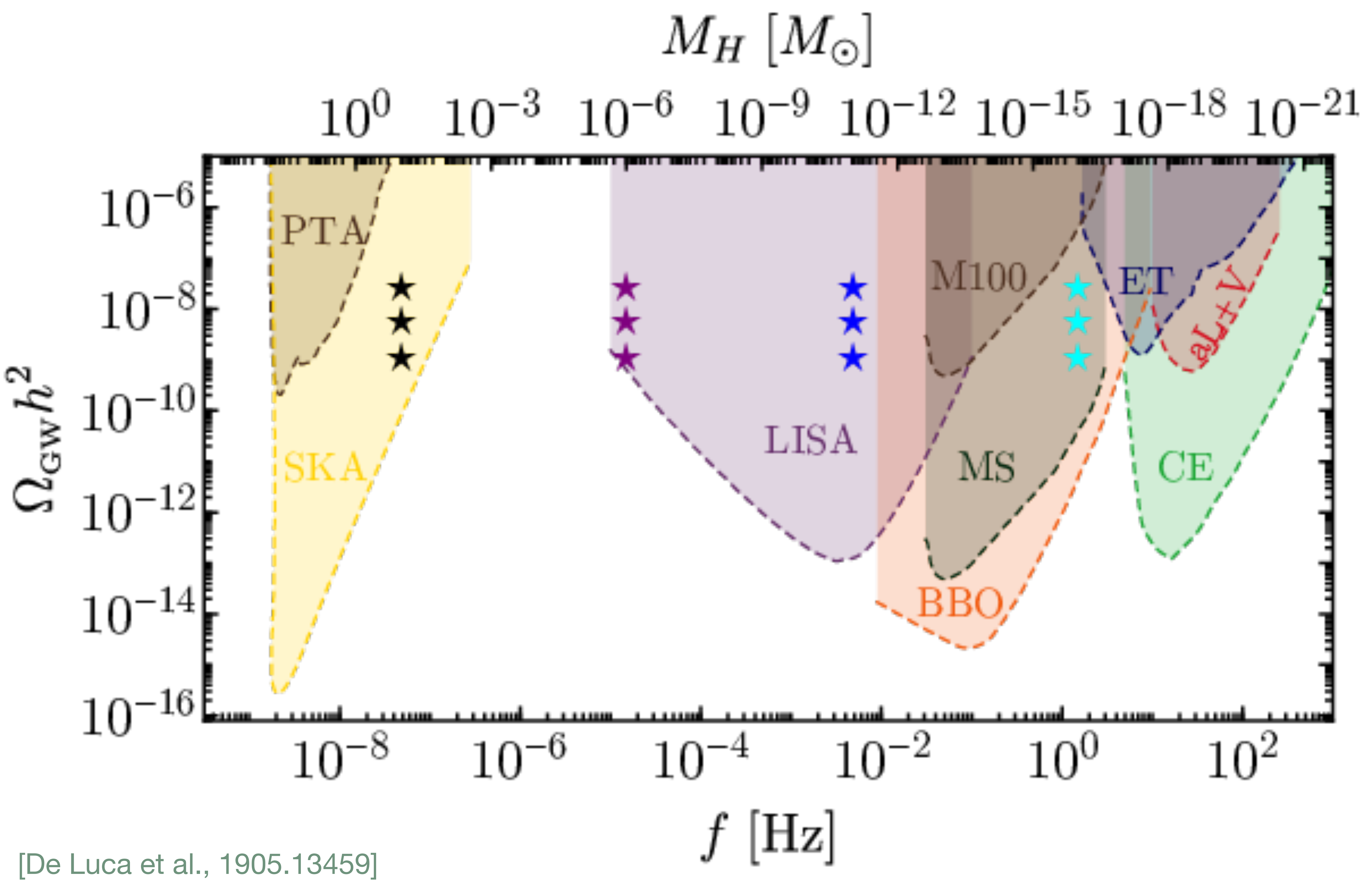


[De Luca et al., 1905.13459]

BIG BANG

VIRGO SUPERCLUSTER SIZE
1.15 SEXTILLION MILES

TO ANDROMEDA GALAXY
15 QUINTILLION MILES

M
620 C

LEY'S COMET MEAN SIZE
7 MILES

SEXTILLIONS

QUINTILLIONS

WAVELENGTH (MILES)

C M B

PULSAR
TIMING ARRAYS

SPACE-BASED
DETECTORS

TERRESTRIAL
DETECTORS

Credit: NASA's Goddard Space Flight Center Conceptual Image Lab

Gravitational Waves induced by Curvature Perturbations

— Highlights of their use in Cosmology —

Significance of Induced Gravitational Waves

(Scalar-)induced GWs = Gravitational waves induced by curvature perturbations

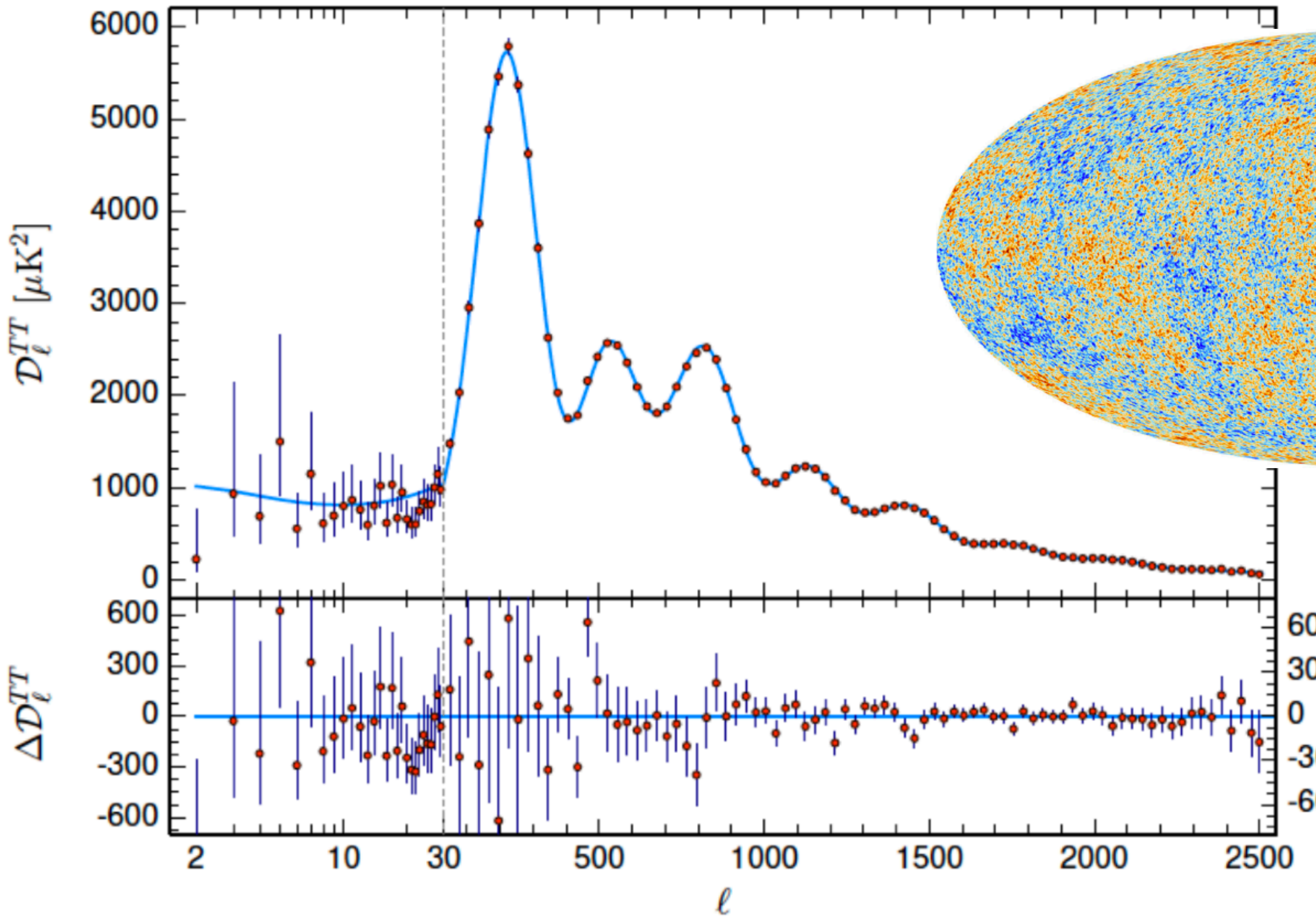
(More details later)

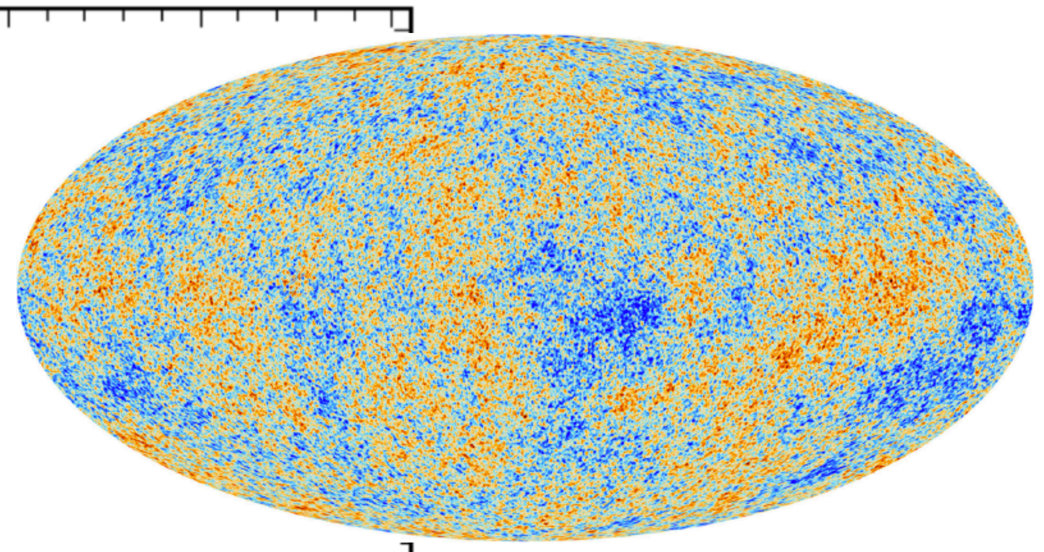
Why are they important?

Using them, we can

- probe inflation
- probe thermal history (equation of state)
- explain the pulsar timing array (PTA) data
- test the primordial black hole (PBH) scenario
- test ideas in quantum gravity

[Planck 2018 results. X. Constraints on Inflation, 1807.06211]

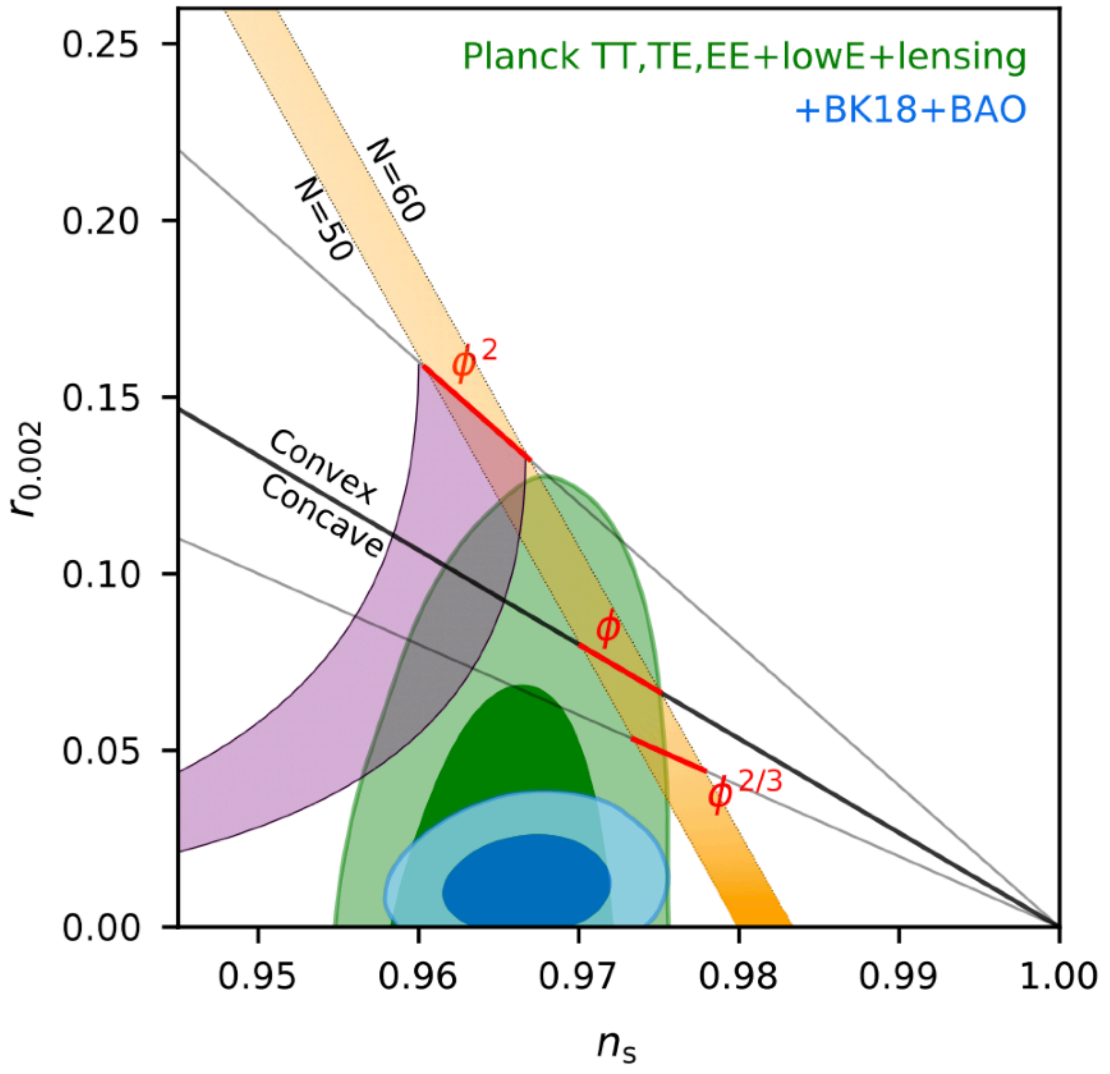


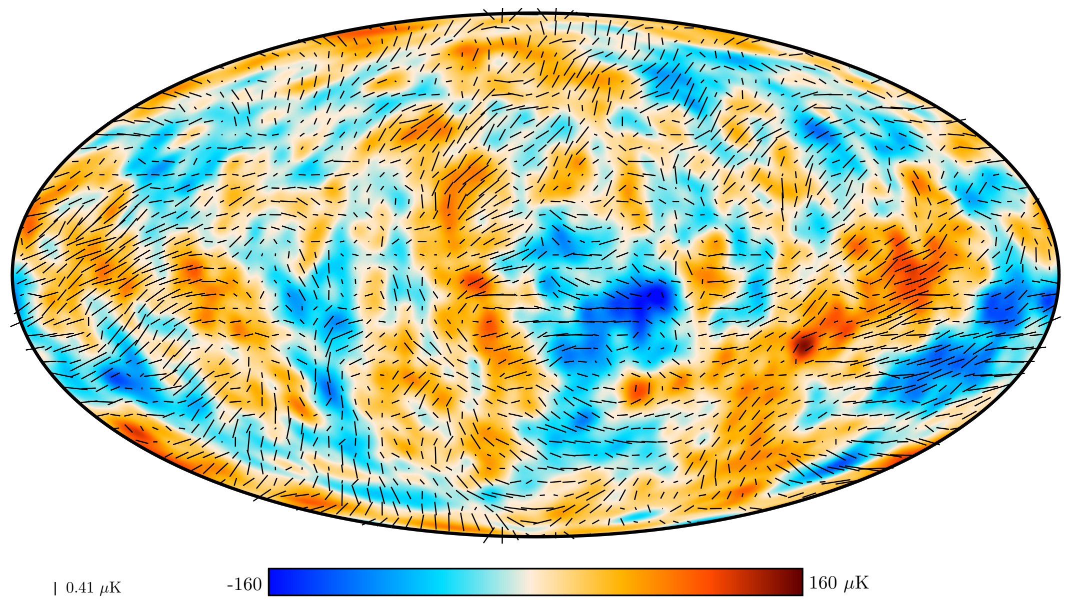


ESA and the Planck Collaboration

fit by only 6 parameters of the ΛCDM model!!

[BICEP/Keck, 2110.00483]





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Primordial GWs have not been detected but upper-bounded.

$$\zeta^{(1)}$$

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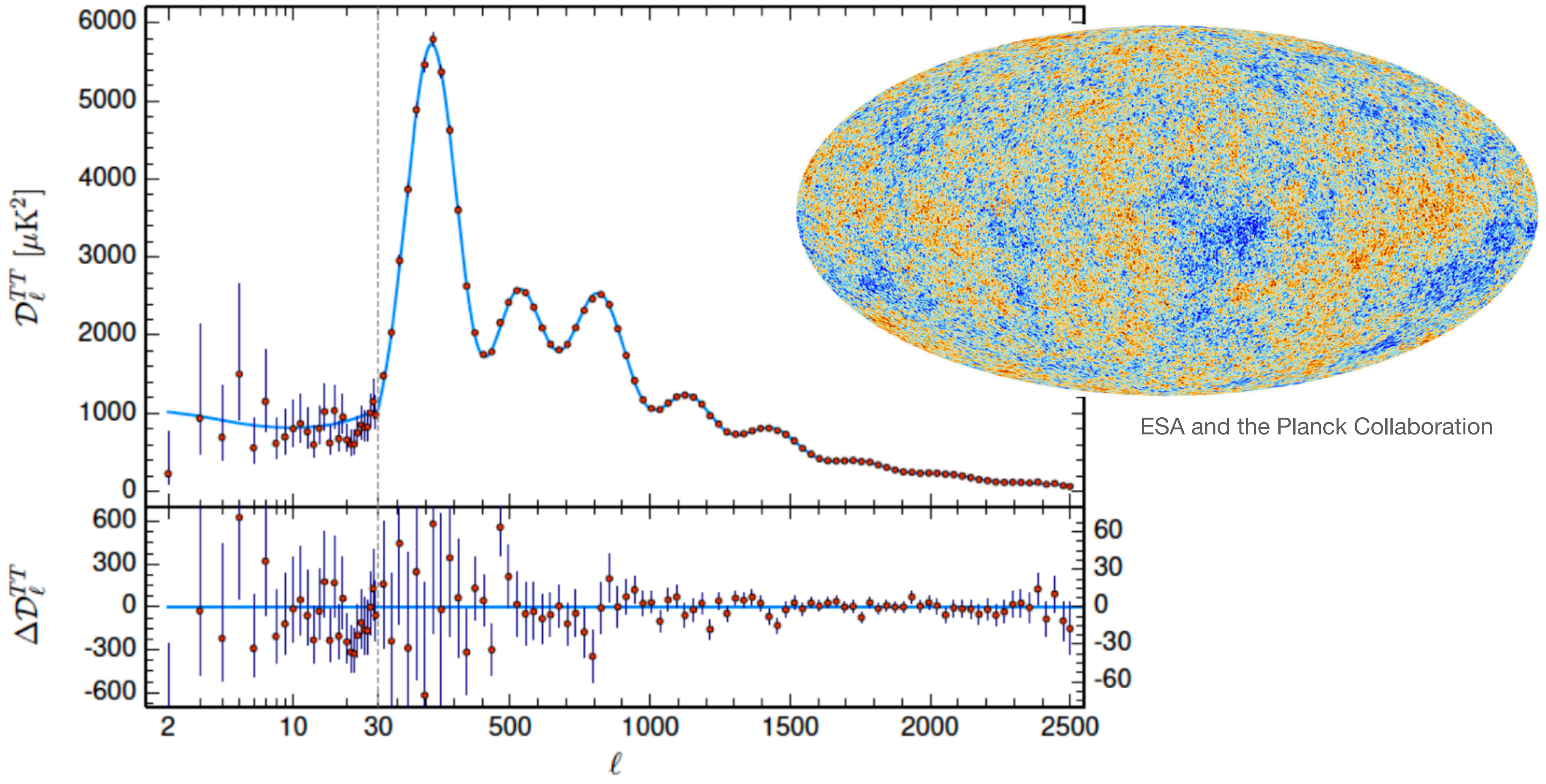

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$$h_{\mu\nu}^{(1)}$$

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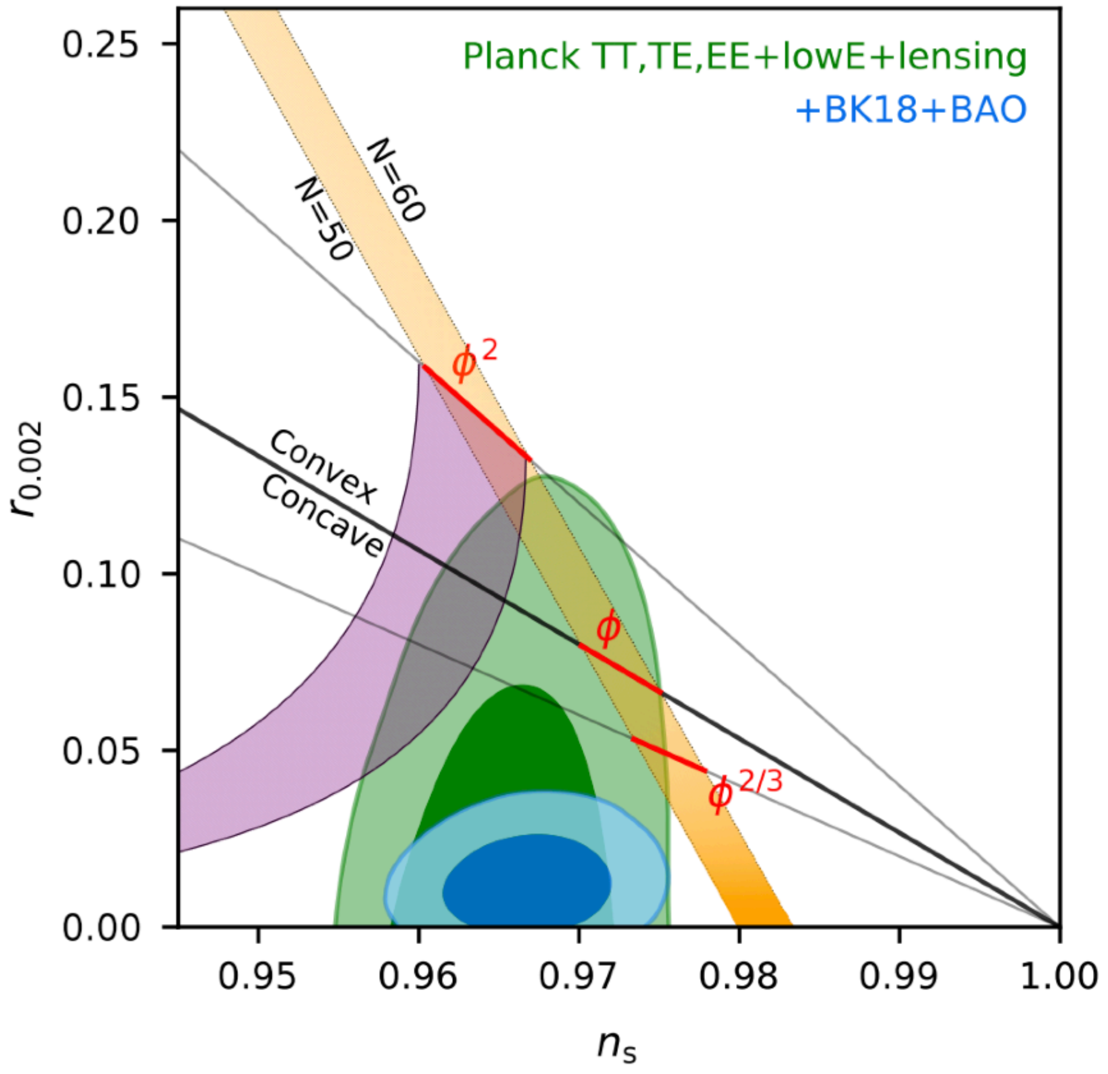

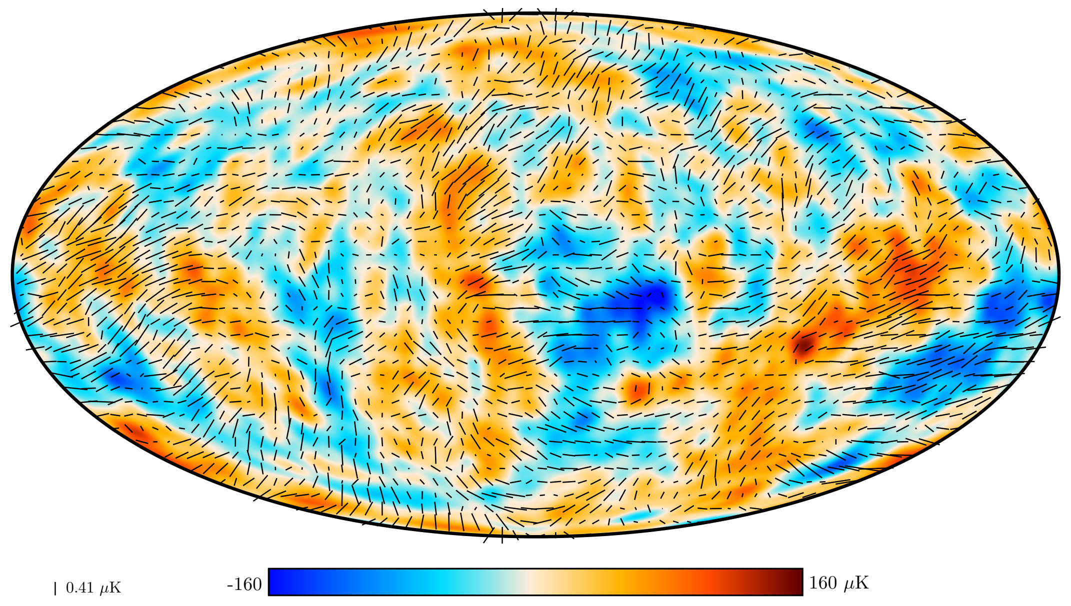

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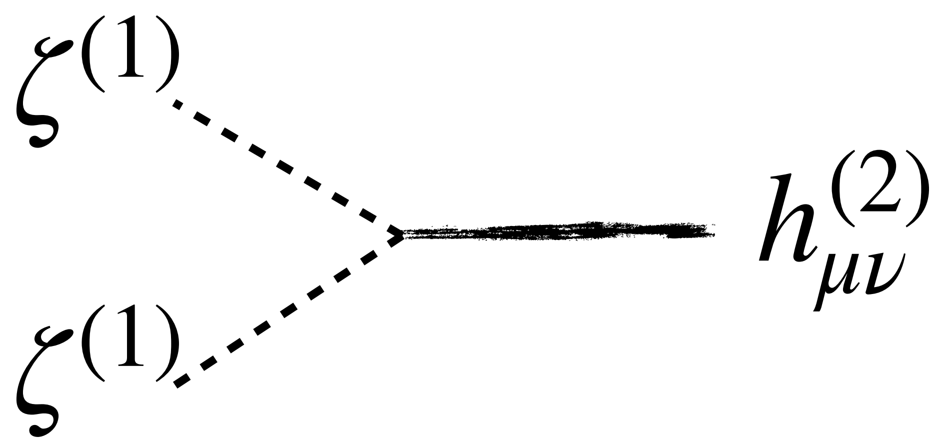

$$\zeta^{(1)}$$

- Nonlinear but perturbative effect
- Potentially important on small scales

$$h^{(2)}_{\mu\nu}$$

$$\sim \mathcal{O}(\partial_\mu \zeta^{(1)} \partial_\nu \zeta^{(1)})$$

$$h^{(1)}_{\mu\nu}$$

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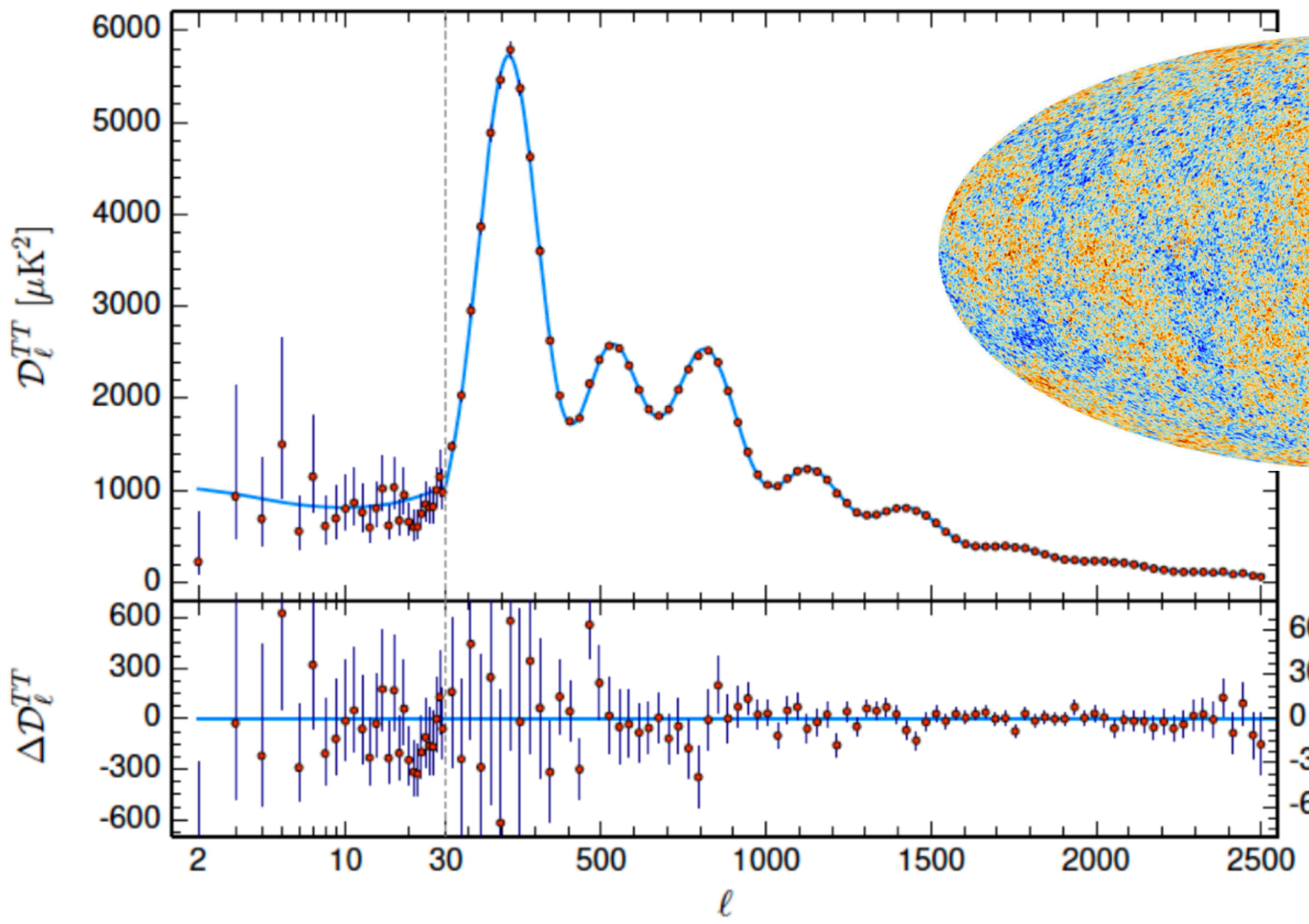

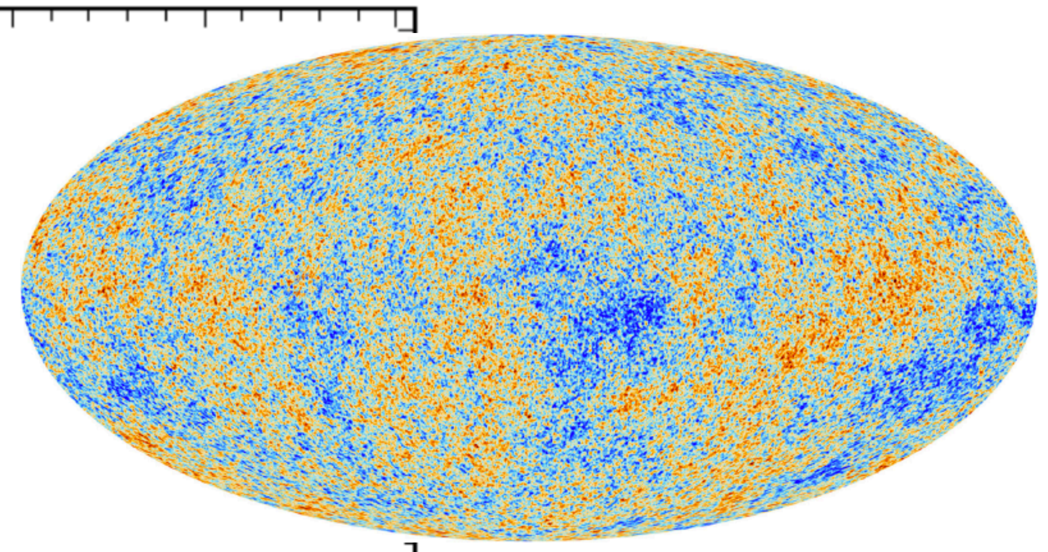

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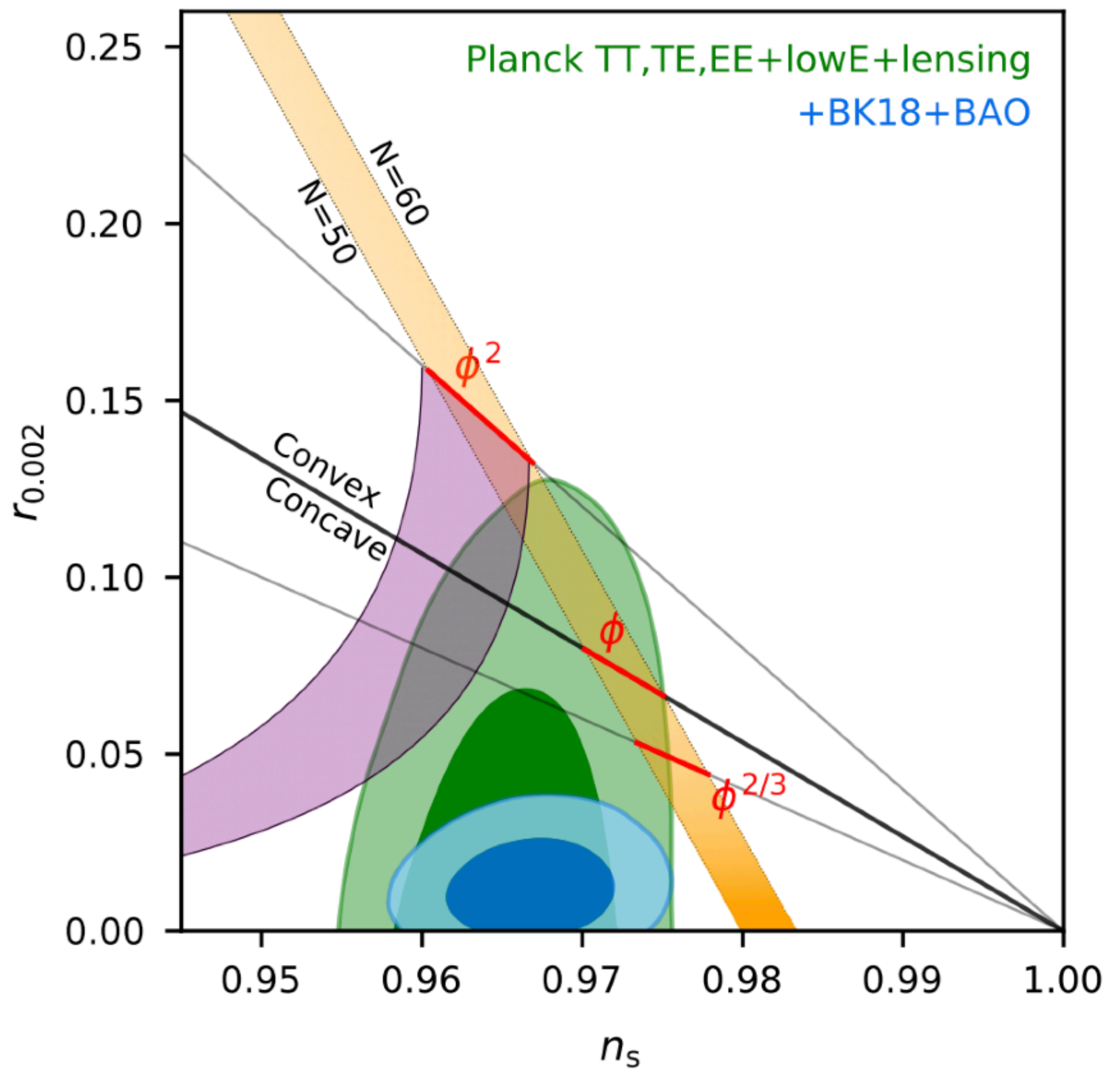

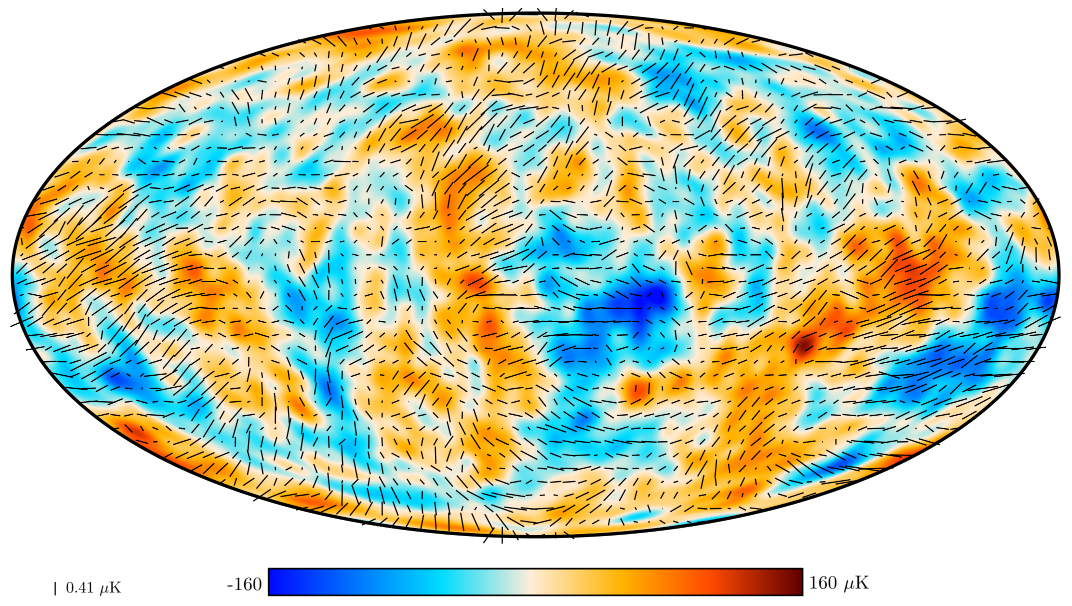

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Small-Scale Perturbations Probing Early Universe

Large curvature/density perturbations ⇒

- primordial black holes (PBHs)
- induced gravitational waves (**induced GWs**)

[Saito, Yokoyama, 0812.4339; 0912.5317]

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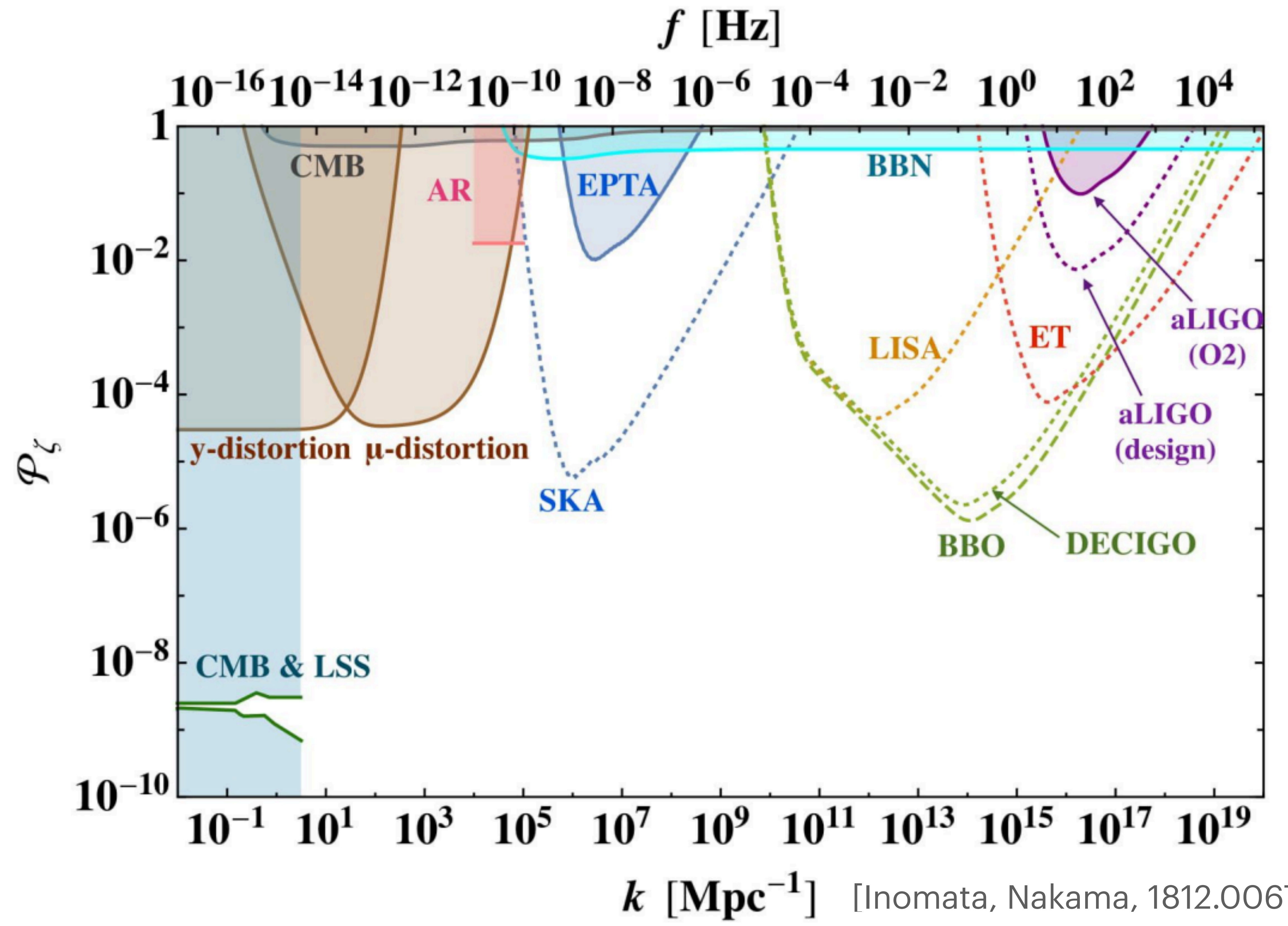
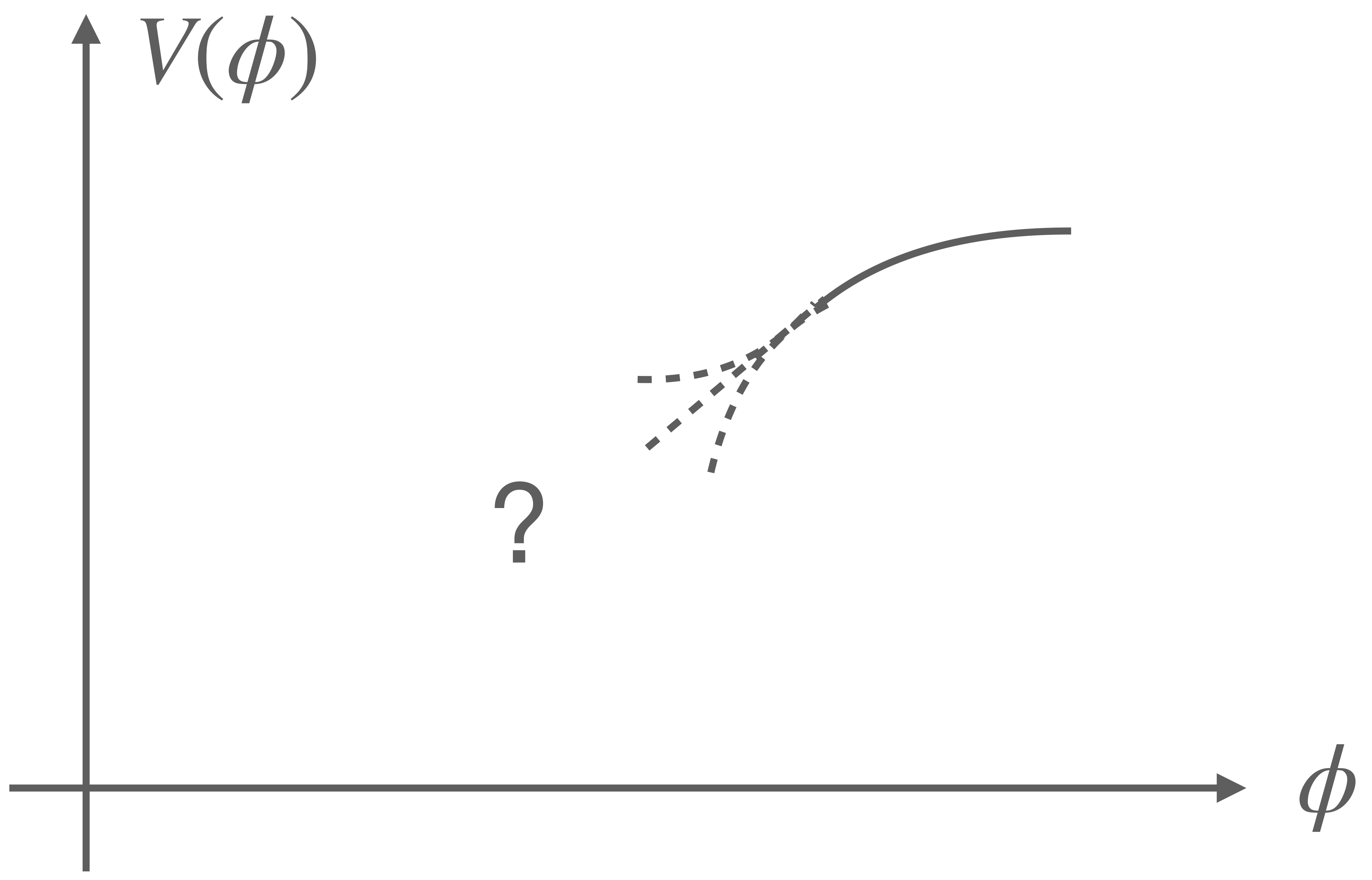


[Inomata, Nakama, 1812.00674]



Scalar-Induced Gravitational Waves

(Induced GWs, Second-order GWs, Secondary GWs, …)

[Ananda, Clarkson, Wands, gr-qc/0612013], [Baumann, Steinhardt, Takahashi, Ichiki, hep-th/0703290] For reviews, see [Yuan, Huang, 2103.04739], [Domènech, 2109.01398].

Conformal Newtonian gauge (There are some variants of its name.)

$$\mathrm{d}s^2 = -a^2(1+2\Phi)\mathrm{d}\eta^2 + a^2\left((1-2\Psi)\delta_{ij} + \frac{1}{2}h_{ij}\right)\mathrm{d}x^i\mathrm{d}x^j$$

Gravitational potential Curvature perturbations GW (tensor mode)

(In the absence of anisotropic stress, $\Phi = \Psi$.)

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Graviton mode expansion

$$h_{ij}(\eta,\mathbf{x}) = \int \frac{\mathrm{d}^3k}{(2\pi)^{3/2}}\left(e^{+}_{ij}(\mathbf{k})h^{+}_{\mathbf{k}}(\eta) + e^{\times}_{ij}(\mathbf{k})h^{\times}_{\mathbf{k}}(\eta)\right)e^{i\mathbf{k}\cdot\mathbf{x}}$$

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Equation of motion

$$h''_{\mathbf{k}}(\eta) + 2\mathscr{H}h'_{\mathbf{k}}(\eta) + k^2h_{\mathbf{k}}(\eta) = 4S_{\mathbf{k}}(\eta)$$

where $\mathscr{H} = aH$ is the conformal Hubble, and the source term is $S_{\mathbf{k}} = \int \frac{\mathrm{d}^3q}{(2\pi)^{3/2}}e_{ij}(\mathbf{k})q_iq_j\left(2\Phi_{\mathbf{q}}\Phi_{\mathbf{k-q}} + \frac{4}{3(1+w)}(\mathscr{H}^{-1}\Phi'_{\mathbf{q}} + \Phi_{\mathbf{q}})(\mathscr{H}^{-1}\Phi'_{\mathbf{k-q}} + \Phi_{\mathbf{k-q}})\right)$

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GW energy density on subhorizon scales [Maggiore, gr-qc/9909001]

$$\rho_{\rm GW}(\eta) = \frac{1}{16a^2}\langle\overline{h'_{ij}h'_{ij}}\rangle = \frac{1}{16a^2}\langle\overline{h_{ij,k}h_{ij,k}}\rangle = \int \mathrm{d}\ln k\, \rho_{\rm GW}(\eta, k)$$

where an overline denotes the oscillation average.

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where an overline denotes the oscillation average.

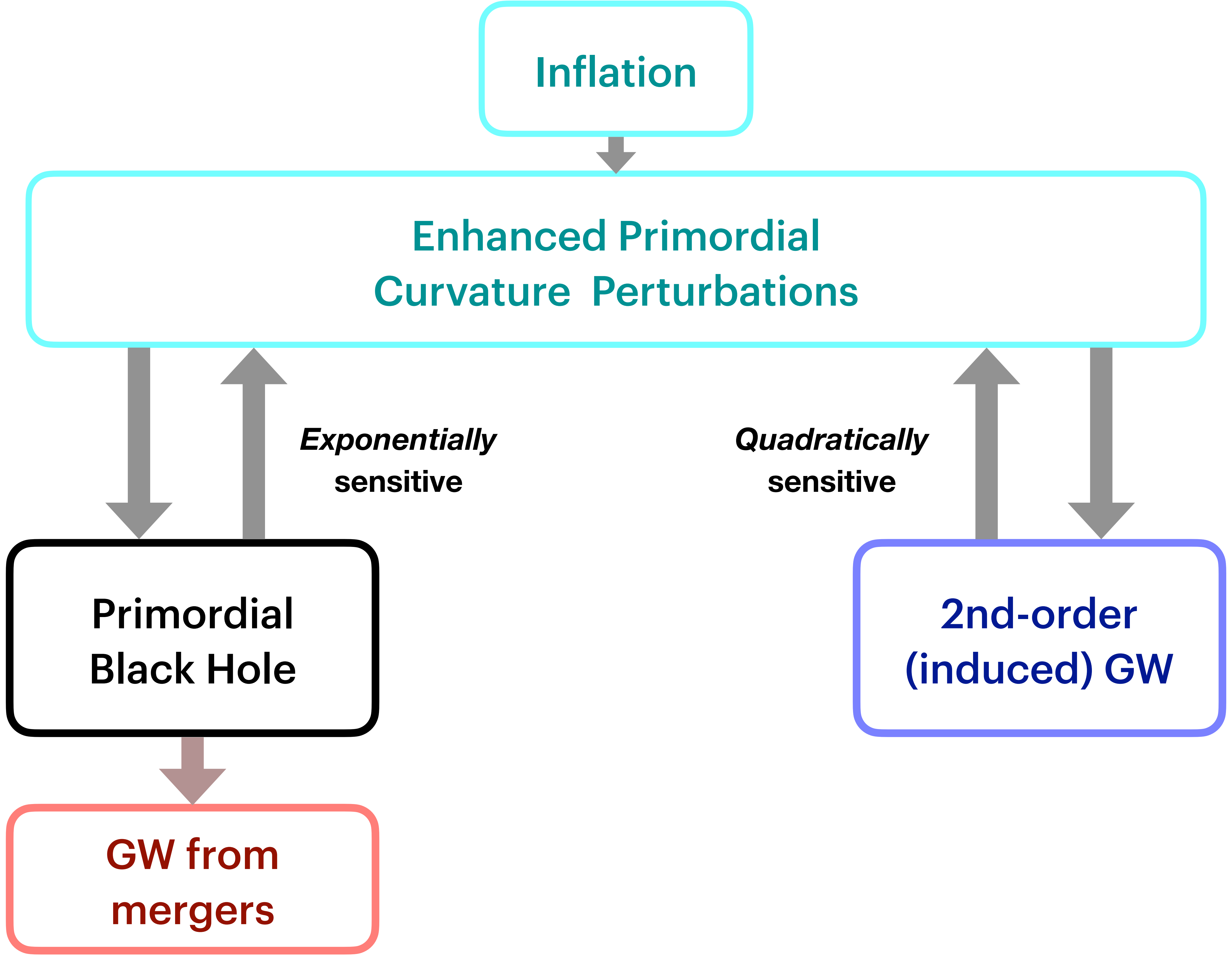
Dimensionless Power spectrum

$$\langle h^{\lambda}_{\mathbf{k}}(\eta)h^{\lambda'}_{\mathbf{k}'}(\eta)\rangle = \delta_{\lambda\lambda'}\delta^3(\mathbf{k}+\mathbf{k}')\frac{2\pi^2}{k^3}\mathscr{P}_h(\eta,k)$$

$$\Omega_{\mathrm{GW}}(\eta,k) = \frac{\rho_{\mathrm{GW}}(\eta,k)}{\rho_{\mathrm{total}}(\eta)} = \frac{1}{24}\left(\frac{k}{aH}\right)^2\overline{\mathscr{P}_h(\eta,k)}$$

Relation to Primordial Black Holes

[Saito, Yokoyama, 0812.4339; 0912.5317]



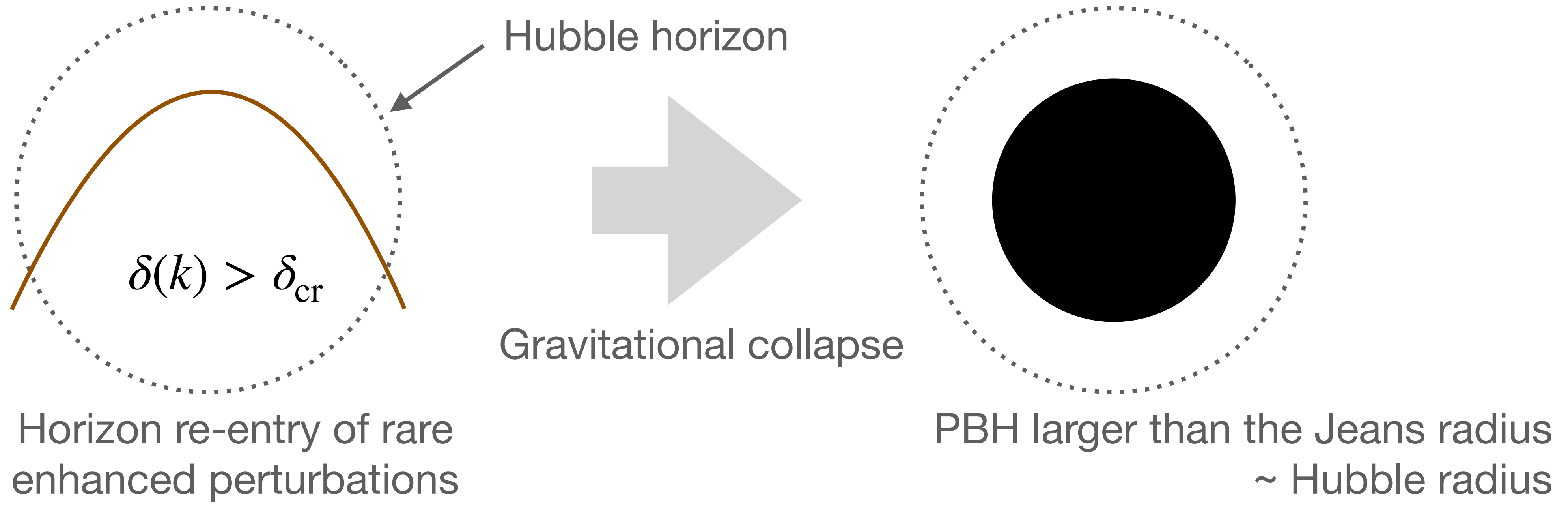
$$
\begin{aligned}
M &= 8.5 \times 10^{-4} M_\odot \left(\frac{\gamma}{0.2}\right) \left(\frac{g_*(T)}{10.75}\right)^{-1/6} \left(\frac{6.5 \times 10^7\,\mathrm{Mpc}^{-1}}{k}\right)^2 \\
&= 8.5 \times 10^{-4} M_\odot \left(\frac{\gamma}{0.2}\right) \left(\frac{g_*(T)}{10.75}\right)^{-1/6} \left(\frac{1.0 \times 10^{-7}\,\mathrm{Hz}}{f}\right)^2
\end{aligned}
$$

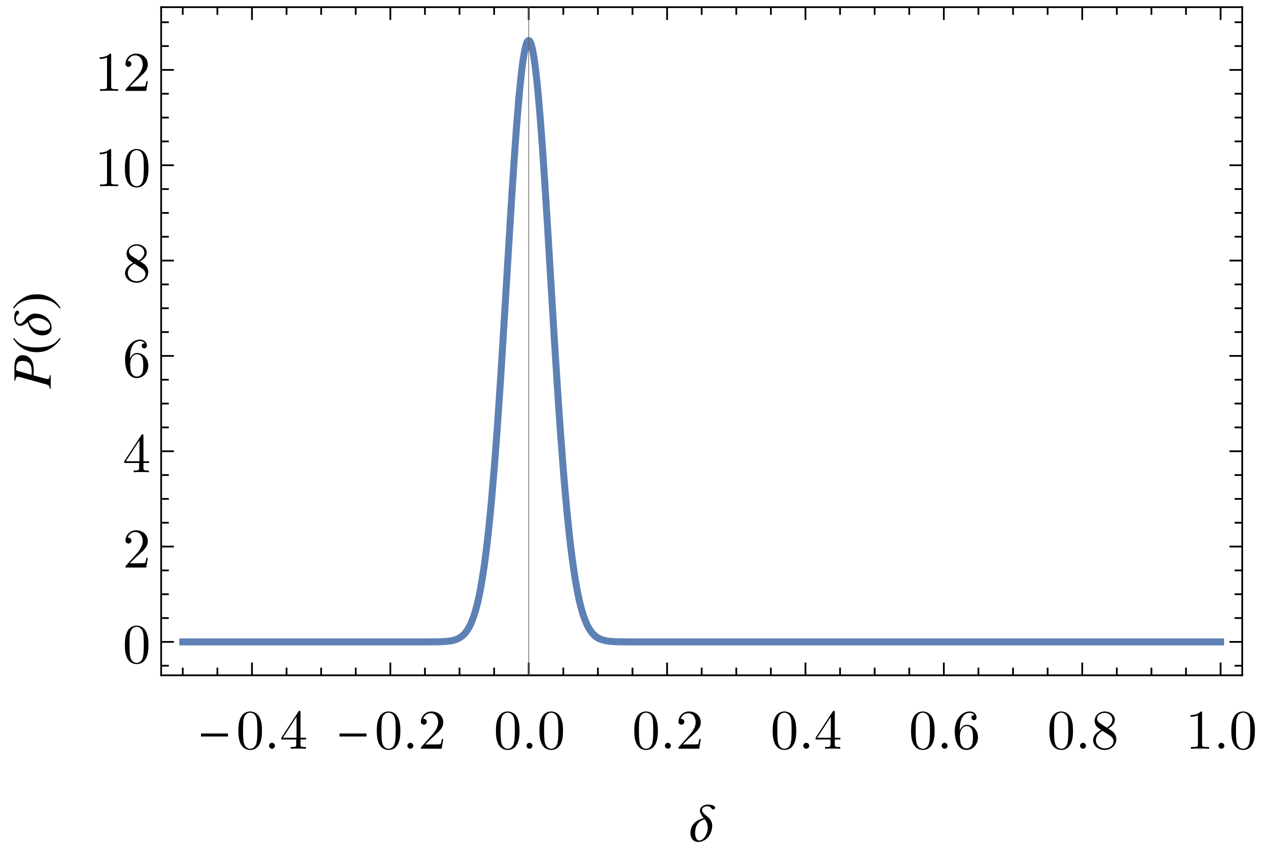
PBH in a nutshell

Production mechanism

The most popular mechanism is the direct gravitational collapse of the horizon-sized overdensity.

$$M \sim \rho \times H^{-3} \sim \frac{M_\mathrm{P}^2}{H}$$





a Gaussian probability distribution of overdensity

$$\beta \equiv \left.\frac{\rho_\mathrm{PBH}}{\rho}\right|_\mathrm{form} \sim \exp\left(-\frac{\delta_c^2}{2\sigma^2}\right)$$

PBH formation is a VERY rare process.

Behaves like cold DM

$$\rho_\mathrm{PBH} \sim a^{-3}$$

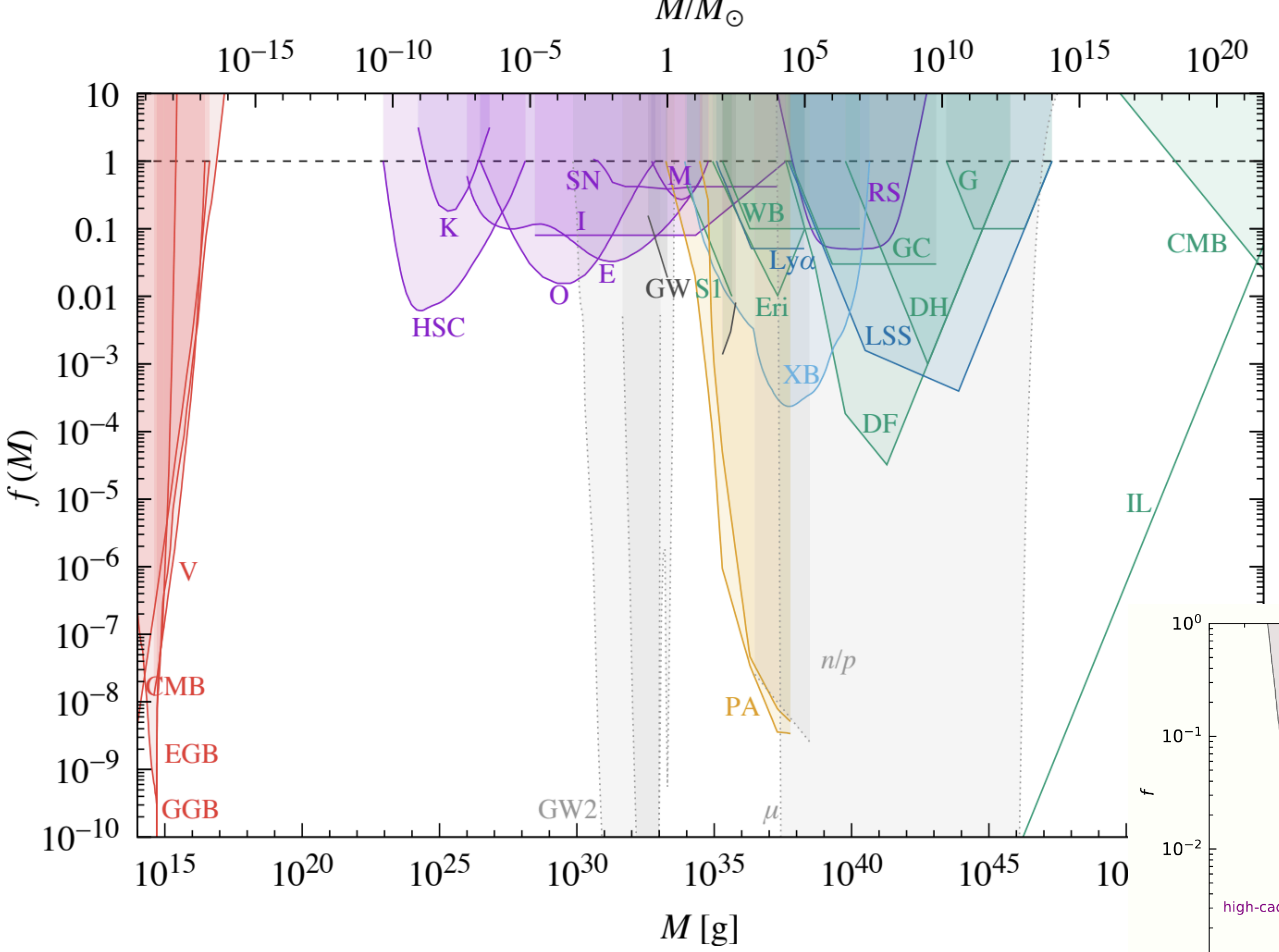
Long lifetime (for heavy PBHs)

$$\tau \sim \frac{M^3}{M_\mathrm{P}^2}$$

PBH DM parameter space

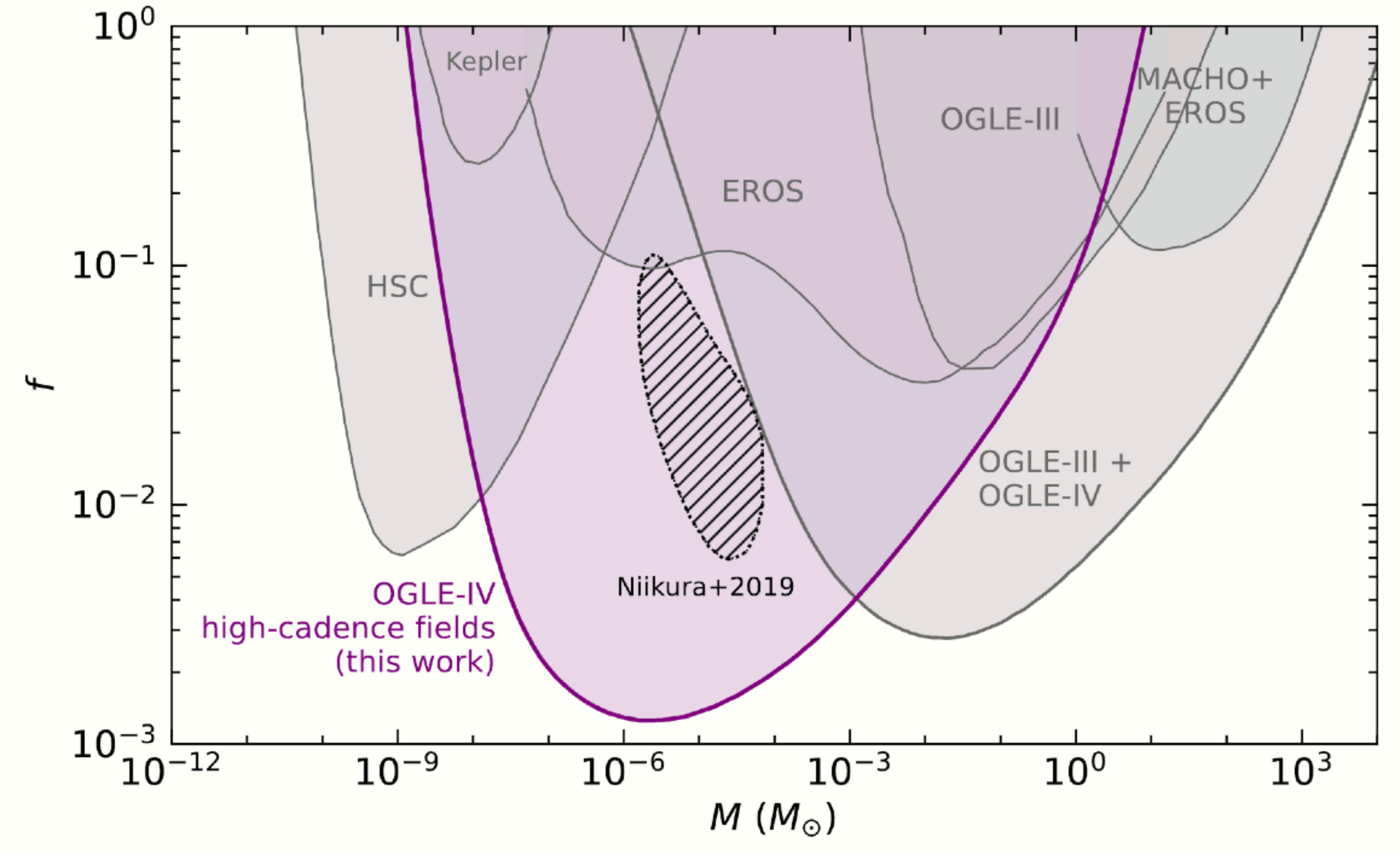
$1M_\odot = 2 \times 10^{30}\,\mathrm{kg}$

$$f(M) \equiv \frac{\rho_{\mathrm{PBH}}}{\rho_{\mathrm{DM}}}$$



[Carr, Kohri, Sendouda, Yokoyama, 2002.12778]

A new constraint
[Mróz et al, 2410.06251]



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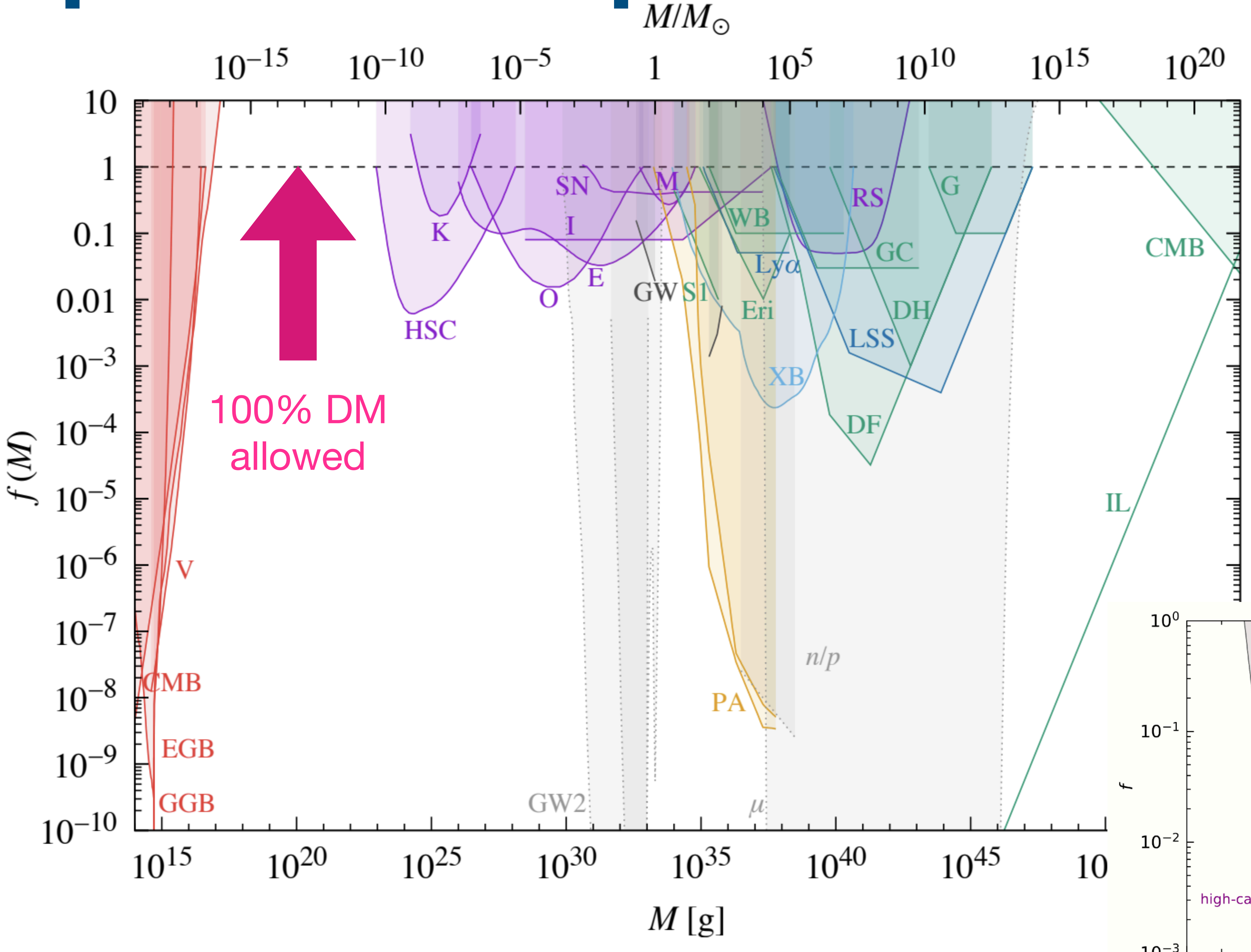

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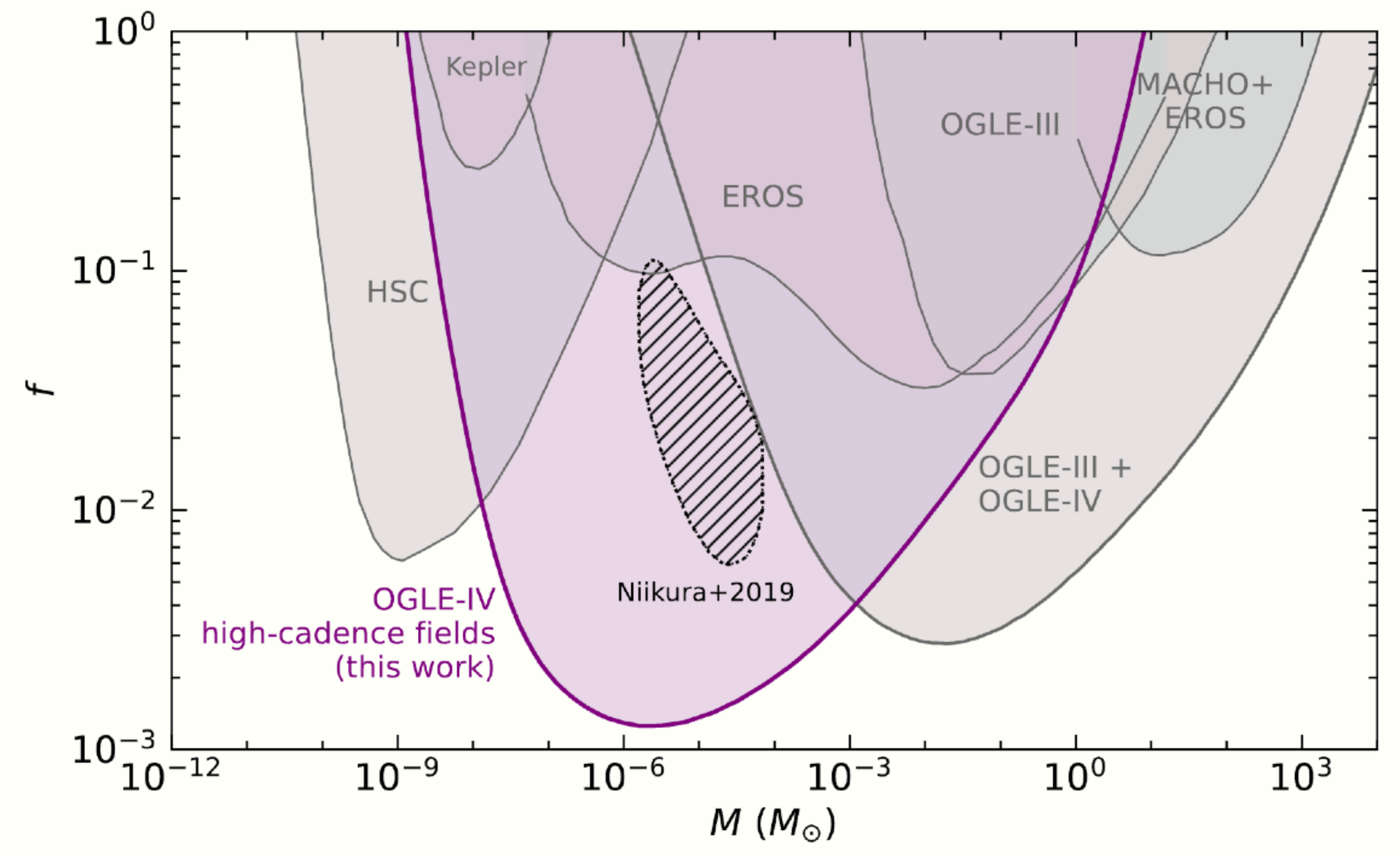

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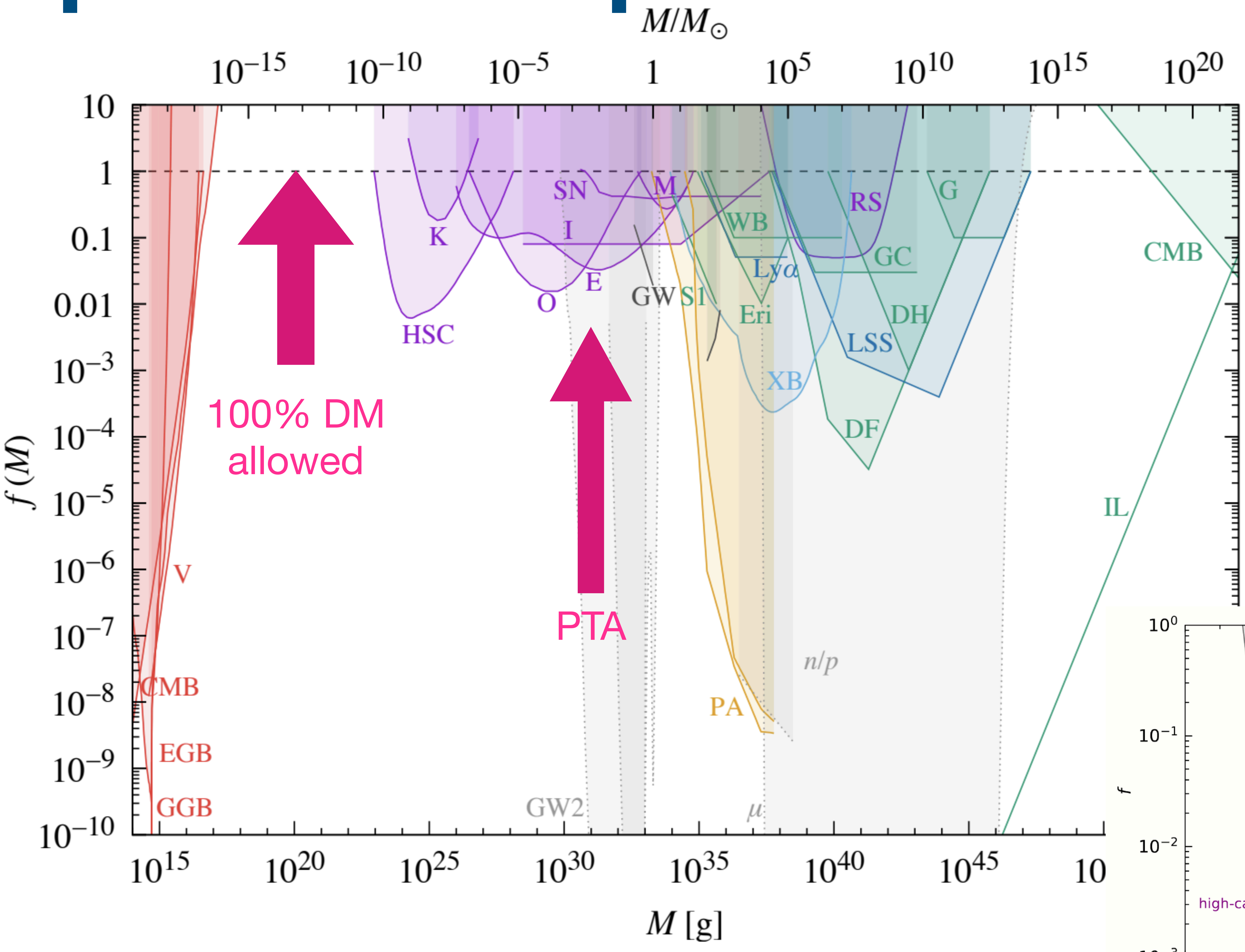

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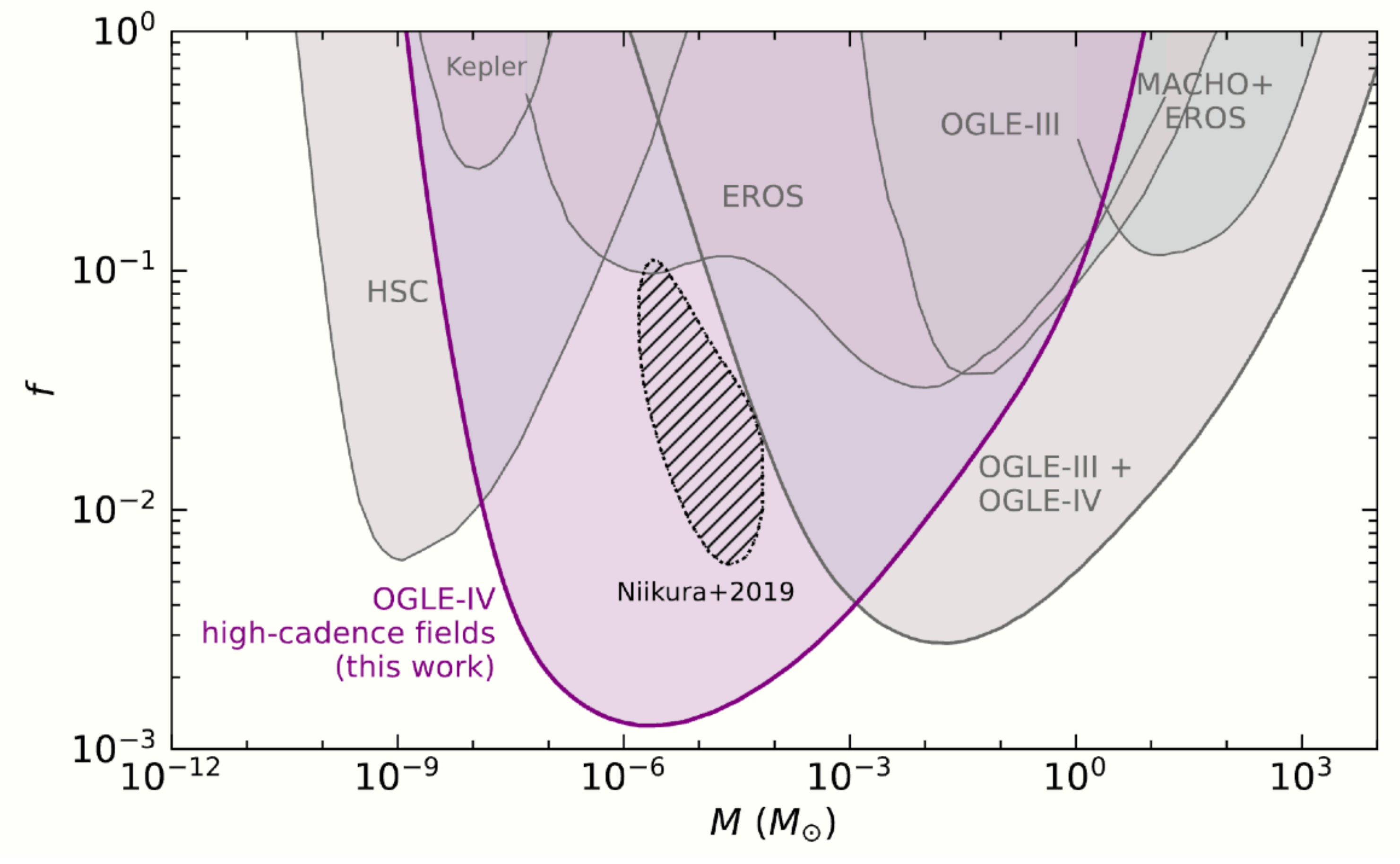

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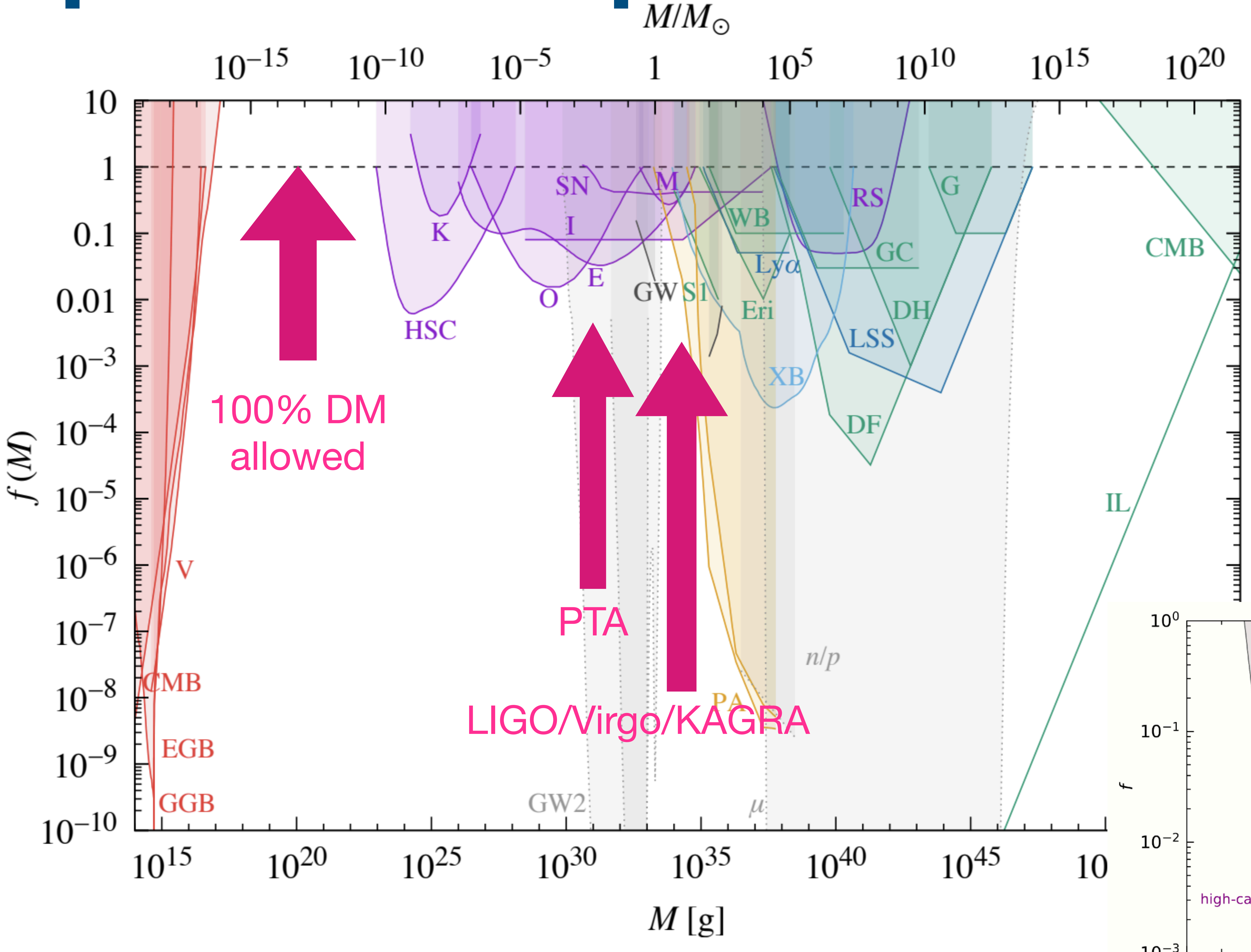

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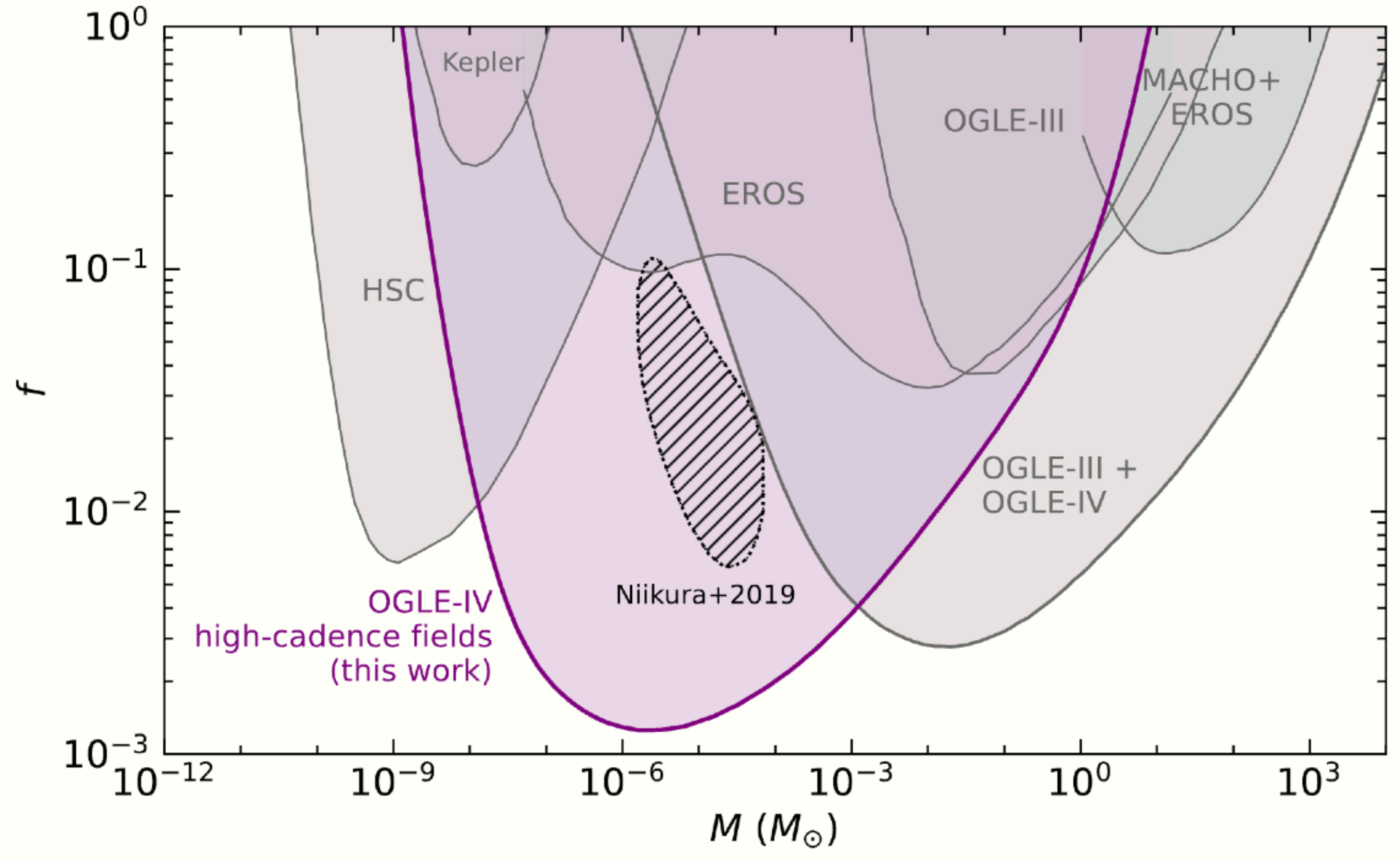

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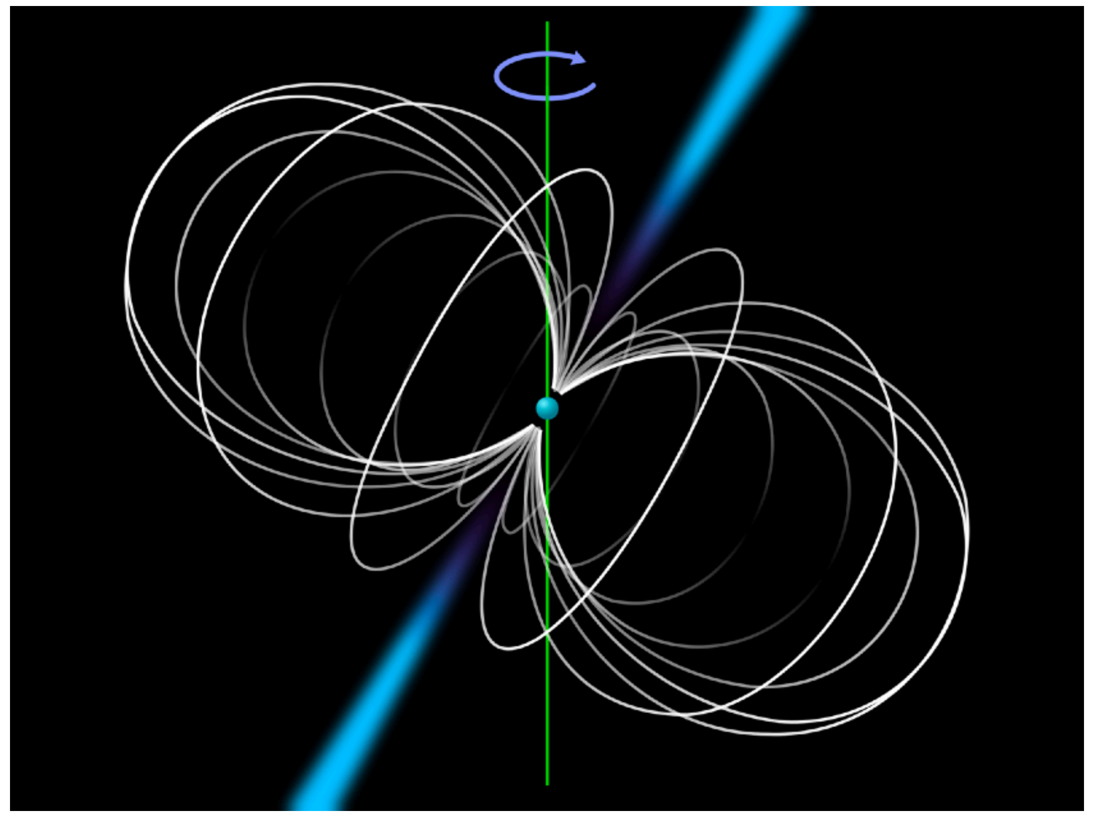
- test ideas in quantum gravity

Pulsar Timing Array



Pulsar: one of the best clocks in Nature

- **supposed to be a spinning neutron star emitting jets**
- **regular periods (milliseconds ~ seconds)**

Uncertainty in the Time Of Arrival (TOA) $\sigma_{\mathrm{TOA}} = 200\,\mathrm{ns}$

[from talk by David Nice, NANOGrav PFC; Fall 2017]

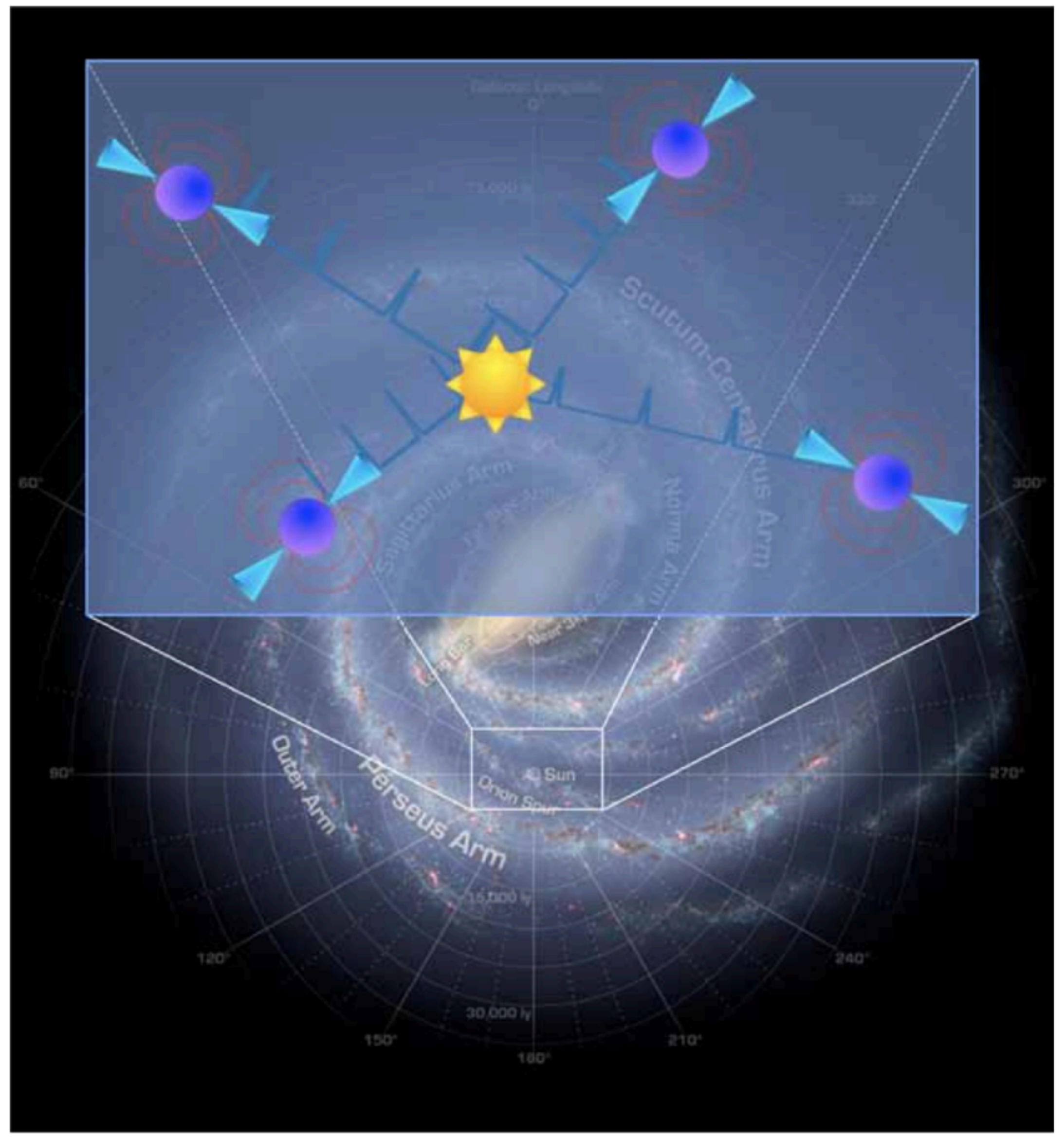


Figure 3: Illustration of lines-of-sight in our Galaxy to four of the 70+ pulsars used in NANOGrav for gravitational wave detection. Each line-of-sight, spanning thousands of light years, is like a detector arm of LIGO. Courtesy Jeffrey Hazboun / NASA / JPL-Caltech / R. Hurt (SSC-Caltech)

pulsar timing

= long-term monitoring of pulsars

O(10) mins. observation per a month $f_{\mathrm{yr}} \approx 32\,\mathrm{nHz}$

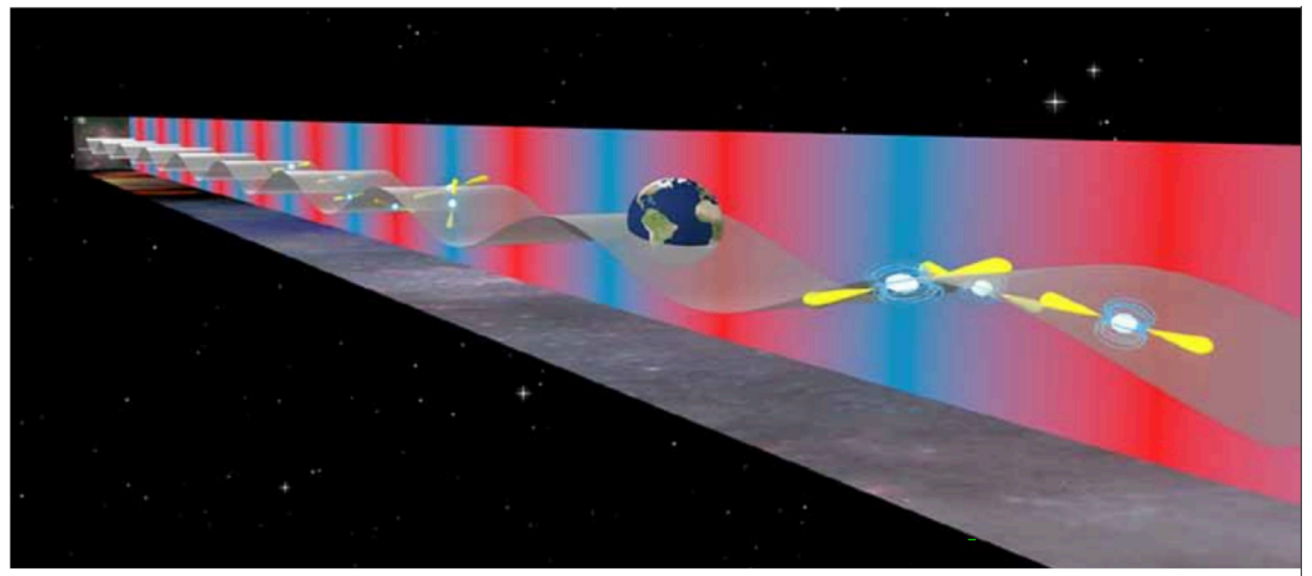
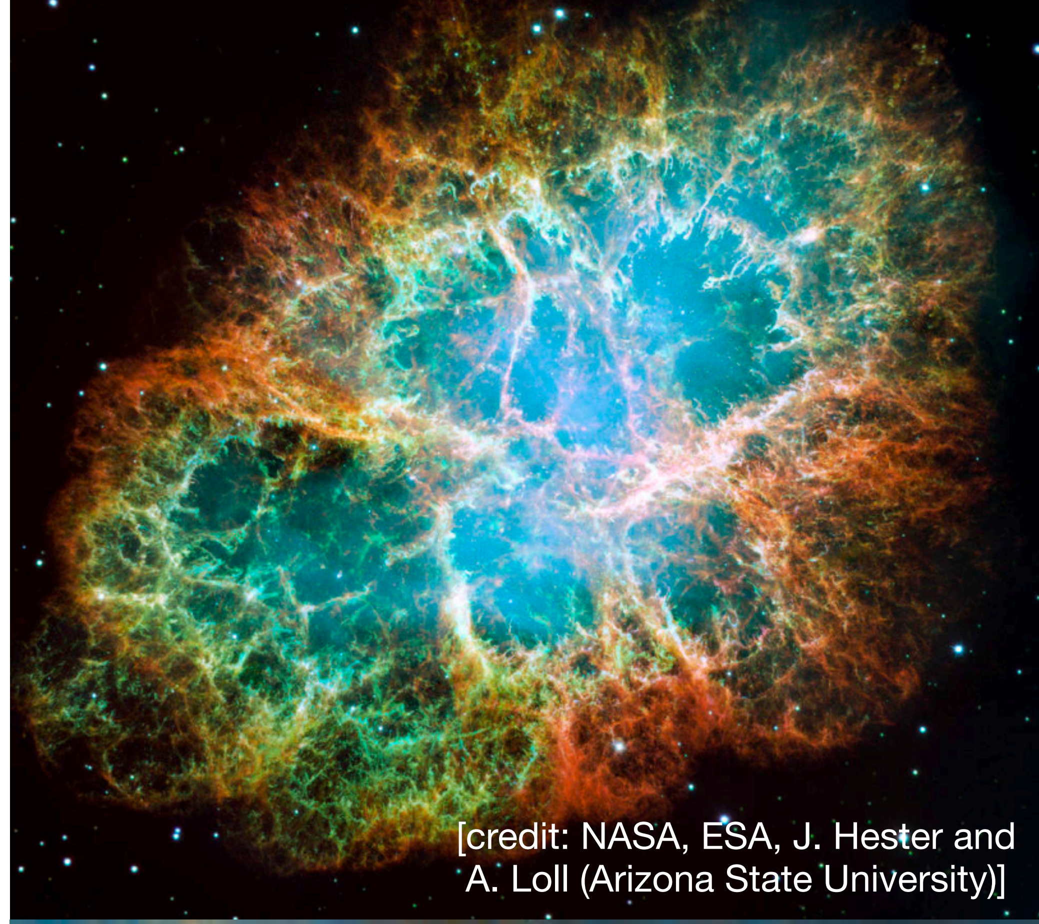


Figure 4: Continuous gravitational waves perturbing the positions of the earth and of several radio pulsars. Credit: B. Saxton (NRAO/AUI/NSF)

[from AstroBeats by Timothy Dolch]

pulse timing modulated by

- scattering with interstellar medium
- binary rotation of a pulsar
- instrumental noises
- solar-system motion (cf. ephemeris uncertainty)
- **gravitational waves**



Pulsar Timing Array Collaborations

Main Radio Telescopes used by NANOGrav

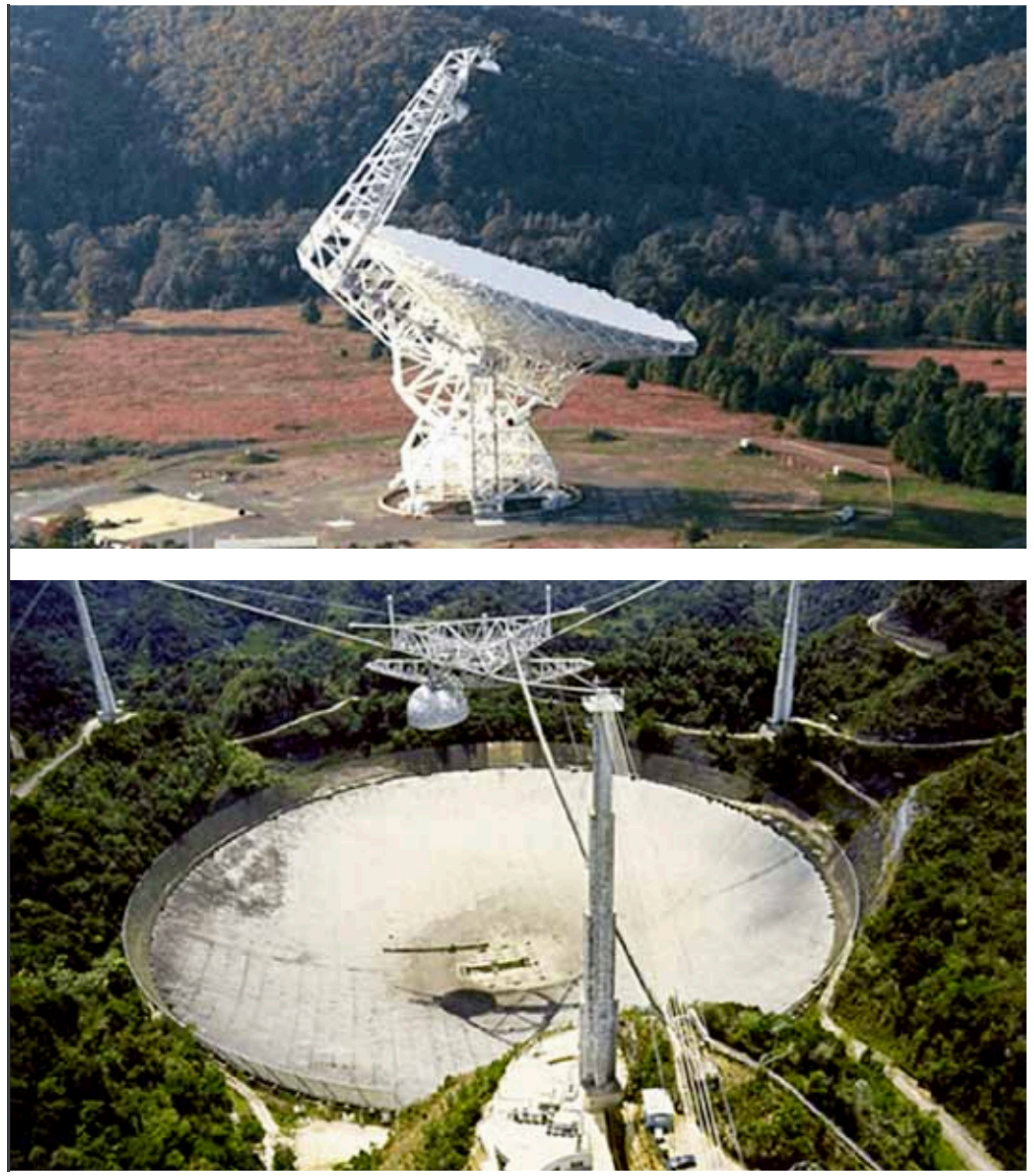
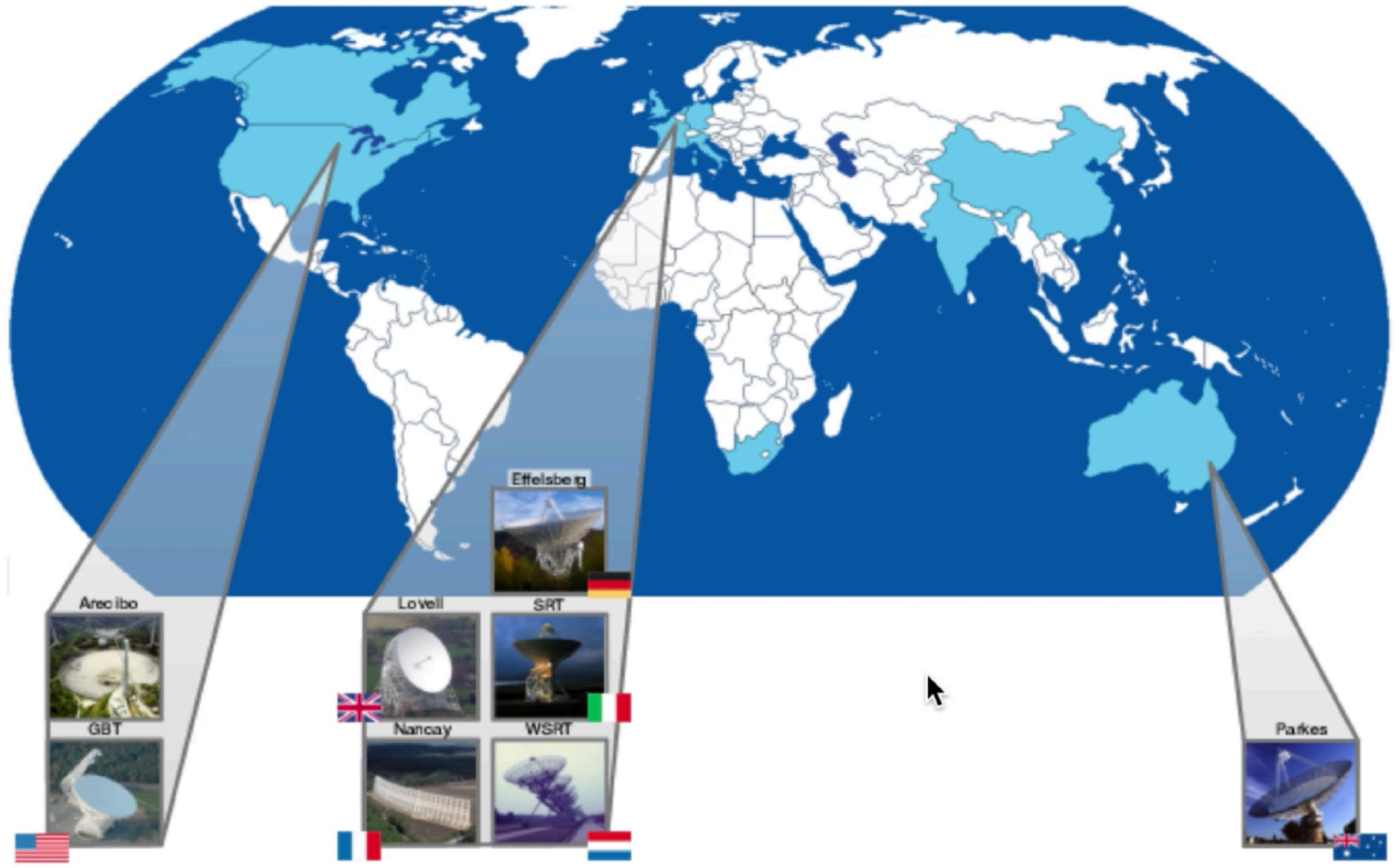


Figure 1: The NRAO Robert C. Byrd Green Bank Telescope in West Virginia (top) and the 305-m William E. Gordon Telescope at Arecibo Observatory in Puerto Rico (bottom). From greenbankobservatory.org

[from AstroBeats by Timothy Dolch]

International Collaborations



Telescopes contributing data to IPTA Data Release 2.

[from slides by David Nice, NANOGrav PFC; Fall 2017]

International PTA (IPTA) now includes

- NANOGrav
- Parkes PTA (PPTA)
- European PTA (EPTA)
- Indian PTA (InPTA)

There are also

- Chinese PTA (CPTA)
- MeerKAT PTA (South Africa)

In future,

- Square Kilometer Array (SKA)

Hellings-Downs Curve

[Hellings, Downs, Astrophys. J. 265, L39 (1983)]

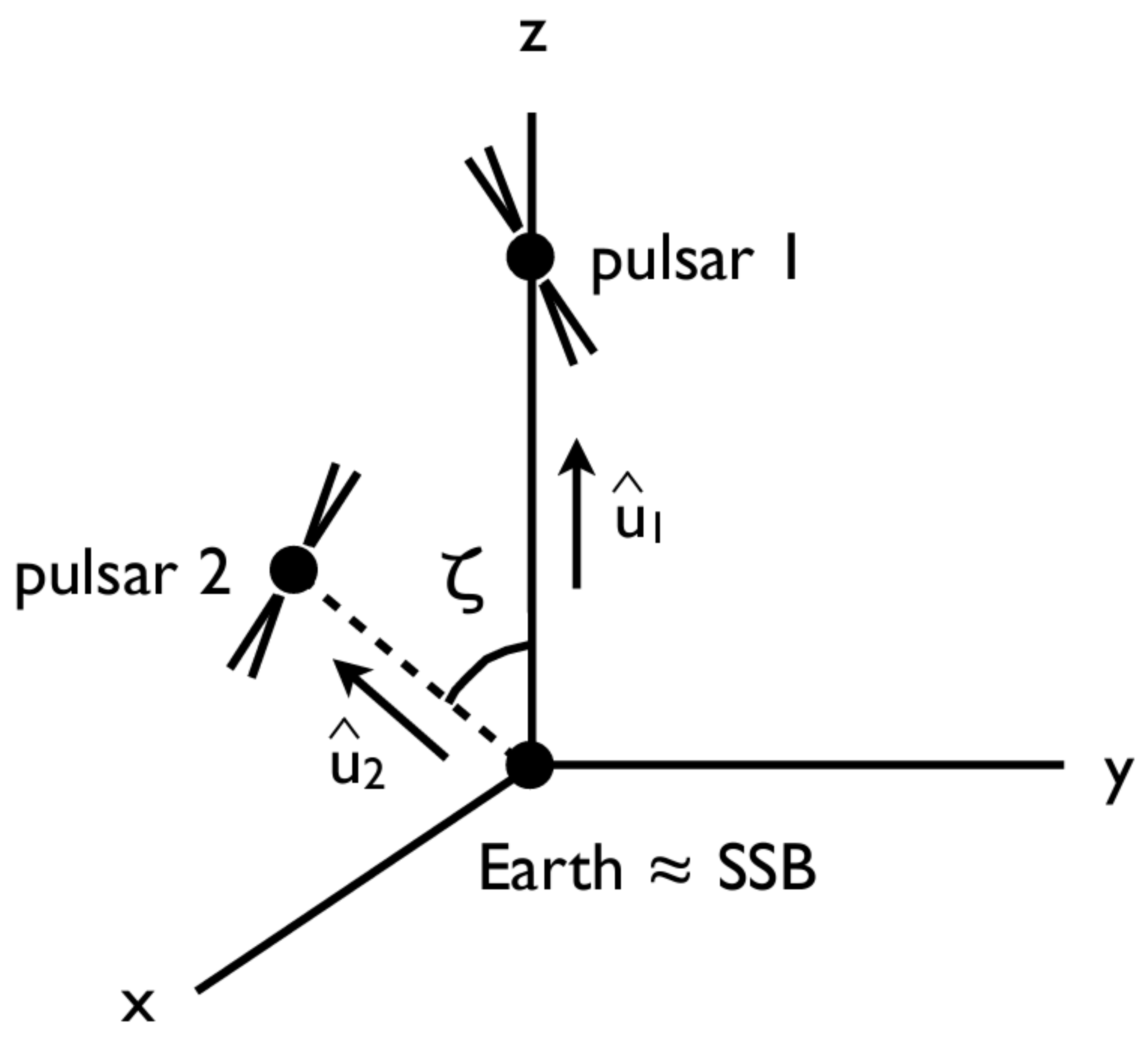


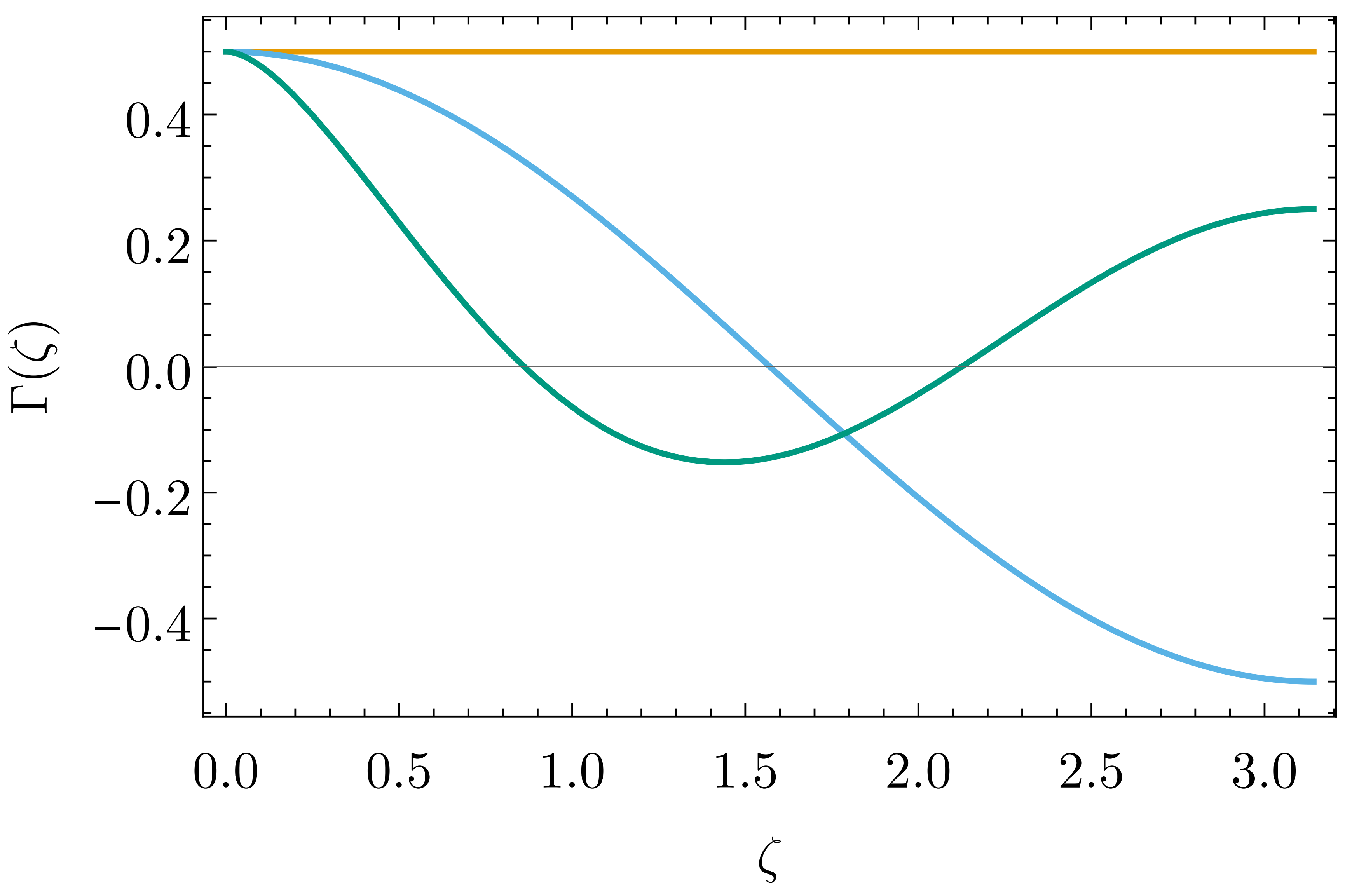
Figure from [Jenet, Romano, 1412.1142]

Assumptions in a pedagogical derivation in [Janet, Romano, 1412.1142]

- Isotropic
- Stationary
- Stochastic (random)
- Spin 2
- unpolarized (sum over polarization)

→ Smoking-gun signal of Stochastic Gravitational Wave Background (SGWB)!

(in the normalization with which $\Gamma(0) = 1/2$.)



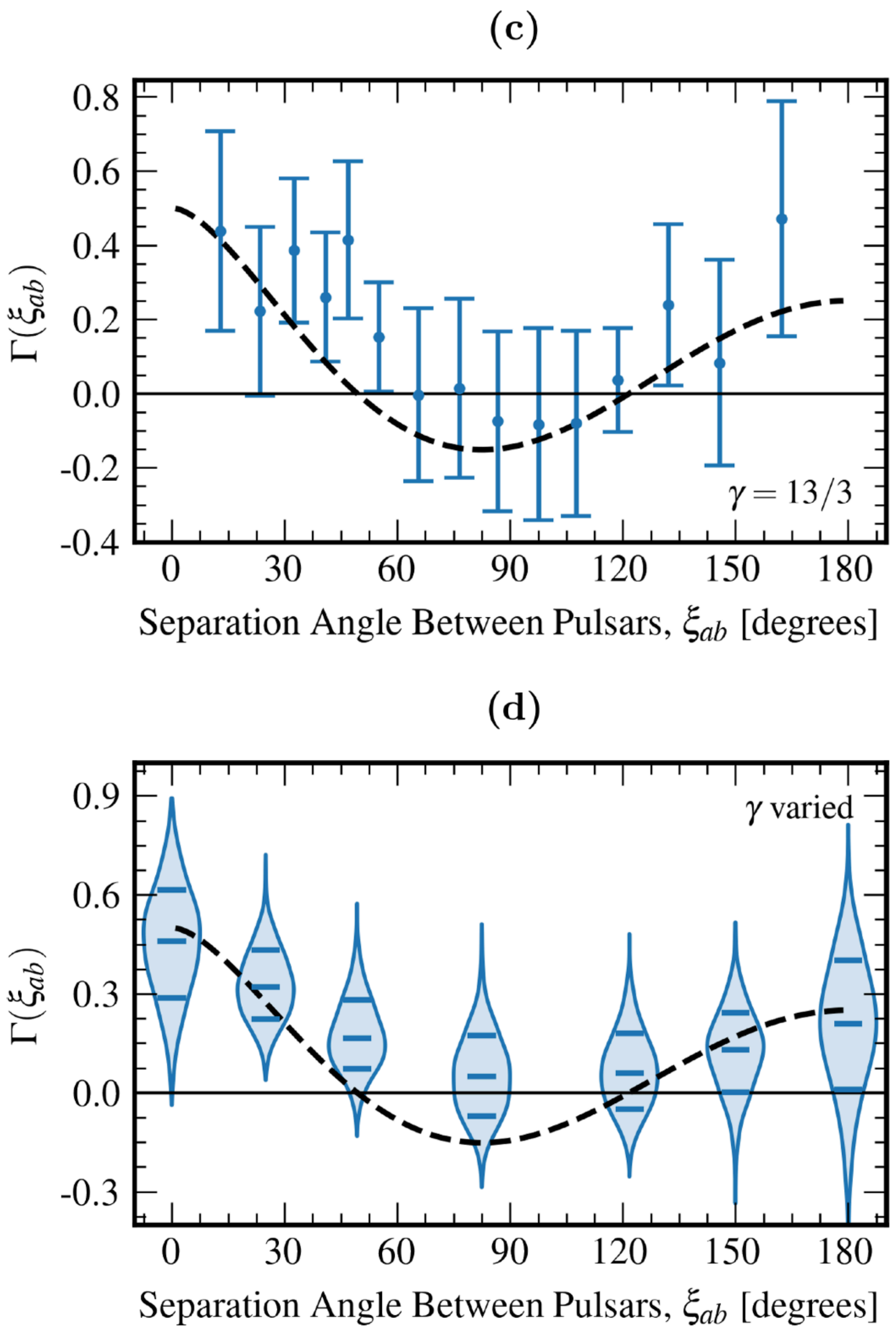
Spin 0: $\Gamma(\zeta) \sim 1$

Spin 2: $\Gamma(\zeta) = \frac{1}{2} - \frac{1}{4}\sin^2\left(\frac{\zeta}{2}\right) + \frac{3}{2}\sin^2\left(\frac{\zeta}{2}\right)\ln\left(\sin^2\frac{\zeta}{2}\right)$

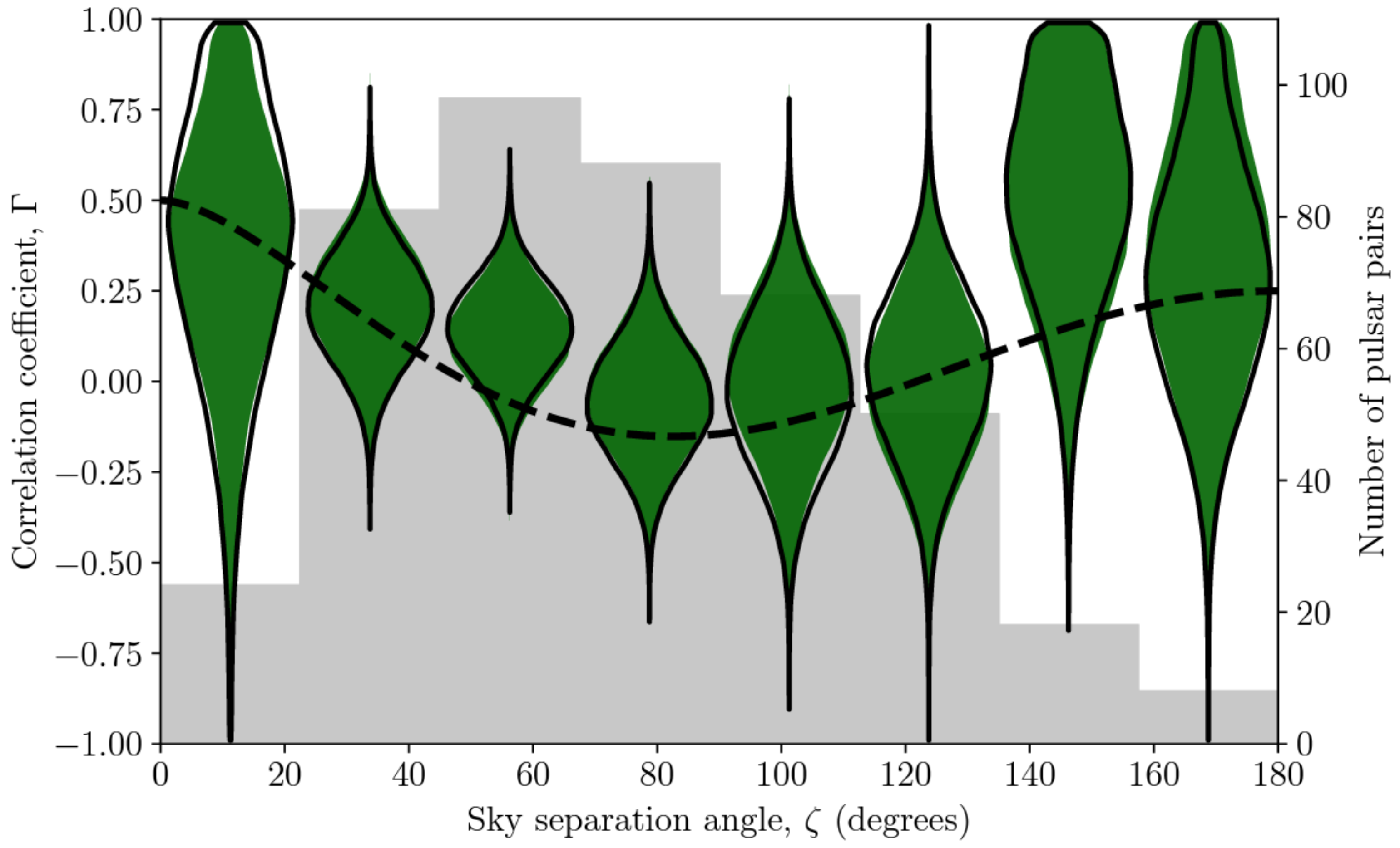
Spin 1: $\Gamma(\zeta) \sim \cos\zeta$

Hellings-Downs Curve

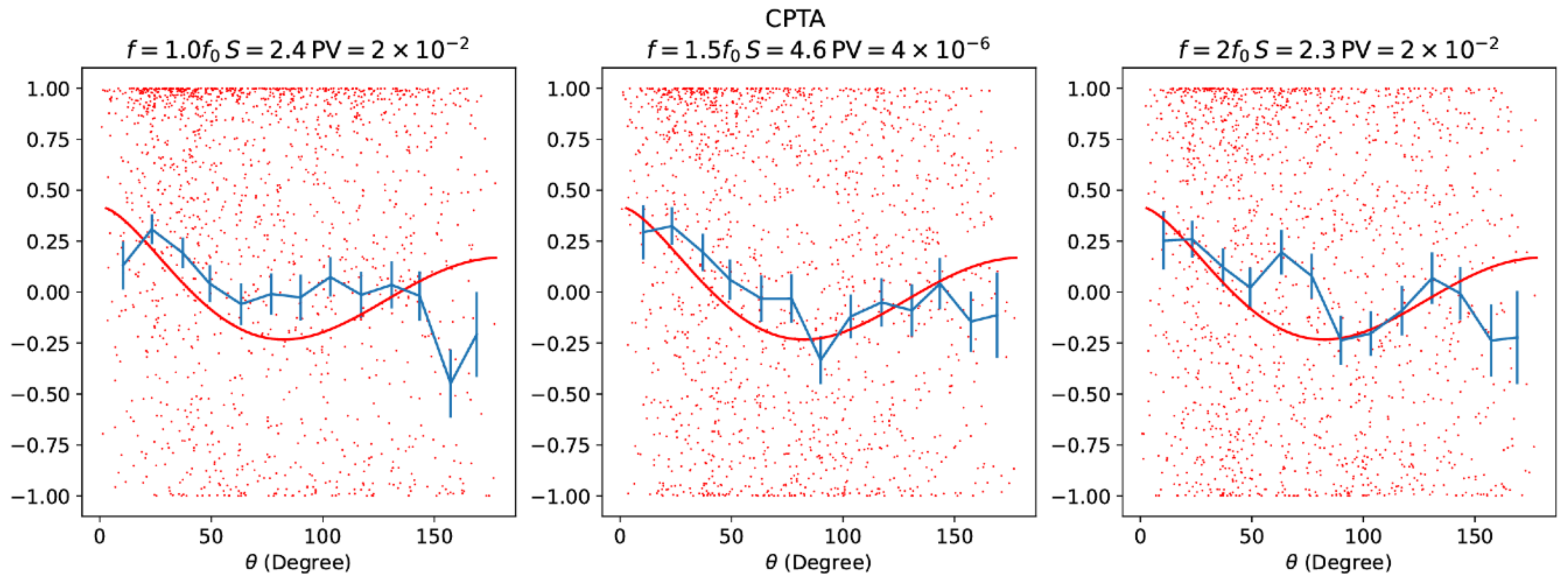
[NANOGrav, 2306.16213]



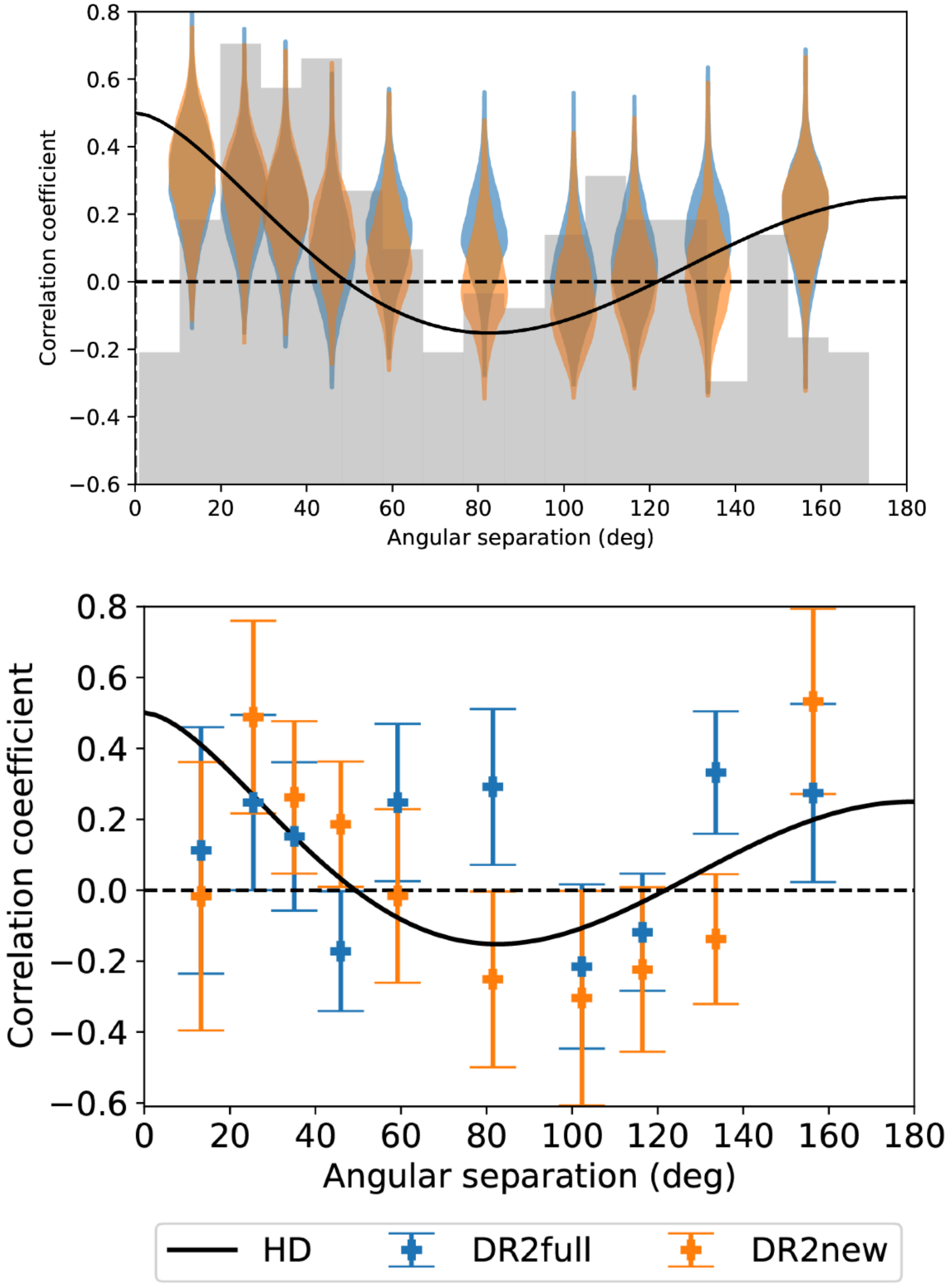
[PPTA, 2306.16215]



[CPTA, 2306.16216]

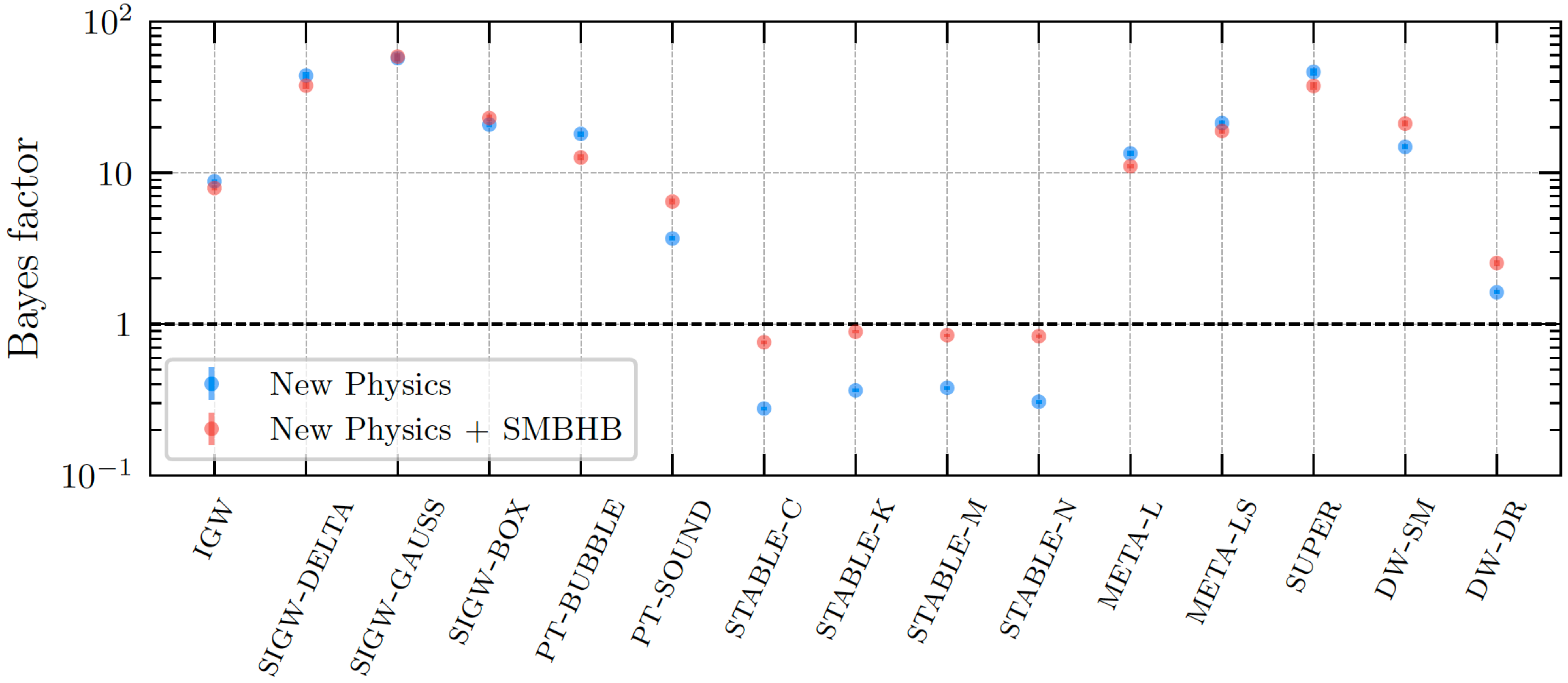


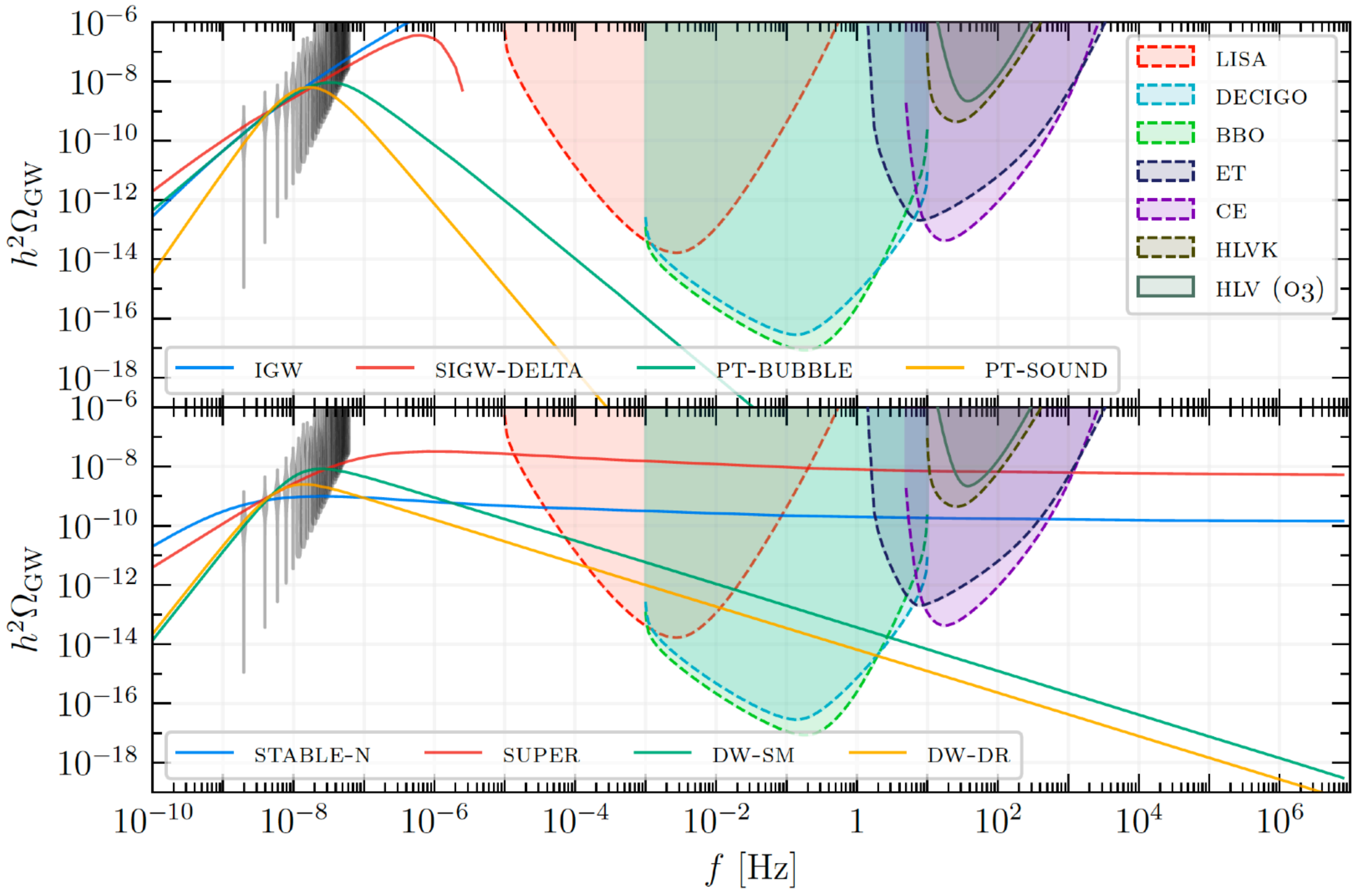
[EPTA/InPTA, 2306.16214]



New Physics Interpretations [NANOGrav, 2306.16219]

See also [EPTA/InPTA, 2306.16227], [Bian et al., 2307.02376], [Figueroa, 2307.02399], and [Ellis et al, 2308.08546].





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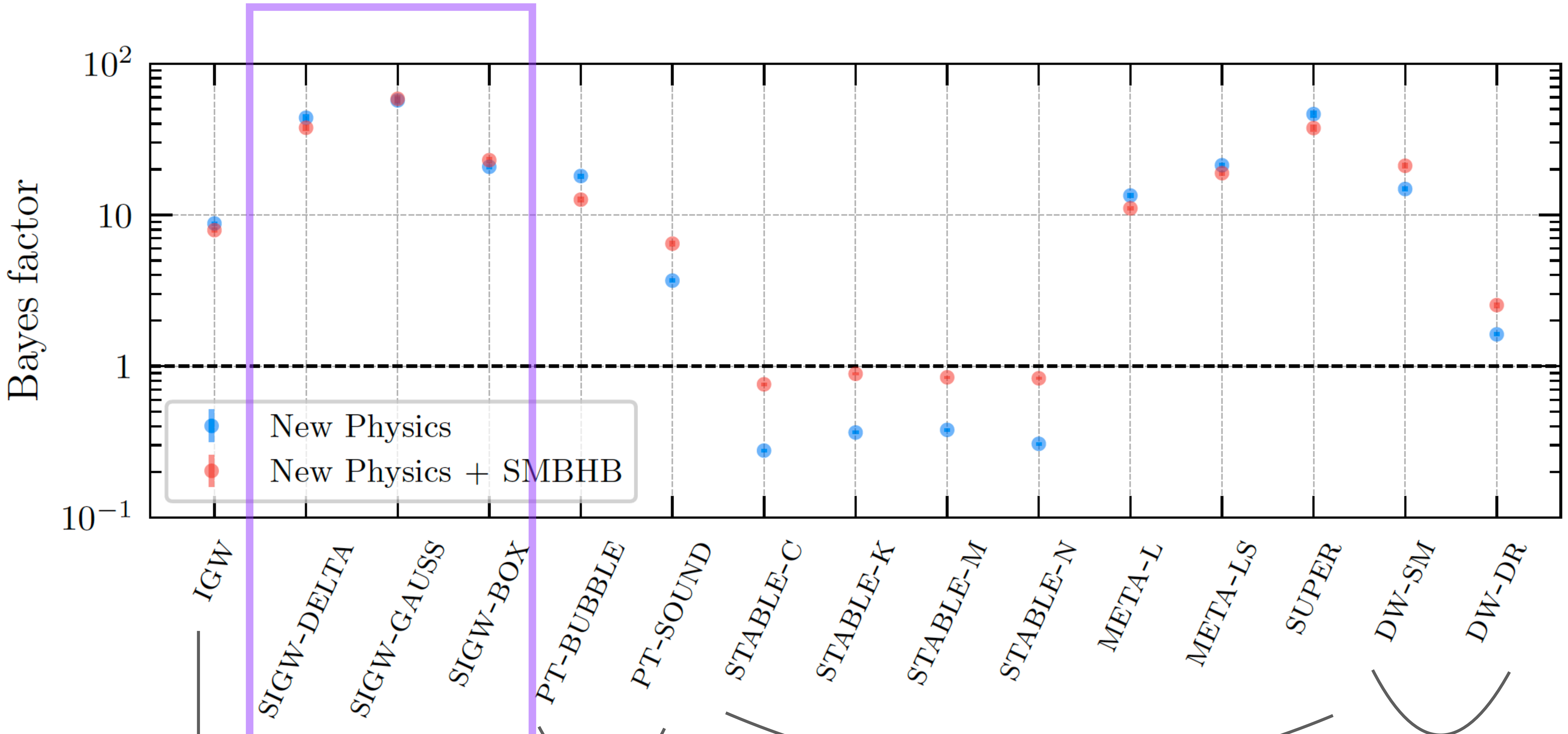


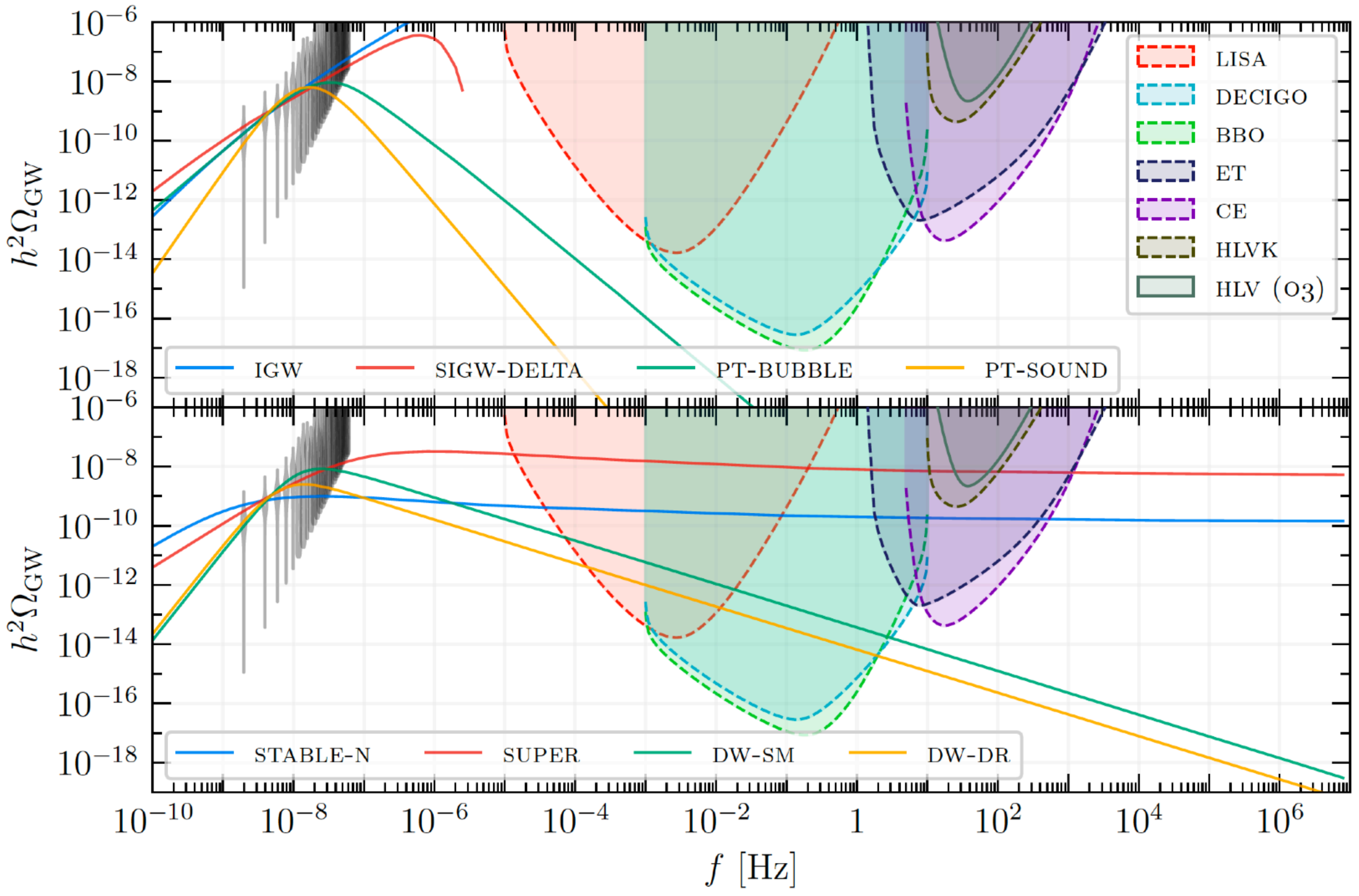
Domain Wall

Phase Transition

Cosmic Strings

Scalar-Induced GW (= Induced GW = Secondary GW = Second-order GW)

Inflationary GW



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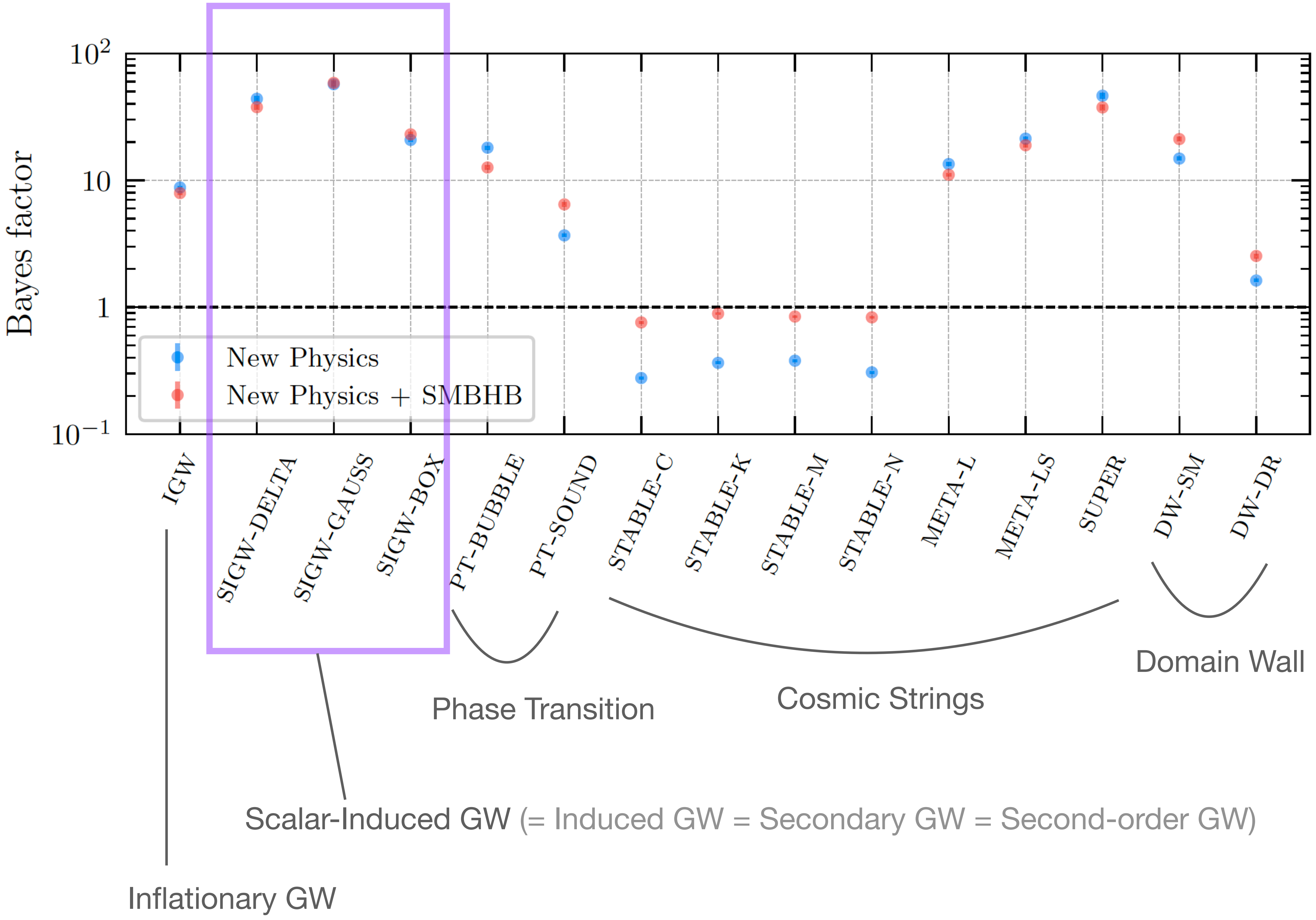
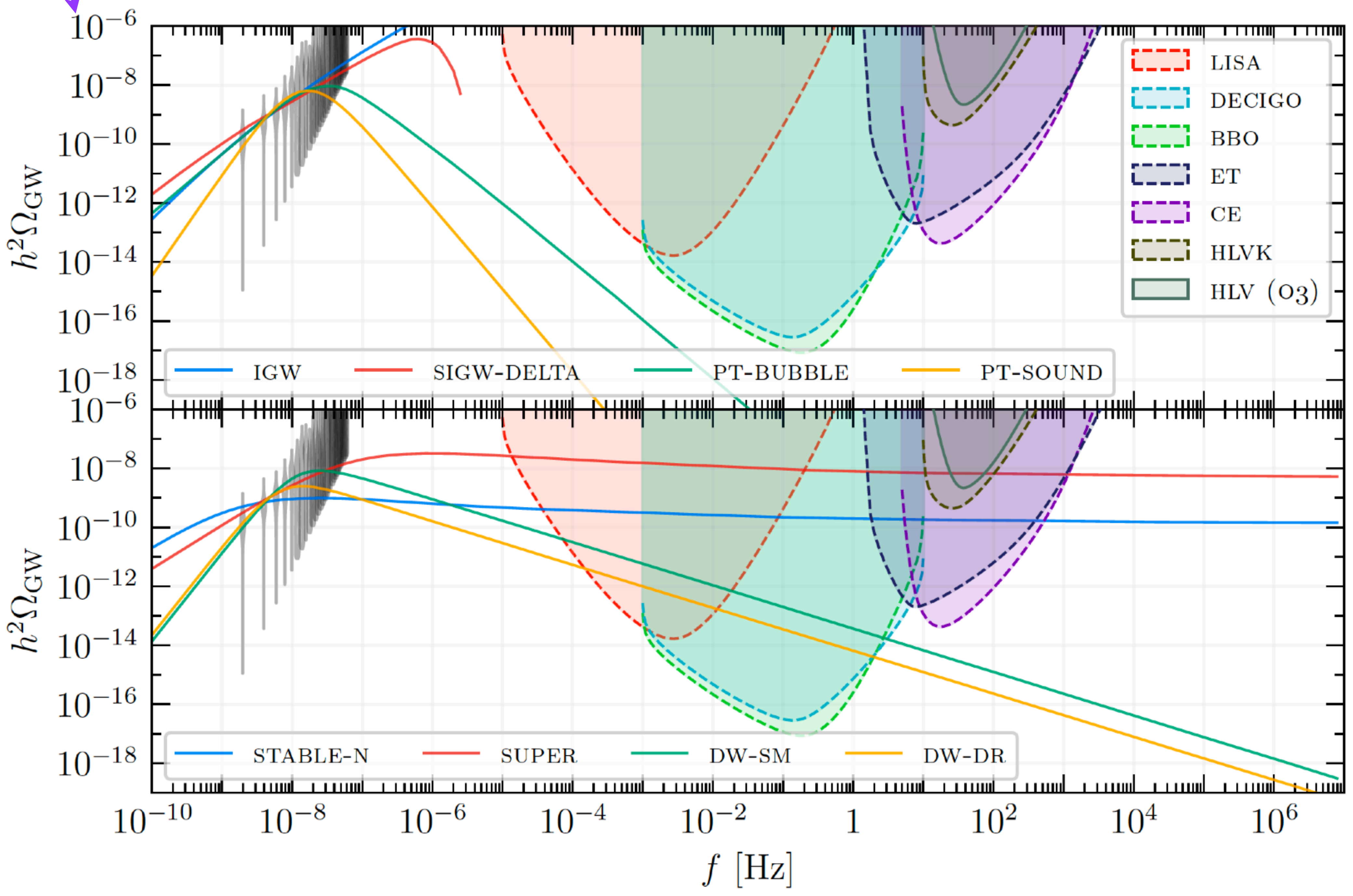
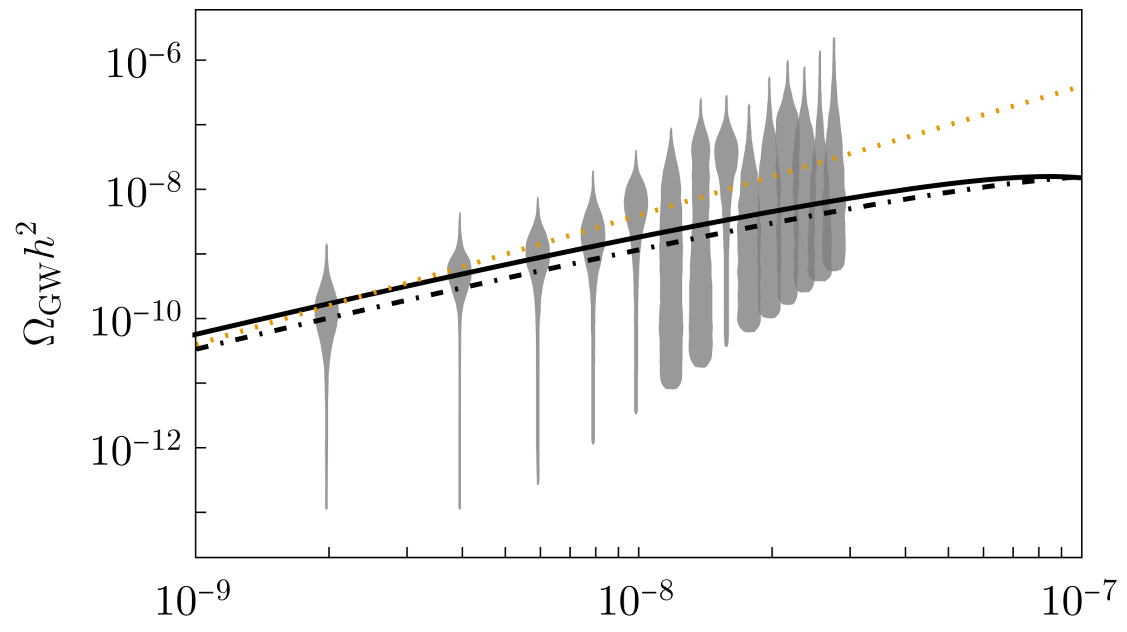


Note on the dark-radiation bound from BBN $\Omega_{\rm GW}h^2 < 1.8\times10^{-6}$.
[Kohri, Terada, 1804.08577]

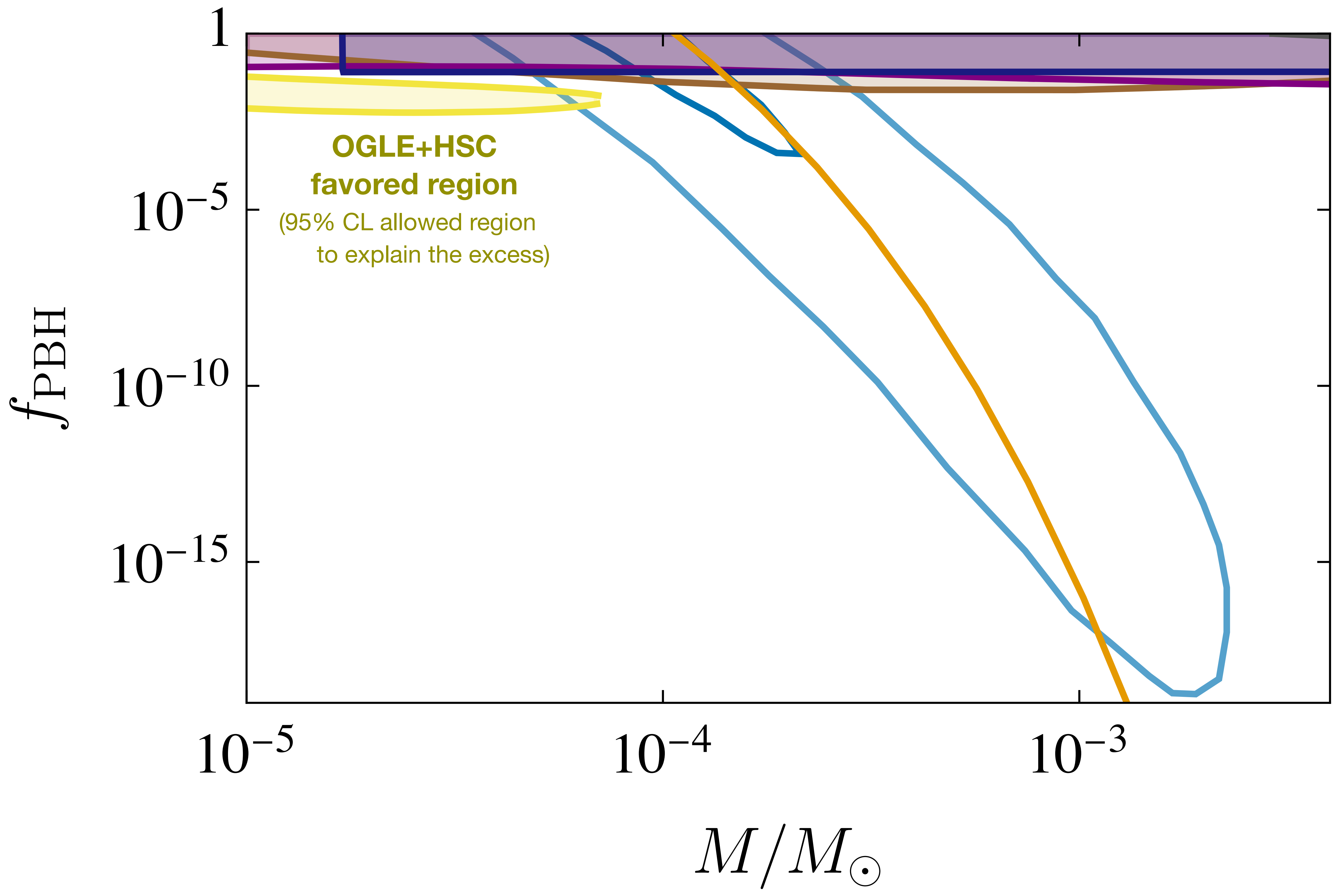


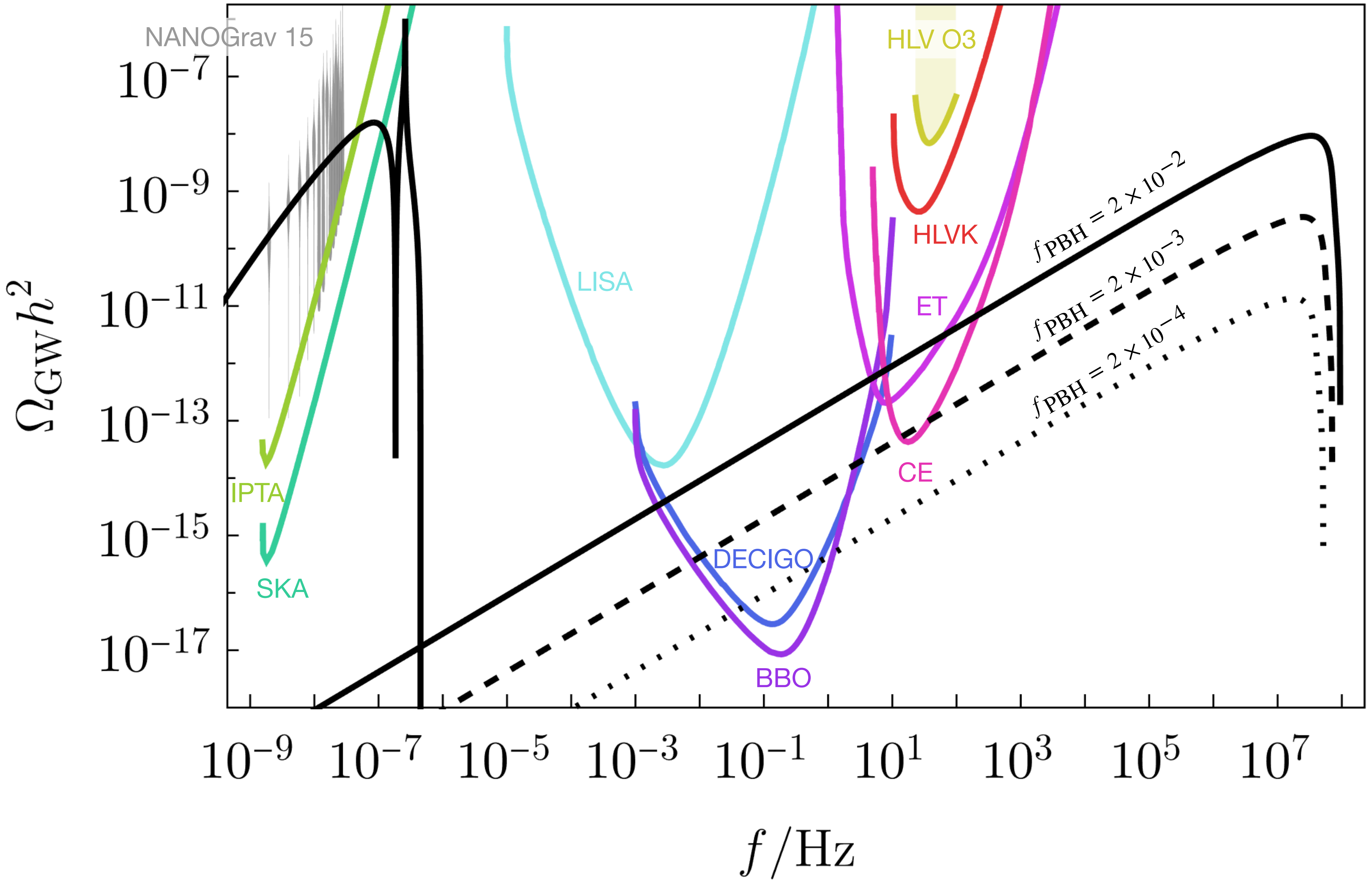
Implications for Primordial Black Holes

[Inomata, Kohri, Terada, 2306.17834]
See also [Franciolini et al., 2306.17149]



The excess events of microlensing at OGLE: [Mróz et al., 1707.07634]
Interpretation by PBHs: [Niikura et al., 1901.07120]





$M/M_\odot = 1.2\times10^{-4}, 1.6\times10^{-4}$, and 2.2×10^{-4} from top to bottom.
The sensitivity curves were taken from [Schmitz, 2002.04615].
The HLV O3 constraint is from [Abbott et al. (LIGO-Virgo-KAGRA), 2101.12130].

Significance of Induced Gravitational Waves

(Scalar-)induced GWs = Gravitational waves induced by curvature perturbations

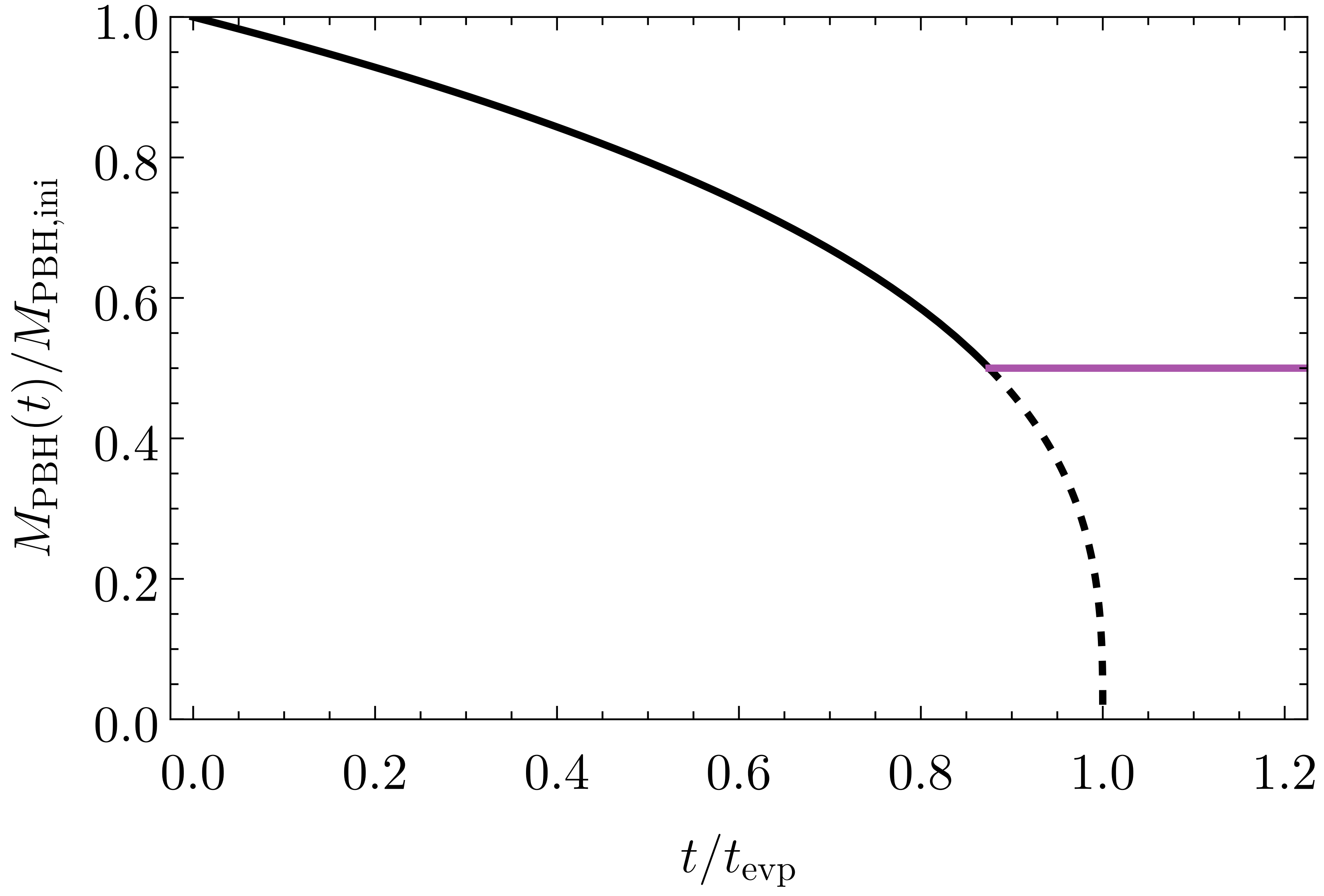
Why are they important?

Using them, we can

- ✓ probe inflation
- probe thermal history (equation of state)
- ✓ explain the pulsar timing array (PTA) data
- ✓ test the primordial black hole (PBH) scenario
- ➡ test ideas in quantum gravity

Memory Burden Effect Opens Light PBH DM Window

A conjectured effect that effectively stabilizes a complex system storing much information (memory). [Dvali, 1810.02336], [Dvali, Eisemann, Michel, and Zell, 2006.00011]

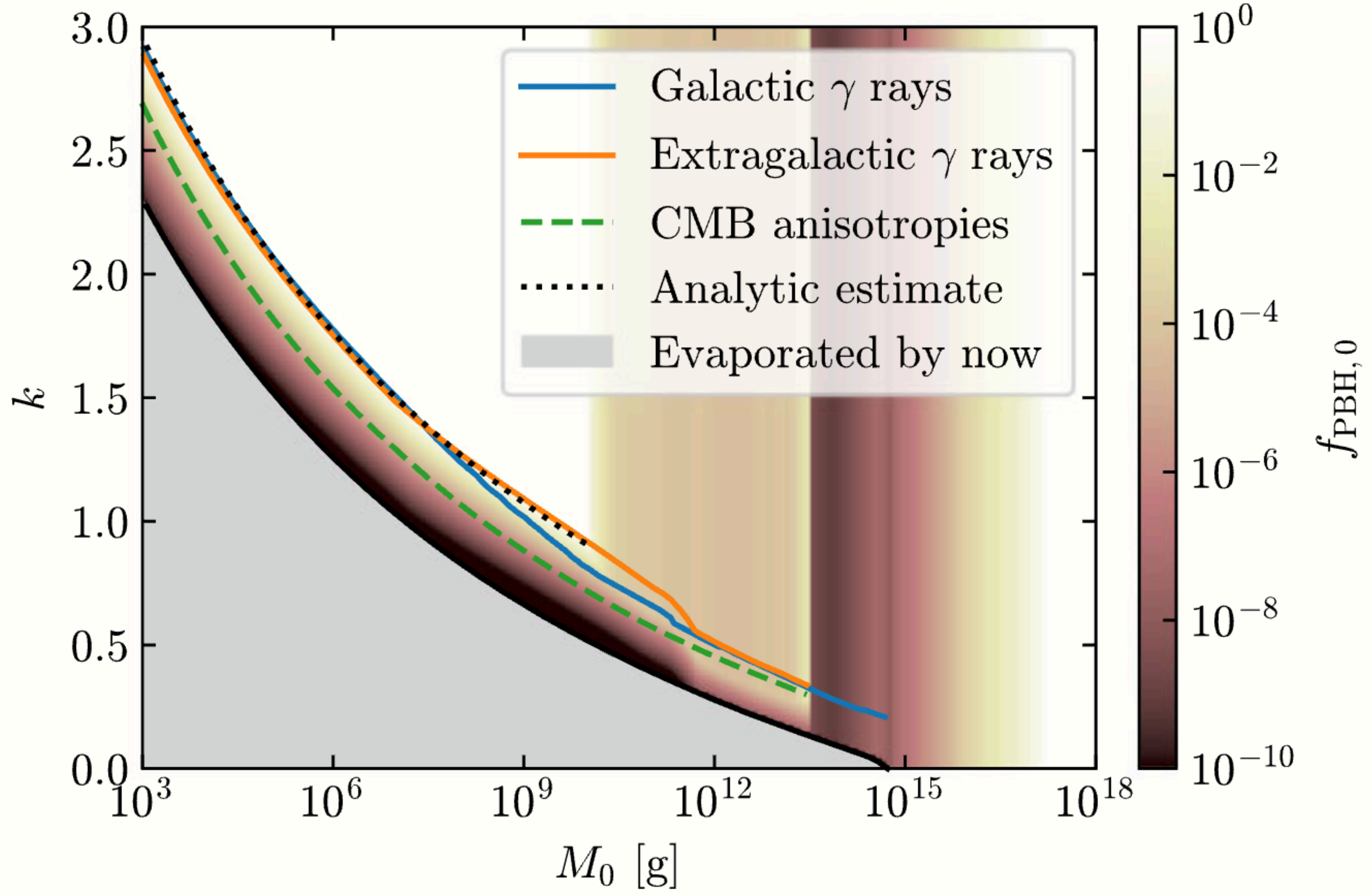


With the memory burden effect,

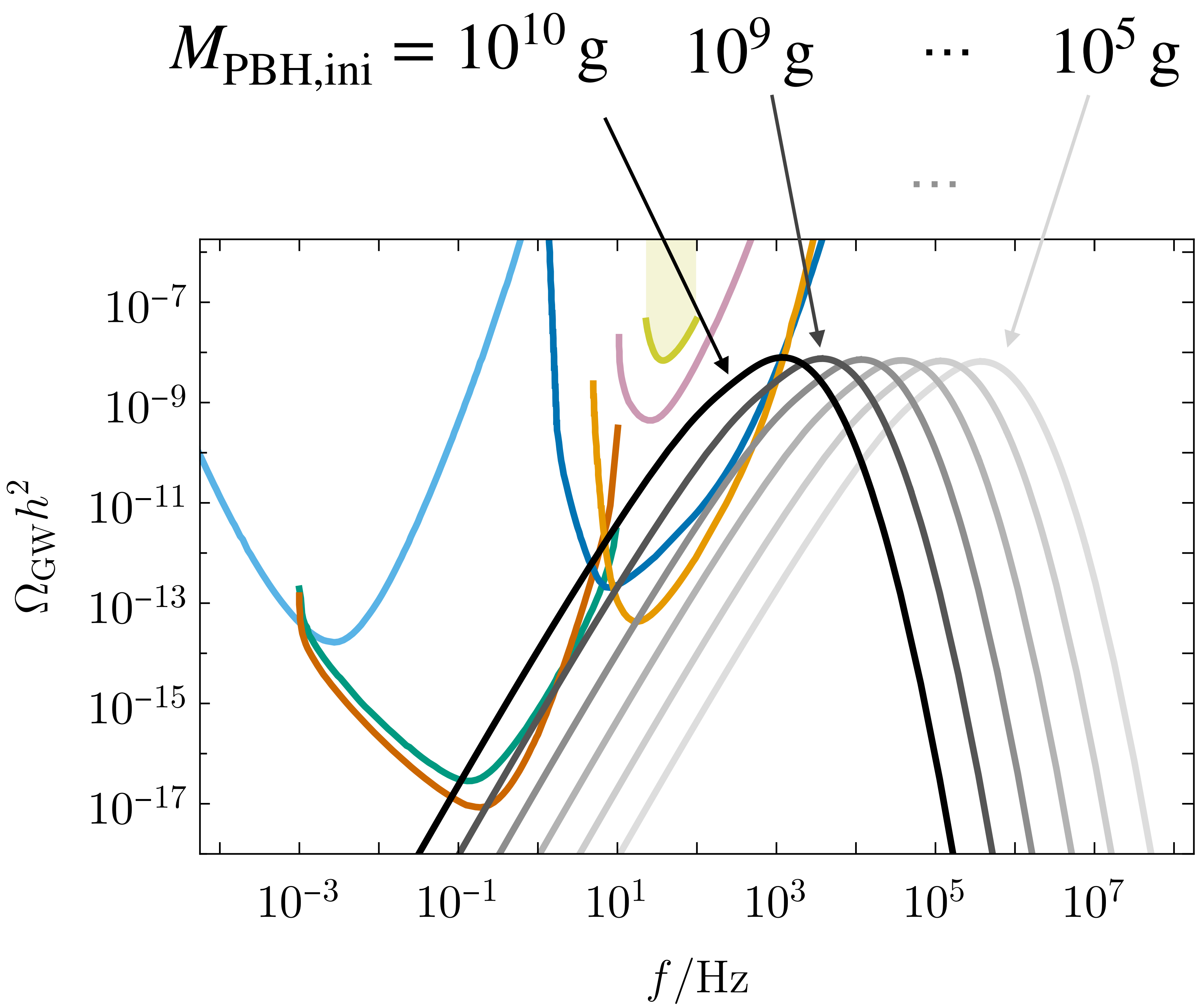
PBHs lighter than 10^{10}g can be dark matter.

[Thoss, Burkert, and Kohri, 2402.17823]

See also [Alexandre, Dvali, and Koutsangelas, 2402.14069]



To Exclude the Memory Burden Effect



$$\mathscr{P}_\zeta(k) = \frac{A_\zeta}{\sqrt{2\pi\Delta^2}} \exp\left(- \frac{\left(\ln \frac{k}{k_0}\right)^2}{2\Delta^2} \right) \qquad \Delta = 1$$

If the GW signal is stronger than the predicted one for the PBHs as the totality of dark matter, it would overproduce dark matter.

(Without the memory burden effect, light PBHs have evaporated away.)

Therefore, **we can exclude the memory burden effect.**

[Kohri, Terada, and Yanagida, 2409.06365]

Matter-Dominated Era and Induced Gravitational Waves

Early Matter-Dominated Era

Heavy particles: gravitino, modulino, etc.

Bose-Einstein condensates: inflaton, curvaton, moduli, dilaton, etc.

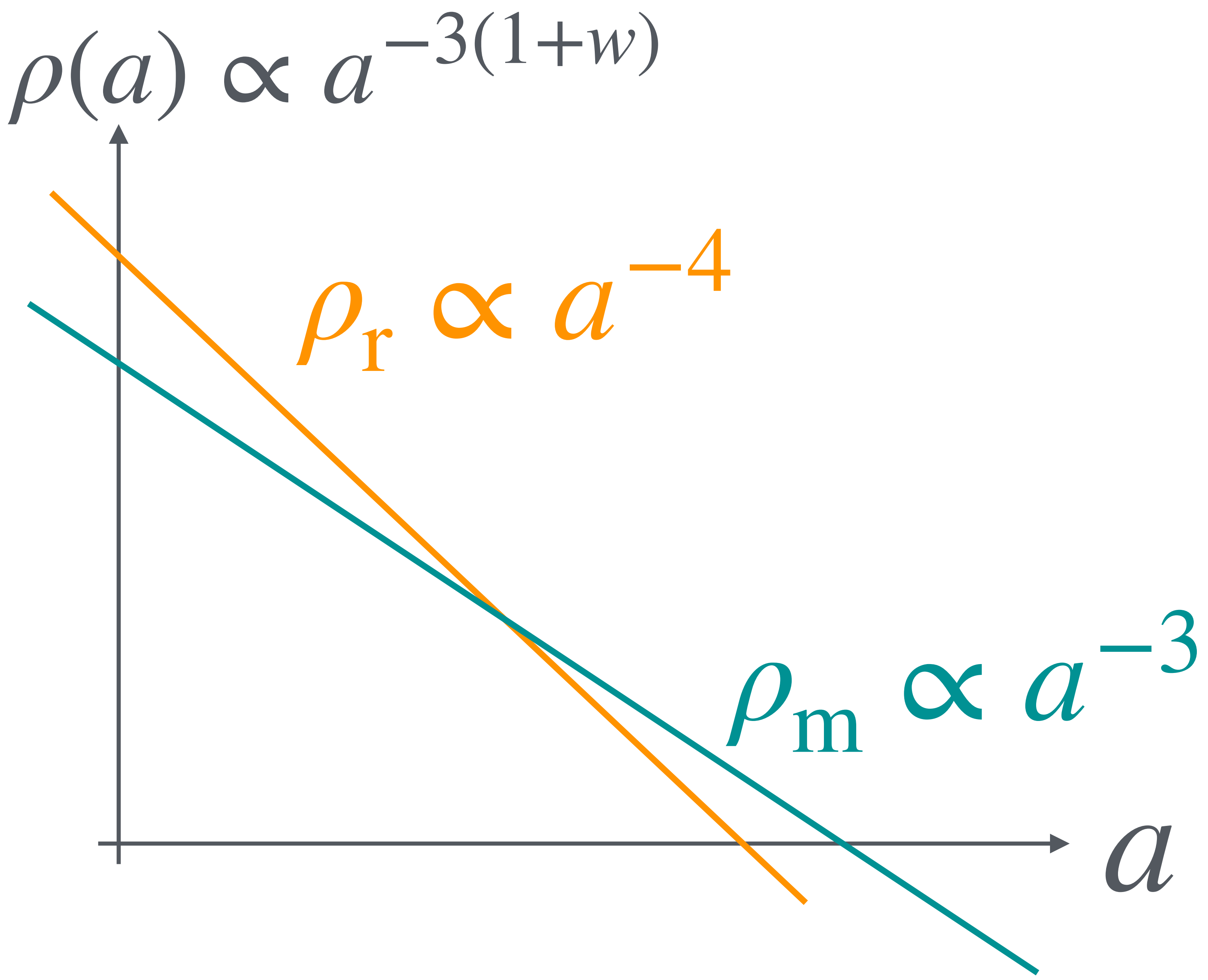
Macroscopic objects: primordial black hole (PBH), oscillon, Q-ball, etc.

For a canonically normalized homogeneous scalar field ϕ,

$$w = \frac{P}{\rho} = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)} \simeq \cos(2mt) \quad \Rightarrow \quad w_{\rm avg} = 0$$

Generically quadratic around the minimum

$$V(\phi) \simeq \frac{1}{2}m^2\phi^2, \quad \phi \simeq \phi_0 \cos(mt)/a(t)^{3/2}$$



MD Era: Enhancement Effect?

[Assadullahi, Wands, 0901.0989]
[Baumann, Steinhardt, Takahashi, Ichiki, hep-th/0703290]

$$\frac{\delta\rho}{\rho} \propto a(\eta) \quad \Longleftrightarrow \quad \Phi = \text{const.}$$

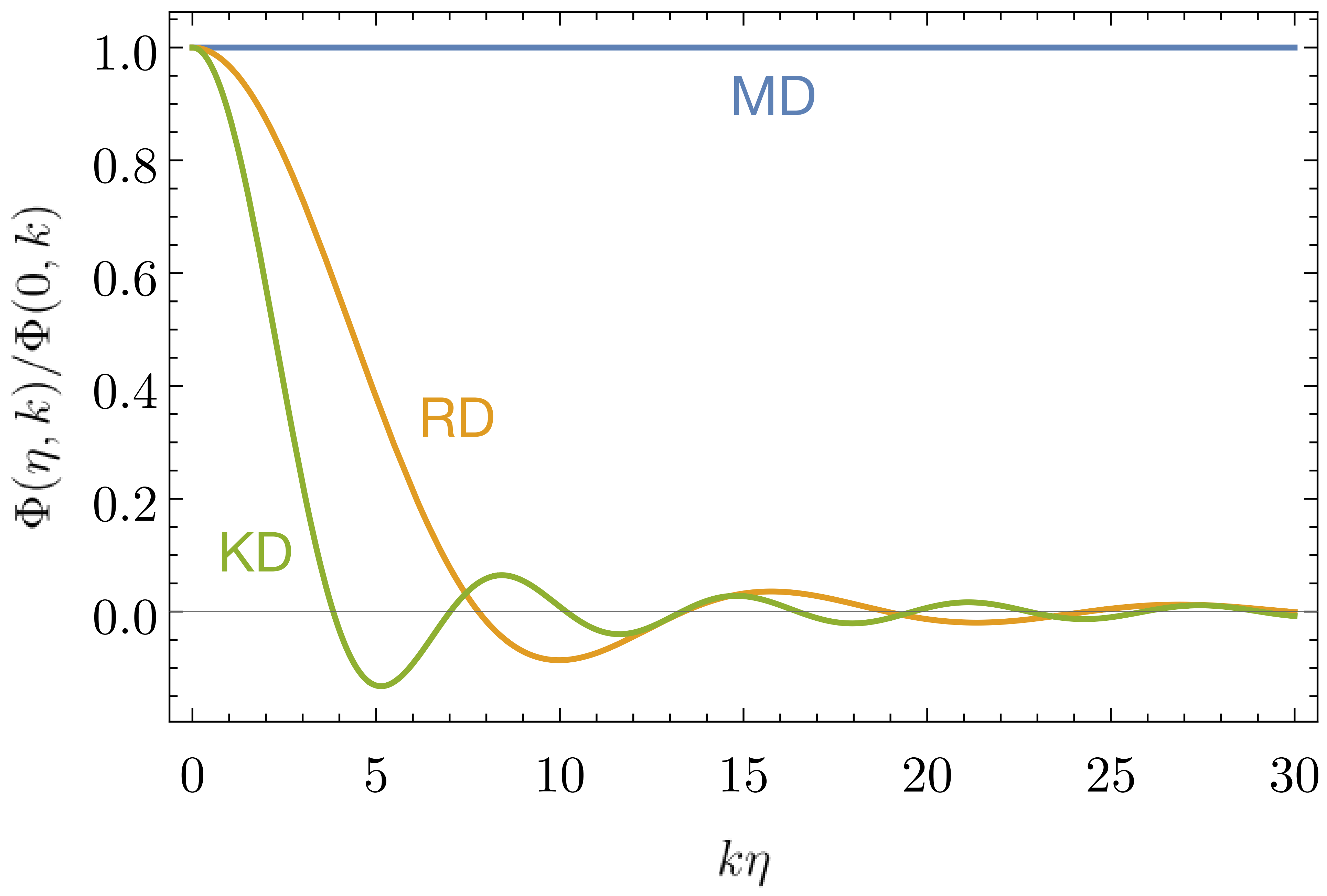
This makes the source term constant.

$$\cancel{h''_{\mathbf{k}}(\eta)} + \cancel{2\mathcal{H}h'_{\mathbf{k}}(\eta)} + k^2 h_{\mathbf{k}}(\eta) = 4S_{\mathbf{k}}(\eta)$$

Constant metric distortion ~ Potential energy
(+ decaying mode)

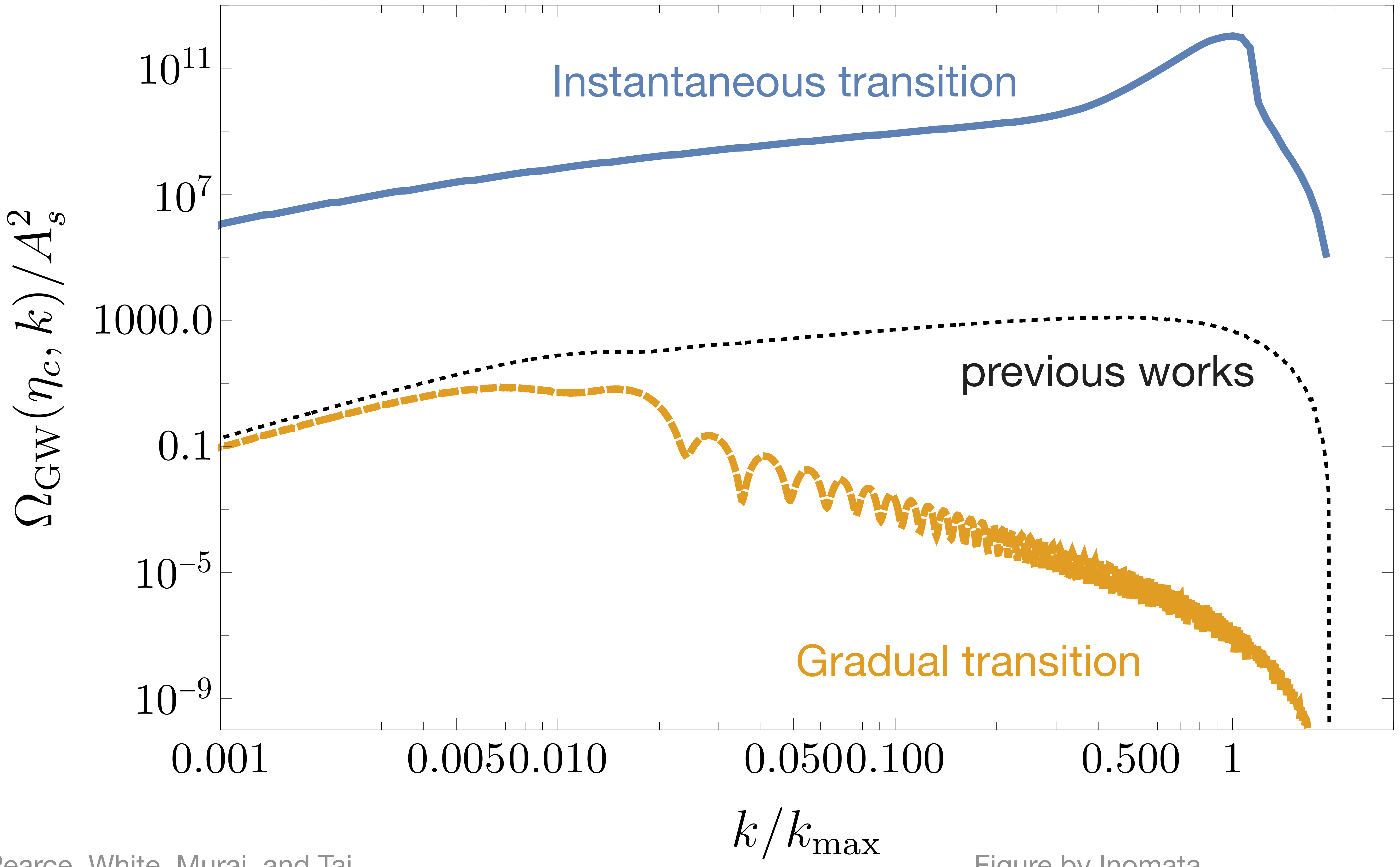
After MD ⇒ **Propagating GW**

How efficient is the conversion?



Importance of Transition Speed

It depends sensitively on the time scale of the transition (MD to RD).



$\mathscr{P}_\zeta = A_s$ with cutoff(s)

$k_{\rm max} = 450/\eta_{\rm R}$

$(k_{\rm min} = 30/\eta_{\rm eq})$

Thanks to Pearce, Balasz, Pearce, White, Murai, and Tai for pointing out errors in the previous version.

Figure by Inomata based on [Inomata, Kohri, Nakama, and Terada, 1904.12878; 1904.12879]

Gradual Reheating Transition

[Inomata, Kohri, Nakama, Terada, 1904.12878]

<u>This is the standard case.</u>

Suppose a constant decay rate Γ
of the field dominating the MD era.

Decay becomes effective when $H \sim \Gamma$.
The time scale of the decay

$$\Gamma^{-1} \sim H^{-1} \gg f^{-1}$$

f : frequency of the GW mode
on subhorizon scales

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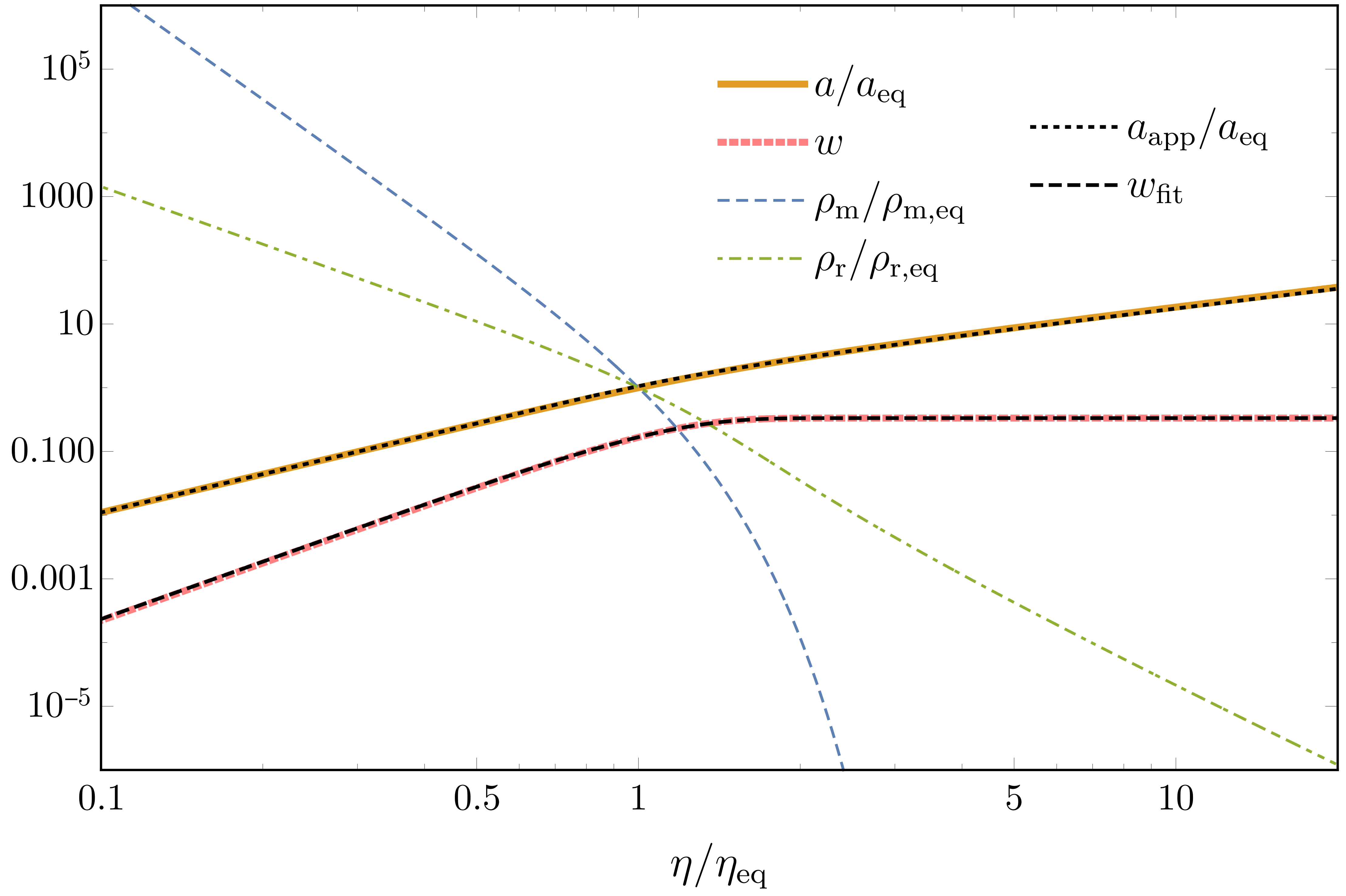
Background evolution



Equations for background quantities

$$\mathscr{H}^2 = \frac{a^2}{3}\rho_{\rm tot}$$
$$\rho_m' + 3\mathscr{H}\rho_m = -a\Gamma\rho_m$$
$$\rho_r' + 4\mathscr{H}\rho_r = a\Gamma\rho_m$$

Good approximations:

$$\frac{a_{\rm app}(\eta)}{a_{\rm app}(\eta_R)} = \begin{cases} (\eta/\eta_R)^2 & (\eta < \eta_R) \\ 2(\eta/\eta_R) - 1 & (\eta \geq \eta_R) \end{cases}$$

with $\eta_R \equiv 0.83\eta_{\rm eq}$

$$w_{\rm fit} = \frac{1}{3}\left(1 - \exp\left(-0.7\left(\eta/\eta_R\right)^3\right)\right)$$

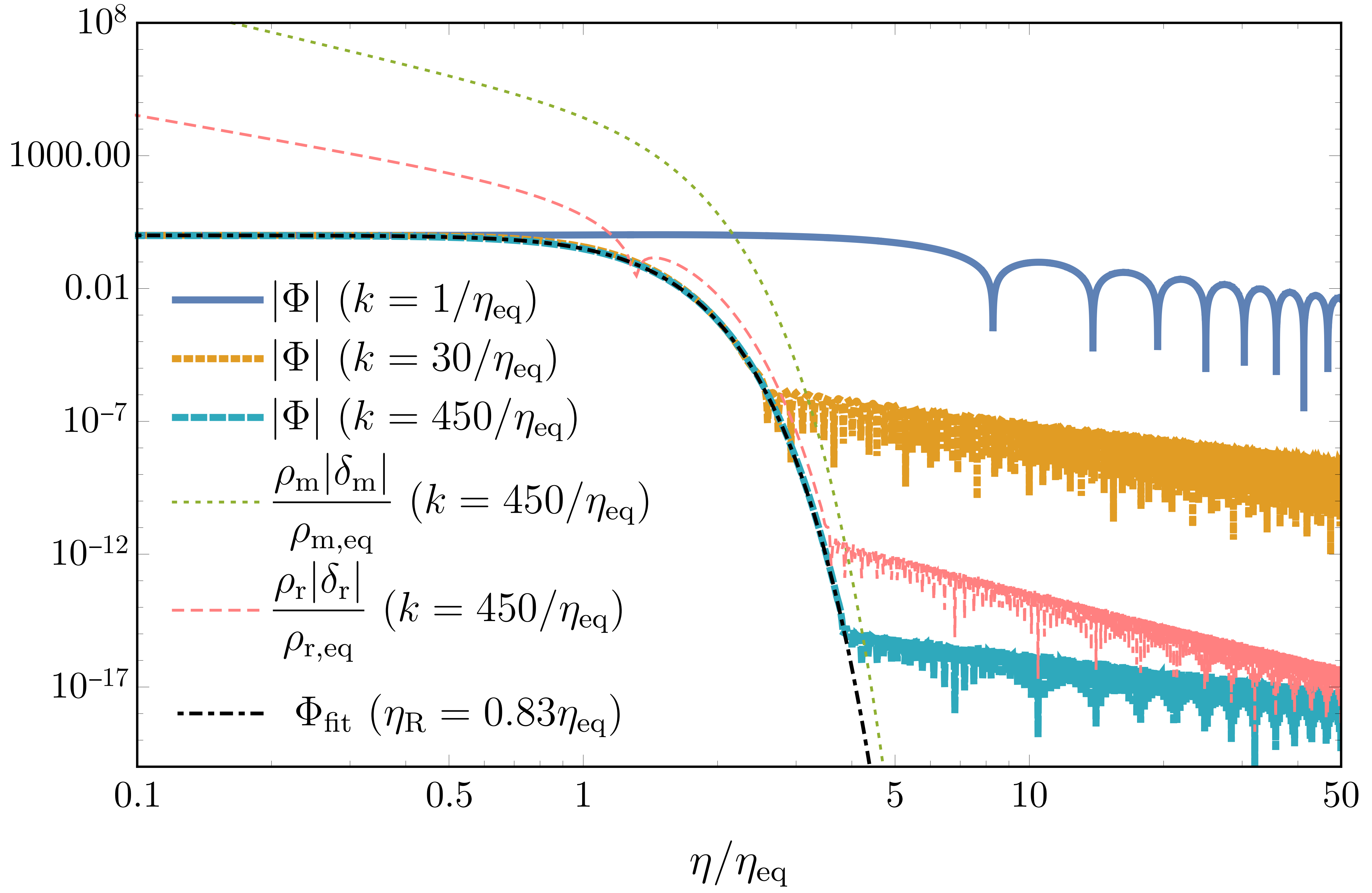
Gradual Reheating Transition

[Inomata, Kohri, Nakama, Terada, 1904.12878]

[Poulin, Serpico, Lesgourgues, 1606.02073]

[textbook by Mukhanov, Cambridge, 2005]

Perturbation evolution



Equations for perturbation quantities

density contrast $\delta_x \equiv \frac{\delta\rho_x}{\rho_x}$

velocity divergence $\theta_x = \frac{1}{a}\nabla \cdot u_x$

$$\delta_m' = -\theta_m + 3\Phi' - a\Gamma\Phi$$

$$\theta_m' = -\mathscr{H}\theta_m + k^2\Phi$$

$$\delta_r' = -\frac{4}{3}(\theta_r - 3\Phi') + a\Gamma\frac{\rho_m}{\rho_r}(\delta_m - \delta_r + \Phi)$$

$$\theta_r' = \frac{k^2}{4}\delta_r + k^2\Phi - a\Gamma\frac{3\rho_m}{4\rho_r}\left(\frac{4}{3}\theta_r - \theta_m\right)$$

$$\Phi' = -\frac{k^2\Phi + 3\mathscr{H}^2\Phi + \frac{3}{2}\mathscr{H}^2\left(\frac{\rho_m}{\rho_{\rm tot}}\delta_m + \frac{\rho_r}{\rho_{\rm tot}}\delta_r\right)}{3\mathscr{H}}$$

Adiabatic initial conditions

$$\delta_m = -2\Phi, \quad \delta_r = \frac{4}{3}\delta_m, \quad \theta_m = \theta_r = \frac{k^2\eta}{3}\Phi$$

Gradual Reheating Transition

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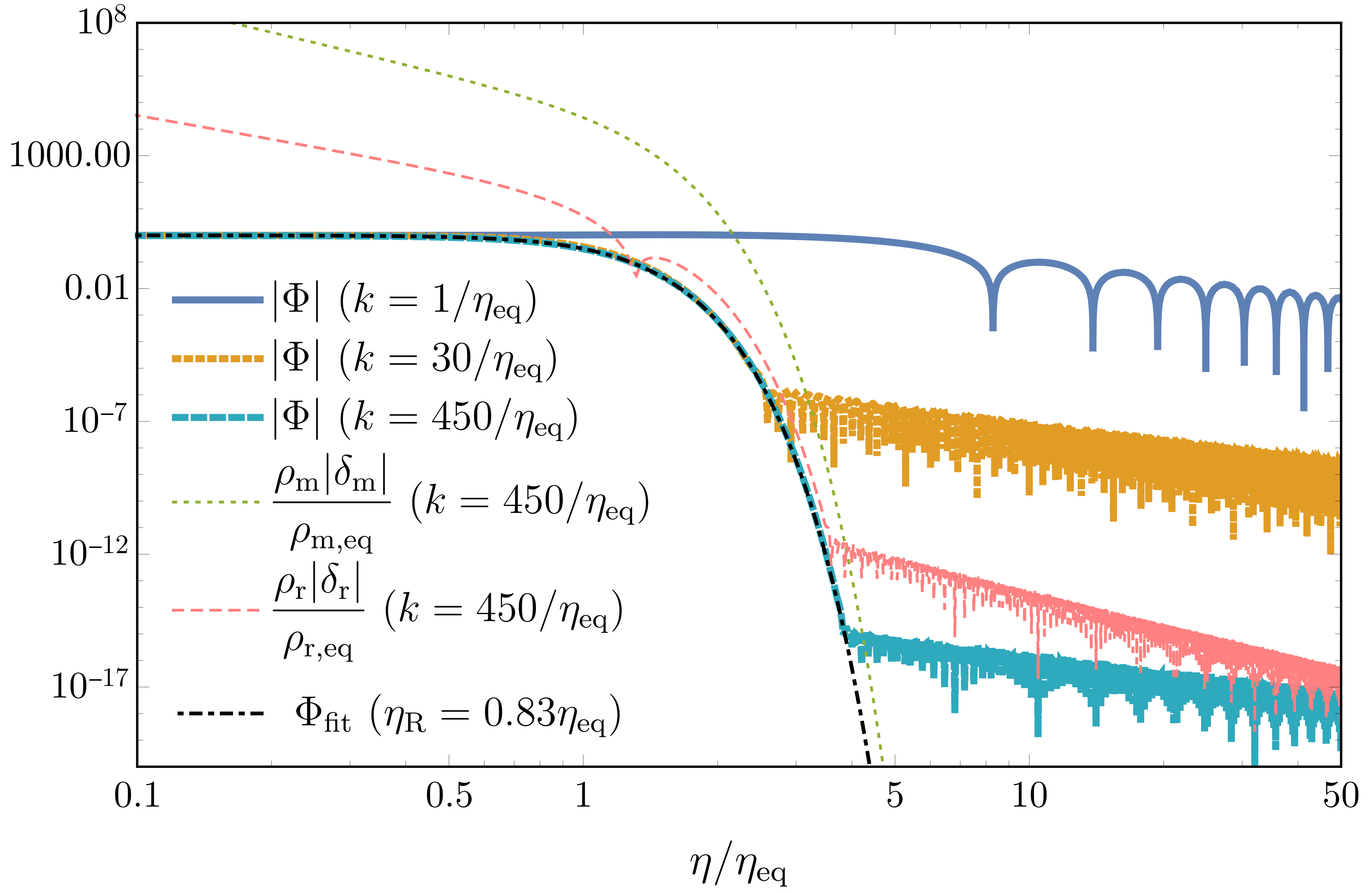

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Adiabatic initial conditions

$$\delta_m = -2\Phi, \quad \delta_r = \frac{4}{3}\delta_m, \quad \theta_m = \theta_r = \frac{k^2\eta}{3}\Phi$$

Because of the growth of matter density perturbations,
$\delta\rho_m > \delta\rho_r$ *even after the equality* ($\rho_m = \rho_r$), so the decay of $\delta\rho_m$ controls Φ.

$\Phi_{\rm fit} = \exp(-\Gamma t)$

Suppression of induced-GWs

$\Omega_{\rm GW} \sim \Phi^4 \sim e^{-4\Gamma t}$

Sudden Reheating Transition

[Inomata, Kohri, Nakama, Terada, 1904.12879]

comparison of the behavior of Φ in sudden/gradual transitions

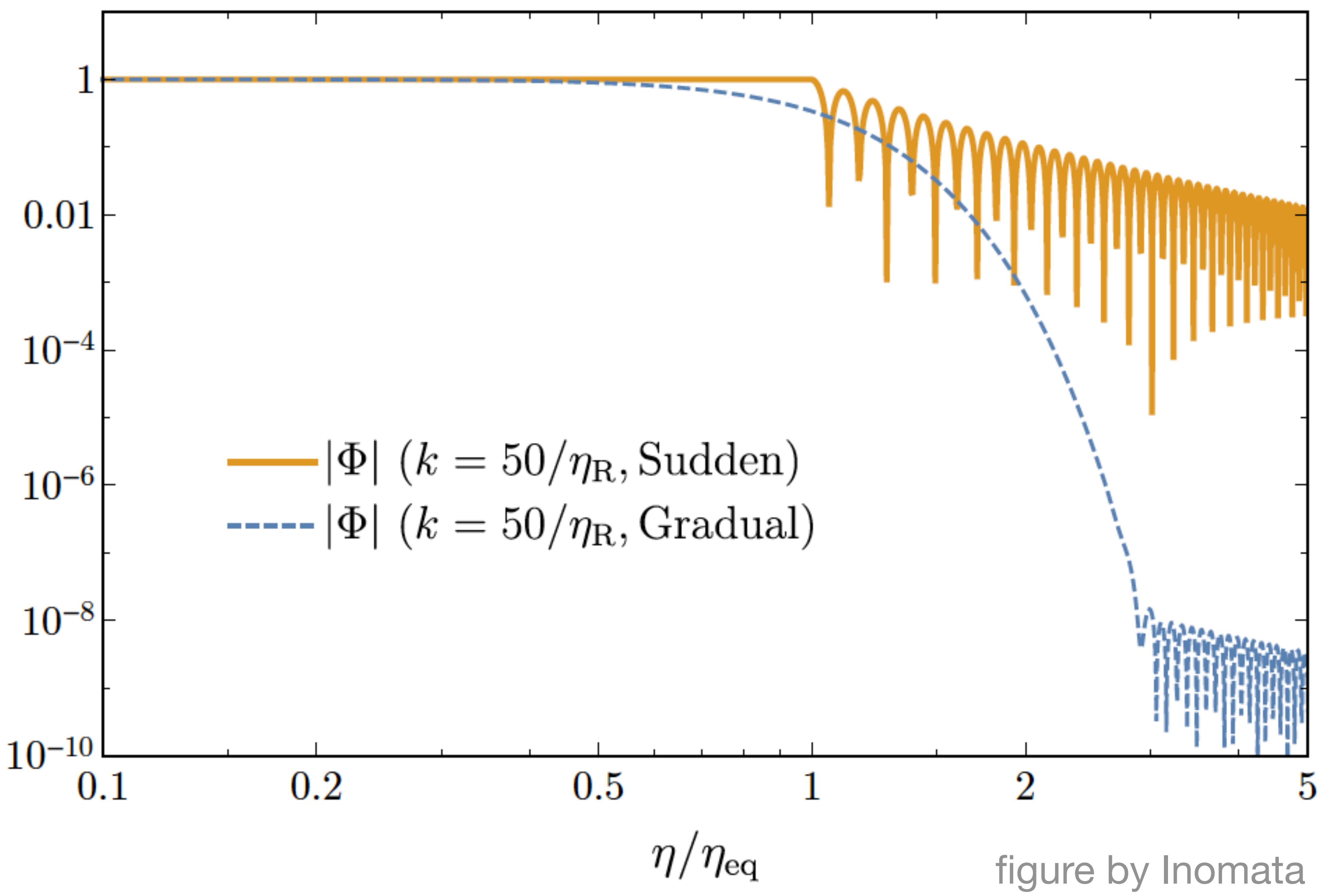

figure by Inomata

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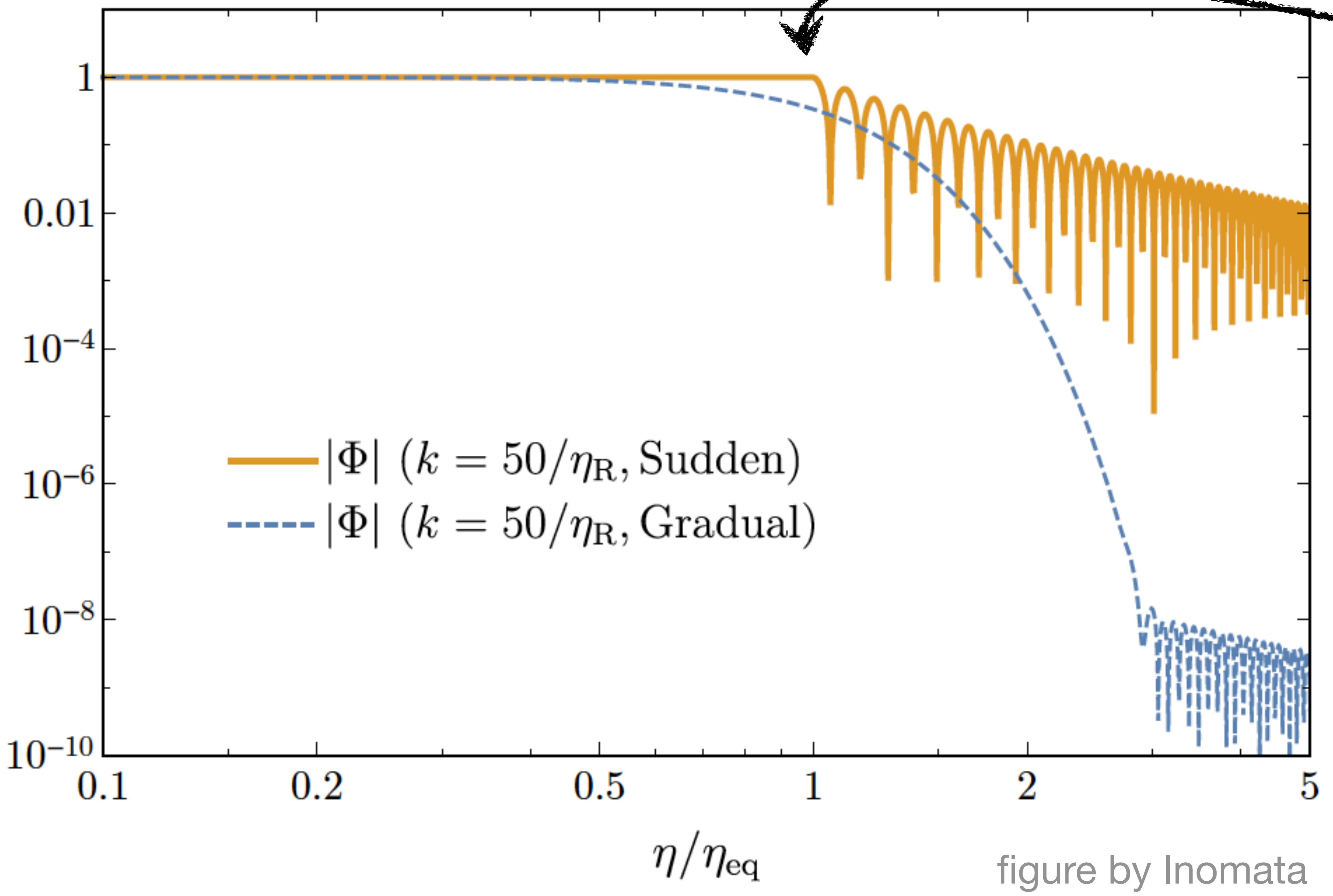

figure by Inomata

No Decay until reheating

Enhanced contribution of GWs from the MD era (known effect).

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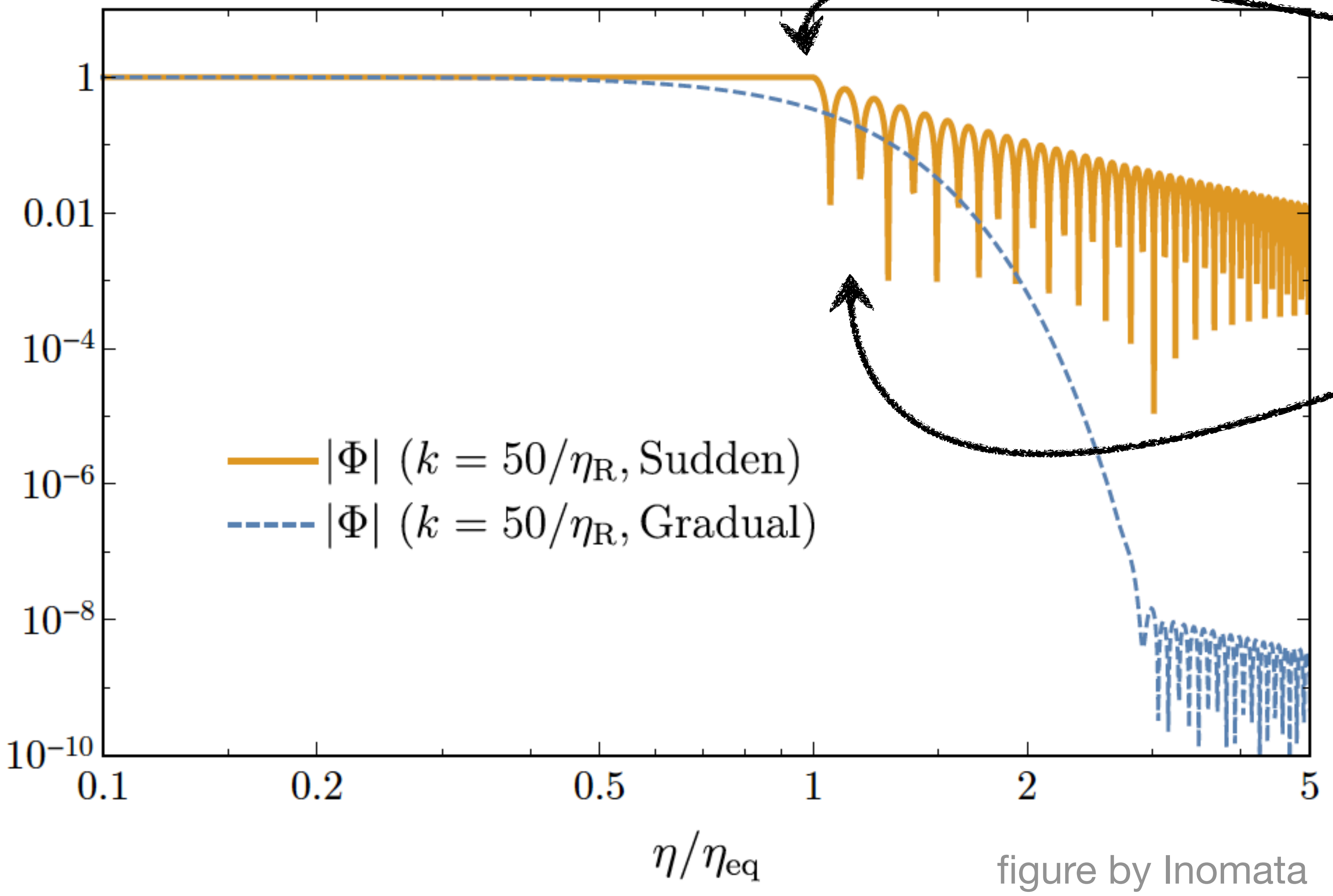

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Fast oscillations due to a hierarchical ratio

$$\frac{k}{\mathcal{H}(\eta)} \gg 1$$

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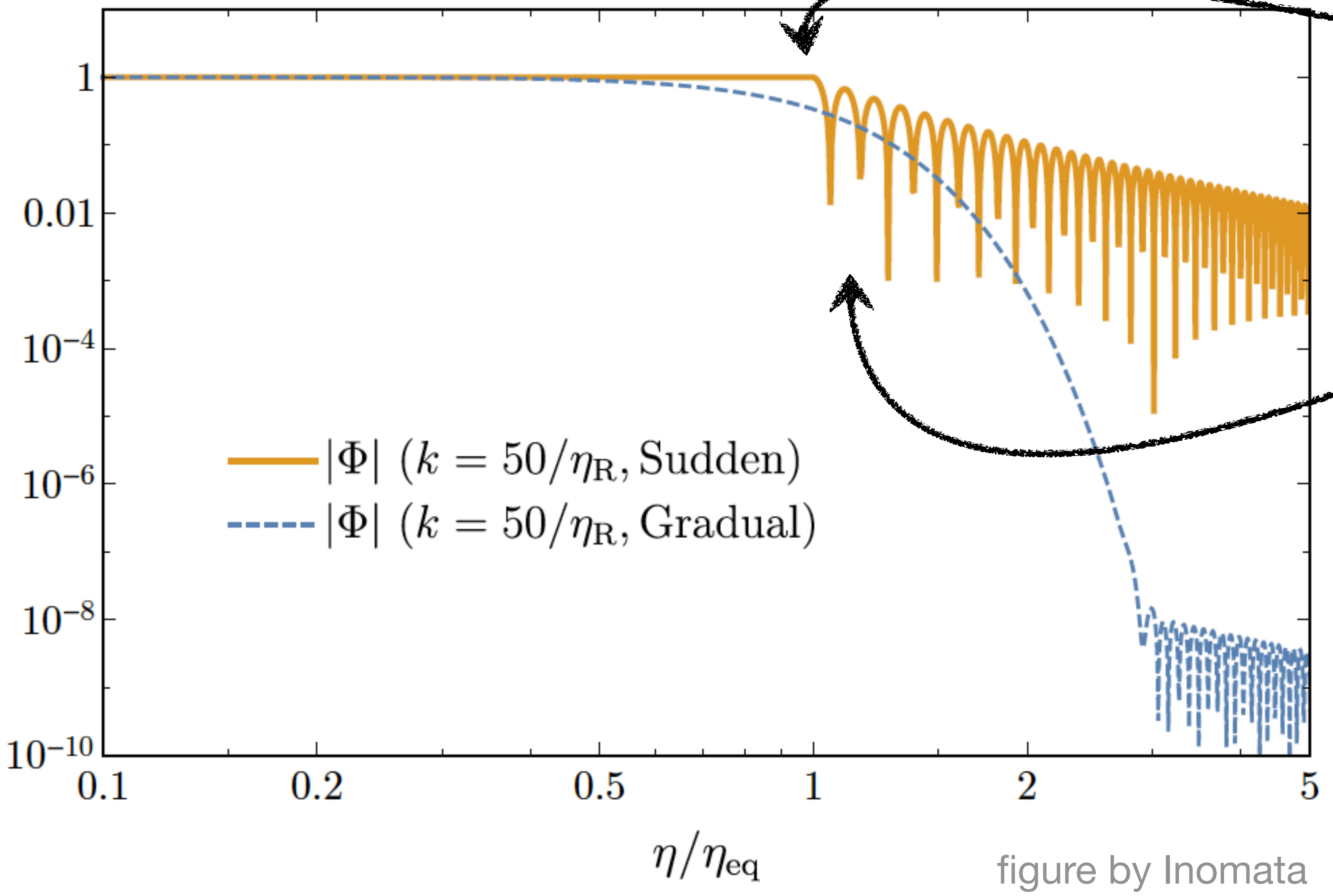

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(Quasi-) Sudden Reheating Examples

- "Triggeron" model (kinematic blocking)
- Dark sector 1st-order phase transition
  [Inomata, Kohri, Nakama, Terada, 1904.12879]
- Black-hole domination & evaporation
  [Inomata, Kawasaki, Mukaida, Terada, Yanagida, 2003.10455]
- Q-ball domination & evaporation
  [White, Pearce, Vagie, Kusenko, 2105.11655]
  [Kasuya, Kawasaki, Murai, 2212.13370]
- I-ball/oscillon domination & evaporation
  [Lozanov, Takhistov, 2204.07152]
- Reheating by some higher-dim. operators
  [Co, Gonzalez, Harigaya, 2007.04328]

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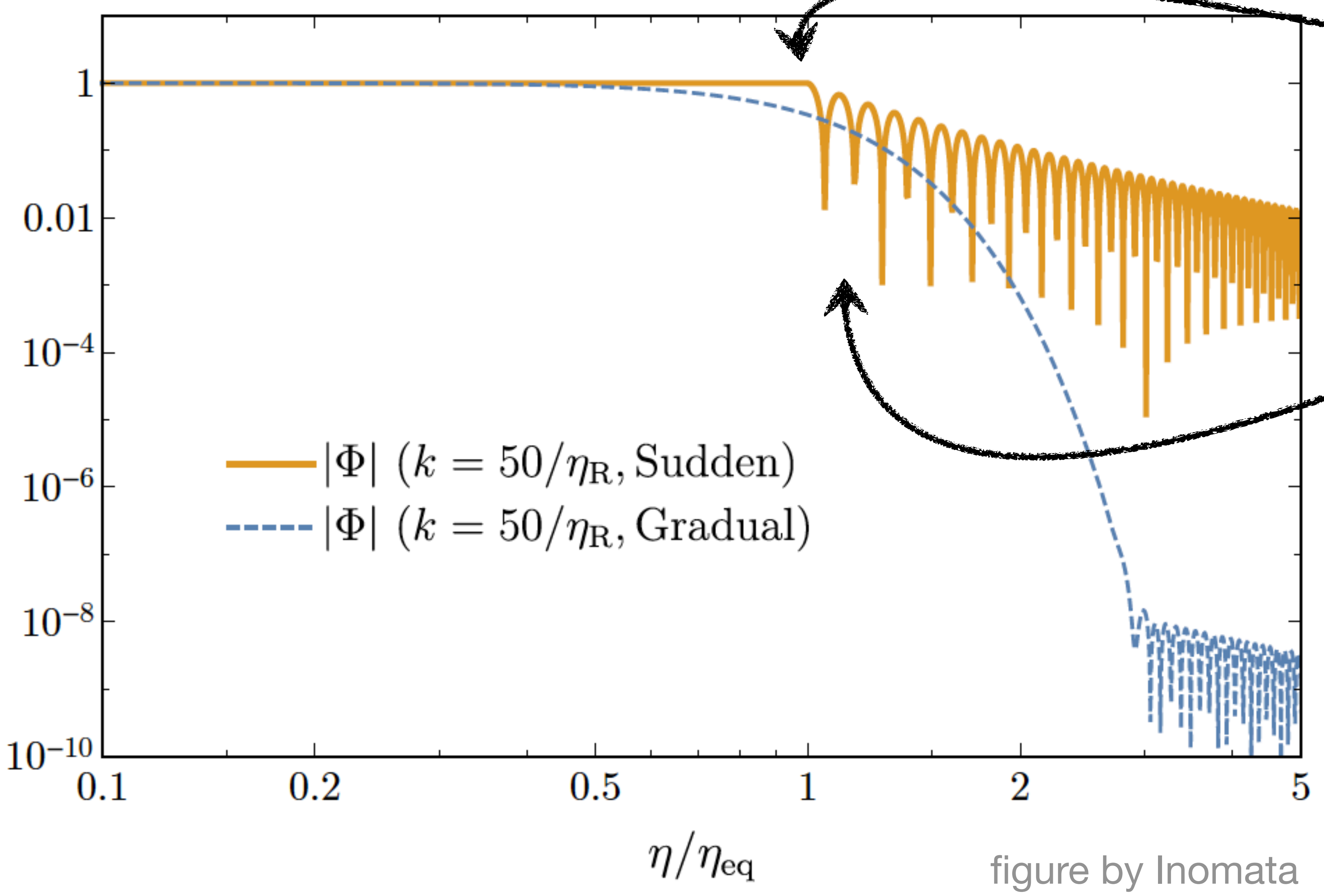

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  [Lozanov, Takhistov, 2204.07152]
- Reheating by some higher-dim. operators
  [Co, Gonzalez, Harigaya, 2007.04328]

Poltergeist

mechanism for gravitational wave production

Poltergeist

Article Talk

From Wikipedia, the free encyclopedia

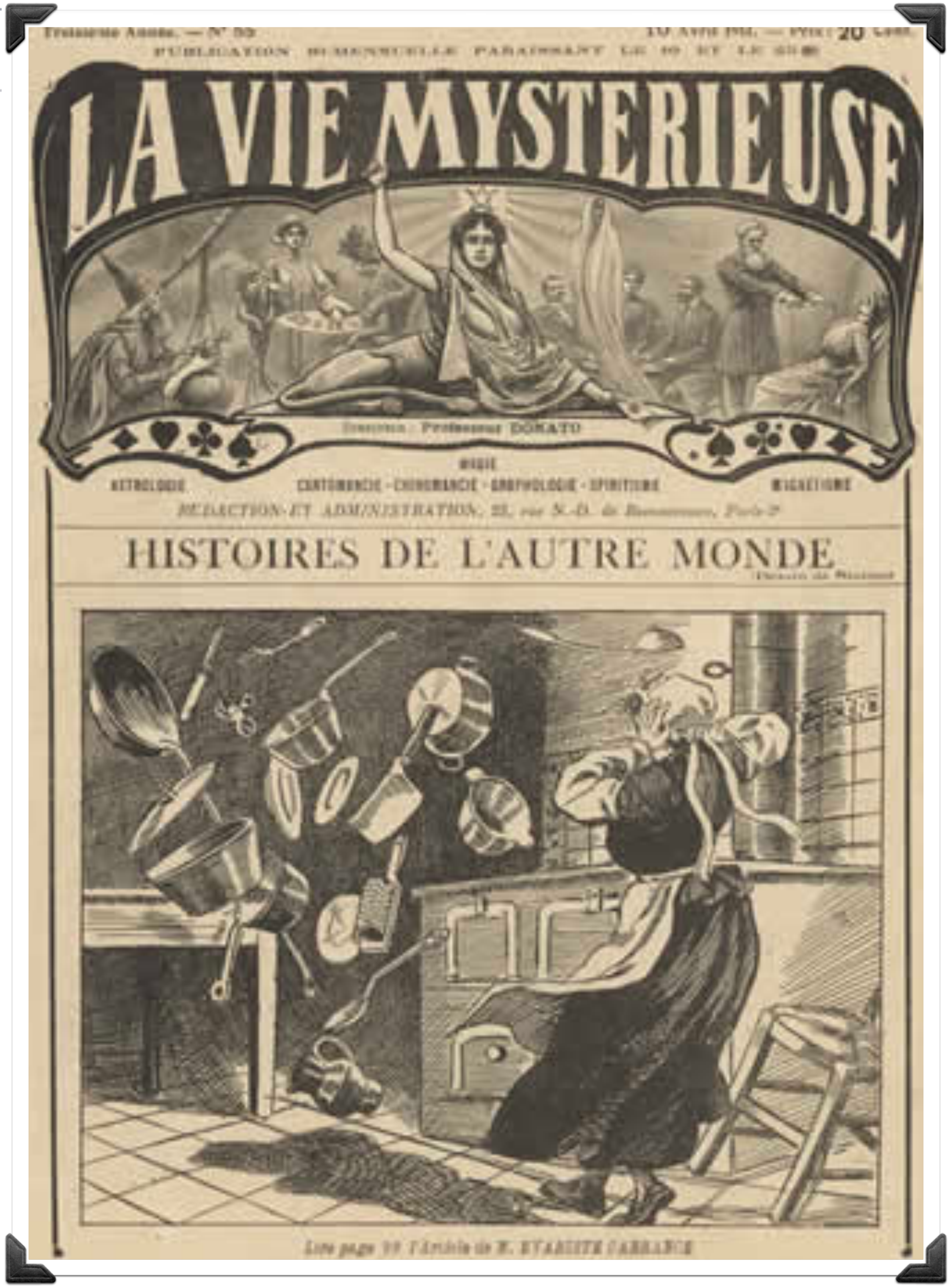
For other uses, see Poltergeist (disambiguation).

In ghostlore, a **poltergeist** (/ˈpoʊltərˌɡaɪst/ or /ˈpɒltərˌɡaɪst/; German for "rumbling ghost" or "noisy spirit") is a type of ghost or spirit that is responsible for physical disturbances, such as loud noises and objects being moved or destroyed. Most claims or fictional descriptions of poltergeists show them as being capable of pinching, biting, hitting, and tripping people. They are also depicted as capable of the movement or levitation of objects such as furniture and cutlery, or noises such as knocking on doors. Foul smells are also associated with poltergeist occurrences, as well as spontaneous fires and different electrical issues such as flickering lights. [1]

They have traditionally been described as troublesome spirits who haunt a particular person instead of a specific location. Some variation of poltergeist folklore is found in many different cultures. Early claims of spirits that supposedly harass and torment their victims date back to the 1st century, but references to poltergeists became more common in the early 17th century.

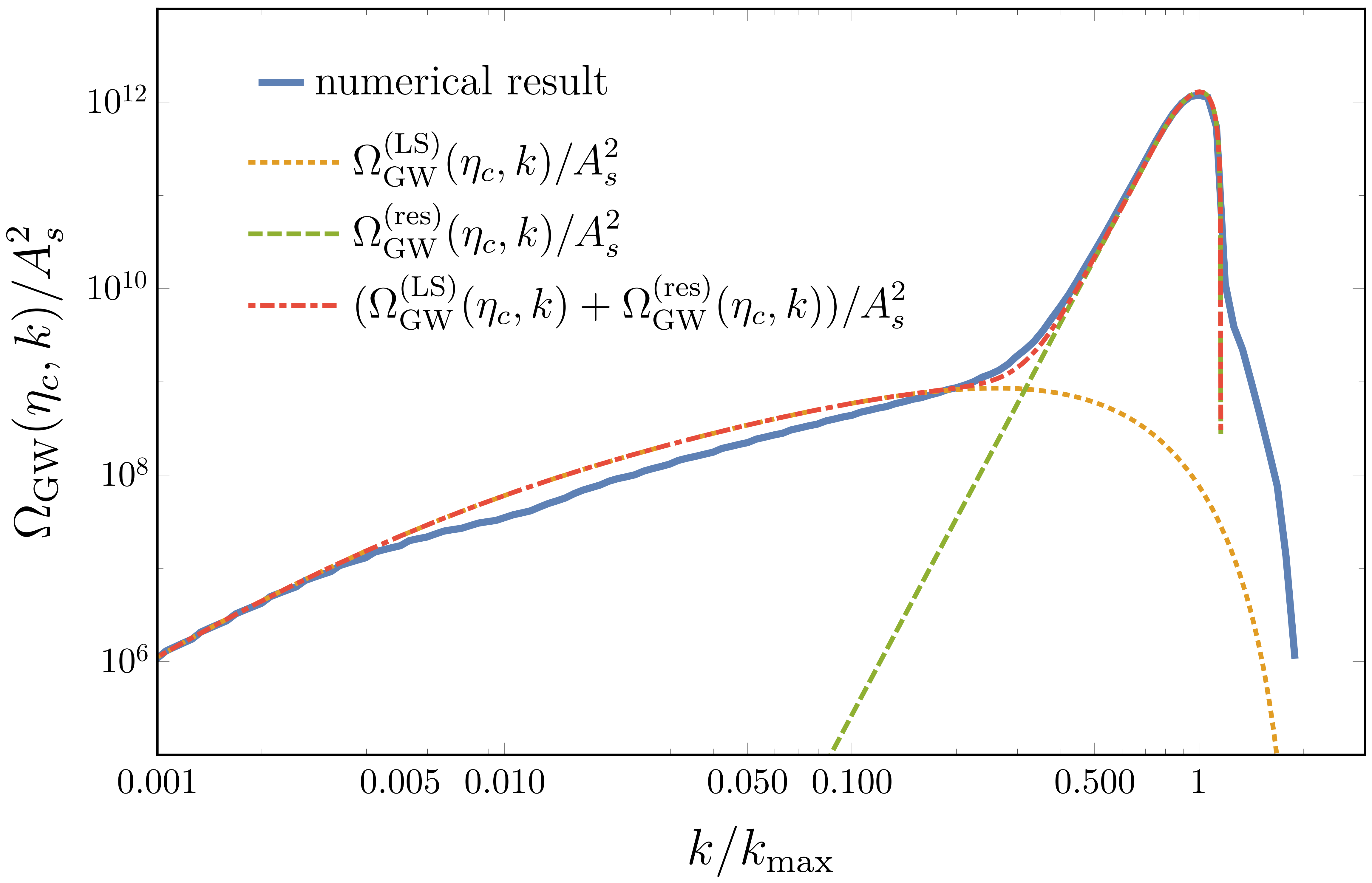
as if "ghost" of matter
is shaking the bath

Matter for MD era → reheating → Sound waves in Thermal Bath → Induced GWs



Sudden Reheating Transition

[Inomata, Kohri, Nakama, Terada, 1904.12879]



$k_{\rm max}$: a cutoff scale to avoid the nonlinear regime ($\delta_m(\eta_{\rm R},k_{\rm max})\sim 1$).

$$\Omega_{\rm GW}(\eta_{\rm c},k)\simeq\Omega_{\rm GW}^{\rm (LS)}(\eta_{\rm c},k)+\Omega_{\rm GW}^{\rm (res)}(\eta_{\rm c},k)$$

$$\Omega_{\rm GW,RD}^{\rm (LS)}(\eta_c,k)\simeq\frac{4{\rm Ci}\left(\frac{x_{\rm R}}{2}\right)^2+\left(\pi-2{\rm Si}\left(\frac{x_{\rm R}}{2}\right)\right)^2}{86016000000}A_{\rm s}^2x_{\rm R}^3x_{\rm max,R}^5\times$$
$$\left(\Theta(x_{\rm max,R}-x_{\rm R})\left(5376-17640\tilde{k}+23760\tilde{k}^2-16425\tilde{k}^3+5825\tilde{k}^4-847\tilde{k}^5\right)\right.$$
$$\left.+\Theta(x_{\rm R}-x_{\rm max,R})\tilde{k}^{-5}\left(2-\tilde{k}\right)^6\left(4-8\tilde{k}-9\tilde{k}^2+13\tilde{k}^3+49\tilde{k}^4\right)\right),$$

$$\Omega_{\rm GW}^{\rm (res)}(\eta_{\rm c},k)\simeq Y\frac{2.30285}{102400000}\sqrt{3}A_{\rm s}^2x_{\rm R}^7s_0(x_{\rm R})\left(15-10s_0^2(x_{\rm R})+3s_0^4(x_{\rm R})\right)$$

$Y=2.3$: fudge factor

$$s_0(x_{\rm R})=\begin{cases}1 & x_{\rm R}\le\frac{2x_{\rm max,R}}{1+\sqrt{3}}\\ 2\frac{x_{\rm max,R}}{x_{\rm R}}-\sqrt{3} & \frac{2x_{\rm max,R}}{1+\sqrt{3}}\le x_{\rm R}\le\frac{2x_{\rm max,R}}{\sqrt{3}}\\ 0 & \frac{2x_{\rm max,R}}{\sqrt{3}}\le x_{\rm R}\end{cases}$$

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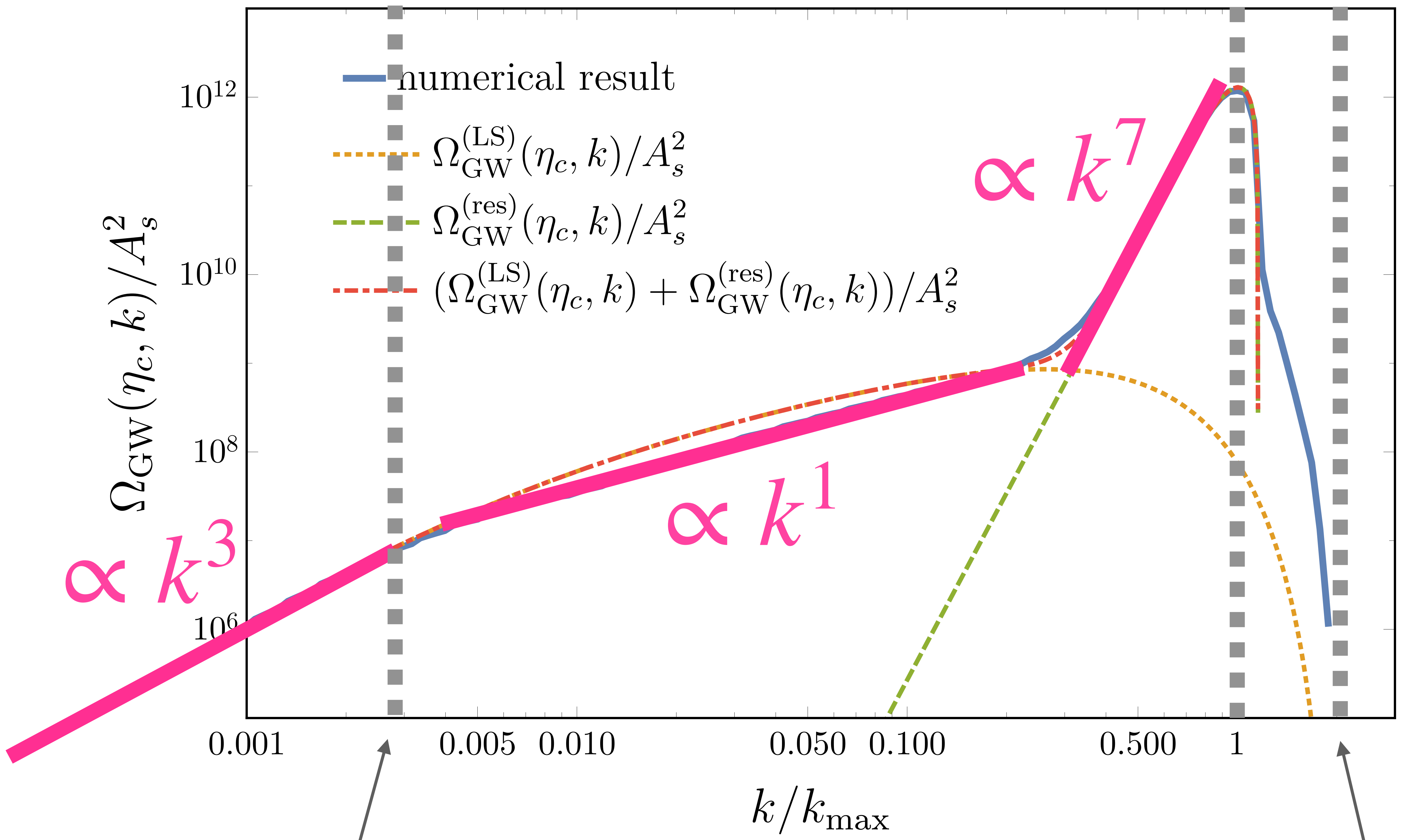

$k_{\rm max}$: a cutoff scale to avoid the nonlinear regime ($\delta_m(\eta_{\rm R}, k_{\rm max}) \sim 1$).

Scale of
Reheating Transition

$k = 2k_{\rm max}$
(second-order effect)

$$\Omega_{\rm GW}(\eta_{\rm c}, k) \simeq \Omega_{\rm GW}^{\rm (LS)}(\eta_{\rm c}, k) + \Omega_{\rm GW}^{\rm (res)}(\eta_{\rm c}, k)$$

$$\begin{aligned}\Omega^{\rm (LS)}_{\rm GW,RD}(\eta_c, k) \simeq & \frac{4{\rm Ci}\left(\frac{x_{\rm R}}{2}\right)^2 + \left(\pi - 2{\rm Si}\left(\frac{x_{\rm R}}{2}\right)\right)^2}{86016000000} A_{\rm s}^2 x_{\rm R}^3 x_{\rm max,R}^5 \times \\ & \left(\Theta(x_{\rm max,R} - x_{\rm R})\left(5376 - 17640\tilde{k} + 23760\tilde{k}^2 - 16425\tilde{k}^3 + 5825\tilde{k}^4 - 847\tilde{k}^5\right)\right. \\ & \left. + \Theta(x_{\rm R} - x_{\rm max,R})\tilde{k}^{-5}\left(2-\tilde{k}\right)^6\left(4 - 8\tilde{k} - 9\tilde{k}^2 + 13\tilde{k}^3 + 49\tilde{k}^4\right)\right),\end{aligned}$$

$$\Omega^{\rm (res)}_{\rm GW}(\eta_{\rm c}, k) \simeq Y \frac{2.30285}{102400000}\sqrt{3} A_{\rm s}^2 x_{\rm R}^7 s_0(x_{\rm R})\left(15 - 10 s_0^2(x_{\rm R}) + 3 s_0^4(x_{\rm R})\right)$$

$Y = 2.3$: fudge factor

$$s_0(x_{\rm R}) = \begin{cases} 1 & x_{\rm R} \leq \frac{2x_{\rm max,R}}{1+\sqrt{3}} \\ 2\frac{x_{\rm max,R}}{x_{\rm R}} - \sqrt{3} & \frac{2x_{\rm max,R}}{1+\sqrt{3}} \leq x_{\rm R} \leq \frac{2x_{\rm max,R}}{\sqrt{3}} \\ 0 & \frac{2x_{\rm max,R}}{\sqrt{3}} \leq x_{\rm R} \end{cases}$$

Poltergeist

mechanism for gravitational wave production

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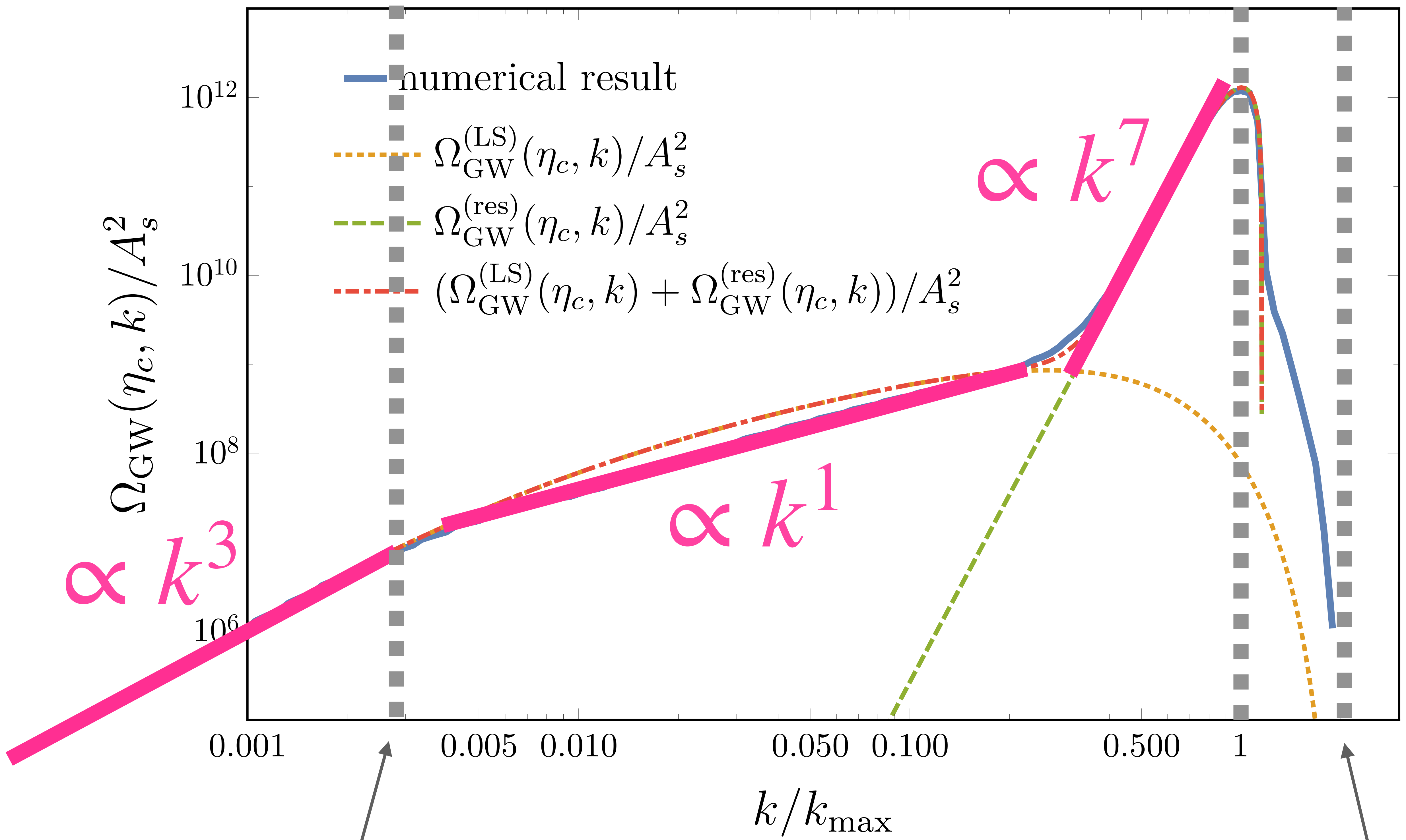

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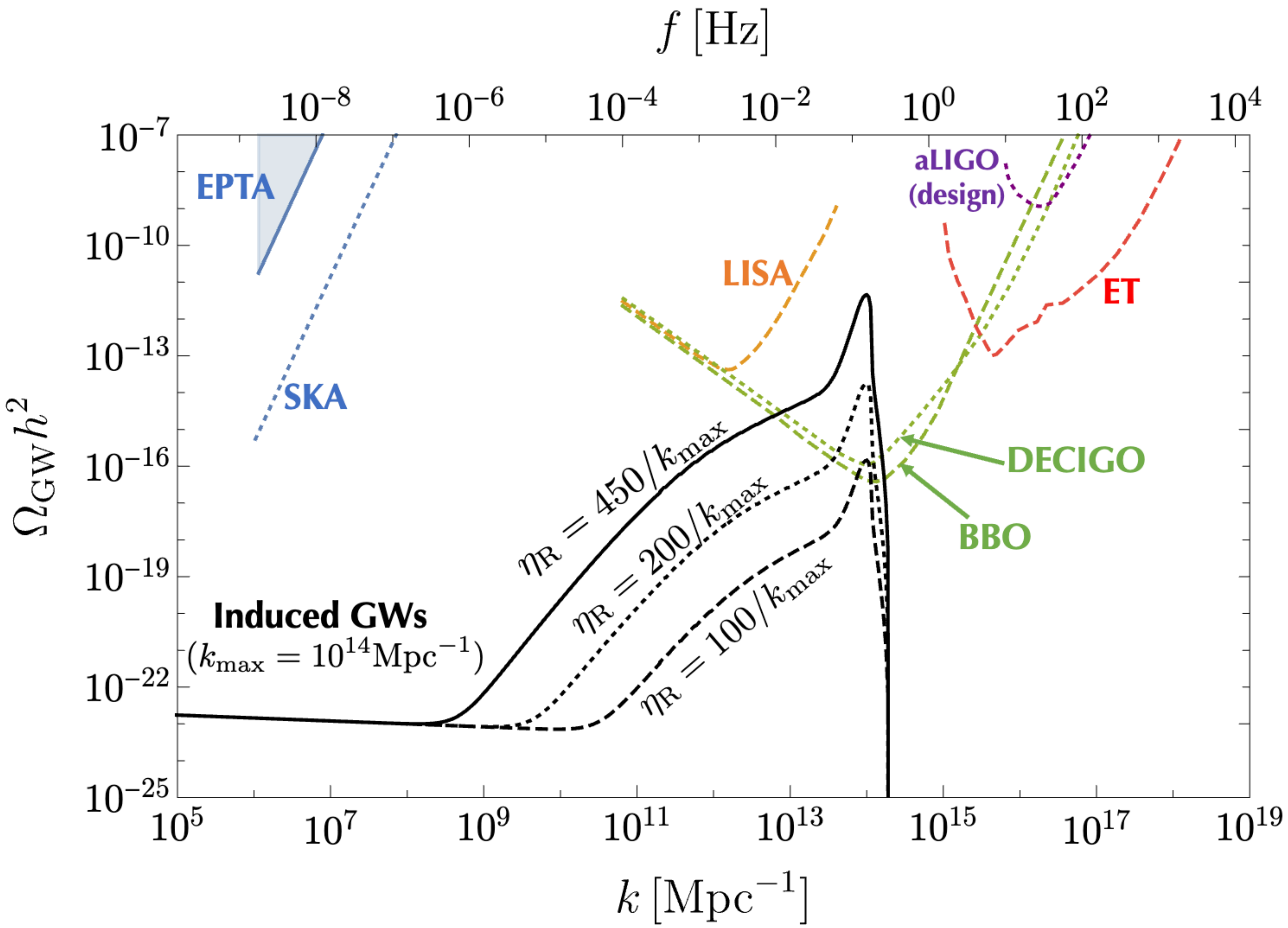

Poltergeist

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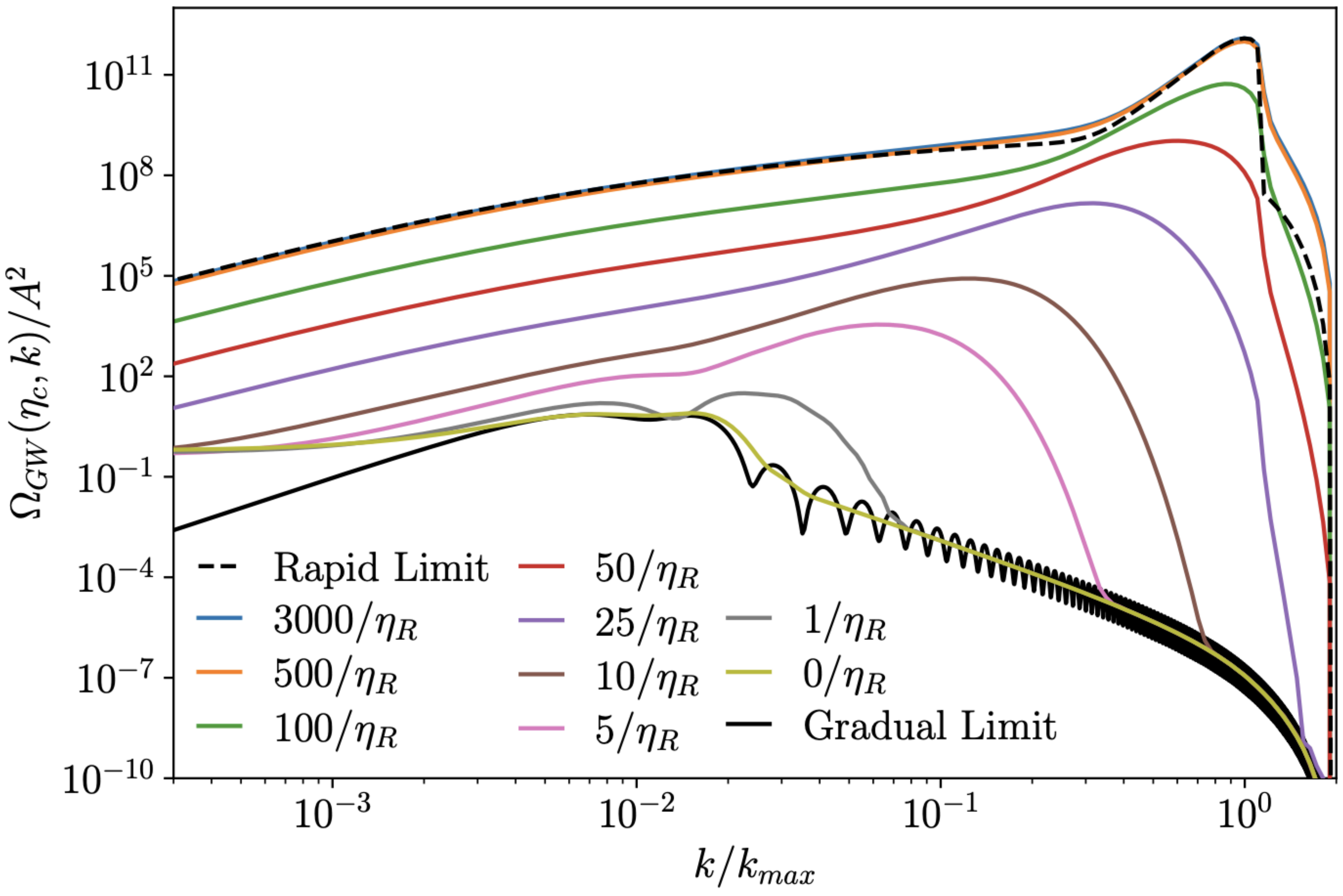
Intermediate Transition-Speed Cases

[Pearce, Pearce, White, Balazs, 2311.12340]



$$\Gamma(\eta) = \Gamma_{\max}(\tanh(\beta(\eta - \eta_\Gamma)) + 1)/2$$

- Our results have been confirmed and refined.
- In particular, the big enhancement effect is not an artifact due to the infinite transition speed.

More recent improvement with relative velocity perturbations [Kumar, Tai, Wang, 2410.17291]

Examples of quasi-sudden transitions

- Domination and evaporation of tiny PBHs
- Axion rotation scenario

Evaporation of Tiny Black Holes [Inomata, Kawasaki, Mukaida, Terada, Yanagida, 2003.10455]

The scenario

- PBHs are produced in the early universe.
- They dominate the universe and realize a MD era (PBH-dominated era).
- They evaporate via Hawking radiation and reheat the universe before BBN.

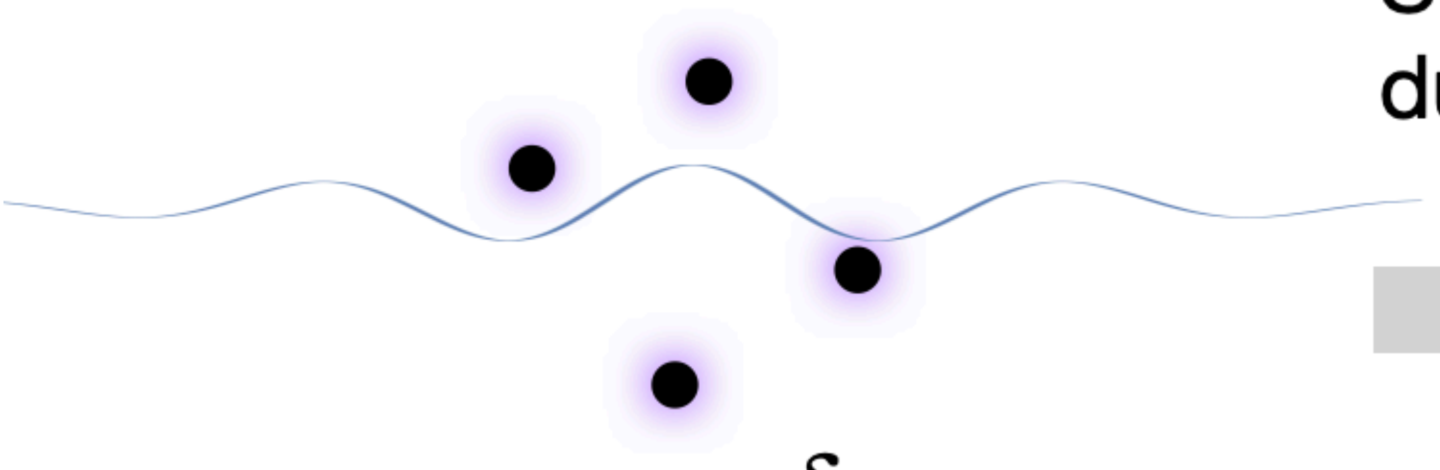
$$\Gamma \equiv -\frac{\mathrm{d}\ln M_{\mathrm{PBH}}}{\mathrm{d}t} = \frac{1}{3(t_{\mathrm{evp}} - t)}$$

Assumption

For the signal not to be suppressed, we need to assume

- small width of mass distribution
- small width of spin distribution

$$M_{\mathrm{PBH}} \propto (t_{\mathrm{evp}} - t)^{1/3}$$

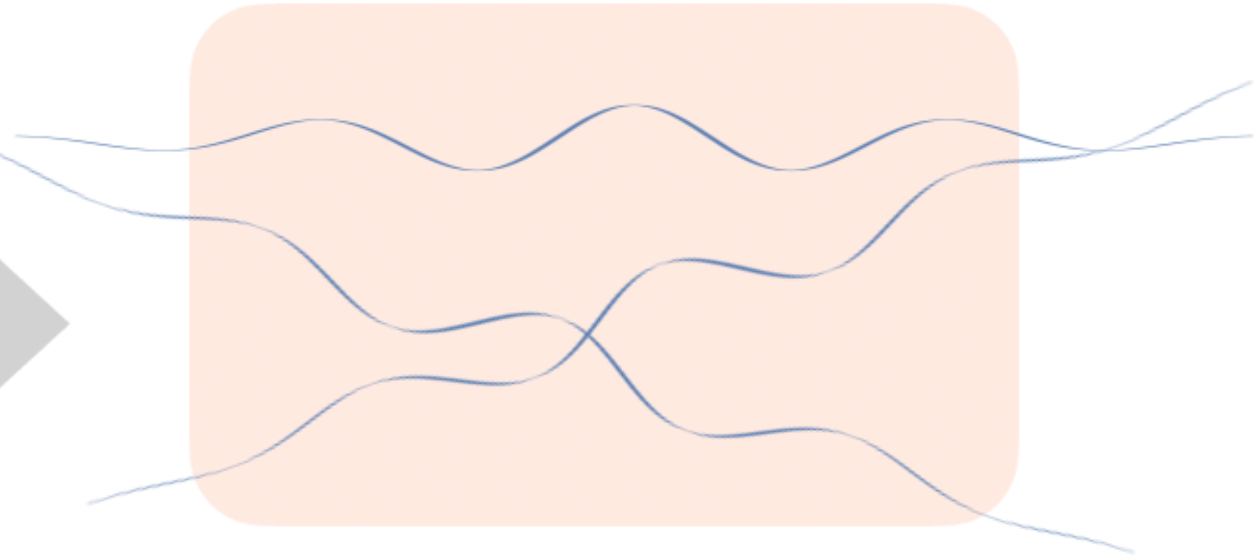


$$\frac{\delta n_{\mathrm{PBH}}}{n_{\mathrm{PBH}}} \propto a$$

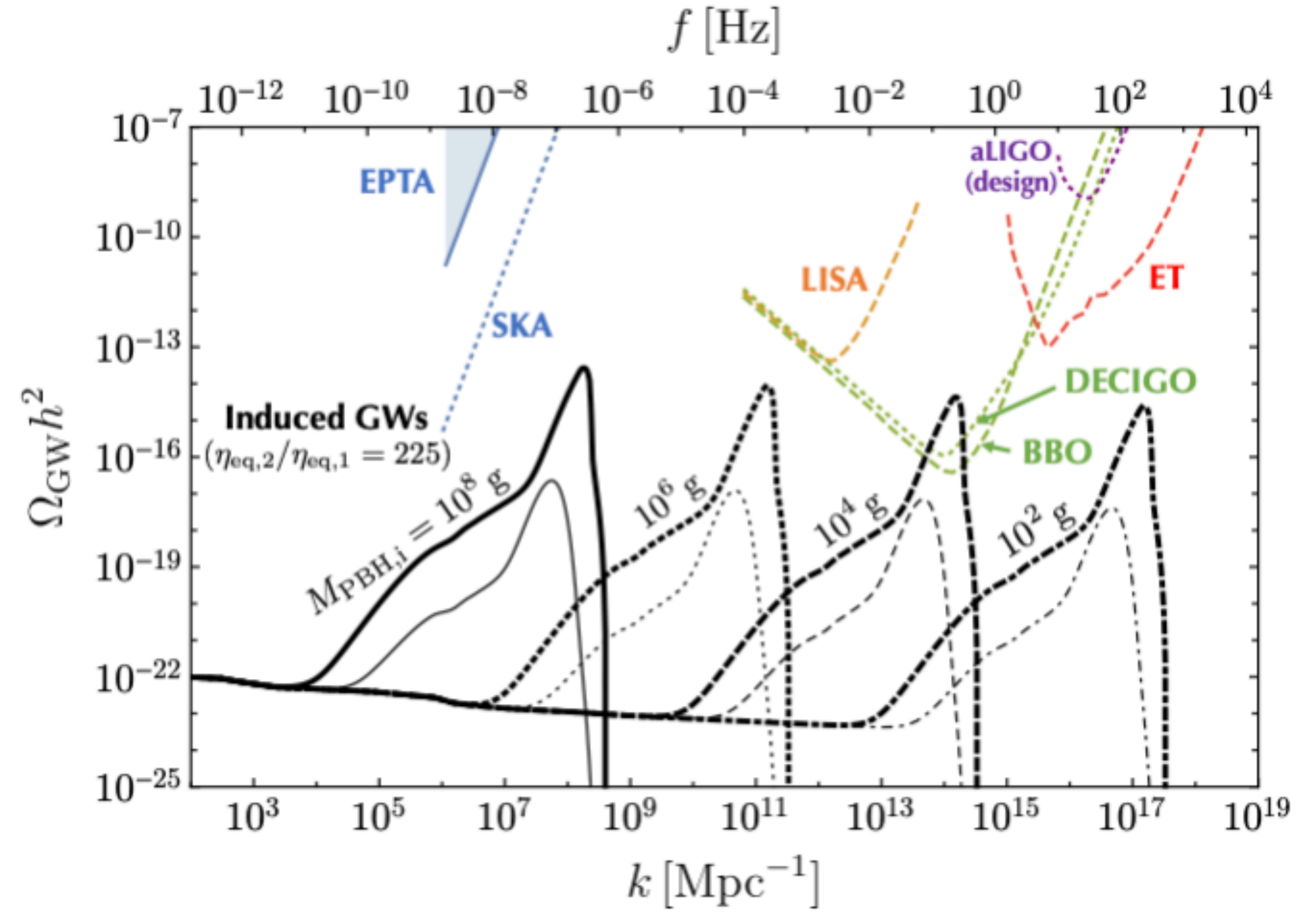
Sudden transition due to evaporation

$$\Gamma \sim \frac{1}{t_{\mathrm{evp}} - t}$$

Sound waves in thermal bath

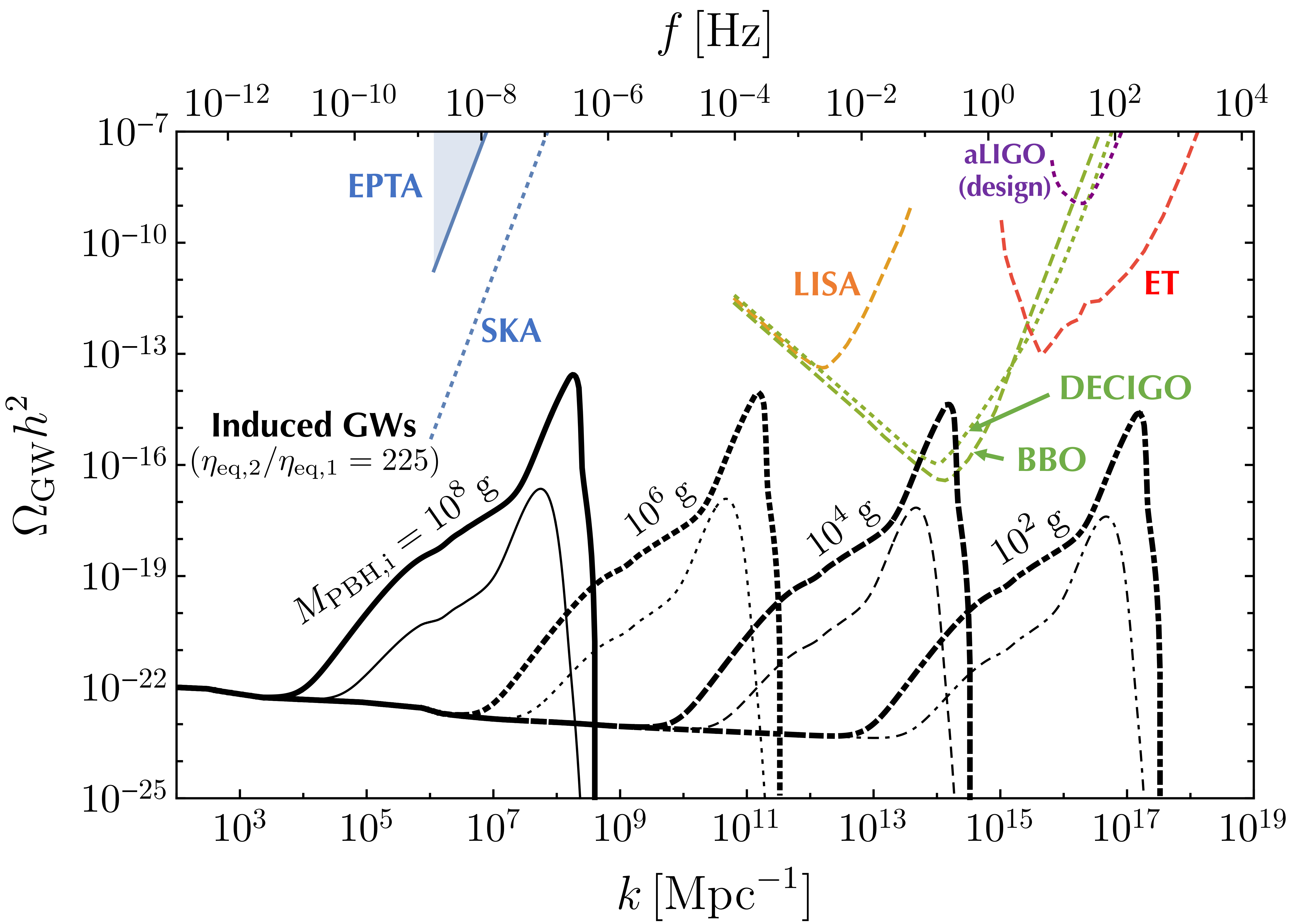


Fast oscillation of Φ_k



(Resonant) production of 2nd-order GWs

Evaporation of Tiny Black Holes [Inomata, Kawasaki, Mukaida, Terada, Yanagida, 2003.10455]



Primordial curvature perturbations

$$\mathscr{P}_\zeta(k) = A_s \left(\frac{k}{k_*}\right)^{n_s - 1} \theta(k_{\rm NL} - k)$$

Finite width of mass (distribution) function

$$\rho_{\rm PBH,i}(M_{\rm PBH,i}) = \frac{\rho_{\rm PBH,i}}{\sqrt{2\pi}\sigma} \exp\left(-\frac{\left(\ln(M_{\rm PBH}/M_{\rm PBH,0})\right)^2}{2\sigma^2}\right)$$

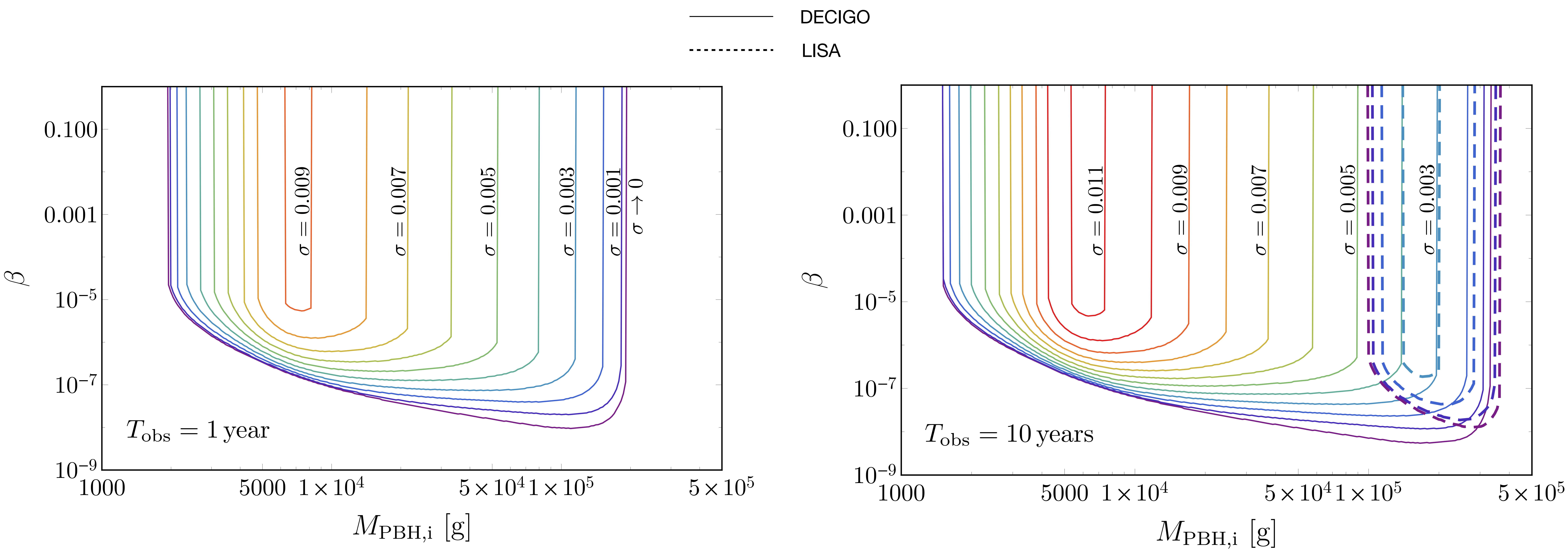
$\sigma \to 0$ (thick line)

$\sigma = 0.01$ (thin line)

Further enhancement in the non-linear regime?

[Papanikolaou, Vennin, Langlois, 2010.11573]
[Domènech, Lin, Sasaki, 2012.08151]
[Domènech, Takhistove, Sasaki, 2105.06816]

Evaporation of Tiny Black Holes [Inomata, Kawasaki, Mukaida, Terada, Yanagida, 2003.10455]



$\beta \equiv \dfrac{\rho_{\rm PBH,i}}{\rho_{\rm tot,i}} \gtrsim \mathcal{O}(10^{-7})$ for $\mathcal{O}(10^3)\,\mathrm{g} \lesssim M_{\rm PBH,i} \lesssim \mathcal{O}(10^5)\,\mathrm{g}$ can be probed by future observations.

Evaporation of Tiny Black Holes

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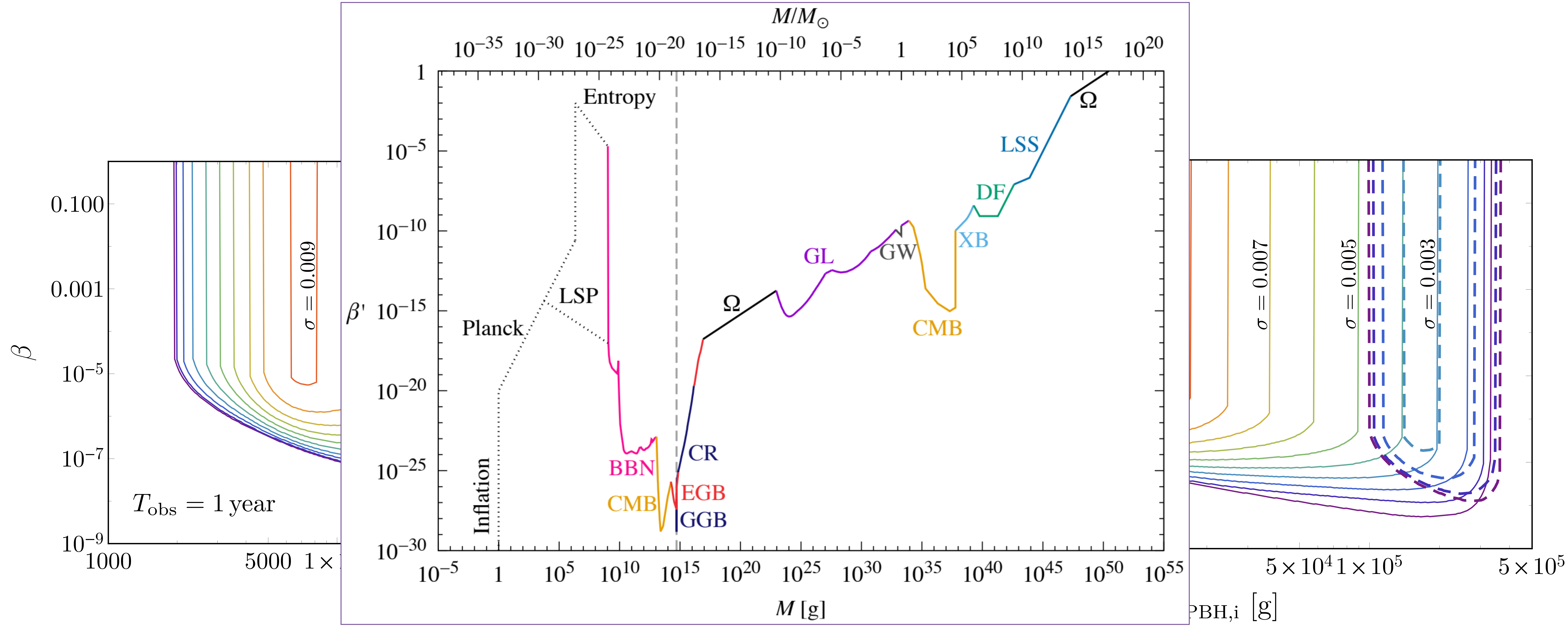

[Carr, Kohri, Sendouda, Yokoyama, 2002.12778]

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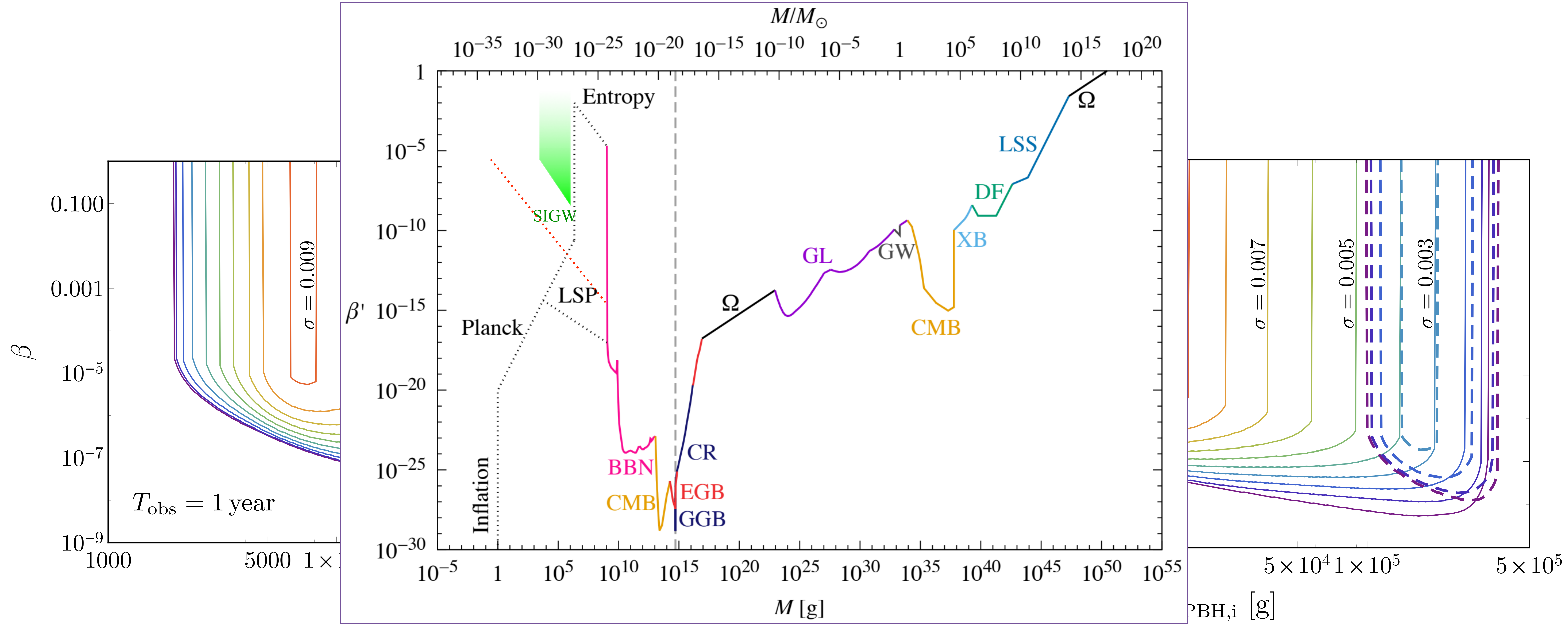


Modification of [Carr, Kohri, Sendouda, Yokoyama, 2002.12778]

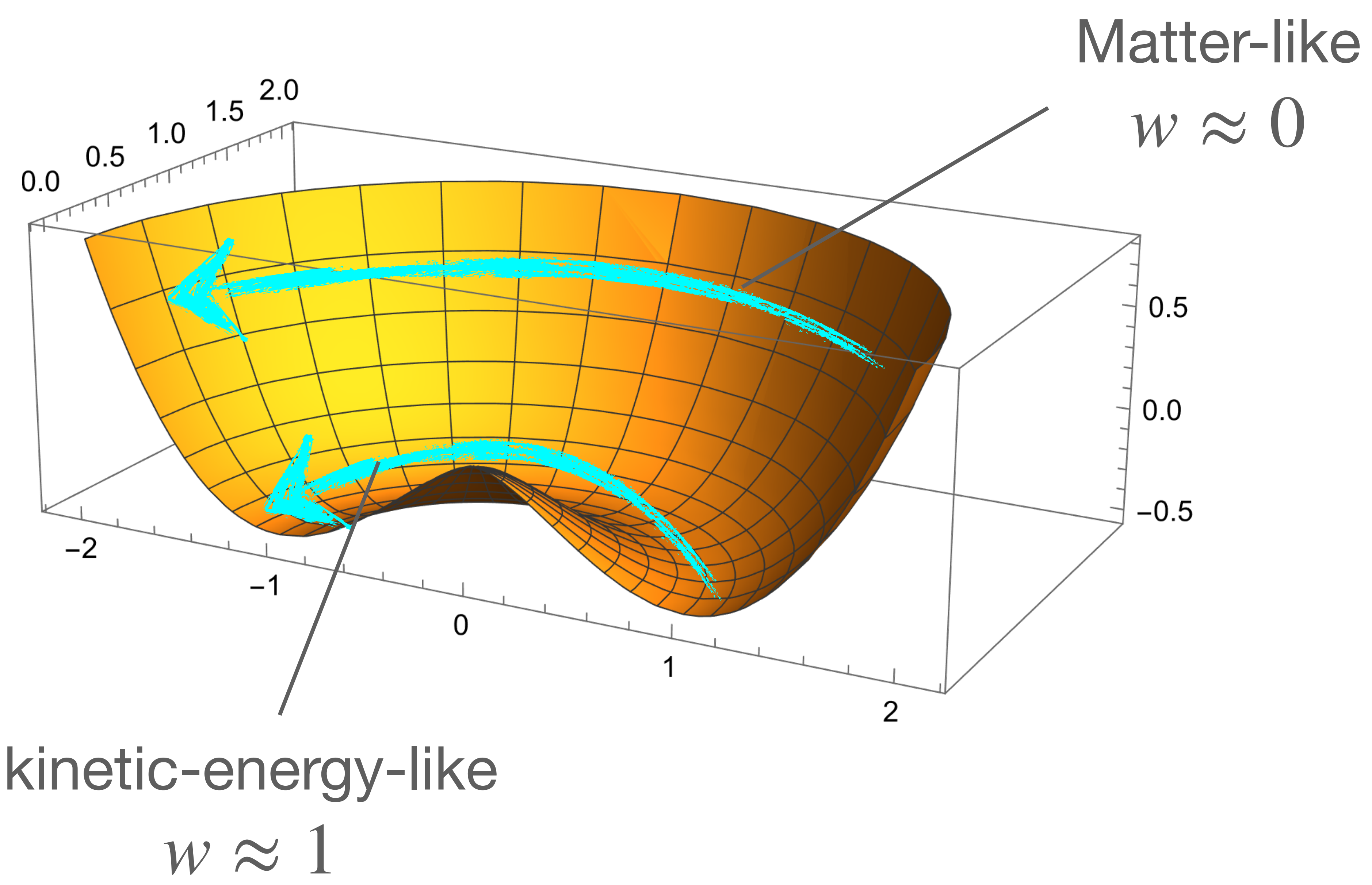
$$\beta \equiv \frac{\rho_{\rm PBH,i}}{\rho_{\rm tot,i}} \gtrsim \mathcal{O}(10^{-7}) \text{ for } \mathcal{O}(10^3)\,{\rm g} \lesssim M_{\rm PBH,i} \lesssim \mathcal{O}(10^5)\,{\rm g}$$ can be probed by future observations.

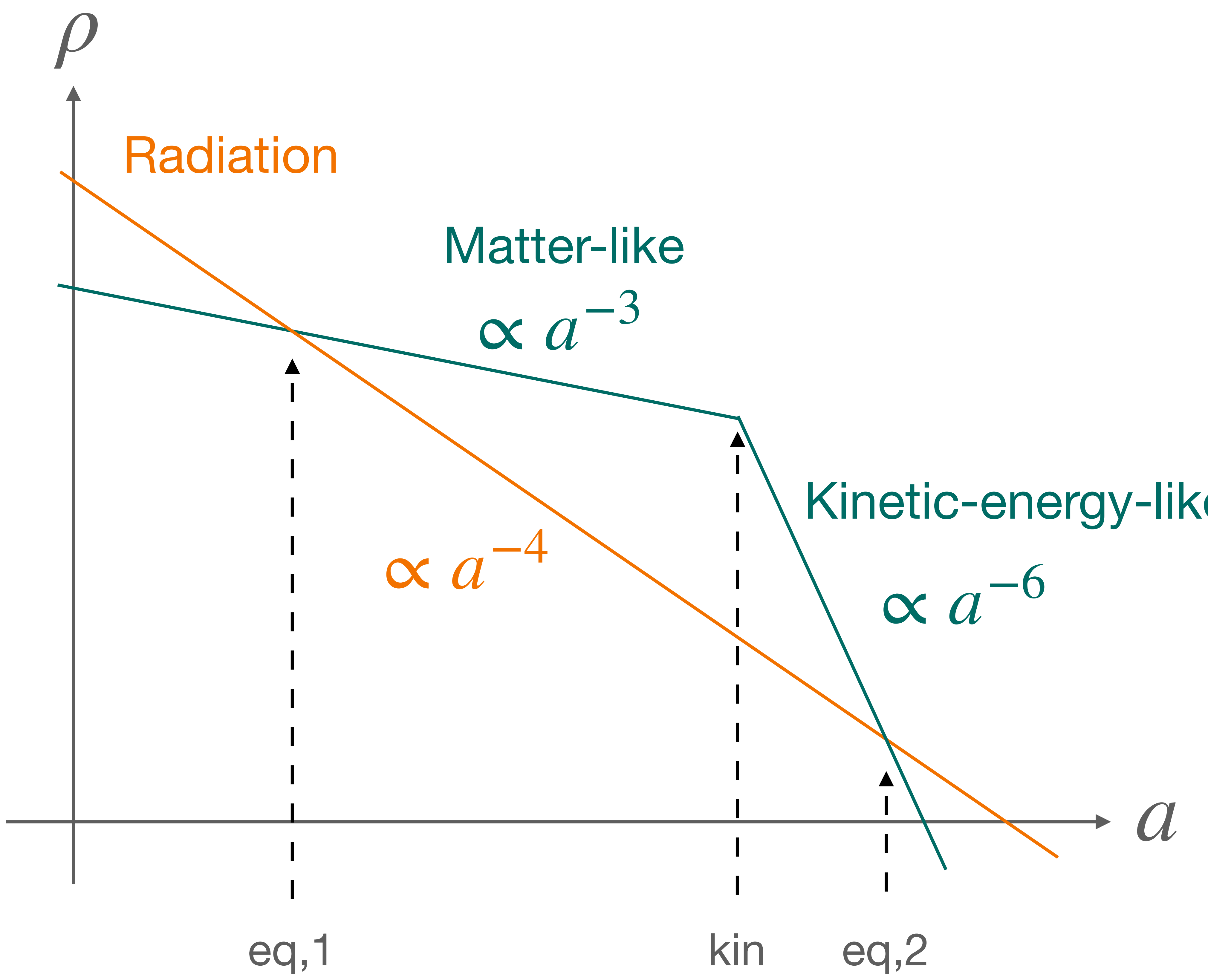
Axion Rotation Cosmology with Kination

[Co, Dunsky, Fernandez, Ghalsasi, Hall, Harigaya, Shelton, 2108.09299]
[Gouttenoire, Servant, Simakachorn, 2108.10328; 2111.01150]

Example:

$$V(P) = m^2 |P|^2 \left(\log\left(\frac{2|P|^2}{f_a^2} \right) - 1 \right)$$





No Entropy Production neither at “kin” nor at “eq,2”!

Axion Model Setup

[Co, Dunsky, Fernandez, Ghalsasi, Hall, Harigaya, Shelton, 2108.09299]

Axion Model Setup

[Co, Dunsky, Fernandez, Ghalsasi, Hall, Harigaya, Shelton, 2108.09299]

(Supersymmetric) Two-field model

Kähler potential $K = |P|^2 + |\bar{P}|^2 + |X|^2$

Superpotential $W = X(P\bar{P} - v_P^2)$

Soft supersymmetry breaking mass terms $(v_P \gg m_P, m_{\bar{P}})$ $V_{\rm soft} = m_P^2 |P|^2 + m_{\bar{P}}^2 |\bar{P}|^2$

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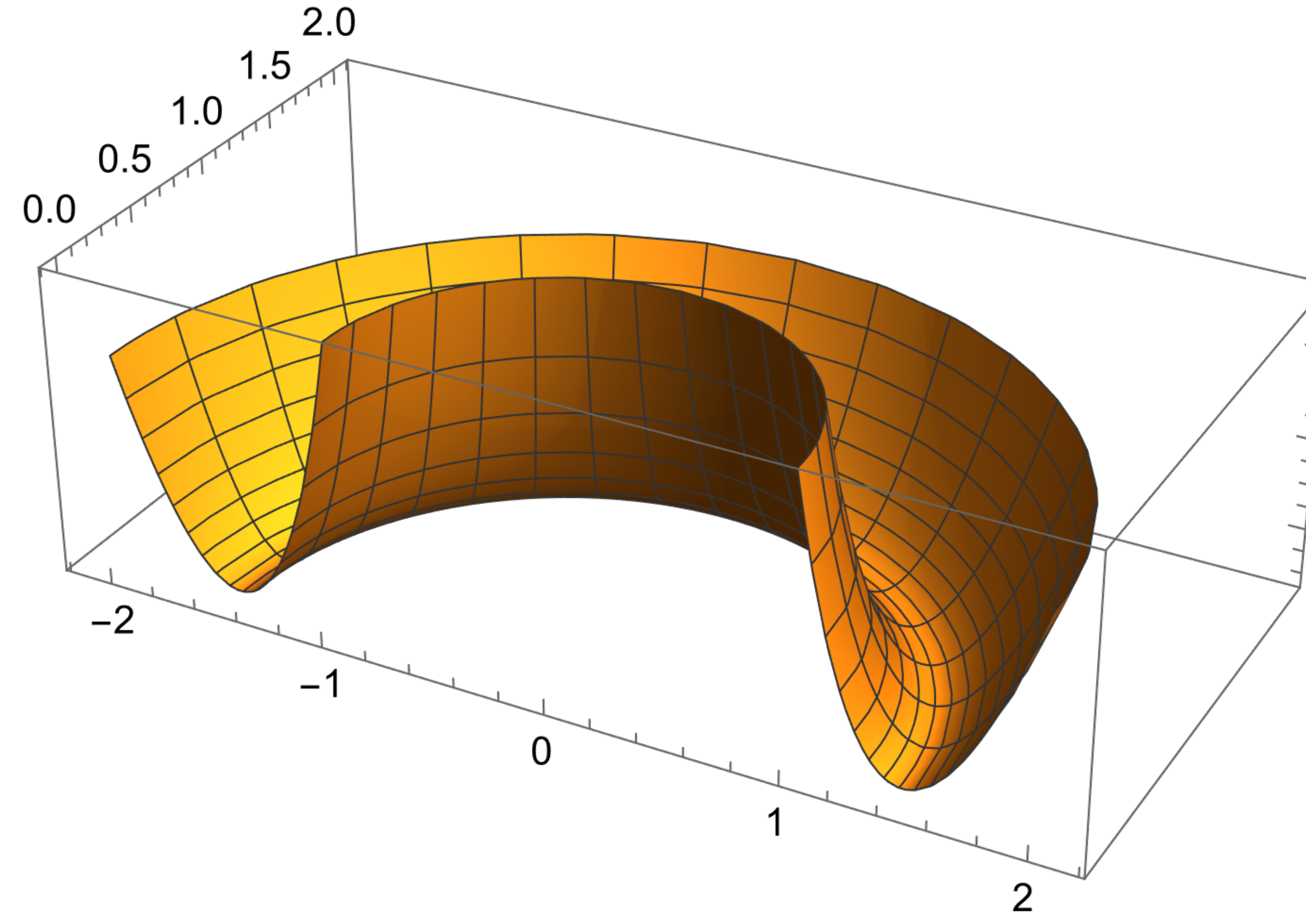
X and a linear combination of P and $\bar{P}$ become heavy and can be integrated out.

$$\mathscr{L} = -\left(1 + \frac{f_a^4}{16|P|^4}\right) g^{\mu\nu}\partial_\mu P^\dagger \partial_\nu P - \left(1 - \frac{(1+d)f_a^2}{4|P|^2}\right)^2 m_P^2 |P|^2$$



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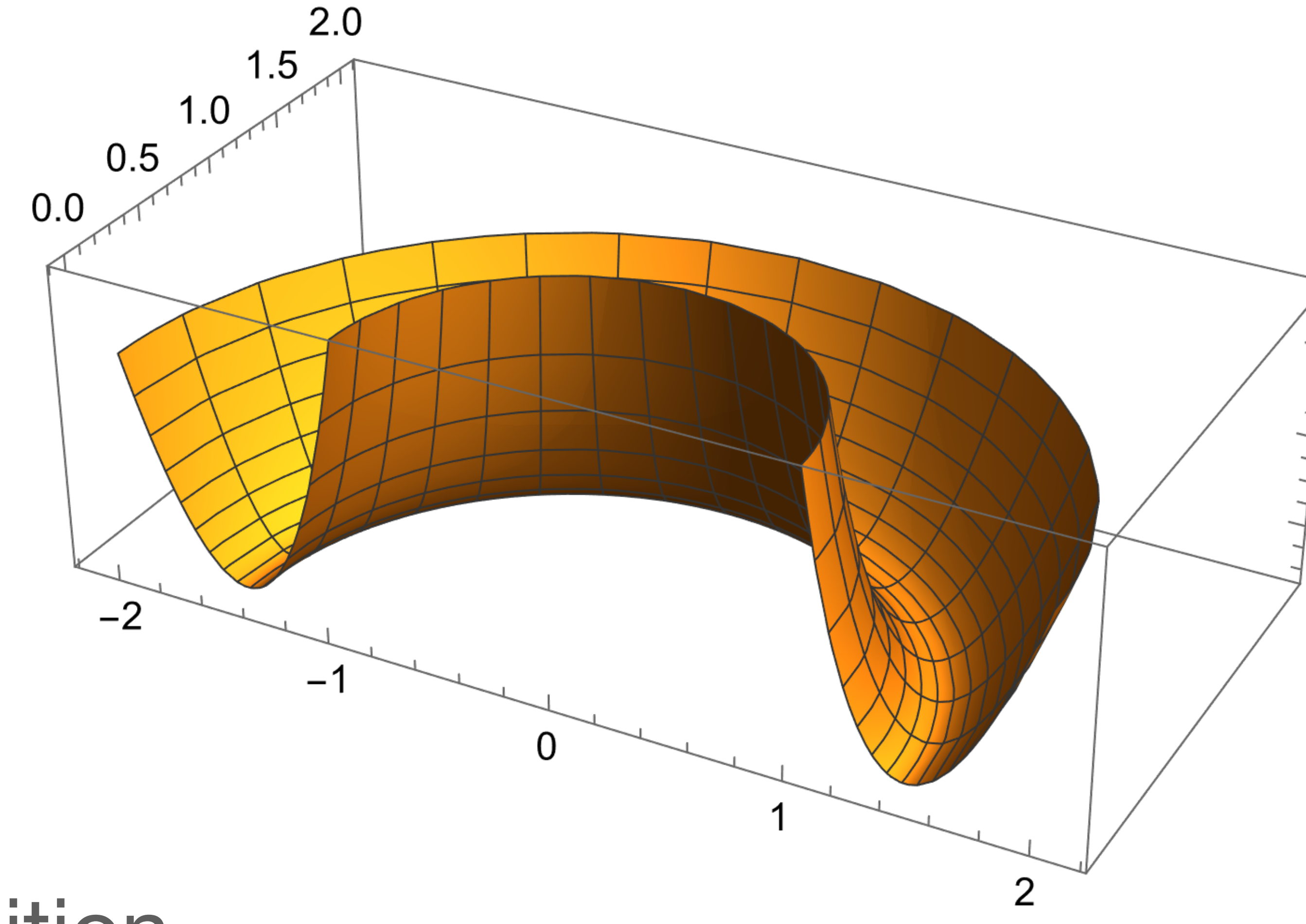

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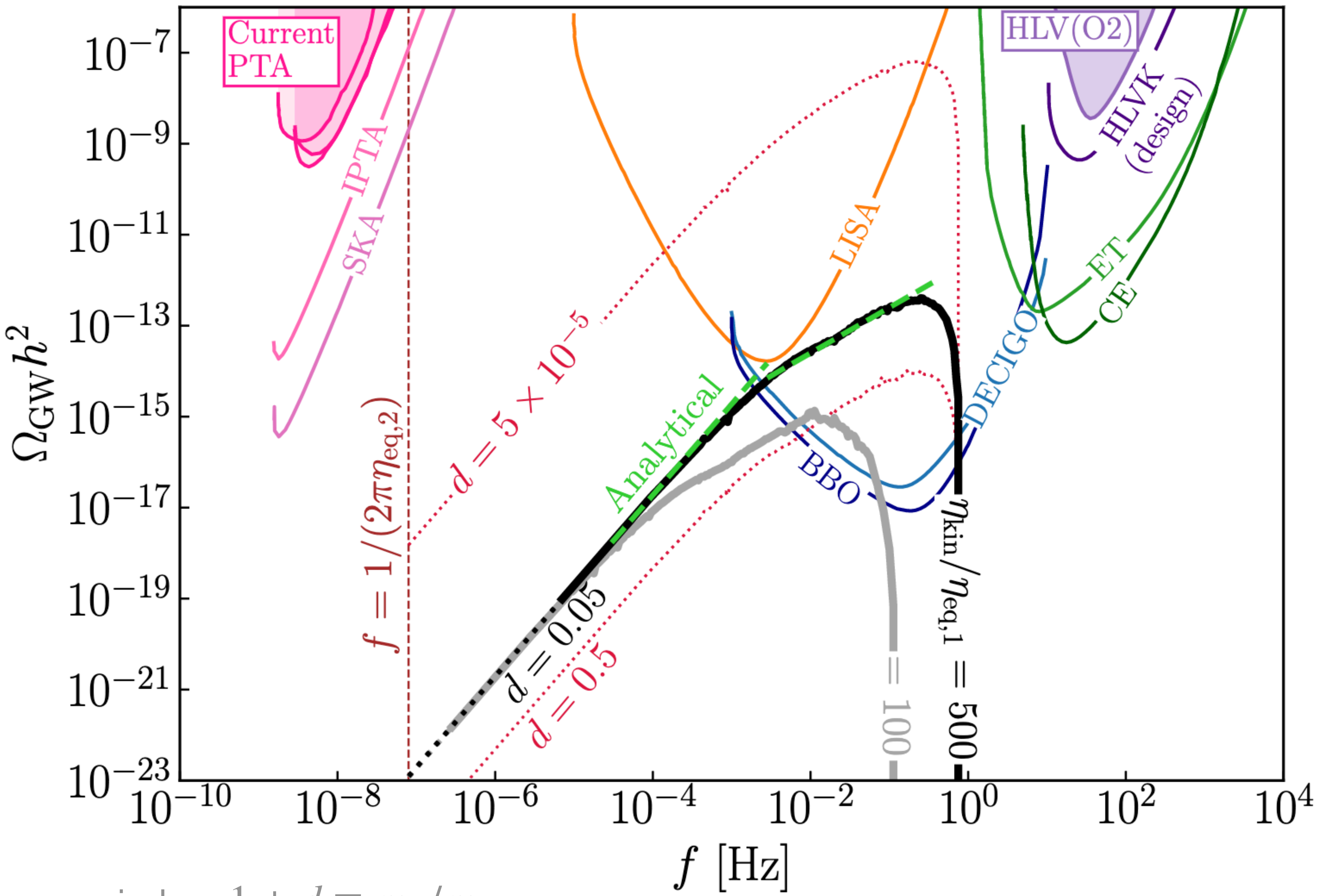
The approximately $\mathbb{Z}_2$ symmetric limit $d \equiv (m_{\bar{P}} - m_P)/m_P \ll 1$ realizes a sudden transition.

Axion-Induced Gravitational Waves [Harigaya, Inomata, Terada, 2305.14242]

See our Supplemental Material, [Domènech, 1912.05583], and [Domènech, Pi, Sasaki, 2005.12314] for Induced GWs with a general equation-of-state parameter w.



reminder: $1 + d \equiv m_{\bar{P}}/m_P$

The dominant contribution (from the KD era by the Poltergeist mechanism) is plotted.

Assumed Curvature Spectrum:

$$\mathscr{P}_\zeta = A\,\Theta(k_{\mathrm{max}} - k)\,\Theta(k - 1/\eta_{\mathrm{kin}})$$

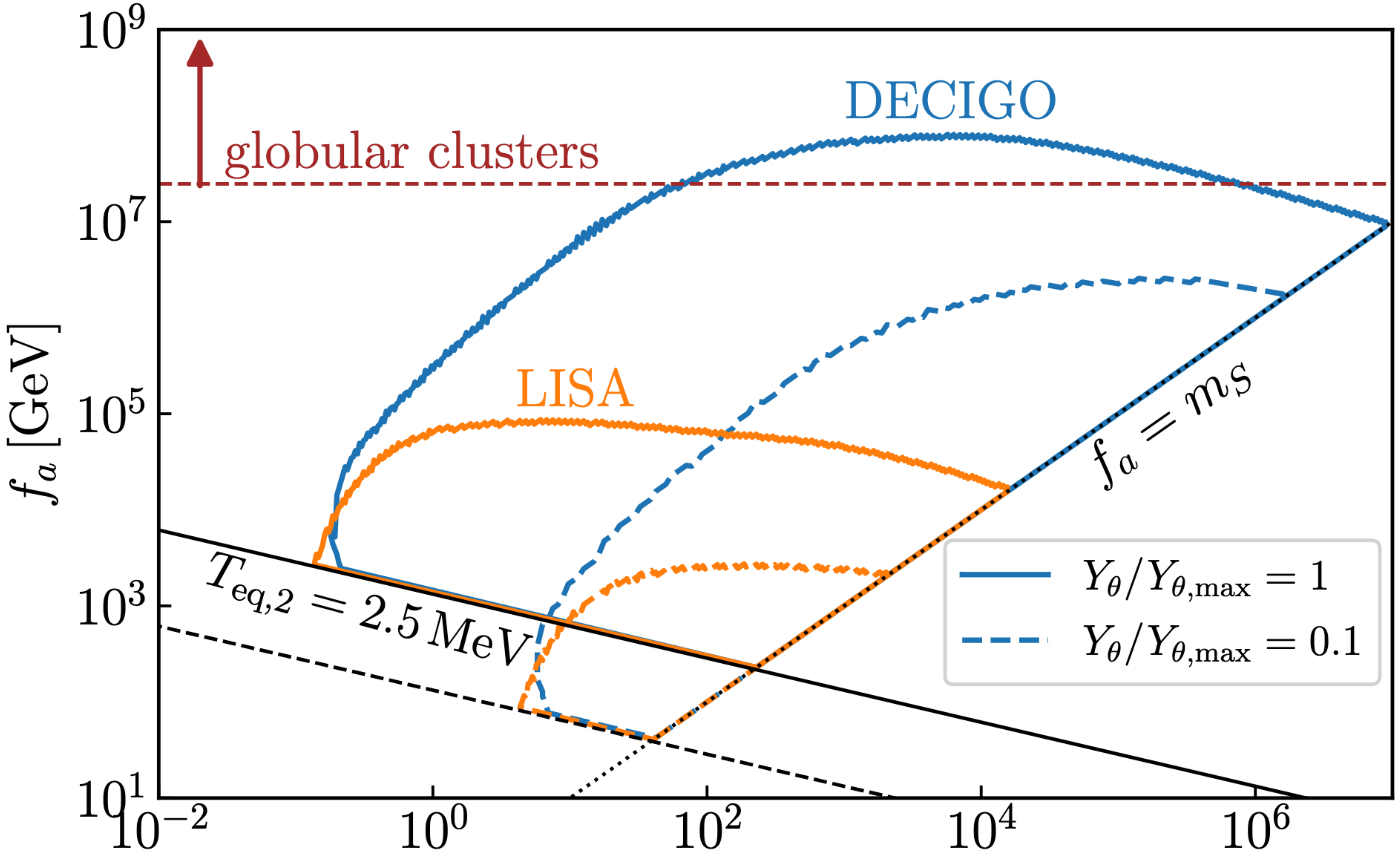
with $A = 2.1 \times 10^{-9}$, and k_{max} the nonlinear scale.

Analytic formula:

$$\Omega_{\mathrm{GW}}h^2 \simeq 2 \times 10^{-11} A^2 Q^4 \frac{\eta_{\mathrm{kin}}^2}{\eta_{\mathrm{eq},1}^2}(k_* \eta_{\mathrm{kin}})^5 \begin{cases} k\eta_{\mathrm{kin}} & (k\eta_{\mathrm{kin}} > 4) \\ 0.535 \times (k\eta_{\mathrm{kin}})^2 & (k\eta_{\mathrm{kin}} < 4) \end{cases}$$

where $k_* \equiv \min[k_{\mathrm{max}}, 1/\eta_{\mathrm{eq},1}]$ and $Q \sim \Phi_{k_*}(\eta_{\mathrm{kin}})/\Phi_{k_*}(\eta_{\mathrm{eq},1} \ll \eta \ll \eta_{\mathrm{kin}})$.

Probable Parameter Space 1: Saxion



($= m_P$; SUSY breaking scale)

$d = 0.05$ and $b = 0.1$

U(1) charge density

$$Y_\theta \equiv \frac{n_\theta}{s}, \quad n_\theta = 2\left(1 + \frac{f_a^4}{16|P|^4}\right)|P|^2\dot\theta$$

$$Y_{\theta,\mathrm{max}} = 100\left(\frac{m_S}{8.7\times 10^8\,\mathrm{GeV}}\right)^{-1/3}\left(\frac{b}{0.1}\right)^{1/3}$$

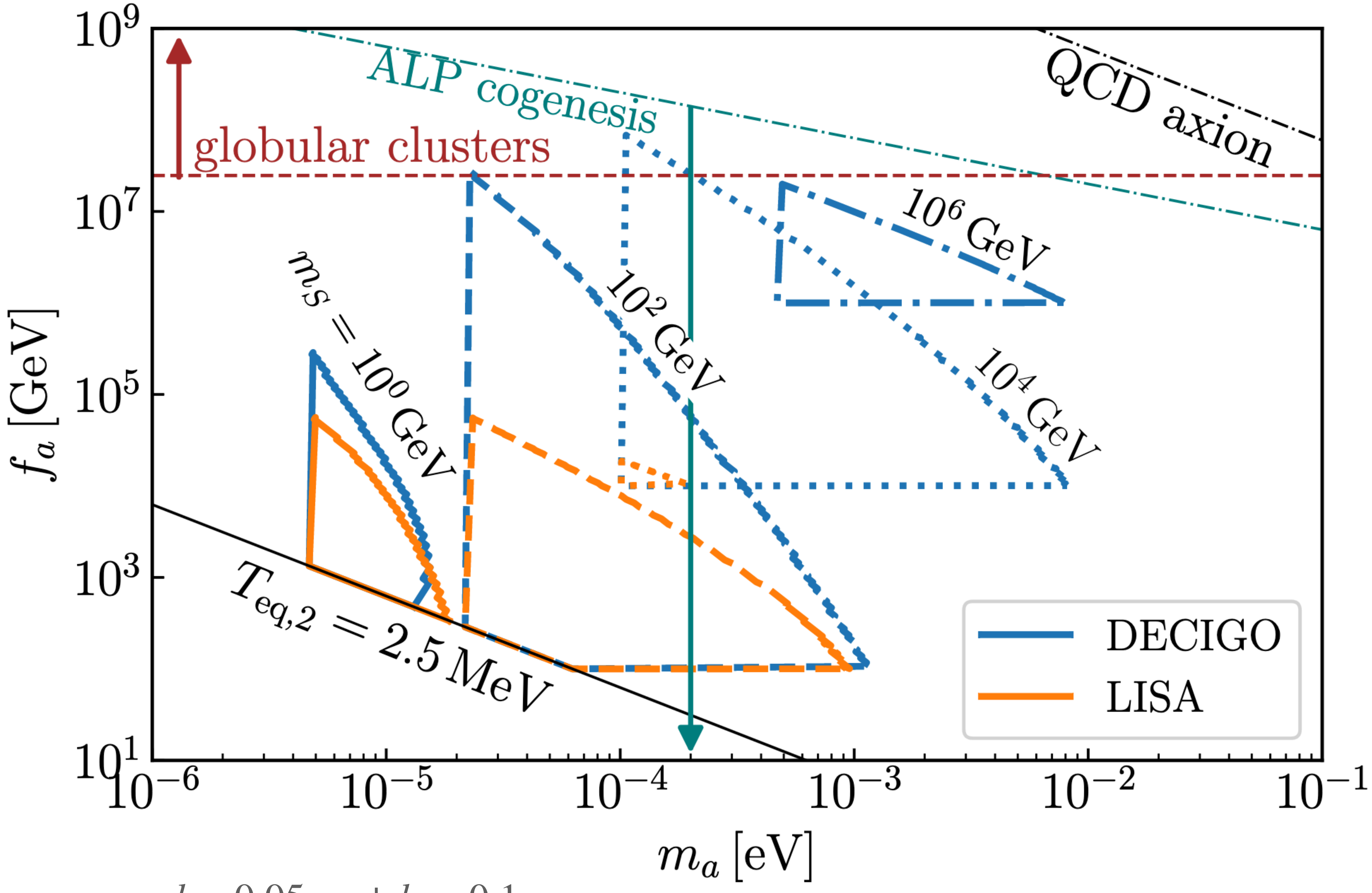
b: parametrizing the efficiency of thermalization

[Dolan, Hiskens, Volkas, 2207.03102]

Globular cluster constraint assumes

$$g_{a\gamma\gamma} = \frac{\alpha}{2\pi f_a}.$$

Probable Parameter Space 2: Axion



$d = 0.05$ and $b = 0.1$

Axion 100% dark matter is assumed.

$$m_a Y_\theta \simeq 0.44\,\mathrm{eV}$$

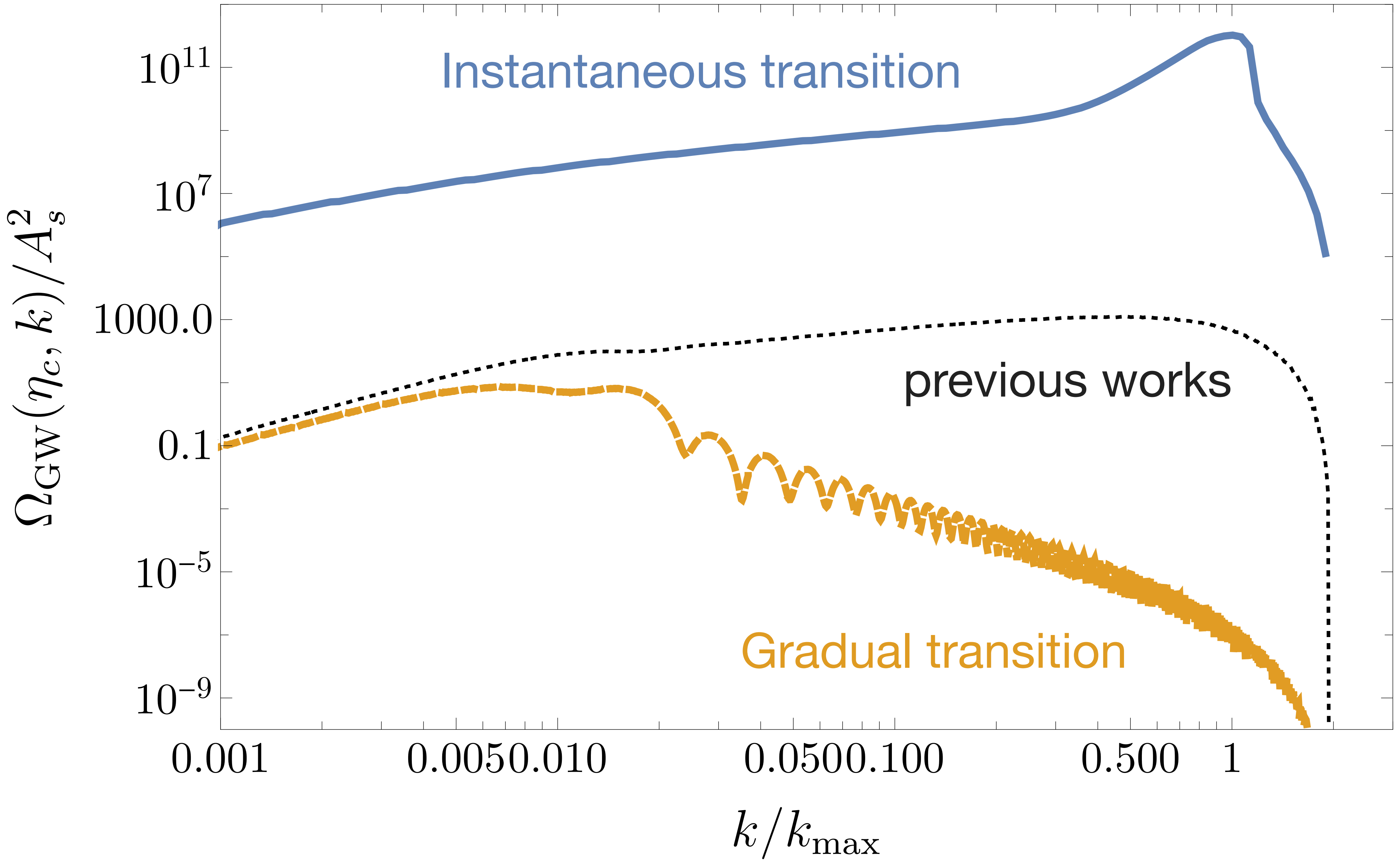
kinetic misalignment [Co, Hall, Harigaya, 1910.14152]

ALP cogenesis [Co, Hall, Harigaya, 2006.04809]

Summary and Conclusions

Conclusions

GWs induced by curvature perturbations are a useful probe of the early Universe.
Applications include inflation, PBHs, and PTAs.



Induced GWs are sensitive to the rapidness of the end of the (early) matter-dominated era.

Gradual transition: the induced GWs are suppressed.
Rapid transition: the induced GWs are enhanced.

Key point in the enhancement mechanism: An interplay between the MD era and RD/KD era.

- A hierarchy $(k/aH) \gg 1$ of the subhorizon modes of Φ that entered deep in the MD era
- Rapid oscillations of Φ, which occurs in the RD/KD era

Appendix

Scalar-Induced Gravitational Waves

Green's function method to solve for $h_{\mathbf{k}}(\eta)$

$$a(\eta)h_{\mathbf{k}}(\eta) = 4\int^{\eta} \mathrm{d}\bar{\eta}\, a(\bar{\eta})G_{\mathbf{k}}(\eta, \bar{\eta})S_{\mathbf{k}}(\bar{\eta})$$

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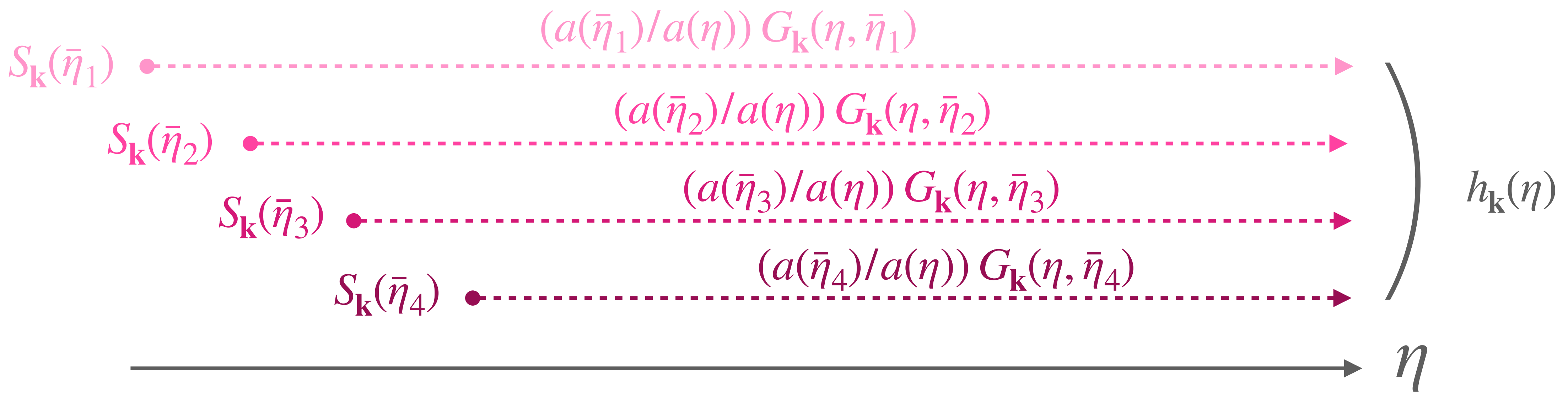

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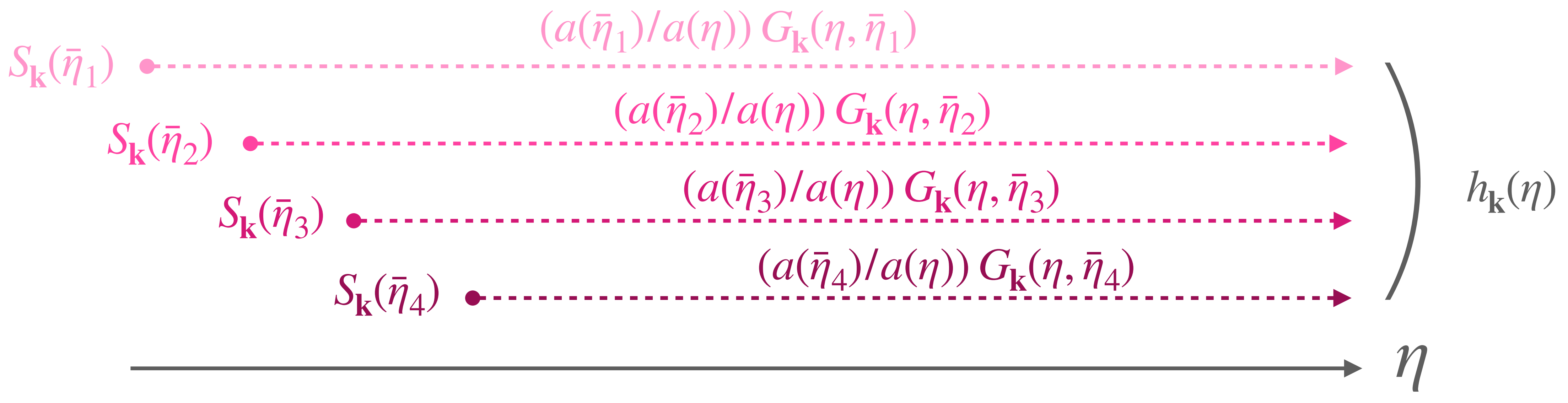

Eq. of motion for Green's function

$$G''_{\mathbf{k}}(\eta,\bar{\eta}) + \left(k^2 - \frac{a''}{a}\right)G_{\mathbf{k}}(\eta,\bar{\eta}) = \delta(\eta-\bar{\eta})$$

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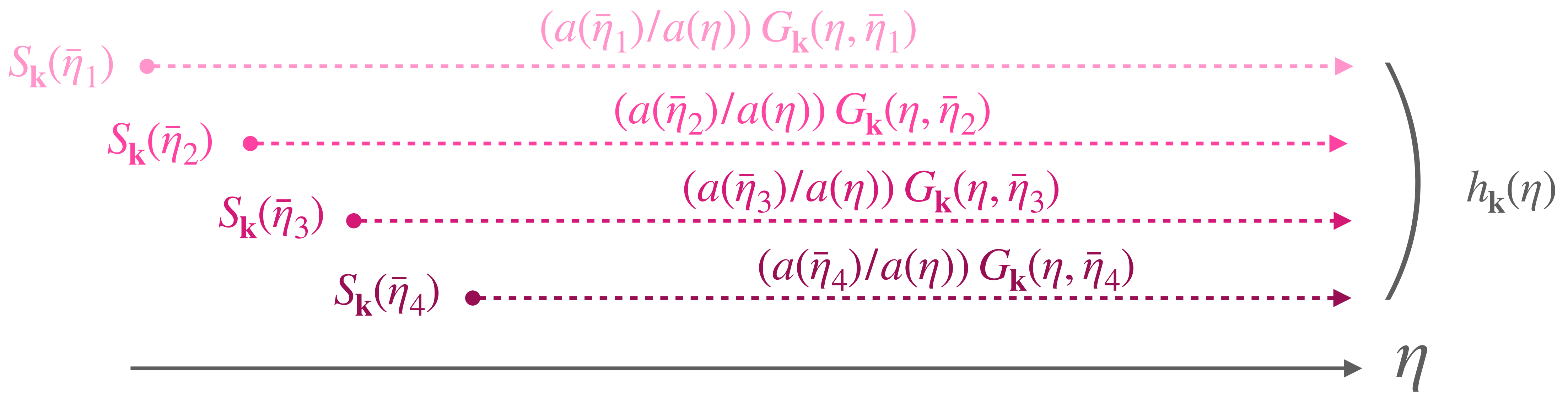

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$$G''_{\mathbf{k}}(\eta,\bar{\eta}) + \left(k^2 - \frac{a''}{a}\right) G_{\mathbf{k}}(\eta,\bar{\eta}) = \delta(\eta - \bar{\eta})$$

Eq. of motion for curvature perturbations

$$\Phi''_{\mathbf{k}} + 3\mathscr{H}(1 + c_{\mathrm{s}}^2)\Phi'_{\mathbf{k}} + (2\mathscr{H}' + (1 + 3c_{\mathrm{s}}^2)\mathscr{H}^2 + c_{\mathrm{s}}^2 k^2)\Phi_{\mathbf{k}} = \frac{a^2}{2}\tau\delta S$$

where $\delta P = c_{\mathrm{s}}^2\delta\rho + \tau\delta S$. In simple cases, $c_{\mathrm{s}}^2 = w$ and $\delta S = 0$.

Scalar-Induced Gravitational Waves

Green's function method to solve for $h_{\mathbf{k}}(\eta)$

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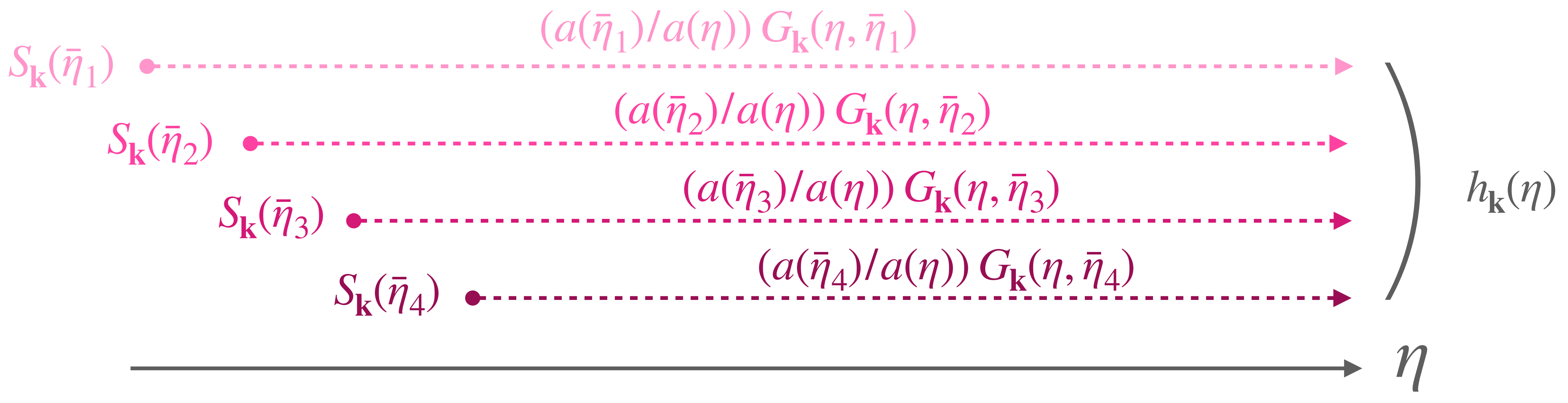

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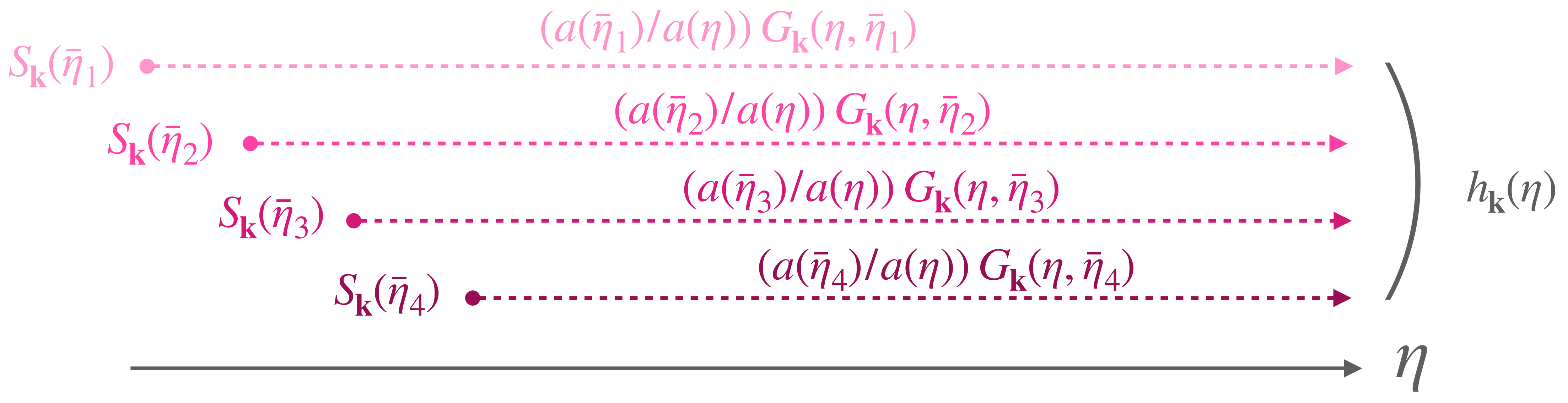
where $\delta P = c_s^2\delta\rho + \tau\delta S$. In simple cases, $c_s^2 = w$ and $\delta S = 0$.

e.g.) $w = 1/3$ (RD era) $\quad \Phi''_{\mathbf{k}} + \frac{4}{\eta}\Phi'_{\mathbf{k}} + \frac{1}{3}k^2\Phi_{\mathbf{k}} = 0$

Scalar-Induced Gravitational Waves

Green's function method to solve for $h_{\mathbf{k}}(\eta)$

$$a(\eta)h_{\mathbf{k}}(\eta) = 4\int^{\eta} \mathrm{d}\bar{\eta}\, a(\bar{\eta})G_{\mathbf{k}}(\eta,\bar{\eta})S_{\mathbf{k}}(\bar{\eta})$$



Eq. of motion for Green's function

$$G''_{\mathbf{k}}(\eta,\bar{\eta}) + \left(k^2 - \frac{a''}{a}\right) G_{\mathbf{k}}(\eta,\bar{\eta}) = \delta(\eta-\bar{\eta})$$

Eq. of motion for curvature perturbations

$$\Phi''_{\mathbf{k}} + 3\mathscr{H}(1+c_{\mathrm{s}}^2)\Phi'_{\mathbf{k}} + (2\mathscr{H}' + (1+3c_{\mathrm{s}}^2)\mathscr{H}^2 + c_{\mathrm{s}}^2k^2)\Phi_{\mathbf{k}} = \frac{a^2}{2}\tau\delta S$$

where $\delta P = c_{\mathrm{s}}^2\delta\rho + \tau\delta S$. In simple cases, $c_{\mathrm{s}}^2 = w$ and $\delta S = 0$.

e.g.) $w = 1/3$ (RD era) $\quad \Phi''_{\mathbf{k}} + \dfrac{4}{\eta}\Phi'_{\mathbf{k}} + \dfrac{1}{3}k^2\Phi_{\mathbf{k}} = 0$

$$\underbrace{\Phi_{\mathbf{k}}(\eta) = \Phi(k\eta)}_{\text{transfer fn.}}\ \phi_{\mathbf{k}} \longleftarrow \text{primordial value}$$

$$\langle \phi_{\mathbf{k}}\phi_{\mathbf{k}'}\rangle = \delta(\mathbf{k}+\mathbf{k}')\frac{2\pi^2}{k^3}\left(\frac{3+3w}{5+3w}\right)^2 \mathscr{P}_{\zeta}(k)$$

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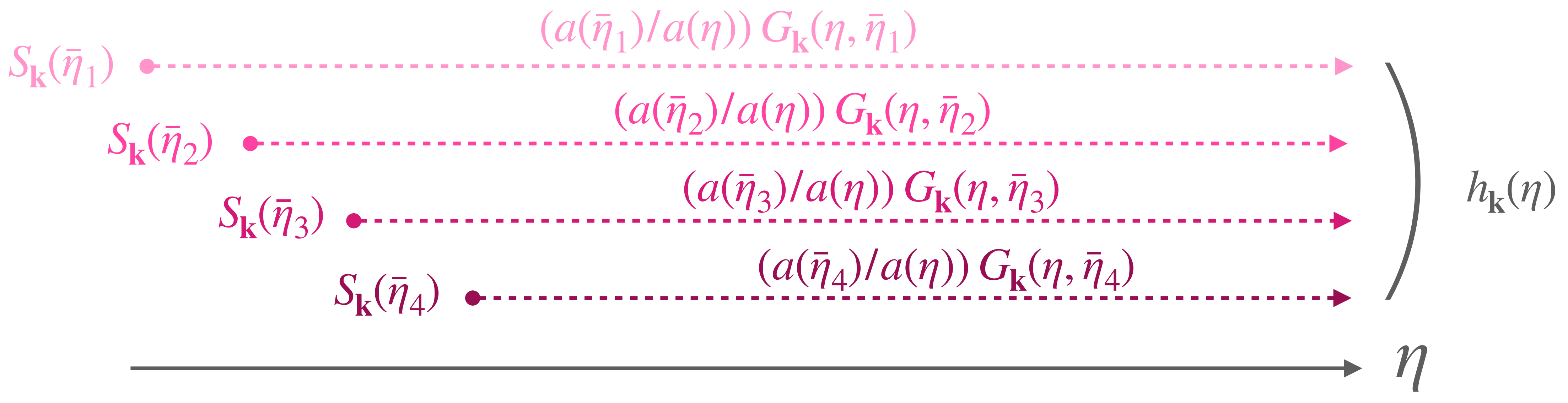

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$$\langle\phi_{\mathbf{k}}\phi_{\mathbf{k}'}\rangle = \delta(\mathbf{k}+\mathbf{k}')\frac{2\pi^2}{k^3}\left(\frac{3+3w}{5+3w}\right)^2\mathscr{P}_\zeta(k)$$

With some more calculations, one obtains

$$\mathscr{P}_h(\eta,k) = 4\int_0^\infty \mathrm{d}v \int_{|1-v|}^{1+v} \mathrm{d}u \left(\frac{4v^2-(1+v^2-u^2)^2}{4uv}\right)^2 I^2(u,v,k\eta)\mathscr{P}_\zeta(uk)\mathscr{P}_\zeta(vk)$$

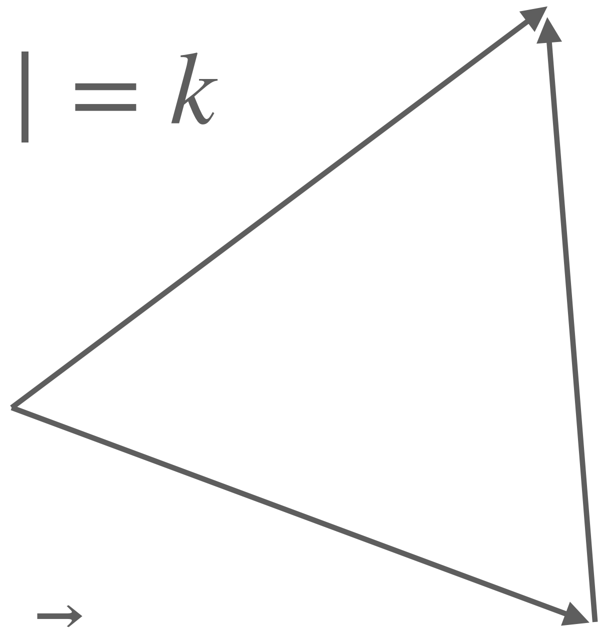
where $I(u,v,k\eta)$ has been calculated in [Espinosa, Racco, Riotto, 1804.07732], [Kohri, Terada, 1804.08577].

Scalar-Induced Gravitational Waves

Intuition from the delta-function case

$$\mathscr{P}_\zeta = A_\zeta\, \delta\left(\ln(k/k_*)\right)$$

$|\vec{k}| = k$



$|\vec{k}_2| = vk$

$|\vec{k}_1| = uk$

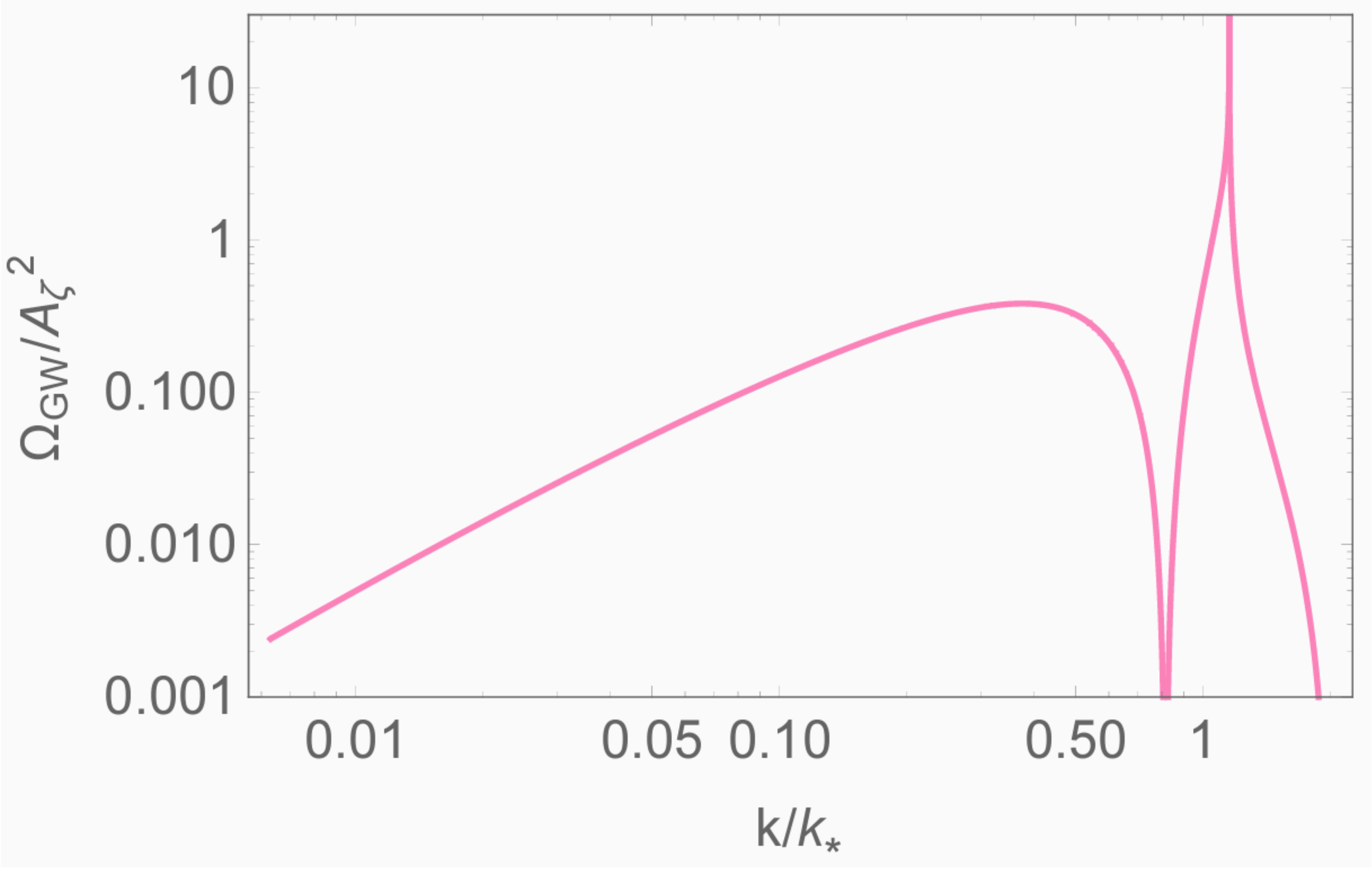
momentum conservation

$$\vec{k} = \vec{k}_1 + \vec{k}_2$$

energy conservation
(resonance condition)

$$|\vec{k}| = c_s\left(|\vec{k}_1| + |\vec{k}_2|\right)$$

induced GWs produced in the RD era



When the GWs are induced?

We adopt the standard inflationary paradigm.

Basically, the induced GWs are dominantly produced around the horizon reentry of each mode.

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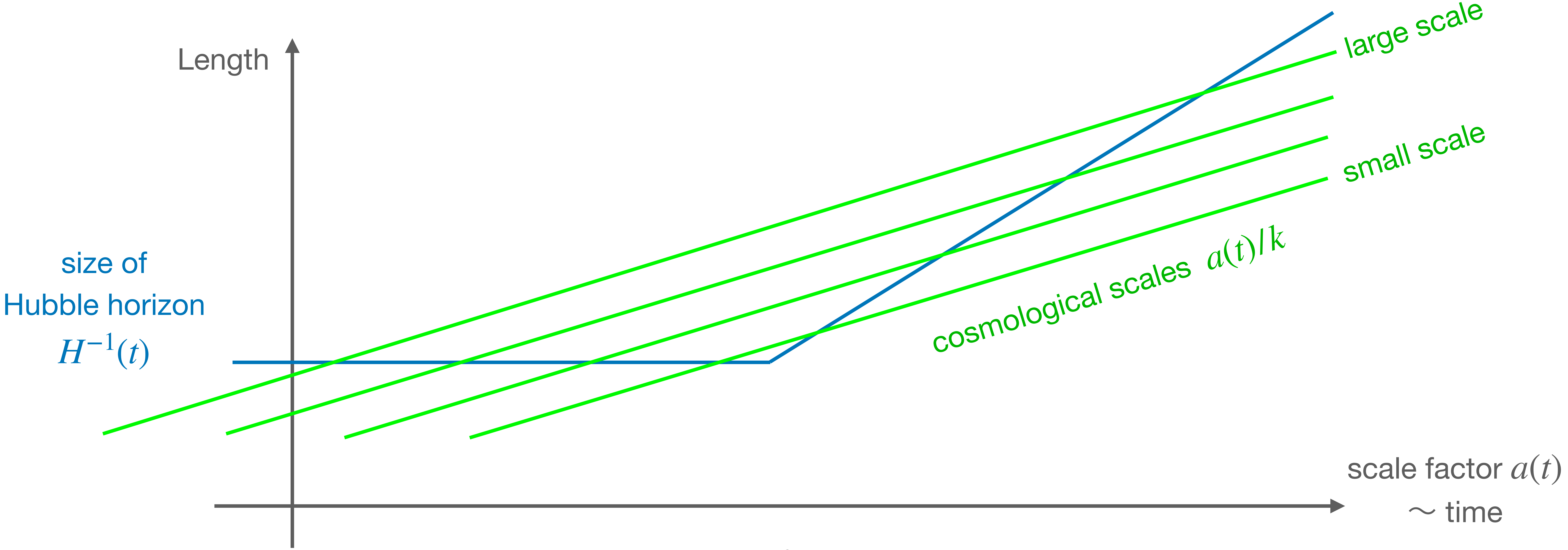
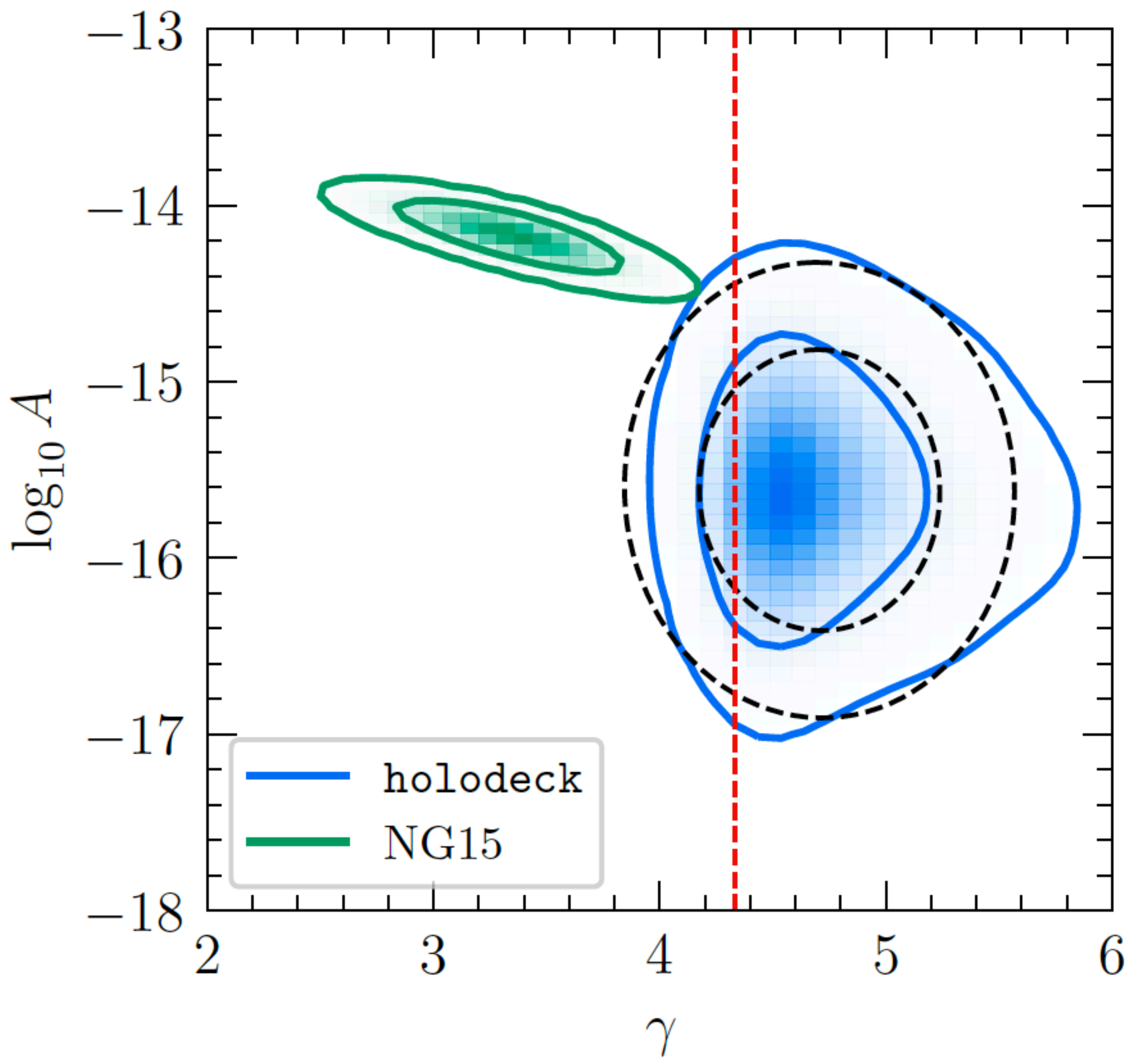


Astrophysical Interpretation

Supermassive Black Hole Binary Mergers



The simplest model doesn't work well.

- Circular orbit
- Energy loss only due to GW emission

Interactions with the environment are important.

Inflation and Cosmic Scales

Inflation solves

- homogeneity puzzle
- flatness puzzle
- monopole puzzle

and seeds the large scale structure of the Universe.

Simple models (single-field slow-roll inflation) are consistent with the CMB observations.

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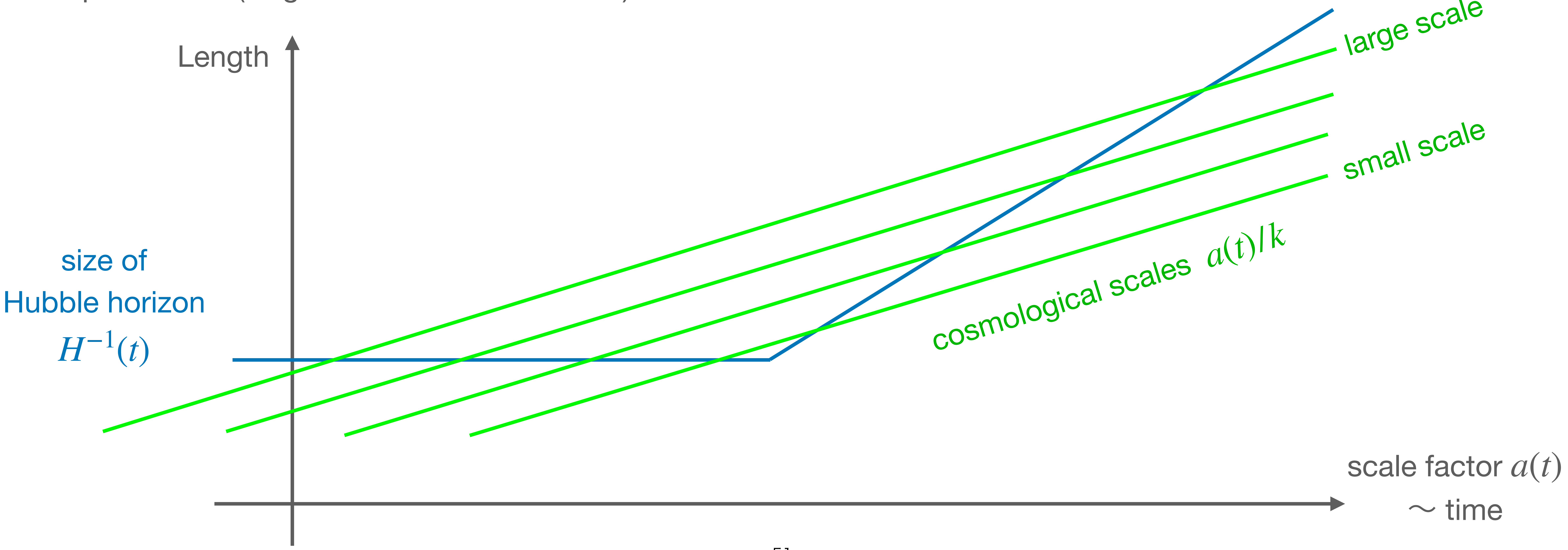


Parameter Relations

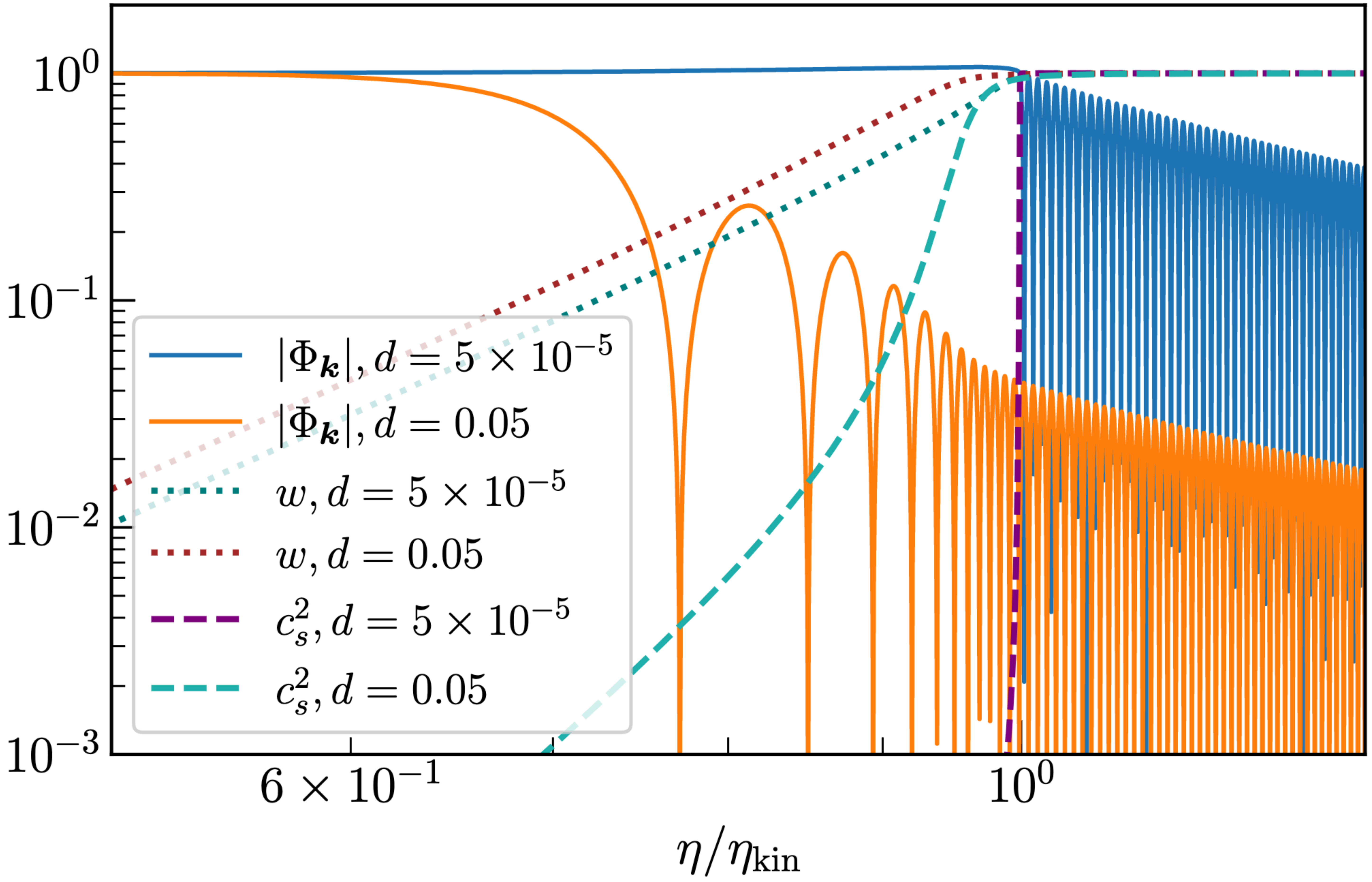
Duration of the MD era: $$\frac{\eta_{\rm kin}}{\eta_{\rm eq,1}} = 3.8 \left(\frac{m_S}{10^5\,\mathrm{GeV}}\right)^{1/3} \left(\frac{f_a}{10^9\,\mathrm{GeV}}\right)^{-1/3} \left(\frac{Y_\theta}{100}\right)^{2/3}$$

Temperature at the second equality: $$T_{\rm eq,2} = 1.8 \times 10^6\,\mathrm{GeV} \left(\frac{f_a}{10^9\,\mathrm{GeV}}\right) \left(\frac{Y_\theta}{100}\right)^{-1}$$

Frequency corresponding to the second equality: $$\frac{1}{2\pi\eta_{\rm eq,2}} = 0.11\,\mathrm{Hz} \left(\frac{T_{\rm eq,2}}{1.8 \times 10^6\,\mathrm{GeV}}\right)$$

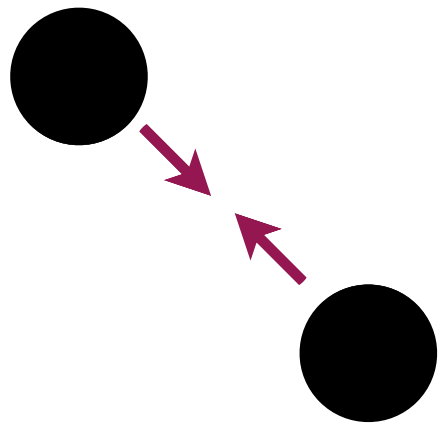
Maximal charge yield: $$Y_{\theta,\rm max} = 100 \left(\frac{m_S}{8.7 \times 10^8\,\mathrm{GeV}}\right)^{-1/3} \left(\frac{b}{0.1}\right)^{1/3}$$

Evolution of $\Phi_{\mathbf{k}}$, w, and c_s^2



GWs from Binary PBH Mergers

Binary formation in the radiation era



[Nakamura, Sasaki, Tanaka, Thorne, 1997]
[Sasaki, Suyama, Tanaka, Yokoyama, 1603.08338]

comoving merger rate

[Sasaki, Suyama, Tanaka, Yokoyama, 1603.08338; 1801.05235]

$$R(z) = \left(\frac{f_{\mathrm{PBH}}\Omega_{\mathrm{CDM}}\rho_{\mathrm{c}}}{M}\right)\frac{\mathrm{d}P_t}{\mathrm{d}t}$$

$$\frac{\mathrm{d}P_t}{\mathrm{d}t} = \begin{cases} \frac{3}{58}\left[-\left(\frac{t}{t_0}\right)^{\frac{3}{8}} + \left(\frac{t}{t_0}\right)^{\frac{3}{37}}\right]\frac{1}{t} & \text{for } t < t_c \\ \frac{3}{58}\left(\frac{t}{t_0}\right)^{\frac{3}{8}}\left[-1 + \left(\frac{t}{t_c}\right)^{-\frac{29}{56}}\left(\frac{4\pi}{3}f_{\mathrm{PBH}}\right)^{-\frac{29}{8}}\right]\frac{1}{t} & \text{for } t \geq t_c, \end{cases}$$

$$t_0 = (3/170)\left\{\bar{x}^4/\left[(GM)^3(4\pi f_{\mathrm{PBH}}/3)^4\right]\right\}$$

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Energy spectrum at the source frame

[Ajith et al., 0710.2335] [Ajith et al., 0909.2867]

$$\frac{\mathrm{d}E}{\mathrm{d}f_{\mathrm{s}}} = \frac{(G\pi)^{2/3}M_c^{5/3}}{3}\begin{cases} f_{\mathrm{s}}^{-1/3} & \text{for } f_{\mathrm{s}} < f_1 \\ w_1 f_{\mathrm{s}}^{2/3} & \text{for } f_1 \leq f_{\mathrm{s}} < f_2 \\ w_2\frac{\sigma^4 f_{\mathrm{s}}^2}{\left(\sigma^2 + 4(f_{\mathrm{s}} - f_2)^2\right)^2} & \text{for } f_2 \leq f_{\mathrm{s}} \leq f_3 \\ 0 & \text{for } f_{\mathrm{s}} > f_3 \end{cases}$$

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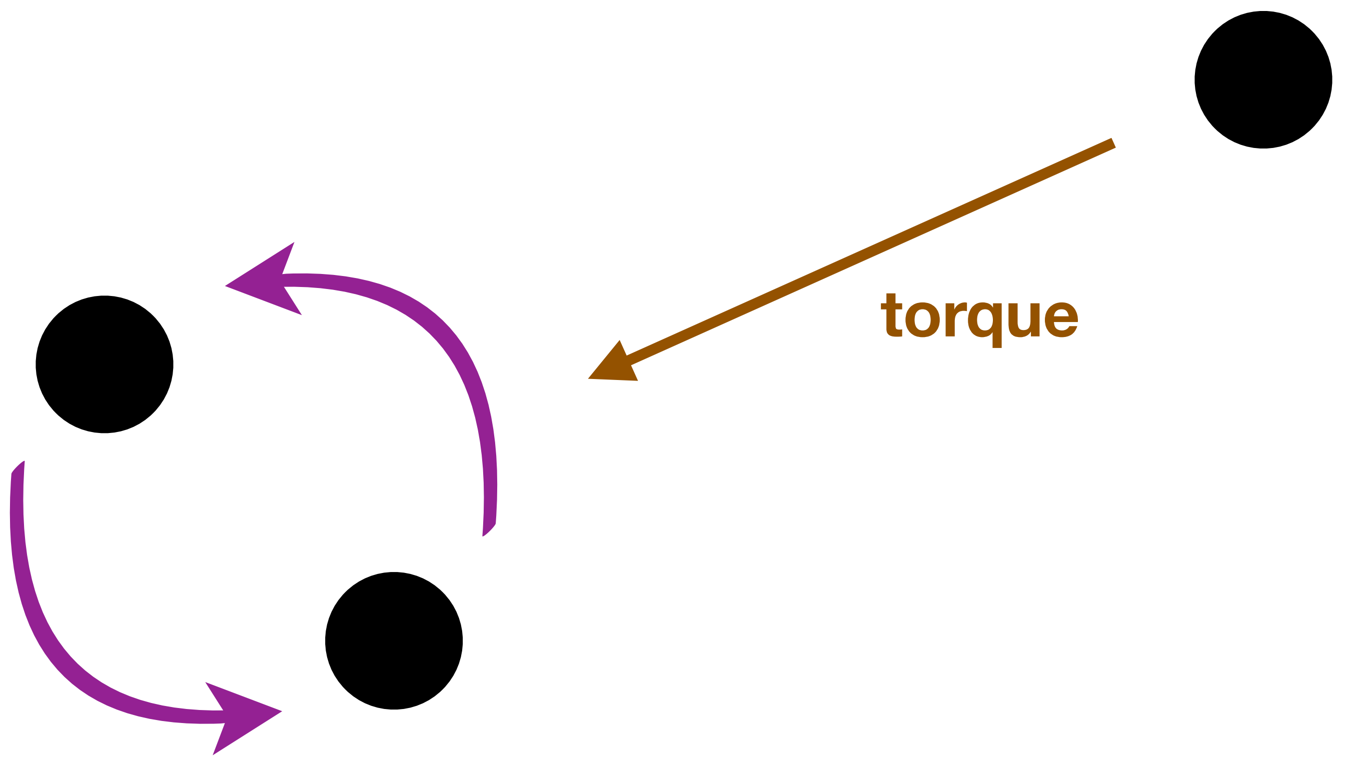

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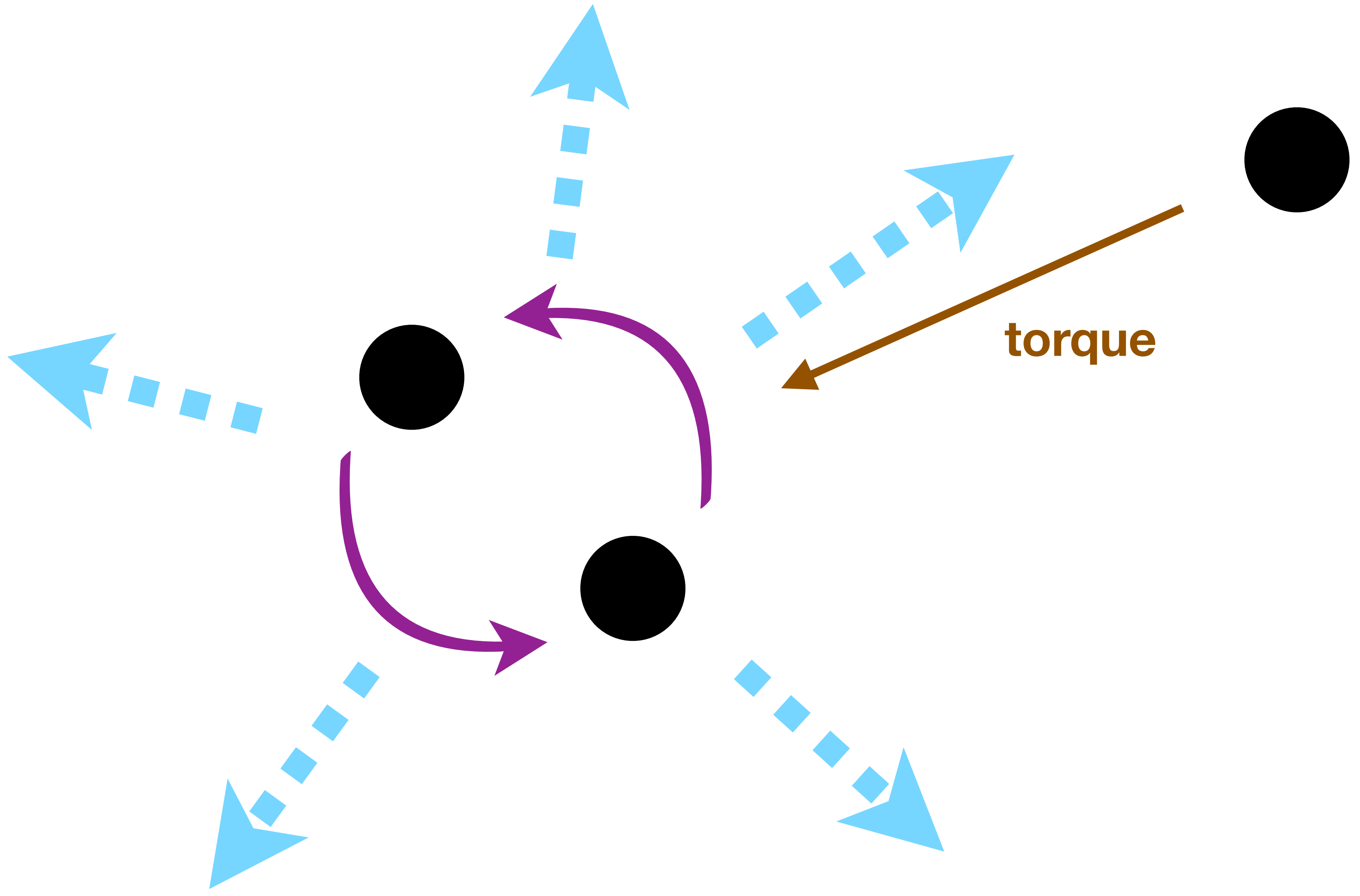

Binary Black Holes loose energy by emitting Gravitational Waves.

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f^2 scaling from Induced GWs

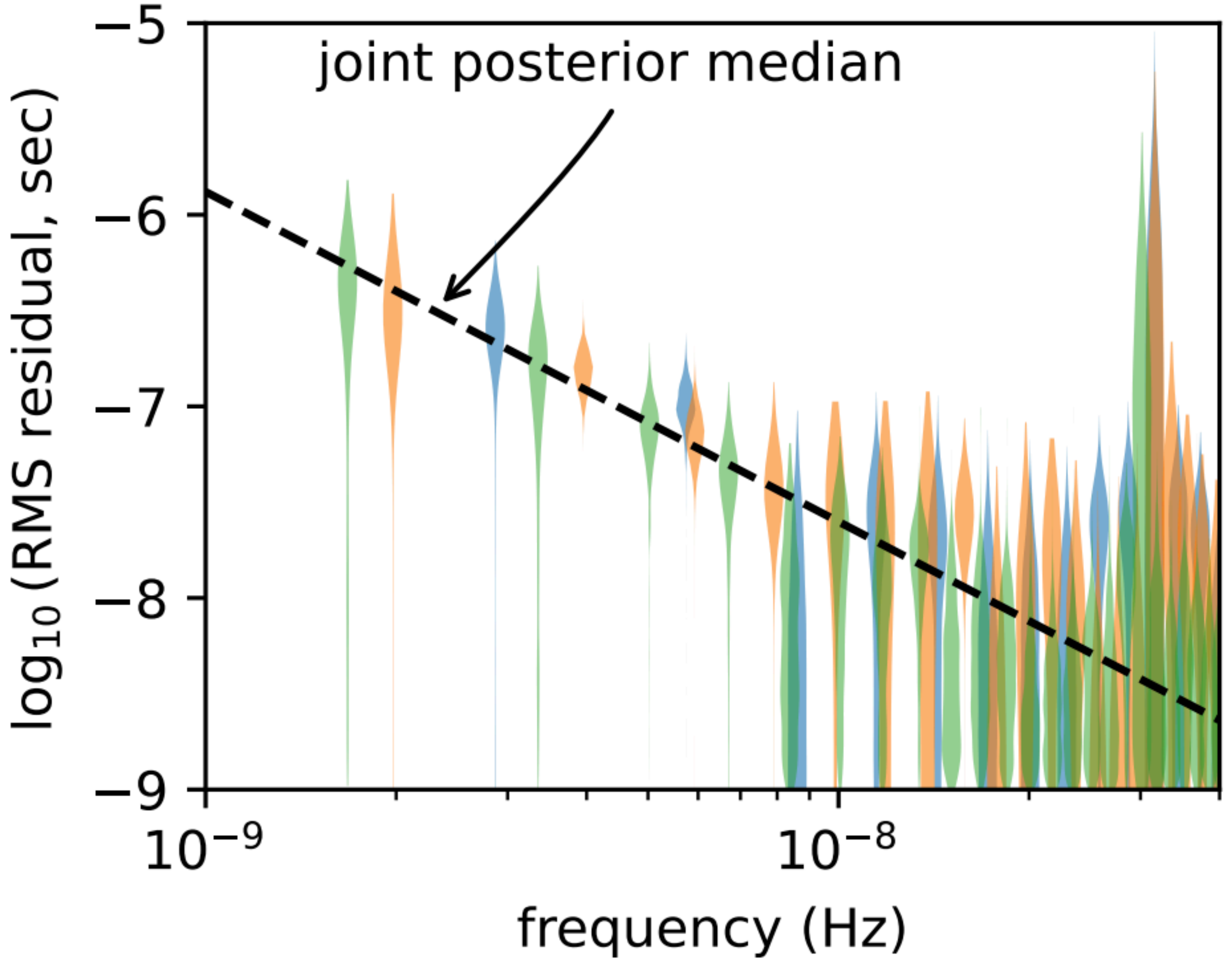
Gravitational-Wave Spectrum

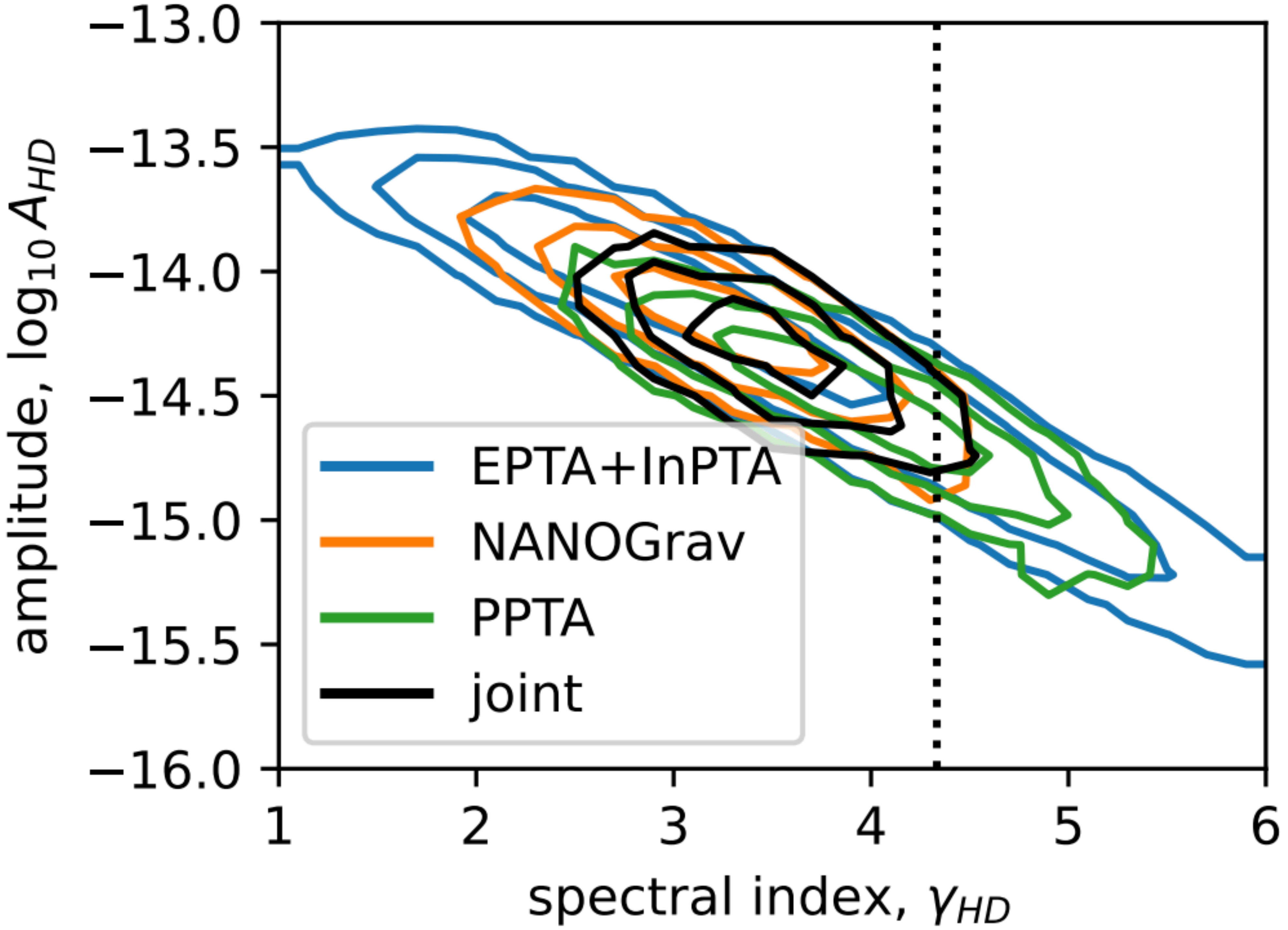
$$\Omega_{\rm GW}(f) = \frac{2\pi^2 f_*^2}{3H_0^2} A_{\rm GWB}^2 \left(\frac{f}{f_*}\right)^{5-\gamma}$$

$5 - \gamma = 1.8 \pm 0.6$ (90% credible region)

[NANOGrav, 2306.16213]

[IPTA, 2309.00693]





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Data may be implying the $\Omega_{\rm GW} \propto f^2$ behavior!?

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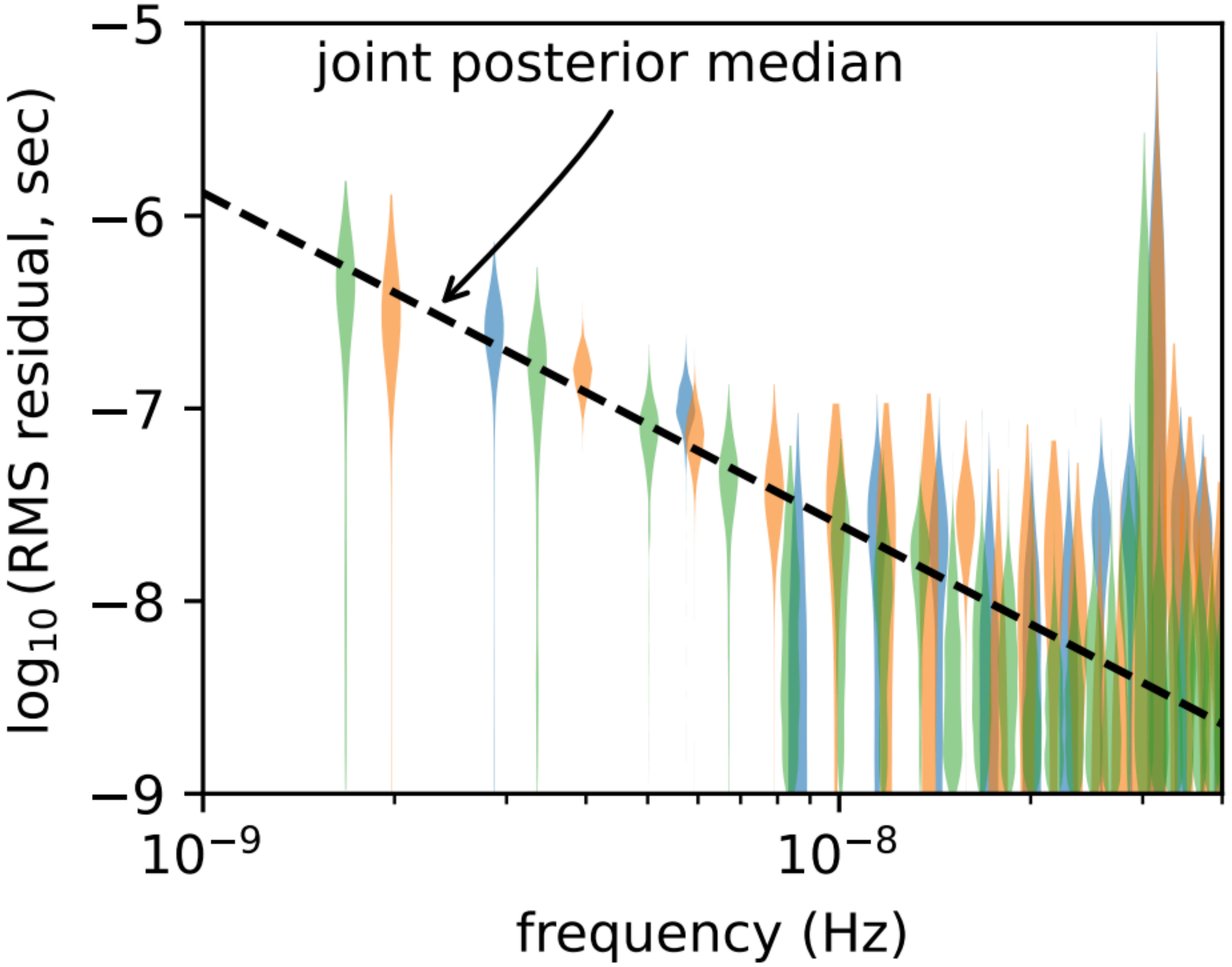
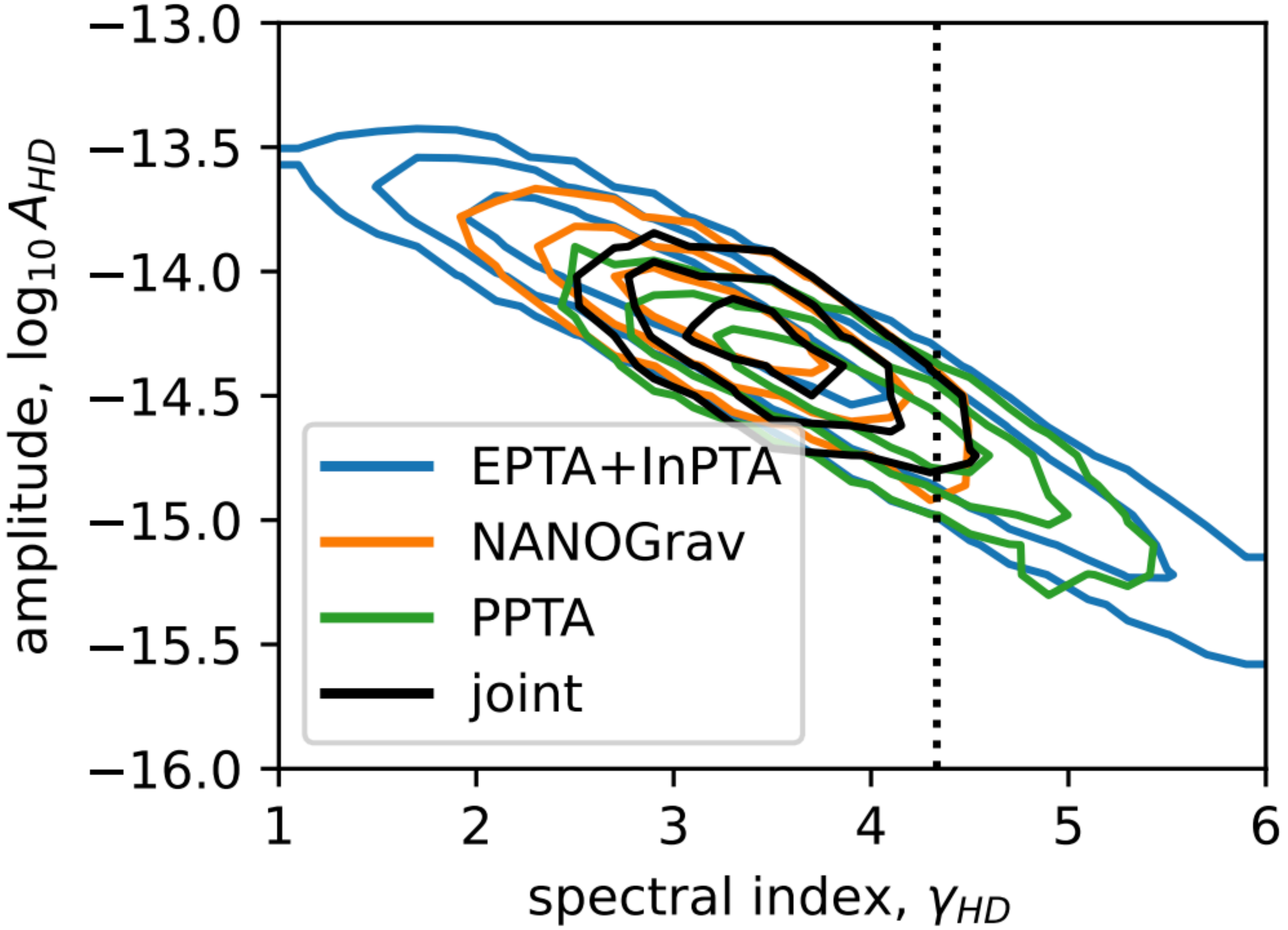
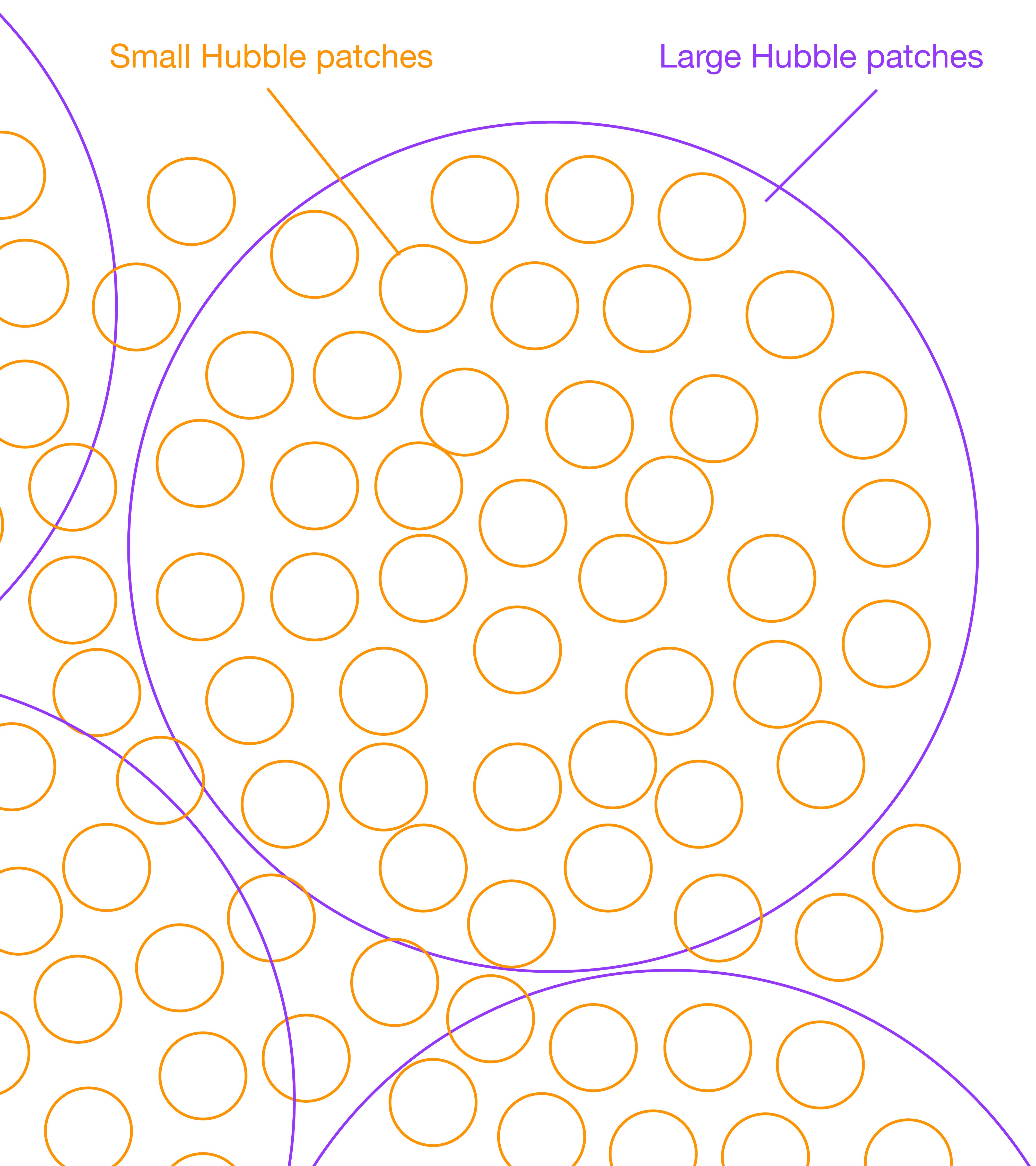




Universal Infrared f^3 scaling

[Cai, Pi, Sasaki, 1909.13728]
[Hook, Marques-Tavares, Racco, 2010.03568]



- Finite duration of GW generation on subhorizon scales

Central Limit Theorem $$\mathscr{P}_h(k_L) \propto \frac{1}{N_{\rm patch}} = \left(\frac{k_L}{k_S}\right)^3$$

- Radiation-dominated era

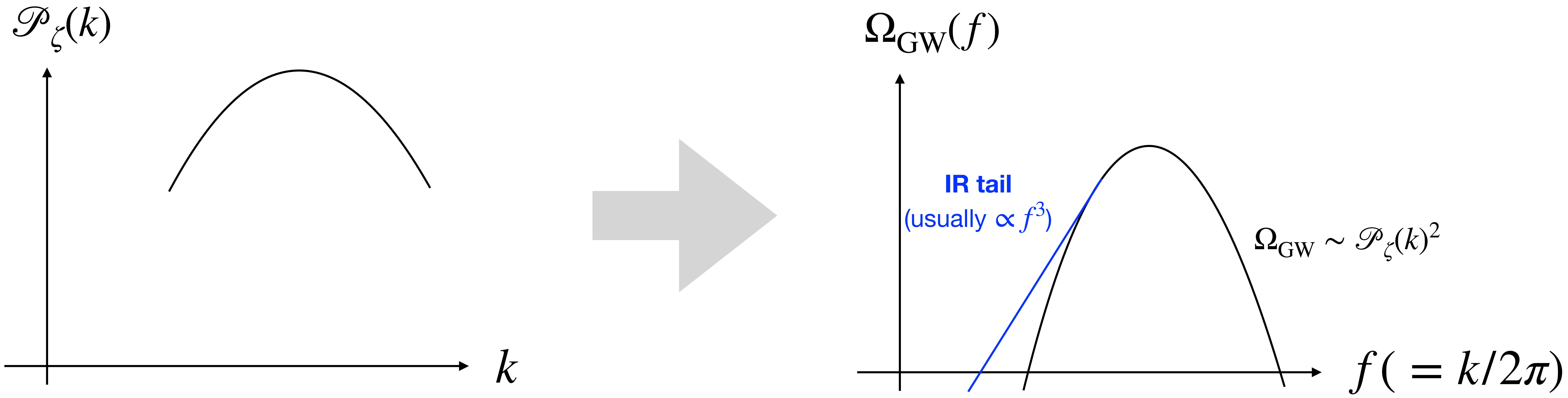
no further redshift factors $\Omega_{\rm GW}(f) \propto f^3$

IR tail of the induced GWs

$$\Omega^{\rm ind}_{\rm GW}(f) = \Omega_{\rm r}\left(\frac{g_*(f)}{g_*^0}\right)\left(\frac{g_{*,s}^0}{g_{*,s}(f)}\right)^{4/3}\bar{\Omega}^{\rm ind}_{\rm GW}(f) \qquad \bar{\Omega}^{\rm ind}_{\rm GW}(f) = \int_0^\infty {\rm d}v \int_{|1-v|}^{1+v} {\rm d}u\, \mathcal{K}(u,v)\,\mathcal{P}_{\mathcal{R}}(uk)\,\mathcal{P}_{\mathcal{R}}(vk)$$

$$\mathcal{K}(u,v) = \frac{3\left(4v^2-(1+v^2-u^2)^2\right)^2\left(u^2+v^2-3\right)^4}{1024\,u^8v^8}\left[\left(\ln\left|\frac{3-(u+v)^2}{3-(u-v)^2}\right| - \frac{4uv}{u^2+v^2-3}\right)^2 + \pi^2\Theta(u+v-\sqrt{3})\right]$$

[Espinosa, Racco, Riotto, 1804.27732]
[Kohri, Terada, 1804.08577]

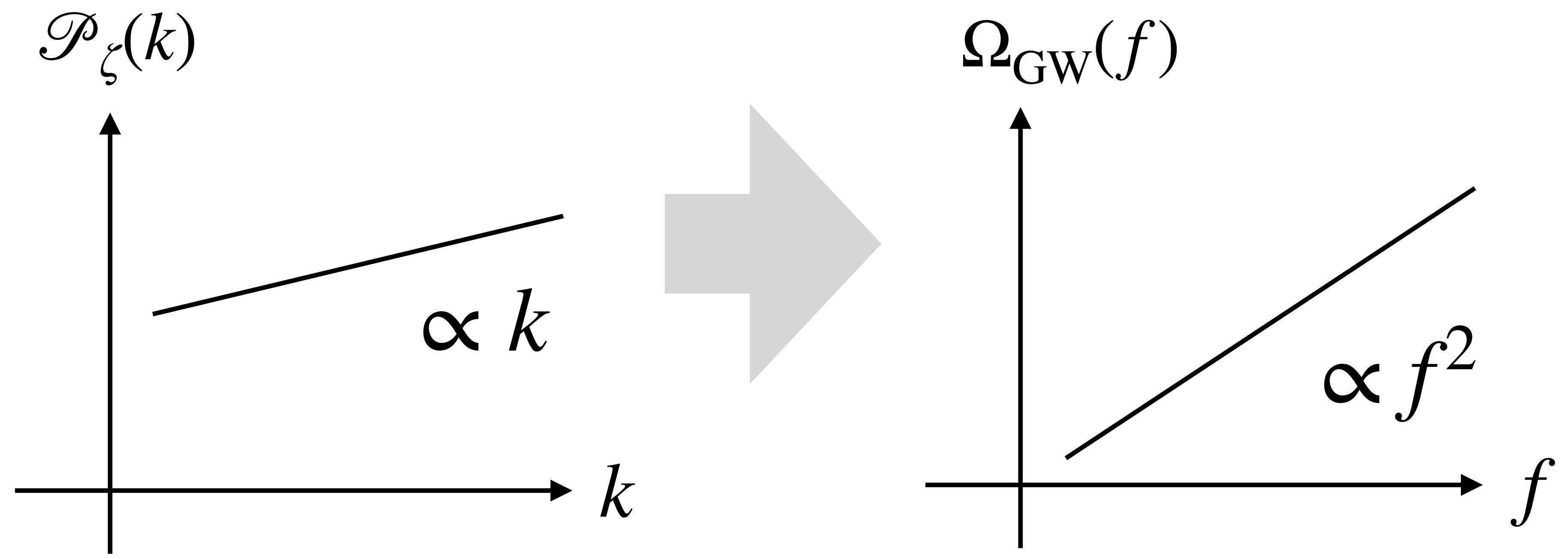


[Cai, Pi, Sasaki, 1909.13728]
[Yuan, Chen, Huang, 1910.09099]
[Domènech, Pi, Sasaki, 2005.12314]

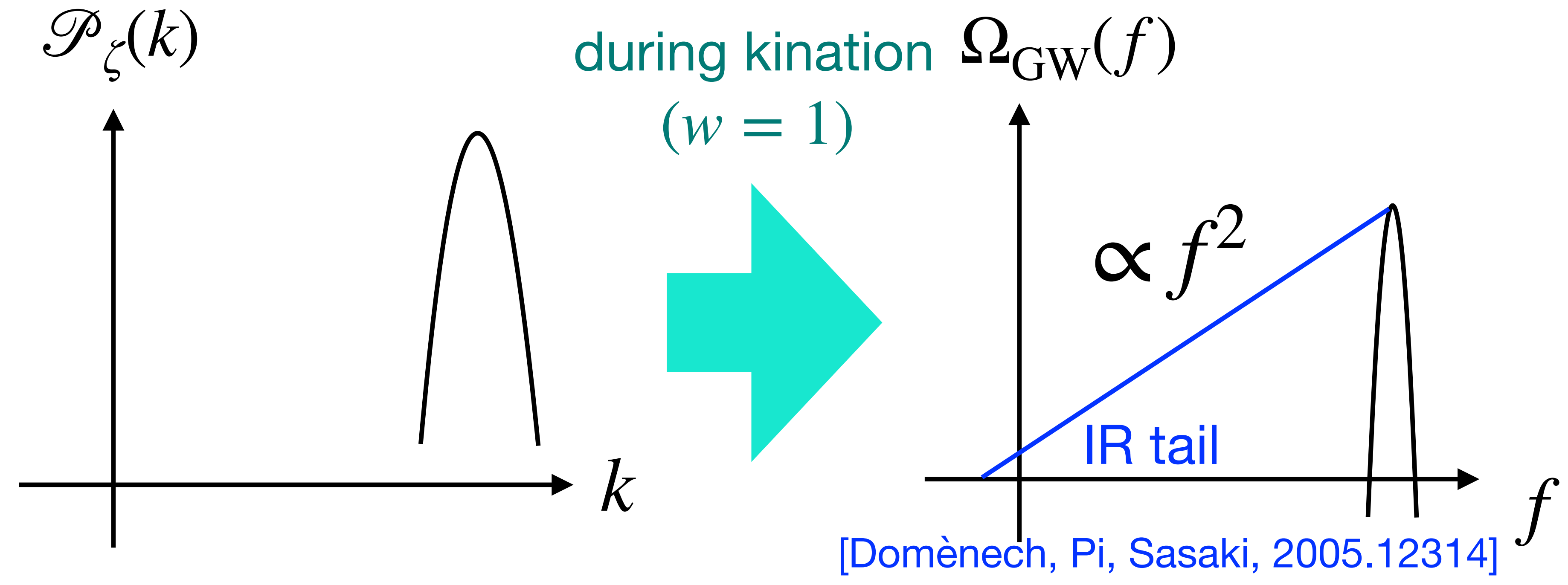
Explaining $\Omega_{\rm GW} \propto f^2$ Scaling

[Inomata, Kohri, Terada, 2306.17834]

Possibility 1

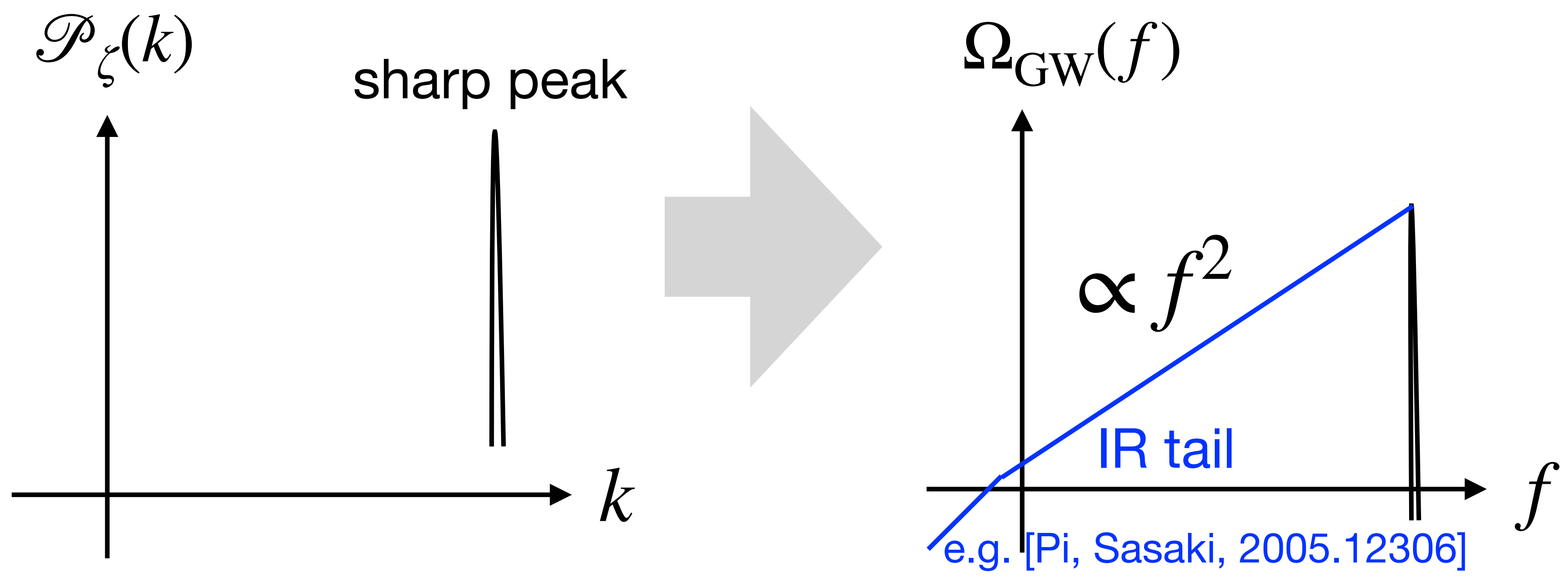


Possibility 2



[Harigaya, Inomata, Terada, 2309.00228]

Possibility 3



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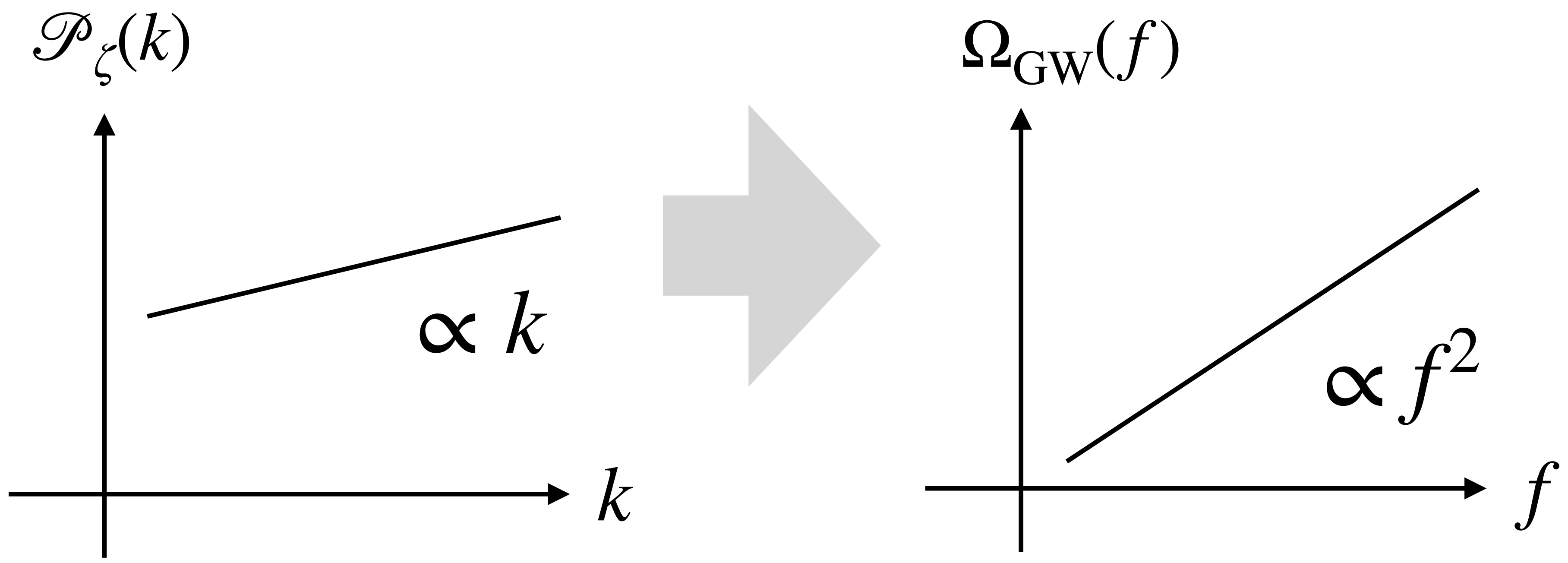

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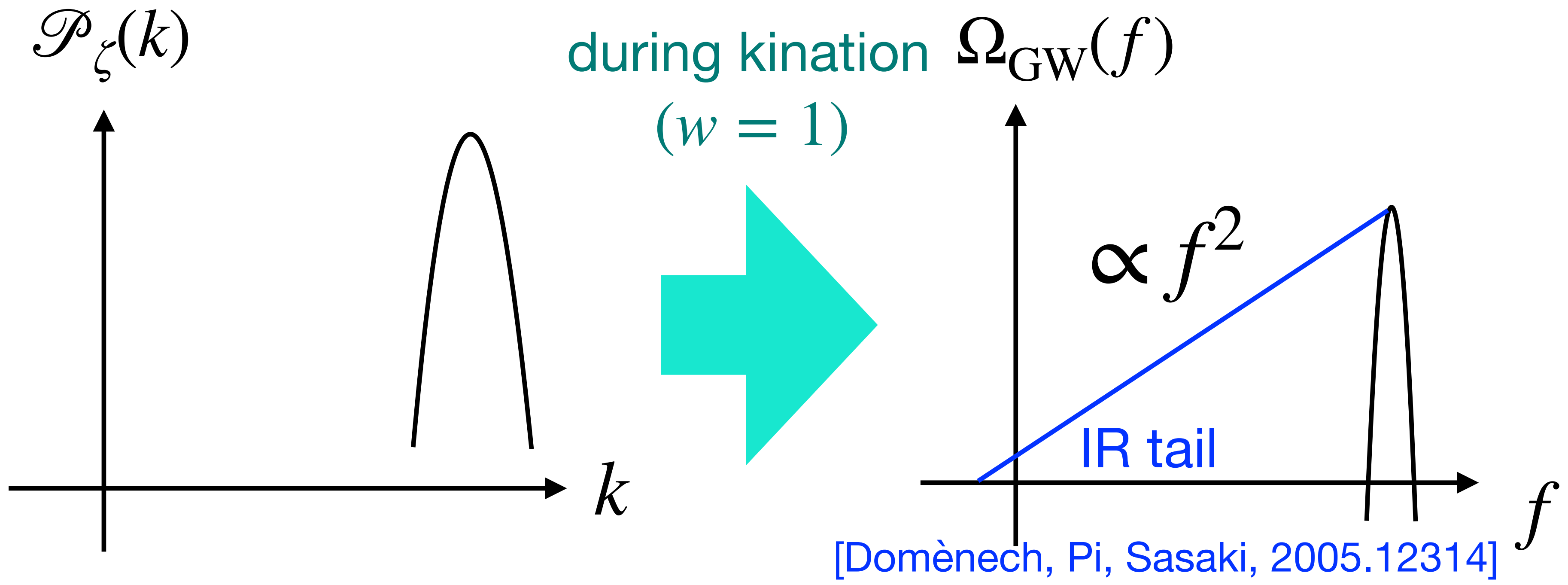

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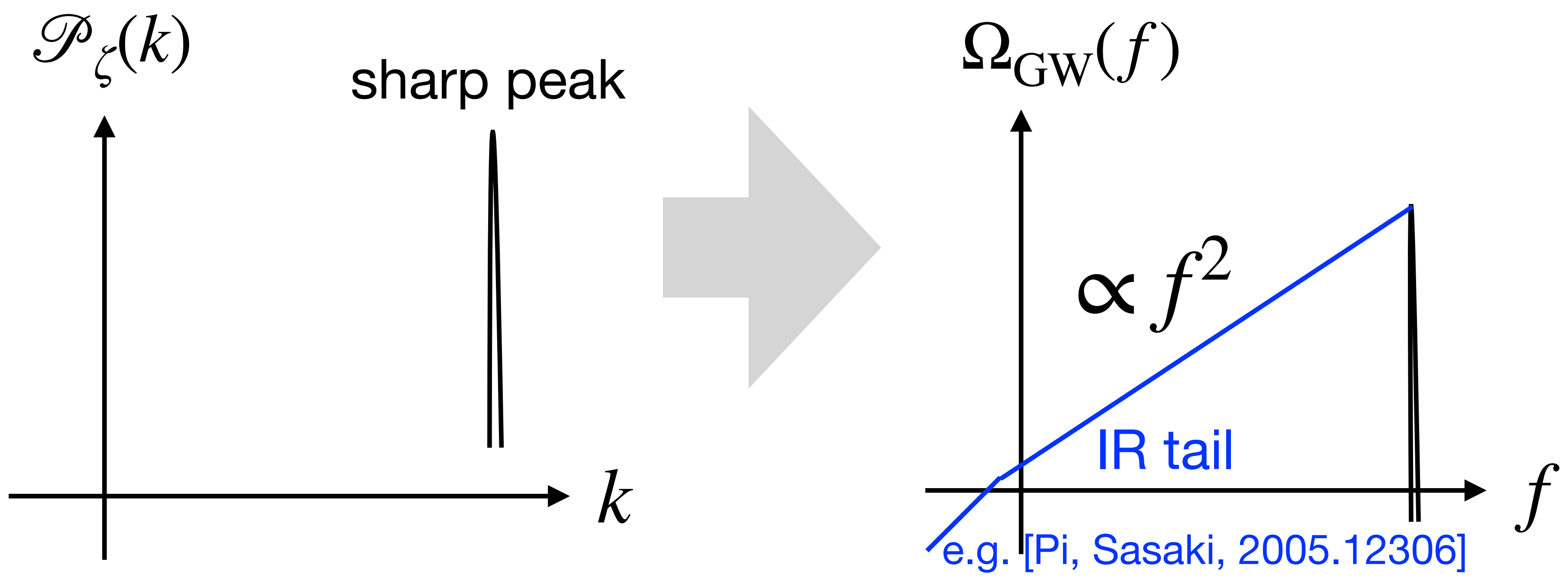

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The peak is on a **smaller scale** than the PTA range.

→ **SUB-solar mass PBHs**

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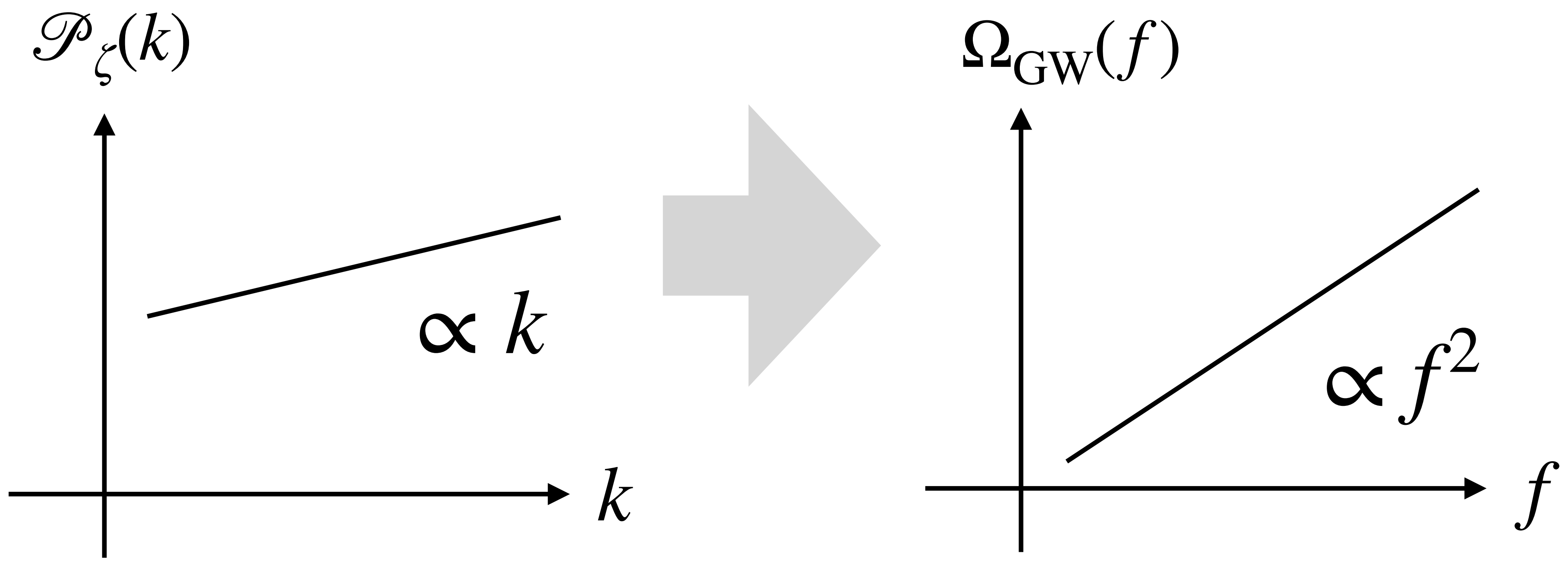

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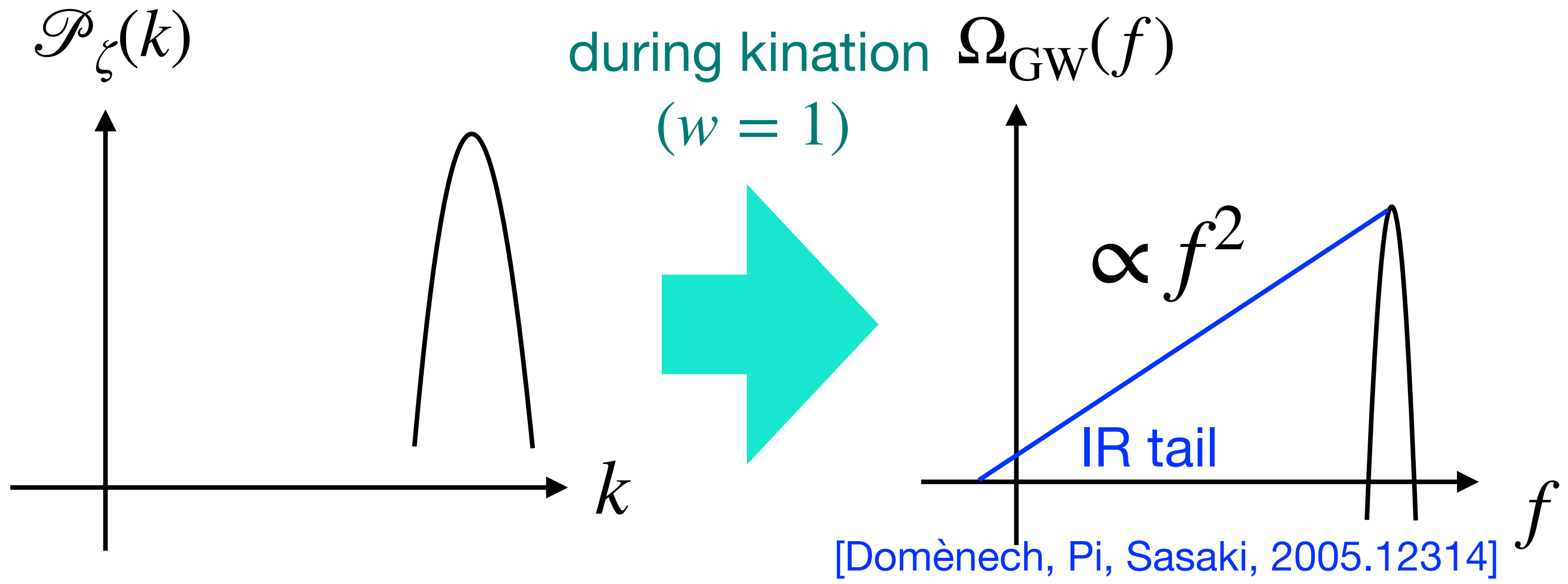

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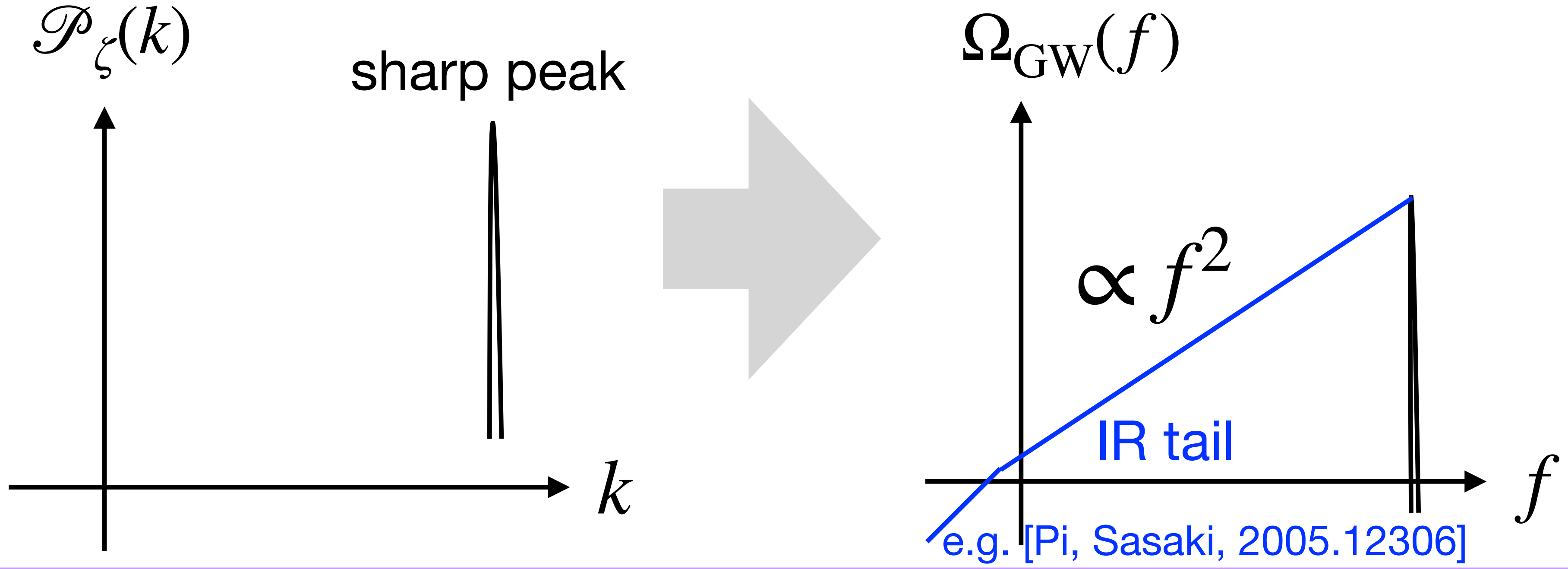

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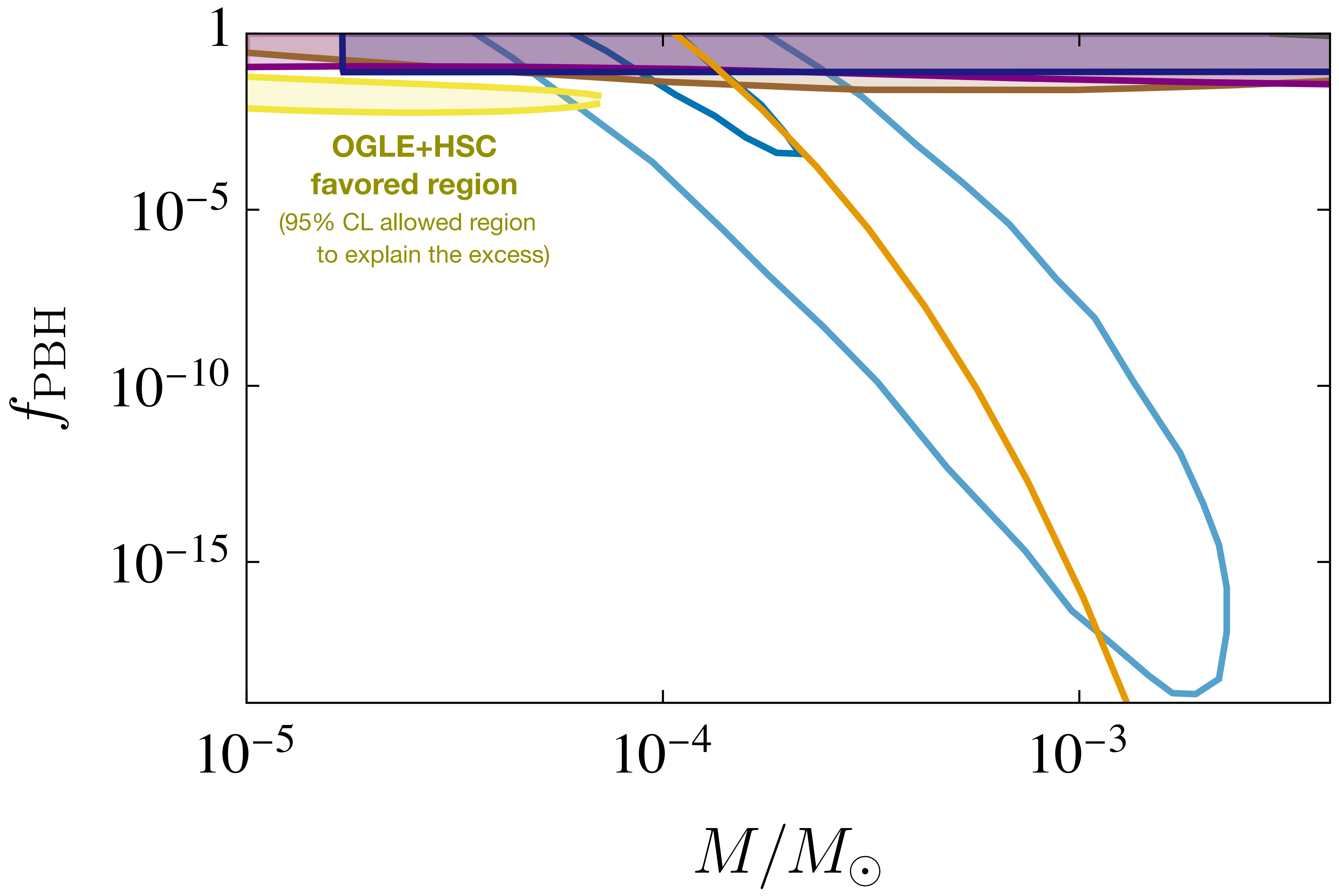
Implications for Primordial Black Holes

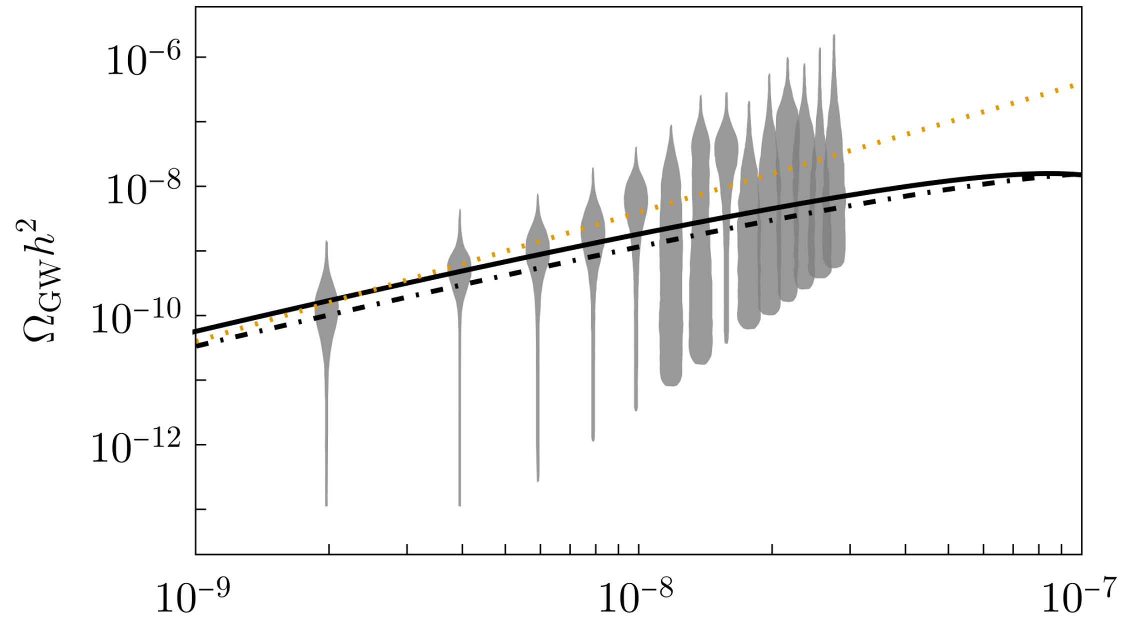
[Inomata, Kohri, Terada, 2306.17834]

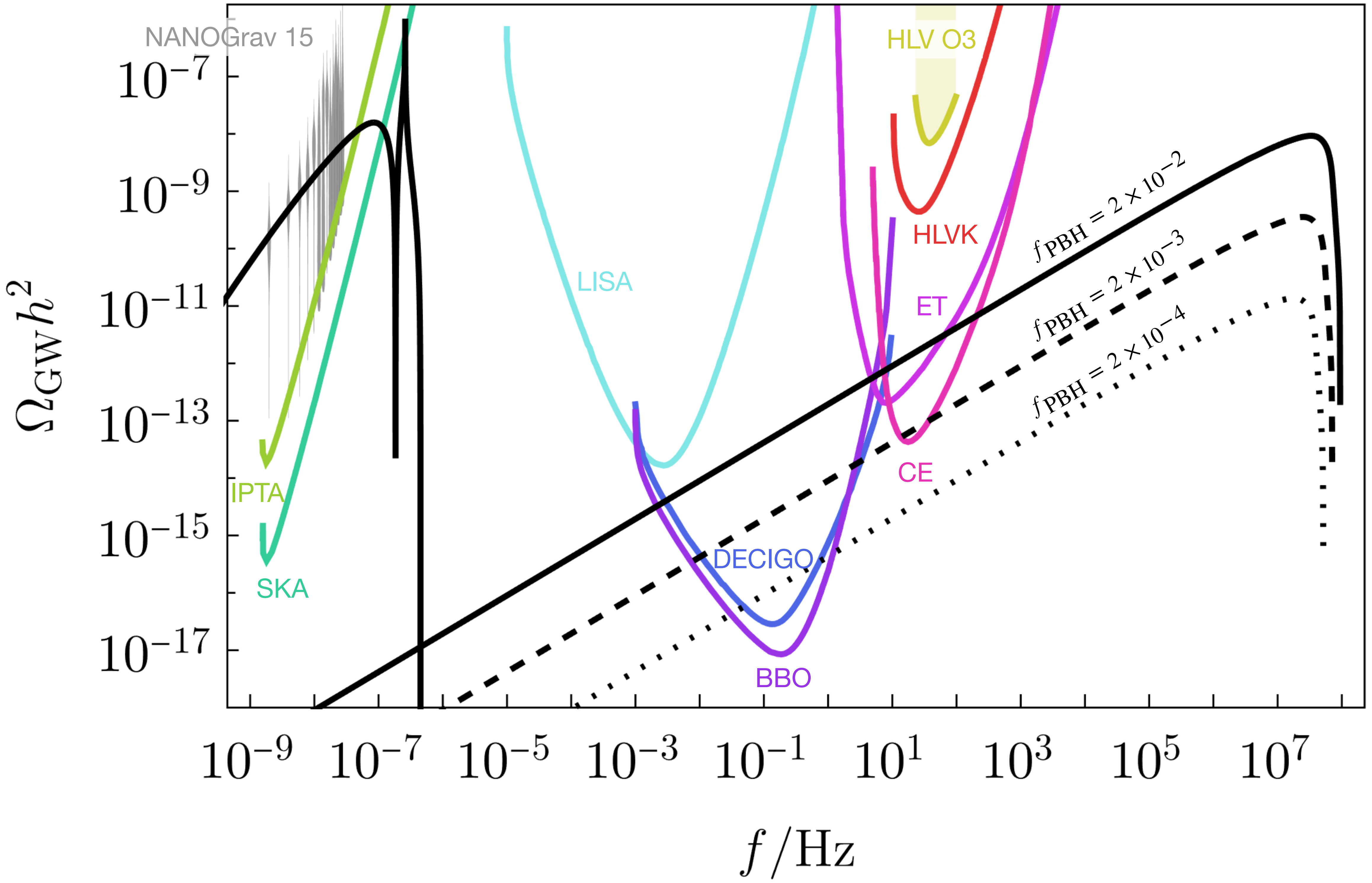
See also [Franciolini et al., 2306.17149]

The excess events of microlensing at OGLE: [Mróz et al., 1707.07634]

Interpretation by PBHs: [Niikura et al., 1901.07120]







$M/M_\odot = 1.2\times10^{-4}, 1.6\times10^{-4}$, and 2.2×10^{-4} from top to bottom.

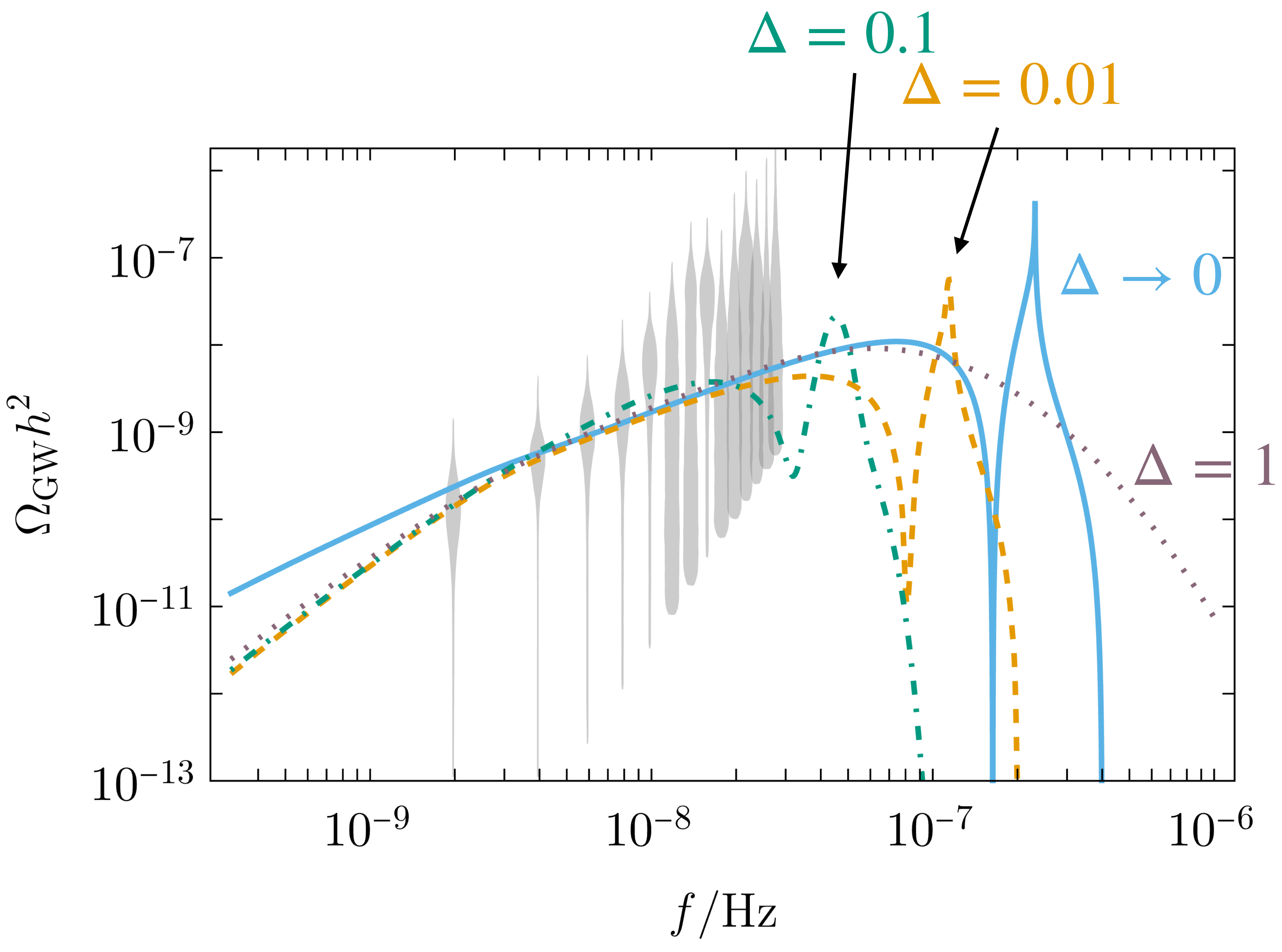
The sensitivity curves were taken from [Schmitz, 2002.04615].

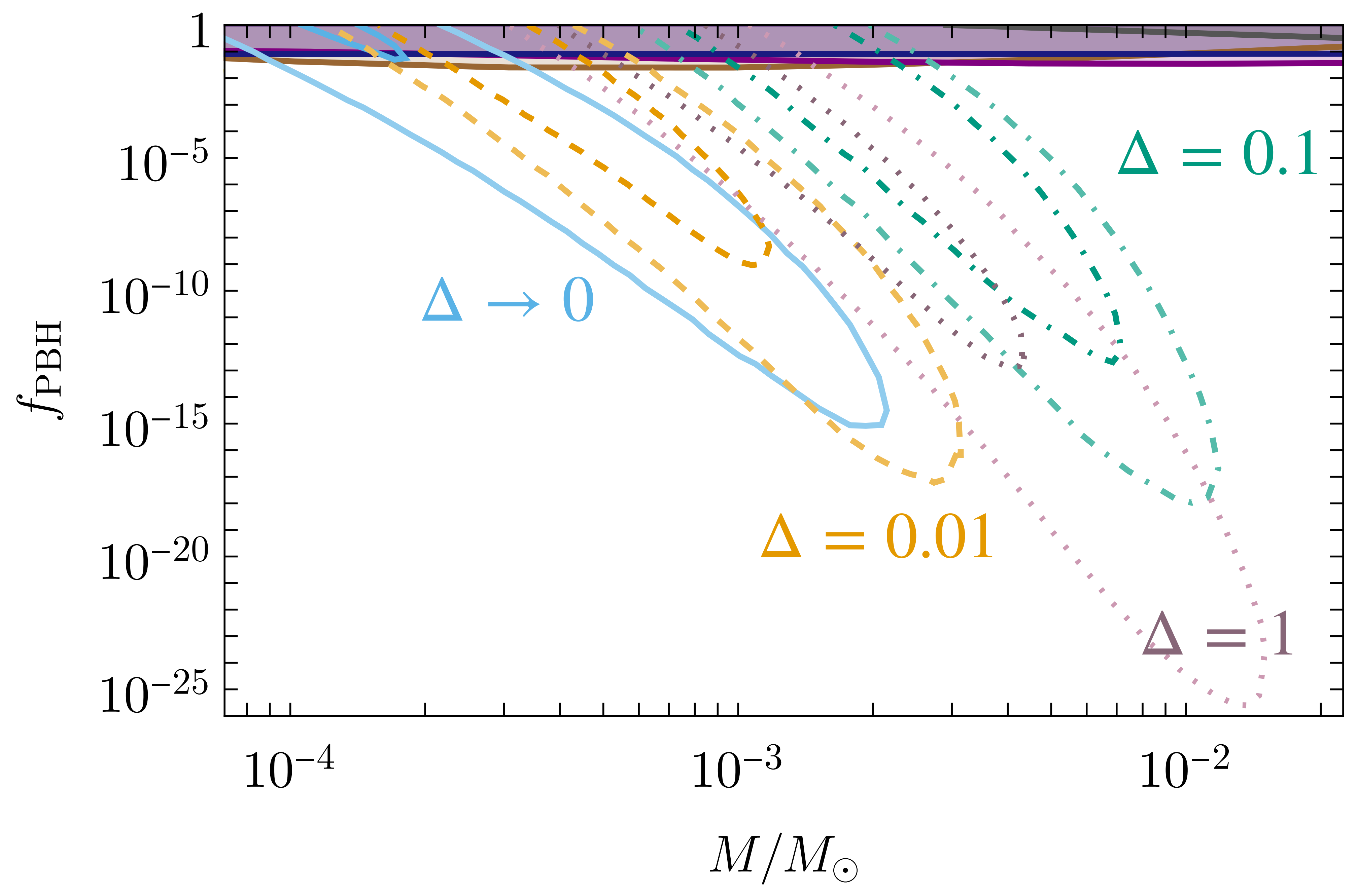
The HLV O3 constraint is from [Abbott et al. (LIGO-Virgo-KAGRA), 2101.12130].

Implications for Primordial Black Holes

Effects of finite width of the power spectrum of the curvature perturbations

$$\mathscr{P}_\zeta(k) = \frac{A_\zeta}{\sqrt{2\pi\Delta^2}} \exp\left(-\frac{(\log(k/k_*))^2}{2\Delta^2}\right)$$

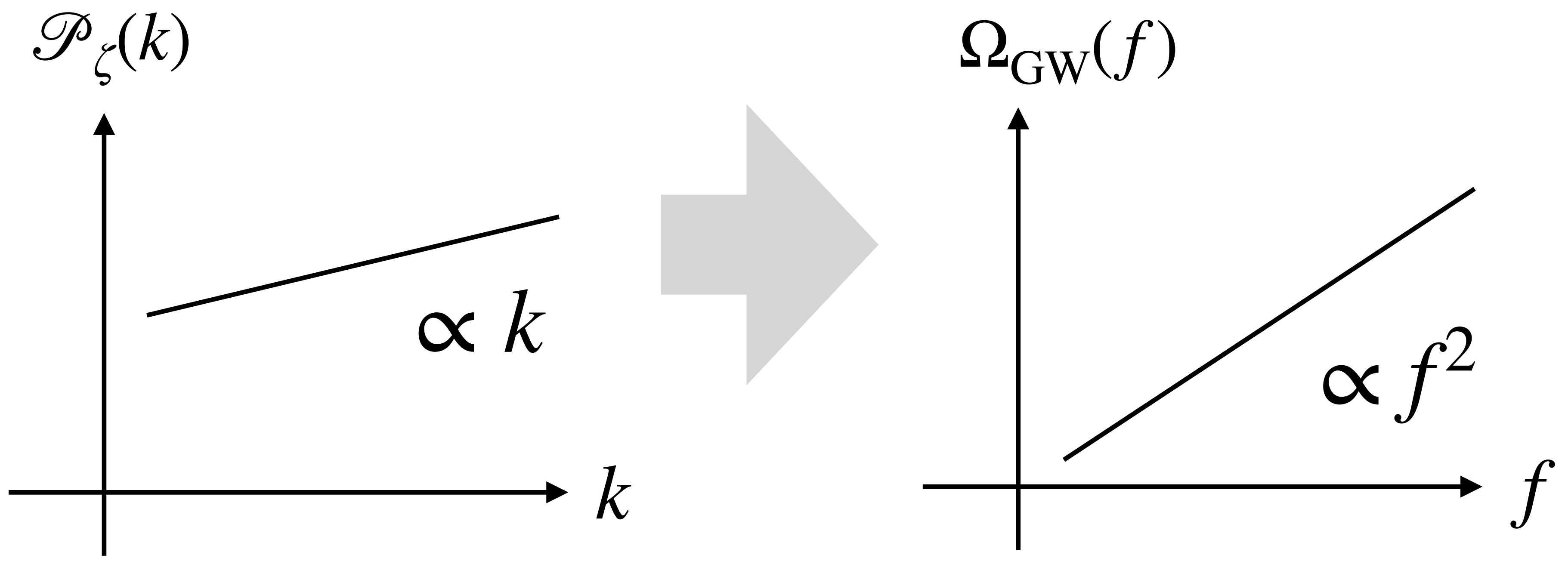




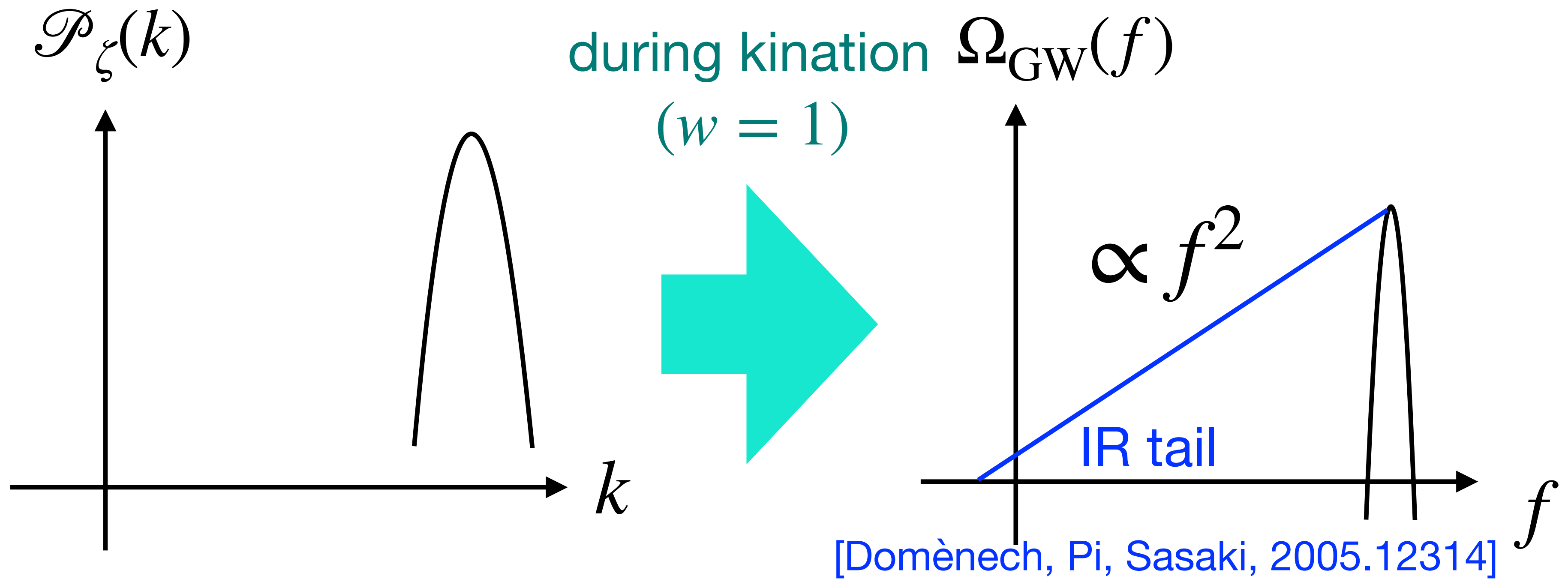
Explaining $\Omega_{\rm GW} \propto f^2$ Scaling

[Inomata, Kohri, Terada, 2306.17834]

Possibility 1

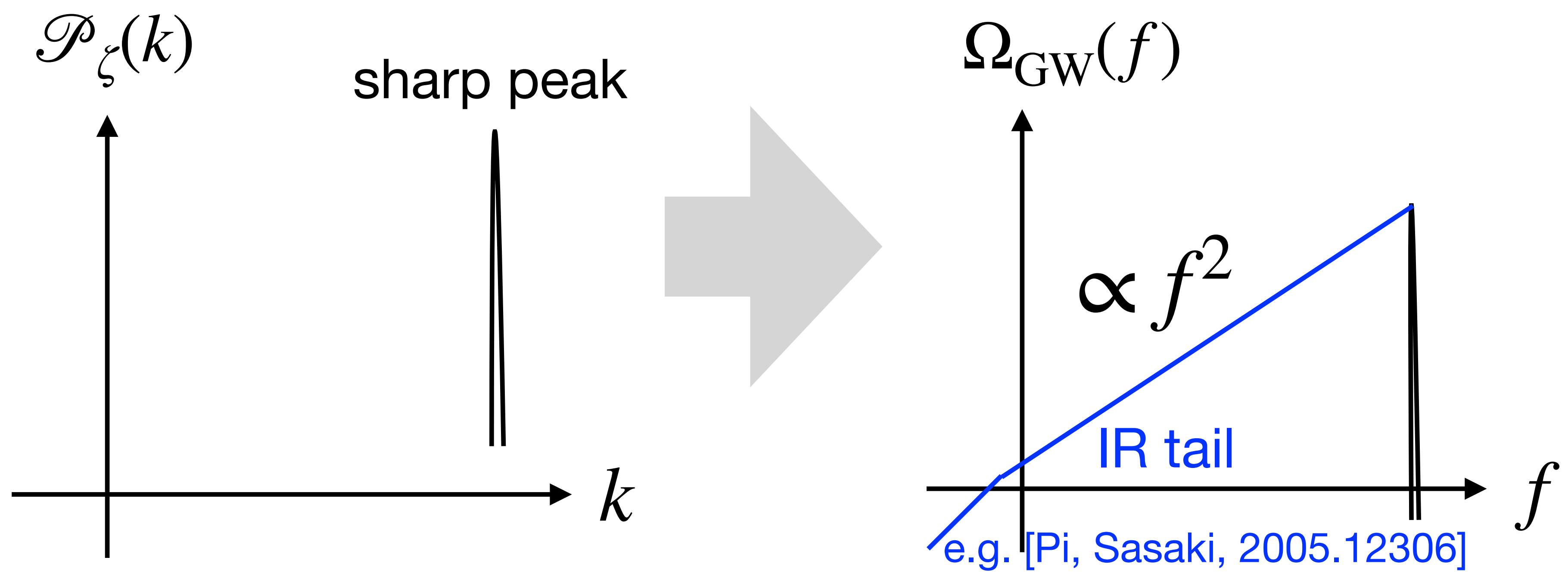


Possibility 2



[Harigaya, Inomata, Terada, 2309.00228]

Possibility 3



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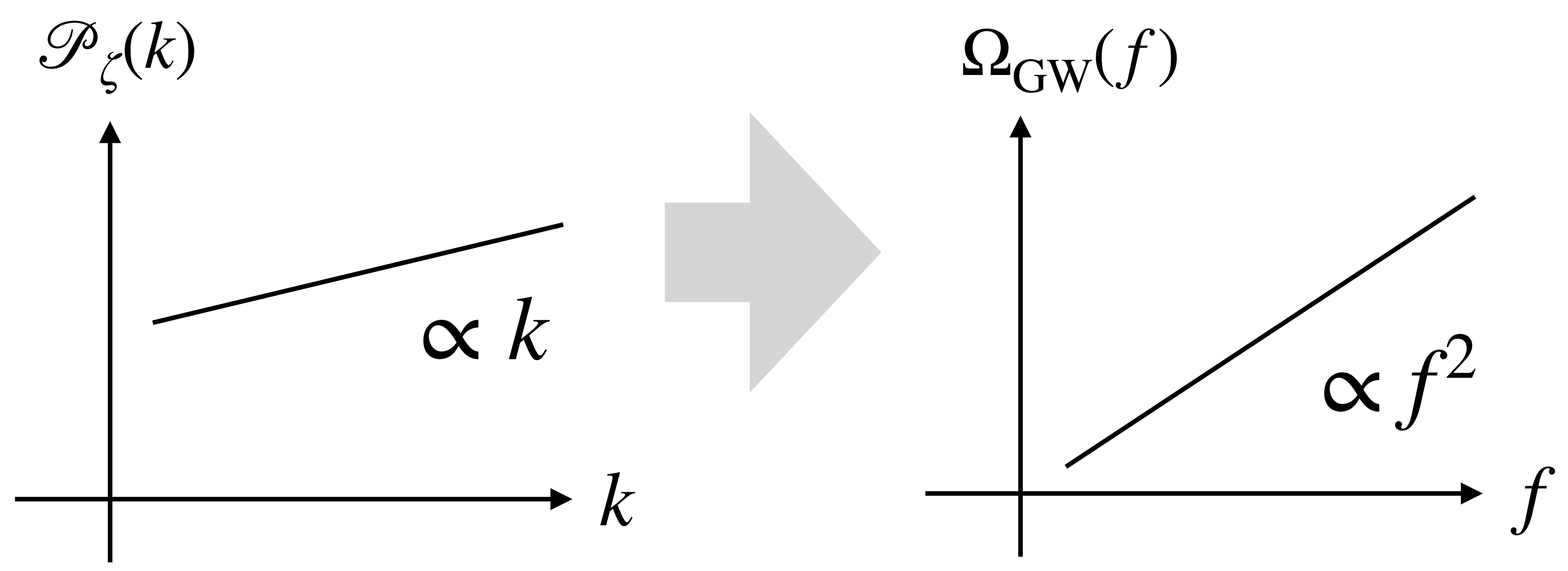
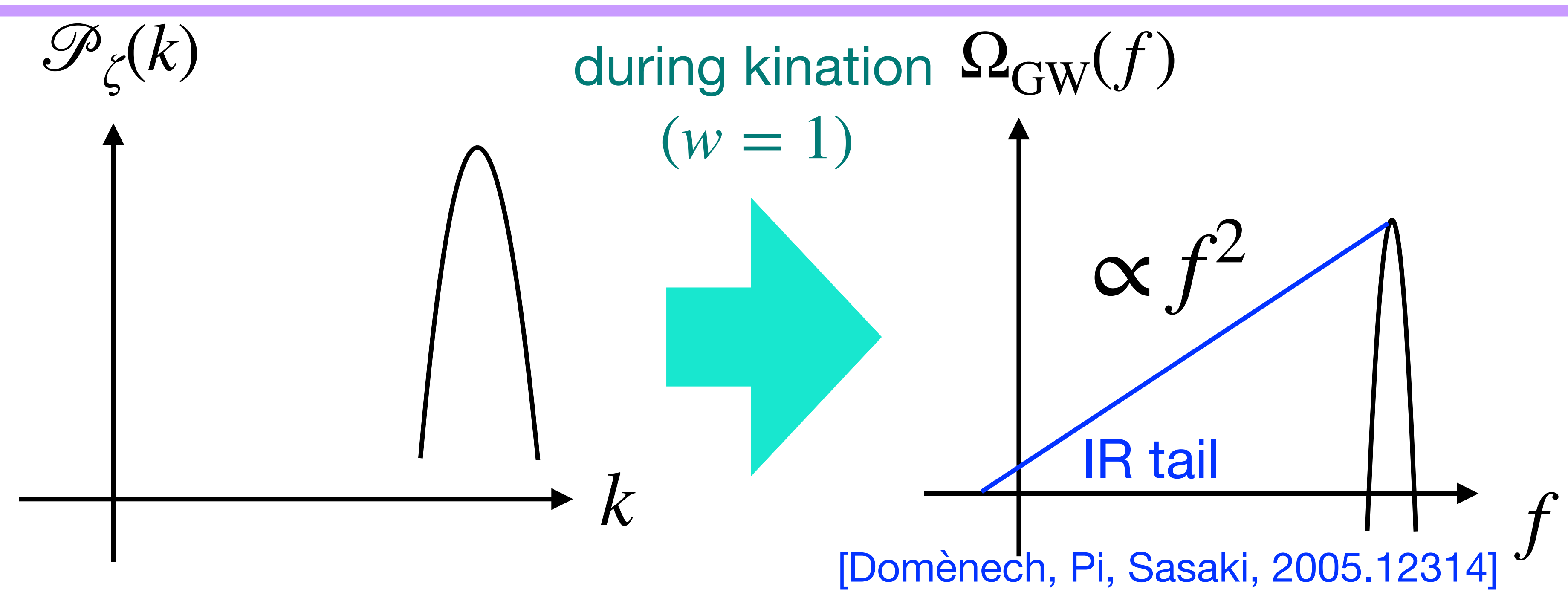
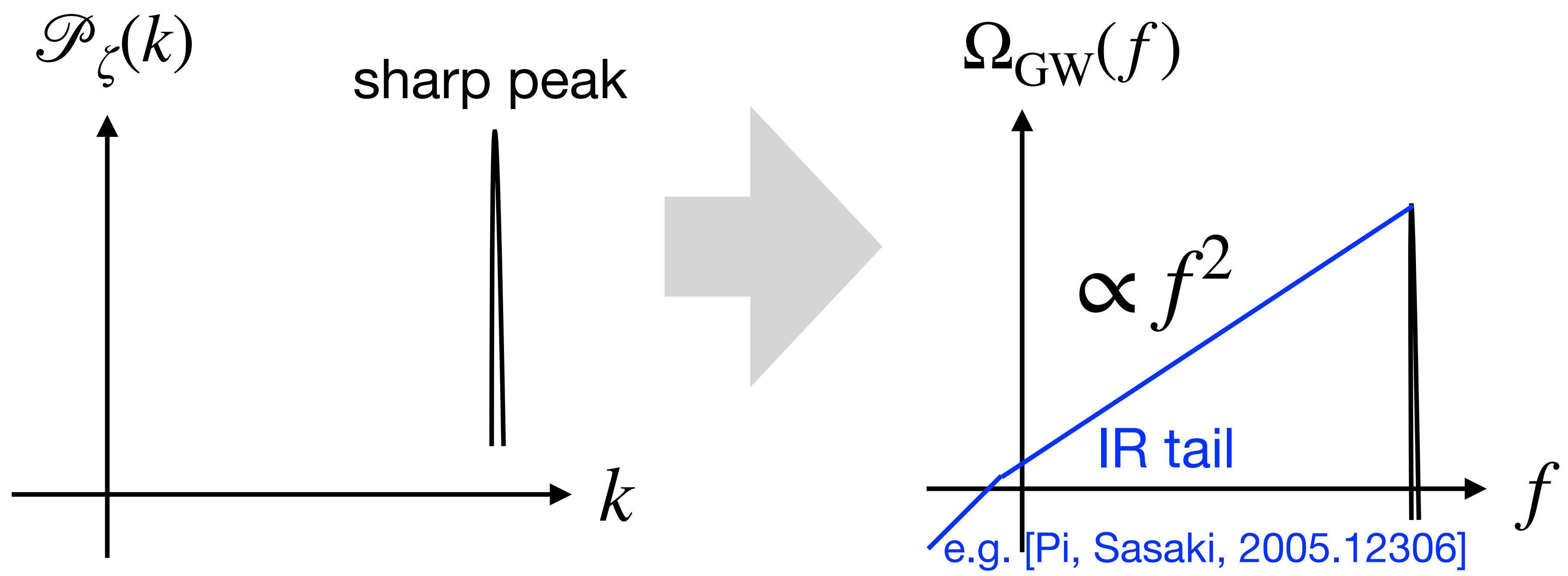


Possibility 2



[Harigaya, Inomata, Terada, 2309.00228]

Possibility 3



[Inomata, Kohri, Terada, 2306.17834]

f^2 Spectrum in the Kination Scenario

$$w := \frac{P}{\rho} = 1 \quad \rho \propto a^{-6}$$

- Growth factor for superhorizon modes from growing subhorizon density perturbations

  The source term decreasing slower than the Hubble scale

  an additional factor of $\left(\frac{a(k)}{a_{\text{fixed}}}\right)^4 \sim f^{-2}$

- Relative redshift factor for subhorizon modes during kination

  an additional factor of $\left(\frac{a_{\text{fixed}}}{a(k)}\right)^2 \sim f$

During an era with $w = 1$,

$$2\pi f = k = \mathcal{H} \propto a^{-2},$$

$$a \propto \eta^{1/2},$$

η: conformal time

$\mathcal{H}$: conformal Hubble parameter

Multiplying the above factors to the standard one (f^3), we obtain $f^3 \cdot f^{-2} \cdot f = f^2$.

More generally, it nontrivially depends on the equation-of-state parameter w:

$$\Omega_{\text{GW}}(f) \sim f^{3-2|(1-3w)/(1+3w)|}$$

for the IR tail part of the spectrum. [Domènech, Pi, Sasaki, 2005.12314]

Induced GW scenario with kination

$$w := \frac{P}{\rho} = 1 \quad \rho \propto a^{-6}$$

The PBH abundance is exponentially suppressed compared to the standard scenario.

$$f_{\rm PBH} \equiv \frac{\rho_{\rm PBH}}{\rho_{\rm DM}} \sim \exp\left(-\frac{\delta_{\rm c}^2}{2\mathscr{P}_\zeta(k(M))}\right)$$

1. Smaller curvature perturbation is required to fit the PTA data.

   This is because the GW fraction is enhanced during kination.

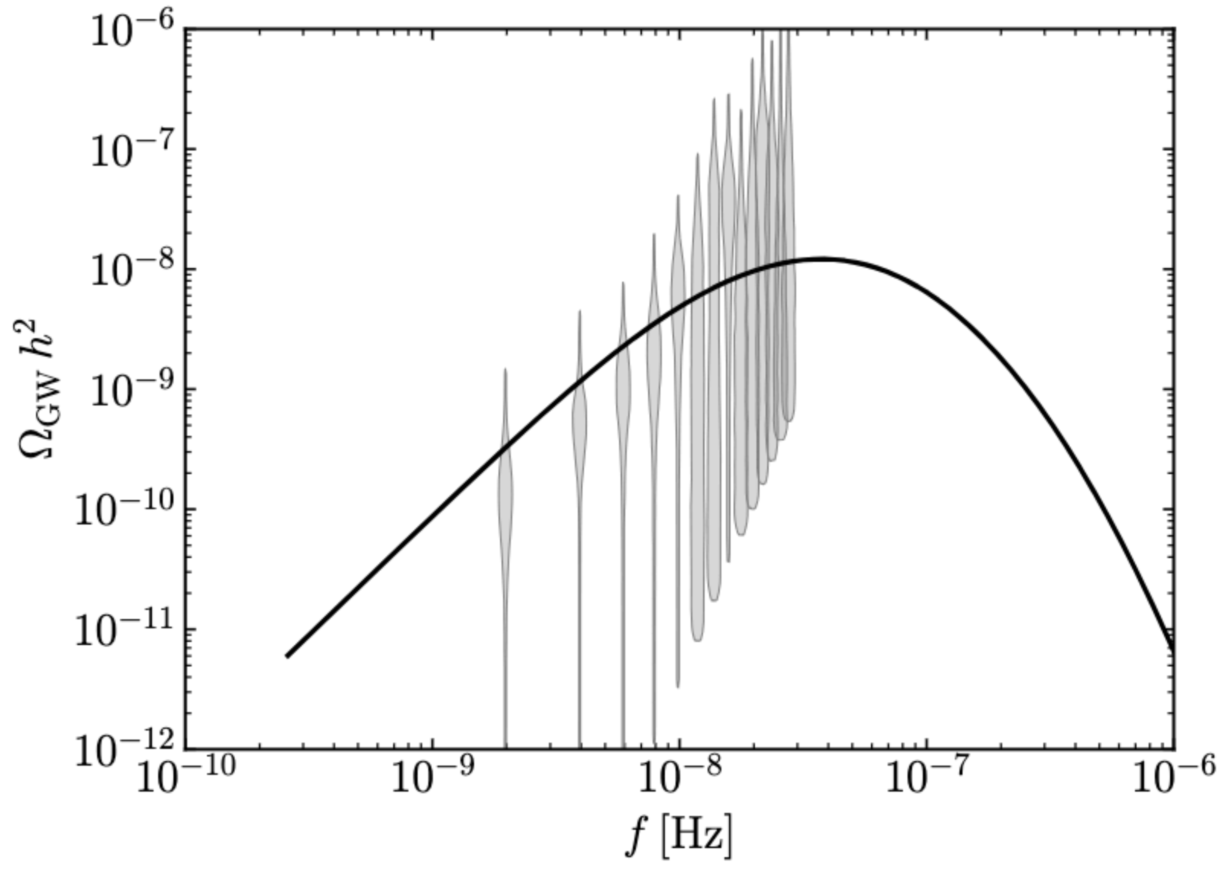
$$\Omega_{\rm GW} \propto a^2$$

2. It will be harder for a PBH to form during kination.

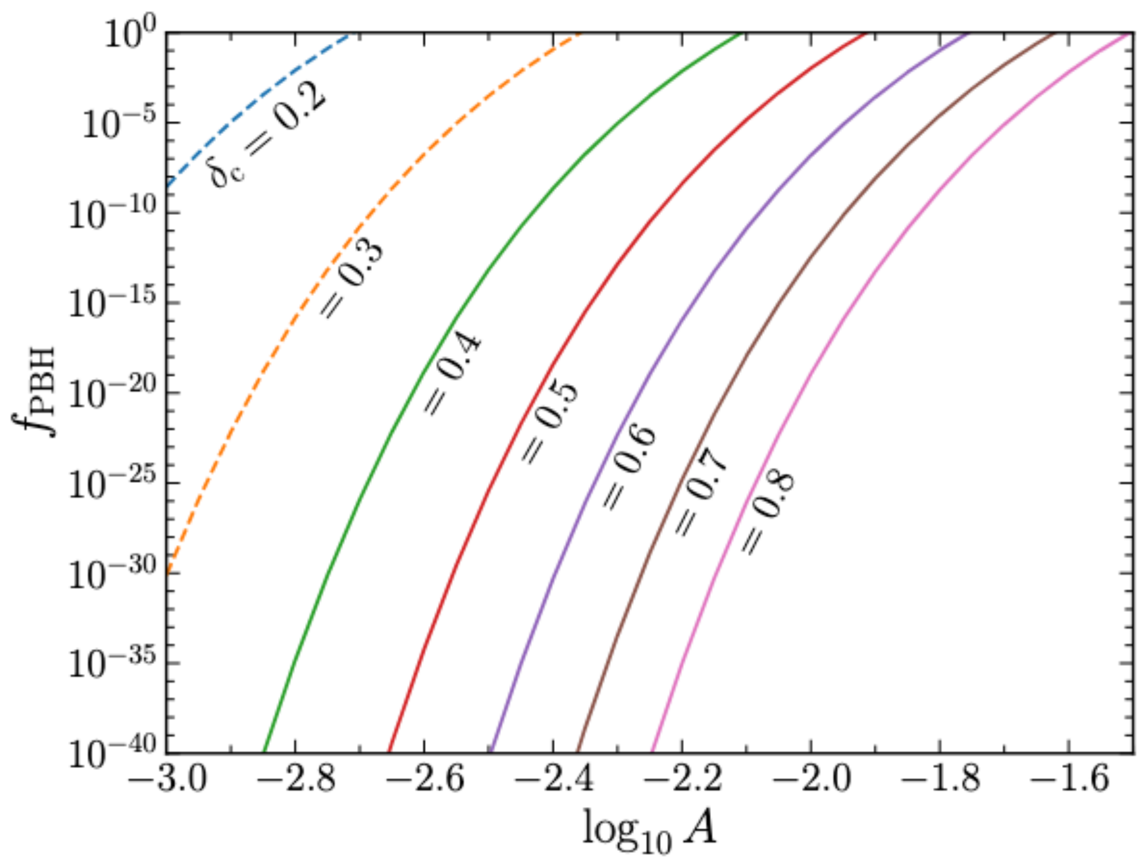
$$\delta_{\rm c} \approx 0.4 - 0.75$$

See, e.g., [Escrivà et al., 2007.05564] and references therein.

Example GW spectrum



PBH abundance



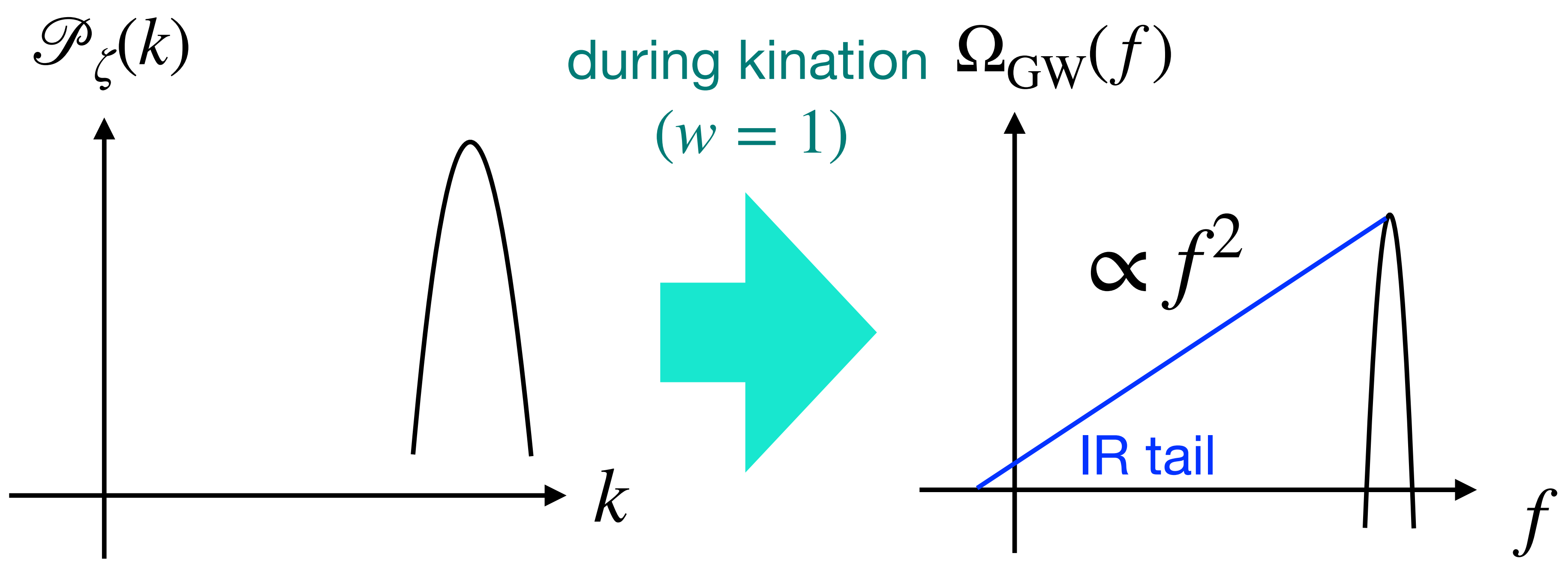
The PTA data can be fit without PBH overproduction.

[Harigaya, Inomata, Terada, 2309.00228]

See also [Balaji et al., 2307.08552] for a similar scenario.

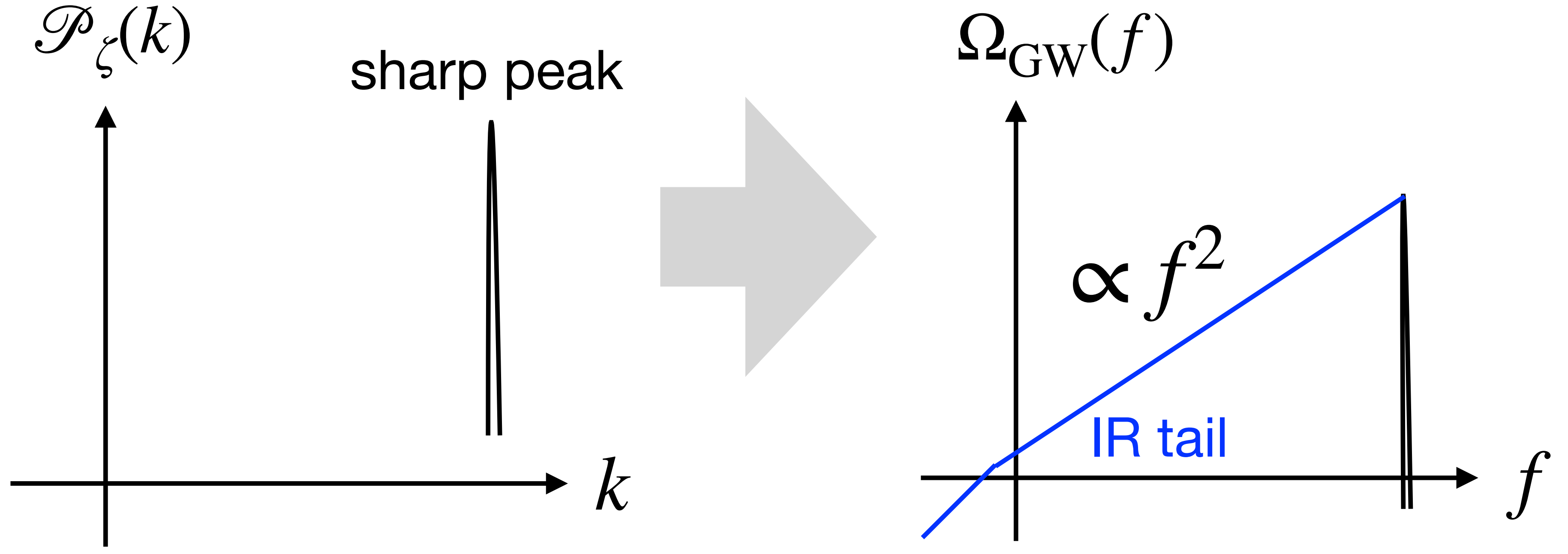
Summary of This Part

The PTA data may be indicating $\Omega_{\rm GW} \propto f^2$ spectrum, which can be interpreted in terms of (the IR tail of) the scalar-induced GWs.



[Harigaya, Inomata, Terada, 2309.00228]

- Fitting the PTA data well.
- No PBH overproduction.



[Inomata, Kohri, Terada, 2306.17834]

- Fitting the PTA data well.
- Associated with $\mathcal{O}(10^{-4})\, M_\odot$ PBHs.
  - Their binary mergers lead to additional GW signals.

Memory Burden Effect

Toy Model of a Black Hole

[Dvali, 1810.02336]
[Dvali, Eisemann, Michel, and Zell, 2006.00011]
see Refs. therein.

Master mode (soft mode)

$$\nabla \sim R^{-1} = (2GM)^{-1}$$

Huge occupation number

Emitted as Hawking Radiation

Interaction

Memory modes (would-be hard modes)

$$\nabla \gg R^{-1}$$

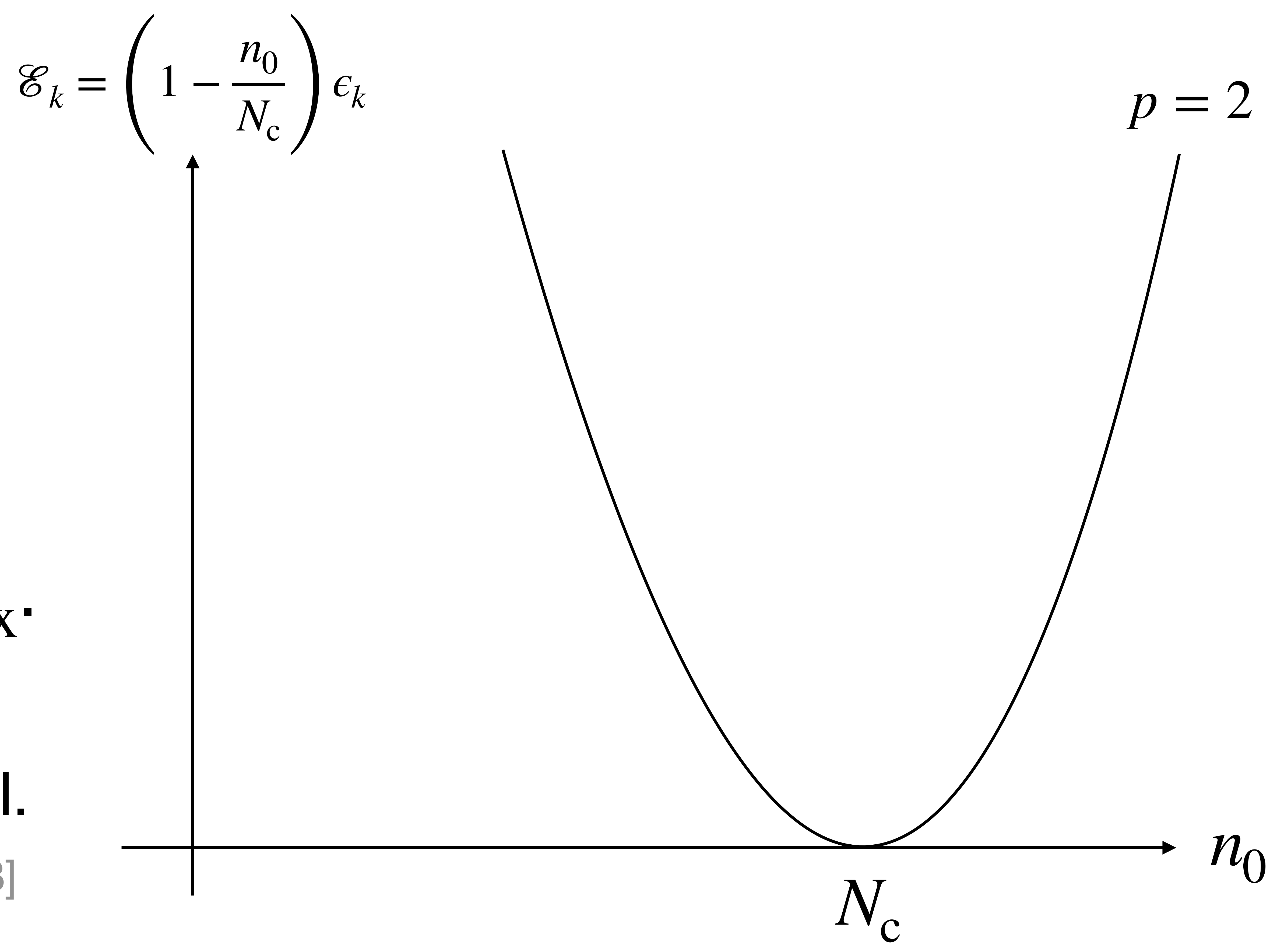
Responsible for the BH entropy

Hardly emitted as Hawking Radiation

Assisted Gaplessness

prototype interaction [Dvali, Eisemann, Michel, and Zell, 2006.00011]

$$\hat{H} = \epsilon_0 \hat{n}_0 + \left(1 - \frac{\hat{n}_0}{N_c}\right)^p \sum_{k=1}^{K} \epsilon_k \hat{n}_k$$



entry

$$|n_0(=N_c), n_1, \ldots, n_K\rangle$$

$$S = K \ln(n_{\max} + 1) \quad \text{if} \quad n_i = 0, \ldots, n_{\max}.$$

It follows an area law in a more specific model.

$K \propto R^{d-1}$ [Dvali, 1712.02233]

Memory Burden

[Dvali, 1810.02336]
[Dvali, Eisemann, Michel, and Zell, 2006.00011]

Extend the system a bit.

$$\hat{H} = \epsilon_0 \hat{n}_0 + \epsilon_0 \hat{m}_0 + \left(1 - \frac{\hat{n}_0}{N_c}\right)^p \sum_{k=1}^{K} \epsilon_k \hat{n}_k + C_0 \left(\hat{a}_0^\dagger \hat{b}_0 + \hat{b}_0^\dagger \hat{a}_0\right)$$

Memory Burden

[Dvali, 1810.02336]
[Dvali, Eisemann, Michel, and Zell, 2006.00011]

Extend the system a bit.

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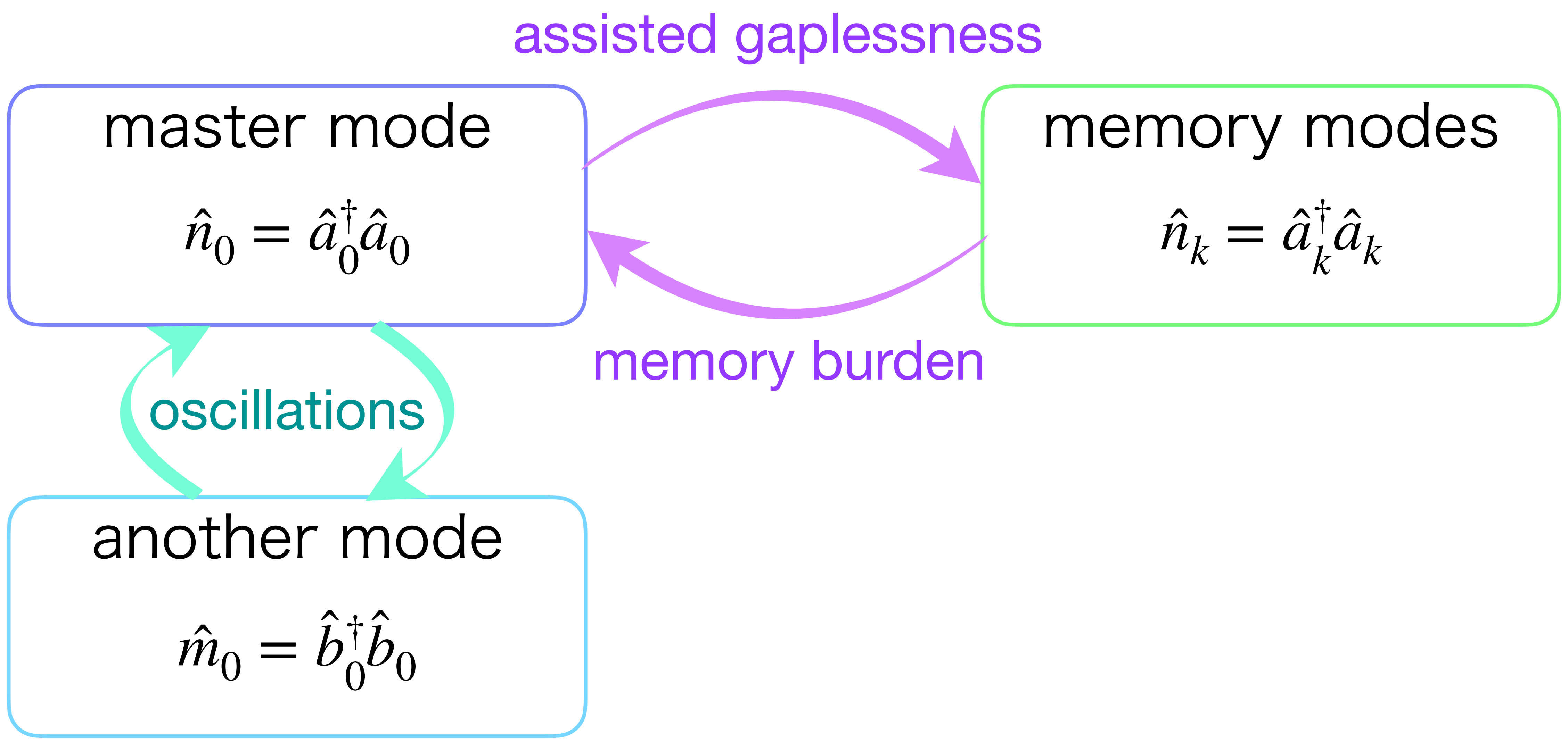

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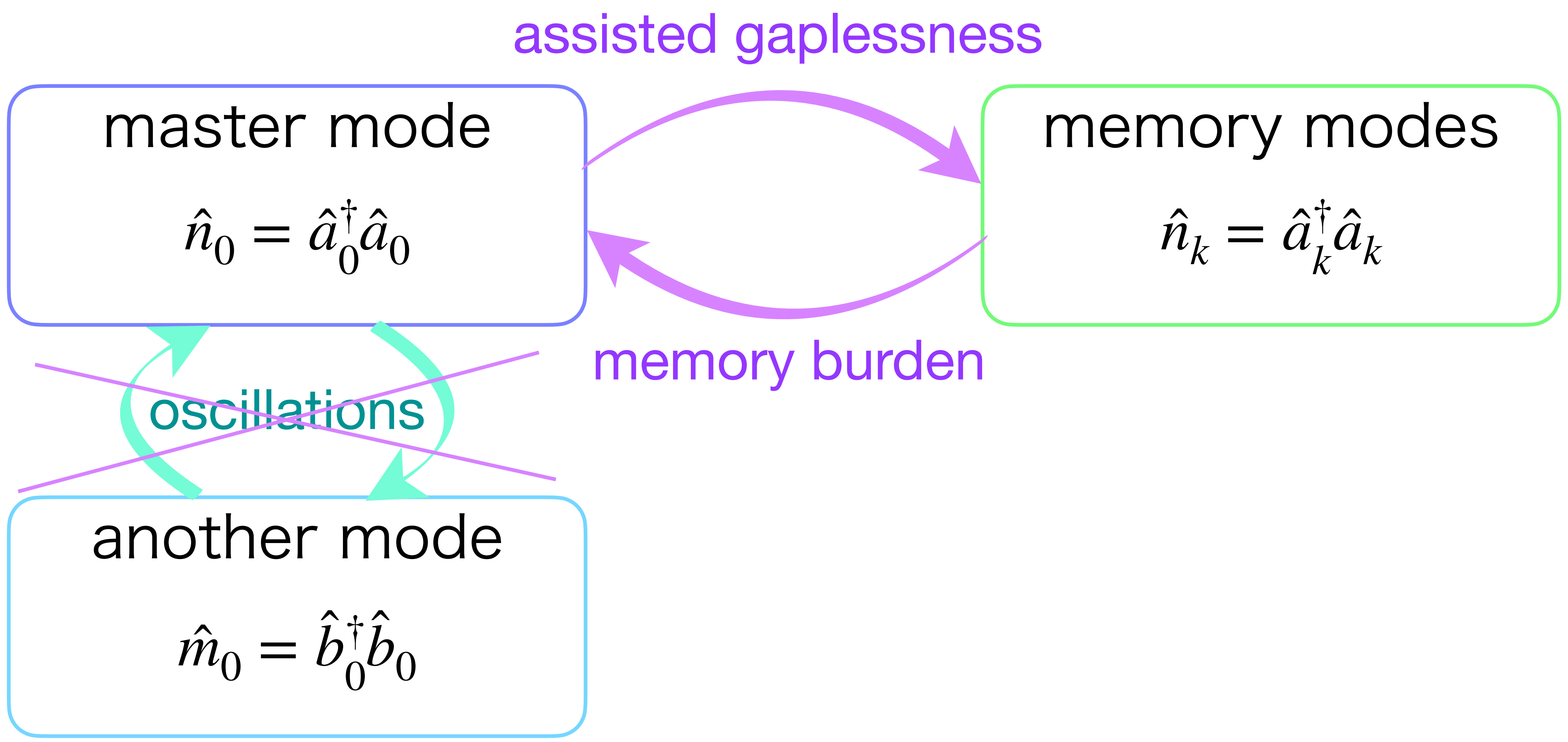
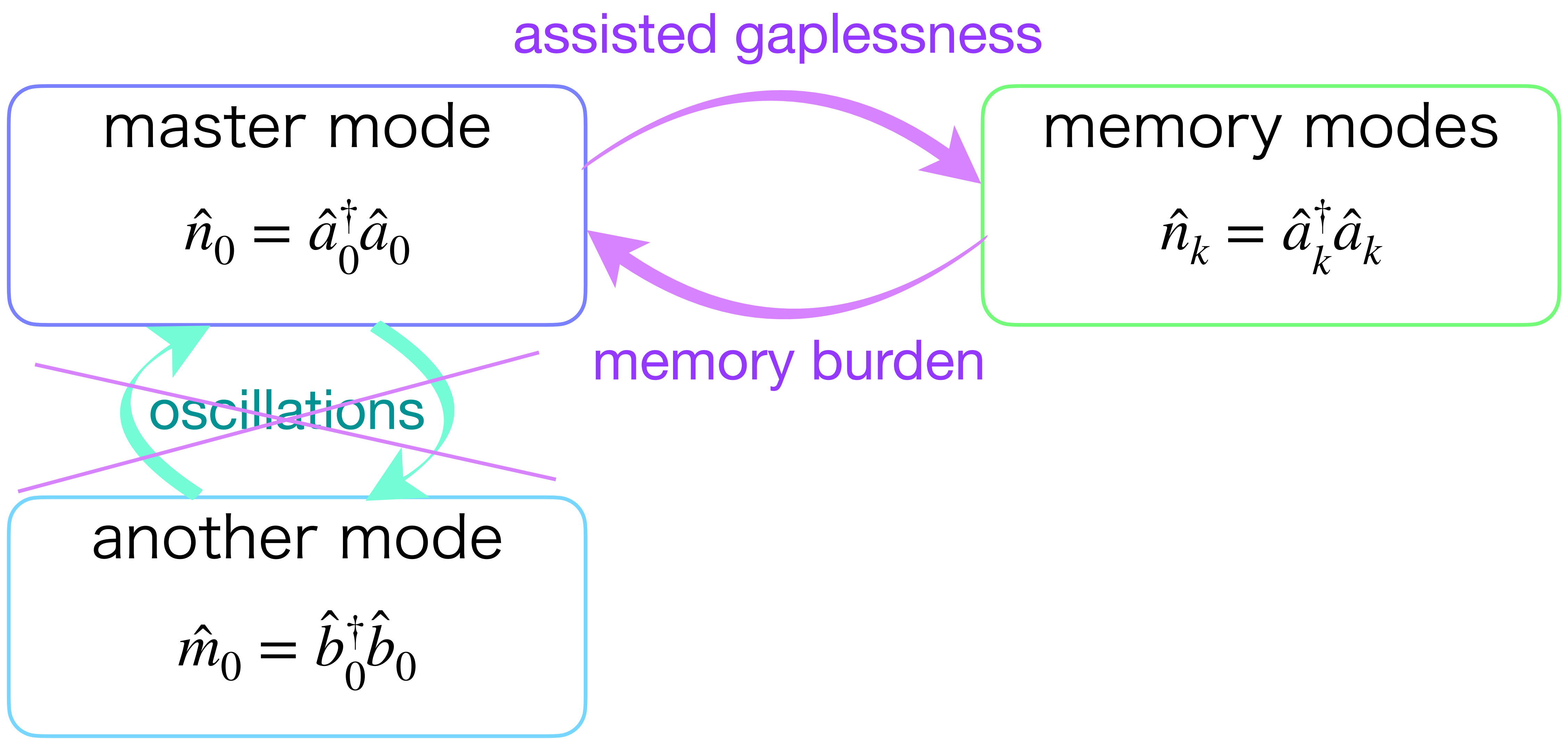


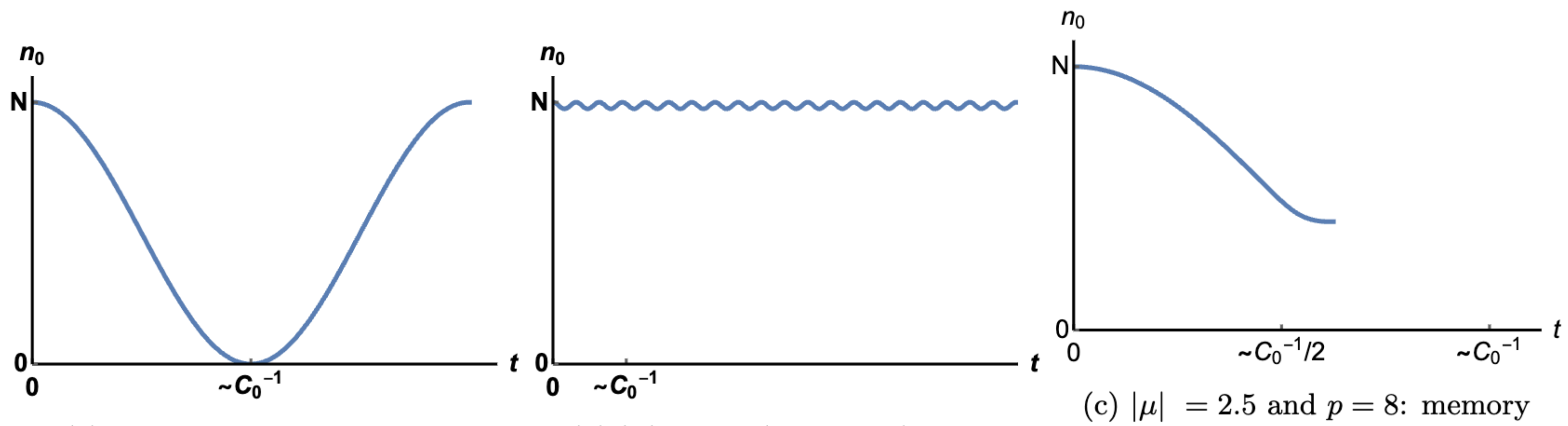
Memory Burden

[Dvali, 1810.02336]
[Dvali, Eisemann, Michel, and Zell, 2006.00011]

Extend the system a bit.

$$\hat{H} = \epsilon_0 \hat{n}_0 \boxed{+ \epsilon_0 \hat{m}_0} + \left(1 - \frac{\hat{n}_0}{N_c}\right)^p \sum_{k=1}^{K} \epsilon_k \hat{n}_k \boxed{+ C_0 \left(\hat{a}_0^\dagger \hat{b}_0 + \hat{b}_0^\dagger \hat{a}_0\right)}$$





(a) $\mu = 0$: absence of memory burden. The expectation value n_0 oscillates freely with full amplitude.

(b) $|\mu| = 2.5$ (and $p = 1$): early backreaction due to memory burden ties n_0 to its initial value.

(c) $|\mu| = 2.5$ and $p = 8$: memory burden can be delayed up to a timescale on the order of half decay. Backreaction sets in at the latest at that point and stabilizes the system.

Figure from [Dvali, Eisemann, Michel, and Zell, 2006.00011]

Memory Burden

[Dvali, 1810.02336]
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Extend the system a bit.

$$\hat{H} = \epsilon_0 \hat{n}_0 + \epsilon_0 \hat{m}_0 + \left(1 - \frac{\hat{n}_0}{N_c}\right)^p \sum_{k=1}^{K} \epsilon_k \hat{n}_k + C_0\left(\hat{a}_0^\dagger \hat{b}_0 + \hat{b}_0^\dagger \hat{a}_0\right)$$

- **Become extremely long lived?**
- A new classical instability?

assisted gaplessness

master mode

$\hat{n}_0 = \hat{a}_0^\dagger \hat{a}_0$

memory modes

$\hat{n}_k = \hat{a}_k^\dagger \hat{a}_k$

memory burden

~~oscillations~~

another mode

$\hat{m}_0 = \hat{b}_0^\dagger \hat{b}_0$

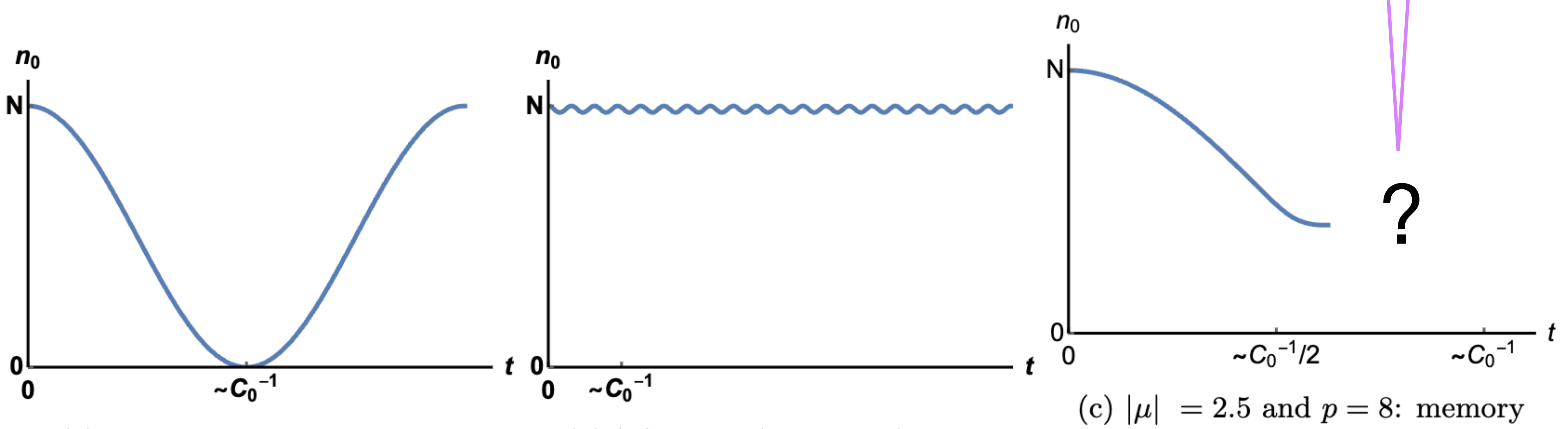

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Figure from [Dvali, Eisemann, Michel, and Zell, 2006.00011]

BH Evaporation Slowed Down by Memory Burden

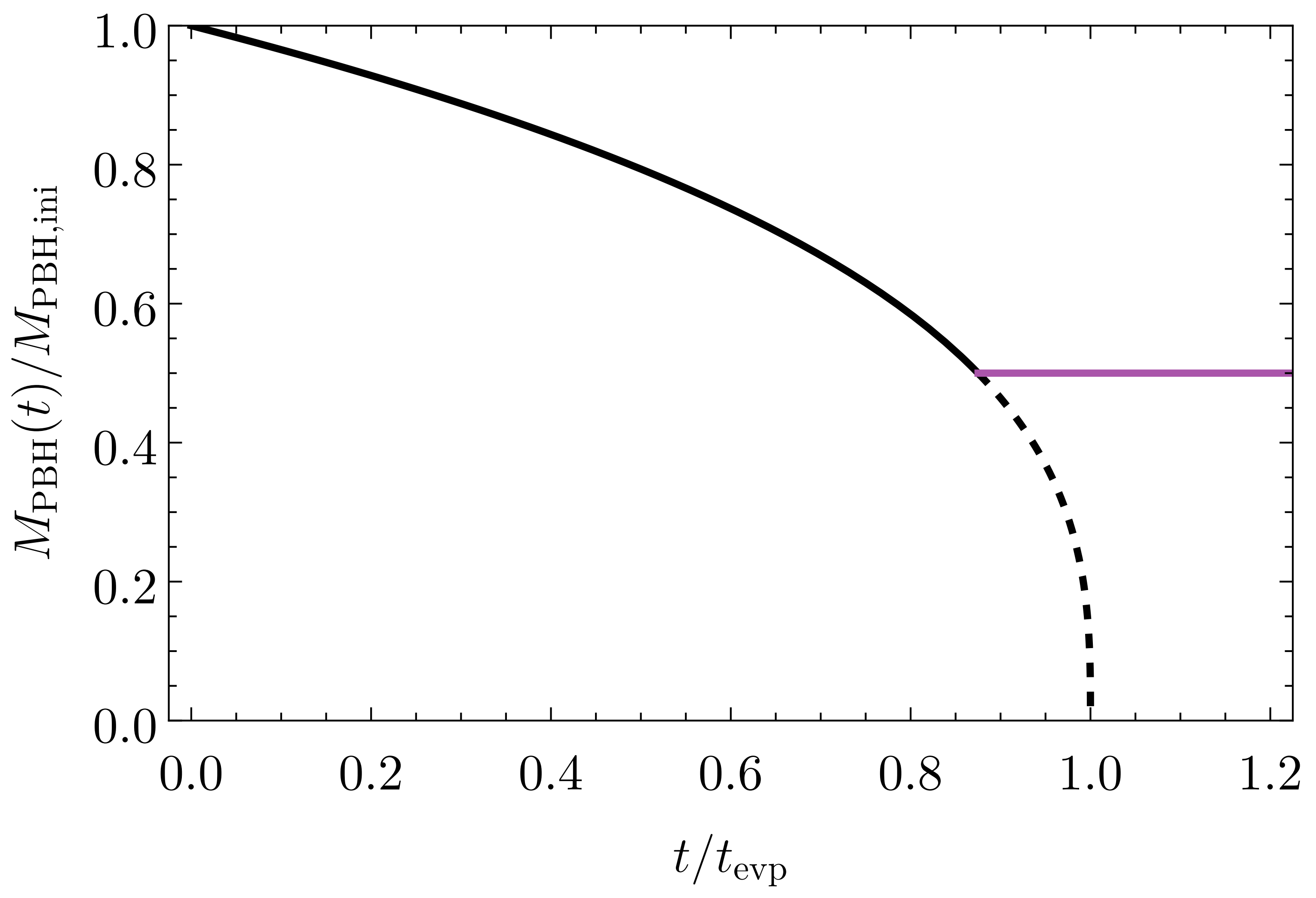
[Hooper, Krnjaic, and McDermott, 1905.01303]

[Alexandre, Dvali, and Koutsangelas, 2402.14069]
[Thoss, Burkert, and Kohri, 2402.17823]
[Balaji, Domènech, Franciolini, Ganz, and Tränkle, 2403.14309]

$$\dot{M}_{\rm PBH} = -\frac{\pi \mathscr{G} g_{*{\rm H}}}{480 M_{\rm PBH}^2} \times \frac{1}{S_{\rm BH}^{n_{\rm MB}}}$$

after the half evaporation

$M_{\rm PBH} \lesssim M_{\rm PBH,ini}/2.$



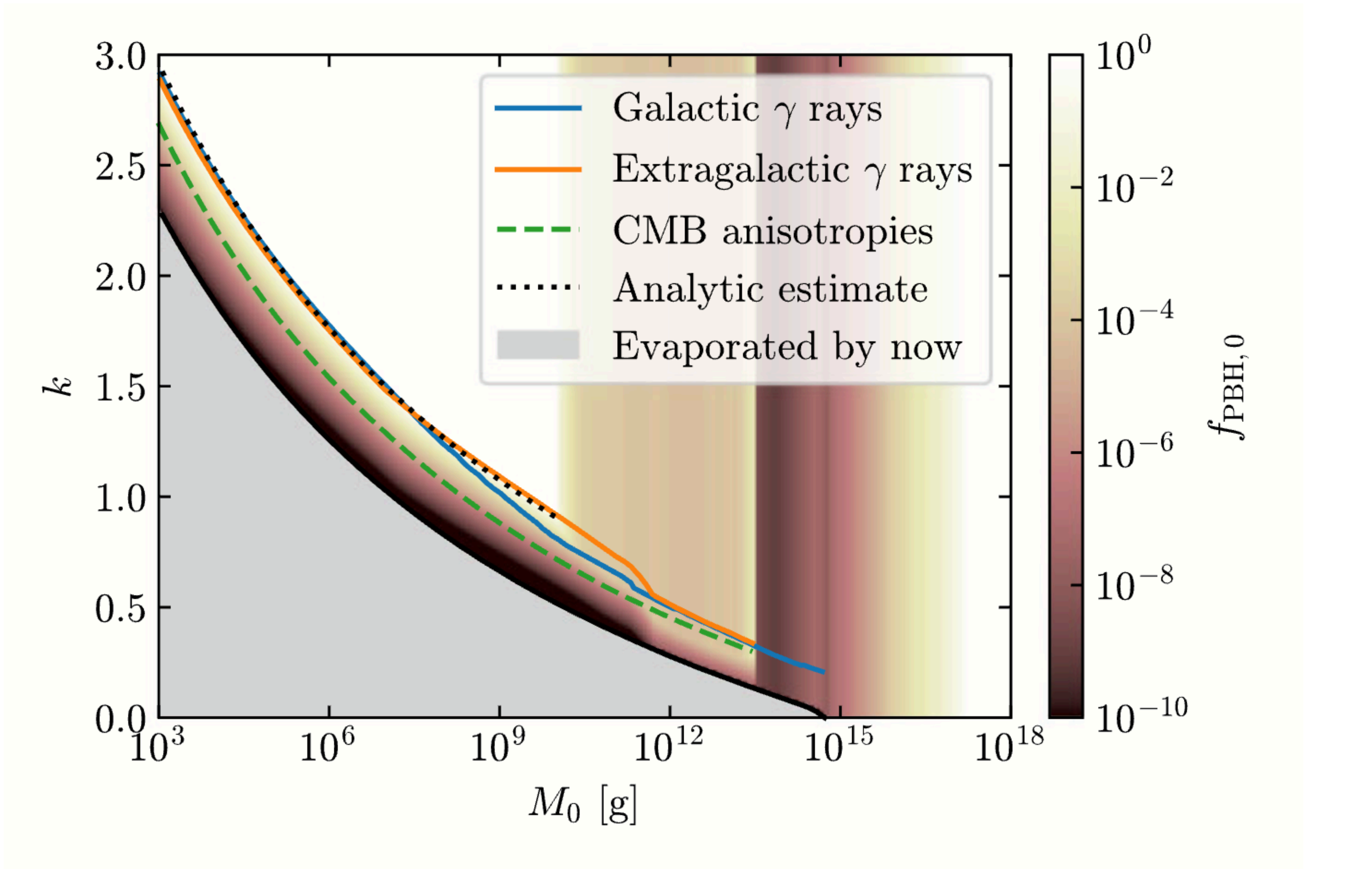
where $\mathscr{G}$: the gray-body factor, $g_{*{\rm H}}$: the effective d.o.f.

$S_{\rm BH} = M_{\rm PBH}^2/2$: the BH entropy

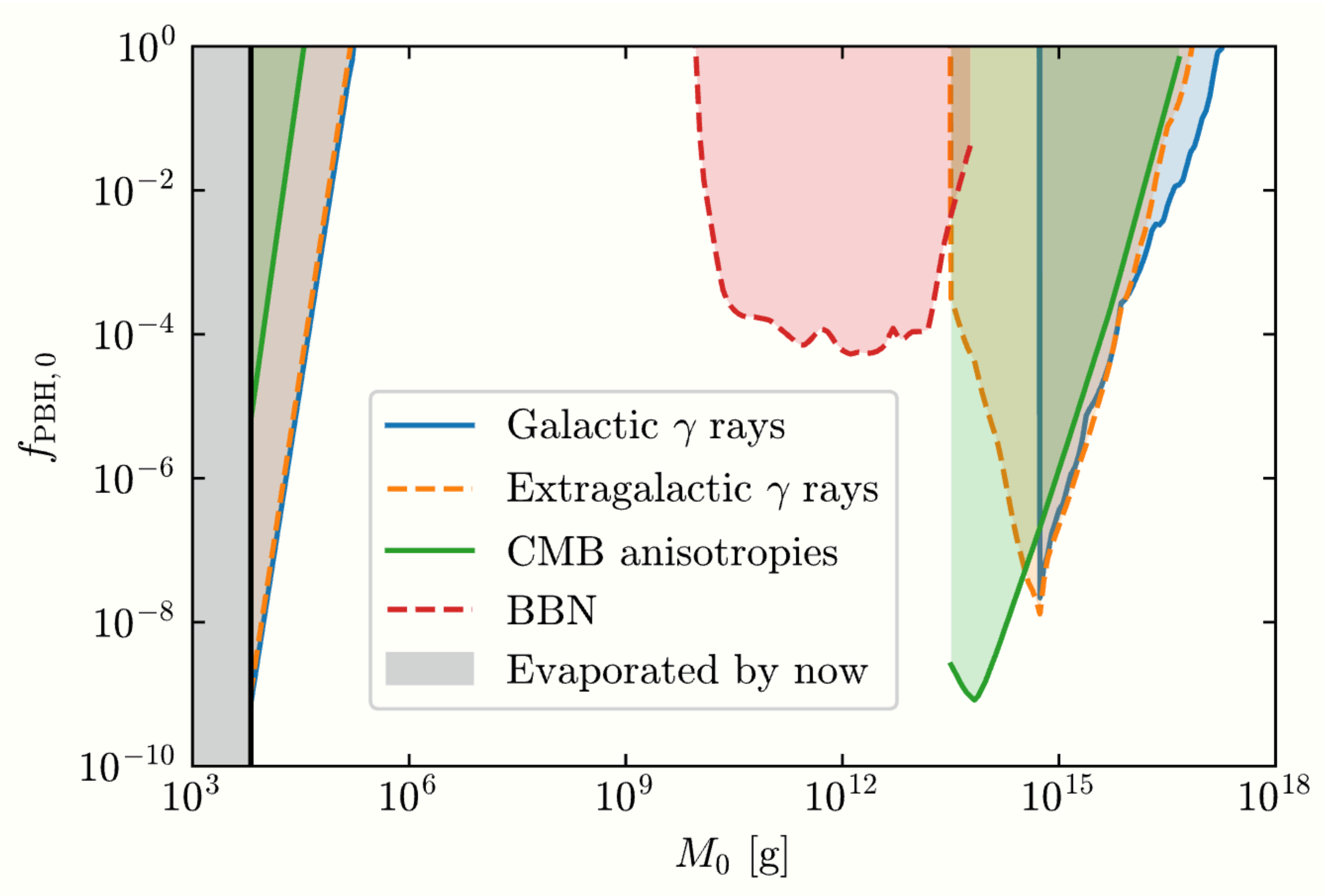
$n_{\rm MB}$: a(n integer) parameter for the memory burden effect.

Light PBH Dark Matter Window Open

[Alexandre, Dvali, and Koutsangelas, 2402.14069], [Thoss, Burkert, and Kohri, 2402.17823]. The figures are from the latter.



$k \equiv n_{\mathrm{MB}},\ M_0 \equiv M_{\mathrm{PBH,ini}},\ f_{\mathrm{PBH},0} \equiv f_{\mathrm{PBH}} \times (M_{\mathrm{PBH,ini}}/M_{\mathrm{PBH}}(t_0))$



Right panel: Case of $n_{\mathrm{MB}} = 2$.

We can consider $0.5\,\mathrm{g} \lesssim M_{\mathrm{PBH,ini}} \lesssim 10^{10}\,\mathrm{g}$ and focus on $10^5\,\mathrm{g} \lesssim M_{\mathrm{PBH,ini}} \lesssim 10^{10}\,\mathrm{g}$.

Big Picture

PBH-GW connection: [Saito and Yokoyama, 0812.4339; 0912.5317]

$$\beta(M_{\mathrm{PBH,ini}})$$

$$f_{\mathrm{PBH}} = 1$$

PBH production

PBH dark matter

Memory Burden

exponential sensitivity

Curvature Perturbations

$$\mathscr{P}_{\zeta}(k)$$

logarithmic sensitivity

quadratic sensitivity

Induced GWs

$$\Omega_{\mathrm{GW}}(f)$$

Big Picture

PBH-GW connection: [Saito and Yokoyama, 0812.4339; 0912.5317]

$$\beta(M_{\rm PBH,ini})$$

PBH production → PBH dark matter

$$f_{\rm PBH} = 1$$

exponential sensitivity

Memory Burden

Curvature Perturbations

$$\mathcal{P}_\zeta(k)$$

logarithmic sensitivity

Exclusion because $f_{\rm PBH} > 1$

quadratic sensitivity

Induced GWs

$$\Omega_{\rm GW}(f)$$

If too strong

Big Picture

PBH-GW connection: [Saito and Yokoyama, 0812.4339; 0912.5317]

$$\beta(M_{\mathrm{PBH,ini}})$$

PBH production → PBH dark matter → GWs from PBH binary merger

$$f_{\mathrm{PBH}} = 1$$

$$\Omega_{\mathrm{GW}}^{\mathrm{(merger)}}(f)$$

Memory Burden

exponential sensitivity

Curvature Perturbations

$$\mathscr{P}_\zeta(k)$$

logarithmic sensitivity

quadratic sensitivity

Induced GWs

$$\Omega_{\mathrm{GW}}(f)$$

PBH-induced Fast Radio Bursts

Exclusion because $f_{\mathrm{PBH}} > 1$

If too strong

Our Prescription for PBH Abundance Calculation

Initial PBH mass

$$M_{\rm PBH,ini} = \frac{4\pi\gamma}{3} H^{-3} \qquad \gamma = \left(1/\sqrt{3}\right)^3$$

PBH formation probability

$$\beta(M) = \frac{1}{2}\mathrm{Erfc}\left(\frac{\delta_{\rm c}}{\sqrt{2\sigma^2(k)}}\right) \qquad \delta_{\rm c} = 0.42$$

Coarse-grained overdensity

$$\sigma^2(k) = \frac{16}{81}\int_{-\infty}^{\infty} \mathrm{d}\ln q \left(\frac{q}{k}\right)^4 W\left(\frac{q}{k}\right)^2 T\left(\frac{q}{k}\right)^2 \mathcal{P}_\zeta(q)$$

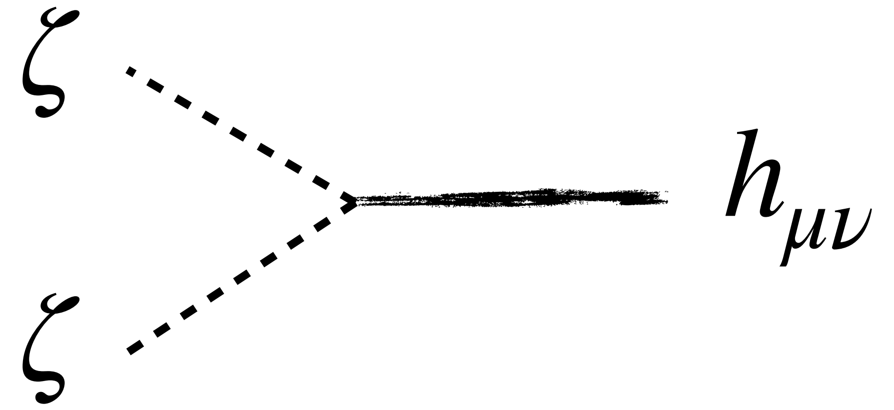
$$W(z) = \exp(-z^2/2)$$

PBH abundance

$$f_{\rm PBH} = \boxed{\frac{1}{2}}\int_{-\infty}^{\infty} \mathrm{d}\ln M \frac{g_*(T(M))}{g_{*,0}} \frac{g_{*s,0}}{g_{*s}(T(M))} \frac{T(M)}{T_{\rm eq}} \gamma\beta(M) \frac{\Omega_{\rm m}}{\Omega_{\rm CDM}}$$

Gravitational Waves Induced by Curvature Perturbations

[Ananda, Clarkson, and Wands, qr-qc/0612013], [Baumann, Steinhardt, Takahashi, and Ichiki, hep-th/0703290]



GWs are induced from ζ beyond the linearized level.

We consider the standard RD era (without PBH domination).

For memory burden in the PBH domination (non-dark matter) scenario, see [Balaji et al., 2403.14309], [Barman et al, 2405.15858], [Bhaumik et al., 2409.04436], [Barman et al., 2409.05953].

Present intensity of the induced GWs

$$\Omega_{\mathrm{GW}}(\eta_0, f)h^2 = D\Omega_{\mathrm{GW}}(\eta_c, f)h^2$$

Dilution factor

$$D = \frac{g_*(T(f))}{g_{*,0}} \left(\frac{g_{*s,0}}{g_{*s}(T(f))} \right)^{4/3} \Omega_r$$

GW Intensity during the RD era

$$\Omega_{\mathrm{GW}}(\eta_c, f) = \frac{1}{6} \int_0^\infty \mathrm{d}t \int_0^1 \mathrm{d}s \left(\frac{t(2+t)(s^2-1)}{(1-s+t)(1+s+t)} \right)^2 \lim_{\eta \to \infty} \overline{(k\eta)^2 I(t, s, k\eta)^2} \mathcal{P}_\zeta(uk) \mathcal{P}_\zeta(vk)$$

The analytic expression of the integration kernel is available.

[Espinosa, Racco, and Riotto, 1804.07732], [Kohri and Terada, 1804.08577]

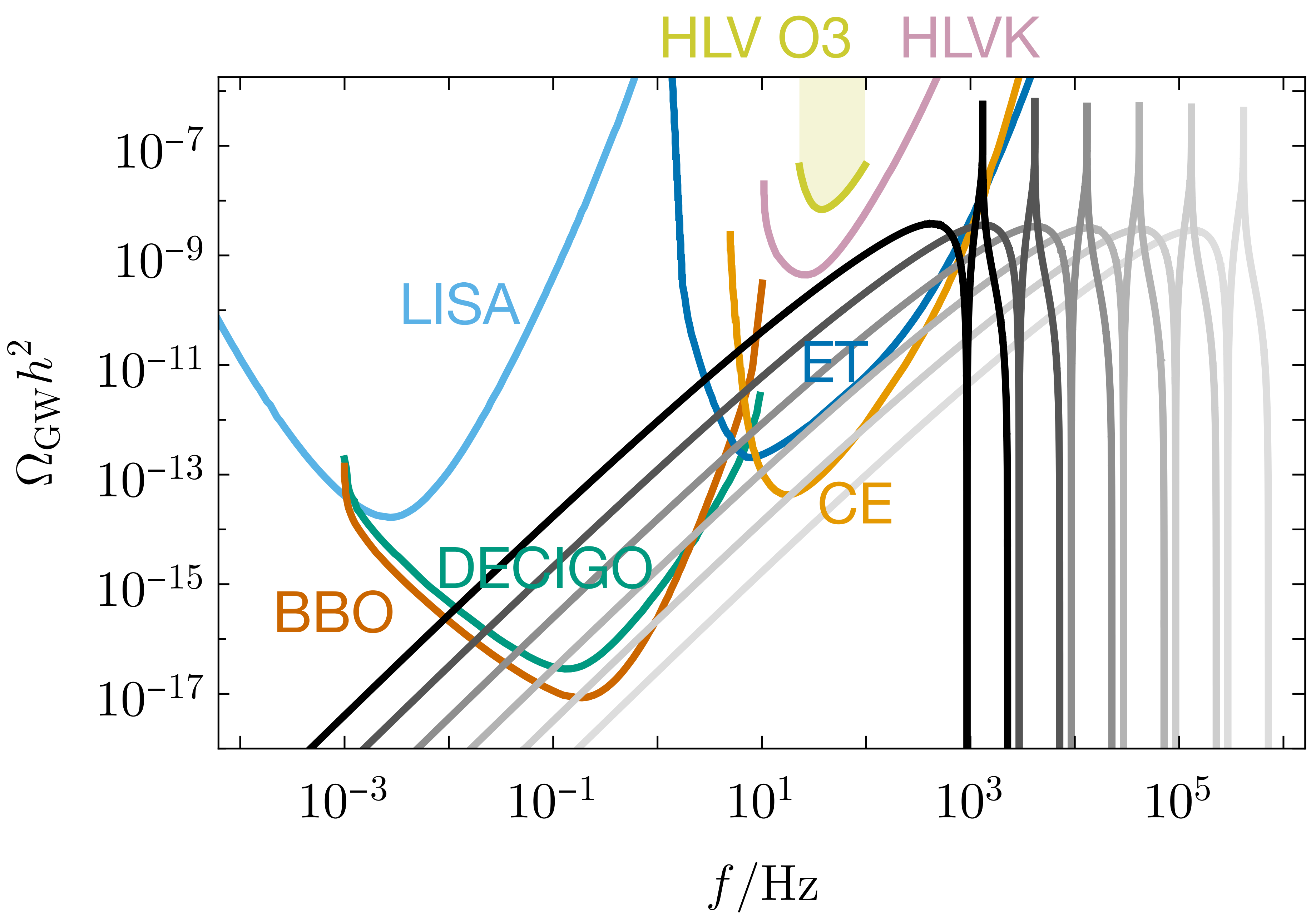
We consider

the lognormal power spectrum.

$$\mathcal{P}_\zeta(k) = \frac{A_\zeta}{\sqrt{2\pi\Delta^2}} \exp\left(-\frac{\left(\ln \frac{k}{k_0} \right)^2}{2\Delta^2} \right)$$

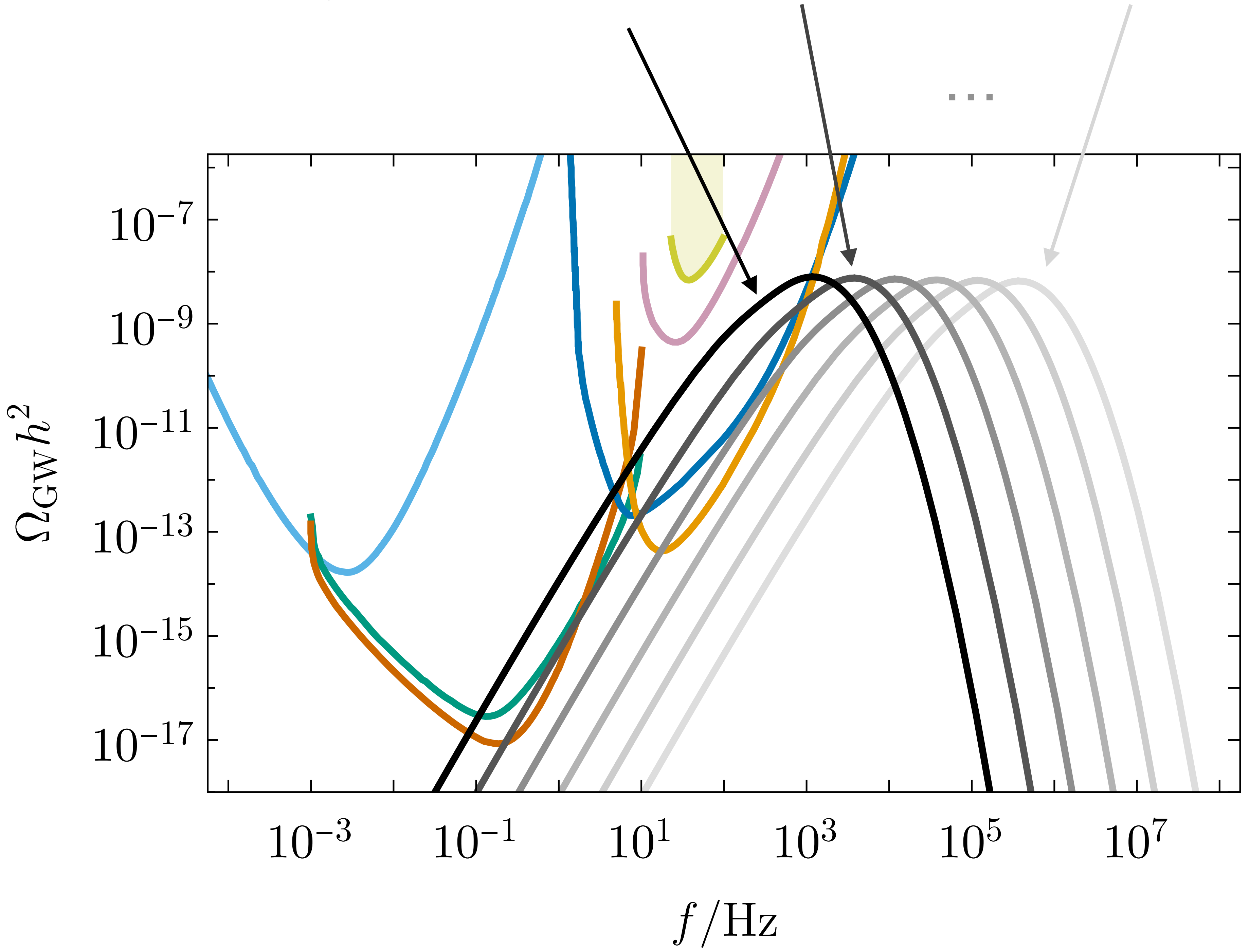
Induced GWs associated with MB PBH DM

$$\mathscr{P}_\zeta(k) = \frac{A_\zeta}{\sqrt{2\pi\Delta^2}} \exp\left(-\frac{\left(\ln \frac{k}{k_0}\right)^2}{2\Delta^2}\right)$$



$\Delta \to 0$

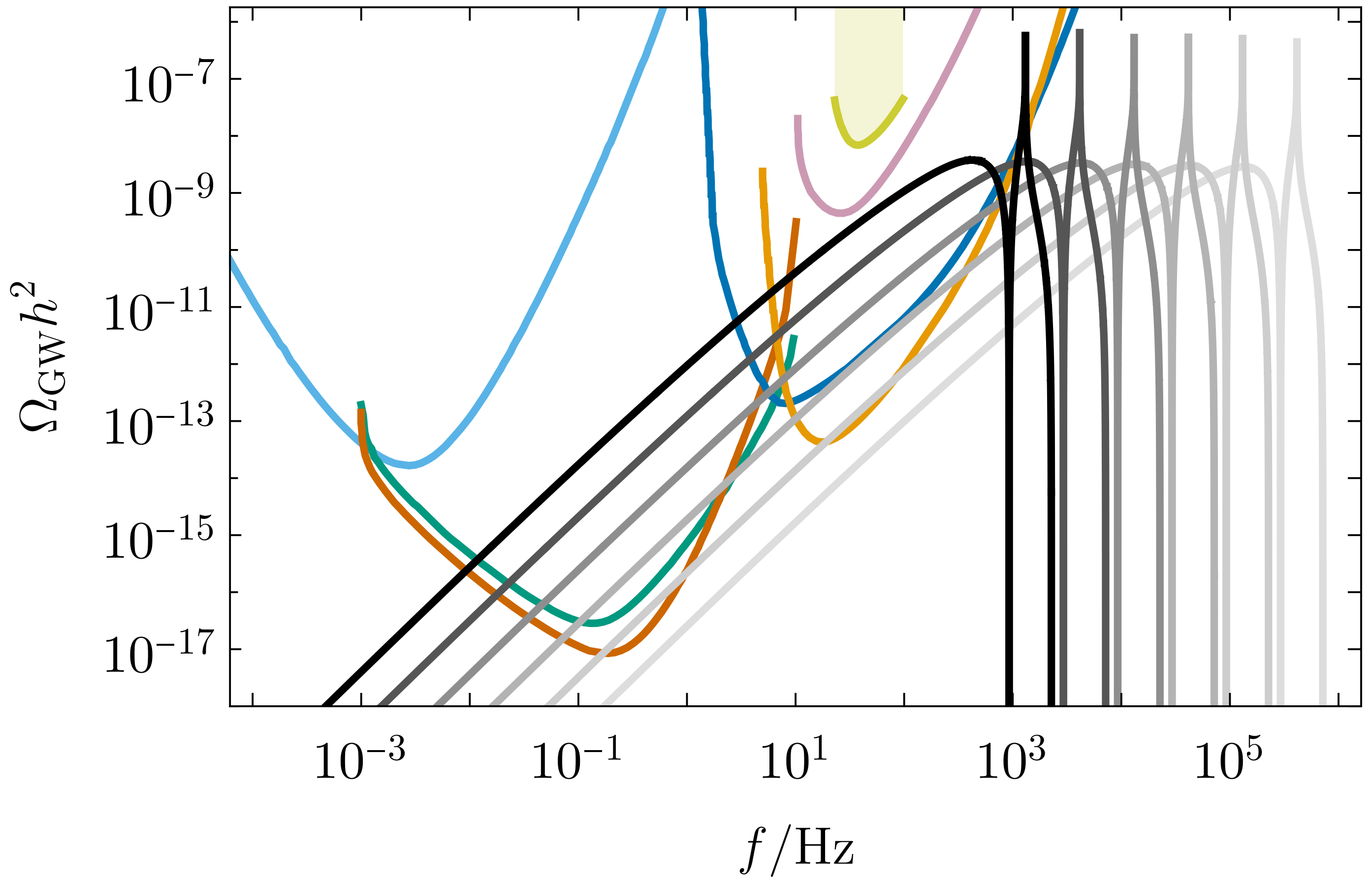
$M_{\rm PBH,ini} = 10^{10}\,{\rm g}$ $\quad 10^9\,{\rm g}$ $\quad \cdots \quad 10^5\,{\rm g}$



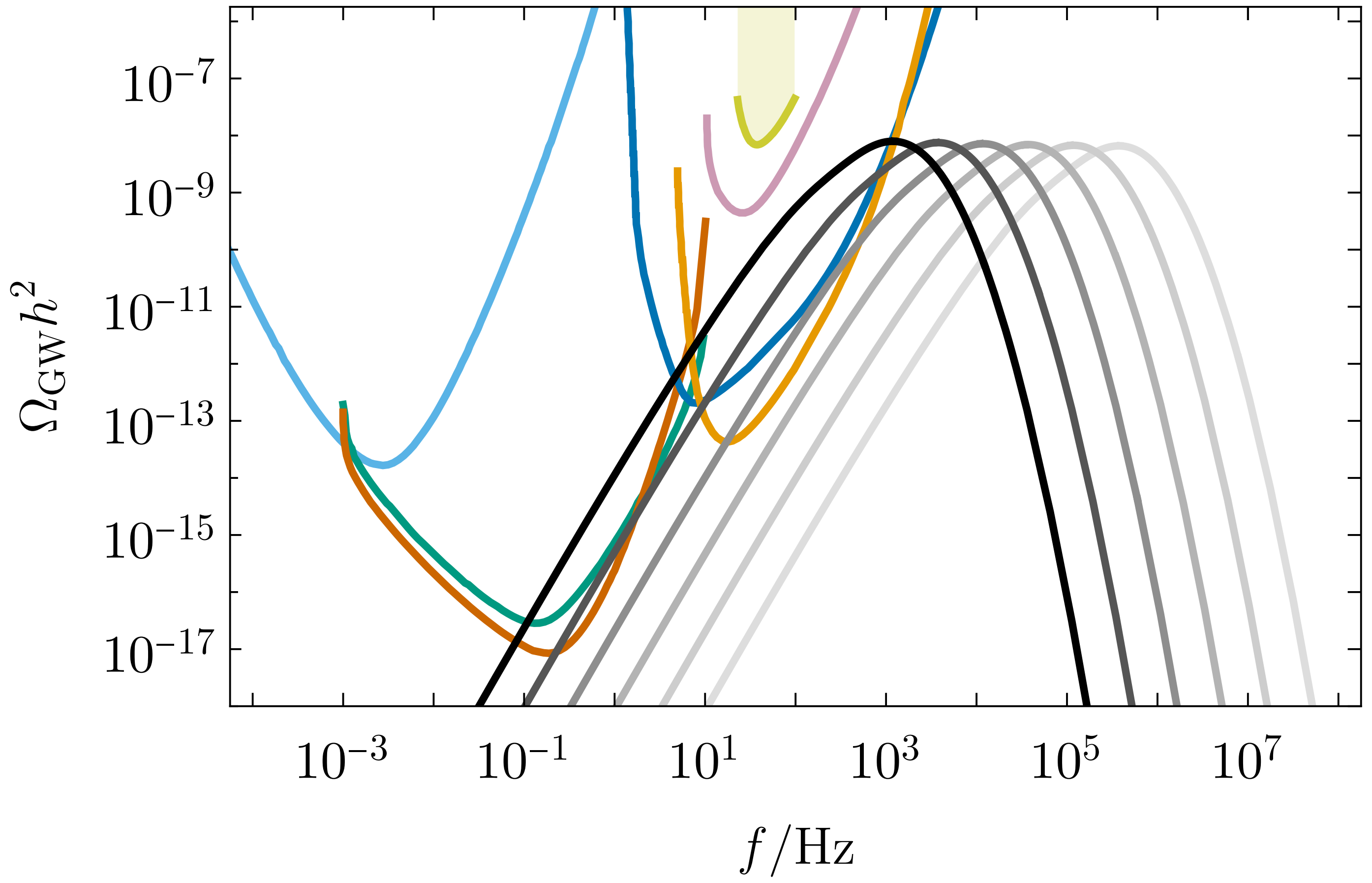
$\Delta = 1$

Induced GWs associated with MB PBH DM

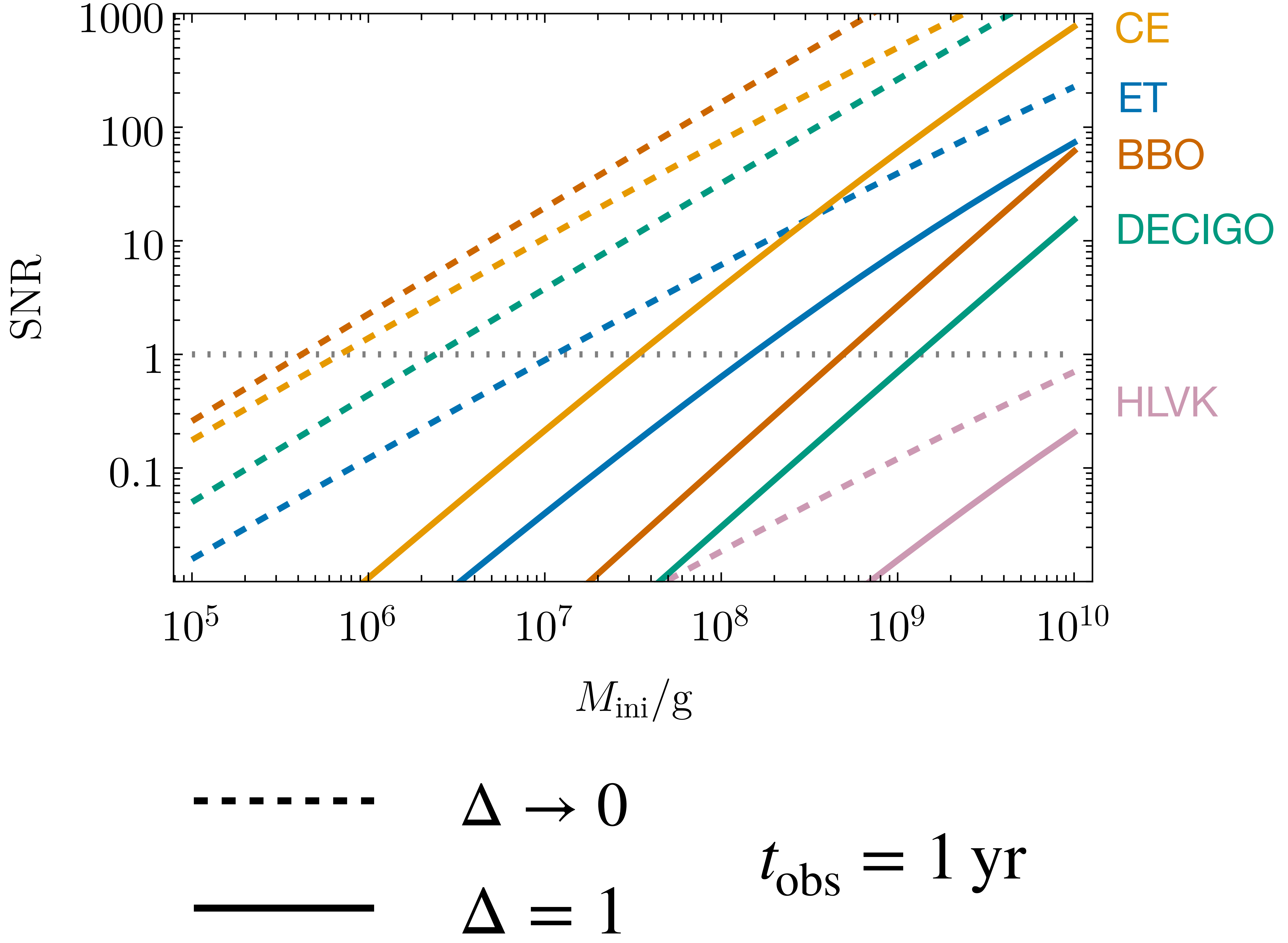
$\Delta \to 0$



$\Delta = 1$



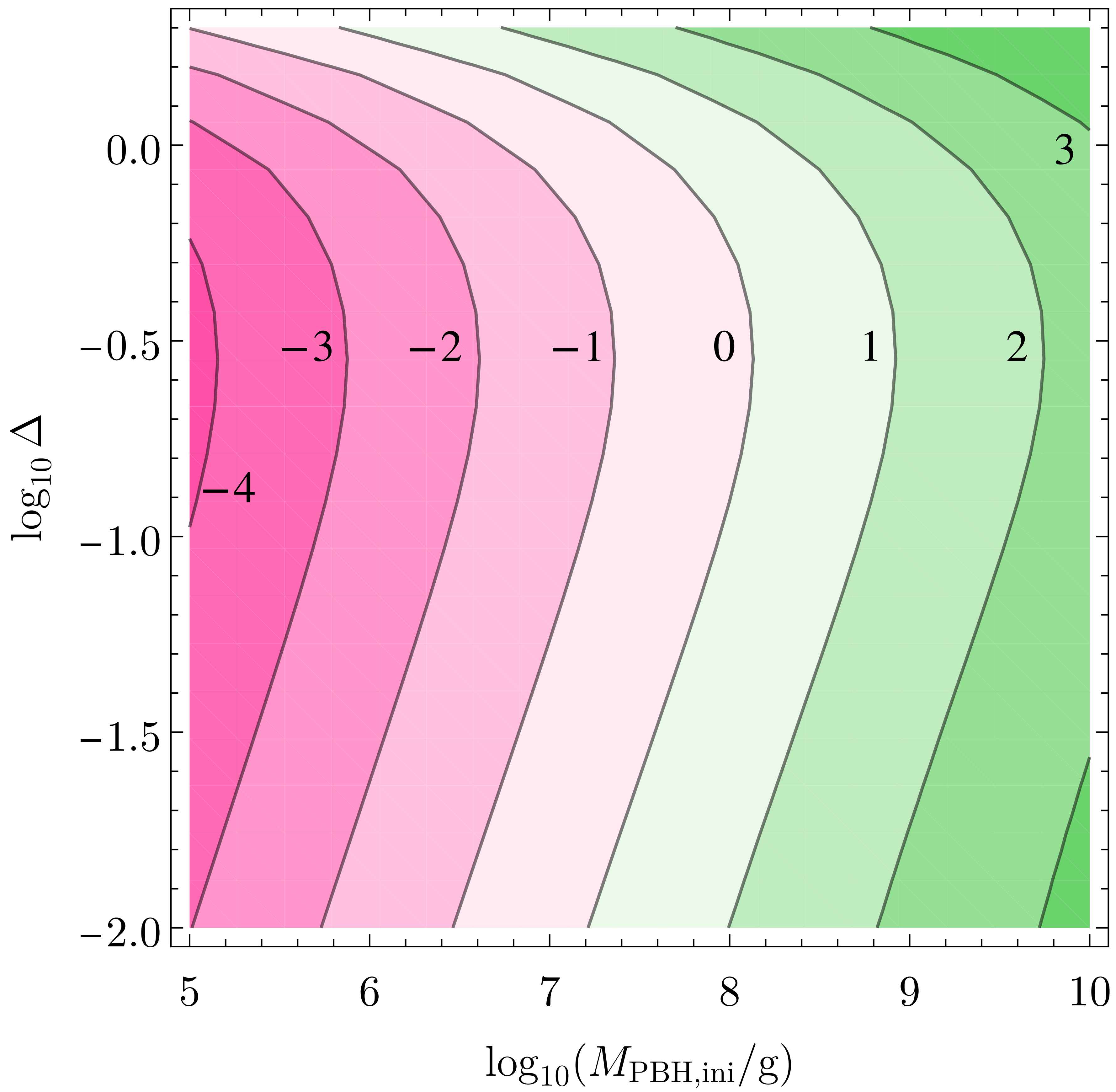
$$\mathrm{SNR} = \sqrt{n_{\rm det} t_{\rm obs} \int_{f_{\rm min}}^{f_{\rm max}} \mathrm{d}f \left(\frac{\Omega_{\rm signal}(f)}{\Omega_{\rm noise}(f)} \right)^2}$$



$t_{\rm obs} = 1\ \mathrm{yr}$

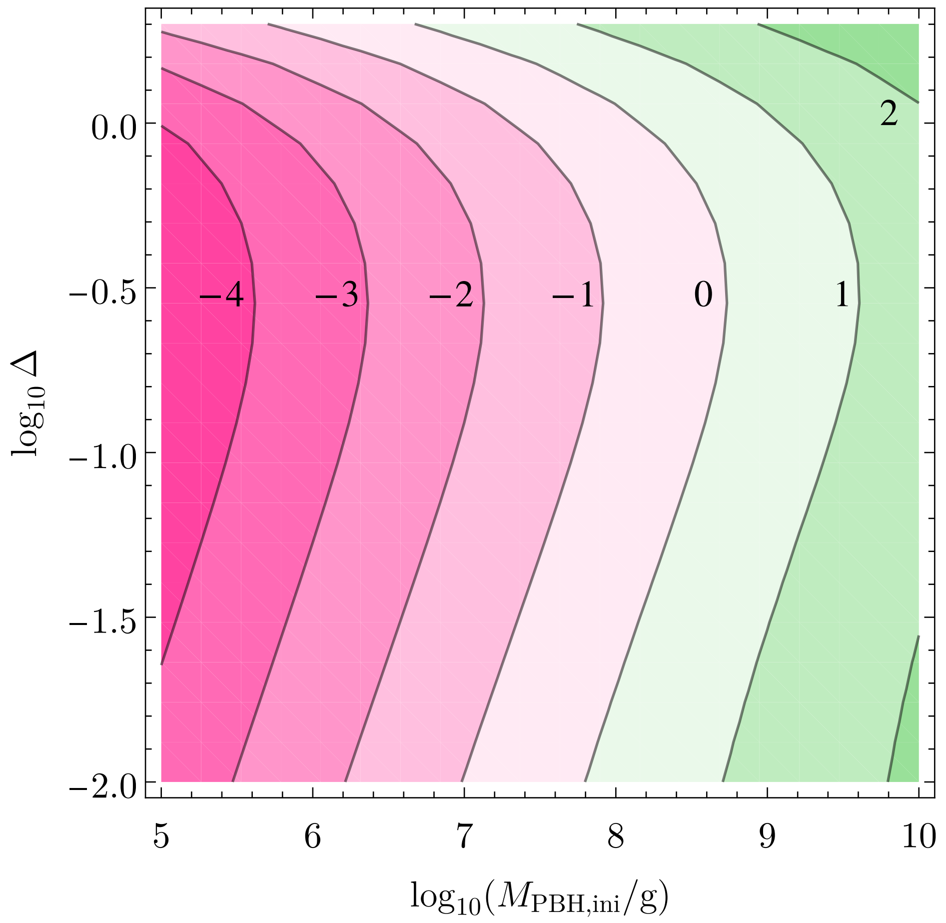
Dependence of $\log_{10}$SNR on the Width

$$\mathscr{P}_\zeta(k) = \frac{A_\zeta}{\sqrt{2\pi\Delta^2}} \exp\left(-\frac{\left(\ln \frac{k}{k_0}\right)^2}{2\Delta^2}\right)$$

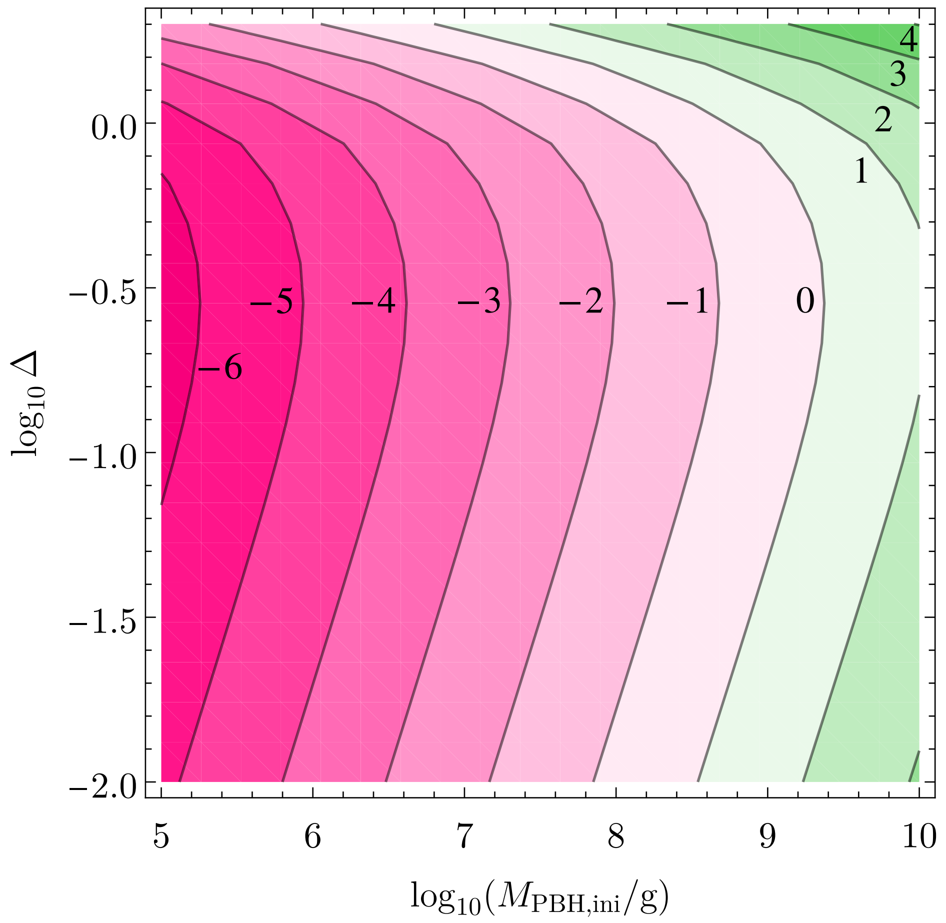


CE

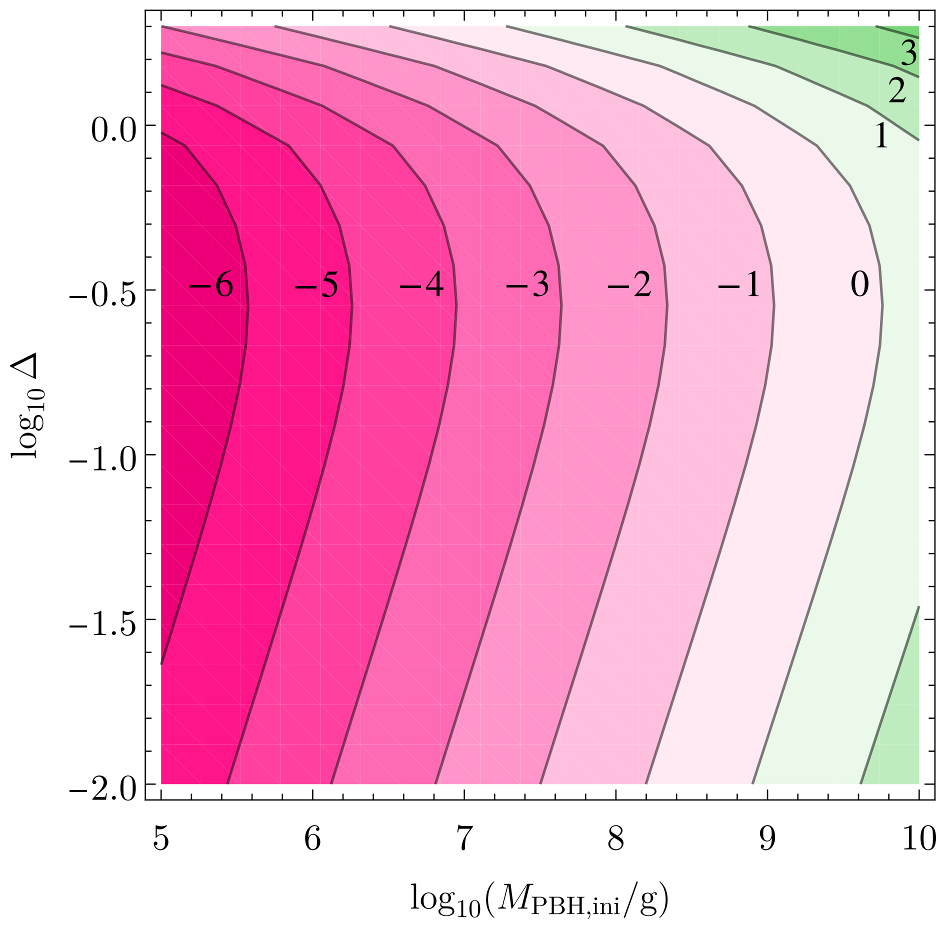
ET



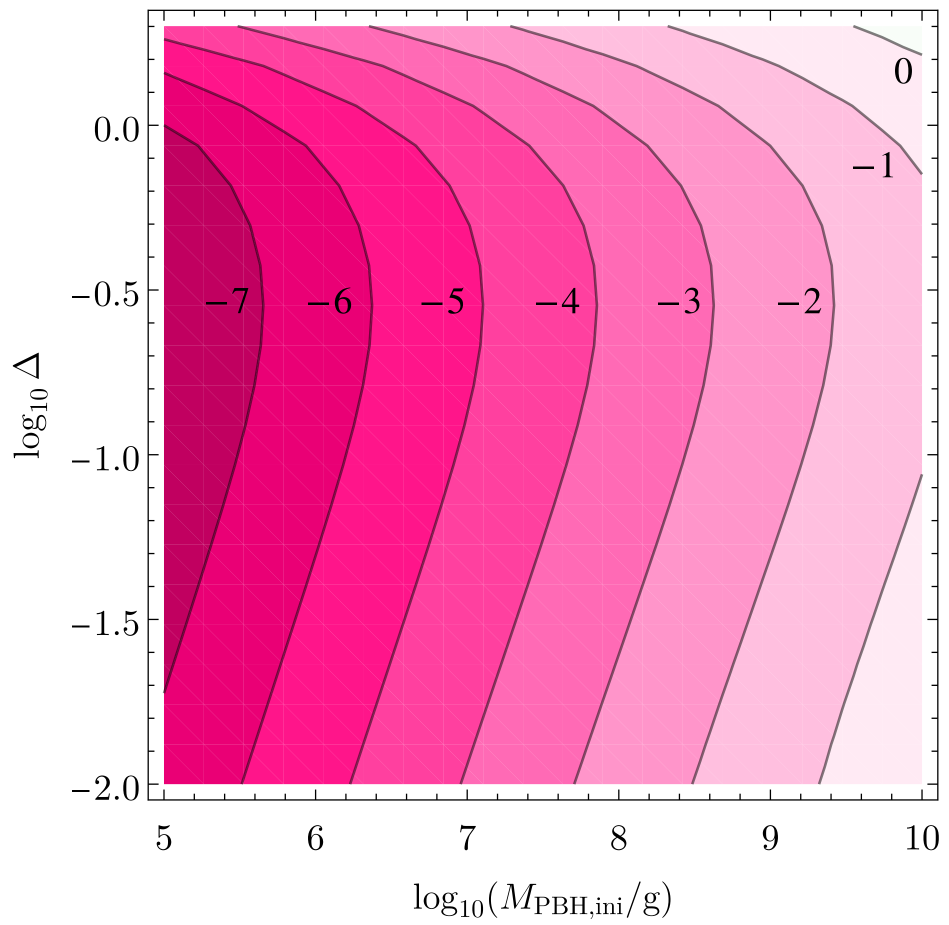
BBO



DECIGO



HLVK



To Exclude the Memory Burden Effect

A few times or dozens of percent stronger induced GWs

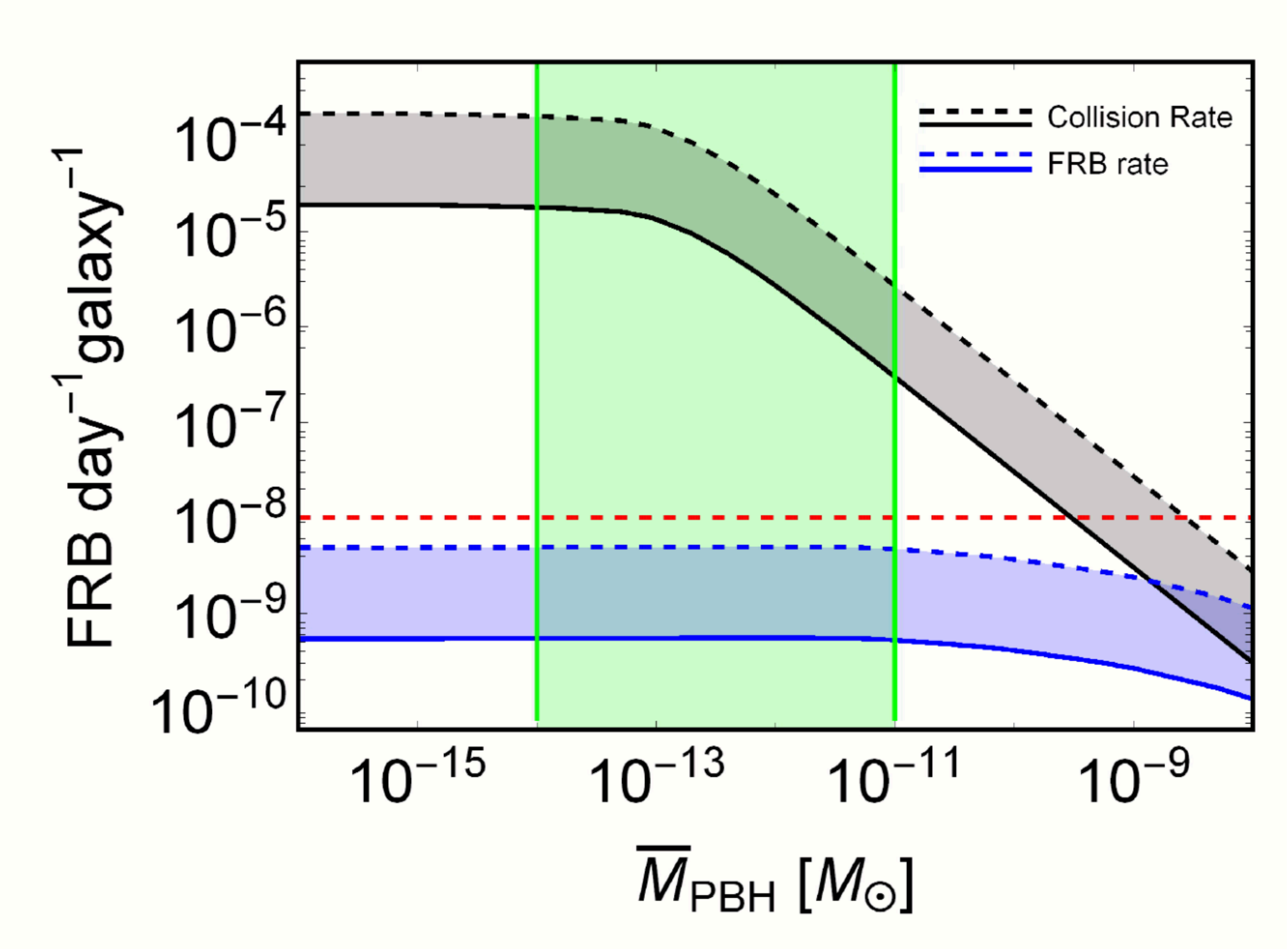
→ Orders of magnitude too many PBH DM.

Thus, the memory burden effect can be observationally excluded.
(Without it, PBHs complete evaporation in the early Universe.)

* Loophole: cosmological dilution (e.g. thermal inflation)
→ but this also constrains the production mechanism of the baryon asymmetry of the Universe

To Confirm the Memory Burden Effect

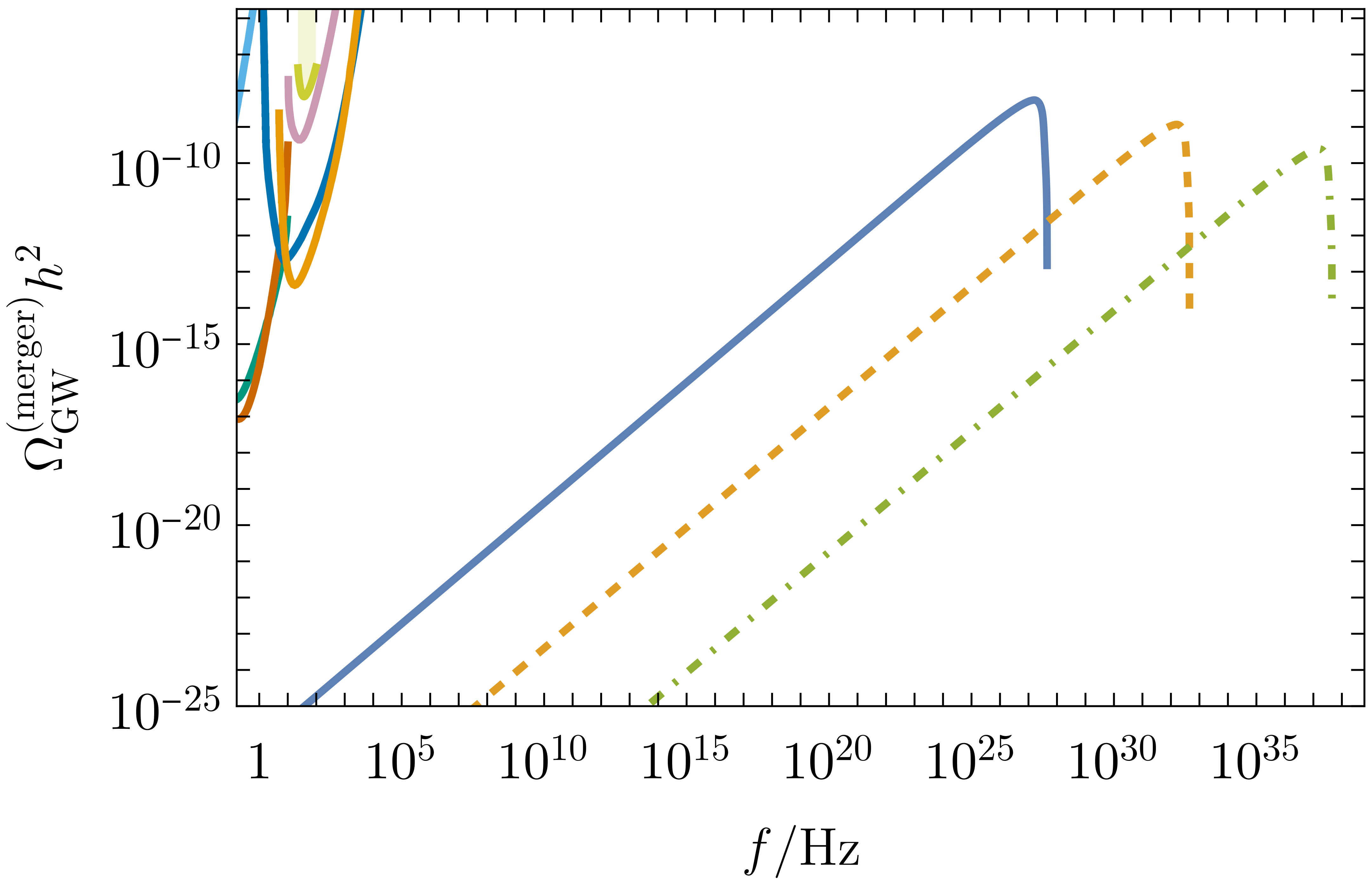
Kainulainen, Nurmi, Schiappacasse, and Yanagida, [2108.08717]



FRB rate may be explained by PBHs colliding with neutron stars.

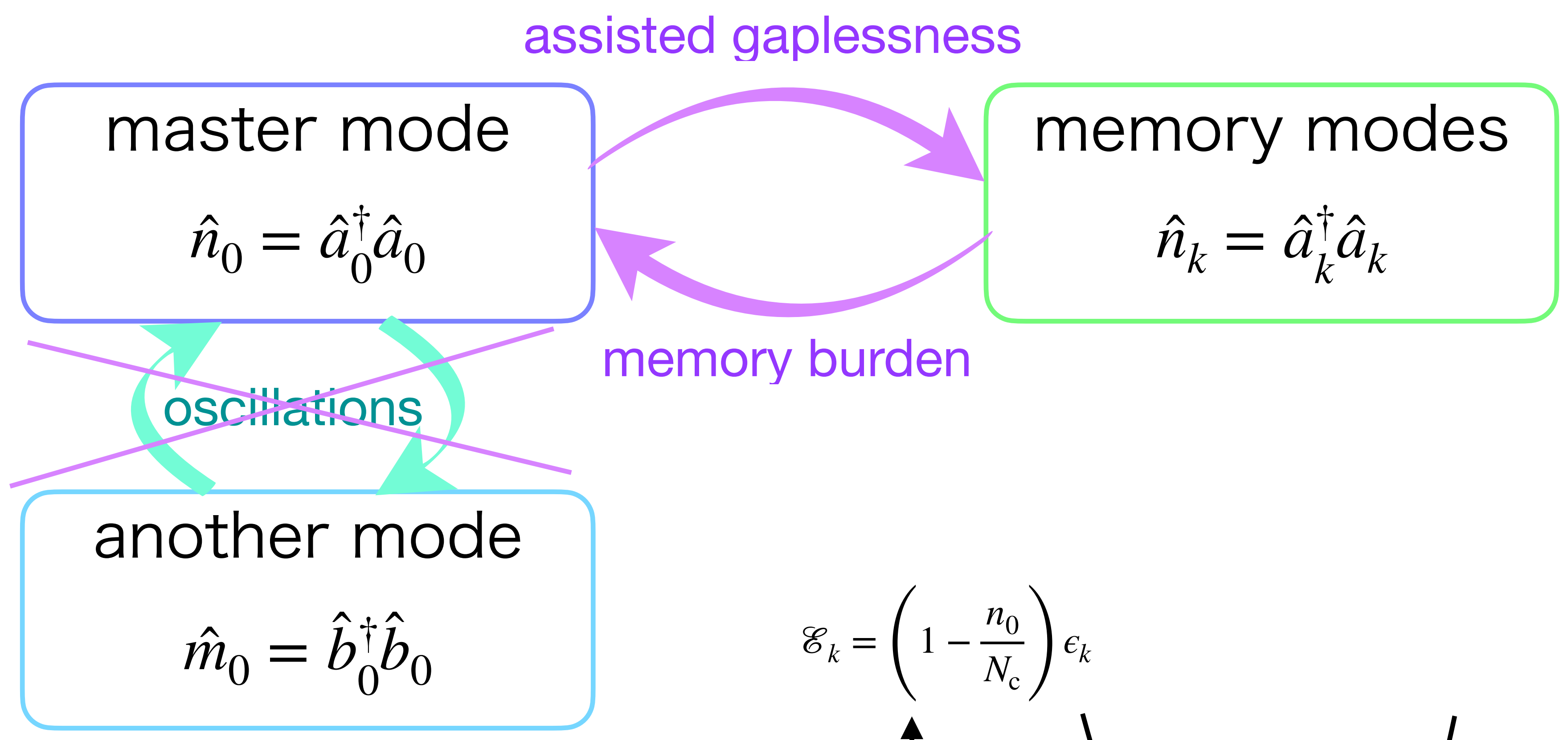
cf. induced/merger GWs connection:

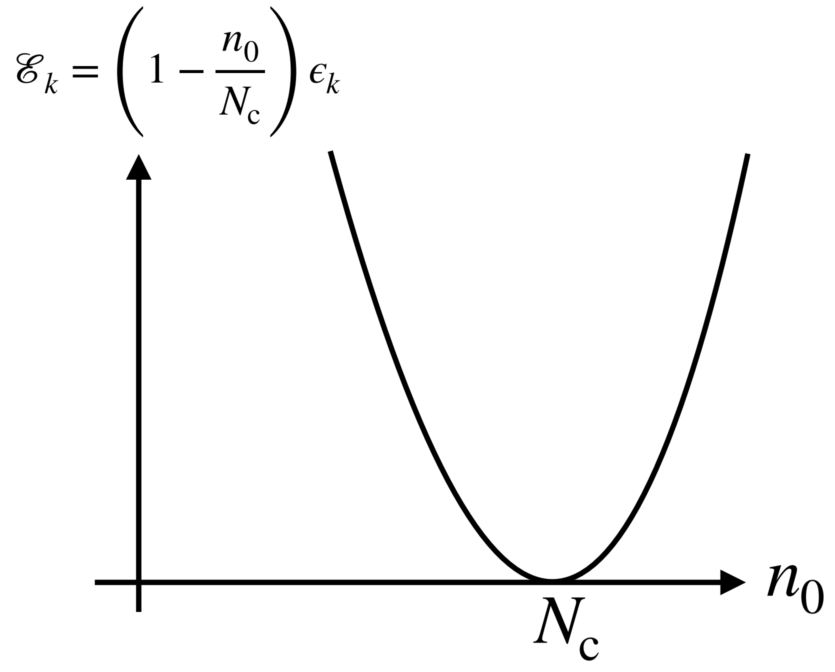
[Wang, Terada, and Kohri, 1903.05924]



Extremely high-frequency GWs from mergers of PBH binaries are predicted.

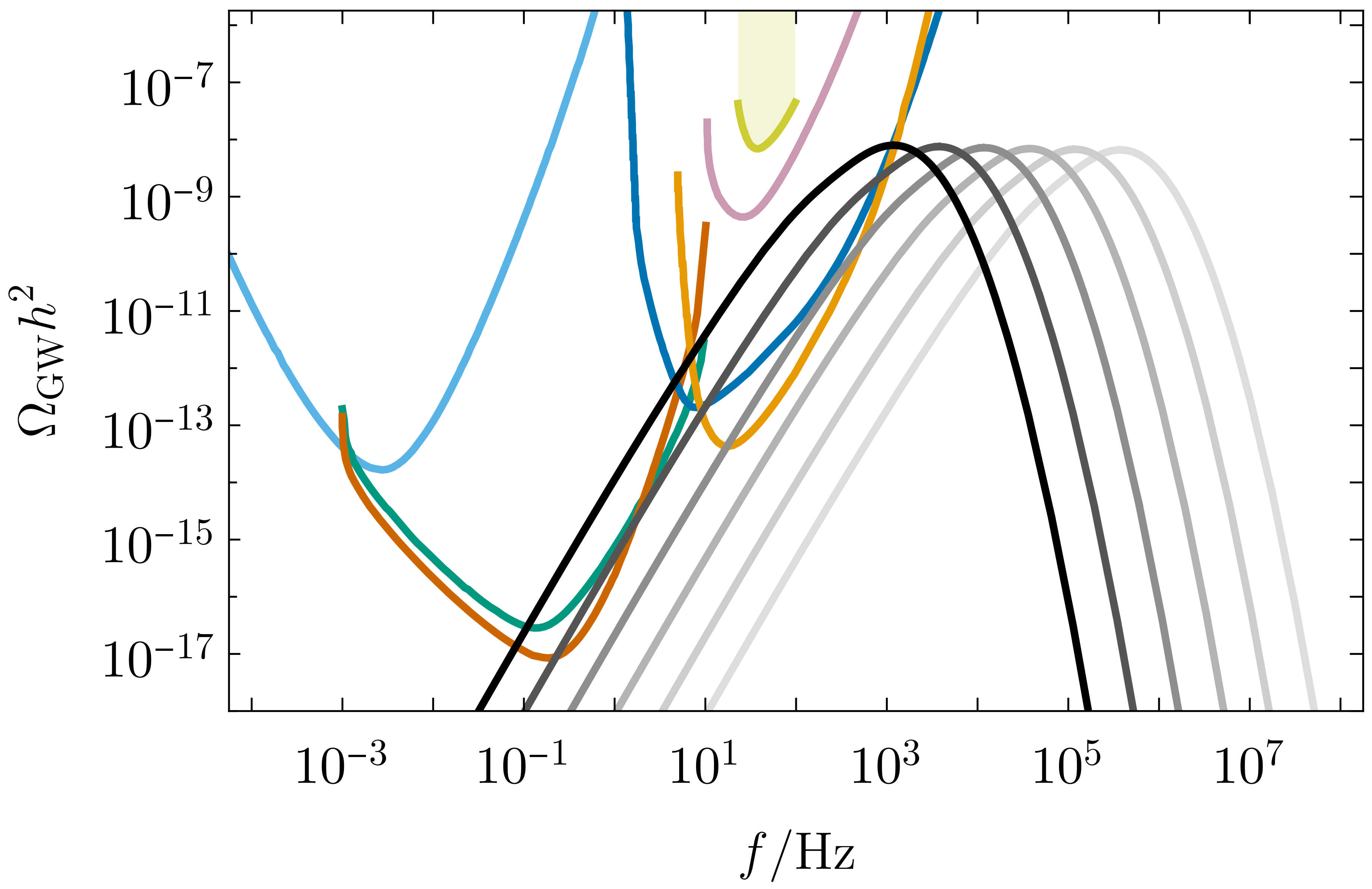
Summary & Conclusion





Memory Burden (MB) effect is a stabilization mechanism due to backreaction from memory modes.

It opens the very light PBH dark matter window $M_{\rm PBH} \lesssim 10^{10}\,\mathrm{g}$.



Future observations such as Cosmic Explorer can test the induced GWs associated with the MB PBH DM with $M_{\rm PBH} \geq 3.5 \times 10^7\,\mathrm{g}$.

We have discussed how to confirm or exclude the MB effect.