

# Pulsar Timing Arrays and ultra-low-frequency Gravitational Waves

Seminar at Institut d'Astrophysique de Paris  
19 March, 2025

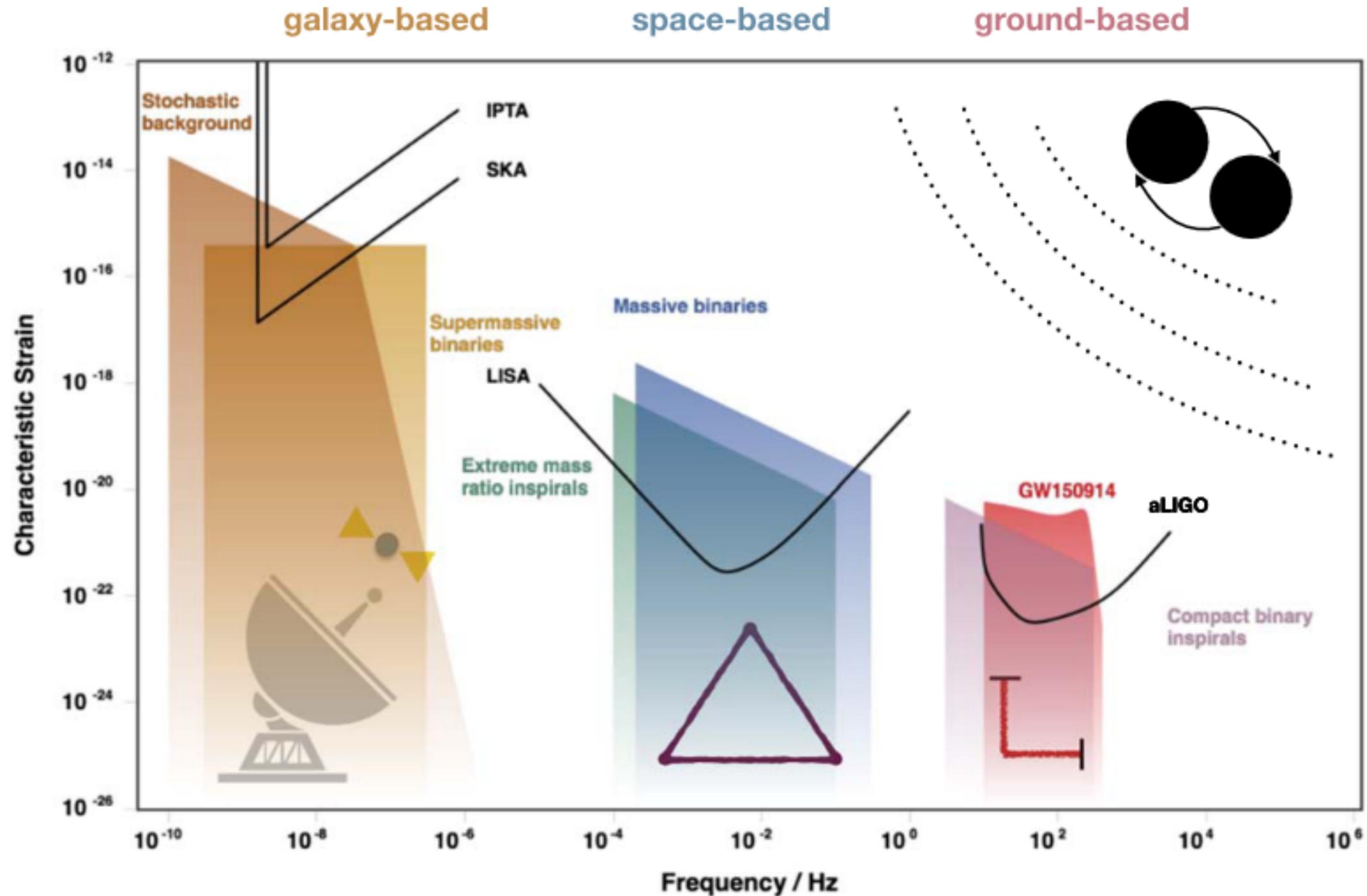
Prerna Rana  
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# Outline

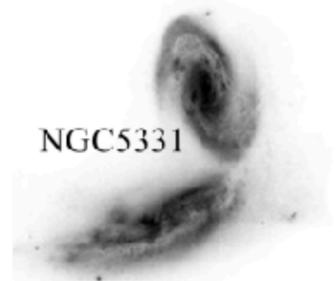
- Pulsar Timing Arrays
- Indian Pulsar Timing Array (InPTA) and data releases
- Recent results
- Future directions

# Astrophysical GW spectrum



# SMBHBs and galaxy evolution

## Galaxy Merger



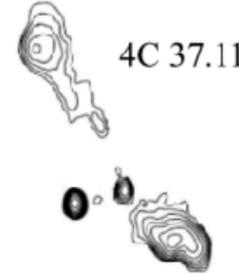
NGC 5331  
Dynamical friction drives massive objects to central positions

## Stellar Core Merger



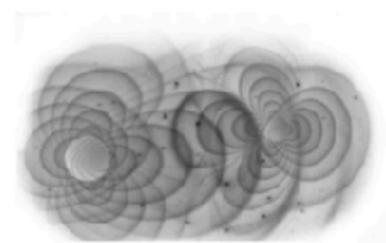
NGC 17  
Dynamical friction less efficient as SMBHs form a binary.

## Binary Formation



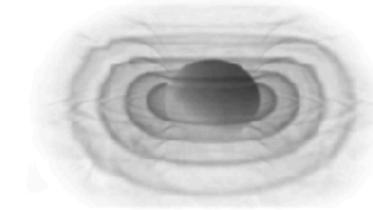
4C 37.11  
Stellar and gas interactions may dominate binary inspiral?

## Continuous GWs



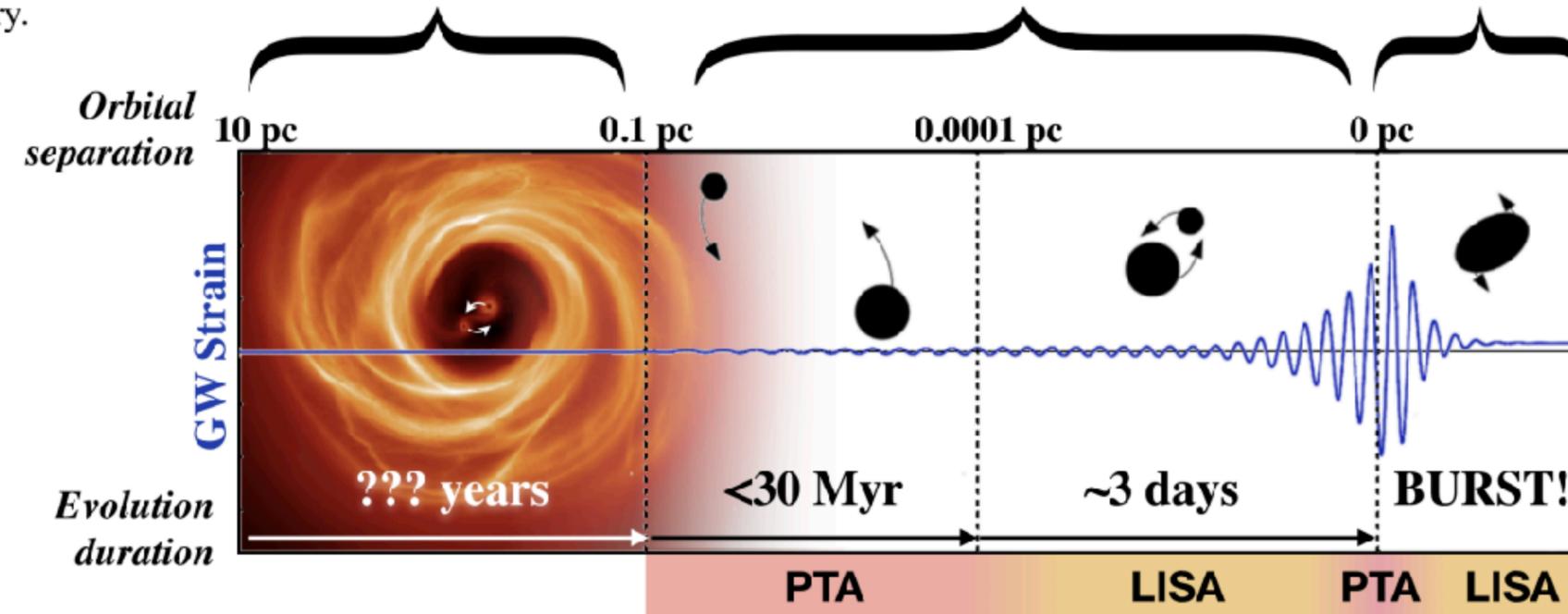
Gravitational radiation provides efficient inspiral. Circumbinary disk may track shrinking orbit.

## Coalescence, Memory & Recoil



Post-coalescence system may experience gravitational recoil.

**The Lifecycle of Binary Supermassive Black Holes**



Ref: Sarah Burke-Spolaor, et al. 2019, The Astronomy and Astrophysics Review, 27:5

# Source of Nano-hertz GWs - SMBHB

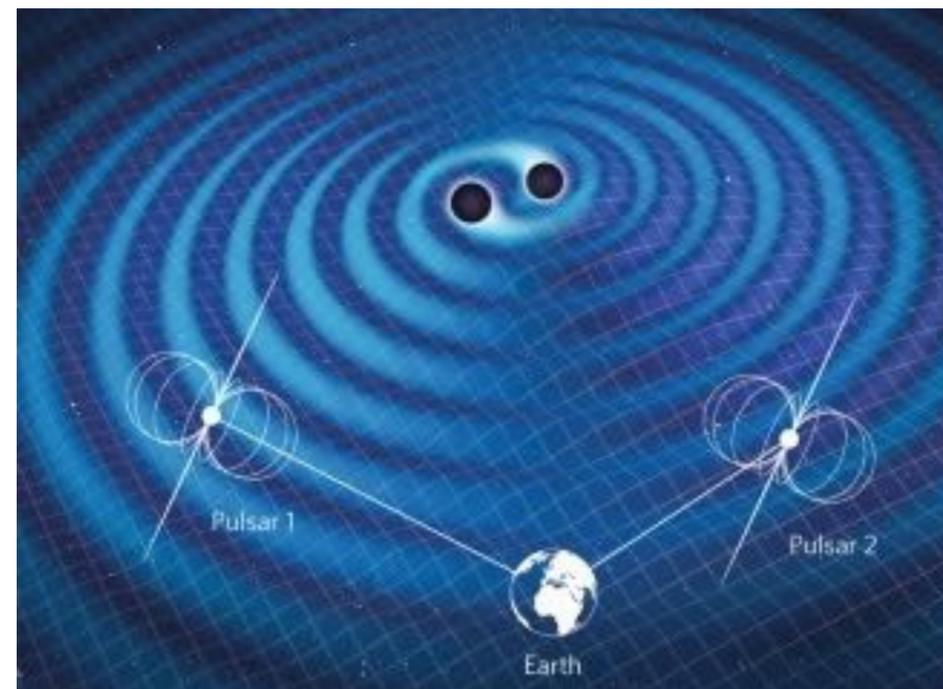
- Supermassive black hole binaries (SMBHB) can emit nHz GWs with amplitudes  $\sim 10^{-15} - 10^{-14}$ .

$$\omega = 2 \times 10^{-8} \text{ s}^{-1} \left( \frac{200M}{R_0} \right)^{3/2} \left( \frac{10^{10} M_\odot}{M} \right)$$

$$A \sim 5 \times 10^{-14} \left( \frac{200M}{R_0} \right) \left( \frac{M}{10^{10} M_\odot} \right) \left( \frac{10^{10} \text{ lt-yr}}{r} \right).$$

Ref: Detweiler, S. (1979), ApJ, 234:1100-1104

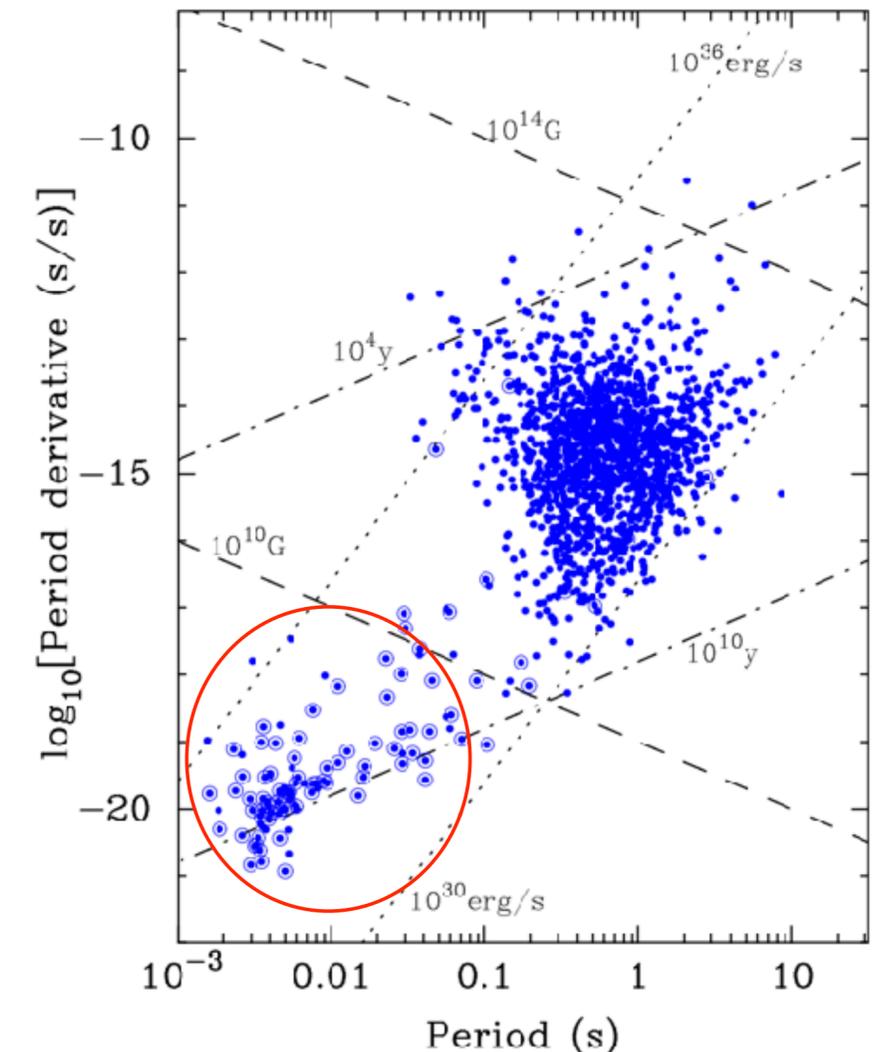
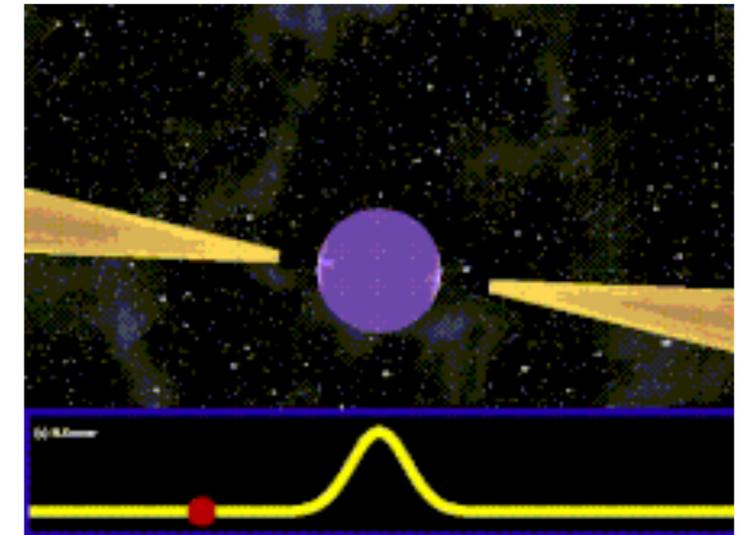
- We need a galactic-size GW detector!
- Ensemble of highly stable celestial clocks - **Millisecond pulsars** - can make one!



Ref: Andrea N. Lommen, Nature Astronomy, volume 1, 809–811 (2017)

# MSPs - Tool of a PTA experiment

- A Pulsar Timing Array (PTA) experiment regularly monitors multiple **MSPs** using radio telescope(s).
- Millisecond pulsars (MSPs)
  - Rotating Neutron stars.
  - Period  $\sim$  a few to 10 ms,  $B \sim 10^9$  G. Pulses arriving at predictable and regular intervals.
  - **Period is stable  $\sim$  1 part in 100 quintillion. High quality galactic clocks!**
  - Time of arrival (ToA) of pulse can be precisely modelled.
  - **Timing Residual** = Measured ToA - Predicted ToA
    - **Data**  $\uparrow$  Measured ToA
    - **Timing Model**  $\uparrow$  Predicted ToA
  - Obtaining accurate Pulsar timing model  $\longrightarrow$  Art of **Pulsar Timing**
  - **Years of observations is used to obtain a more precise timing model.**



# Pulsar Timing Method

- Once a large number of ToAs are accumulated, simple model is sufficient to predict ToAs of subsequent pulses.

$$\phi(\mathcal{T}) = \phi_0 + (\mathcal{T} - \mathcal{T}_0)\Omega_0 + \frac{1}{2}(\mathcal{T} - \mathcal{T}_0)^2\dot{\Omega}_0 + \dots$$

- Using an initial estimate of period, period derivative, and position of pulsar (part of timing model), timing residuals can be calculated.
- Measured ToA:  $t_{obs}, t_{err}$  ; predicted ToA from timing model:  $t_p$  ;

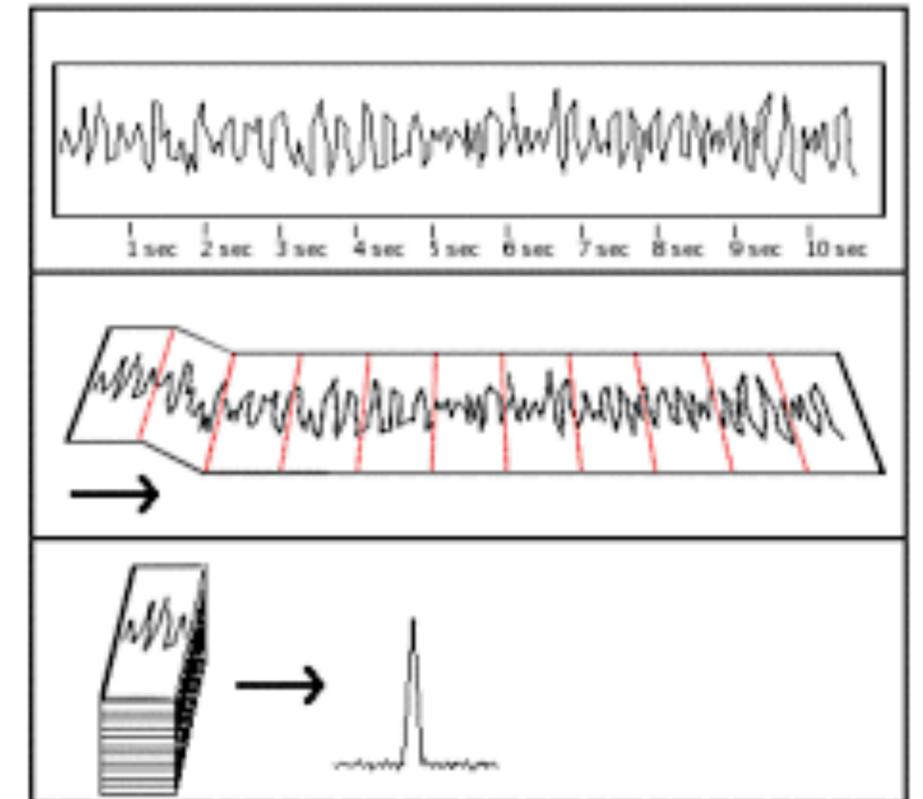
$$\text{Timing residuals} = t_{obs} - t_p$$

Regression analysis is used to refine “timing model” parameters

$$\chi^2 = \sum_i \left( \frac{t_{obs} - t_p}{t_{err}} \right)^2$$

This allows high precision estimates of

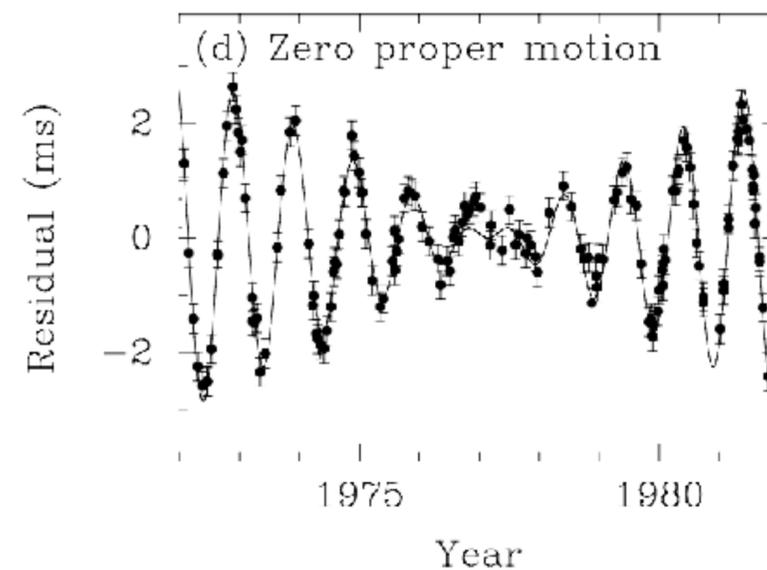
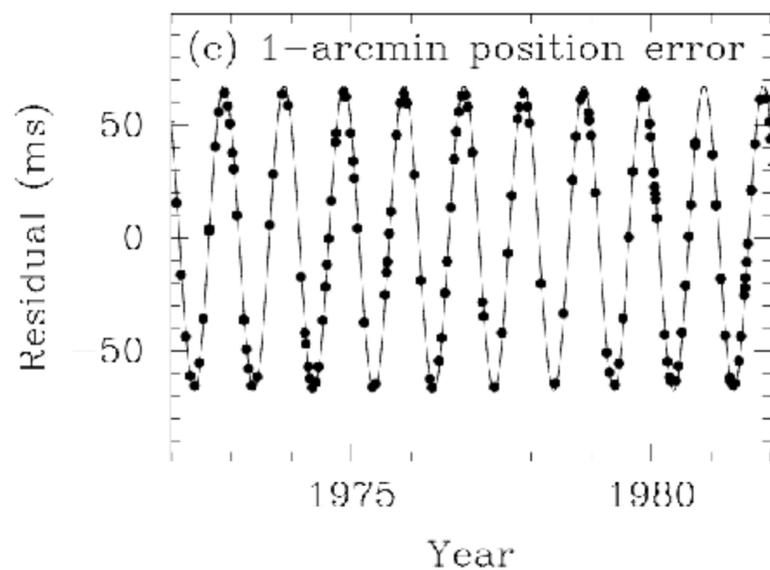
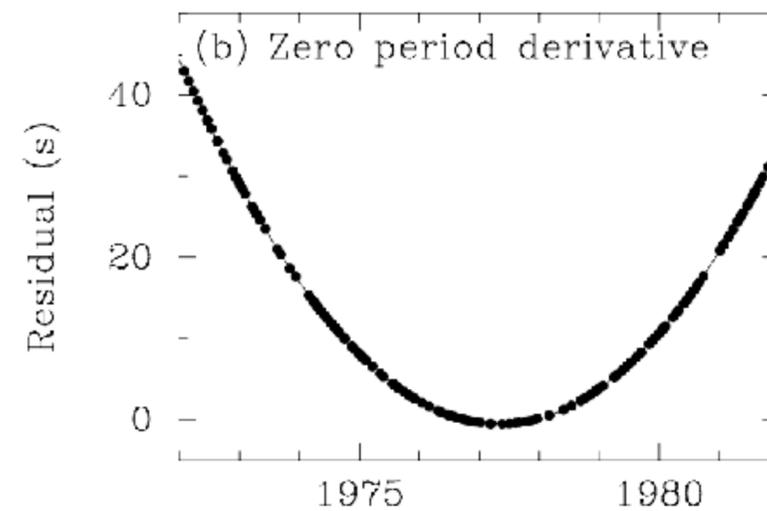
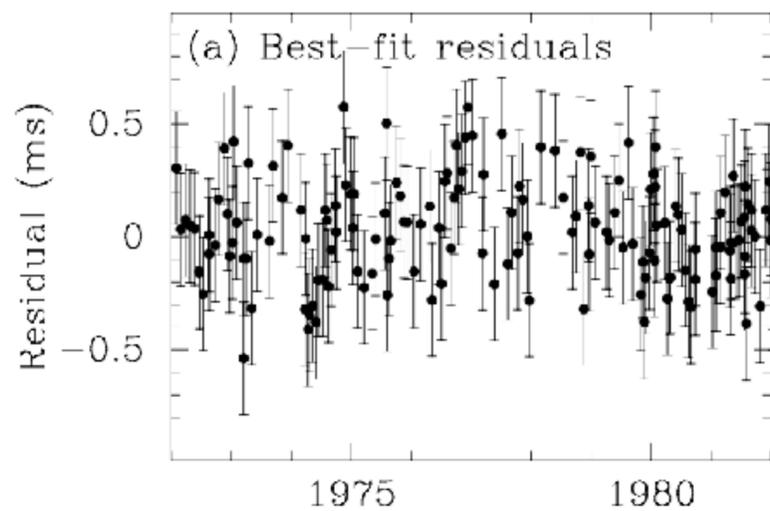
- Pulsar spin period, its derivatives,
- Position, proper motion, parallax,
- Pulsar binary parameters,
- Dispersion measure (ISM).



**Stable integrated pulse profile**

# Pulsar Timing Method

- A good timing model will give “white noise” like timing residuals.



# Imprints of GWs on Timing residuals

- A passing GW, travelling in z-direction, induces a fractional change in the observed pulse frequency,

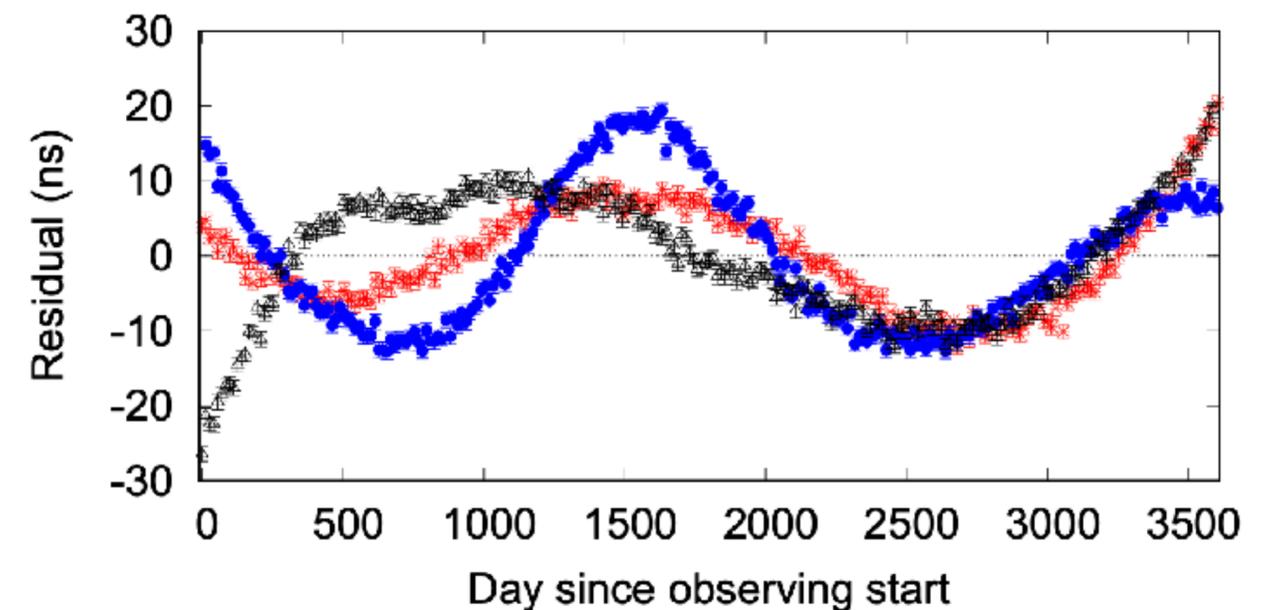
$$\frac{\nu_0 - \nu(t)}{\nu_0} = \frac{\alpha^2 - \beta^2}{2(1 + \gamma)} \Delta h_+(t) + \frac{\alpha\beta}{1 + \gamma} \Delta h_\times(t)$$

Ref: Detweiler, S. (1979), ApJ, 234:1100-1104

where  $\alpha$ ,  $\beta$ ,  $\gamma$  are pulsar's direction cosines to x, y, z axes.

- A time-varying influence of the GW on the local space-time of the pulsar and Earth.
- GWs should appear as systematic trends in timing residuals.

$$R(t) = \int_0^t \frac{\nu_0 - \nu(t')}{\nu_0} \delta t'$$



Systematic trends due to GW background

Ref: Sarah Burke-Spolaor, et al. 2019, The Astronomy and Astrophysics Review, 27:5

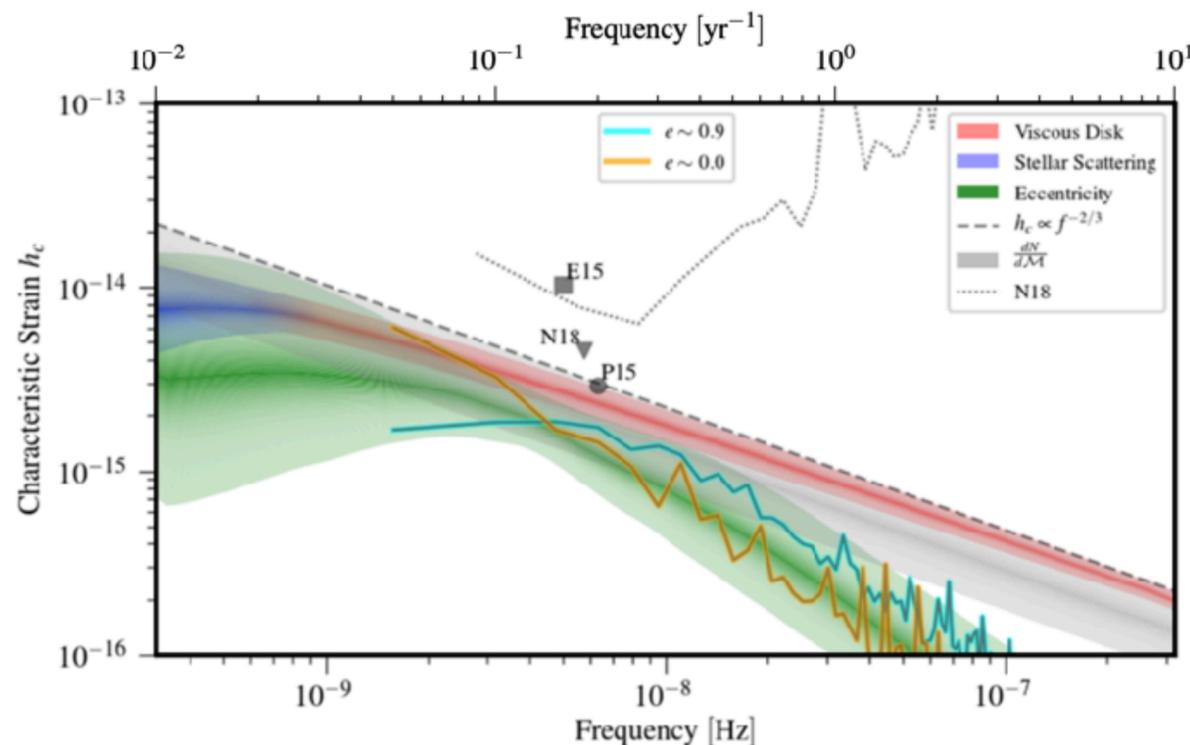
# Stochastic nano-Hz Gravitational Wave Background

- Superposition of GWs from a large population of SMBHB can produce a stochastic background.
- A detection of the SMBHB background would confirm the consensus view that SMBHs reside in most or all massive galaxies.

- Characteristic strain spectrum of GWB from circular SMBHB

$$h_c(f) = A \left( \frac{f}{f_{\text{yr}}} \right)^\alpha \quad \text{where } \alpha = -2/3$$

Ref: [Phinney 2001, arXiv:astro-ph/0108028](#)



- Grey region - uncertainty in the overall distribution of SMBHB in the universe.
- Green region - Distribution of SMBHB in eccentric orbits.
- Red, Blue regions - impact of the direct SMBHB environment.

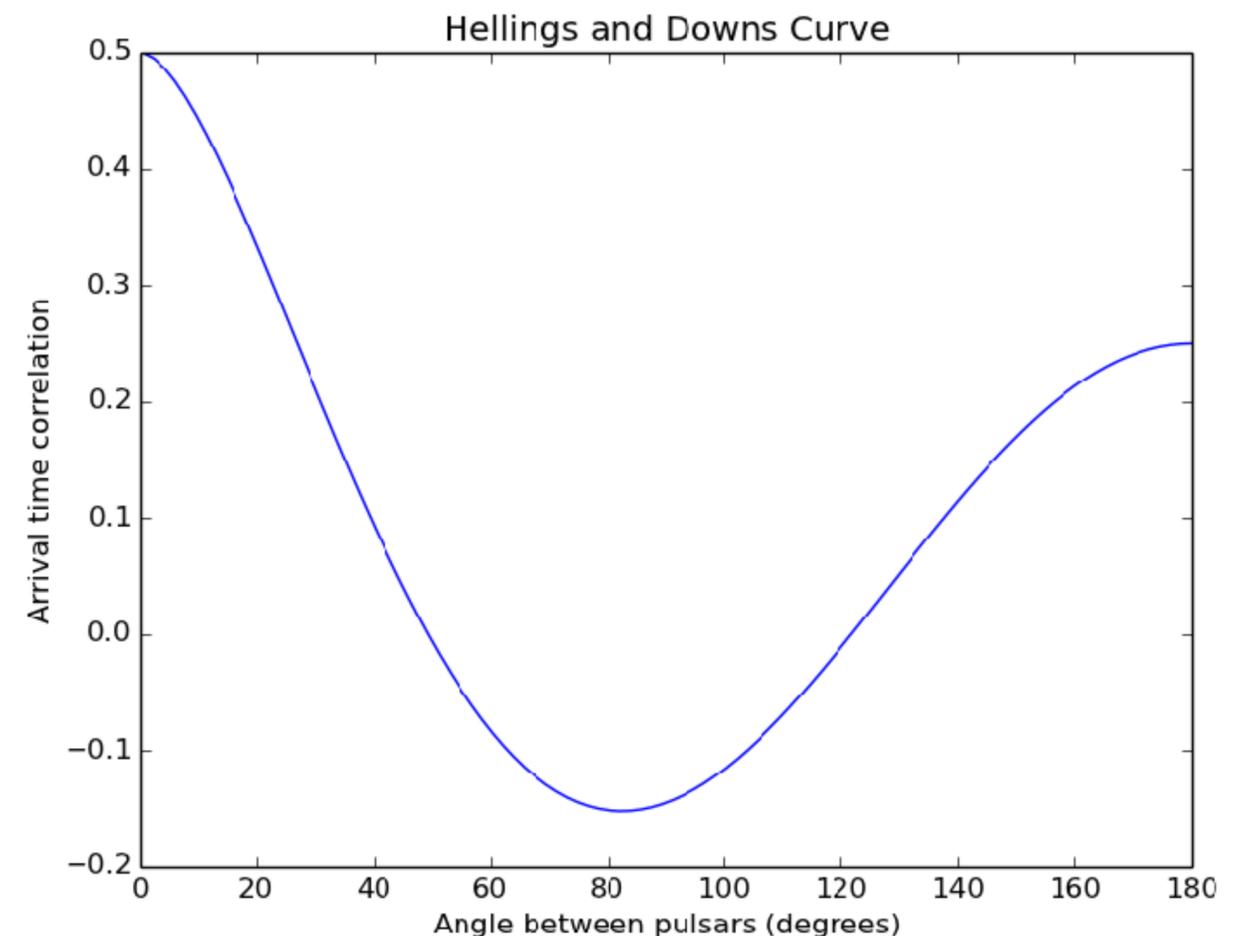
# Stochastic nano-Hz Gravitational Wave Background

- GWB will appear as a “common red-noise” in Fourier power spectra of PTA data.

$$P(f|A, \gamma) = \Gamma(\zeta_{ab}) \frac{A^2}{12\pi^2} \left( \frac{f}{\text{yr}^{-1}} \right)^{-\gamma} \longrightarrow \mathbf{13/3}$$

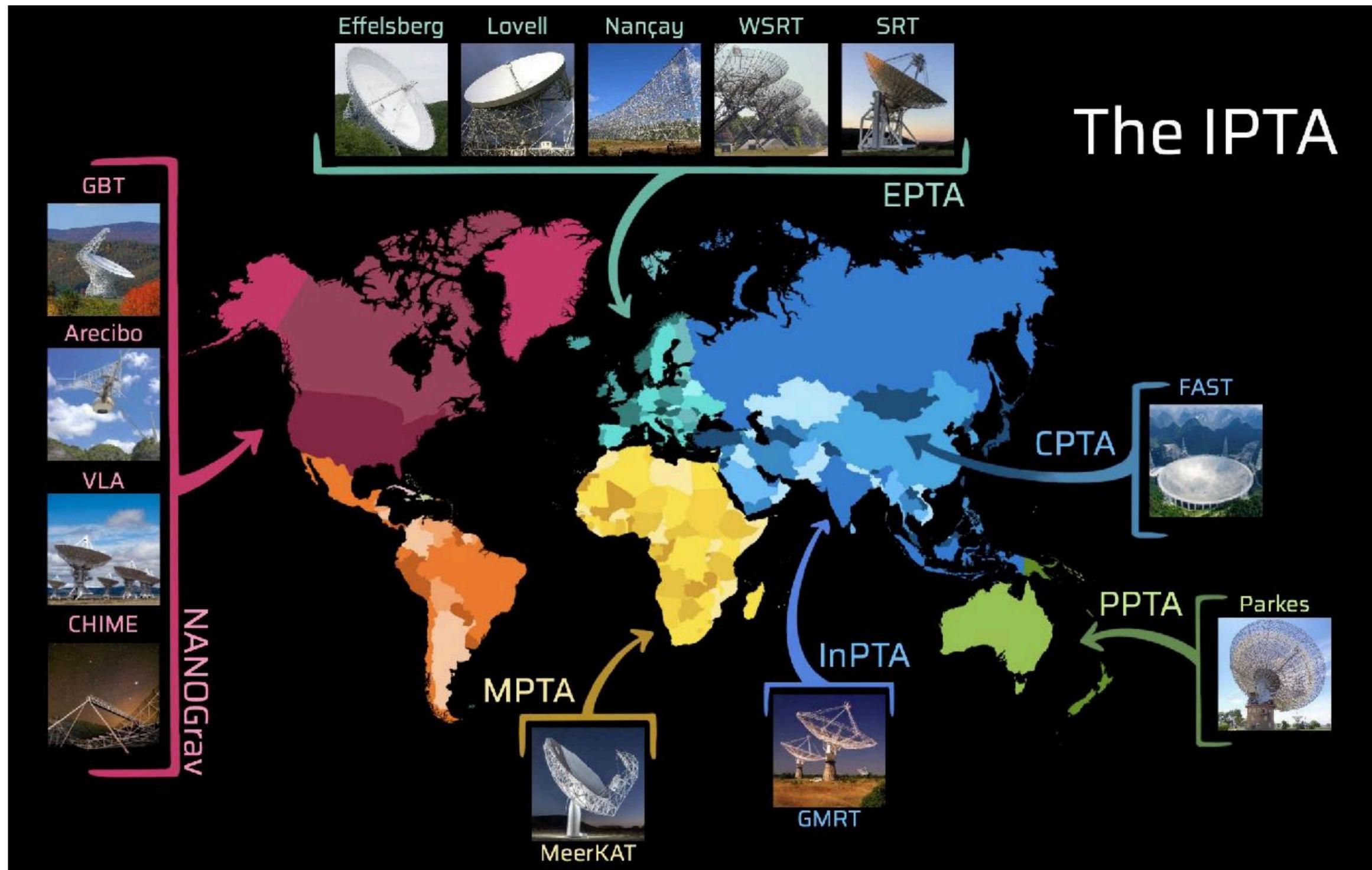
Correlation function

- Pulsar residuals due to SMBHB GWB are correlated with quadrupolar nature. The correlations encodes the distribution of gravitational-wave power on the sky.
- Checklist of GWB detection for a PTA experiment:
  - Common red noise.
  - H-D Correlation (if GWB is isotropic).



Ref: Hellings & Downs 1983, ApJ 265:L39-L42

# Pulsar Timing Array experiments on globe



**The International  
PTA**

**EPTA+InPTA+  
NANOGrav+PPTA  
+MPTA**

**CPTA plans to  
join soon.**

# Indian Pulsar Timing Array (InPTA)

# Indian Pulsar Timing Array



- Indo-Japanese collaboration.
- The InPTA experiment started in 2015.
- Presently observing 22 IPTA pulsars.
- Cadence: 10-14 days

## Unique strength of InPTA uGMRT data:

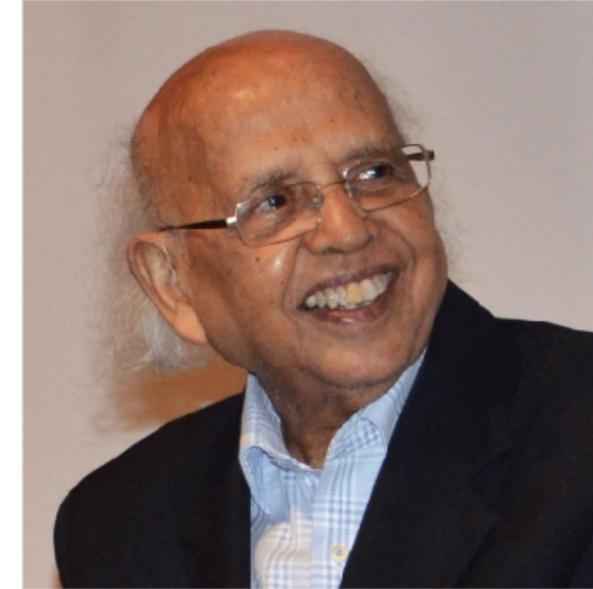
- High sensitivity at low-radio frequencies.
- Concurrent observations in two bands covering large frequency range (300 - 500 MHz, 1260 - 1460 MHz).
- Ideal for studying radio frequency dependent effects dominant at low frequencies.

**Website:**

**<https://inpta.iitr.ac.in/>**



- Located near Khodad town, about 80 km north of Pune, Maharashtra, India.
- **High-sensitivity low-frequency Interferometer** - 30 fully steerable gigantic parabolic dishes of **45m** diameter.
- Spread over distances of upto **25 km** - max baseline.
- Has capability of dividing antennas into subarrays to do multi-band observations.



Govind Swarup (1929-2020). Credit: NCRA

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## The Giant Metre-wave Radio Telescope

*G. Swarup, S. Ananthkrishnan, V. K. Kapahi, A. P. Rao, C. R. Subrahmanya and V. K. Kulkarni*

*The Giant Metre-wave Radio Telescope, an aperture-synthesis array consisting of 30 fully steerable parabolic dishes of 45-m diameter each, is being set up about 80 km north of Pune as a national facility for frontline research in radio astronomy in the frequency range 38 MHz to 1420 MHz. The new and novel design of a low-solidity dish for metre-wave operation, in which a thin wire mesh (varying in size from 10 mm × 10 mm to 20 mm × 20 mm and made of 0.55 mm diameter stainless-steel wire), which constitutes the reflecting surface, is stretched over a parabolic surface formed by rope trusses, has made it possible to build a large collecting area (total effective area of about 30,000 m<sup>2</sup>, over three times that of the Very Large Array in the USA) at modest cost. It will be a major new instrument designed to fill the existing world-wide gap in powerful radio telescopes operating at metre wavelengths, where there are many exciting and challenging astrophysical problems and phenomena to be investigated. Two of the primary scientific objectives of the telescope are to detect the highly redshifted '21-cm' line of neutral hydrogen from protoclusters or protogalaxies in the early epochs of the Universe before galaxy formation and to detect and study a large number of millisecond pulsars in an attempt to detect the primordial background of gravitational radiation.*

**Ref: G. Swarup et al. 1991, Current Science, Vol. 60, pp. 95-105**

# InPTA observational history

**Table 1.** Pulsar sample of the InPTA. The sample labeled ‘Classic InPTA’ was observed from 2018–2022, whereas the sample labeled ‘Exploratory InPTA’ was observed only in 2018–2019. Likewise, the sample labeled ‘Expanded InPTA’ was mostly observed only after 2021 with high sensitivity. The ‘Exploratory InPTA’ sample also included all pulsars listed in the ‘Classic InPTA’ and ‘Expanded InPTA’ samples. The ‘Classic InPTA’ and the ‘Expanded InPTA’ pulsars are being observed using bands 3 and 5 with 200 MHz bandwidth, whereas ‘Exploratory InPTA’ sample was observed with bands 3, 4 and 5 with 100 MHz bandwidth.

	Name	Period (s)	DM (pc cm <sup>-3</sup> )	Median S/N 1460	Median S/N 500	Obs time (min)
Exploratory InPTA	J0645+5158	0.008853	18.25	7	52	30
	J1024–0719	0.005162	6.49	13	17	30
	J1455–3330	0.007987	13.57	9	19	30
	J1614–2230	0.003151	34.49	12	18	30
	J1640+2224	0.003163	18.43	20	34	30
	J1730–2304	0.008123	9.62	61	56	30
	J1738+0333	0.005850	33.77	7	10	30
	J2317+1439	0.003445	21.91	8	21	30
	J2302+4442	0.005192	13.79	9	24	30
	J1643–1224	0.004622	62.41	129	423	50
	J1713+0747	0.004570	15.91	254	174	50
	J1857+0943	0.005362	13.31	74	134	30
	Classic InPTA	J1909–3744	0.002947	10.39	77	289
J1939+2134		0.001558	71.02	159	323	20
J2145–0750		0.016052	9.00	103	946	50
J2124–3358		0.004931	4.60	39	208	50
J0437–4715		0.005757	2.64	958	2343	15
J0613–0200		0.003061	38.77	61	119	50
J0751+1807		0.003479	30.25	32	59	30
Expanded InPTA	J1012+5307	0.005255	9.02	259	359	30
	J1022+1001	0.016452	10.25	38	417	20
	J1600–3053	0.003597	52.32	71	67	50
	J1744–1134	0.004075	3.14	72	343	50

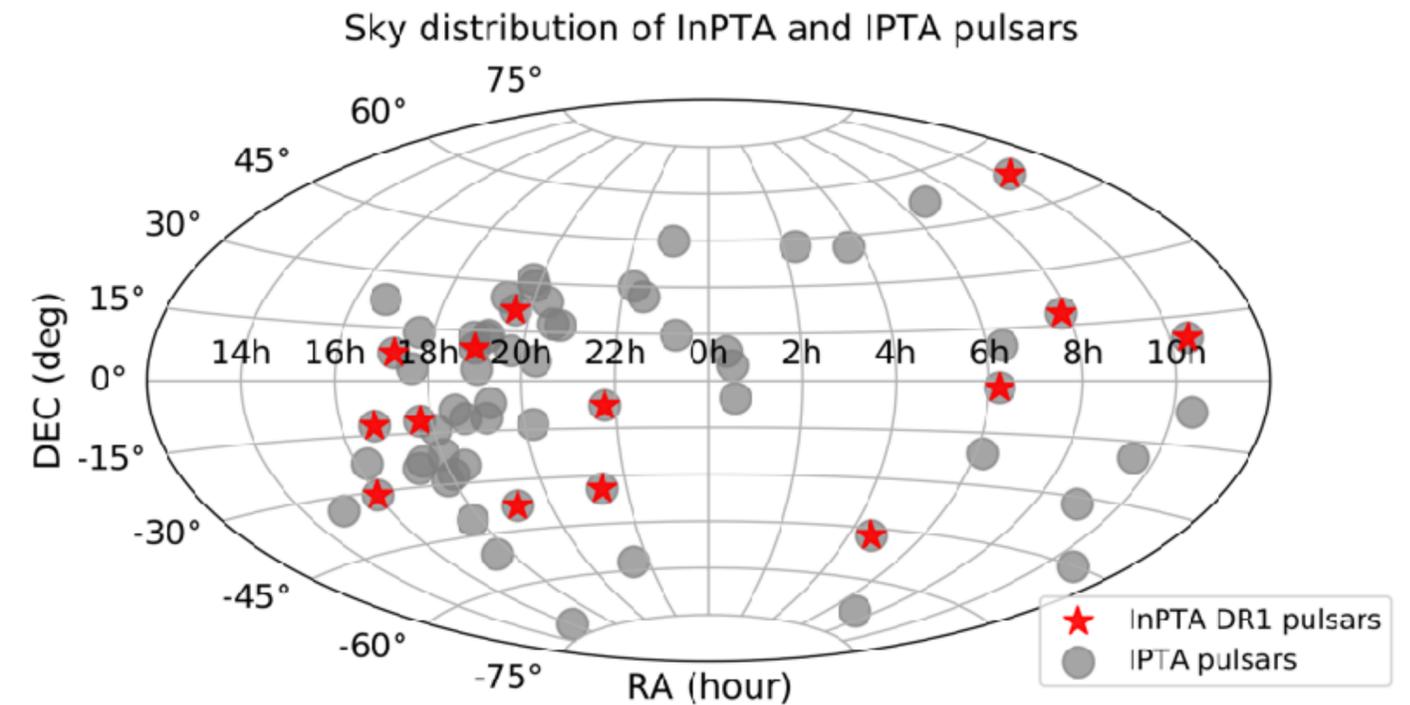
- The experiment started as a pilot experiment in 2014 using the Ooty Radio Telescope (ORT) for observing two pulsars, PSRs J1713+0747 and J1909–3744.

9 pulsars were observed using both the legacy GMRT and ORT in 2015, to test the feasibility of a PTA with both the instruments for the first time.

- Since 2018, InPTA is observing 5 to 22 pulsars from uGMRT simultaneously in multiple bands.
- Multiple phased arrays using different groups of antennas, called sub-arrays.
- Currently, we use two sub-arrays with 10 and 15 antennas in bands 3 and 5 with 200 MHz BW each is used.

# InPTA first data release: InPTA-DR1

- |              |              |
|--------------|--------------|
| ● J0613-0200 | ● J0437-4715 |
| ● J1012+5307 | ● J0751+1807 |
| ● J1600-3053 | ● J1022+1001 |
| ● J1713+0747 | ● J1643-1224 |
| ● J1857+0943 | ● J1744-1134 |
| ● J1939+2134 | ● J1909-3744 |
| ● J2145-0750 | ● J2124-3358 |



- 3.5 years of data, 2018-2021, of 14 MSPs.
- **Band 3 (300-500 MHz)** + Band 5 (1260-1460 MHz), Observing bandwidth = 200MHz
- **Data:**
  - Time of Arrival (ToA) of pulse.
  - Timing residuals and timing model.
  - Dispersion measure → [Key importance](#).

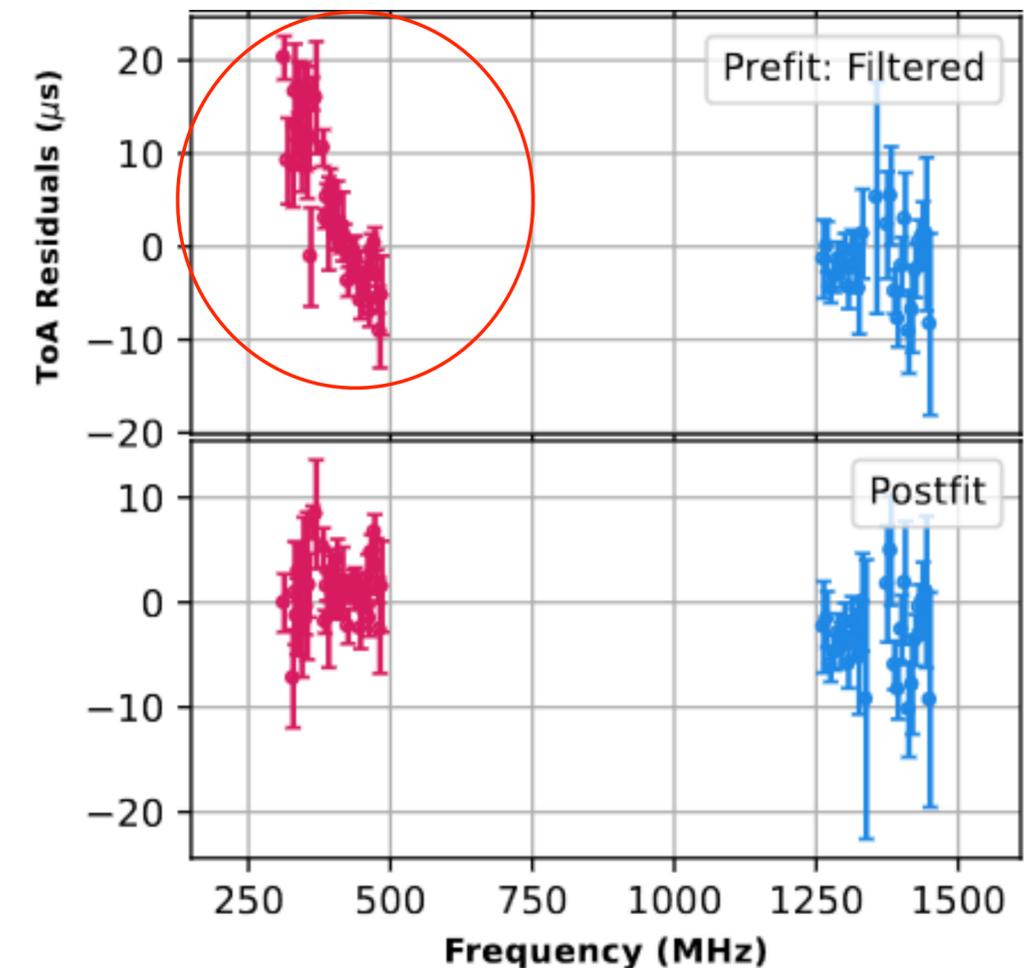
Ref: P. Tarafdar, et al. 2022, *PASA*, 39, e053

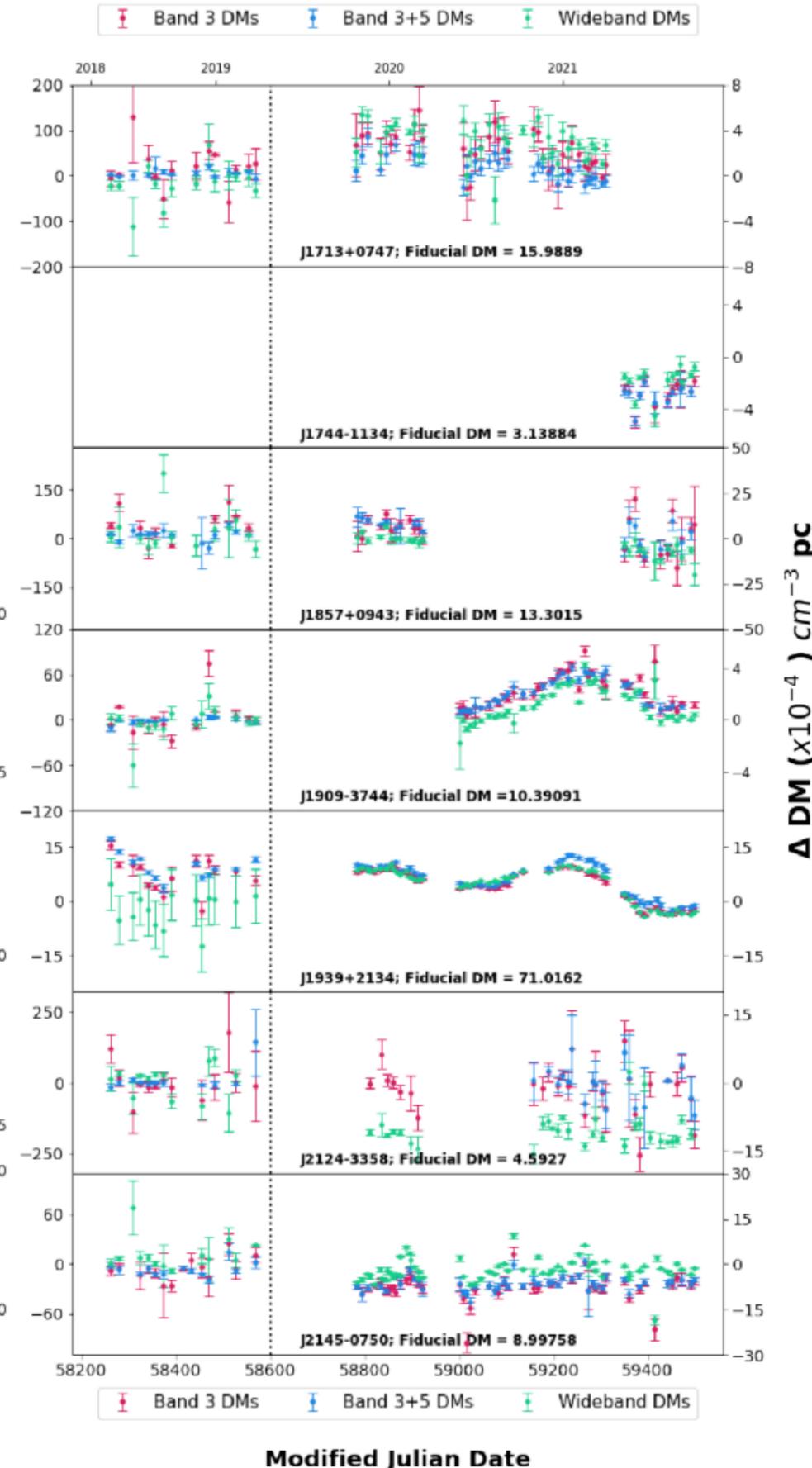
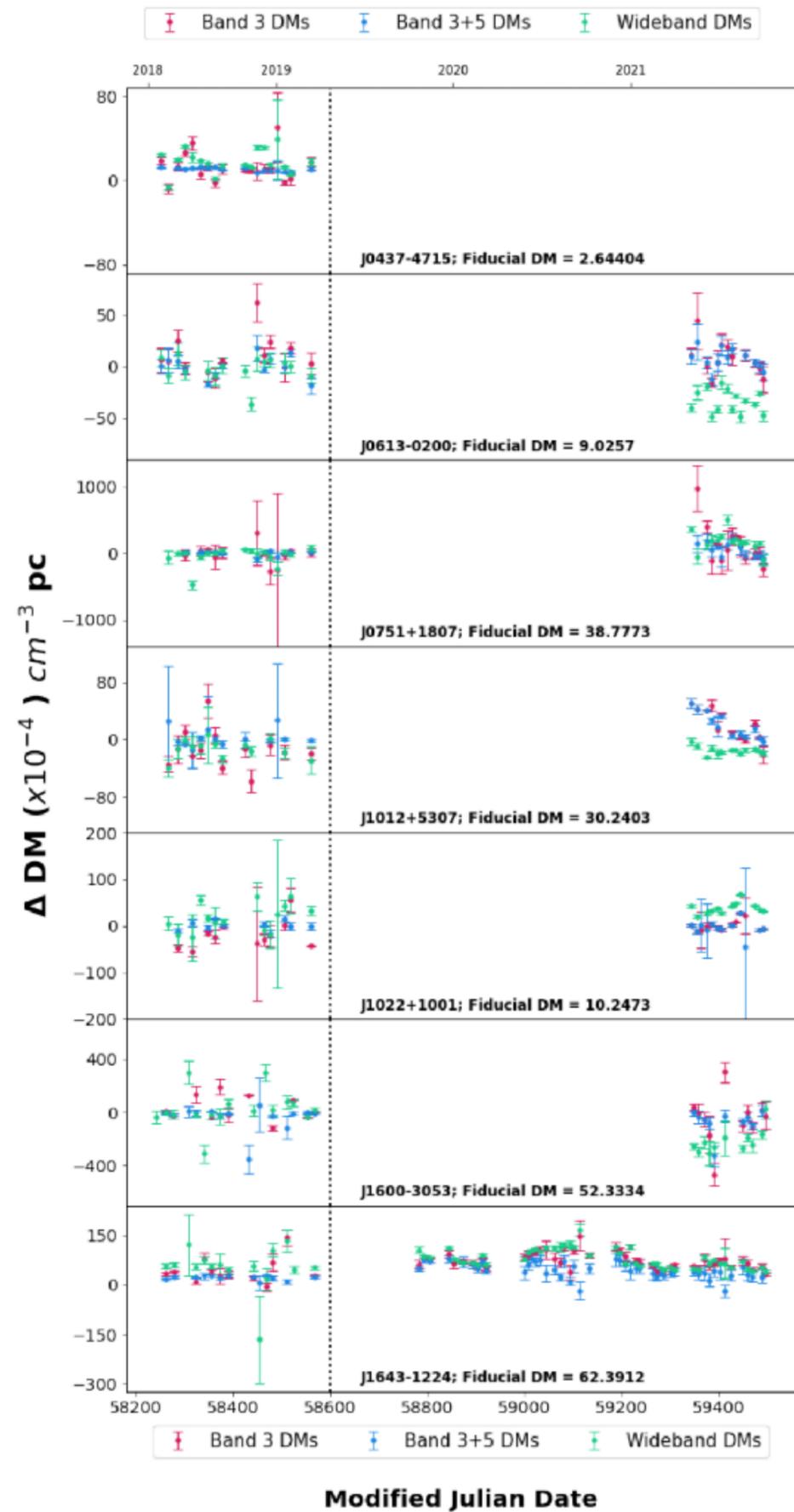
# Dispersion measure

$$\Delta t \simeq 4.15 \times 10^6 \text{ ms} \times (f_1^{-2} - f_2^{-2}) \times \text{DM.}$$

- ISM b/w Pulsar and Earth (ionised gas with free electrons) affect ToAs significantly at low-radio frequencies.
- This causes radio frequency dependent time delay.
- Low-radio frequency waves travel slower.
- Important to mitigate this —> **De-dispersion required.**
- Fitting timing residuals (Band3 + 5) gives us high-precision DM estimates —> **Unique strength of InPTA.**
- DM varies from epoch to epoch.

↓  
**Integrated electron column density**





## DM time series for 14 InPTA DR1 pulsars

Unprecedented DM precision

$$10^{-4} - 10^{-6} \text{ pc cm}^{-3}$$

Ref: P. Tarafdar, et al. 2022, PASA, 39, e053

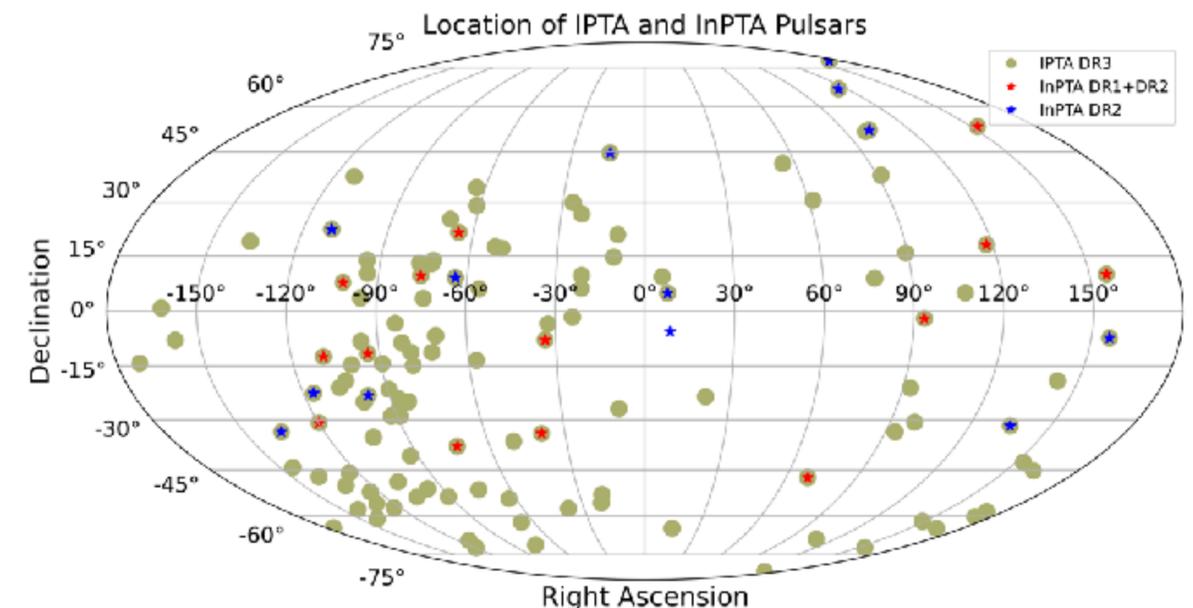
# Second data release of InPTA (InPTA-DR2)

14 InPTA-DR1 MSPs +

- J0030+0451
- J0034-0534
- J0645+5158
- J0740+6620
- J0900-3144
- J1024-0719
- J1125+7819
- J1455-3330
- J1614-2230
- J1640+2224
- J1730-2304
- J1944+0907
- J2302+4442

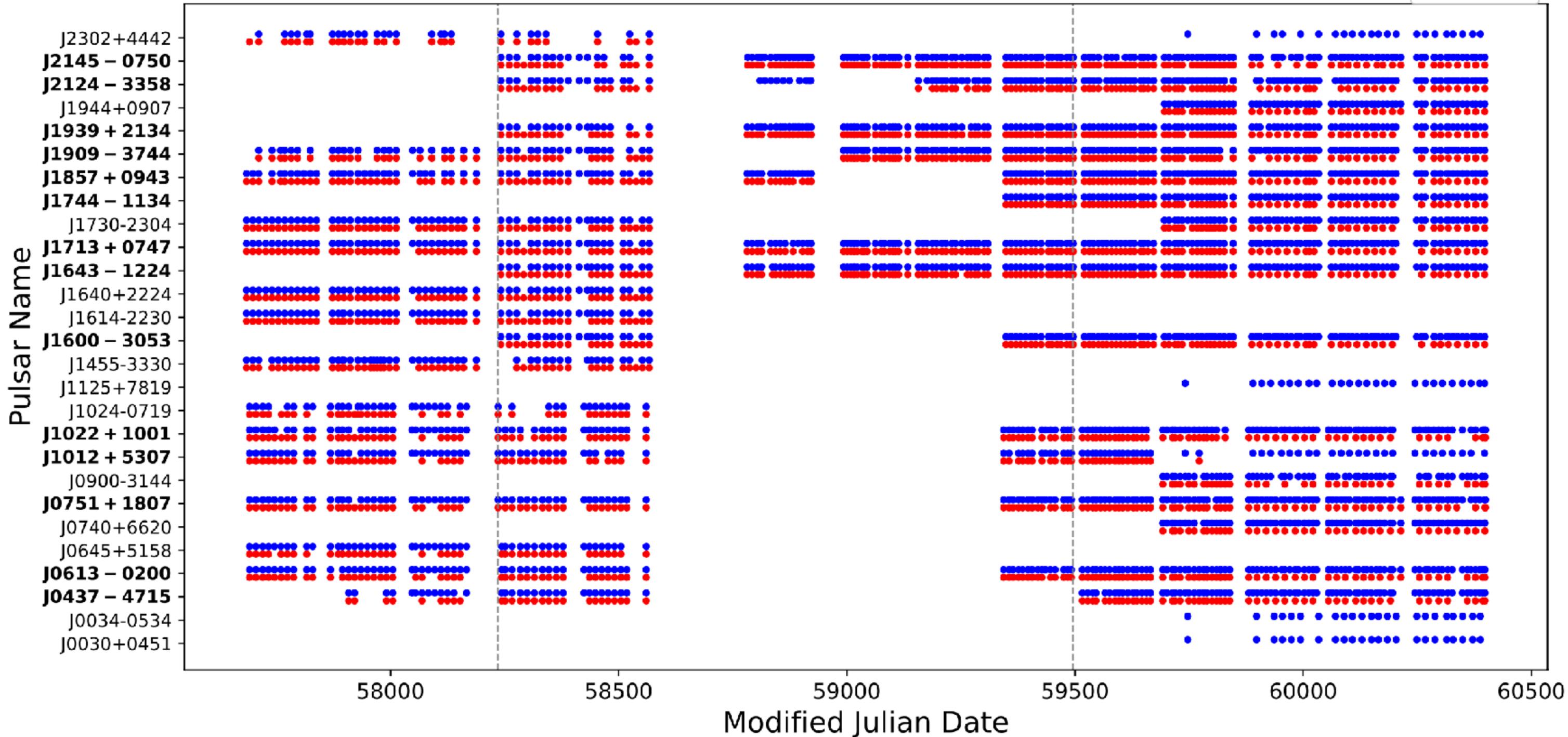
= Total 27 pulsars

- InPTA-DR2 includes dataset from early 2017 to early 2024.
- Includes concurrent observations of two sub-arrays: **Band 3 (300-500 MHz)** and **Band 5 (1260-1460 MHz)** of the uGMRT.
- High-precision Dispersion measure (DM) and Narrowband sub-banded Time-of-Arrivals (ToAs).
- Timing models obtained using ~ 7 years time baseline data.



Ref: P. Rana et al. 2025, under revision in PASA

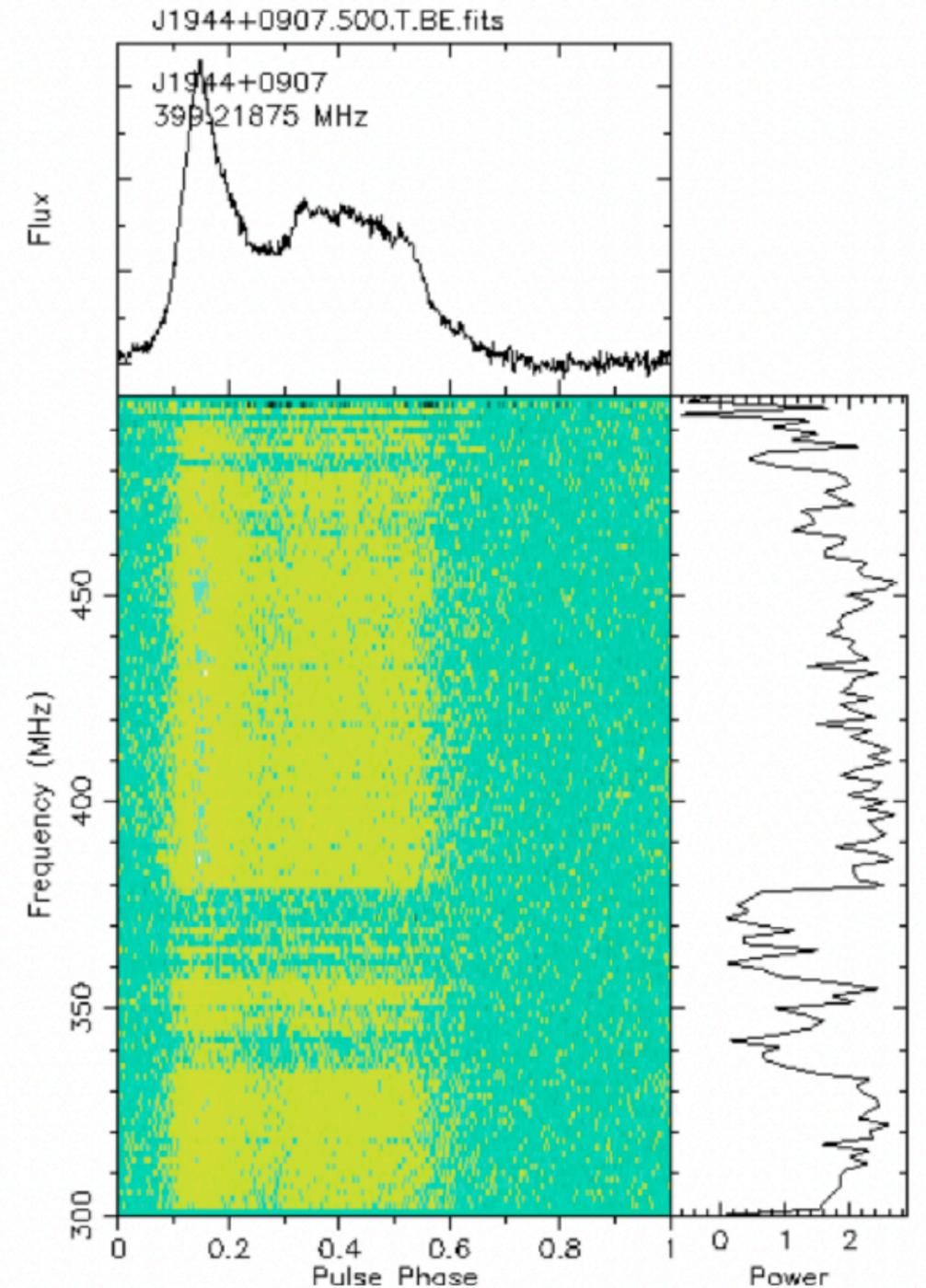
# Cadence Plot of Pulsar Observation

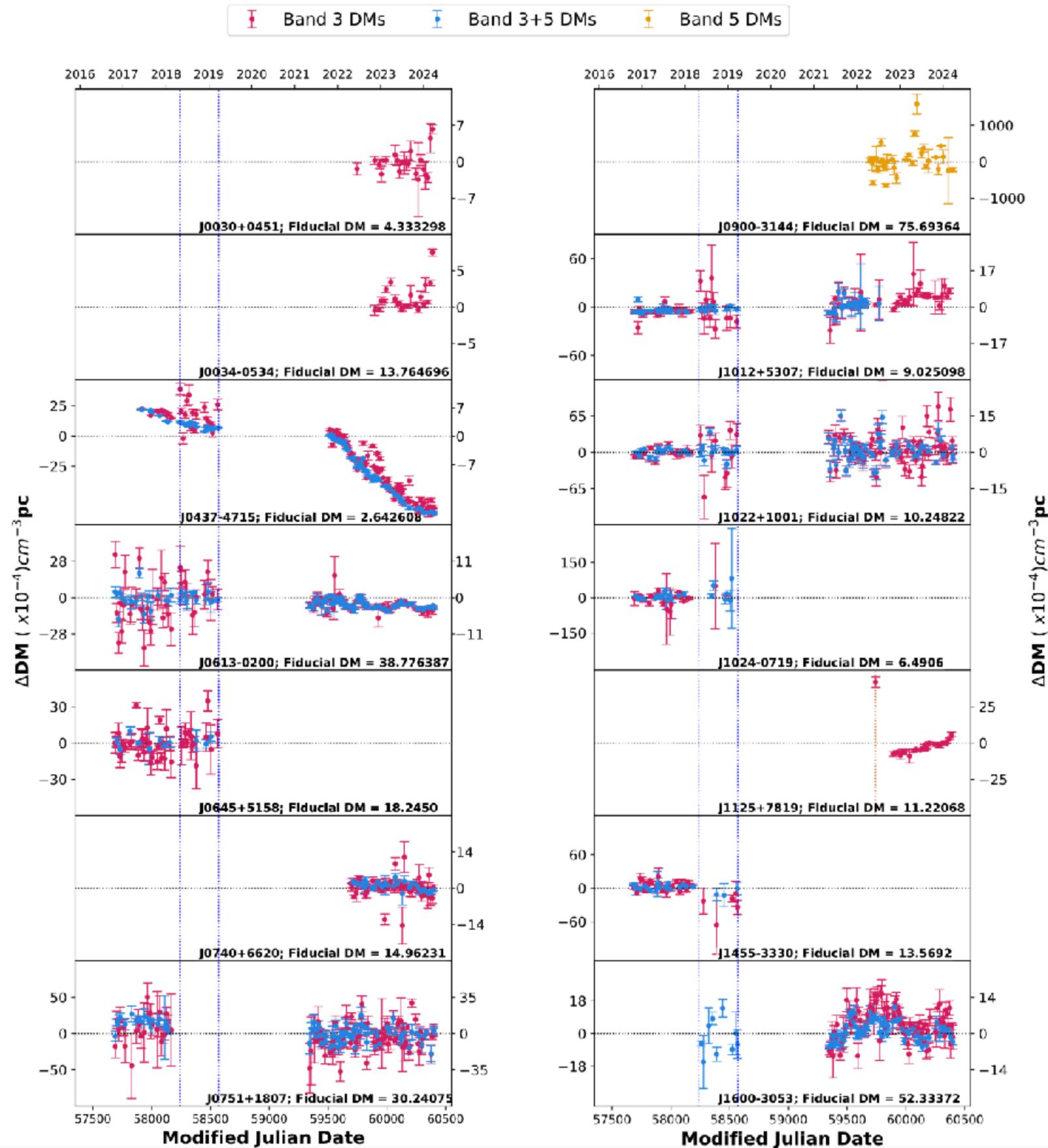


Ref: P. Rana et al. 2025, under revision in PASA

# Preparing initial ingredients

- Template profile:
  - A recorded pulse profile can be related to a standard profile called the template profile:  $p(t) = a + b s(t - \tau) + g(t)$
  - A pulse profile is cross-correlated with template in Fourier domain to obtain ToA.
  - We choose **high-SNR, simultaneous Band3+Band5 observation** to make a template.
- Optimising frequency sub-bands in each band:
  - Pulsar profile shape varies or evolves with radio frequency.
  - We carefully optimise frequency sub-bands used in each band to:
    - (i) retain enough S/N in each sub-band,
    - (ii) Minimal profile shape evolution in adjacent sub-bands.



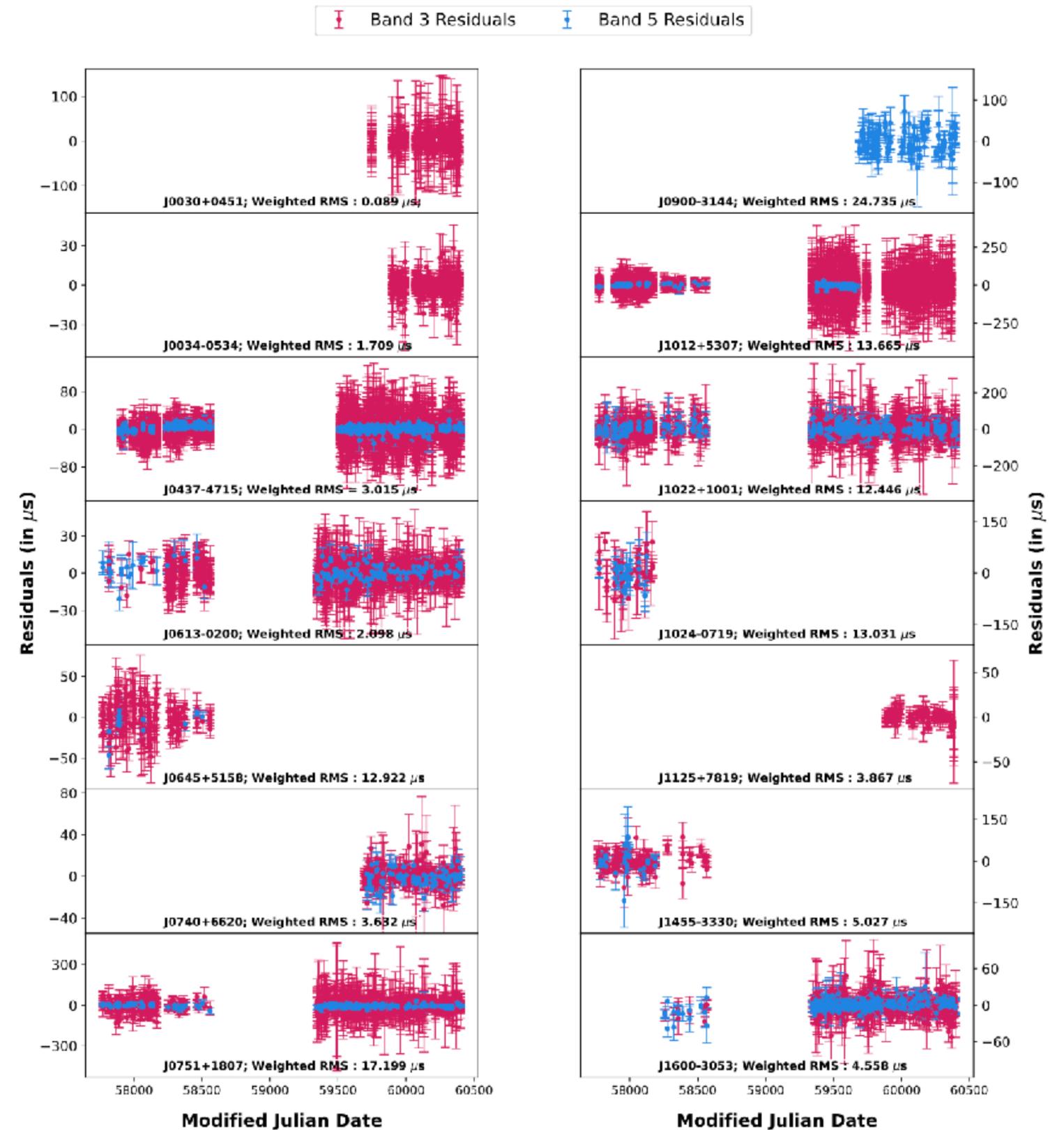


- InPTA-DR2 DM time series.
- PSRs J0034-0534, J0613-0200, show interesting annual trends due to solar winds.
- PSR J1125+7819 show interesting DM event having  $\Delta DM$  value  $\sim 0.0042 \text{ pc cm}^{-3}$

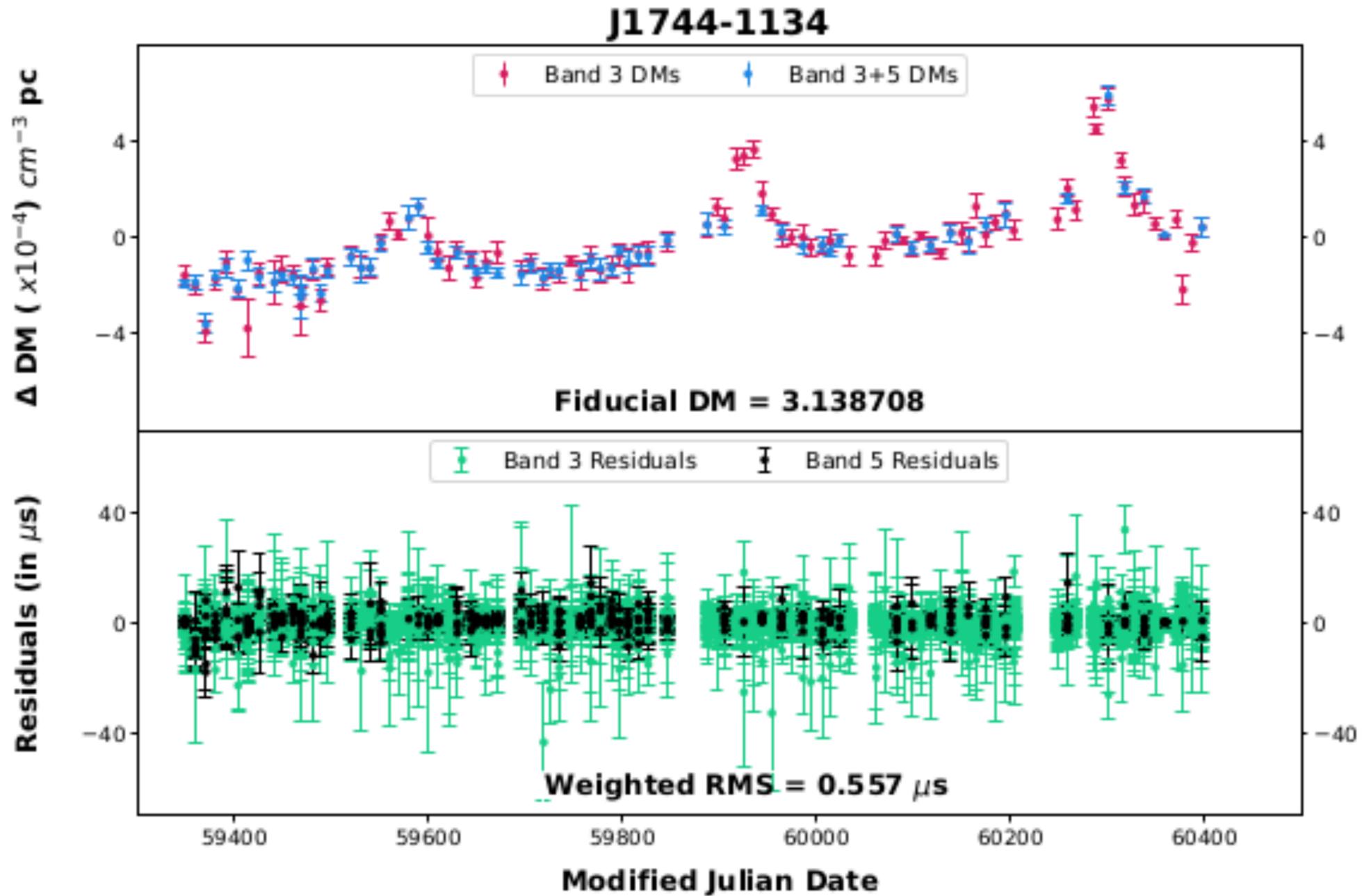
Ref: P. Rana et al. 2025, under revision in PASA

# Timing Analysis

- We use estimated DMs in timing analysis to mitigate ISM propagation delays.
- ToA outlier removal:
  - First, ToAs with  $> 100 \mu\text{s}$  ToA error are removed
  - Typically, ToAs with  $> 10 \times$  median ToA error are removed.
  - ToAs associated with RFI.
  - Timing model is obtained by fitting pulsar parameters in the order of their variation with the available time baseline.



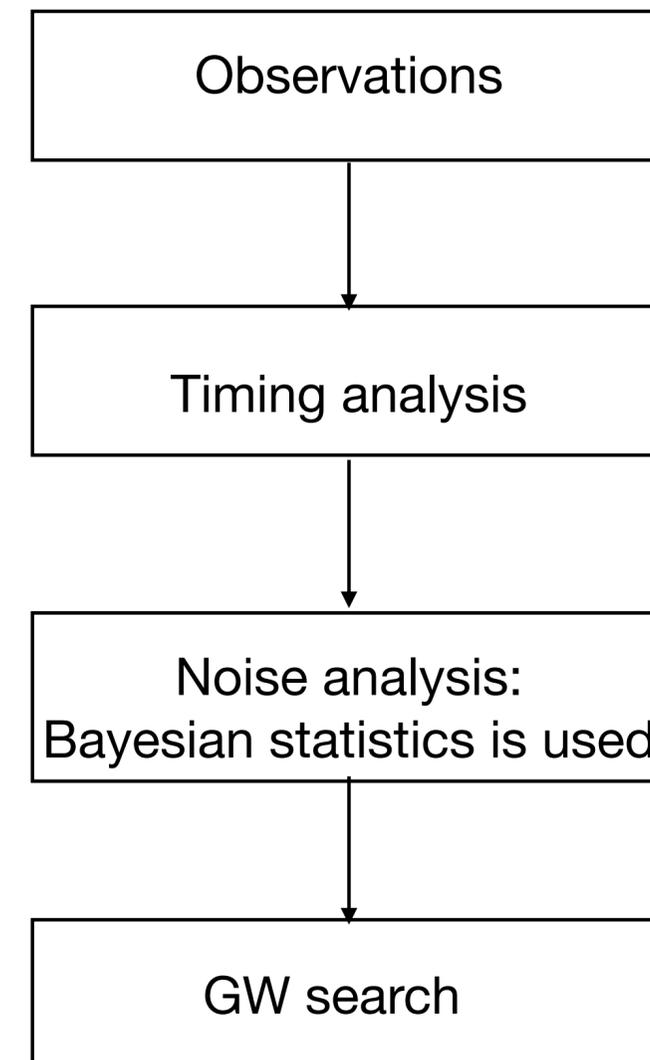
# Effect of Solar wind



# Noise analysis

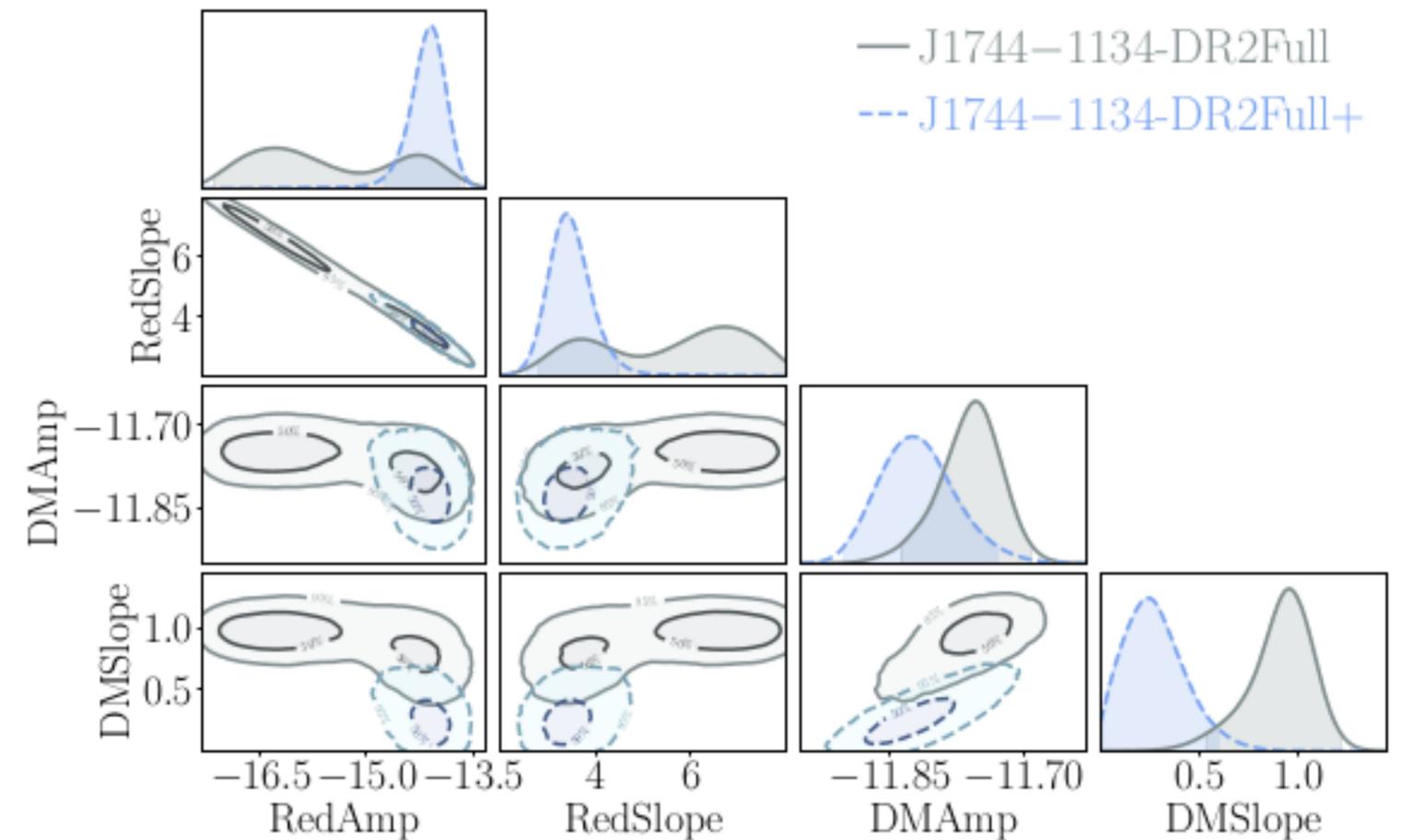
- **Four kinds of noises in PTA dataset:**
  - White noise: due to underestimation of ToA uncertainties.
  - Achromatic red noise: Intrinsic instabilities in the evolution of the pulsar's spin period, usually low-amplitude red-noise in MSPs. Modelled as a power-law.
  - Chromatic red noise: Radio-frequency dependent, non-deterministic variations in pulse arrival times. Modelled as a power-law. **In PTA data is of importance.**
  - Achromatic Correlated noise: GWs
- Careful modelling of first three is required to detect GWs. **Current assumption is that red-noise is a stationary-Gaussian process following power-law.**

## PTA data analysis

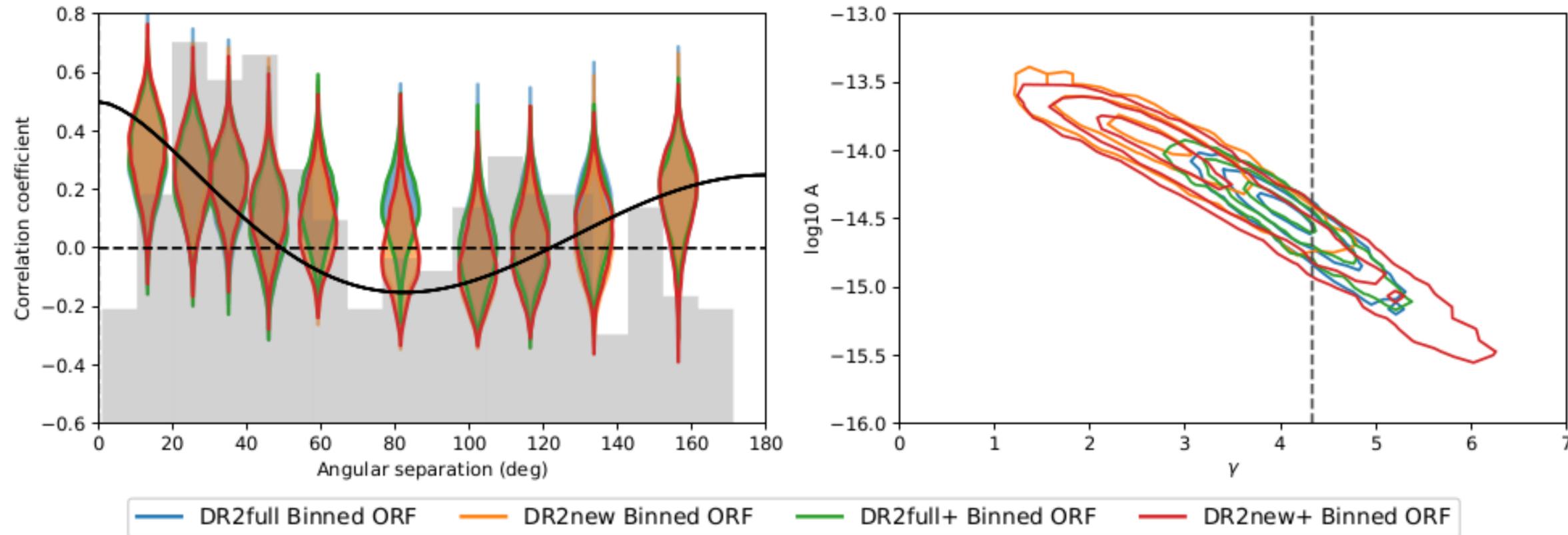


# Global efforts

- EPTA and InPTA joined hands in year 2023 to search GWB on [EPTA-DR2+InPTA-DR1](#)
- [EPTA-DR2 full](#) - 25 pulsars, 24.7 years of dataset.  
InPTA-DR1 - 10 common pulsars, 3.5 years of dataset.
- Addition of InPTA data showed better constraints on noise model parameters.
- Removing bimodality and putting tighter constraints in achromatic noise for PSR J1744-1134.



# EPTA+InPTA



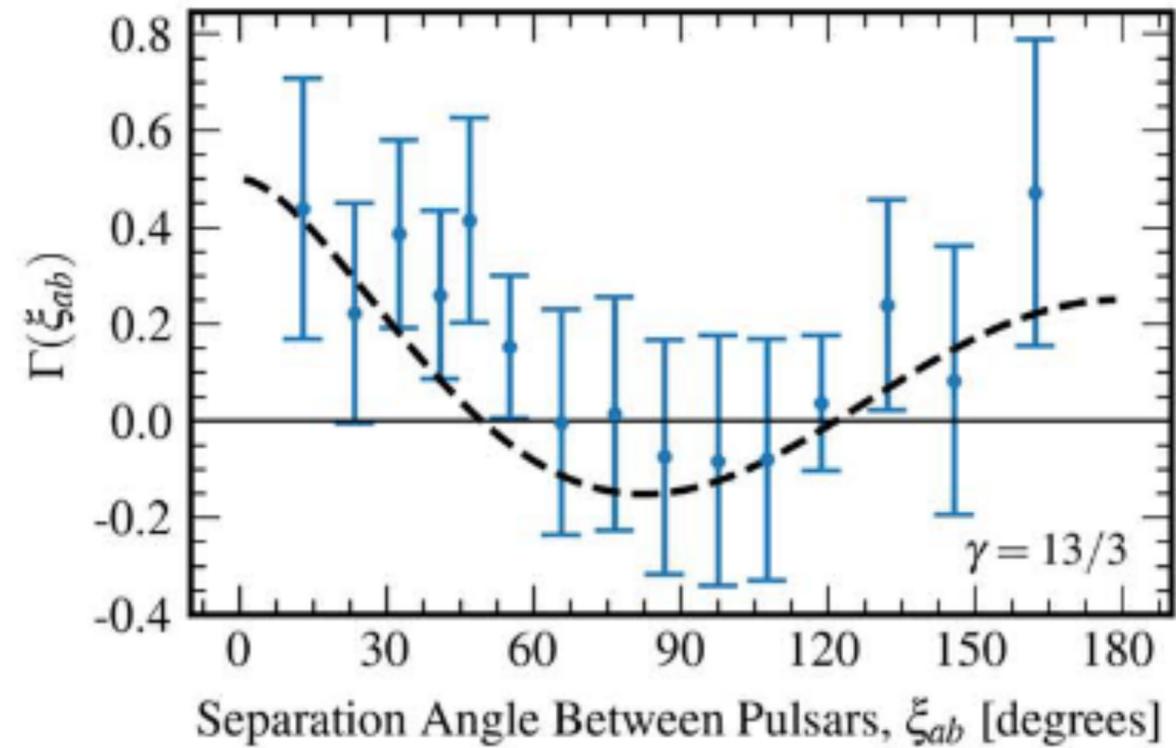
- EPTA DR2 full - 24.7 years of dataset.
- EPTA DR2 new - 10.3 years of dataset using new instrumentation.
- Implications: better instruments are required to constraint HD correlation.

Software + Model	DR2full+		DR2new+	
	log <sub>10</sub> A <sub>CRS</sub>	γ <sub>CRS</sub>	log <sub>10</sub> A <sub>CRS</sub>	γ <sub>CRS</sub>
ENTERPRISE + CURN	-14.44 <sup>+0.17</sup> <sub>-0.20</sub>	3.98 <sup>+0.40</sup> <sub>-0.37</sub>	-14.30 <sup>+0.33</sup> <sub>-0.52</sub>	3.53 <sup>+1.12</sup> <sub>-0.84</sub>
ENTERPRISE + GWB	-14.48 <sup>+0.18</sup> <sub>-0.20</sub>	4.06 <sup>+0.39</sup> <sub>-0.40</sub>	-14.10 <sup>+0.25</sup> <sub>-0.44</sub>	3.03 <sup>+1.02</sup> <sub>-0.67</sub>

90% credible regions for constraints on power-law parameters.

# NANOGrav

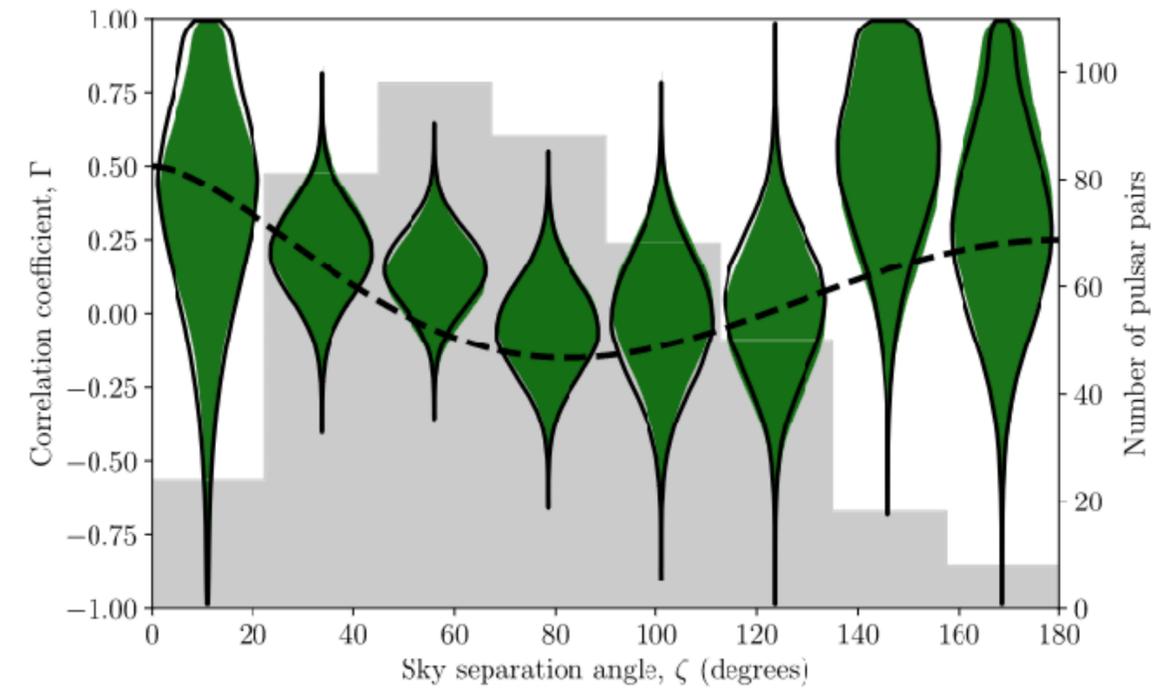
- 67 Pulsars
- 15 years of dataset



Ref: Gabriella Agazie et. al. 2023, ApJL, 951:L8 (24pp)

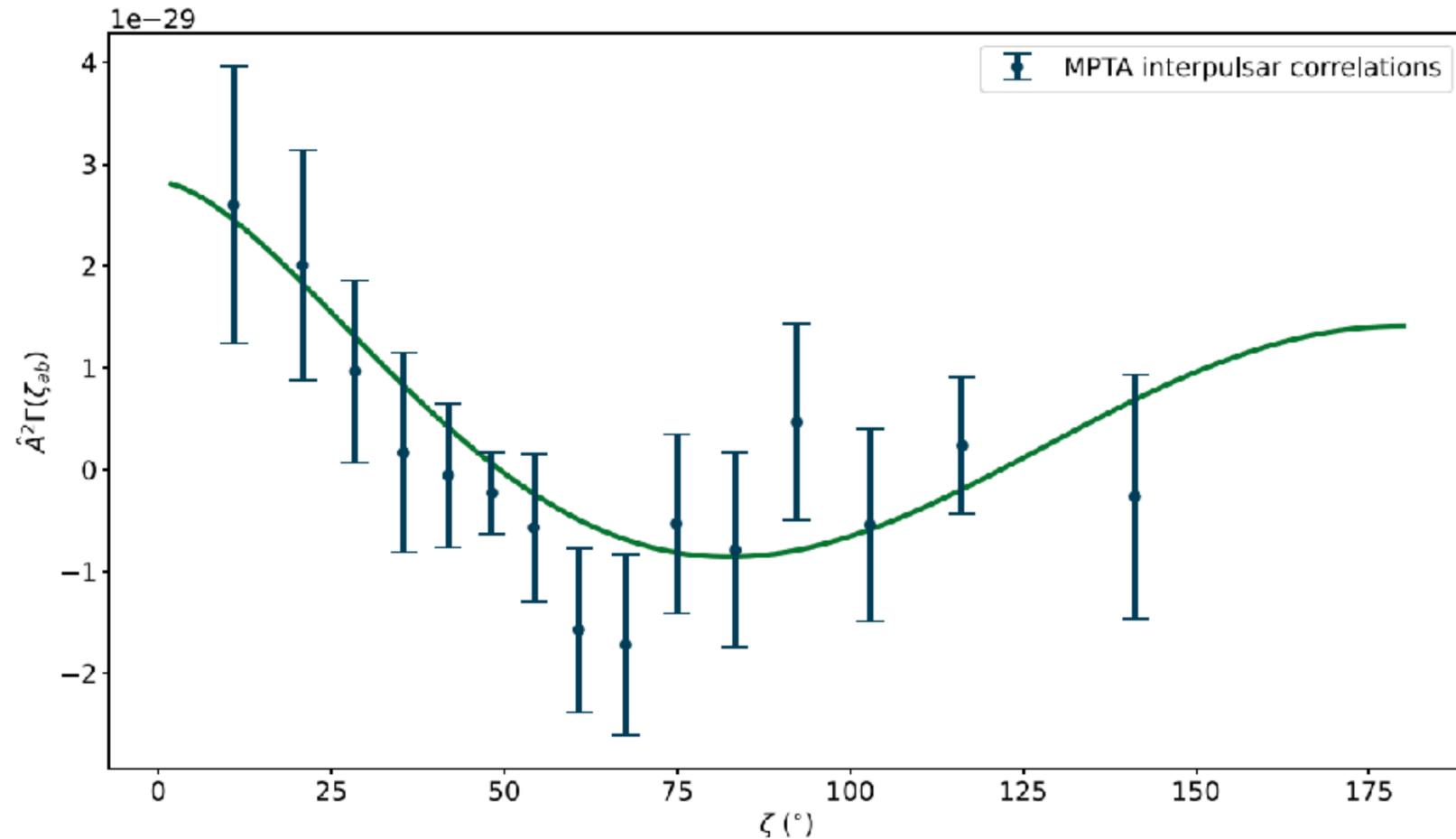
# PPTA

- 30 Pulsars
- 18 years dataset



Ref: Daniel J. Reardon et al 2023 ApJL 951 L6

# MeerKAT PTA



- MPTA-DR2 includes 4.5 years of data for 83 MSPs observed with MeerKAT telescope.
- They see 3 to 3.4 sigma significance for HD correlations which is shown to be highly dependent on the choice of noise models.
- $\log_{10}A_{HD} = -14.28^{+0.23}_{-0.3}$
- $\gamma_{HD} = 4.5^{+1.0}_{-0.93}$  (close to 13/3 expected from circular SMBHB)

Ref: Matthew T. Miles *et al* 2025 *MNRAS* 536, 1489–1500  
Matthew T. Miles *et al* 2025 *MNRAS* 536, 1467–1488

# Future directions

- InPTA is conducting advance noise modelling and GWB search on InPTA-DR2 dataset (*Nobleson et al. 2025 in prep.*; *Tahbilda et al. 2025 in prep.*).

- The International PTA (IPTA) is working on its DR3. Include ~120 pulsars including [InPTA](#), [EPTA](#), [PPTA](#), [NANOGrav](#), [MPTA](#) dataset with ~ 25 years of time baseline.

Signal-to-noise ratio of the GWB in the data

$$S/N \propto N \sqrt{T} \left( \frac{A \sqrt{C}}{\sigma} \right)^{3/13}$$

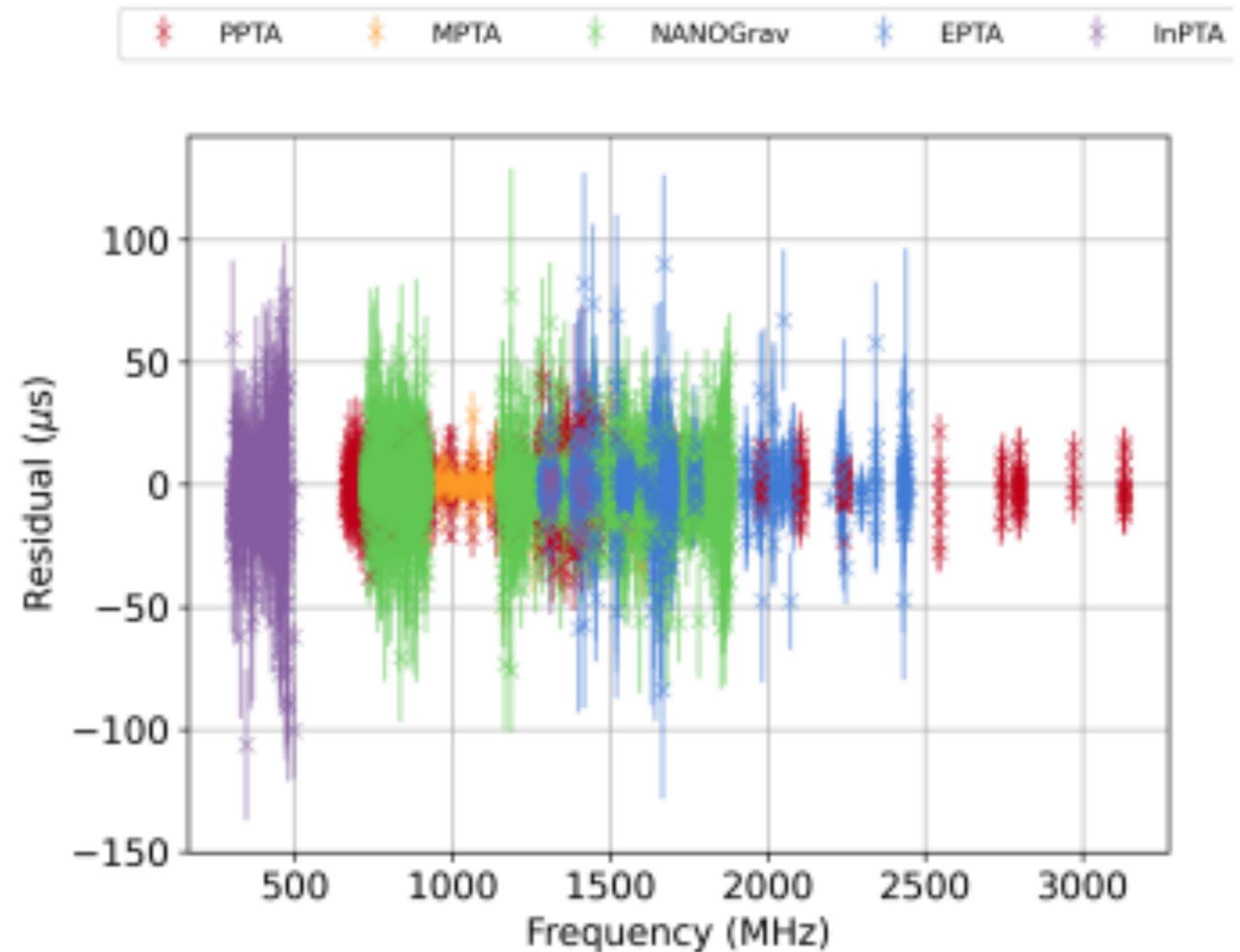
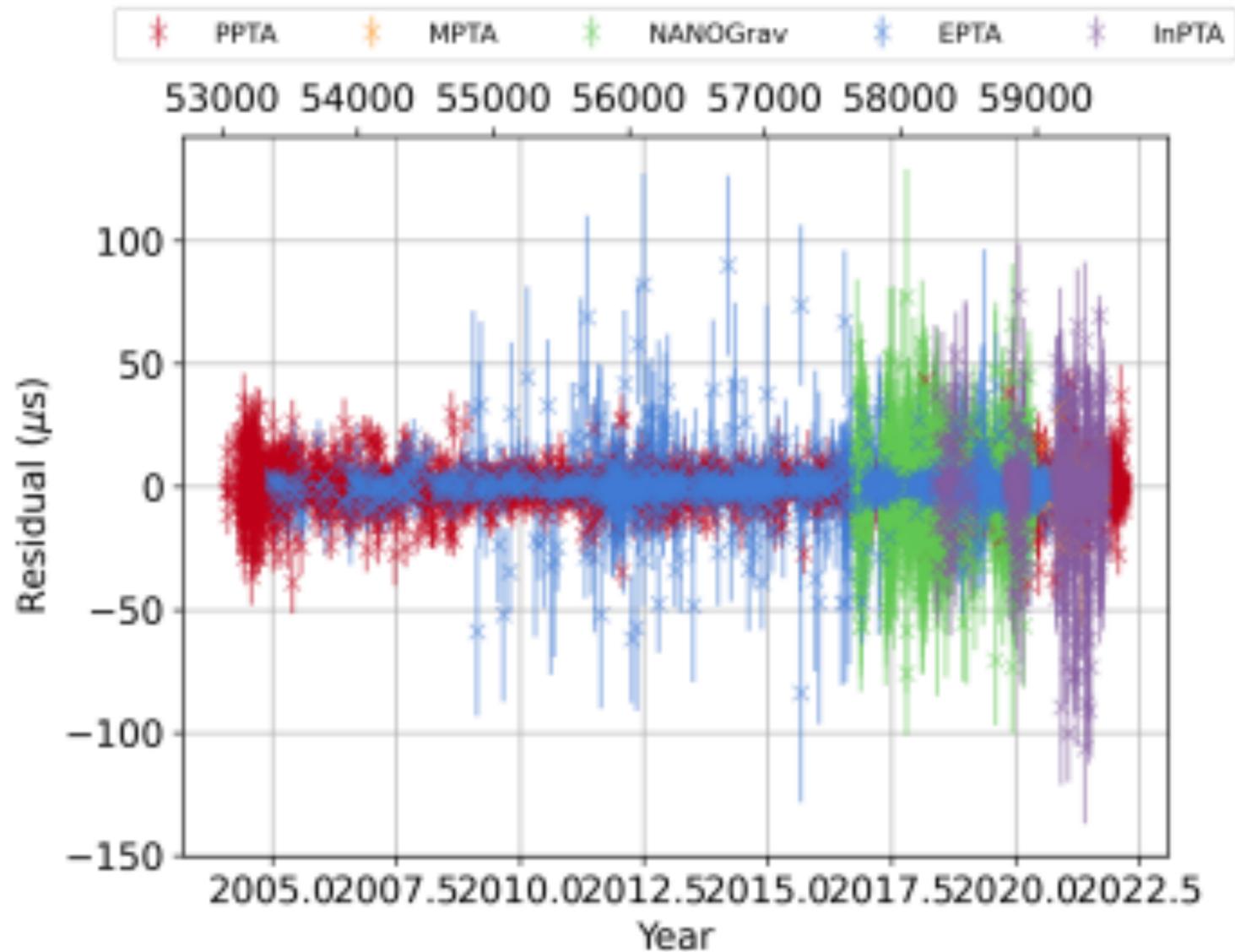
Ref: Siemens X et al, 2013  
Classical Quantum Gravity;  
30:224015

- The IPTA is expecting > 5 sigma detection of HD correlations of GWB (hopefully!).
- Efforts are on to search the first individual SMBHB in the IPTA data, transient signals like burst with memory.
- Anisotropies in GWB should give insights on galaxy SMBHB distribution.

# A glance at International PTA combined dataset

- PSR J2124-3358

*Preliminary*



**THANK YOU!**