Cosmological perturbation theory of compact sources in FLRW

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Based on 2601.01967 with Geoffrey Compère

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Overview of cosmological perturbation theory

- Cosmological perturbation theory is the building block of modern theoretical and observational cosmology.

- Will focus on linearized perturbations.

- LHS of Linearized Einstein equation is gauge invariant for maximally symmetric background, but not for FLRW.

$$\delta g_{\mu\nu} \to \delta g_{\mu\nu} + \mathcal{L}_\xi \bar{g}_{\mu\nu} \quad ; \quad \delta G_{\mu\nu} \to \delta G_{\mu\nu} + \mathcal{L}_\xi \bar{G}_{\mu\nu}$$

- Standard cosmological perturbation theory constructs gauge-invariant variables.

Lifshitz, Khalantikof - 1946-63

Hawking, Olson - 1966, 76

James M. Bardeen - 1980

- Einstein equations can be written in gauge-invariant variables.

Overview of cosmological perturbation theory

- The perturbed spatially flat FLRW metric -

$$dS^2 = a^2[-(1+2A)d\eta^2 + 2N_i d\eta dx^i + (\delta_{ij} + h_{ij})dx^i dx^j]$$

- The perturbations can be split into

$$\begin{aligned} N_i &= \partial_i B + B_i, \\ h_{ij} &= 2C\delta_{ij} + 2\partial_{\langle i}\partial_{j\rangle}E + 2\partial_{(i}E_{j)} + 2h_{ij}^{\mathrm{TT}}. \end{aligned}$$

with

$$\partial^i E_i = 0 = \partial^i B_i, \quad \partial^i h_{ij}^{\mathrm{TT}} = 0 = \delta^{ij} h_{ij}^{TT}$$

- 10 dof of metric is decomposed into 4+4+2 SVT dof.

Downsides of SVT decomposition

- SVT decomposition is non-local in position space! SVT is more convenient in decomposing perturbation in Fourier modes.

- Fourier domain is convenient for the study of the fundamental stochastic GW background.

- Given field variables, it is not straight forward to construct SVT variables. One needs to solve Poisson equations.

- It is not also straightforward to reverse engineer the gauge-independent formalism to a particular coordinate (harmonic or Bondi-Sachs)!

R. Brandenberger, R. Khan, W. H. Press, Phys Rev D 28, 8, 1983.

- SVT decomposition obscures the relation between gravitational field and multipolar structure of the individual compact source.

Questions...

- Relation of linearized gravitational fields in terms of multipolar structure of individual compact sources in FLRW?

  In Minkowski background this has been achieved in PN/PM methods in position space using STF tensors.

- Position space decoupled EOM in FLRW?

- Definition of compact sources in FLRW geometry?

- Definition of multipole moments in FLRW?

A detour to linearized gravity in flat case

Linearized inhomogeneous wave equation

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad \tilde{h}_{\mu\nu} := h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h$$

$$\Box\tilde{h}_{\mu\nu} = -16\pi T_{\mu\nu}$$ with gauge, $$\partial^{\mu}\tilde{h}_{\mu\nu} = 0$$

Solve in Green's function

$$\tilde{h}_{ij} = 4G\int d^3x' \frac{1}{|x-x'|}T_{ij}(t-|x-x'|, x')$$

Relate source integral in terms of source moments

$$\int d^3x' T_{ij} = \frac{1}{2}\ddot{Q}_{ij}$$

Field in terms of quadrupole moments

$$h_{ij} = \frac{2G}{r}\ddot{Q}_{ij}$$

Background FLRW geometry

- Spatially flat FLRW background in conformal coordinates

$$\bar{g}_{\mu\nu}dx^{\mu}dx^{\nu} = a^2(\eta)(-d\eta^2 + \delta_{ij}dx^i dx^j). , \quad a d\eta = dt$$

- Decelaration parameter: $q = 1 - \dfrac{a\ddot{a}}{\dot{a}^2}$

- FLRW equations:

$$\begin{aligned}\left(\frac{\dot{a}}{a}\right)^2 - \frac{\Lambda}{3}a^2 &= \frac{8\pi G}{3}a^2\epsilon, \\ q\left(\frac{\dot{a}}{a}\right)^2 + \frac{\Lambda}{3}a^2 &= \frac{4\pi G}{3}a^2(\epsilon + 3p).\end{aligned}$$

- Continuity equation: $\dot{\epsilon} + \dfrac{3\dot{a}}{a}(\epsilon + p) = 0.$

Linearized Einstein equation

- Define: $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}, \ \tilde{h}_{\mu\nu} := h_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}h \quad , \quad h := h_{\alpha\beta}\bar{g}^{\alpha\beta}$

EOM:
$$\frac{1}{2}(-\bar{\Box}\tilde{h}_{\mu\nu} + \bar{\nabla}_\mu B_\nu + \bar{\nabla}_\nu B_\mu - \bar{g}_{\mu\nu}(\bar{\nabla}^\alpha B_\alpha)) + \bar{R}_{\mu\alpha\beta\nu}\tilde{h}^{\alpha\beta} - \frac{\bar{R}}{2}\tilde{h}_{\mu\nu}$$
$$+ \frac{1}{2}(\bar{R}_{\mu\alpha}\tilde{h}^\alpha{}_\nu + \bar{R}_{\nu\alpha}\tilde{h}^\alpha{}_\mu + \bar{g}_{\mu\nu}\bar{R}_{\alpha\beta}\tilde{h}^{\alpha\beta}) + \Lambda(\tilde{h}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\tilde{h}) = 8\pi G\delta T_{\mu\nu}.$$

- Linearized equation in FLRW background -

$$\Box\tilde{h}_{\mu\nu} + 2\left(\frac{\dot{a}}{a}\right)\partial_0\tilde{h}_{\mu\nu} - 2\left(\frac{\dot{a}}{a}\right)^2\Bigg\{ - q\delta^0_\mu\delta^0_\nu\tilde{h}_\alpha{}^\alpha - (1-3q)\tilde{h}_{\mu\nu} - q(\delta^0_\mu\tilde{h}_{0\nu} + \delta^0_\nu\tilde{h}_{0\mu})$$
$$+\eta_{\mu\nu}\tilde{h}_{00}(1+q) + \eta_{\mu\nu}\tilde{h}_\alpha{}^\alpha(1-\frac{q}{2})\Bigg\} - 2\Lambda a^2(\tilde{h}_{\mu\nu} - \frac{\eta_{\mu\nu}}{2}\tilde{h}_\alpha{}^\alpha) = -16\pi G a^2\delta T_{\mu\nu},$$

- Generalized harmonic gauge - $\quad B_\mu := \bar{\nabla}_\alpha\tilde{h}^\alpha{}_\mu = -\frac{2\dot{a}}{a^3}\tilde{h}_{0\mu}$

Linearized Einstein equation

- Generalized harmonic gauge $B_\mu := \bar{\nabla}_\alpha \tilde{h}^\alpha{}_\mu = -\frac{2\dot{a}}{a^3}\tilde{h}_{0\mu}$ is crucial for decoupling the equations.

- Generalization of the gauge in de Sitter case: $B_\mu = \frac{2\Lambda}{3}\eta\tilde{h}_{0\mu}$

H. J. De Vega, J. Ramirez, N . G. Sanchez - 1999

A. Ashtekar, B. Bonga, A. Kesavan - 2015

G. Date, J. Hoque - 2015

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- Define: $\tilde{\chi}_{\mu\nu} = a^{-1}\tilde{h}_{\mu\nu}$

$$\Box\tilde{\chi}_{\mu\nu} - 2\left(\frac{\dot{a}}{a}\right)^2\Big\{ - q\delta^0_\mu\delta^0_\nu\tilde{\chi}_\alpha{}^\alpha + \frac{1}{2}(5q-3)\tilde{\chi}_{\mu\nu} - q(\delta^0_\mu\tilde{\chi}_{0\nu} + \delta^0_\nu\tilde{\chi}_{0\mu})$$

$$+\eta_{\mu\nu}\tilde{\chi}_{00}(1+q) + \eta_{\mu\nu}\tilde{\chi}_\alpha{}^\alpha(1-\frac{q}{2})\Big\} - 2\Lambda a^2(\tilde{\chi}_{\mu\nu} - \frac{\eta_{\mu\nu}}{2}\tilde{\chi}_\alpha{}^\alpha) = -16\pi G a\delta T_{\mu\nu}.$$

Decoupled linearized equation

- The spatial trace free part and mixed $0i$ parts decouple

$$\Box\tilde{\chi}_{\langle ij\rangle} + \left(-2\Lambda a^2 + (3-5q)\frac{\dot{a}^2}{a^2}\right)\tilde{\chi}_{\langle ij\rangle} = -16\pi G a\delta T_{\langle ij\rangle},$$

$$\Box\tilde{\chi}_{0i} + \left(-2\Lambda a^2 + 3(1-q)\frac{\dot{a}^2}{a^2}\right)\tilde{\chi}_{0i} = -16\pi G a\delta T_{0i}.$$

- Scalar equations remain coupled in general

$$\Box\begin{pmatrix}\tilde{\chi}_{00}\\ \tilde{\chi}_{ii}\end{pmatrix} + \begin{pmatrix}-\Lambda a^2 + \frac{3\dot{a}^2}{a^2} & -\Lambda a^2 + \frac{\dot{a}^2}{a^2}(2+q)\\ -3\Lambda a^2 - \frac{9\dot{a}^2}{a^2}q & \Lambda a^2 - \frac{\dot{a}^2}{a^2}(3+2q)\end{pmatrix}\begin{pmatrix}\tilde{\chi}_{00}\\ \tilde{\chi}_{ii}\end{pmatrix} = -16\pi G a\begin{pmatrix}\delta T_{00}\\ \delta T_{ii}\end{pmatrix}.$$

- We will further decouple these equations.

Conservation of linear stress-energy in FLRW

- Before decoupling the equations further, let us look into the conservation equations.

- Conservation equations $\nabla^\mu T_{\mu\nu} = 0$, to the linear order

$$\bar{\nabla}^\mu \delta T_{\mu\nu} - \bar{g}^{\mu\rho}(\delta\Gamma^\lambda{}_{\rho\nu}\bar{T}_{\lambda\mu} + \delta\Gamma^\lambda{}_{\rho\mu}\bar{T}_{\lambda\nu}) - h^{\mu\rho}\bar{\nabla}_\rho \bar{T}_{\mu\nu} = 0.$$

- Linearized stress-energy tensor is not conserved by its own unlike maximally symmetric background!

Compact sources in FLRW!

- In generalized harmonic gauge, conservation equations become

$$\partial_\eta(a\delta\tilde{T}_{00}) - \partial_i(a\delta\tilde{T}_{0i}) + \dot{a}\delta\tilde{T}_{kk} = -\frac{1}{4}\partial_\eta(a^2(p+\epsilon))(\tilde{\chi}_{00}+\tilde{\chi}_{kk}),$$

$$(\partial_\eta + \frac{\dot{a}}{a})(a\delta\tilde{T}_{0i}) - \partial_j(a\delta\tilde{T}_{ij}) = 0.$$

with effective stress-energy components

$$\delta\tilde{T}_{00} \equiv \delta T_{00} + a\frac{\epsilon - p}{4}(\tilde{\chi}_{00}+\tilde{\chi}_{kk}),$$

$$\delta\tilde{T}_{0i} \equiv \delta T_{0i} - pa\tilde{\chi}_{0i},$$

$$\delta\tilde{T}_{ij} \equiv \delta T_{ij} - pa\tilde{\chi}_{ij} - \delta_{ij}a\left(\frac{\epsilon - p}{4}\tilde{\chi}_{kk} + \frac{\epsilon + 3p}{4}\tilde{\chi}_{00}\right).$$

Compact sources in FLRW!

- The concept of compact sources, $\delta T_{\mu\nu} = 0$, for Minkowski or de Sitter background does not go through for FLRW!

- Nearly compact sources - $$\delta \tilde{T}_{0i}(x) = 0 = \delta \tilde{T}_{ij}(x)$$

  outside a localized volume V,

  but $$\delta \tilde{T}_{00}(x) \neq 0$$

- The choice of $\delta \tilde{T}_{00}(x) = 0$ is inconsistent with generalized harmonic gauge.

Decoupled linearized equation

- Given the effective stress-energy tensor, we will write down the EOM.

$$
\begin{aligned}
\mathcal{O}^- \tilde{\chi}_{\langle ij \rangle} &= -16\pi G a \delta \tilde{T}_{\langle ij \rangle}, \\
\mathcal{O}^+ \tilde{\chi}_{0i} &= -16\pi G a \delta \tilde{T}_{0i}, \\
\mathcal{O}^+ (\tilde{\chi}_{00} + \tilde{\chi}_{ii}) &= -16\pi G a (\delta \tilde{T}_{00} + \delta \tilde{T}_{ii}), \\
\mathcal{O}^- \tilde{\chi}_{ii} &= -16\pi G a \delta \tilde{T}_{ii}.
\end{aligned}
$$

where operators $\mathcal{O}^\pm \equiv \Box + \left(\frac{\dot{a}}{a}\right)^2 (1 \pm q).$

- Decoupling of inhomogeneous perturbation equations in generalized harmonic gauge without SVT decomposition.

Linearized equation for power-law cosmologies

- For power-law cosmologies: $a(\eta) \sim |\eta|^{\alpha}$; $\dfrac{\dot{a}}{a} = \dfrac{\alpha}{\eta}, q = \dfrac{1}{\alpha}$

- EOM for power-law cosmologies:

$$(\Box + \frac{\alpha(\alpha-1)}{\eta^2})\tilde{\chi}_{\langle ij\rangle} = -16\pi G a \delta\tilde{T}_{\langle ij\rangle},$$

$$(\Box + \frac{\alpha(\alpha+1)}{\eta^2})\chi_{0i} = -16\pi G a \delta\tilde{T}_{0i},$$

$$(\Box + \frac{\alpha(\alpha+1)}{\eta^2})(\tilde{\chi}_{00} + \tilde{\chi}_{ii}) = -16\pi a(\delta\tilde{T}_{00} + \delta\tilde{T}_{ii}),$$

$$(\Box + \frac{\alpha(\alpha-1)}{\eta^2})\tilde{\chi}_{ii} = -16\pi G a \delta\tilde{T}_{ii}.$$

- For RD and MD universe $\alpha = 1, 2$ respectively.

Hadamard ansatz

- Can we solve the inhomogeneous wave equation in terms of Green's function for power-law cosmologies?

- For power-law cosmologies, wave equations have a generic form

$$\left(\Box + \frac{C}{\eta^2}\right)\psi(x) = -4\pi\mu(x).$$

$$\Longrightarrow \psi(x) = \int G_+(x,x')\mu(x')\sqrt{-g'}d^4x'.$$

- Use the Hadamard construction of Green's function:

$$G_+(x,x') = U(x,x')\delta_+(\sigma) + V(x,x')\theta_+(-\sigma).$$

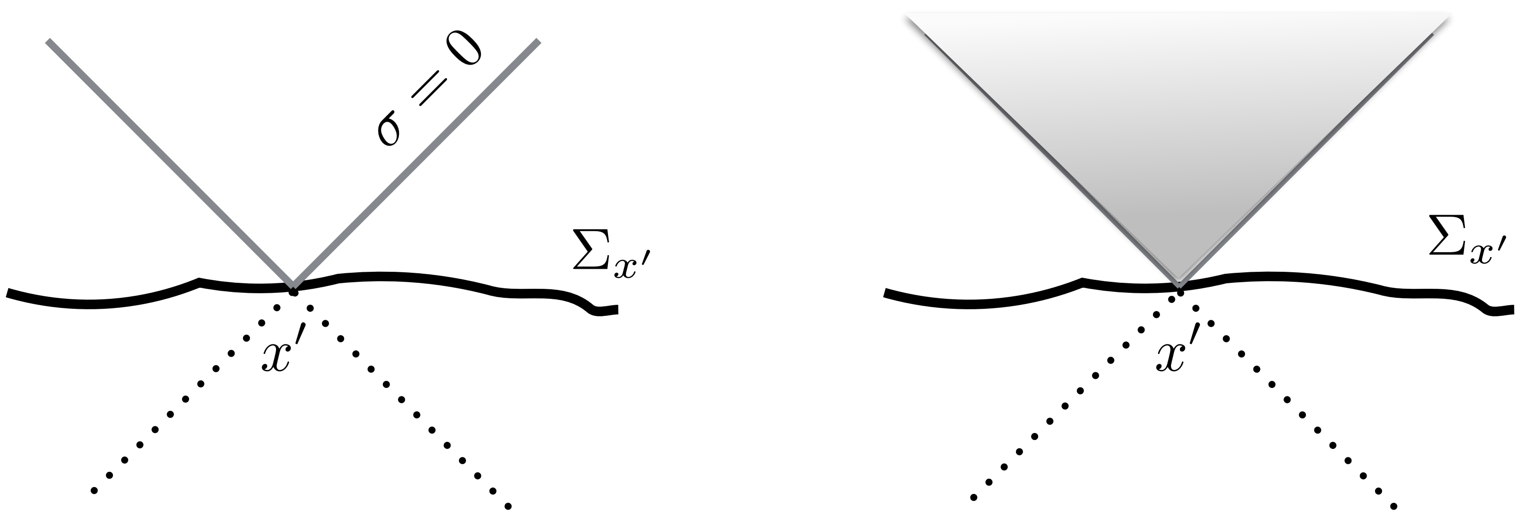
σ - half of the geodesic distance squared between x' and x .

- This ansatz is generic for wave equation in curved background.

Solution of Hadamard ansatz

- Plug the Hadamard ansatz in the wave equation, and solve for U and V.

- For Minkowski wave-operator $U(x, x') = 1;\ V(x, x')$ depends on potential term.

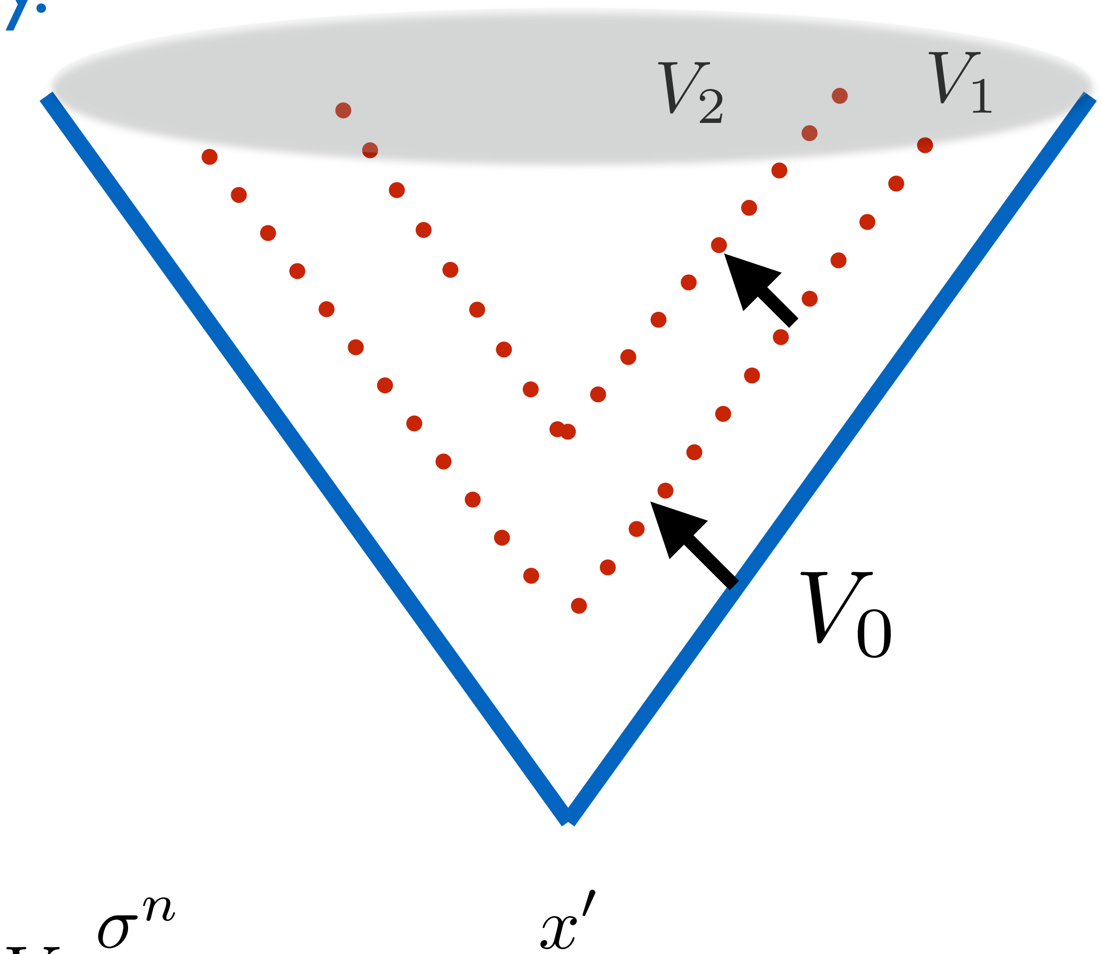


- V - signature of tail term. Field not only depends on the response of the source at retarded time but on the entire past history.

Solution of V

- V is the solution of the characteristic initial value problem which satisfies both equations simultaneously.

$$\begin{aligned}\lambda\frac{d\check{V}}{d\lambda} + \check{V} - \frac{C}{2\eta^2} &= 0,\\ (\Box + \frac{C}{\eta^2})V(x,x') &= 0.\end{aligned}$$



- The problem can be solved by a series -

$$V = \sum_{n=0}^{\infty} V_n \frac{\sigma^n}{n!}.$$

Thanks to Abraham Harte

Book - F. G. Friedlandler

- One obtains a transport equation for V -

$$(\partial^\alpha V_n)(\partial_\alpha \sigma) + (n+1) = -\frac{1}{2}P(V_{n-1})$$

Solution of $\mathbf{V}$

- A generic solution for V_n

$$V_n = -\left(-\frac{1}{2\eta'\eta}\right)^{n+1} \prod_{i=0}^{n} \frac{C - i(i+1)}{i+1}.$$

- This series solution of V can also expressed in terms of hypergeometric function.

- With $C = \alpha(\alpha \pm 1)$, green's function for power-law cosmologies -

$$G_+(x, x') = \delta_+(\sigma) + \frac{\alpha(\alpha \pm 1)}{2\eta\eta'}\ {}_2F_1\big(2 \pm \alpha, 1 \mp \alpha; 2; \frac{\sigma}{2\eta\eta'}\big)\theta_+(-\sigma).$$

Tensor inhomogeneous solution

$$\left(\Box + \frac{\alpha(\alpha-1)}{\eta^2}\right)\tilde{\chi}_{\langle ij\rangle} = -16\pi a\delta\tilde{T}_{\langle ij\rangle}.$$

$$\tilde{\chi}_{\langle ij\rangle} = 4G\int d^3x' \frac{a\delta T_{\langle ij\rangle}(\eta - |\bar{x}-\bar{x}'|, \bar{x}')}{|\bar{x}-\bar{x}'|} + \sum_{n=0}^{\infty}\int d^3x' \int_{\eta_i}^{\eta-|\bar{x}-\bar{x}'|} d\eta' \frac{V_n\sigma^n}{n!} a\delta T_{\langle ij\rangle}(\eta', \bar{x}').$$

- First term is light-cone term, second term is tail term.

$$\int_{\eta_i}^{\eta-|\bar{x}-\bar{x}'|} f(\eta')d\eta' = \int_{\eta_i}^{\eta-\rho} f(\eta')d\eta' + \int_0^{\rho-|\bar{x}-\bar{x}'|} f(\eta'+\eta_{\rm ret})d\eta',$$
$$:= T_1 + T_2$$

- T_2 generates instantaneous tail term, while T_1 in general depends on entire past history of the source.

Source moments

- Having discussed the wave equation in FLRW, we need two more things to write source integrals in terms of moment.
  - Source moments
  - Conservation of stress-energy tensor in FLRW background

Mass moment: $$Q_L^{(\rho)}(\eta) := \int d^3\bar{x}\delta\tilde{T}_{\bar{0}\bar{0}}\bar{x}_L = \int d^3x a^{\ell+1}(\eta)\delta\tilde{T}_{00}x_L,$$

Momentum moment: $$P_{i|L}(\eta) := \int d^3\bar{x}\delta\tilde{T}_{\bar{0}\bar{i}}\bar{x}_L = \int d^3x a^{\ell+1}(\eta)\delta\tilde{T}_{0i}x_L,$$

Stress moment: $$S_{ij|L}(\eta) := \int d^3\bar{x}\delta\tilde{T}_{\bar{i}\bar{j}}\bar{x}_L = \int d^3x a^{\ell+1}(\eta)\delta\tilde{T}_{ij}x_L.$$

$L = i_1 i_2 \cdots i_\ell$ - multi-index, $x_L = x_{i_1}x_{i_2}\cdots x_{i_\ell}$

Pressure moment: $$Q_L^{(p)}(\eta) = \int d^3\bar{x}\eta^{\bar{i}\bar{j}}\delta\tilde{T}_{\bar{i}\bar{j}}\bar{x}_L = \int d^3x a^{\ell+1}(\eta)\delta_{ij}\delta\tilde{T}_{ij}x_L = S_{ii|L}.$$

$$Q_{kl}^{(p)}(\eta) = \int d^3x a^3(\eta) T_{ij}\delta^{ij}x_k x_l$$

Conservation equations in terms of moments

- Conservation equations:

$$\partial_\eta(a\delta\tilde{T}_{00}) - \partial_i(a\delta\tilde{T}_{0i}) + \dot{a}\delta\tilde{T}_{kk} = -\frac{1}{4}\partial_\eta(a^2(p+\epsilon))(\tilde{\chi}_{00} + \tilde{\chi}_{kk}),$$

$$(\partial_\eta + \frac{\dot{a}}{a})(a\delta\tilde{T}_{0i}) - \partial_j(a\delta\tilde{T}_{ij}) = 0.$$

- Conservation of linearized stress-energy in terms of moments

$$\partial_t Q_L^{(\rho)} = H(\ell Q_L^{(\rho)} - Q_L^{(p)}) - \ell P_{(i_1|i_2\cdots i_\ell)} + s(\eta)\hat{\chi}_L \;,$$

$$\partial_t P_{i|L} = (\ell-1)HP_{i|L} - \ell S_{i(i_1|i_2\cdots i_\ell)}.$$

- In Particular,

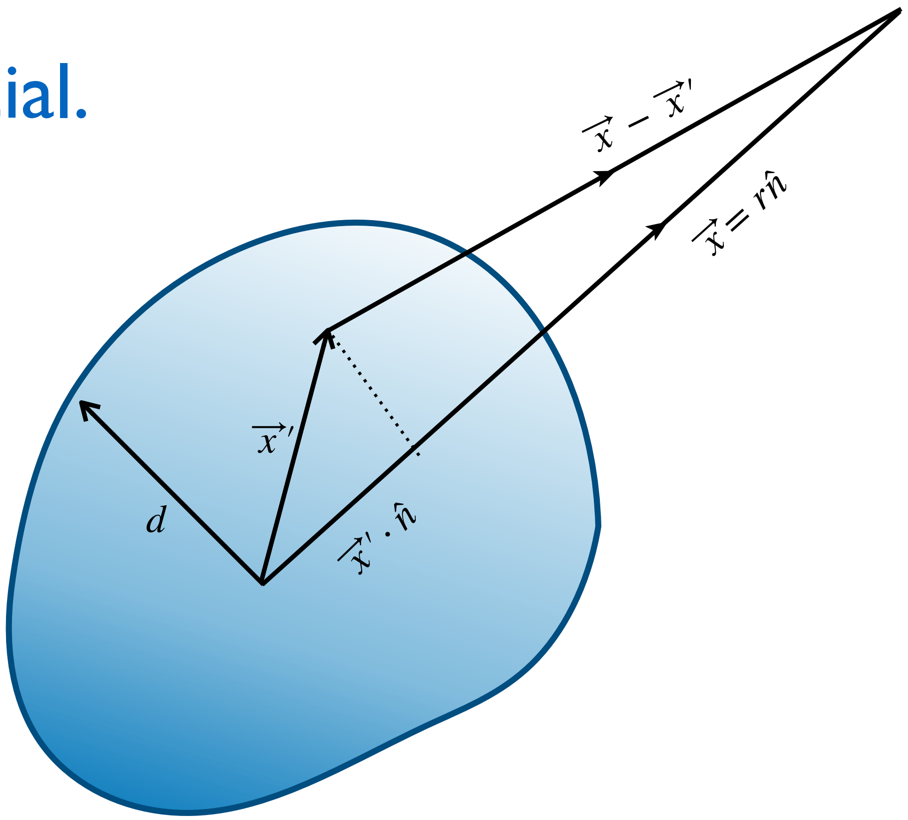
$$S_{ij}(\eta) = \int d^3x' a(\eta) T_{ij}(\eta, x') = \frac{1}{2}\partial_t\left(\partial_t Q_{ij}^{(\rho)} - 2HQ_{ij}^{(\rho)} + HQ_{ij}^{(p)} - s\hat{\chi}_{ij}\right).$$

Consistence quadrupolar truncation

- Higher order term in Taylor expansion is related to the lower order term via conservation equation. Distinct feature even in de Sitter!!

$$\int \frac{d^3x'}{|x-x'|} T_{ij}(\eta - |x-x'|, x') = \int d^3x' \frac{T_{ij}(\eta_{\rm ret}, x') + \vec{n}\cdot\vec{x}'\, T^{(1)}_{ij}(\eta_{\rm ret}, x') + \frac{1}{2}(\vec{n}\cdot\vec{x}')^2 T^{(2)}_{ij}(\eta_{\rm ret}, x') + \dots}{\rho}$$

A consistence quadrupolar truncation is crucial.



$$\int d^3x\, a^{\ell+1} T_{\mu\nu} x^L = 0, \qquad \forall \ell > 2.$$

Conservation equations imply

$$P_{(i|jk)} = 0, \qquad P_{i|jkl} = 0, \qquad S_{i(j|kl)} = 0.$$

Consistence quadrupolar truncation

- Solution in terms of SO(3) irreducible tensors

$$
\begin{aligned}
P_{i|jk} &= \frac{1}{2}\epsilon_{li(j}J_{k)l} - \frac{1}{2}\delta_{i(k}P_{j)|ll} + \frac{1}{2}\delta_{jk}P_{i|ll},\\
S_{ij|k} &= \frac{1}{2}\epsilon_{kl(i}K_{j)l} - \frac{1}{2}\delta_{k(i}Q^{(p)}_{j)} + \frac{1}{2}\delta_{ij}Q^{(p)}_{k},\\
S_{ij|kl} &= \delta_{ij}Q^{(p)}_{kl} - (\delta_{i(k}Q^{(p)}_{l)j} + \delta_{j(k}Q^{(p)}_{l)i}) + Q^{(p)}_{ij}\delta_{kl} - \frac{1}{2}\delta_{ij}\delta_{kl}Q^{(p)}_{mm} + \frac{1}{2}\delta_{i(k}\delta_{l)j}Q^{(p)}_{mm},
\end{aligned}
$$

where

$$
J_{ij} := \frac{4}{3}P_{k|l(i}\epsilon_{j)kl}, \qquad (\partial_t - H)P_{i|jj} = Q^{(p)}_i,
$$

$$
K_{ij} := \frac{4}{3}\epsilon_{kl(i}S_{j)k|l}. \qquad (\partial_t - H)J_{ij} = -K_{ij}.
$$

- This is even crucial for de Sitter case.

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Differs from

G. Date, J. Hoque - 2015

A. Ashtekar, B. Bonga, A. Kesavan -2015

Tensor inhomogeneous solution

$$\tilde{\chi}_{\langle ij\rangle}(\eta,\rho,n_i) = L_{c\,\langle ij\rangle} + T_{1\,\langle ij\rangle} + T_{2\,\langle ij\rangle}$$

$$\begin{aligned}
L_{c\,\langle ij\rangle} &= \frac{4G}{\rho}\Bigg(S_{\langle ij\rangle} + n_k(\partial_t S_{\langle ij\rangle|k} - H S_{\langle ij\rangle|k}) + \frac{1}{2}n_k n_l(\partial_t^2 S_{\langle ij\rangle|kl} - 3H\partial_t S_{\langle ij\rangle|kl} + 2H^2 S_{\langle ij\rangle|kl} \\
&\quad + 2H^2(1+q)S_{\langle ij\rangle|kl}) + \frac{1}{\rho a(\eta_{\rm ret})}\Bigg(n_k S_{\langle ij\rangle|k} + \frac{1}{2}(3n_k n_l - \delta_{kl})(\partial_t S_{\langle ij\rangle|kl} - 2H S_{\langle ij\rangle|kl})\Bigg) \\
&\quad + \frac{1}{2\rho^2 a^2(\eta_{\rm ret})}(3n_k n_l - \delta_{kl})S_{\langle ij\rangle|kl}\Bigg), \qquad (1) \\
T_{1\,\langle ij\rangle} &= 4G\int_{\eta_i}^{\eta-\rho} d\eta' \frac{\alpha(\alpha-1)}{2\eta\eta'}\Bigg[\, {}_2F_1\Big(2-\alpha, 1+\alpha; 2; \frac{p_\rho}{2}\Big) S_{\langle ij\rangle}(\eta') \\
&\quad - \frac{1}{a(\eta')}\frac{\rho}{\eta\eta'}\frac{d}{dp_\rho}\Bigg({}_2F_1\Big(2-\alpha, 1+\alpha; 2; \frac{p_\rho}{2}\Big)\Bigg) n_k S_{\langle ij\rangle|k}(\eta') \\
&\quad + \frac{1}{a^2(\eta')}\frac{1}{2}\frac{1}{\eta\eta'}\frac{d}{dp_\rho}\Bigg({}_2F_1\Big(2-\alpha, 1+\alpha; 2; \frac{p_\rho}{2}\Big)\Bigg)\delta_{kl} S_{\langle ij\rangle|kl}(\eta') \\
&\quad + \frac{1}{a^2(\eta')}\frac{1}{2}\frac{\rho^2}{(\eta\eta')^2}\frac{d^2}{d^2 p_\rho}\Bigg({}_2F_1\Big(2-\alpha, 1+\alpha; 2; \frac{p_\rho}{2}\Big)\Bigg) n_k n_l S_{\langle ij\rangle|kl}(\eta')\Bigg], \qquad (2) \\
T_{2\,\langle ij\rangle} &= 4G\Bigg(\frac{\alpha(\alpha-1)}{2}\frac{1}{\eta\eta_{\rm ret}}\Bigg(\frac{1}{a(\eta')}S_{\langle ij\rangle|k}(\eta')n_k - \frac{1}{2\rho a^2(\eta')}S_{\langle ij\rangle|kl}(\eta')(\delta_{kl} - n_k n_l)\Bigg)\Bigg|_{\eta'=\eta_{\rm ret}} \\
&\quad + \frac{1}{2}\frac{\alpha(\alpha-1)}{2}\frac{1}{\eta\eta_{\rm ret}}\Bigg(\frac{1}{a(\eta')}(\partial_t - 2H)S_{\langle ij\rangle|kl}(\eta')n_k n_l\Bigg)\Bigg|_{\eta'=\eta_{\rm ret}} \\
&\quad + \frac{1}{2}\Bigg(-\frac{\alpha(\alpha-1)}{2\eta\eta_{\rm ret}^2} + \frac{\alpha(\alpha-1)(\alpha(\alpha-1)-2)}{8}\Bigg(\frac{1}{\eta\eta_{\rm ret}}\Bigg)^2\rho\Bigg)\Bigg(\frac{1}{a^2(\eta')}S_{\langle ij\rangle|kl}(\eta')n_k n_l\Bigg)\Bigg|_{\eta'=\eta_{\rm ret}}\Bigg). \qquad (3)
\end{aligned}$$

Matter dominate case

- For matter dominated case

$$T_{1\,\langle ij\rangle} \quad = 4G \int_{\eta_i}^{\eta-\rho} \frac{d\eta'}{\eta\eta'} S_{\langle ij\rangle}(\eta').$$

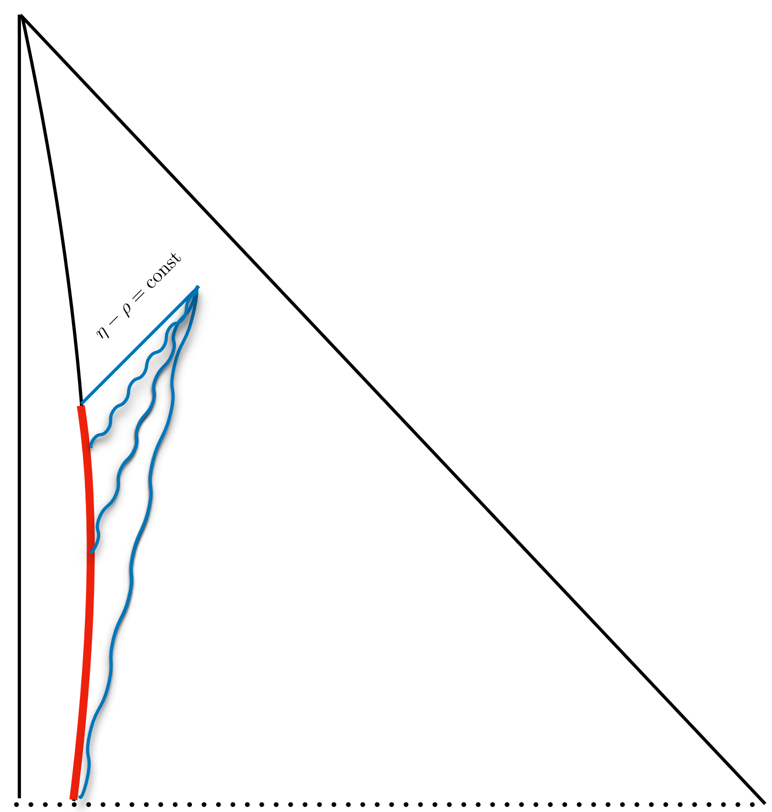
- Using the conservation equation, $S_{ij} = -\partial_t P_{i|j}$

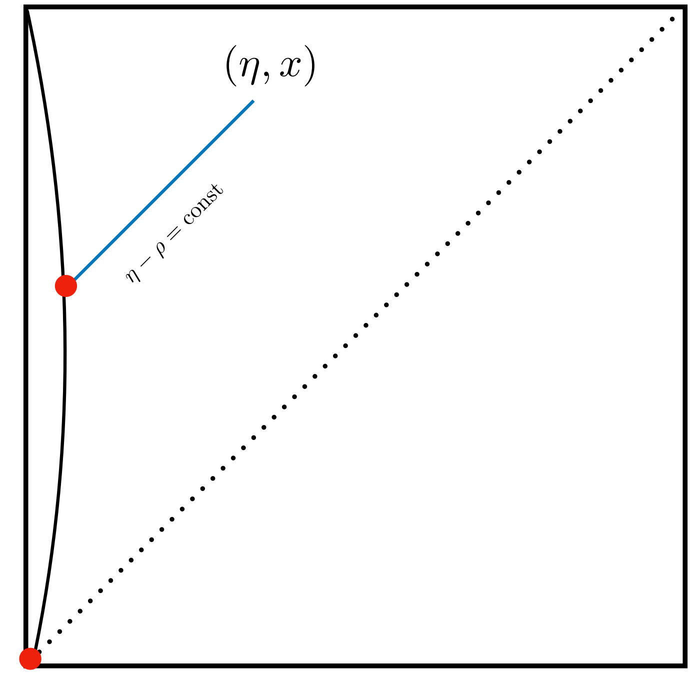
$$\begin{aligned} T_{1\langle ij\rangle} &= -\frac{4G}{\alpha\eta}\left[HP_{\langle i|j\rangle}\right]_{\eta_i}^{\eta_{\rm ret}} + \frac{4G}{\alpha\eta}\int_{\eta_i}^{\eta_{\rm ret}} d\eta' \partial_{\eta'} H(\eta') P_{\langle i|j\rangle}(\eta') \\ &= -\frac{4G}{\alpha\eta}\left[HP_{\langle i|j\rangle}\right]_{\eta_i}^{\eta_{\rm ret}} - \frac{4G}{\alpha\eta}\int_{t_i}^{t_{\rm ret}} dt' H^2(1+q) P_{\langle i|j\rangle}(t'). \end{aligned}$$

- Tail term truncated to V_0

- Order ρ term vanishes in tail term.

de Sitter case





- Tail integral can be performed exactly - leaving instantaneous term and a boundary term.
- dS limit reproduces our previous result. G. Compère, J. Hoque, E. S. Kutluk - 2023
- Similarly, vector and scalar perturbations can be obtained.

Summary and outlook

- We decouple linearized perturbations around spatially flat FLRW background in a generalized harmonic gauge - without performing the SVT decompsition.

- Obtain green's function for power-law cosmologies.

- Linearized perturbations are written in terms of multipolar moments of the effective stress-energy tensor upto the quadrupolar order.

- Our result reproduces correct dS limit which differs from Date-JH, ABK et. al

- An important line of the study will be construction of cosmological observables in our framework.

Thank you

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