The mystery of the missing accretion disc physics

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1. Introduction

2. Standard THIN DISC model of accretion

3. MORE LUMINOUS than standard disc model
   - SLIM DISC

4. LESS LUMINOUS than standard disc model
   - ION TORUS

5. Summary
X-rays are only produced in the most energetic events.

Many sources are persistently bright. These can only be explained by considering accretion of matter onto a compact object.
1. Introduction: Compact sources

Stellar evolution: As stars age they reach a point where they can no longer support their self-gravity by burning $\Rightarrow$ collapse $\Rightarrow$ compact object

New equilibrium stages (listed for increasing progenitor mass):

→ **White Dwarf**: e- degenerate, Fermi pressure (Chandrasekhar limit !)

→ **Neutron Star**: nuclei degenerate, repulsive nuclear forces (TOV limit !)

→ **Quark Star / Strange Star / Boson Star**: quark degeneracy (hypothetical !)

→ **Black Hole**: “ongoing gravitational collapse”
1. Introduction: Compact sources

**Galaxy evolution**: The formation process of supermassive black holes is still a matter of debate. We know currently of two plausible ways.

→ Collapse of primordial gas cloud → quasi star → IMBH → merging/accretion

→ Formation from early Pop.III stars → BH → merging/accretion
1. Compact sources: Rotating black holes

Rotating black holes are creatures conjured by Albert Einstein's General Theory of Relativity.

Stationary black holes are completely described by 2 parameters

* the MASS
* the SPIN

They are found, one reckons, where GRAVITY and ROTATION coalesce in a confined space.

Where these 2 primordial properties of the universe meet - extreme conditions (curvature) arise.

They are beasts akin to the smile of the Cheshire cat.

They are enormous stars that have winked out but are still there.

(C. Sagan, 1973)
1. Compact sources: X-ray binaries

Low-mass XRB:
- Roche lobe overflow

High-mass XRB:
- Wind accretion
Disc radiation:

A multicolour disc blackbody, produced by blackbody emission over multiple disc annuli.

The hottest blackbody component comes from the annulus closest to the black hole.

The piling of matter induces all kinds of stresses that heat up the gas and trigger energy release via radiation, so that a part of the initial angular momentum is lost and matter can make way by moving slightly inward.
2. Accretion discs: Observational evidence

The spectral appearance changes as a function of luminosity.

Evidence of disc in the Ultra-Soft (thermal) state...

Done, Gierliński & Kubota 2007
2. Accretion discs: Simple model

\[ F(R) = \frac{3GM\dot{M}}{8\pi R^3} \left( 1 - \sqrt{\frac{R_{\text{in}}}{R}} \right) \]

\[ T(R) = T_0 \left( \frac{R_{\text{in}}}{R} \right)^{3/4} \left( 1 - \sqrt{\frac{R_{\text{in}}}{R}} \right)^{1/4} \]

\[ T_0 = \left( \frac{3GM\dot{M}}{8\pi \sigma R_{\text{in}}^3} \right)^{1/4} \quad , \quad L_{\text{disk}} = \frac{GM\dot{M}}{2R_{\text{in}}} \]

\[ R_{\text{in}} = \sqrt{\frac{3}{4\pi \sigma}} \frac{L_{\text{disk}}^{1/2}}{T_0^2} \]

Shakura & Sunyaev, 1973
2. Accretion discs: BH properties

\[ L_\star = 4\pi D^2 F_{\text{obs}} = 4\pi R_\star^2 \sigma T^4 \]

* M, D, i from ground-based observations

* R_in = R_isco

\[ \rightarrow a^* = \frac{Jc}{GM^2} \]
2. Troubles with high $L$ spectra

The spin is not constant for all luminosity? → Go back to the drawing board!
Supplement: The Eddington limit

The theoretical limit at which the force generated by radiation pressure of a light-emitting body equals its gravitational attraction. A star emitting radiation at greater than the Eddington limit would break up.

\[
\frac{L\sigma_T}{4\pi r^2 c} = \frac{GM m_p}{r^2}
\]

\[\Rightarrow L_{Edd} = \frac{4\pi c G M m_p}{\sigma_T} = 1.2 \times 10^{38} \left(\frac{M}{M_{\text{Sun}}}\right) \text{ erg/s}\]

The Eddington mass accretion rate:

\[L_{Edd} = \eta c^2 \dot{M}_{Edd}\]
Supplement: Modelling accretion discs

(Abramowicz & Straub, Scholarpedia, 2013)
3. Luminous accretion: Slim disc equations

1) mass conservation
\[ \dot{M} = -2\pi \Sigma \Delta^{1/2} \frac{V}{\sqrt{1 - V^2}} \]

V, gas radial velocity

2) radial momentum conservation
\[ \frac{V}{1 - V^2} \frac{dV}{dr} = -\frac{MA}{r^2 \Delta \Omega^+ \Omega^-} (\Omega - \Omega^+)(\Omega - \Omega^-) \frac{A \Delta \gamma}{1 - \Omega^2 K R} \]

3) angular momentum conservation
\[ \frac{\dot{M}}{2\pi} \left( \mathcal{L} - \mathcal{L}_{in} \right) = \frac{\sqrt{A \Delta \gamma}}{r} \alpha P \]

\( \mathcal{P} = 2\text{Hp}, \) vertically integr. pressure \( \alpha = 0.1 \)

4) energy conservation
\[ F = \frac{3GM^2}{8\pi R^3} + F_{adv} \]

5) vertical equilibrium
\[ \frac{P}{\Sigma H^2} = \frac{\mathcal{L}^2 - \alpha^2 (\epsilon^2 - 1)}{2r^4} \]

Lasota (1994); Abramowicz et al. (1996, 1997); Sadowski (2009);
3. **Luminous accretion: Advection**

Slim discs are cooled by advection.

Advection sweeps some of the emitted energy along with the flow because photons are trapped in the optically thick disc.

The photons can be released again at lower radii as the material accelerates towards the black hole.

(A. Sadowski)
3. Luminous accretion: What is different?

**THIN discs**

- Keplerian rotation

- Inner disk terminates at the ISCO

- All locally released energy (photons) is radiated away

Shakura & Sunyaev (1973)

**SLIM discs**

- Super-Keplerian rotation in the innermost part of the disk, otherwise sub-Keplerian

- Inner disk edge can be closer to the BH than the ISCO (for higher accretion rates)

- A part of the dissipated energy is captured in the flow and advected inwards

Abramowicz et al. (1988)
3. Luminous accretion: Flux
3. Luminous accretion: Advection

The spin is STILL not constant for all luminosity? → Go back to the drawing board!
The spin is STILL not constant for all luminosity? → Go back to the drawing board!

→ change value of the viscosity parameter $\alpha$
from $\alpha = 0.1$ to $\alpha = 0.01$

3. Luminous accretion: Advection
3. Luminous accretion: Viscosity

\[ \alpha = \alpha_* \left( \frac{\sigma}{\omega} \right) \]

\[ \alpha = \alpha_{**} \left( \frac{1}{\gamma^2} \right) \]

(Penna et al., 2013)
3. Luminous accretion: ULX

Great Nebula in Andromeda (M 31 - NGC 224)

Type: Galaxy

Magnitude: 14.65

RA/DE (J2000): 0h42m42.0s/+41°16'00.0"

RA/DE (of date): 0h43m25s/+41°20'16"

Hour angle/DE: 6h37m14s/+41°20'16" (geometric)

Hour angle/DE: 6h37m10s/+41°21'21" (apparent)

Az/Alt: +300°46'53"/+30°24'31" (geometric)

Az/Alt: +300°46'53"/+30°25'52" (apparent)

Size: +2°50'00"
3. Luminous accretion: ULX

CXOM31 J004253.1+411422 (M31 ULX-1 hereafter)

1. \( L > 10^{39} \) erg/s
   (roughly \( L_{\text{Edd}} \) of a BH of 20 M\(_{\odot}\))

2. Distinct from centre of galaxy

3. Not coincident with background AGN or QSOs

4. Often found in star-forming regions
   (Gao et al. 2003)

The fact that ULXs have Eddington luminosities larger than that of stellar mass objects implies that they may be different from normal X-ray binaries. - There are several models for ULXs, and it is likely that different models apply for different sources.
Compact sources: ULX

Solution 1: Super-Eddington accretion onto stellar mass BHs

=> This would mean that Eddington accretion can be readily observed

Solution 2: Sub-Eddington accretion onto ‘Intermediate mass BHs’

=> Need new physics for their formation (potentially seen already in GCs: Maccarone & Servillat 2008)
3. Luminous accretion: ULX

How can we distinguish between the two solutions?

Axiom of accretion physics: **Timing and spectral properties scale with mass**

* Timing: Features of PDS shifted in frequency due to mass
* Spectra:

(XRB spectra: Done et al., 2007)

(AGN spectra: Done et al., 2011)
3. Luminous accretion: ULX

(XRB spectra: Done et al., 2007)

(ULX-1 spectra: Middleton et al., 2012)
Interpreting observational data
ULX model – better, can incorporate intrinsic absorption

With decreasing luminosity the disc gets hotter and more advection dominated, the ‘wind’ component gets hotter and less important.

Best fit with 2 model components.
BUT: more parameters = more degrees of freedom! \(\rightarrow\) can fit ANYTHING
3. **Phenomenological vs physical models**

- synthetic SLIM disc spectrum
- disc model (DISKBB)
- Compton model (COMPTT)
3. Luminous accretion: ULX

Result: Something is amiss with the low luminosity models

![Graph showing energy distribution](image)
3. Luminous accretion: Conclusion

It has long been assumed that in the trans-Eddington luminosity regime two effects could become increasingly important for accretion disc models,

(i) advection (Abramowicz et al. 1988; Mineshige et al. 2000) and/or
(ii) outflows (Shakura & Sunyaev 1973; Poutanen et al. 2007; Ohsuga et al. 2009).

→ Advection makes very little difference to spectra below the Eddington luminosity.

→ Model/Disc comparison show exactly opposite behaviour with respect to luminosity than expected from either winds, advection, bulk turbulence and/or inhomogeneities in the disc.

→ Instead variable alpha and most likely a missing physical process(es) ...

Solution? → Find a better alpha prescription & include e.g. magnetic fields
Supplement: Modelling accretion discs

(Abramowicz & Straub, Scholarpedia, 2012)
4. Journey into the heart of the galaxy

A few 100 pc - optical
4. Journey into the heart of the galaxy

A few 100 pc - infrared
4. Journey into the heart of the galaxy

150 pc = 1° (Chandra)
4. Journey into the heart of the galaxy

Visible Light

Infrared Light

few 10' (Chandra)
4. Journey into the heart of the galaxy

1" (VLT)

→ to resolve the horizon of Sgr A* we need MICRO ARCSECONDS
4. Journey into the heart of the galaxy

(EHT, 2020)
4. Geometrically thick disc: Doughnut family

construction of a geometrically THICK disc model: The Polish Doughnut.

* Euler equation of a perfect fluid
  \[ A := -u_t \]

\[
\frac{\nabla_\mu p}{p + \rho} = -\nabla_\mu \ln A + \frac{l \nabla_\mu \Omega}{1 - \Omega l}.
\]

* Von Zeipel condition
  → simplest solution: \( l = \text{const.} \)

\[ p = p(\rho) \iff l = l(\Omega) \]

→ define potential function:

\[ \nabla_\mu W \equiv -\nabla_\mu \ln A + \frac{l \nabla_\mu \Omega}{1 - \Omega l} \]

→ equipotential surfaces
4. Geometrically thick disc: Doughnut family

→ torus surface is the self-crossing equipotential surface

→ depending on choice of angular momentum (l=const.) we get different size tori
4. Torus model vs MHD simulations

→ Analytical structure, few simple parameters
→ Reproduces basic features of sophisticated GRMHD simulations

(Qian et al., 2009)
4. Optically thin disc: Ion torus family

* ION TORI are optically thin and advection dominated like ADAFs.

→ magnetic pressure \( p_{\text{tot}} = p_{\text{gas}} + p_{\text{mag}} \) with \( p_{\text{mag}} = \beta p_{\text{tot}} \)

\[ p_{\text{gas}} = \text{perfect gas law for electrons \& ions} \]

→ EOS for total gas pressure: polytropic

\[ p_{\text{tot}} = \kappa (\rho c^2)^{1+1/n} \]

→ express thermodynamic quantities in terms of the potential function W

→ radiative processes: Bremstrahlung \& synchrotron emission, their inverse Compton scattering (ADAF model by Narayan \& Yi, 1995)

* radiative emission coefficient \( j \) given by

\[ dE = j_\nu \, dV \, d\Omega \, dt \, d\nu \]

→ \( j_\nu = j_\nu^{br} + j_\nu^{sy} + j_\nu^{br,c} + j_\nu^{sy,c} \)
4. Optically thin disc: Radiative transfer

The transfer equation for radiation in the emitter's frame is

\[ \frac{dI_\nu}{ds} = -\alpha_\nu I_\nu + j_\nu \]

Intensity in the emitter's frame

To solve the equation one requires the emission and absorption coefficients

\[ j_\nu = j_{\nu}^{br} + j_{\nu}^{sy} + j_{\nu}^{br,c} + j_{\nu}^{sy,c} \]

\[ \alpha_\nu = \frac{j_\nu}{B_\nu} \] (negligible at ion torus temperatures)

This equation is integrated by a ray-tracing code to get the intensity that is transported to the observer by each photon (expressed in the emitter's frame).
4. Ray-tracing basics: Geodesic equation

Ray-tracing means integrating the null geodesics of photons from a distant observer's screen to the source.

A geodesic is given by the geodesic equation:

\[
\ddot{x}^\alpha + \Gamma^\alpha_{\mu\nu} \dot{x}^\mu \dot{x}^\nu = 0
\]

(derivative with respect to affine parameter \(\lambda\) where

\[
\Gamma^\alpha_{\mu\nu} = \frac{1}{2} g^{\alpha\sigma} (\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu})
\]

are the Christoffel symbols.)
4. Ray-tracing basics: Constants of motion

There are 3 obvious constants of (geodesic) motion:

\[
\begin{align*}
\mathbf{m}^2 & = -\mathbf{p} \cdot \mathbf{p} & \text{the photon's mass (scalar prod. between 4-vect)} \\
E & = -p_t & \text{the photon's energy} \\
L & = p_\phi & \text{the photon's } \phi \text{-component of the angular mom.}
\end{align*}
\]

where \( p^\mu = \dot{x}^\mu \) is the photon's 4-momentum (E, L seen at infinity).

And there is the 4th constant: Carter's Constant (in Kerr)

\[
Q = p_\theta^2 + \cos^2 \theta \left[ a^2 (m^2 - p_t^2) + \frac{p_\phi}{\sin^2 \theta} \right]
\]

These 4 constants allow us to rewrite the geodesic equation (in terms of the 4 constants) and get a system of 4 differential equations...
4. Ray-tracing basics

\[
\begin{align*}
\Sigma \frac{dr}{d\lambda} &= \pm \sqrt{R} \\
\Sigma \frac{d\theta}{d\lambda} &= \pm \sqrt{\Theta} \\
\Sigma \frac{d\phi}{d\lambda} &= -(aE - \frac{L}{\sin^2 \theta}) + \frac{a}{\Delta} P \\
\Sigma \frac{dt}{d\lambda} &= -a(aE \sin^2 \theta - L) + \frac{r^2 + a^2}{\Delta} P
\end{align*}
\]

Ray-tracing codes integrate the above system over \( \lambda \) in order to determine the photon trajectory from its initial position, velocity \((x_0^\mu, p_0^\mu)\).

→ All ray-tracing codes follow these steps (formulations may vary, though).
4. Radiative transfer equation

To get the intensity in the observer's frame we use the frame invariant \( \mathcal{I} = I_\nu / \nu^3 \)

and define \( g \equiv \frac{\nu^{obs}}{\nu^{em}} = \frac{p^{obs} \cdot u^{obs}}{p^{em} \cdot u^{em}} \)

where \( p^{obs}, p^{em} \) is the photon's tangent vector to the geodesic in each frame and \( u^{obs}, u^{em} \) is the respective 4-velocity.

The intensity measured by the observer is then \( I^{obs}_\nu = g^3 I_\nu \)
4. Radiative transfer equation

A map of specific intensity in the observer's frame is called an image.

The observer's screen has N pixels. Each pixel is associated to one direction of incidence of the photons.

The flux is given by the integral over solid angle of the specific intensity. Computing the flux for a range of frequencies gives the spectrum.

$$dF_{\nu_{\text{obs}}} = I_{\nu}^{\text{obs}} \cdot \cos \theta \delta \Omega$$

GYOTO code: A pixel on the screen corresponds to a direction on the sky, just like a pixel on a camera detector. Flux is given by the sum over all pixels. (F. Vincent et al., 2011)
4. Dim accretion: Spectra

\[ a^* = 0, 0.5, 0.9 \]
\[ \lambda = 0.3 \]
\[ \text{inc} = 80^\circ \]
4. Dim accretion: Spectra

Add 2nd synchrotron component to account for electrons created in p-p interaction

\[ a^* = 0, 0.5, 0.9 \]
\[ \lambda = 0.3 \]
\[ \text{inc} = 80^\circ \]
4. Dim accretion: Images

- $a^* = 0.5$
- $\lambda = 0.3$
- $\text{inc} = 80^\circ$
4. Dim accretion: Images

\[ a^* = 0.9 \]

\[ \text{lambda} = 0.3 \]

\[ \text{inc} = 80^\circ \]
4. Dim accretion: Images

- $a^* = 0.5$
- $\lambda = 0.3$
- inc = $40^\circ$
4. **Dim accretion: Images**

- $a^* = 0.5$
- $\lambda = 0.7$
- $\text{inc} = 80^\circ$
4. Dim accretion: Images

Spin constraint?
Future: measuring the black hole’s silhouette

Using VLBI measurements: Doeleman et al., Broderick et al., Dexter et al. ...
4. New telescopes

* GRAVITY beam combiner, resolution: 10 – 100 μas (ca 2014)

* Event Horizon Telescope, resolution: 1 – 10 μas (ca 2020)
Standard **THIN DISC** model of accretion

**LESS LUMINOUS** than standard disc model

**ION TORUS**

**MORE LUMINOUS** than standard disc model

**SLIM DISC**
The End