Using general relativity to study condensed matter

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Outline

A. Review general relativity and black holes

B. Gauge/gravity duality

C. Using general relativity to study superconductivity

General Relativity (Einstein, 1915)

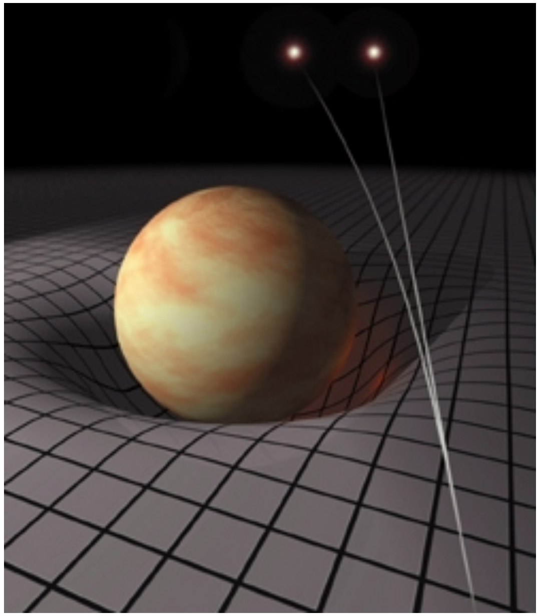
Main idea of general relativity: Since gravity is universal, identify it with something else which is universal - the geometry of spacetime.

Einstein's equation:

curvature of spacetime = distribution of matter

Particles travel along geodesics - the straightest possible path in the curved spacetime.

GR predicts light is bent by massive objects.



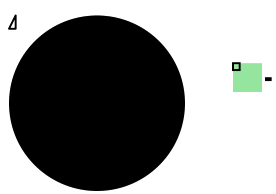
This has now been confirmed to an accuracy 2×10^{-5} by bouncing radio waves off the Cassini spacecraft (Bertotti et al, 2003).

Classical black holes (1970's)

Uniqueness theorem: The <u>only</u> stationary (vacuum) black hole solution is the Kerr solution with two free parameters M, J.

"Black holes have no hair." Wheeler

Surface gravity of a black hole κ: The force at infinity required to hold a unit mass at rest as it approaches the horizon of a black hole.



Laws of black hole mechanics

(Carter, Bardeen, Hawking, 1973)

0) For stationary black holes, the surface gravity κ is constant on the horizon

1) Under a small perturbation: $dM = \frac{\kappa}{8\pi G} dA + \Omega dJ$

2) The area of the event horizon always increases

Semiclassical black holes

Hawking coupled quantum matter fields to a classical black hole, and showed that they emit black body radiation with a temperature

$$T = \frac{\hbar\kappa}{2\pi}$$

This implies black holes have an entropy (Bekenstein)

$$S_{BH} = \frac{A}{4\hbar G}$$

Claim:

In addition to describing gravitational phenomena (black holes, gravitational waves, etc.) general relativity can also describe other fields of physics including aspects of condensed matter.

Gauge/gravity duality

Gauge Theories

These are generalizations of electromagnetism in which the U(1) gauge invariance is replaced by e.g. SU(N).

Our standard model of particle physics is based on a gauge theory.

QCD has SU(3) gauge symmetry. The interactions are weak at high energy but become strong at low energy causing quark confinement.

‘t Hooft argued in the 1970’s that a 1/N expansion of an SU(N) gauge theory would resemble a theory of strings.

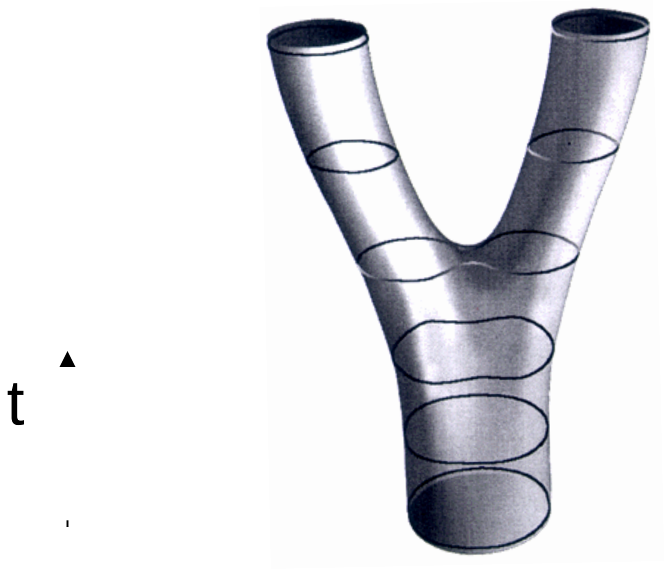
It took more than 20 years for this idea to be made precise.

String Theory

This is a promising candidate for both

1) a complete quantum theory of gravity
2) a unified theory of all forces and particles

It is based on the idea that elementary particles are not pointlike, but excitations of a one dimensional string.



Strings interact with a simple splitting and joining interaction with strength g.

String theory reduces to general relativity (with certain matter) in a classical limit.

Gauge/gravity duality

(Maldacena, 1997)

Under certain boundary conditions, string theory (which includes gravity) is completely equivalent to a (nongravitational) gauge theory living on the boundary at infinity.

When string theory is weakly coupled, gauge theory is strongly coupled, and vice versa.

Shows that quantum gravity is holographic

(‘tHooft, Susskind)

The boundary condition that is required is that the spacetime must approach constant negative curvature. This is called anti-de Sitter (AdS) spacetime. The metric looks like

$$ds^2 = r^2(-dt^2 + d\vec{x}^2) + \frac{dr^2}{r^2}$$

Metric of special relativity

Rescaling r -> a r, (t, x) -> (t/a, x/a) leaves the metric invariant. Small radius corresponds to large distance (low energy) in gauge theory.

Traditional applications of gauge/gravity duality

Gain new insight into strongly coupled gauge theories, e.g., geometric picture of confinement.

Gain new insight into quantum gravity, e.g., quantum properties of black holes

Quantum Black Holes

- What is the origin of black hole entropy?
- Does black hole evaporation lose information? Does it violate quantum mechanics?

Answers from Gauge/Gravity Duality

The gauge theory has enough microstates to reproduce the entropy of black holes.

The formation and evaporation of small black holes can be described by ordinary Hamiltonian evolution in the gauge theory. It does not violate quantum mechanics.

After thirty years, Hawking finally conceded this point in 2004.

In a certain limit, all stringy and quantum effects are suppressed and gravity theory is just general relativity

(with asymptotically anti-de Sitter boundary conditions).

New application of gauge/gravity duality: Condensed matter

In 2007, a few condensed matter effects (e.g. the Hall effect) were reproduced using general relativity. (Hartnoll, Herzog, Kovtun, Sachdev, Son)

This duality allows one to compute dynamical transport properties of strongly coupled systems at nonzero temperature.

Theoretical physicists have very few other tools to do this.

Basic Ingredients of the Duality

A state of thermal equilibrium at temperature T is dual to a black hole with temperature T.

Fields in spacetime are dual to operators in the boundary theory.

Local properties of the gauge theory are related to the asymptotic behavior of the gravity solution.

Superconductivity

Superconductivity 101

In conventional superconductors (Al, Nb, Pb, …) pairs of elections with opposite spin can bind to form a charged boson called a Cooper pair.

Below a critical temperature T_c, there is a second order phase transition and these bosons condense.

The DC conductivity becomes infinite.

This is well described by BCS theory.

The new high T_c superconductors were discovered in 1986. These cuprates (e.g. YBaCuO) are layered and superconductivity is along CuO_2 planes.

Highest T_c today (HgBaCuO) is T_c = 134K

New superconductors based on iron and not copper (FeAs) discovered in 2008 have T_c = 56K.

The pairing mechanism is not well understood. Unlike BCS theory, it is not weakly coupled.

Use gauge/gravity duality to try to gain insight into these high T_c superconductors.

Gravity dual of a superconductor

(Hartnoll, Herzog, and G.H., 2008)

Gravity	Superconductor
Black hole	Temperature
Charged scalar field	Condensate

Need to find a black hole that has scalar hair at low temperatures, but no hair at high temperatures.

This is not an easy task.

Gubser (2008) argued that a charged scalar field around a charged black hole would have the desired property. A charged scalar field has

$$\mathcal{L} = -|\partial\psi - iqA\psi|^2 - m^2|\psi|^2$$

For an electrically charged black hole, the effective mass is

$$m^2_{eff} = m^2 + q^2 g^{tt} A_t^2$$

But the last term is negative. This produces scalar hair at low temperature.

At large radius, the vector potential and charged scalar behave as

$$A_t = \mu - \frac{\rho}{r}, \qquad \psi = \frac{\psi^{(2)}}{r^2}$$

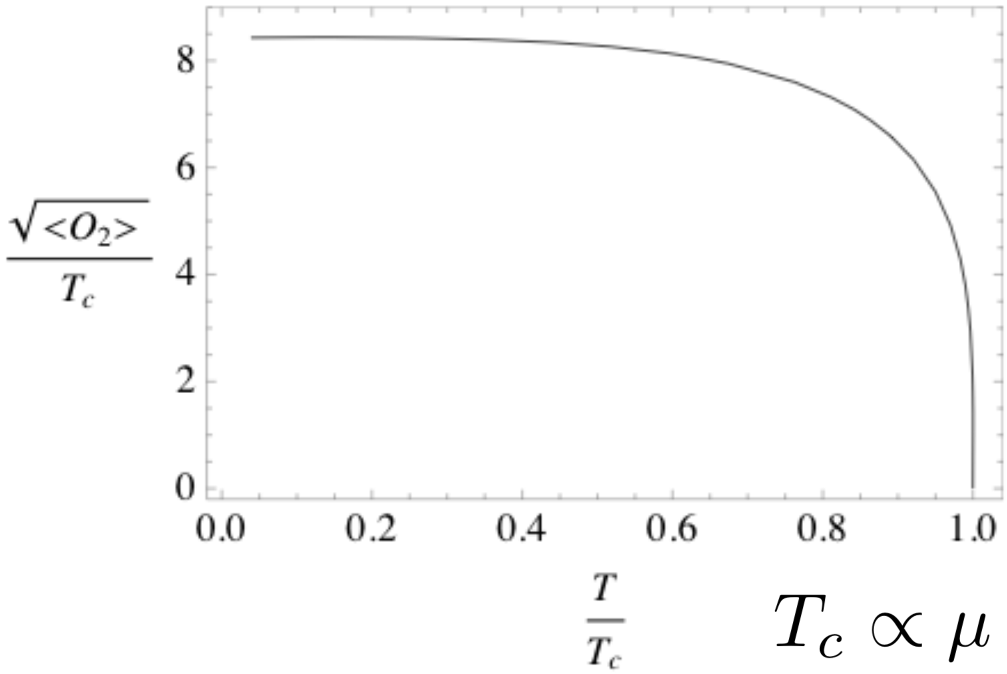
Gauge/gravity duality relates these constants to properties of the dual field theory:

μ = chemical potential, ρ = charge density

There is an operator O_2 dual to ψ, and

$$\langle O_2 \rangle = \psi^{(2)}$$

Condensate (hair) as a function of T



$T_c \propto \mu$

As T · > 0, the horizon area vanishes, consistent with a unique ground state.

Conductivity

We want to compute the conductivity as a function of frequency. Start by perturbing the black hole solution.

Assume time dependence $e^{-i\omega t}$ and impose ingoing wave boundary conditions at the horizon.

The asymptotic behavior is

$$A_x = A_x^{(0)} + \frac{A_x^{(1)}}{r} + \cdots$$

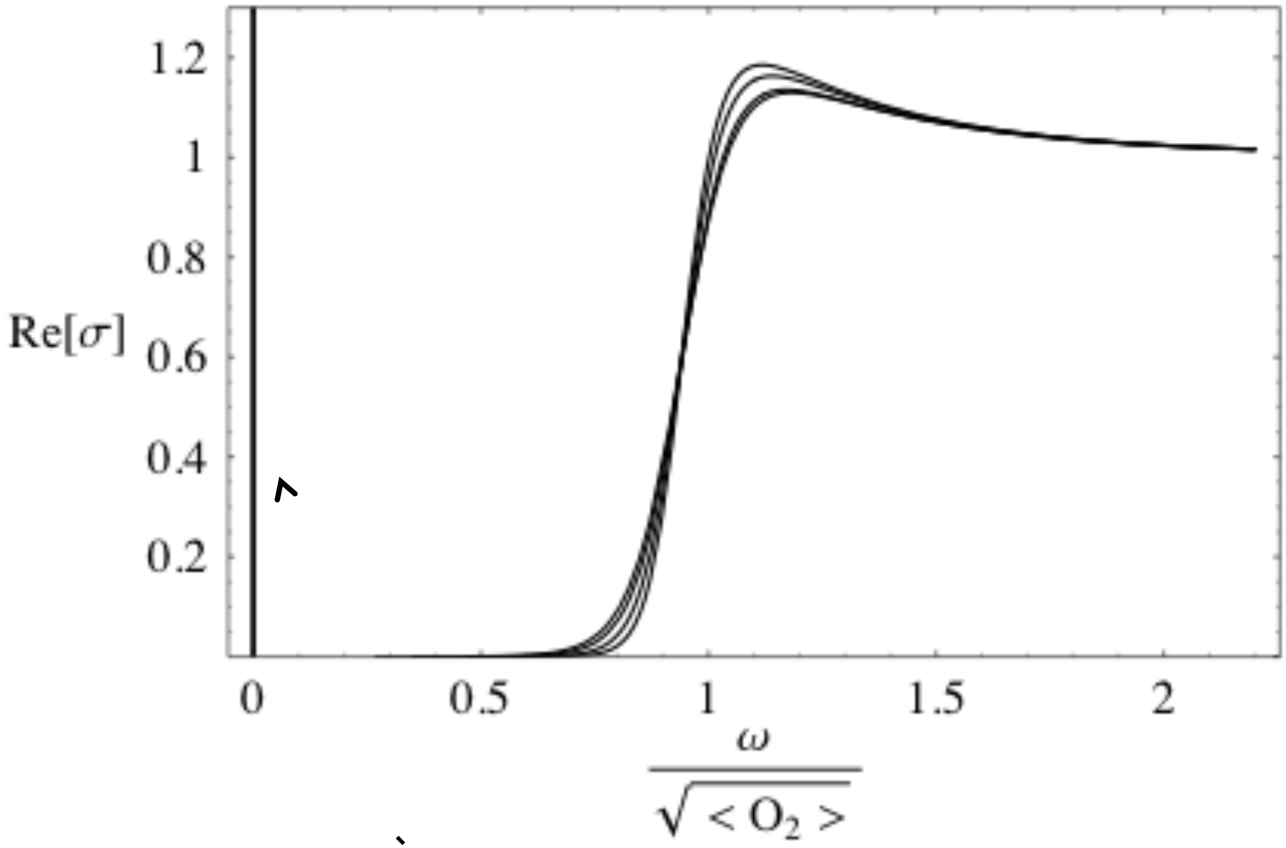
The gauge/gravity duality dictionary says

$$E_x = i\omega A_x^{(0)}, \quad J_x = A_x^{(1)}$$

We obtain the conductivity from Ohm’s law

$$\sigma(\omega) = \frac{J_x}{E_x} = \frac{A_x^{(1)}}{i\omega A_x^{(0)}}$$

The conductivity at low T



Delta function at ω = 0

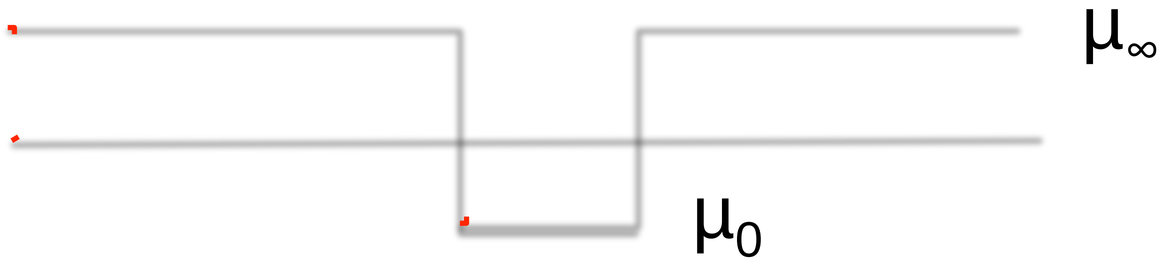
Josephson junctions

A Josephson junction consists of two superconductors separated by a weak link:

Insulator: SIS junctions
Normal conductor: SNS junctions
Narrow superconducting bridge

Josephson predicted that even without a voltage difference across the junction,
$J = J_{max} \sin \gamma$ where γ is the phase difference.

Model this by letting μ be position dependent.
(Santos, Way, G.H., 2011)

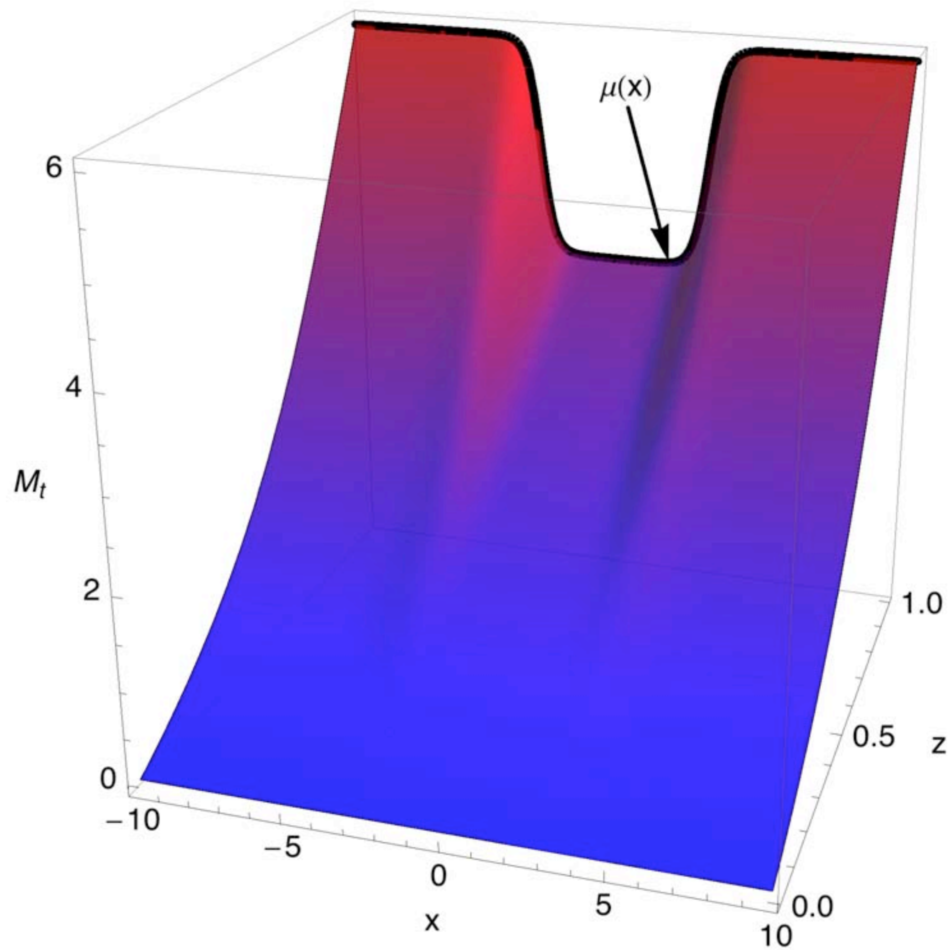


For a range of temperatures, you have two superconductors separated by a normal conductor.

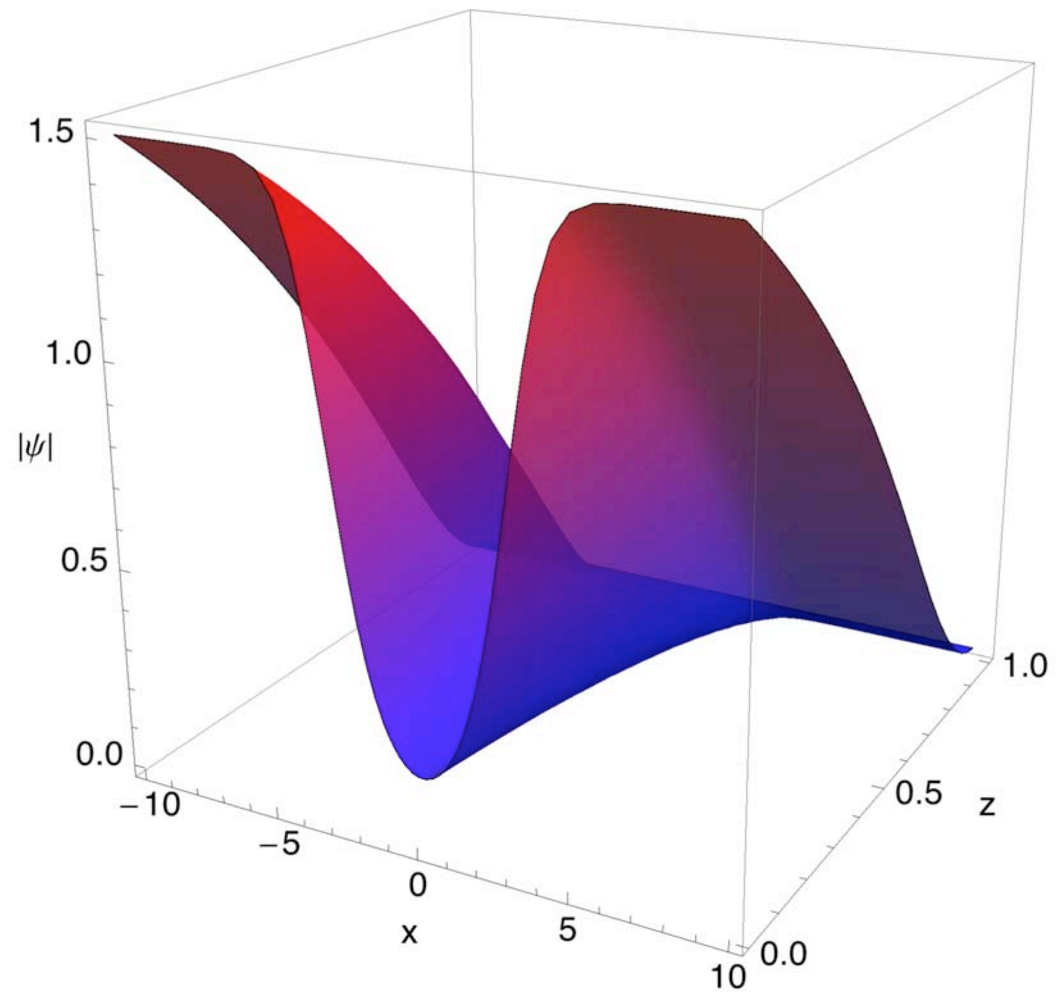
The critical temperature for the junction is the same as the case with constant $\mu = \mu_\infty$

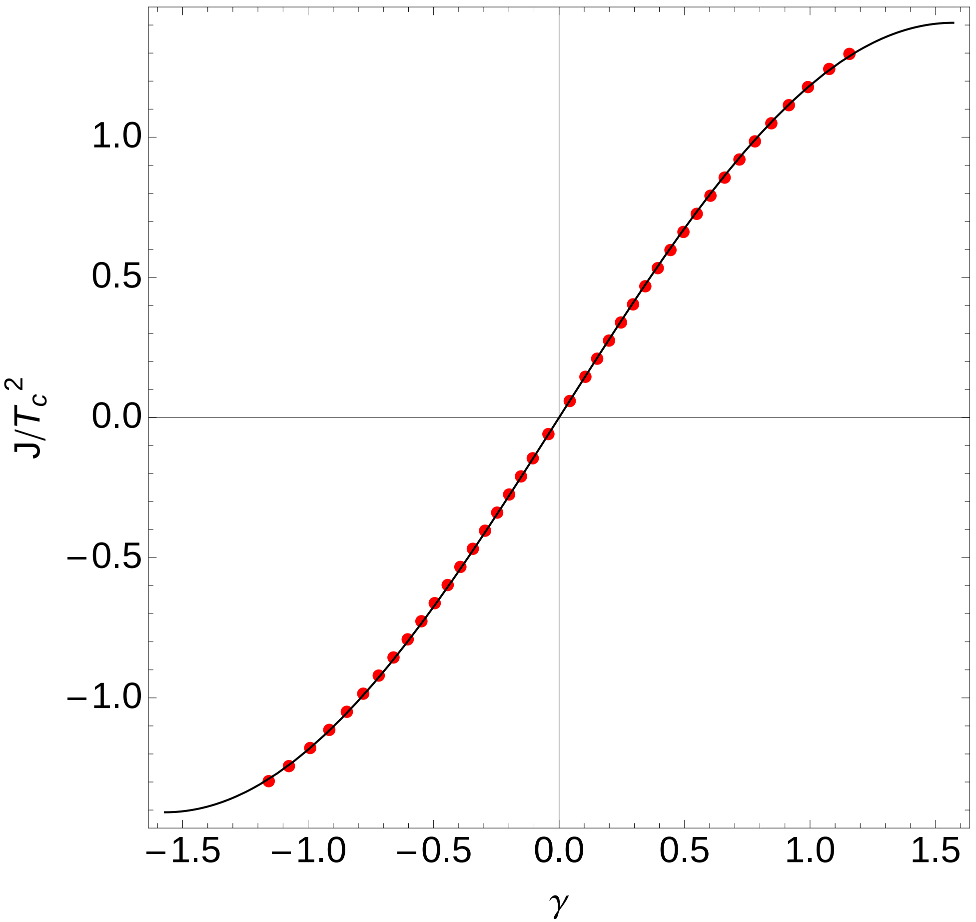
Results

A_t as a function of x and
$z = 1 - r_0/r$



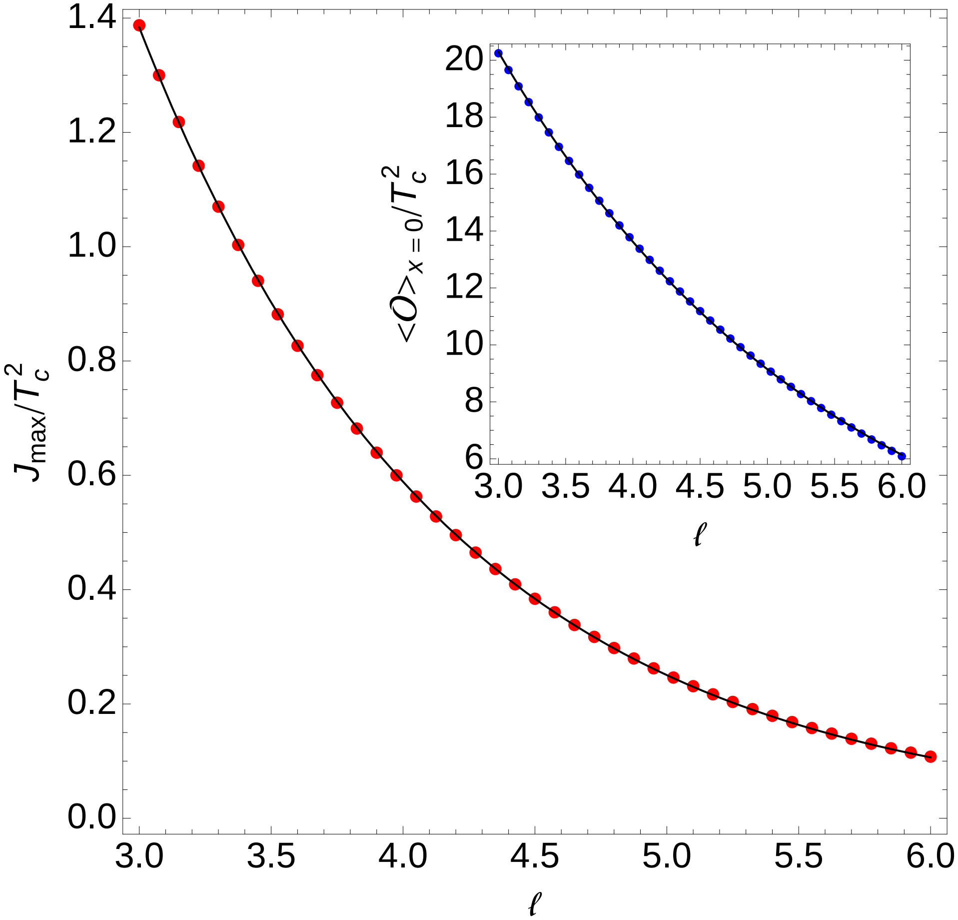
Scalar stays small inside the gap.





Indeed: $J = J_{max} \sin \gamma$

Dependence of J_{max} on width of junction:



Line is fit to

$$J_{\mathrm{max}}/T_c^2 = A_0\, e^{-\frac{\ell}{\xi}}$$

Condensate at center of junction behaves similarly

$$\langle\mathcal{O}\rangle_{x=0}/T_c^2 = A_1\, e^{-\frac{\ell}{2\,\xi}}$$

Can one do more than reproduce qualitative features of condensed matter systems?

Can gauge/gravity duality provide a quantitative explanation of some mysterious property of real materials?

Yes

Add Lattice

(J. Santos, D. Tong, G.H., 2012)

We now take our bulk theory to be just gravity + Maxwell theory (no charged scalar).

Introduce a lattice by making the chemical potential be a periodic function:

$$A_t \to \mu(x) \equiv \bar{\mu}\left[1 + \mathcal{A}_0 \cos(k_0 x)\right]$$

Simple model of a conductor

Suppose electrons in a metal satisfy

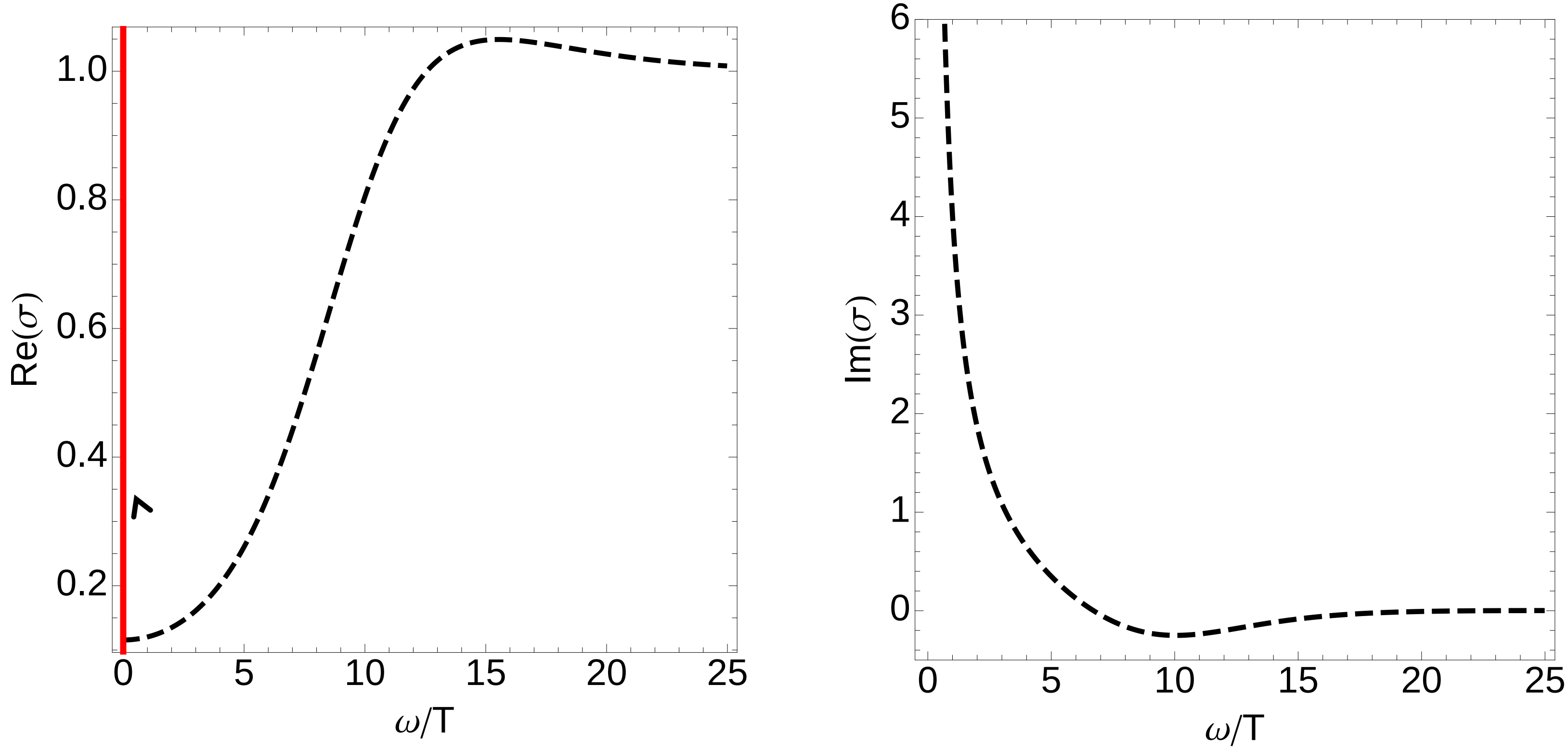
$$m\frac{dv}{dt} = eE - m\frac{v}{\tau}$$

If there are n electrons per unit volume, the current density is J = nev. Letting $E(t) = Ee^{-i\omega t}$, find J = σ E, with

$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau}$$

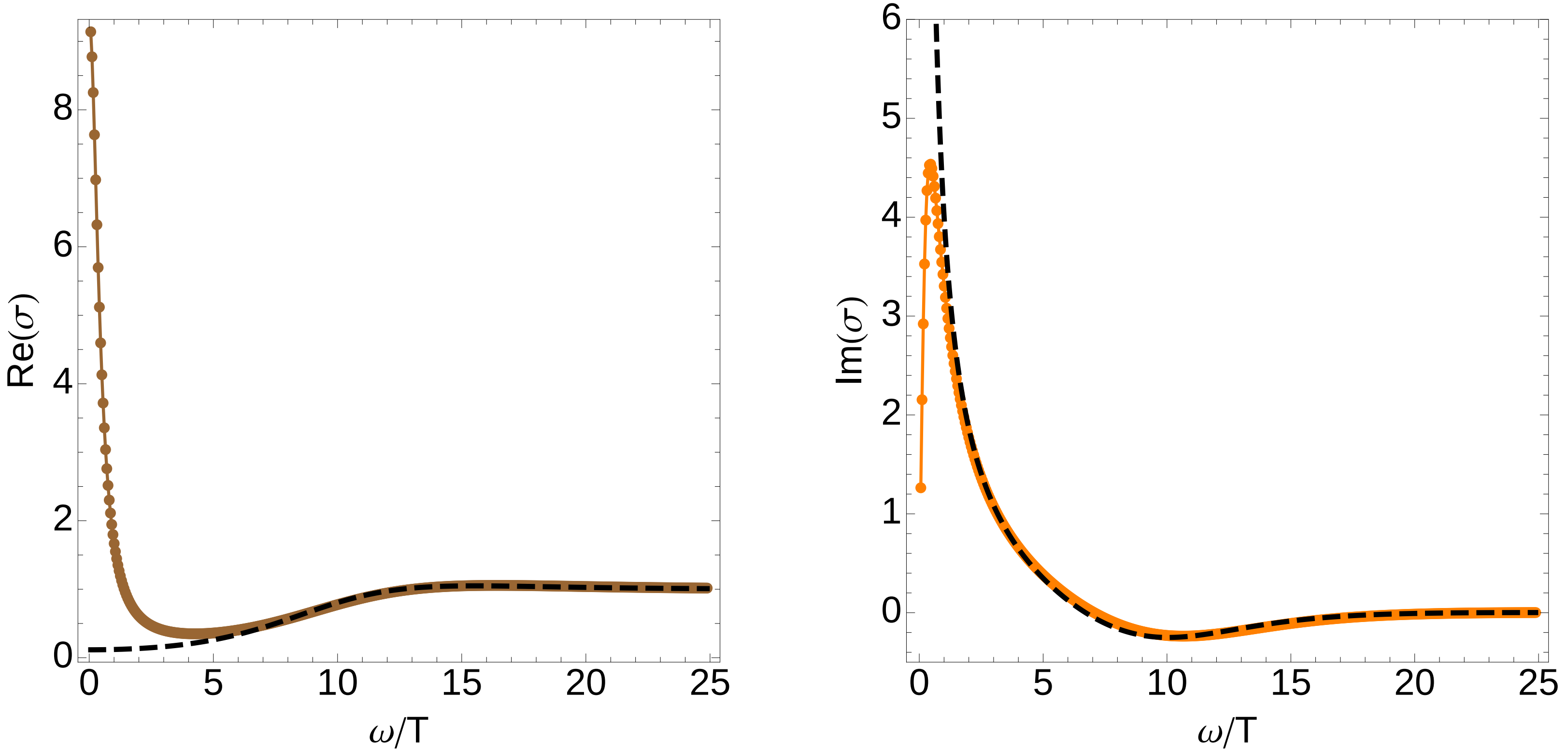
where $K=ne^2/m$. This is the Drude model.

Optical conductivity with no lattice
(T/μ = .115)



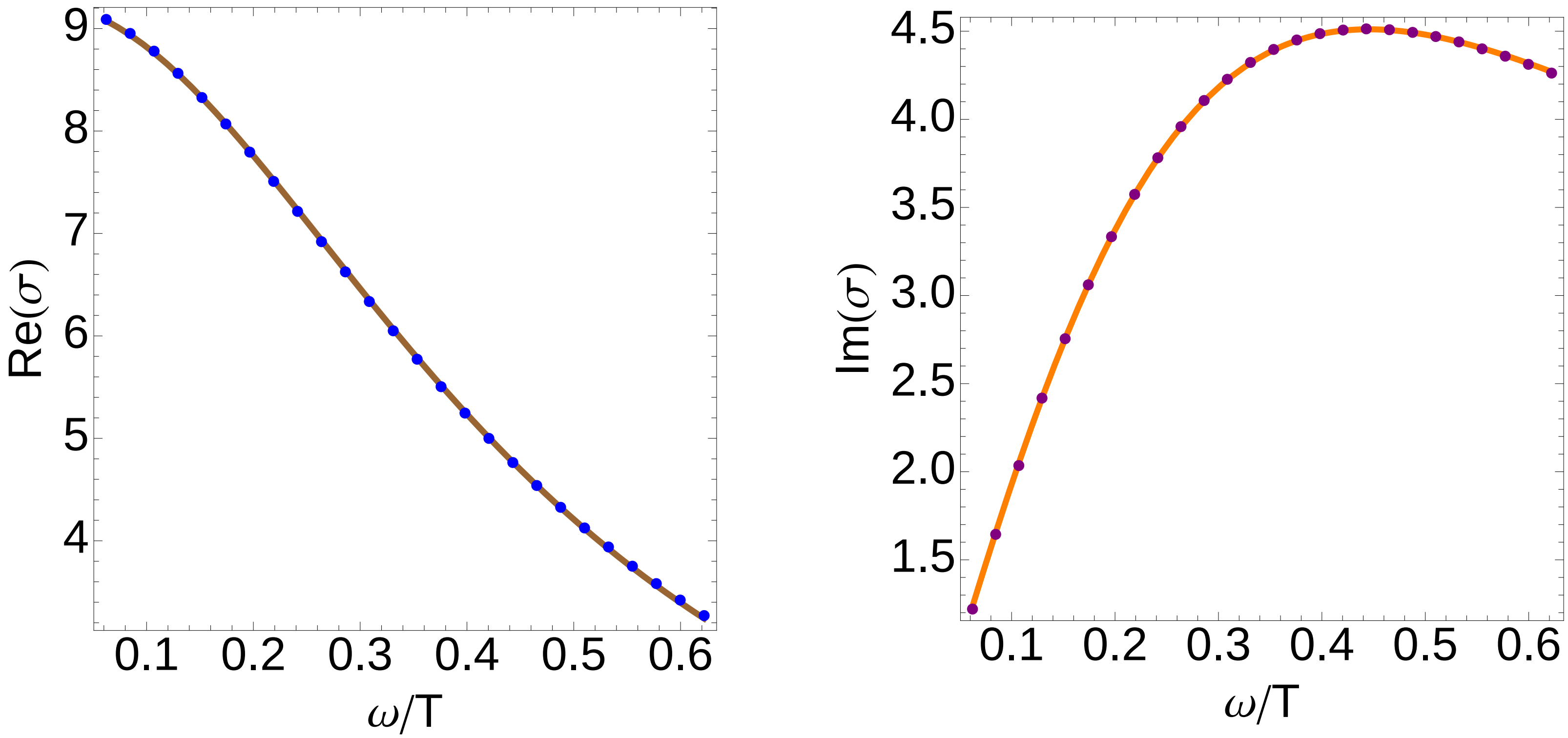
Delta function at ω = 0 due to translation invariance

With the lattice, the delta function is smeared out

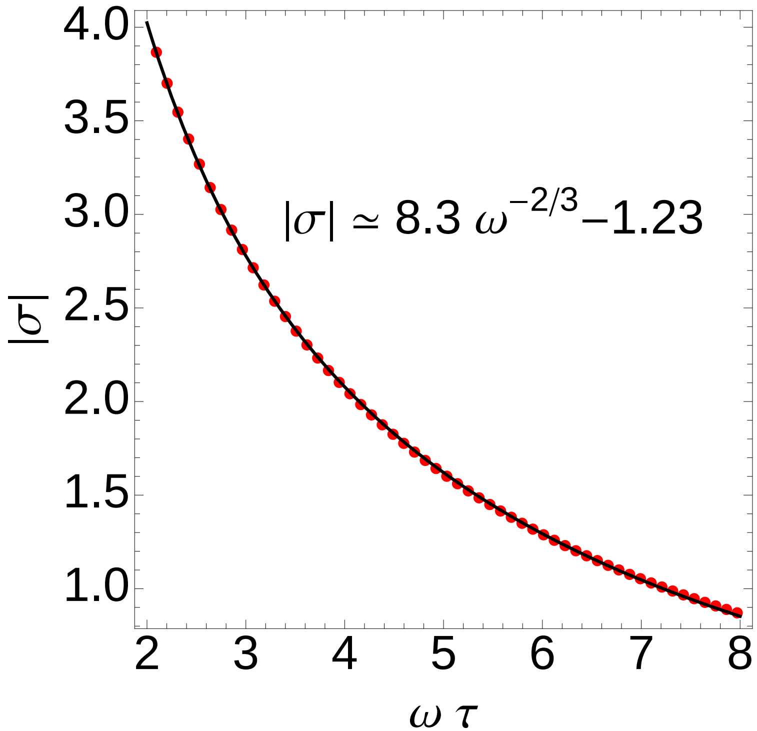


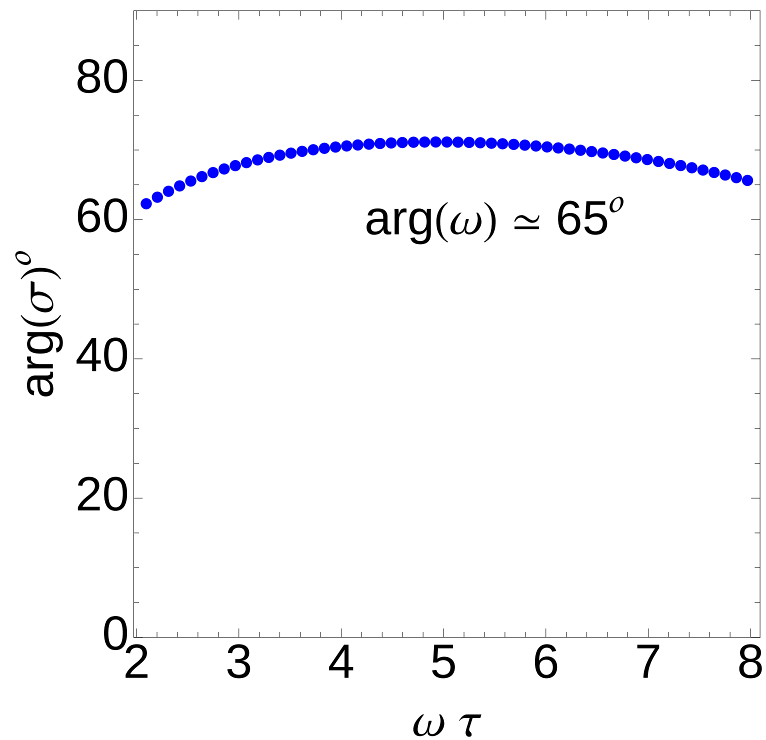
The low frequency conductivity takes the simple Drude form:

$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau}$$



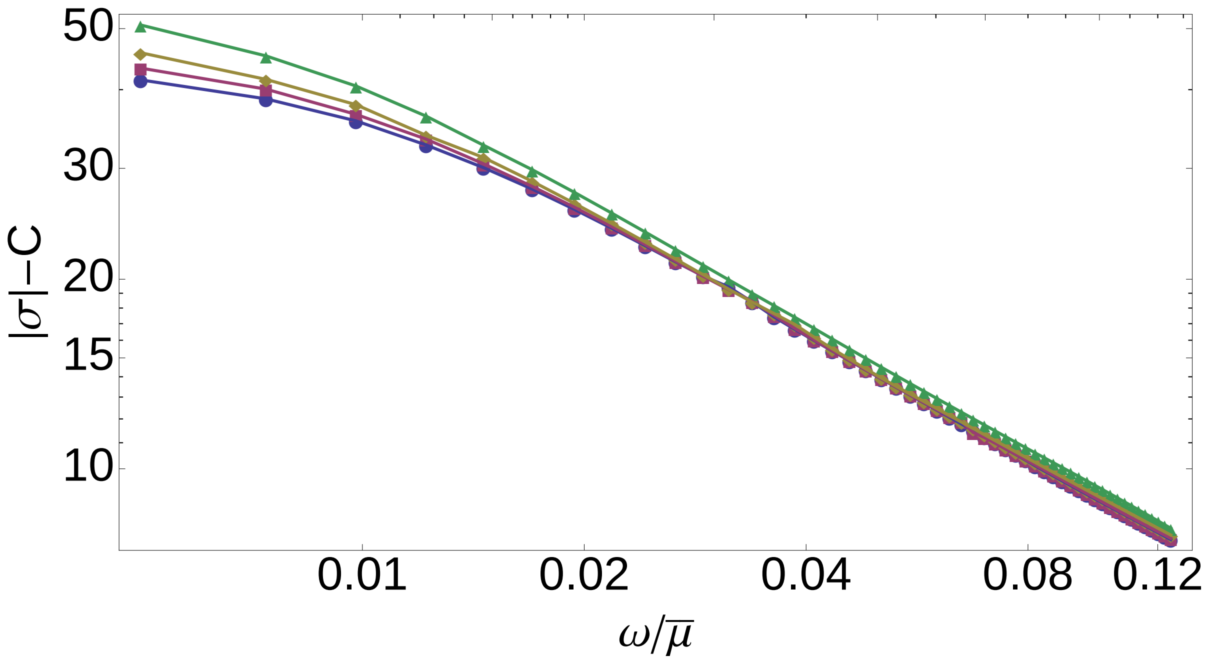
Intermediate frequency shows scaling regime





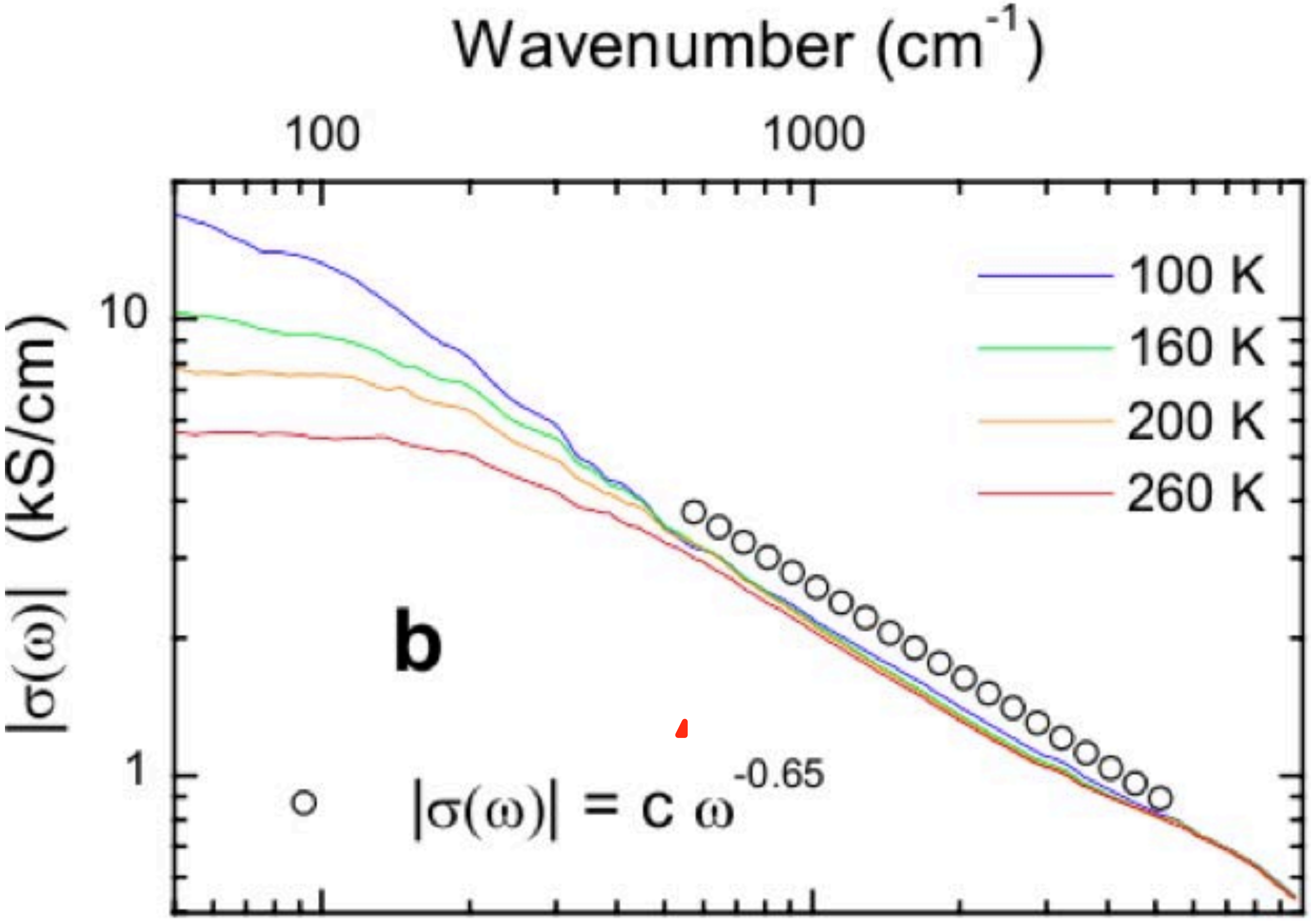
The data is very well fit by $$|\sigma| = \frac{B}{\omega^{2/3}} + C$$

The exponent 2/3 is robust



Lines denote different temperatures
(.033 < T/μ < .055)

Comparison with the cuprates
(van der Marel, et al 2003)



$$Bi_2Sr_2Ca_{0.92}Y_{0.08}Cu_2O_{8+\delta}$$

Superconductor with lattice

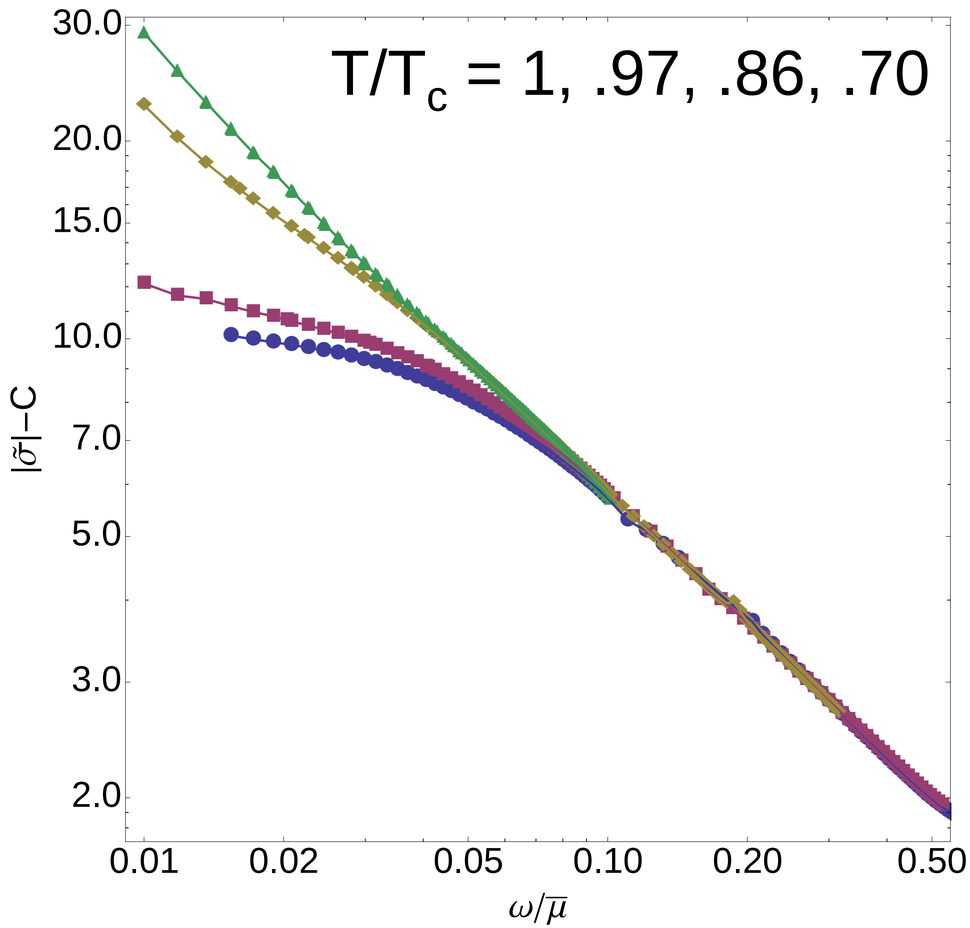
(J. Santos and G.H., 2013)

We now add the charged scalar back into our gravity theory and consider the lattice effects in the superconducting regime.

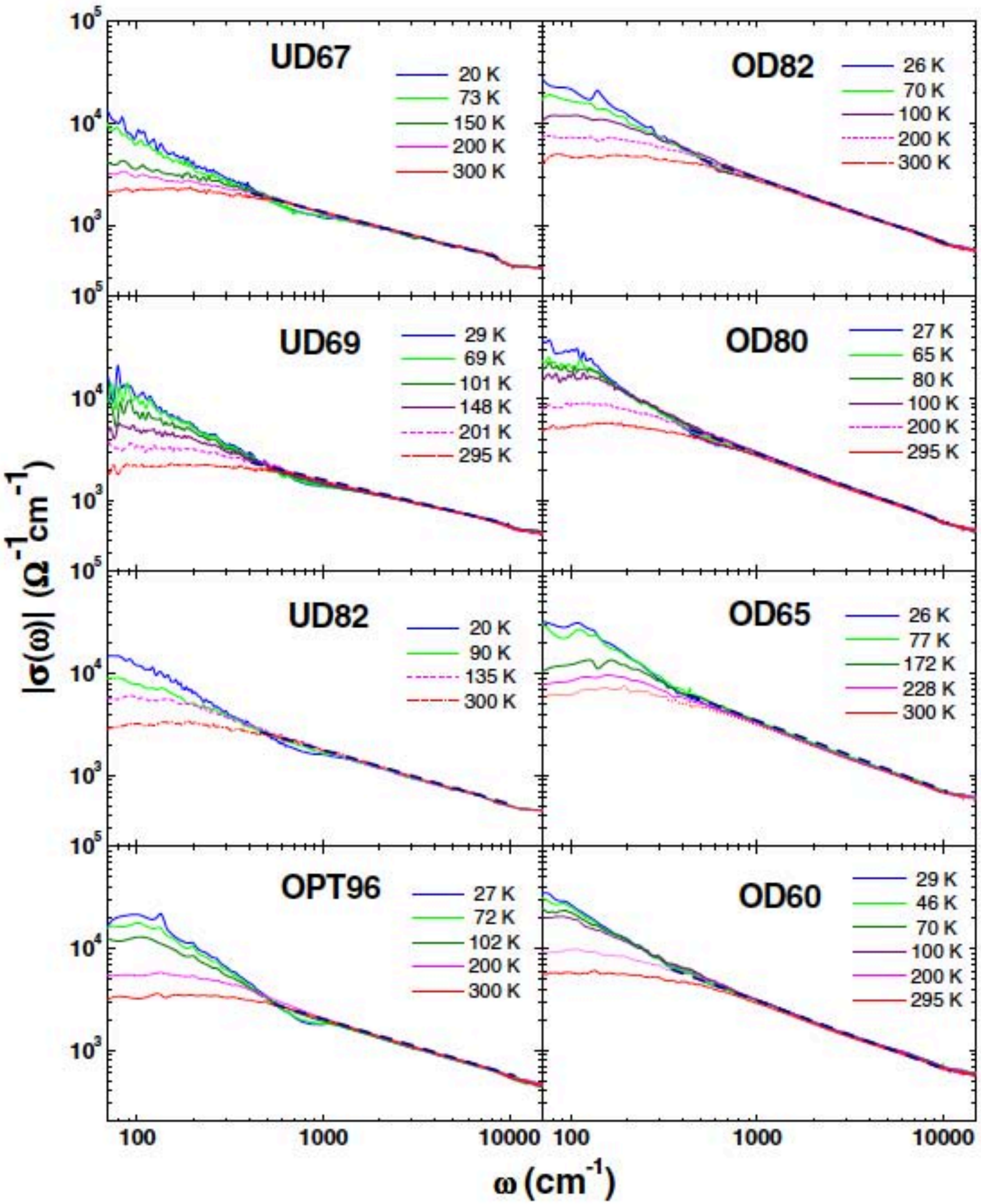
In addition to the superconducting component, there is a normal component to the conductivity which again has Drude behavior at low frequency.

Intermediate frequency conductivity again shows the same power law:

$$|\sigma(\omega)| = \frac{B}{\omega^{2/3}} + C$$



Coefficient B and exponent 2/3 are independent of T and identical to normal phase.



8 samples of BSCCO with different doping.

Each plot includes $T < T_c$ as well as $T > T_c$.

No change in the power law.

(Data from Timusk et al, 2007.)

Summary

1) In addition to describing gravity, gauge/gravity duality predicts general relativity can also describe nongravitational physics.
2) General relativity can indeed describe:
   (a) superconducting phase transitions
   (b) Josephson junctions
   (c) anomalous power laws seen in the optical conductivity of the cuprates