

Constantes fondamentales, gravitation et cosmologie

Développements récents

Jean-Philippe UZAN



Constants

Fundamental constants play an important role in physics

- set the order of magnitude of phenomena;
- allow to forge new concepts;
- linked to the structure of physical theories;
- characterize their domain of validity;

- *gravity*: linked to the equivalence principle;
- *cosmology*: at the heart of reflections on fine-tuning/naturalness/design/multiverse;

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Any parameter not determined by the theories we are using.

It has to be assume constant (no equation/ nothing more fundamental)

Reproductibility of experiments.

One can only measure them.

Reference theoretical framework

The number of physical constants depends on the level of description of the laws of nature.

In our present understanding [*General Relativity + SU(3)xSU(2)xU(1)*]:

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In our present understanding [*General Relativity* + $SU(3) \times SU(2) \times U(1)$]:

- G : Newton constant (**1**)
 - **6** Yukawa coupling for quarks
 - **3** Yukawa coupling for leptons
 - mass and VEV of the Higgs boson: **2**
 - CKM matrix: **4** parameters
 - Non-gravitational coupling constants: **3**
 - Λ_{uv} : **1**
 - c, \hbar : **2**
 - cosmological constant
- 22** constants
19 parameters

Thus number can *increase* or *decrease* with our knowledge of physics

Constant	Symbol	Value
Speed of light	c	$299\,792\,458\text{ m s}^{-1}$
Planck constant (reduced)	\hbar	$1.054\,571\,628(53) \times 10^{-34}\text{ J s}$
Newton constant	G	$6.674\,28(67) \times 10^{-11}\text{ m}^2\text{ kg}^{-1}\text{ s}^{-2}$
Weak coupling constant (at m_Z)	$g_2(m_Z)$	0.6520 ± 0.0001
Strong coupling constant (at m_Z)	$g_3(m_Z)$	1.221 ± 0.022
Weinberg angle	$\sin^2 \theta_W(91.2\text{ GeV})_{\overline{\text{MS}}}$	0.23120 ± 0.00015
Electron Yukawa coupling	h_e	2.94×10^{-6}
Muon Yukawa coupling	h_μ	0.000607
Tauon Yukawa coupling	h_τ	0.0102156
Up Yukawa coupling	h_u	0.000016 ± 0.000007
Down Yukawa coupling	h_d	0.00003 ± 0.00002
Charm Yukawa coupling	h_c	0.0072 ± 0.0006
Strange Yukawa coupling	h_s	0.0006 ± 0.0002
Top Yukawa coupling	h_t	1.002 ± 0.029
Bottom Yukawa coupling	h_b	0.026 ± 0.003
Quark CKM matrix angle	$\sin \theta_{12}$	0.2243 ± 0.0016
	$\sin \theta_{23}$	0.0413 ± 0.0015
	$\sin \theta_{13}$	0.0037 ± 0.0005
Quark CKM matrix phase	δ_{CKM}	1.05 ± 0.24
Higgs potential quadratic coefficient	$\hat{\mu}^2$? $-(250.6 \pm 1.2)\text{ GeV}^2$
Higgs potential quartic coefficient	λ	? 1.015 ± 0.05
QCD vacuum phase	θ_{QCD}	$< 10^{-9}$

$$m_H = (125.3 \pm 0.6)\text{ GeV}$$

$$v = (246.7 \pm 0.2)\text{ GeV}$$

Constants and relativity

« C'est alors, considérant ces faits, qu'il me vint à l'esprit que si l'on supprimait totalement la résistance du milieu, tous les corps descendraient avec la même vitesse. »

Galilée, *in Discours concernant deux sciences nouvelles*, 1638
Traduction de Maurice Clavelin, PUF, 1995.

« Il y a une puissance de la gravité, qui concerne tous les corps, proportionnelle aux différentes quantités de matière qu'ils contiennent. »

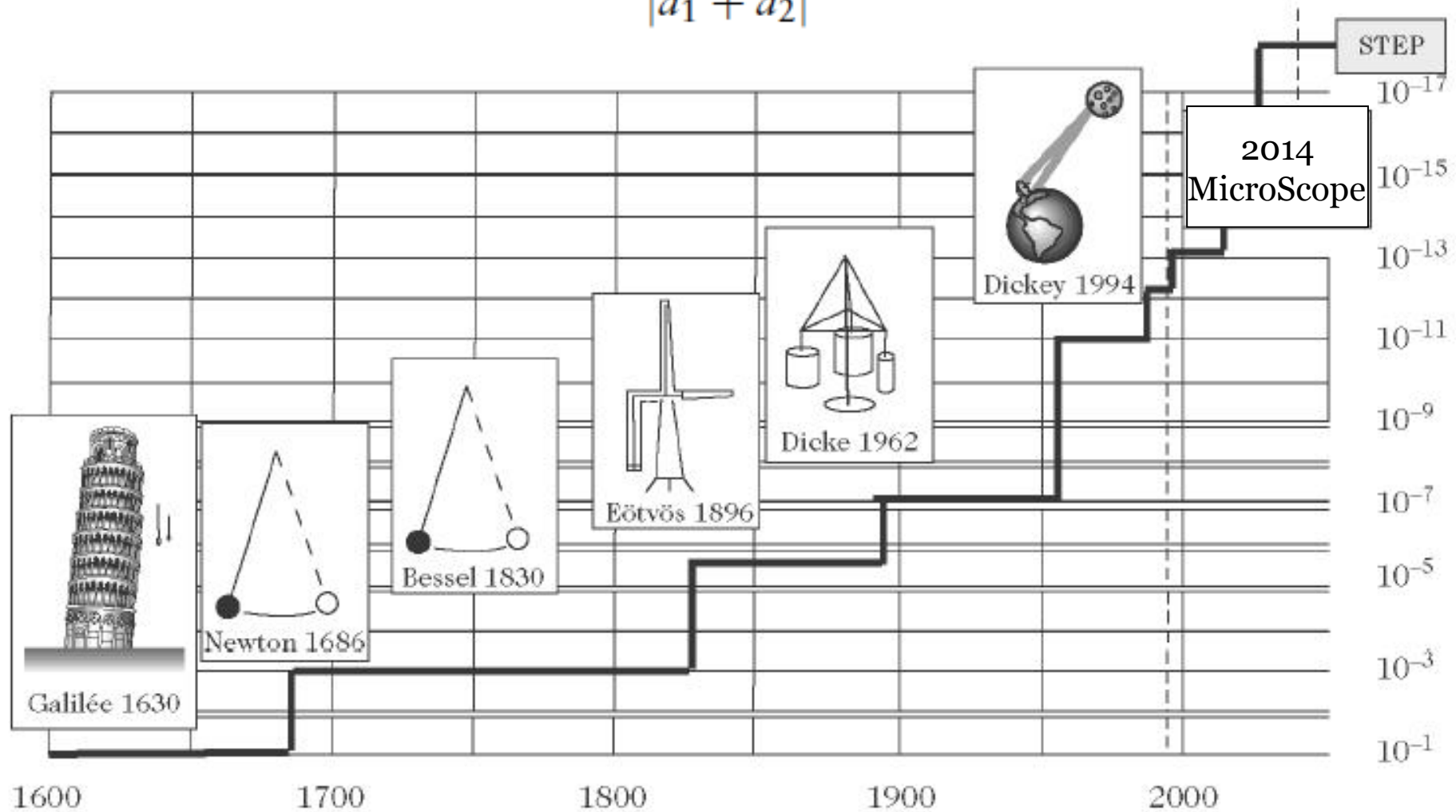
« Cette force est toujours proportionnelle à la quantité de matière des corps, & elle ne diffère de ce qu'on appelle l'inertie de la matière que par la manière de la concevoir. »

« La force de la pesanteur entre les différentes particules de tout corps est inversement proportionnelle au carré des distances des positions des particules. »

Isaac Newton, *in Principia*, Londres, 1687
Traduction d'Émilie du Châtelet, Paris, 1759.

Tests on the universality of free fall

$$\eta \equiv 2 \frac{|a_1 - a_2|}{|a_1 + a_2|}$$



On the basis of general relativity

The equivalence principle takes much more importance in general relativity

It is based on **Einstein equivalence principle**

universality of free fall

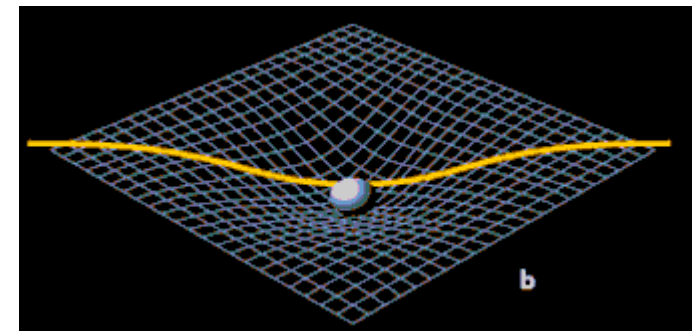
local Lorentz invariance

local position invariance

Not a basic principle of physics but mostly an empirical fact.



If this principle holds then gravity is a consequence of the geometry of spacetime



This principle has been a driving idea in theories of gravity from Galileo to Einstein

GR in a nutshell

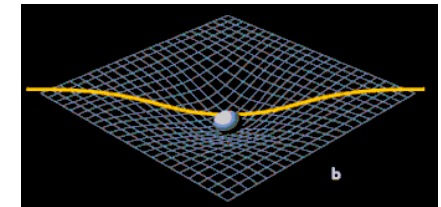
Underlying hypothesis

Equivalence principle

- Universality of free fall
- Local lorentz invariance
- Local position invariance

Physical
metric

$$S_{\text{matter}}(\psi, g_{\mu\nu})$$



GR in a nutshell

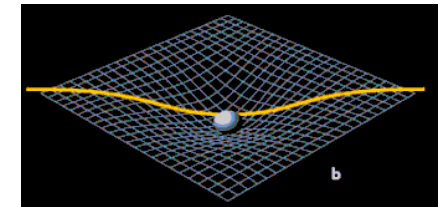
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Physical
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$$S_{matter}(\psi, g_{\mu\nu})$$



gravitational
metric

Dynamics

$$S_{grav} = \frac{c^3}{16\pi G} \int \sqrt{-g_*} R_* d^4x$$

Relativity

$$g_{\mu\nu} = g_{\mu\nu}^*$$

Equivalence principle and constants

In general relativity, any test particle follows a geodesic, which does not depend on the mass or on the chemical composition

Imagine some constants are space-time dependent

1- Local position invariance is violated.



Equivalence principle and constants

In general relativity, any test particle follow a geodesic, which does not depend on the mass or on the chemical composition



Imagine some constants are space-time dependent

1- Local position invariance is violated.

2- Universality of free fall has also to be violated

Mass of test body = mass of its constituents + binding energy

In Newtonian terms, a free motion implies $\frac{d\vec{p}}{dt} = m \frac{d\vec{v}}{dt} = \vec{0}$

But, now

$$\frac{d\vec{p}}{dt} = \vec{0} = m\vec{a} + \underbrace{\frac{dm}{d\alpha} \dot{\alpha} \vec{v}}_{m\vec{a}_{\text{anomalous}}}$$

Varying constants: constructing theories

$$S[\phi, \bar{\psi}, A_\mu, h_{\mu\nu}, \dots; c_1, \dots, c_2]$$

Varying constants: constructing theories

$$S[\phi, \bar{\psi}, A_\mu, h_{\mu\nu}, \dots; c_1, \dots, c_2]$$

If a constant is varying, this implies that it has to be replaced by a dynamical field

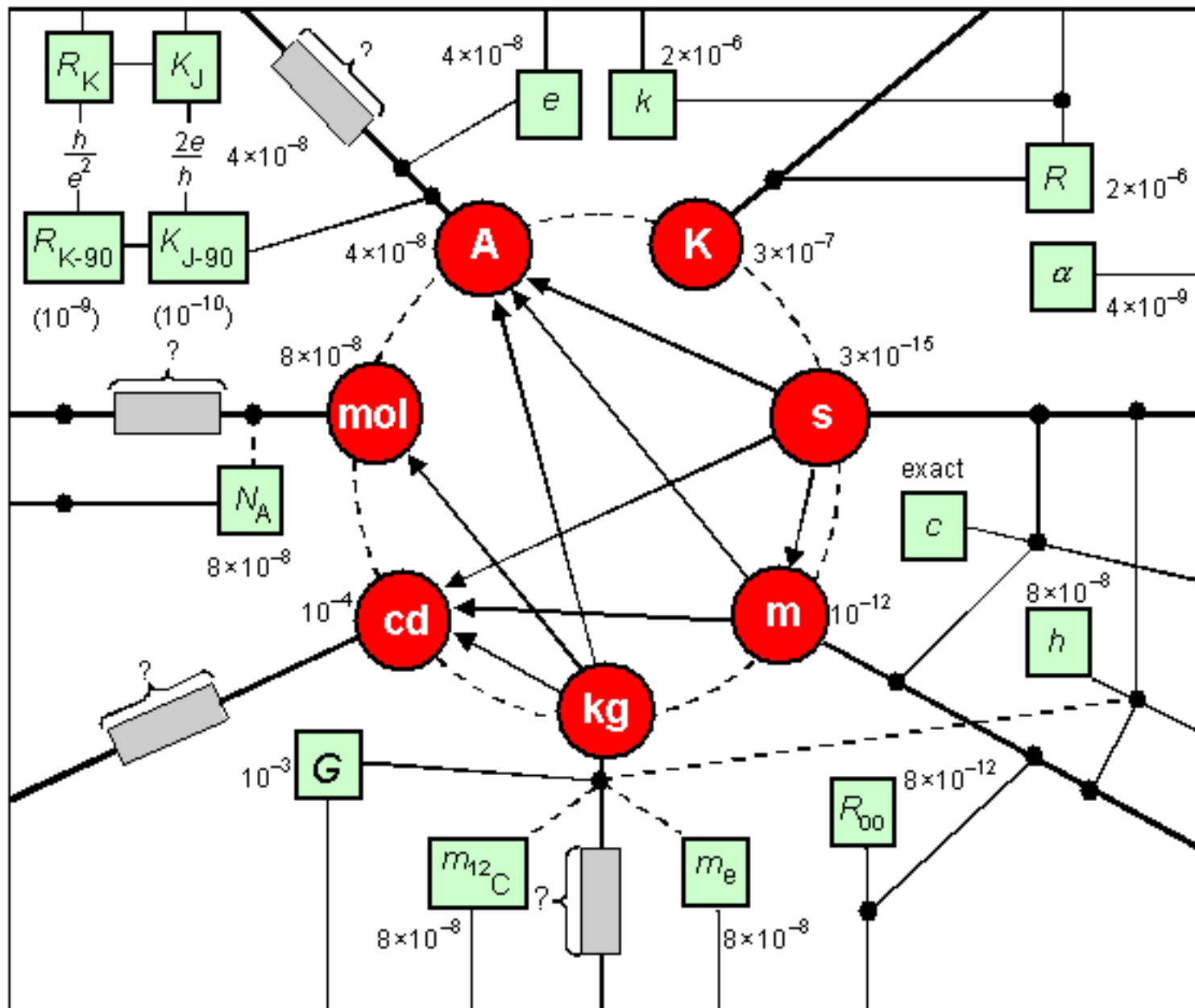
This has 2 consequences:

1- the equations derived with this parameter constant will be modified
one cannot just make it vary in the equations

2- the theory will provide an equation of evolution for this new parameter

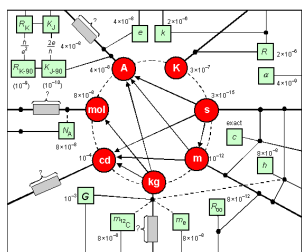
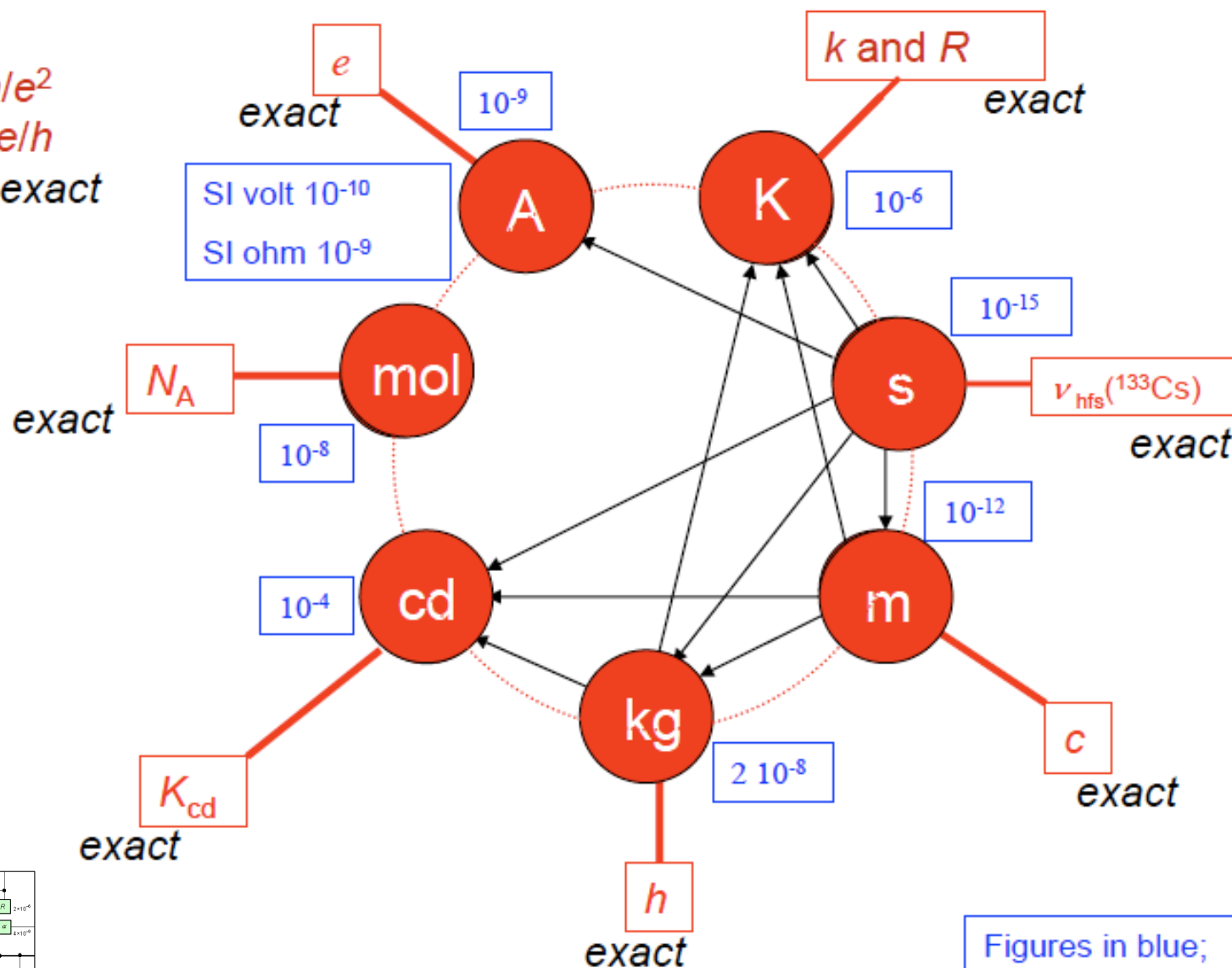
The field responsible for the time variation of the « constant » is also responsible for a long-range (composition-dependent) interaction
i.e. at the origin of the deviation from General Relativity.

Constants and systems of units



Constants and systems of units

$R_K: h/e^2$
 $K_J: 2e/h$
 both exact



Figures in blue;
 approximate relative
 uncertainty of realization

http://www.bipm.org/en/si/new_si/

- **Modelisation of gyromagnetic factors**

[with K. Olive & Fang Luo (2011)]

- **Planck & CMB constraints**

[with S. Galli, O. Fabre, S. Prunet, E. Menegoni, et al. (2013)]

- **Big bang nucleosynthesis**

[with A. Coc, E Vangioni, L. Olive (2007-2013)]

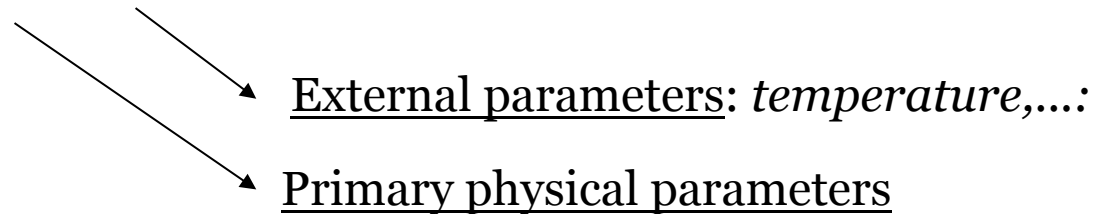
- **Population III stars**

[with A. Coc, E. Vangioni, K. Olive, P. Descouvemont, G. Meynet, S. Ekström (2010)]

Observables and primary constraints

A given physical system gives us an observable quantity

$$O(G_k, X)$$



Step 1:

From a physical model of our system we can deduce the sensitivities to the primary physical parameters

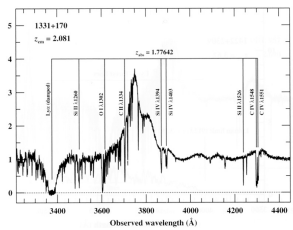
$$\kappa_{G_k} = \frac{\partial \ln O}{\partial \ln G_k}$$

Step 2:

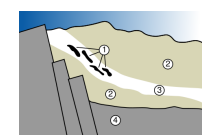
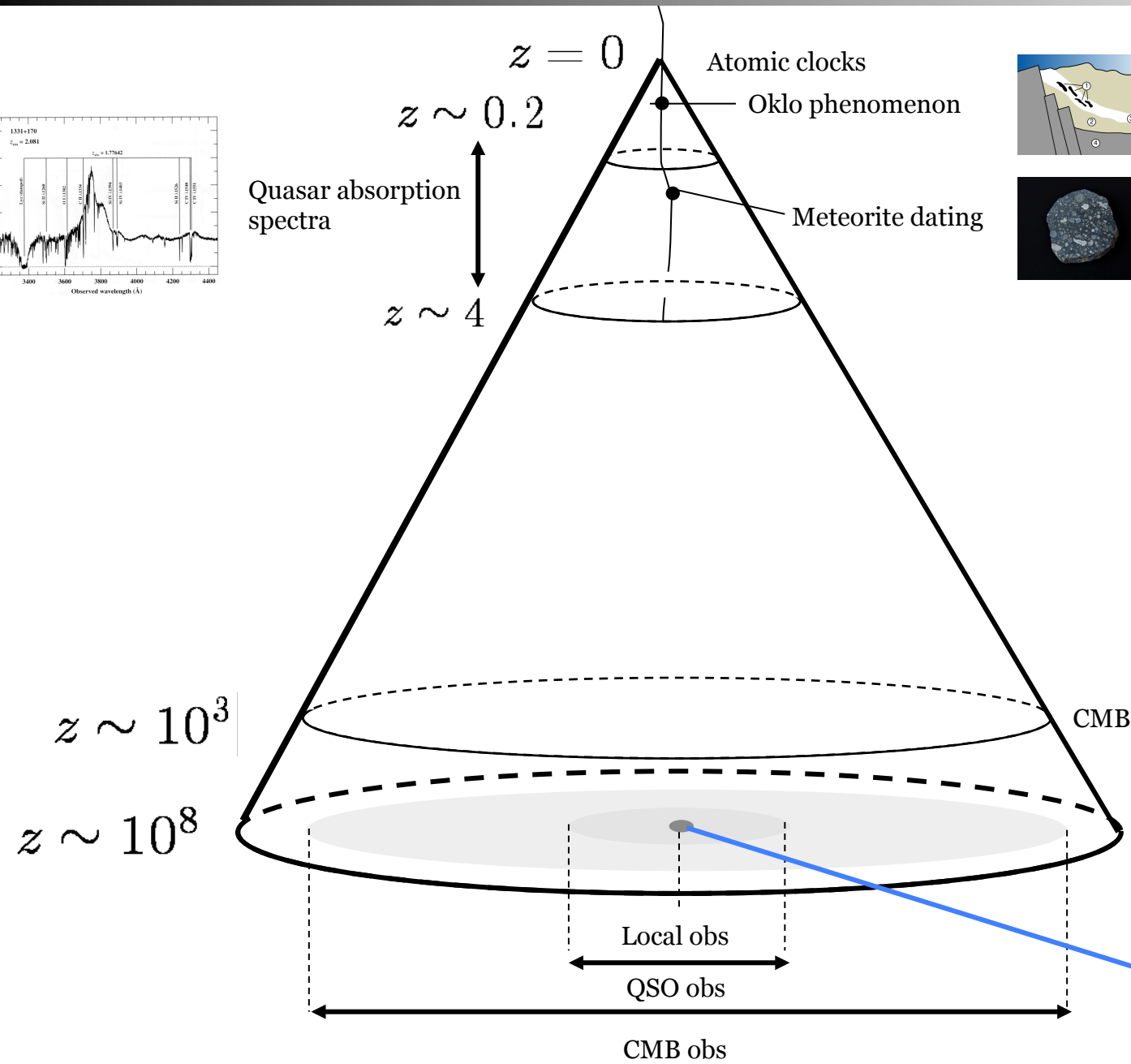
The primary physical parameters are usually not fundamental constants.

$$\Delta \ln G_k = \sum_i d_{ki} \Delta \ln c_i$$

Physical systems



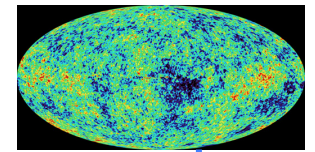
Quasar absorption spectra



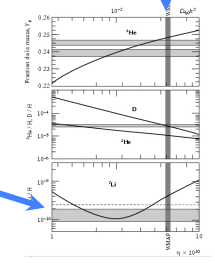
$z = 0.14$



$z = 0.43$

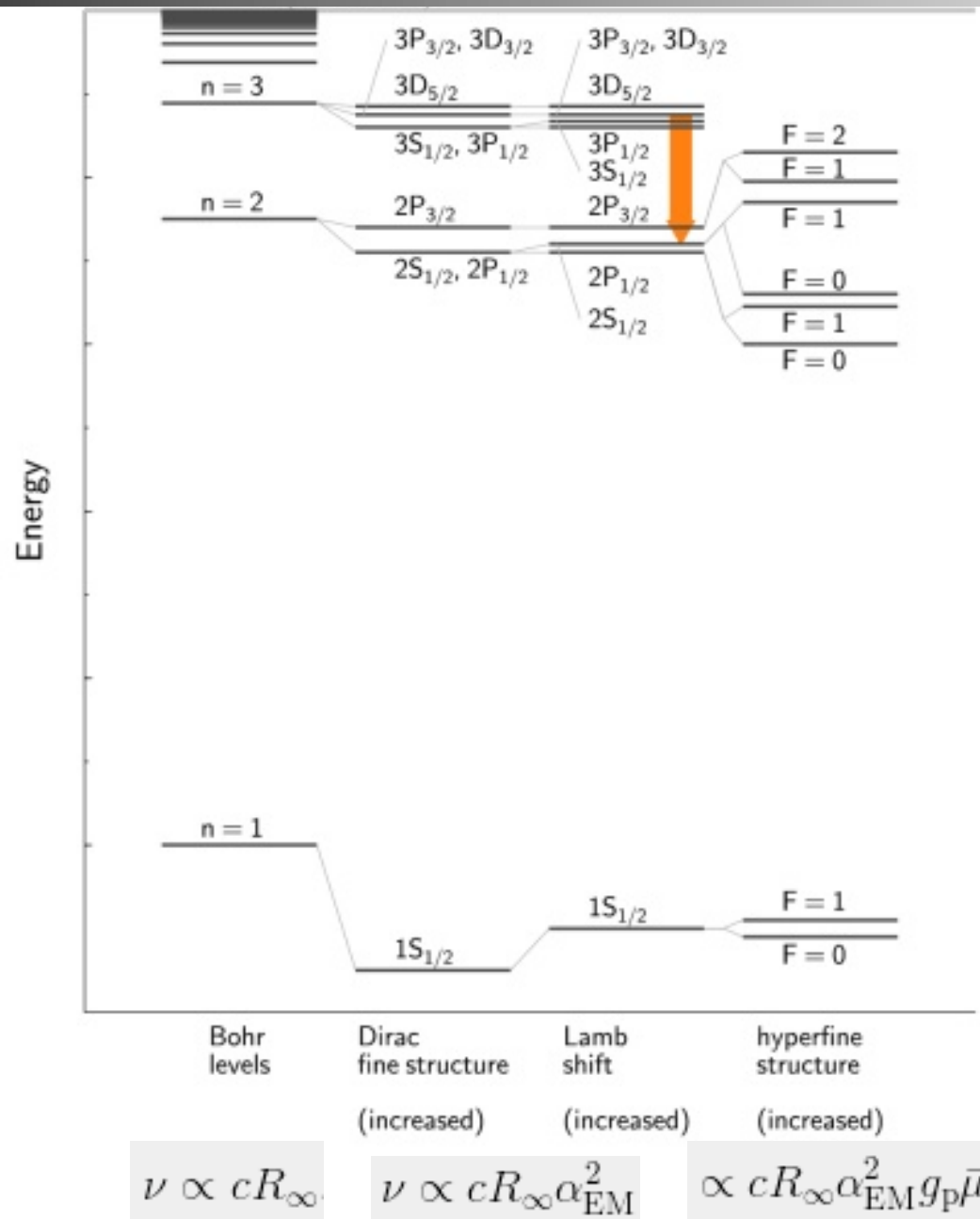


BBN



Atomic clocks
&
modelisation of gyromagnetic
factors

Hydrogen atom



Atomic clocks

General atom

$$\nu_{\text{hfs}} \simeq R_{\infty} c \times A_{\text{hfs}} \times g_i \times \alpha_{\text{EM}}^2 \times \bar{\mu} \times F_{\text{hfs}}(\alpha)$$

$$\nu_{\text{elec}} \simeq R_{\infty} c \times A_{\text{elec}} \times F_{\text{elec}}(Z, \alpha)$$

$$\kappa_{\alpha} \equiv \frac{\partial \ln F}{\partial \ln \alpha_{\text{EM}}}$$

Atom	Transition	sensitivity κ_{α}
^1H	$1s - 2s$	0.00
^{87}Rb	hf	0.34
^{133}Cs	$^2S_{1/2}(F=2) - (F=3)$	0.83
$^{171}\text{Yb}^+$	$^2S_{1/2} - ^2D_{3/2}$	0.9
$^{199}\text{Hg}^+$	$^2S_{1/2} - ^2D_{5/2}$	-3.2
^{87}Sr	$^1S_0 - ^3P_0$	0.06
$^{27}\text{Al}^+$	$^1S_0 - ^3P_0$	0.008

Atomic clocks

Clock 1	Clock 2	Constraint (yr^{-1})	Constants dependence	Reference
	$\frac{d}{dt} \ln \left(\frac{\nu_{\text{clock1}}}{\nu_{\text{clock2}}} \right)$			
^{87}Rb	^{133}Cs	$(0.2 \pm 7.0) \times 10^{-16}$	$\frac{g_{\text{Cs}}}{g_{\text{Rb}}} \alpha_{\text{EM}}^{0.49}$	Marion (2003)
^{87}Rb	^{133}Cs	$(-0.5 \pm 5.3) \times 10^{-16}$		Bize (2003)
^1H	^{133}Cs	$(-32 \pm 63) \times 10^{-16}$	$g_{\text{Cs}} \mu \alpha_{\text{EM}}^{2.83}$	Fischer (2004)
$^{199}\text{Hg}^+$	^{133}Cs	$(0.2 \pm 7) \times 10^{-15}$	$g_{\text{Cs}} \mu \alpha_{\text{EM}}^{6.05}$	Bize (2005)
$^{199}\text{Hg}^+$	^{133}Cs	$(3.7 \pm 3.9) \times 10^{-16}$		Fortier (2007)
$^{171}\text{Yb}^+$	^{133}Cs	$(-1.2 \pm 4.4) \times 10^{-15}$	$g_{\text{Cs}} \mu \alpha_{\text{EM}}^{1.93}$	Peik (2004)
$^{171}\text{Yb}^+$	^{133}Cs	$(-0.78 \pm 1.40) \times 10^{-15}$		Peik (2006)
^{87}Sr	^{133}Cs	$(-1.0 \pm 1.8) \times 10^{-15}$	$g_{\text{Cs}} \mu \alpha_{\text{EM}}^{2.77}$	Blatt (2008)
^{87}Dy	^{87}Dy			Cingöz (2008)
$^{27}\text{Al}^+$	$^{199}\text{Hg}^+$	$(-5.3 \pm 7.9) \times 10^{-17}$	$\alpha_{\text{EM}}^{-3.208}$	Blatt (2008)

Atomic clocks: from observations to constraints

The gyromagnetic factors can be expressed in terms of g_p and g_n (shell model).

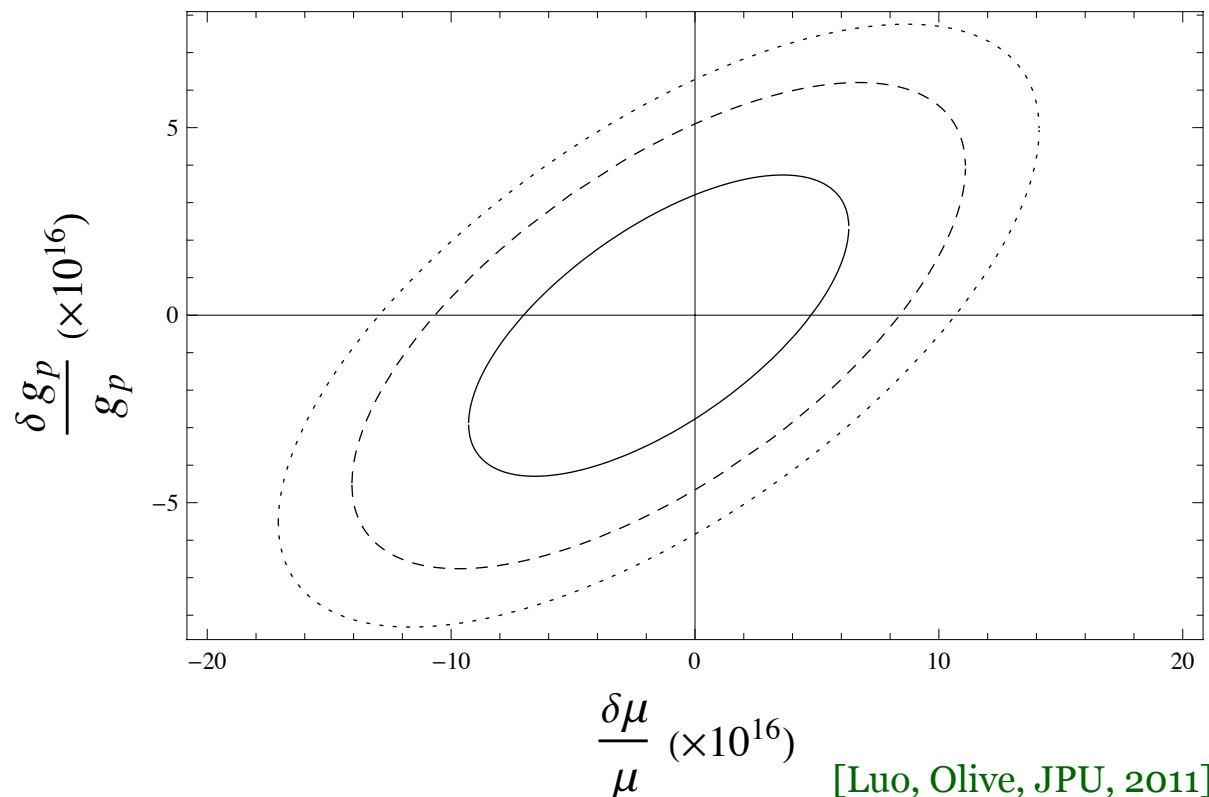
$$\frac{\delta g_{\text{Cs}}}{g_{\text{Cs}}} \sim -1.266 \frac{\delta g_p}{g_p} \quad \frac{\delta g_{\text{Rb}}}{g_{\text{Rb}}} \sim 0.736 \frac{\delta g_p}{g_p}$$

All atomic clock constraints take the form $\frac{\dot{\nu}_{AB}}{\nu_{AB}} = \lambda_{g_p} \frac{\dot{g}_p}{g_p} + \lambda_{\mu} \frac{\dot{\mu}}{\mu} + \lambda_{\alpha} \frac{\dot{\alpha}}{\alpha}$.

Using Al-Hg to constrain α , the combination of other clocks allows to constraint $\{\mu, g_p\}$.

Note: one actually needs to include the effects of the polarization of the non-valence nucleons and spin-spin interaction.

[Flambaum, 0302015,...



Atomic clocks: from observations to constraints

One then needs to express m_p and g_p in terms of the quark masses and Λ_{QCD} as

$$\frac{\delta g_p}{g_p} = \kappa_u \frac{\delta m_u}{m_u} + \kappa_d \frac{\delta m_d}{m_d} + \kappa_s \frac{\delta m_s}{m_s} + \kappa_{\text{QCD}} \frac{\delta \Lambda_{\text{QCD}}}{\Lambda_{\text{QCD}}},$$
$$\frac{\delta m_p}{m_p} = f_{T_u} \frac{\delta m_u}{m_u} + f_{T_d} \frac{\delta m_d}{m_d} + f_{T_s} \frac{\delta m_s}{m_s} + f_{T_g} \frac{\delta \Lambda_{\text{QCD}}}{\Lambda_{\text{QCD}}},$$

$$m_i = h_i v$$

Assuming unification.

$$\frac{\dot{\nu}_{AB}}{\nu_{AB}} = \lambda_{g_p} \frac{\dot{g}_p}{g_p} + \lambda_\mu \frac{\dot{\mu}}{\mu} + \lambda_\alpha \frac{\dot{\alpha}}{\alpha} \longrightarrow \frac{\dot{\nu}_{AB}}{\nu_{AB}} = C_{AB} \frac{\dot{\alpha}}{\alpha}$$

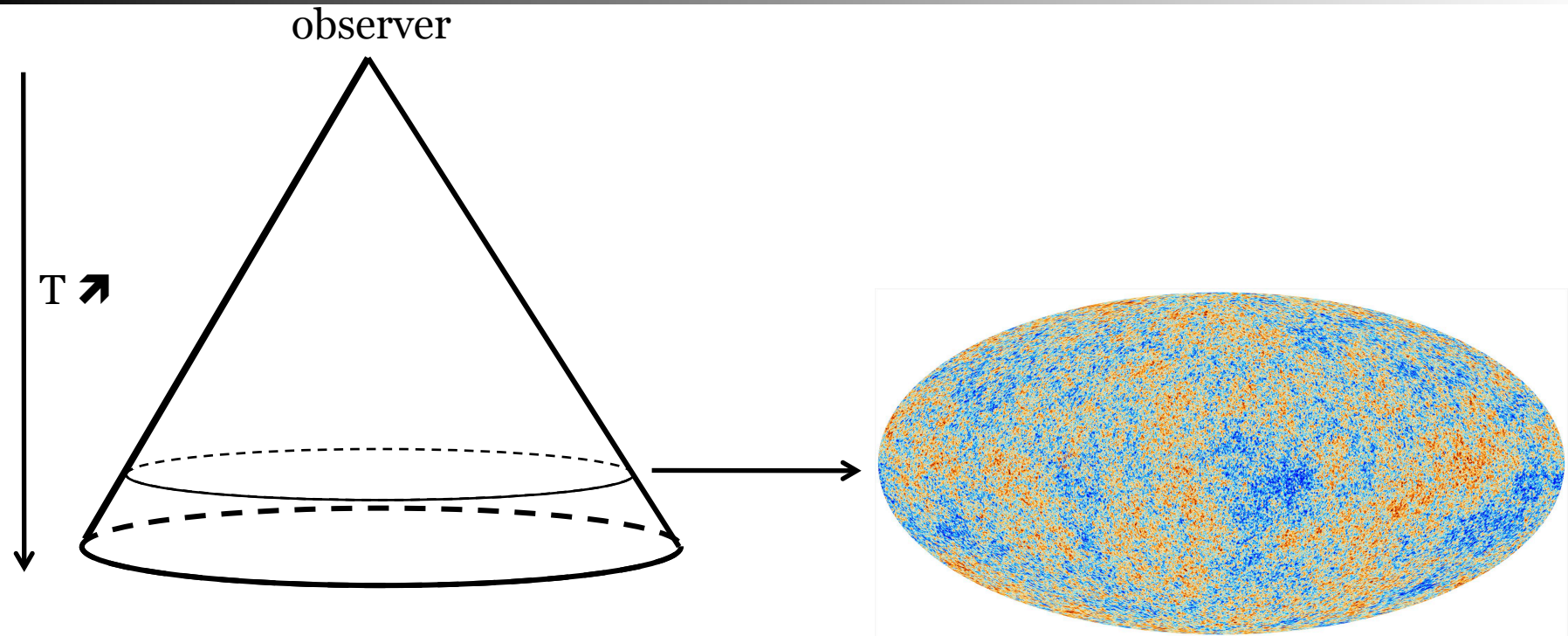
C_{AB} coefficients range from 70 to 0.6 typically.

Model-dependence remains quite large.

Cosmic microwave background

[with S. Galli, O. Fabre, S. Prunet, E. Menegoni, et al. (2013)]

Recombination



Reaction rate $\Gamma_T = n_e \sigma_T$

- 1- Recombination $n_e(t), \dots$
- 2- Decoupling $\Gamma \ll H$
- 3- Last scattering

Out-of-equilibrium process – requires to solve a Boltzmann equation

Dependence on the constants

Recombination of hydrogen and helium

Gravitational dynamics (expansion rate)

predictions depend on G, α, m_e

$$\sigma_T = \frac{8\pi}{3} \frac{\hbar^2}{m_e^2 c^2} \alpha_{\text{EM}}^2$$

We thus consider the parameters:

$$\{\omega_b, \omega_c, H_0, \tau, n_s, A_s, \alpha \text{ or } m_e\}$$

All the dependences of the constants can be included in a CMB code (recombination part: RECFAST):

$E = h\nu$ Binding energies

σ_T Thomson cross-section

σ_n photoionisation cross-sections

α recombination parameters

β photoionisation parameters

K cosmological redshifting of the photons

A Einstein coefficient

Λ_{2s} 2s decay rate by 2γ

$$\nu_i = \nu_{i0} \left(\frac{\alpha}{\alpha_0} \right)^2 \left(\frac{m_e}{m_{e0}} \right)$$

$$\sigma_T = \sigma_{T0} \left(\frac{\alpha}{\alpha_0} \right)^2 \left(\frac{m_e}{m_{e0}} \right)^{-2}$$

$$\sigma_n = \sigma_{n0} \left(\frac{\alpha}{\alpha_0} \right)^{-1} \left(\frac{m_e}{m_{e0}} \right)^{-2}$$

$$\alpha_i = \alpha_{i0} \left(\frac{\alpha}{\alpha_0} \right)^3 \left(\frac{m_e}{m_{e0}} \right)^{-3/2}$$

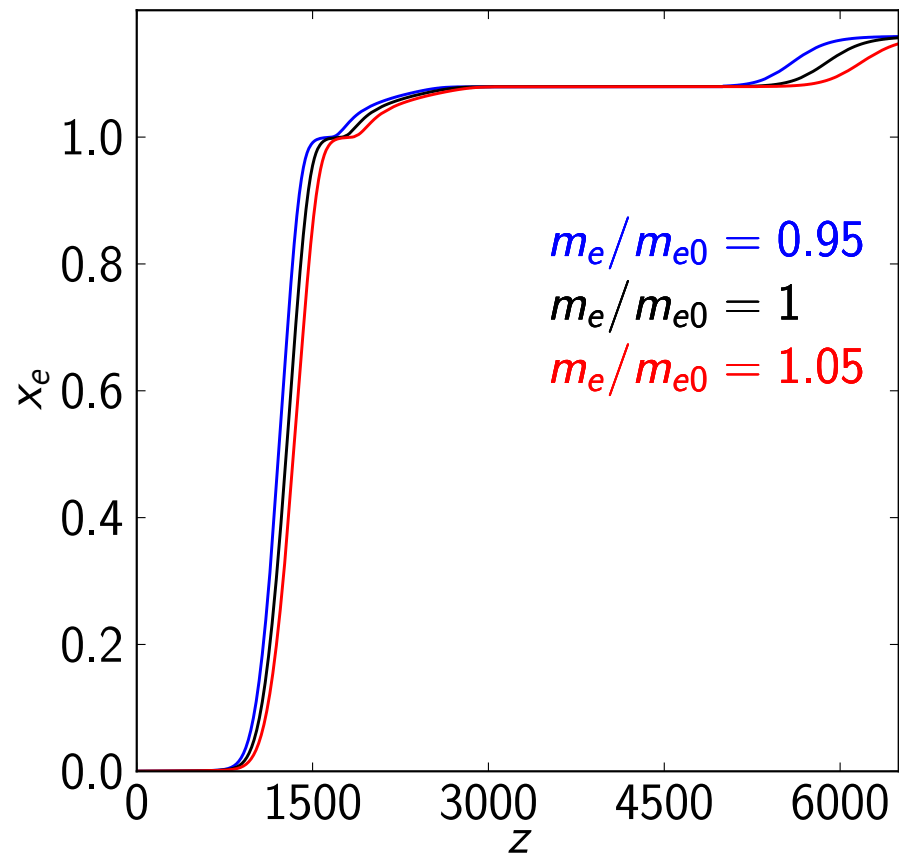
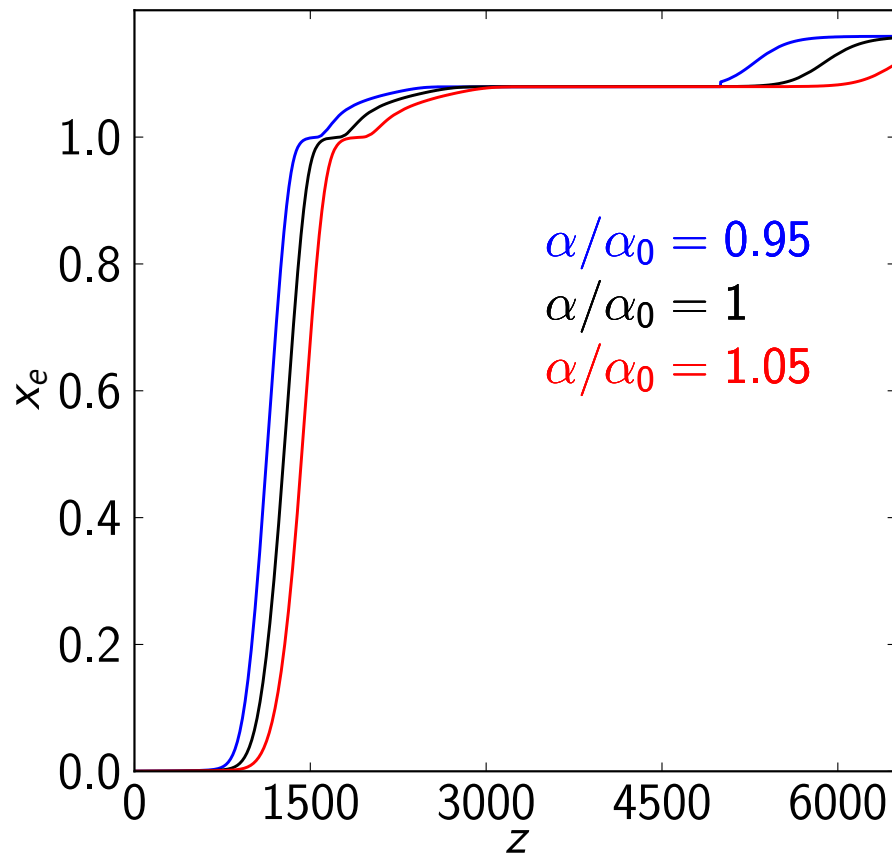
$$\beta_i = \beta_{i0} \left(\frac{\alpha}{\alpha_0} \right)^3$$

$$K_i = K_{i0} \left(\frac{\alpha}{\alpha_0} \right)^{-6} \left(\frac{m_e}{m_{e0}} \right)^{-3}$$

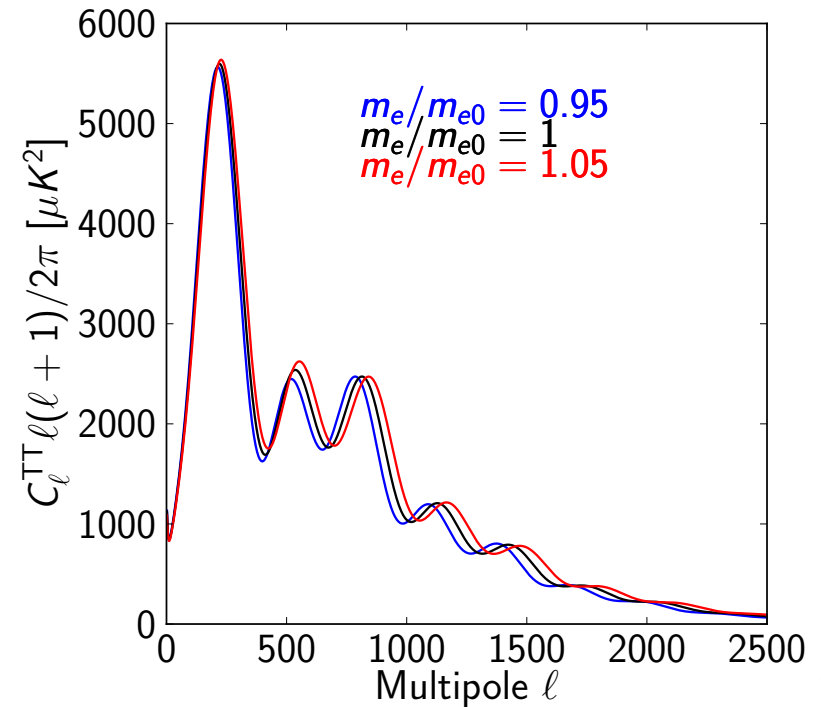
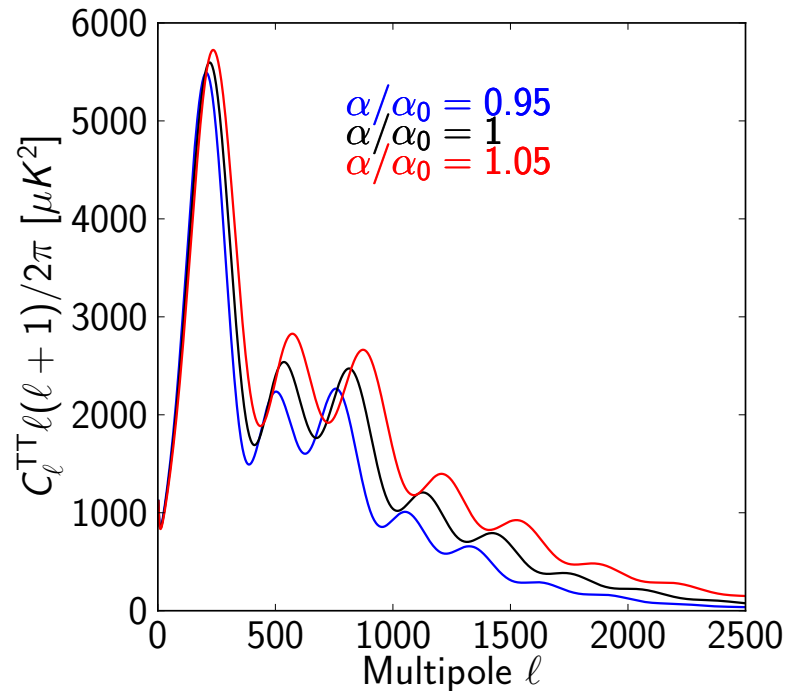
$$A_i = A_{i0} \left(\frac{\alpha}{\alpha_0} \right)^5 \left(\frac{m_e}{m_{e0}} \right)$$

$$\Lambda_i = \Lambda_{i0} \left(\frac{\alpha}{\alpha_0} \right)^8 \left(\frac{m_e}{m_{e0}} \right)$$

Dependence on the constants



Effect on the temperature power spectrum

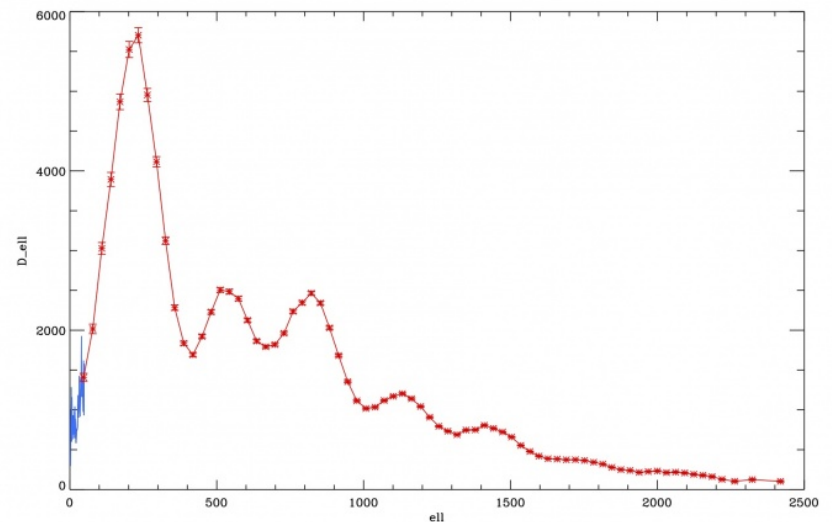


Increase of α induces

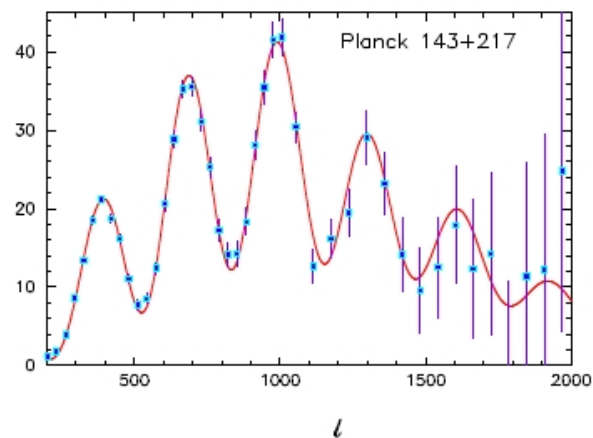
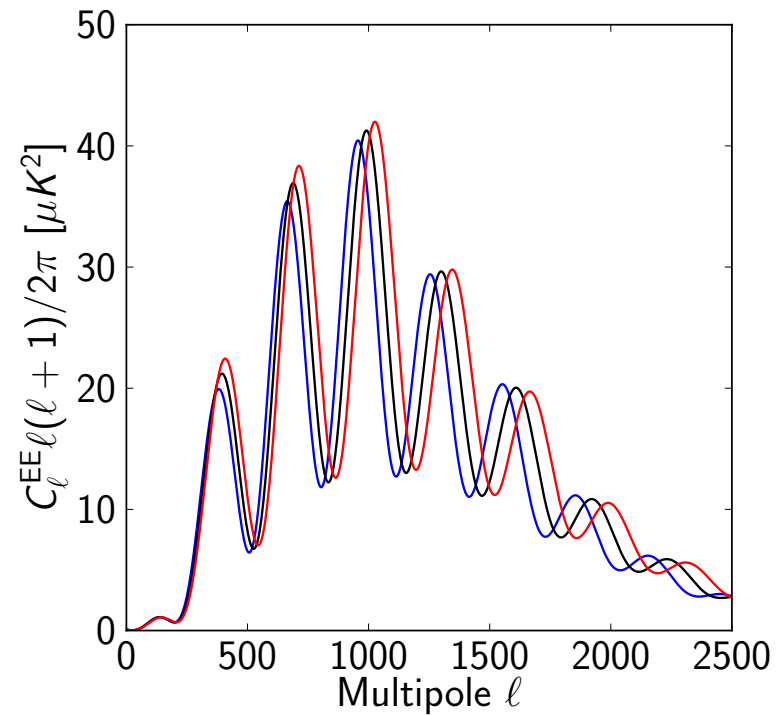
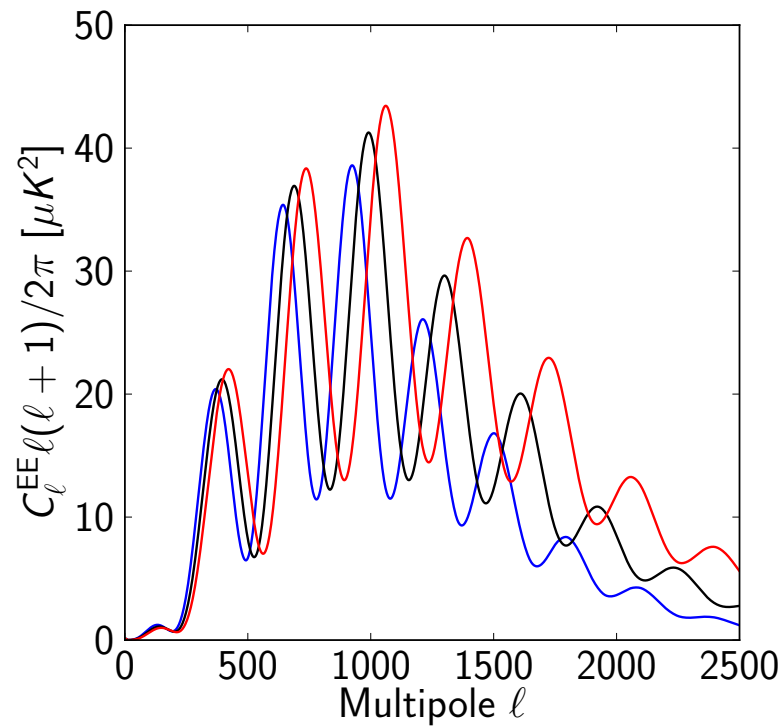
- an earlier decoupling
- smaller sound horizon
- **shift of the peaks to higher multipoles**

- an increase of amplitude of large scale (early ISW)
- an increase of amplitude at small scales (Silk damping)

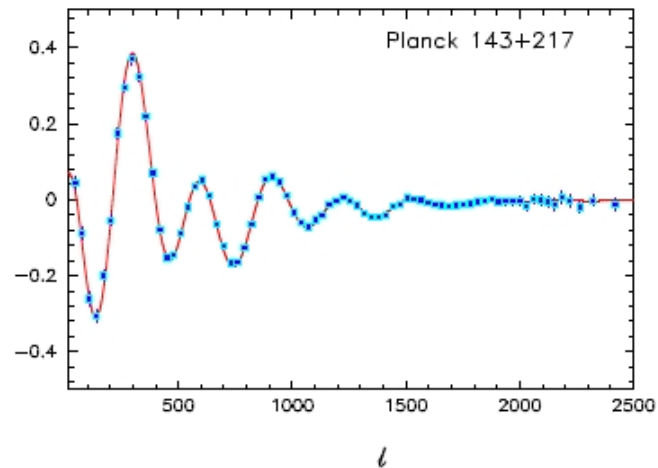
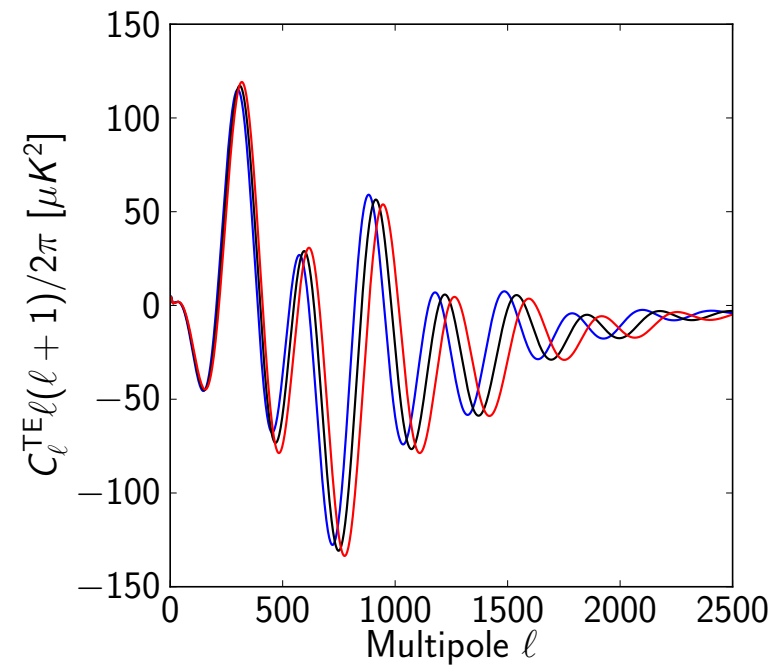
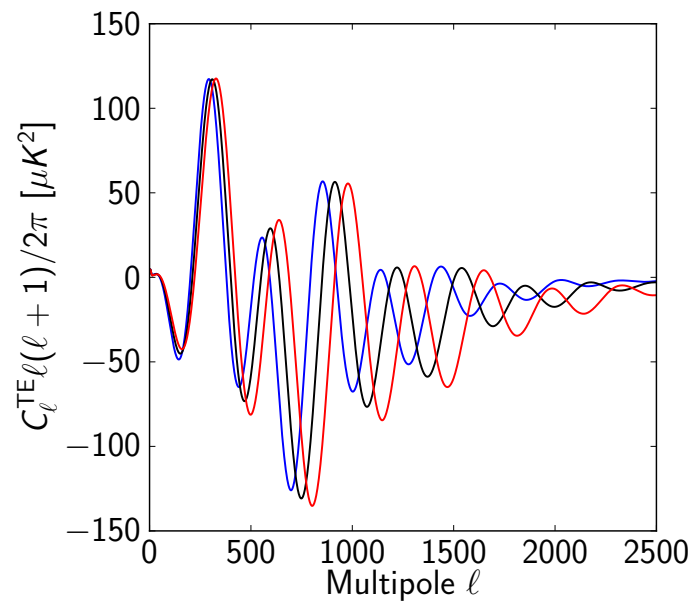
$$\lambda_D^2 = \frac{1}{6} \int_0^{\eta_{dec}} \frac{d\eta}{\sigma_T n_e a} \left[\frac{R^2 + \frac{16}{15}(1+R)}{(1+R)^2} \right] \propto \frac{1}{\sigma_T} \propto \frac{1}{\alpha^2 m_e^{-2}}$$



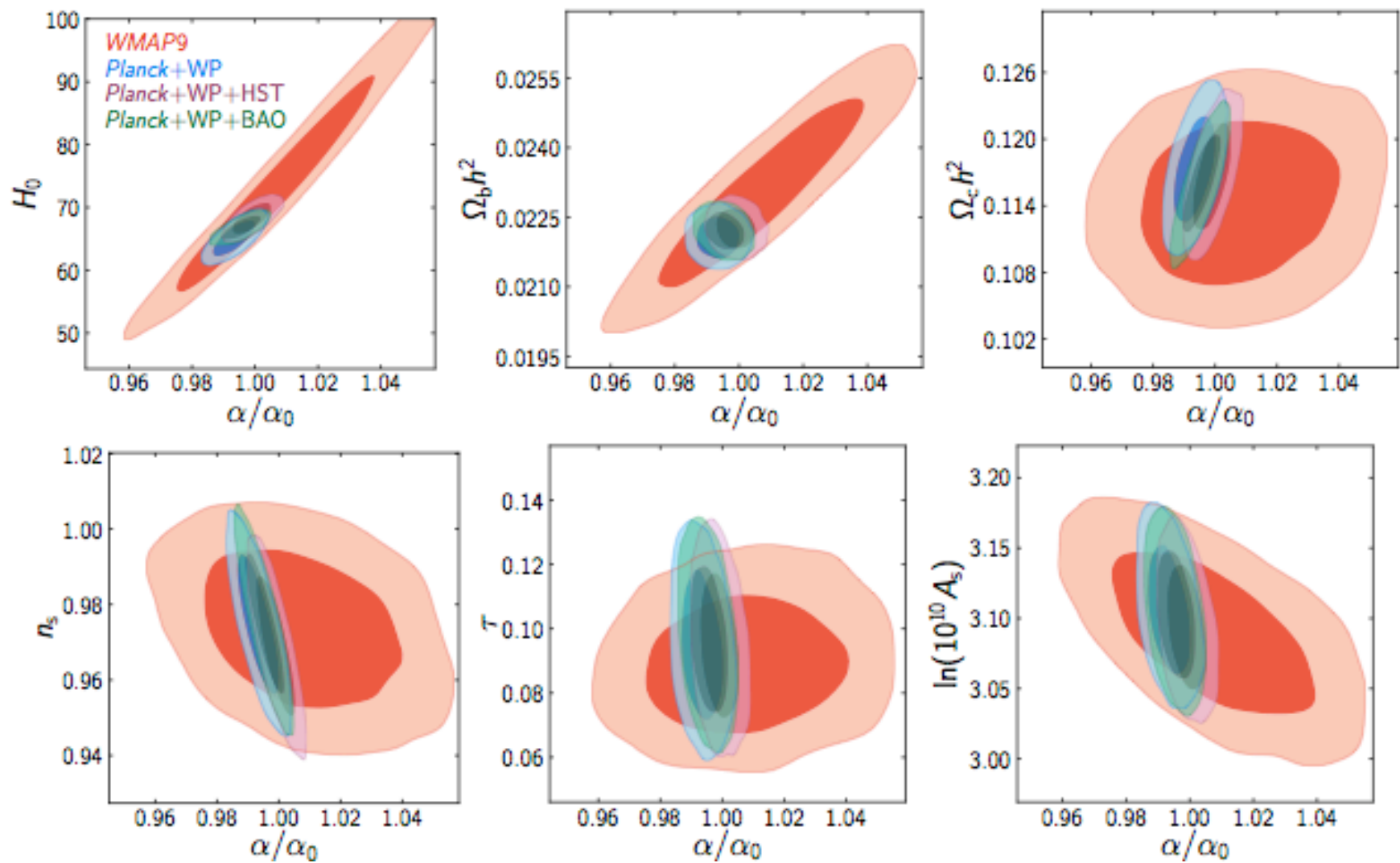
Effect on the polarization power spectrum



Effect on the cross-correlation

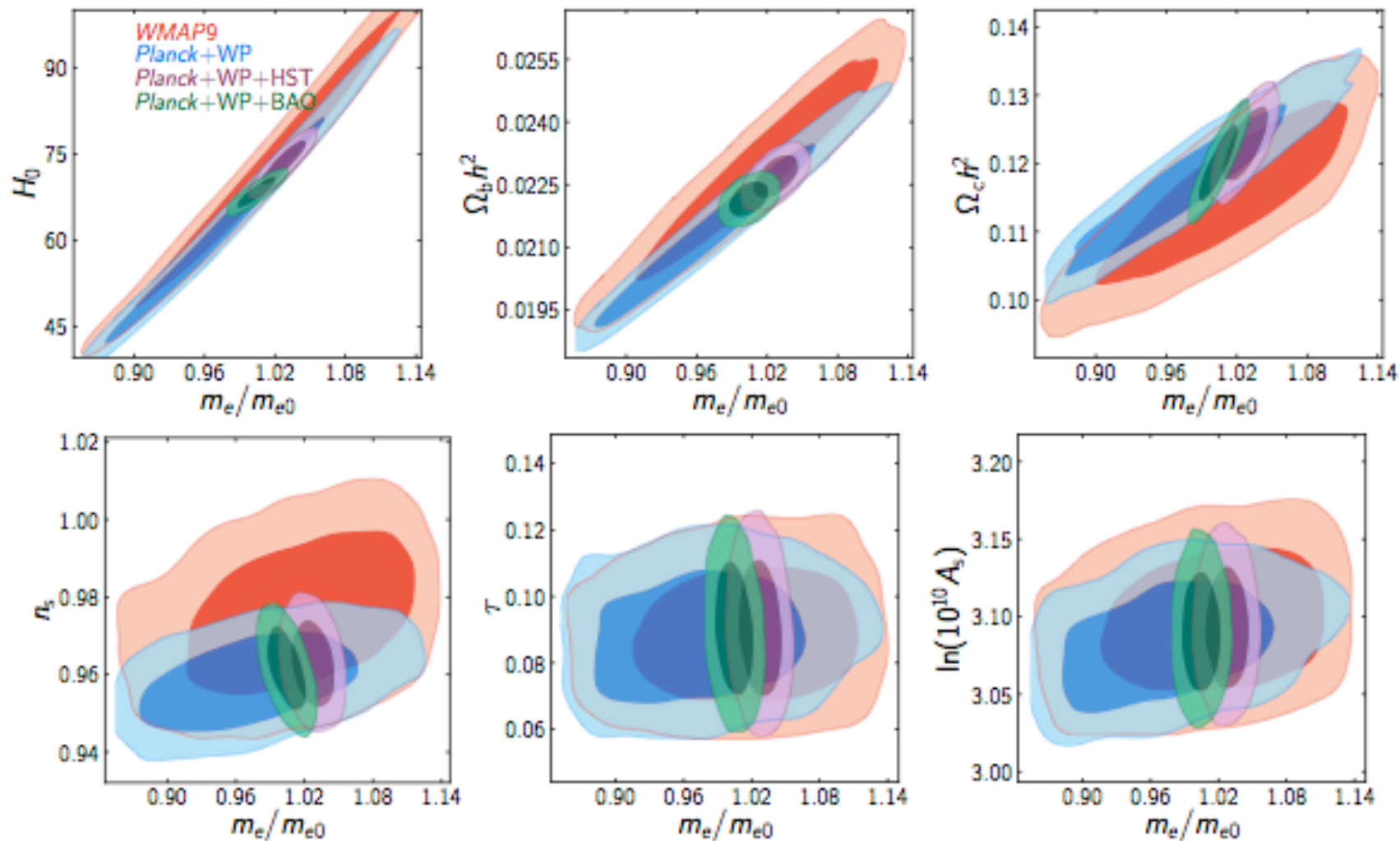


Varying α alone



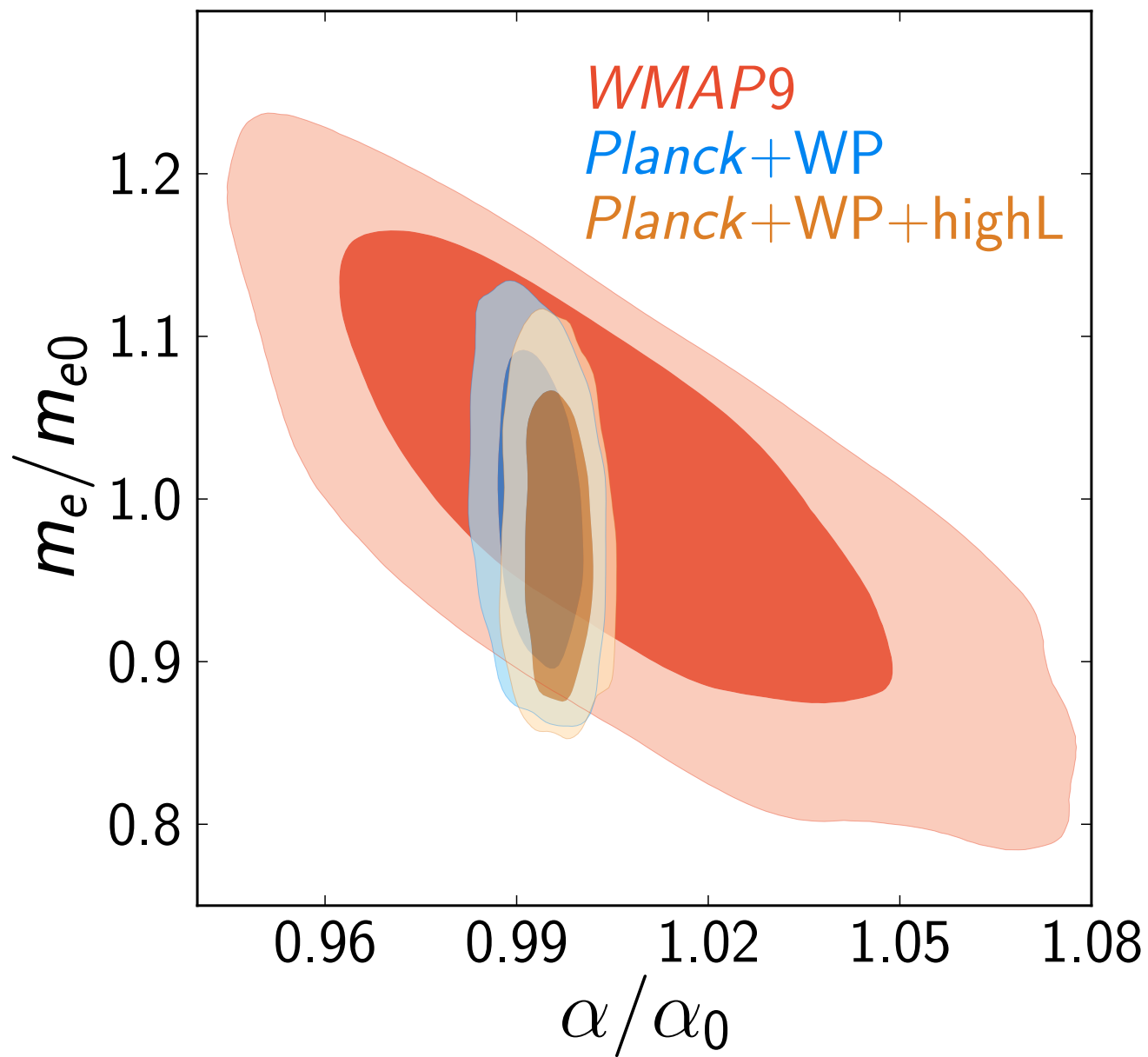
$\{\omega_b, \omega_c, H_0, \tau, n_s, A_s, \alpha \text{ or } m_e\}$

Varying m_e alone

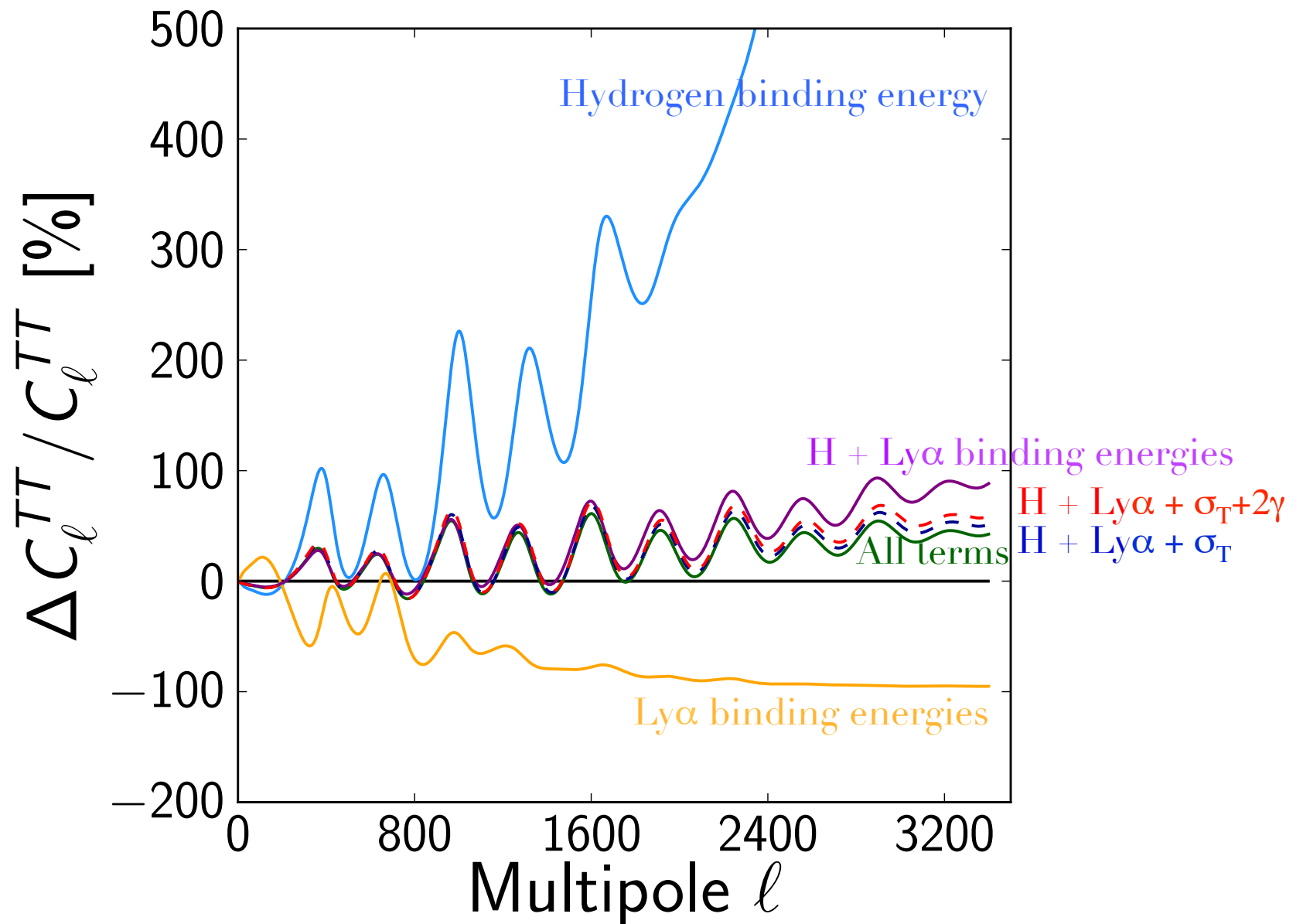


$\{\omega_b, \omega_c, H_0, \tau, n_s, A_s, \alpha \text{ or } m_e\}$

(α, m_e) -degeneracy



Why *Planck* does better

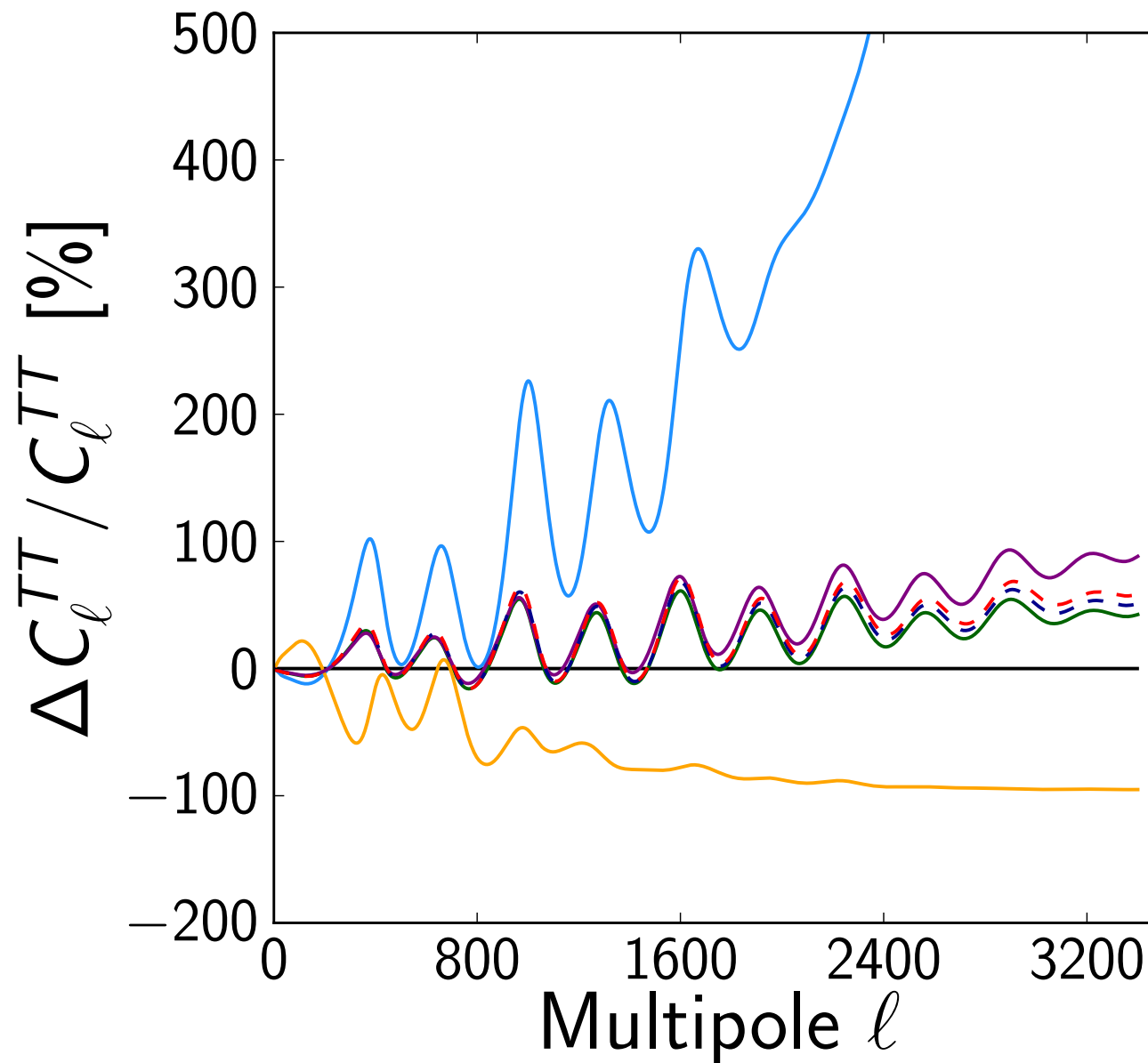


Why Planck does better

$\delta\alpha = 5\%$

$\delta m_e = 10.025\%$

$\alpha^2 m_e$ identical

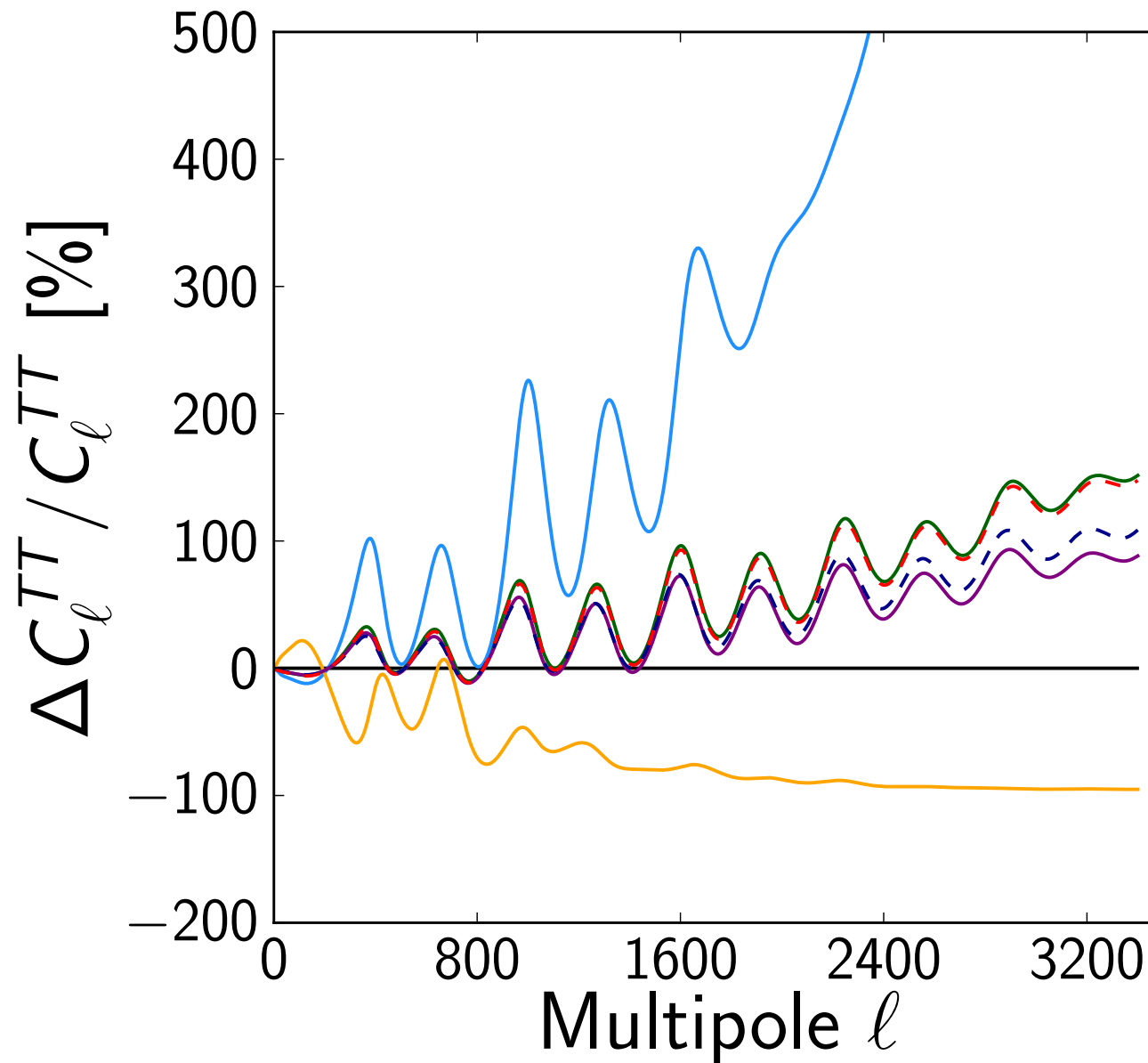


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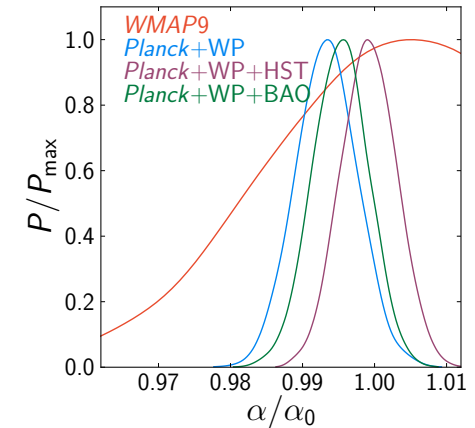


In conclusion

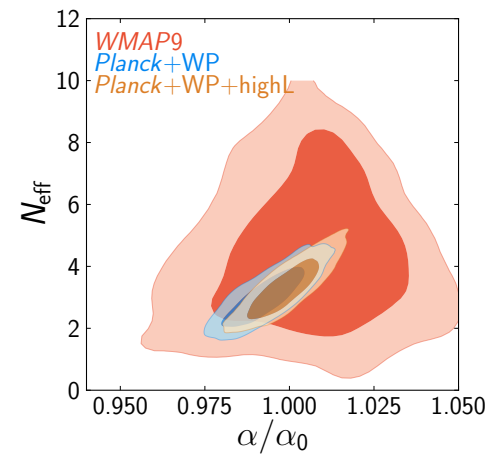
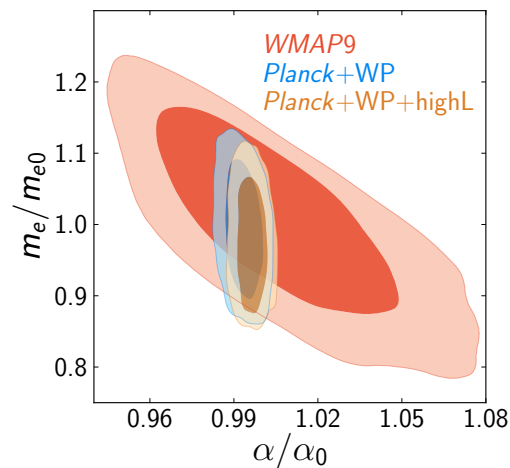
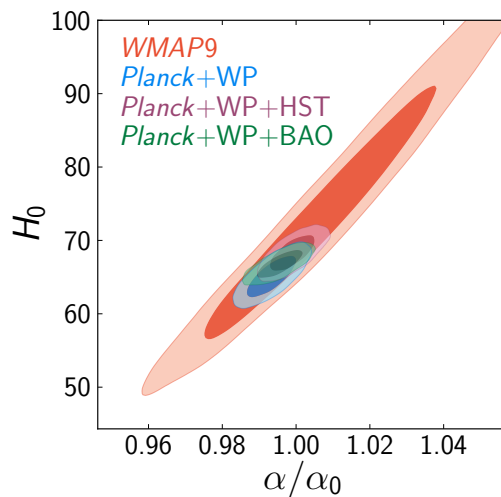
Independent variations of α and m_e are constrained to be

$$\Delta\alpha/\alpha = (3.6 \pm 3.7) \times 10^{-3} \quad \Delta m_e/m_e = (4 \pm 11) \times 10^{-3}$$

This is a factor 5 better compared to WMAP analysis



Planck breaks the degeneracy with H_0 and with m_e and other cosmological parameters (e.g. N_{ν} or helium abundance)

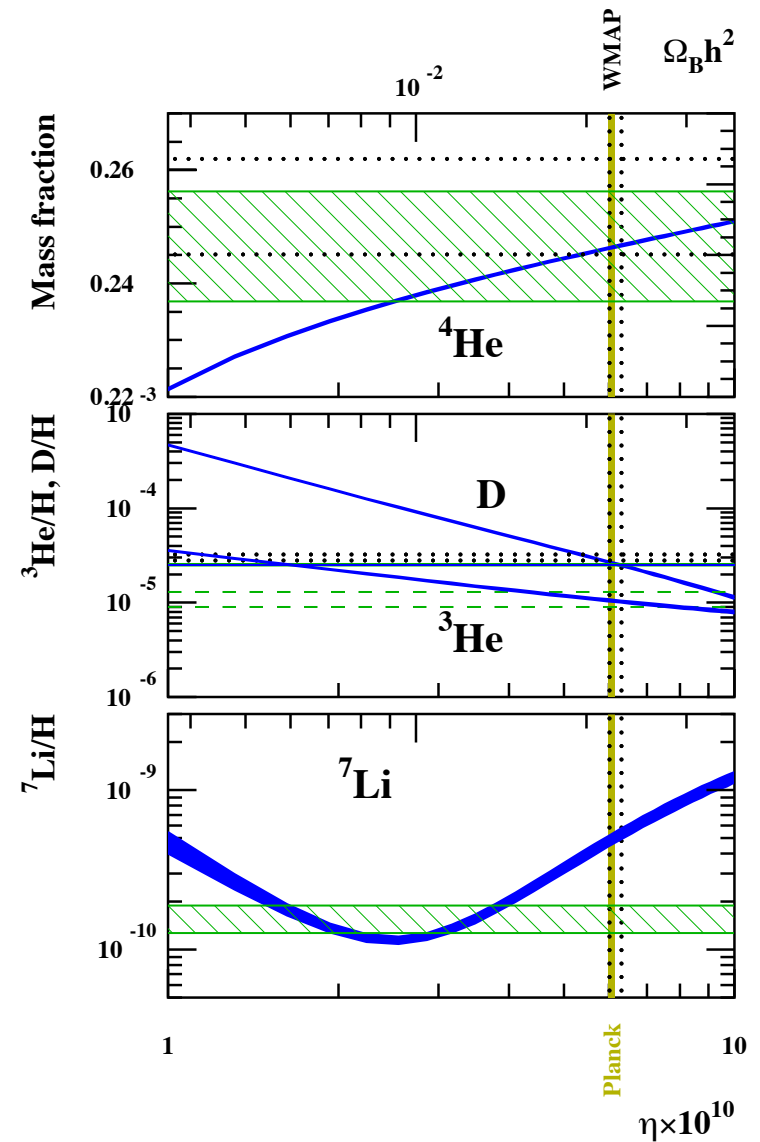
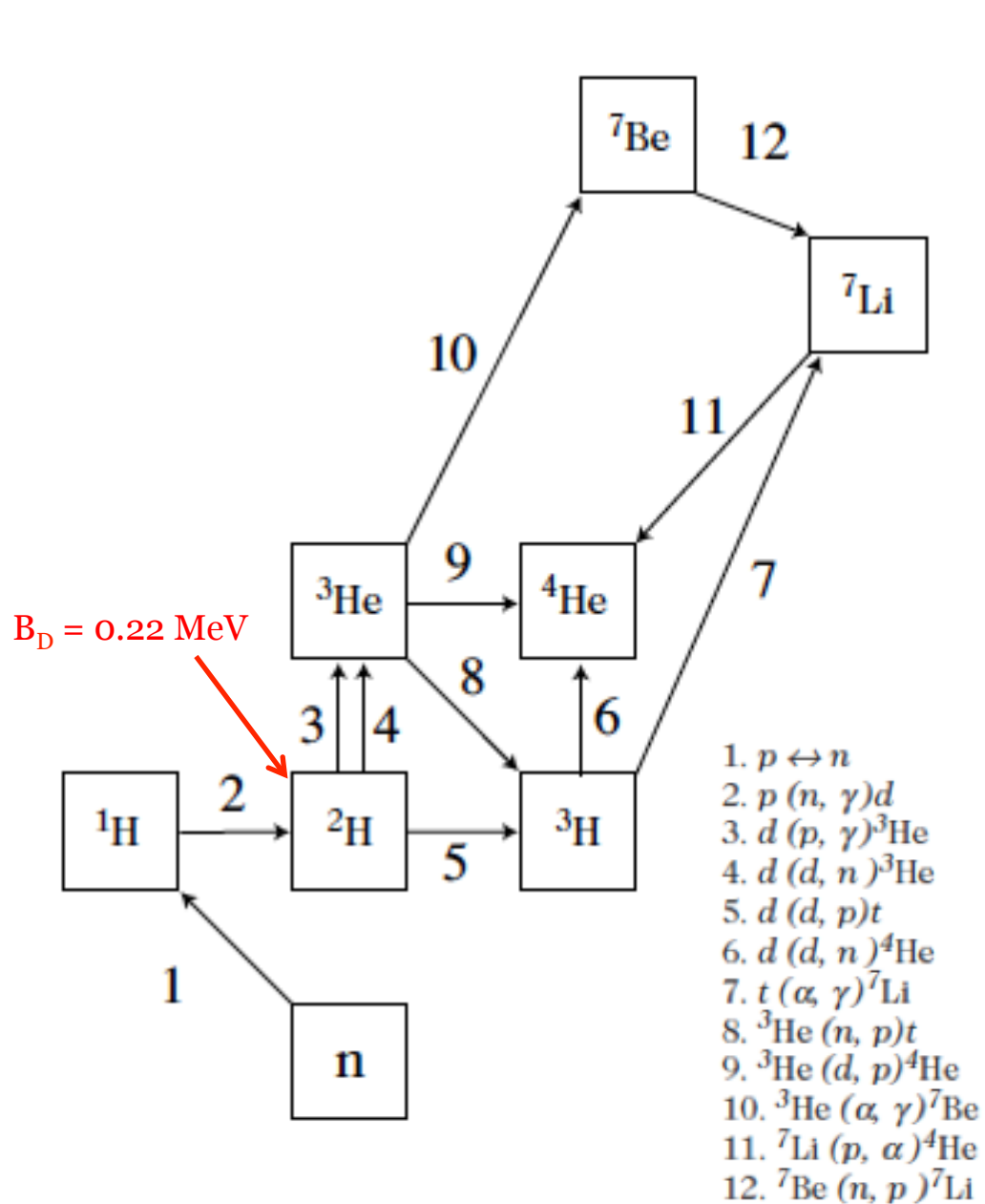


Big bang nucleosynthesis & Population III stars

Nuclear physics at work in the universe

[Coc,Nunes,Olive,JPU,Vangioni 2006
Coc, Descouvemont, Olive, JPU, Vangioni, 2012
Ekström, Coc, Descouvemont, Meynet, Olive, JPU, Vangioni,2009]

BBN: basics



Stellar carbon production

Triple α coincidence (Hoyle)

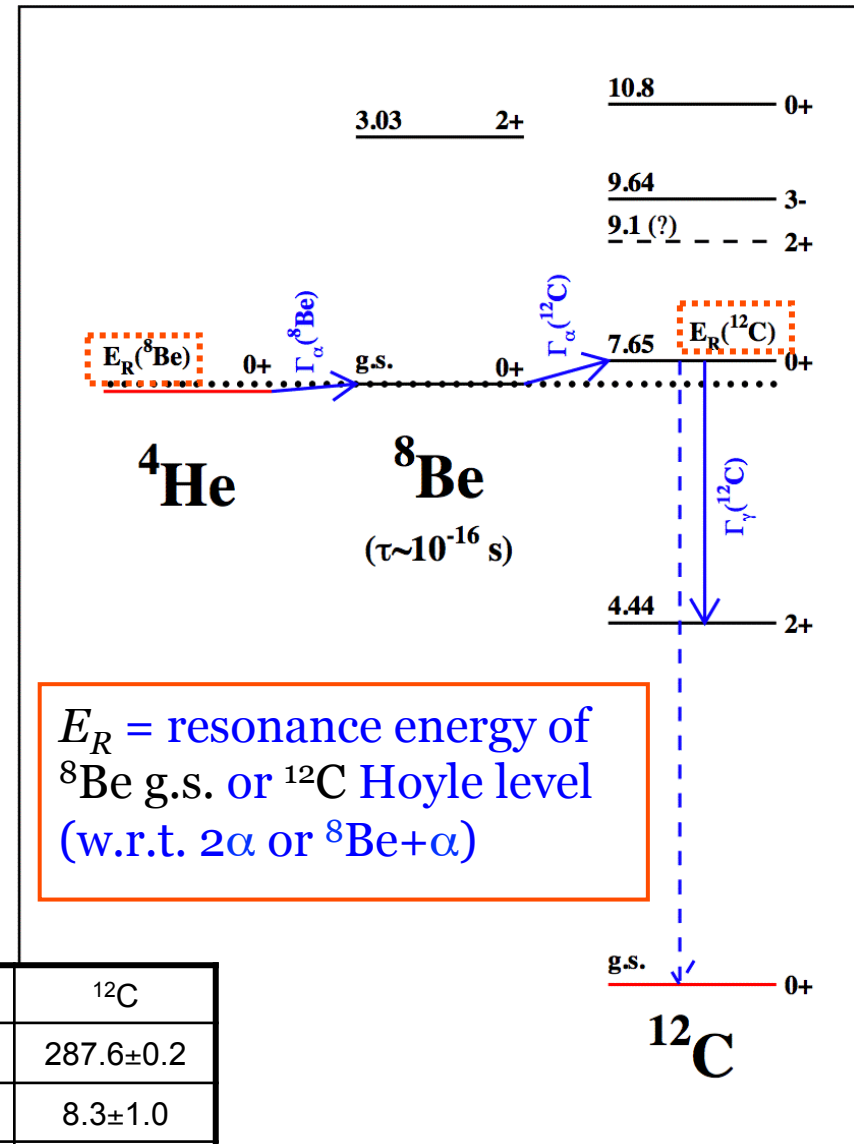
1. Equilibrium between ^4He and the short lived ($\sim 10^{-16}$ s) ^8Be : $\alpha\alpha \leftrightarrow ^8\text{Be}$
2. Resonant capture to the ($l=0, J^\pi=0^+$) Hoyle state: $^8\text{Be} + \alpha \rightarrow ^{12}\text{C}^* (\rightarrow ^{12}\text{C} + \gamma)$

Simple formula used in previous studies

1. Saha equation (thermal equilibrium)
2. Sharp resonance analytic expression:

$$N_A^2 \langle \sigma v \rangle^{\alpha\alpha\alpha} = 3^{3/2} 6 N_A^2 \left(\frac{2\pi}{M_\alpha k_B T} \right)^3 \hbar^5 \gamma \exp\left(\frac{-Q_{\alpha\alpha\alpha}}{k_B T} \right)$$

with $Q_{\alpha\alpha\alpha} = E_R(^8\text{Be}) + E_R(^{12}\text{C})$ and $\gamma \approx \Gamma_\gamma$



Nucleus	^8Be	^{12}C
E_R (keV)	91.84 ± 0.04	287.6 ± 0.2
Γ_α (eV)	5.57 ± 0.25	8.3 ± 1.0
Γ_γ (meV)	-	3.7 ± 0.5

Nuclear physics

Both phenomena involve nuclear physics.

The microphysics involves binding energies / resonance energies / cross-sections

BBN: dependence on constants

Light element abundances mainly based on the balance between

1- expansion rate of the universe

2- weak interaction rate which controls n/p at the onset of BBN

Example: helium production

$$Y = \frac{2(n/p)_N}{1+(n/p)_N}$$

$$(n/p)_f \sim e^{-Q/k_B T_f} \quad (B_D, \eta)$$
$$(n/p)_N \sim (n/p)_f e^{-t_N/\tau_n}$$

freeze-out temperature is roughly given by $G_F^2 (k_B T_f)^5 = \sqrt{GN} (k_B T_f)^2$

Coulomb barrier: $\sigma = \frac{S(E)}{E} e^{-2\pi\alpha Z_1 Z_2 \sqrt{\mu/2E}}$

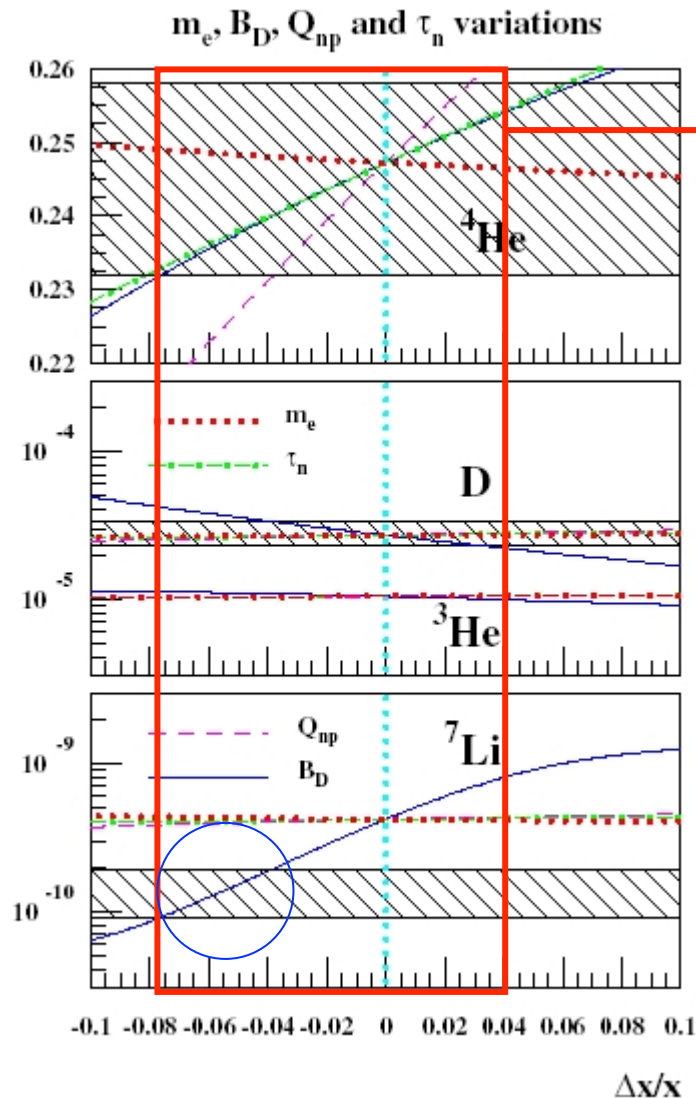
Predictions depend on

$$G_k = (G, \alpha, \tau_n, m_e, Q, B_D, \sigma_i)$$

$$X = (\eta, h, N_\nu, \dots)$$

Sensitivity to the nuclear parameters

Independent variations of the BBN parameters



$$-7.5 \times 10^{-2} < \frac{\Delta B_D}{B_D} < 6.5 \times 10^{-2}$$

$$-8.2 \times 10^{-2} < \frac{\Delta \tau_n}{\tau_n} < 6 \times 10^{-2}$$

$$-4 \times 10^{-2} < \frac{\Delta Q}{Q} < 2.7 \times 10^{-2}$$

Abundances are very sensitive to B_D .
Equilibrium abundance of D and the reaction rate $p(n,\gamma)\text{D}$ depend exponentially on B_D .

These parameters are not independent.

Difficulty: QCD and its role in low energy nuclear reactions.

BBN: fundamental parameters (1)

Neutron-proton mass difference:

$$Q = m_n - m_p = a\alpha\Lambda + (h_d - h_u)v$$

$$\frac{\Delta Q}{Q} = -0.6 \left(\frac{\Delta\alpha}{\alpha} + \frac{\Delta\Lambda}{\Lambda} \right) + 1.6 \left(\frac{\Delta(h_d - h_u)}{h_d - h_u} + \frac{\Delta v}{v} \right)$$

Neutron lifetime:

$$\tau_n^{-1} = G_F^2 m_e^5 f(Q/m_e) \quad m_e = h_e v$$
$$G_F = 1/\sqrt{2}v^2$$

$$\frac{\Delta\tau_n}{\tau_n} = -4.8 \frac{\Delta v}{v} + 1.5 \frac{\Delta h_e}{h_e} - 10.4 \frac{\Delta(h_d - h_u)}{h_d - h_u} + 3.8 \left(\frac{\Delta\alpha}{\alpha} + \frac{\Delta\Lambda}{\Lambda} \right)$$

BBN: fundamental parameters (2)

D binding energy:

Use a potential model $V_{nuc} = \frac{1}{4\pi r}(-g_s^2 e^{-rm_\sigma} + g_v^2 e^{-rm_\omega})$

$$\frac{\Delta B_D}{B_D} = -48 \frac{\Delta m_\sigma}{m_\sigma} + 50 \frac{\Delta m_\omega}{m_\omega} + 6 \frac{\Delta m_N}{m_N}$$

Flambaum, Shuryak 2003

This allows to determine BD as a function of mass of the quarks (u,d,s), Λ_{QCD} , α .

This allows to determine all the primary parameters in terms of (h_i , v , Λ , α)

BBN: assuming GUT

GUT:

The low-energy expression for the QCD scale

$$\Lambda = \mu \left(\frac{m_c m_b m_t}{\mu^3} \right)^{2/27} \exp \left(- \frac{2\pi}{9\alpha_3(\mu)} \right)$$

We deduce

$$\frac{\Delta\Lambda}{\Lambda} = R \frac{\Delta\alpha}{\alpha} + \frac{2}{27} \left(3 \frac{\Delta v}{v} + \sum_{i=c,b,t} \frac{\Delta h_i}{h_i} \right)$$

The value of R depends on the particular GUT theory and particle content which control the value of M_{GUT} and of $\alpha(M_{\text{GUT}})$. Typically $R=36$.

Assume (for simplicity) $h_i=h$

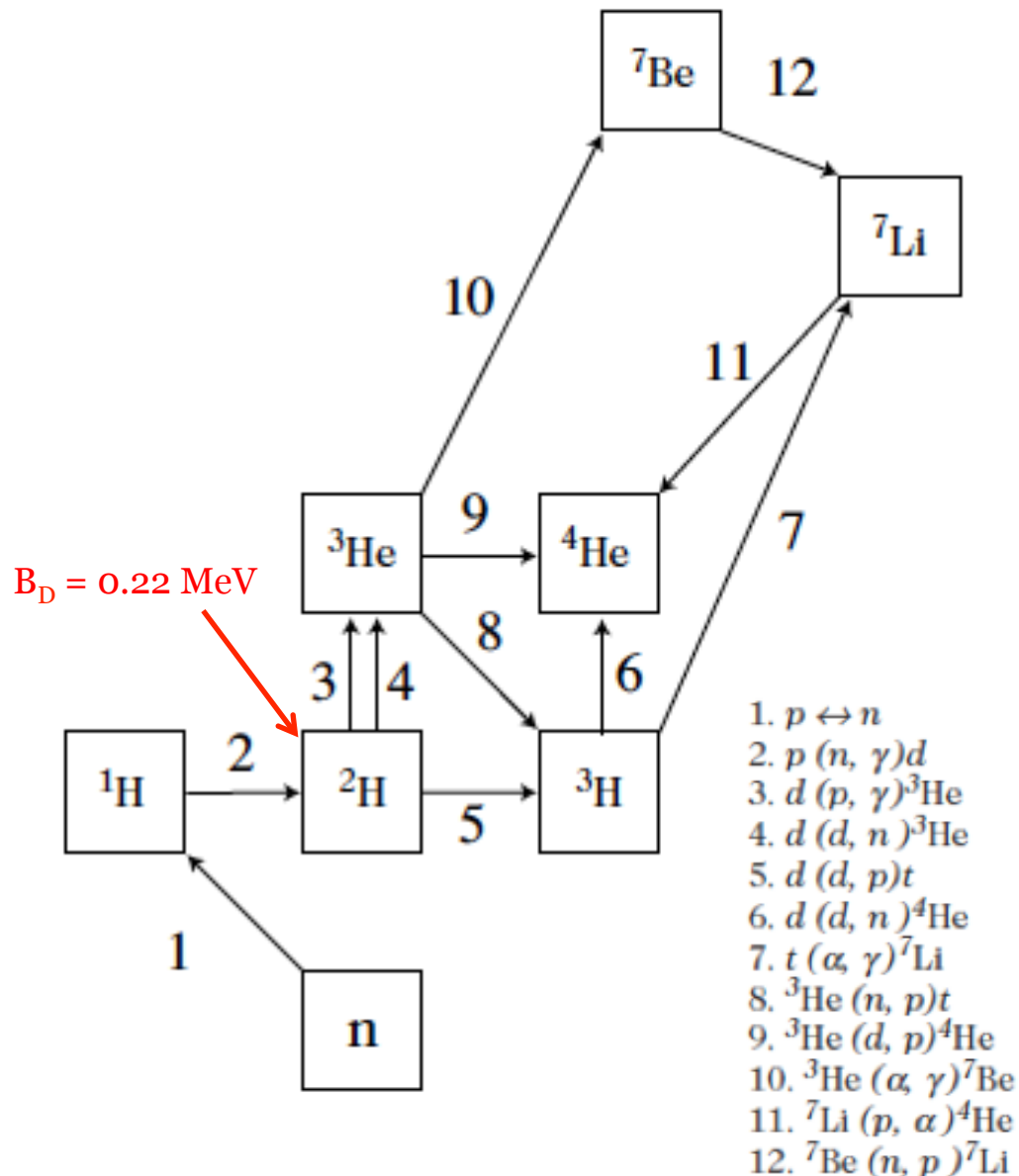
$$\frac{\Delta B_D}{B_D} = -13 \left(\frac{\Delta v}{v} + \frac{\Delta h}{h} \right) + 18 R \frac{\Delta\alpha}{\alpha}$$

$$\frac{\Delta Q}{Q} = 1.5 \left(\frac{\Delta v}{v} + \frac{\Delta h}{h} \right) - 0.6(1+R) \frac{\Delta\alpha}{\alpha}$$

$$\frac{\Delta \tau_n}{\tau_n} = -4 \frac{\Delta v}{v} - 8 \frac{\Delta h}{h} + 3.8(1+R) \frac{\Delta\alpha}{\alpha}$$

$$(\alpha, v, h)$$

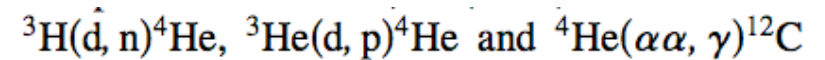
A=5 & A=8



To go further:

- influence on helium-5,
lithium-5, beryllium-8, carbon-12

- cross-sections such as



To that goal, we introduce a
modelisation that will also allow
to study the stellar physics.

Cluster model & δ_{NN}

Cluster approach:

- solve the Schrödinger equation by considering Be8/C12 as clusters of α particle

$$\Psi_{8\text{Be}}^{JM\pi} = \mathcal{A}\phi_\alpha\phi_\alpha g_2^{JM\pi}(\rho)$$

$$\Psi_{12\text{C}}^{JM\pi} = \mathcal{A}\phi_\alpha\phi_\alpha\phi_\alpha g_3^{JM\pi}(\rho, \mathbf{R}),$$

- The Hamiltonian is then given by

$$H = \sum_{i=1}^A T(r_i) + \sum_{i<j=1}^A (V_{Coul.}(r_{ij}) + V_{Nucl.}(r_{ij}))$$

- We assume that

$$V_{ij} = (1 + \delta_\alpha)V_{ij}^C + (1 + \delta_{NN})V_{ij}^N \text{ to obtain } B_D, E_R(^8\text{Be}), E_R(^{12}\text{C})$$

- δ_{NN} is an effective parameter

Cluster model \longleftrightarrow Theoretical analysis

$$\Delta B_D/B_D = 5.716 \times \delta_{NN}.$$

$$\frac{\Delta B_D}{B_D} = 18 \frac{\Delta \Lambda_{\text{QCD}}}{\Lambda_{\text{QCD}}} - 17 \left(\frac{\Delta v}{v} + \frac{\Delta h_s}{h_s} \right).$$

Microscopic calculation

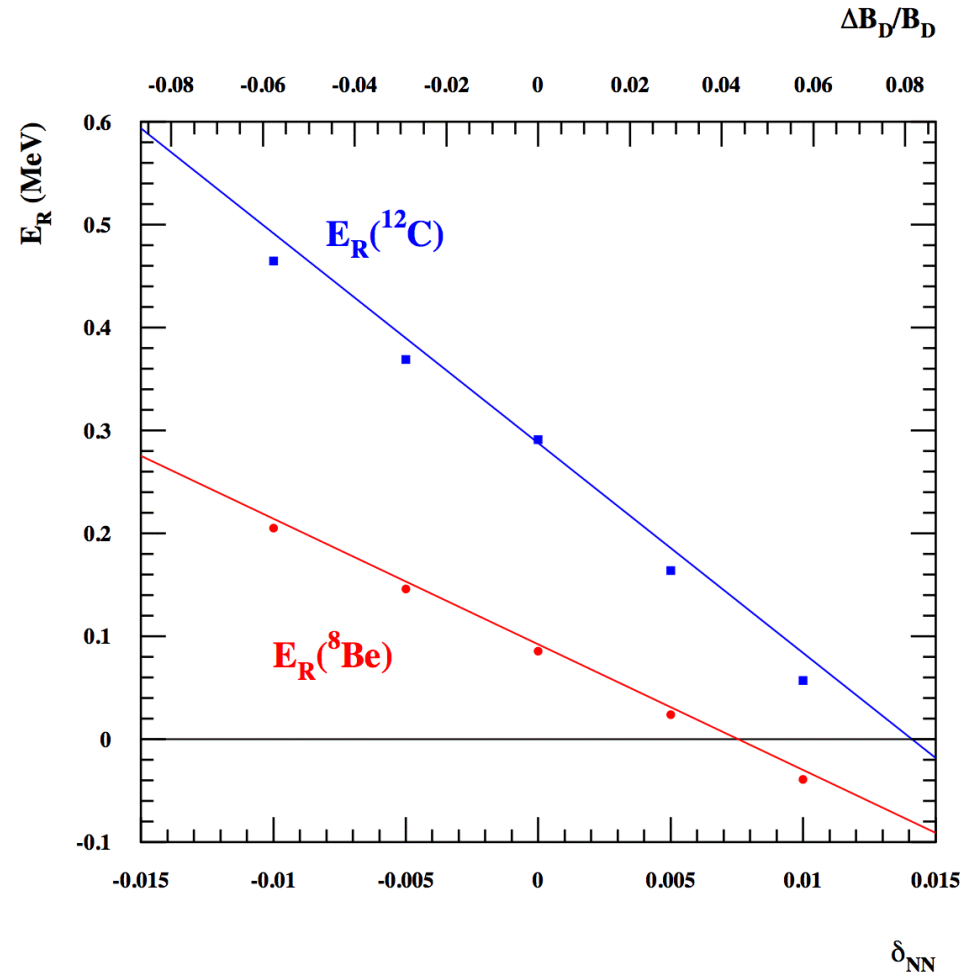
$$\Delta B_D/B_D = 5.716 \times \delta_{NN}.$$

$$E_R(^8\text{Be}) = (0.09184 - 12.208 \times \delta_{NN}) \text{ MeV}$$

$$E_R(^{12}\text{C}) = (0.2876 - 20.412 \times \delta_{NN}) \text{ MeV}$$

Note:

- $\delta_{NN} > 7.52 \times 10^{-3}$, Be8 becomes stable
- $\delta_{NN} > 0.15$, dineutron is stable
- $\delta_{NN} > 0.35$, diproton is stable
- effect of α is subdominant



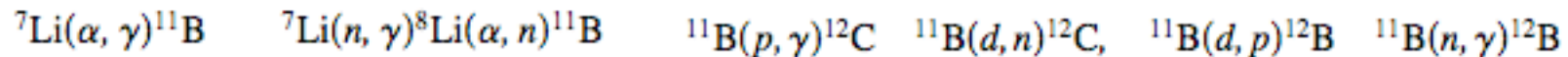
[□ Link to fundamental couplings through \$B_D\$ or \$\delta_{NN}\$](#)

Primordial CNO production

Primordial CNO may affect dynamics of Pop III if $\text{CNO}/\text{H} > 10^{-12} - 10^{-10}$

In standard BBN $\text{CNO}/\text{H} = (0.2-3)10^{-15}$ [Iocco et al (2007); Coc et al. (1012)].

It proceeds as

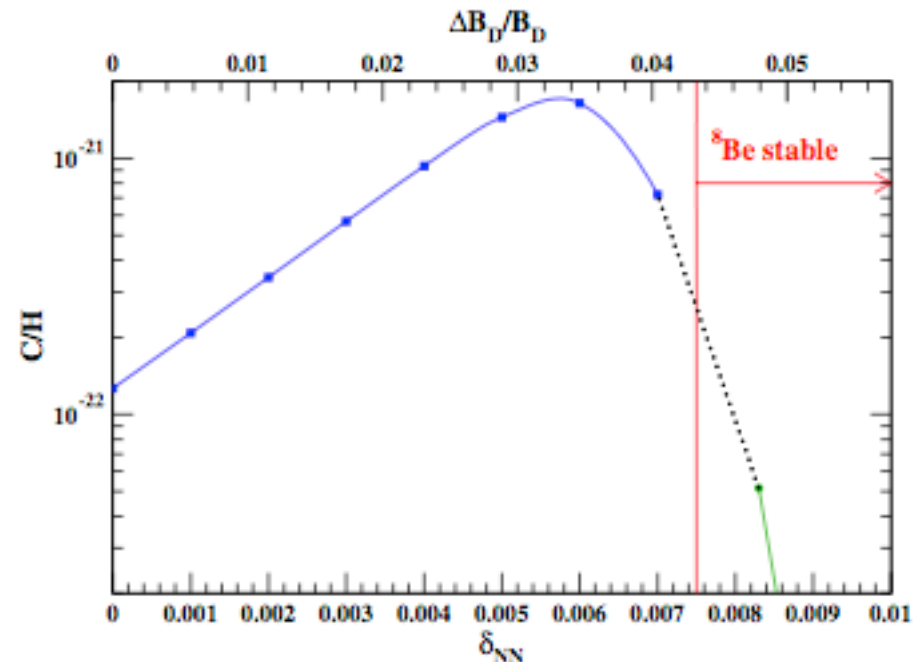


which bridge the gap between $A=7$ and $A=12$.

Just consider the 3α -reactions: *6 orders of magnitude below SBBN.*

Effect on He-5 and Li-5 were also studied.

Stable $A=5$ & $A=8$ do not affect the standard BBN abundances



Constraints

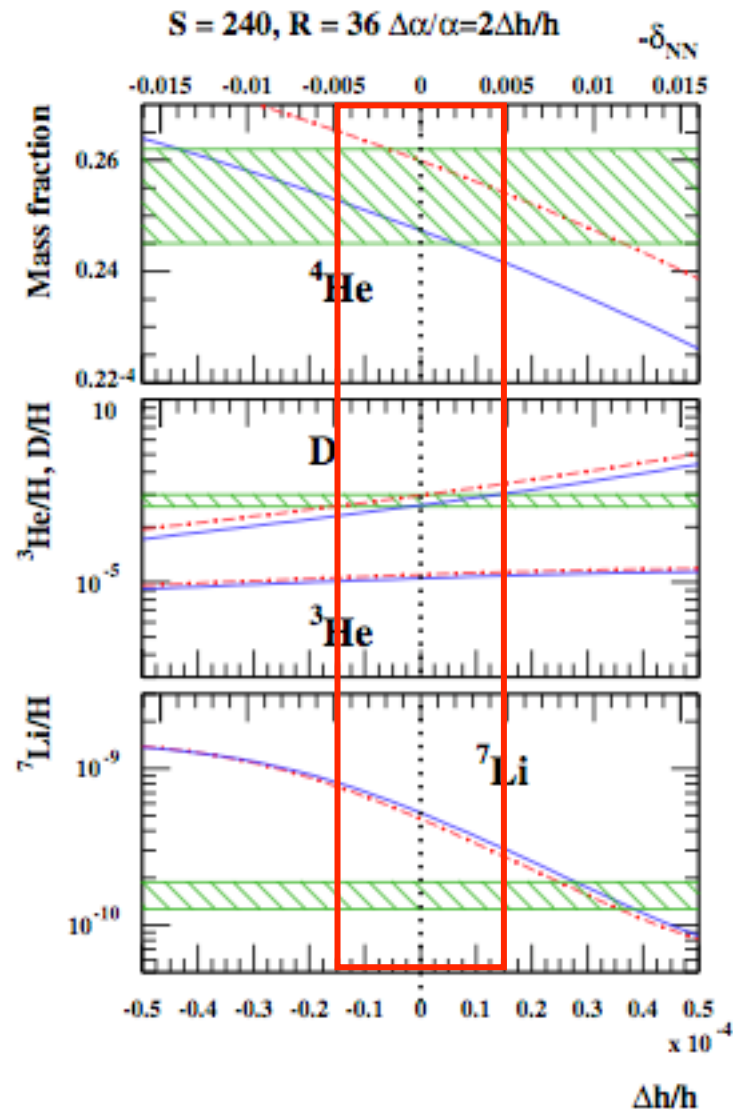


FIG. 12 (color online). Update Fig. 4 of Ref. [22] assuming $S = 240$ and $R = 36$ (solid blue line), using new rates for $^3\text{He}(\alpha, \gamma)^7\text{Li}$ [73] and $^1\text{H}(n, \gamma)\text{D}$ [74] and the Ω_b value from WMAP7 [4]. The top axis is $-\delta_{\text{NN}}$ from Eq. (5.8) (mind the sign) and the dashed red line assumes $N_\nu = 4$.

BBN / Pop III

In the temperature range 0.1 GK -1 GK, the baryon density during BBN changes from 0.1 to 10^{-5} g/cm³.

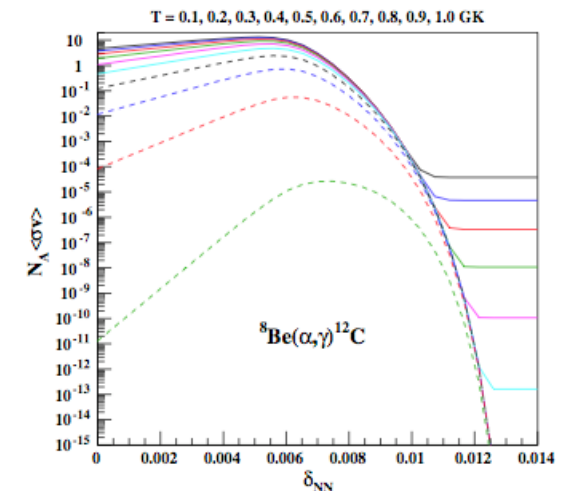
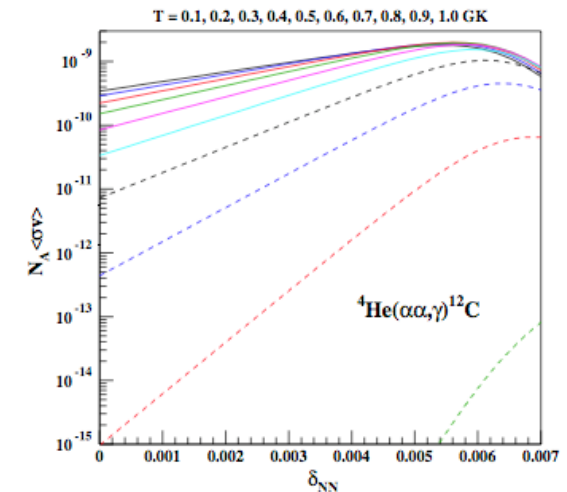
-Variation of the reaction rates is limited at higher T

-3-body reactions are less efficient

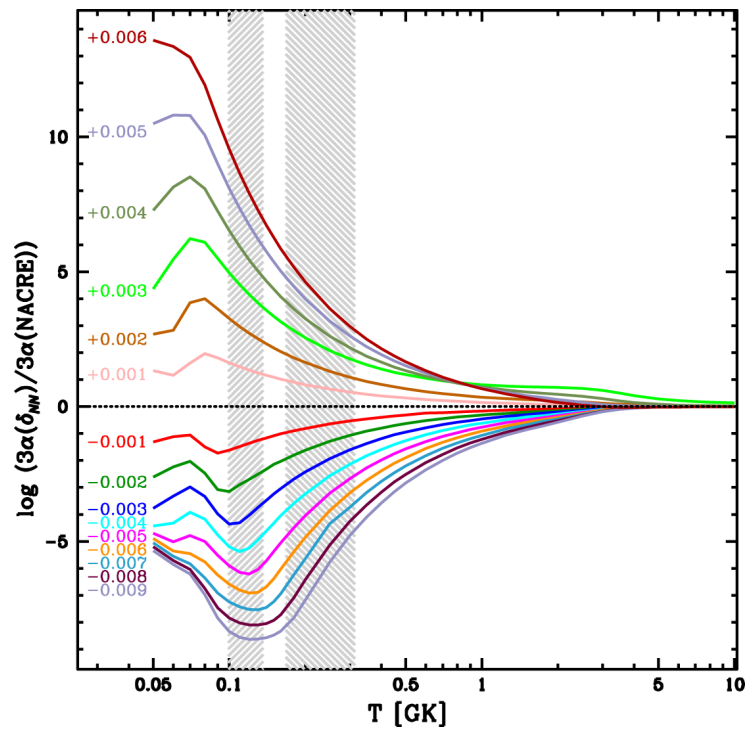
In population III stars, the situation is however different:

- density varies between 30 to 3000 g/cm³,

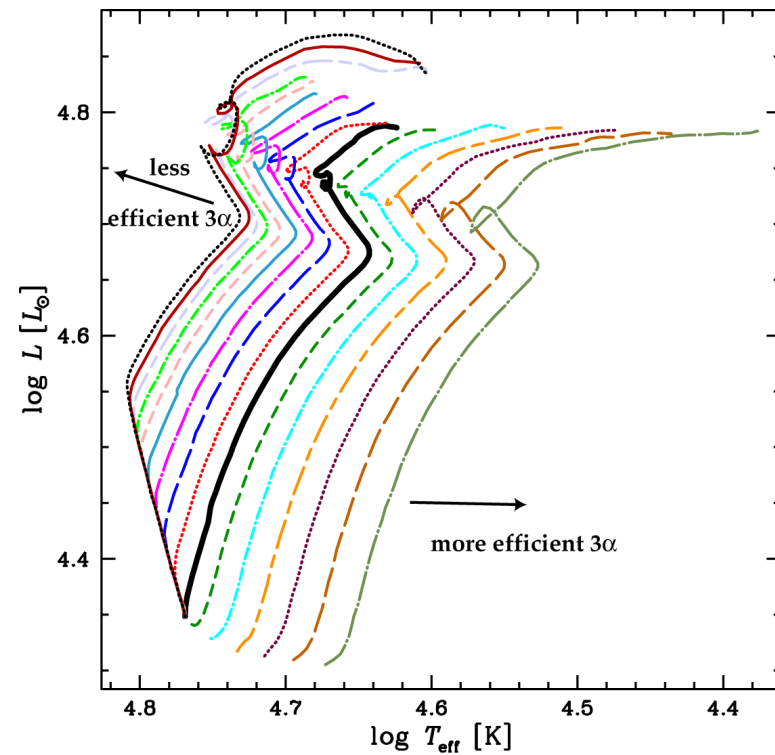
- 3α occurs during the helium burning phase, without significant sources of Li-7, D, p, n so that the 2-body « route » is not effective.



Effects on the stellar evolution



3α -reaction rate

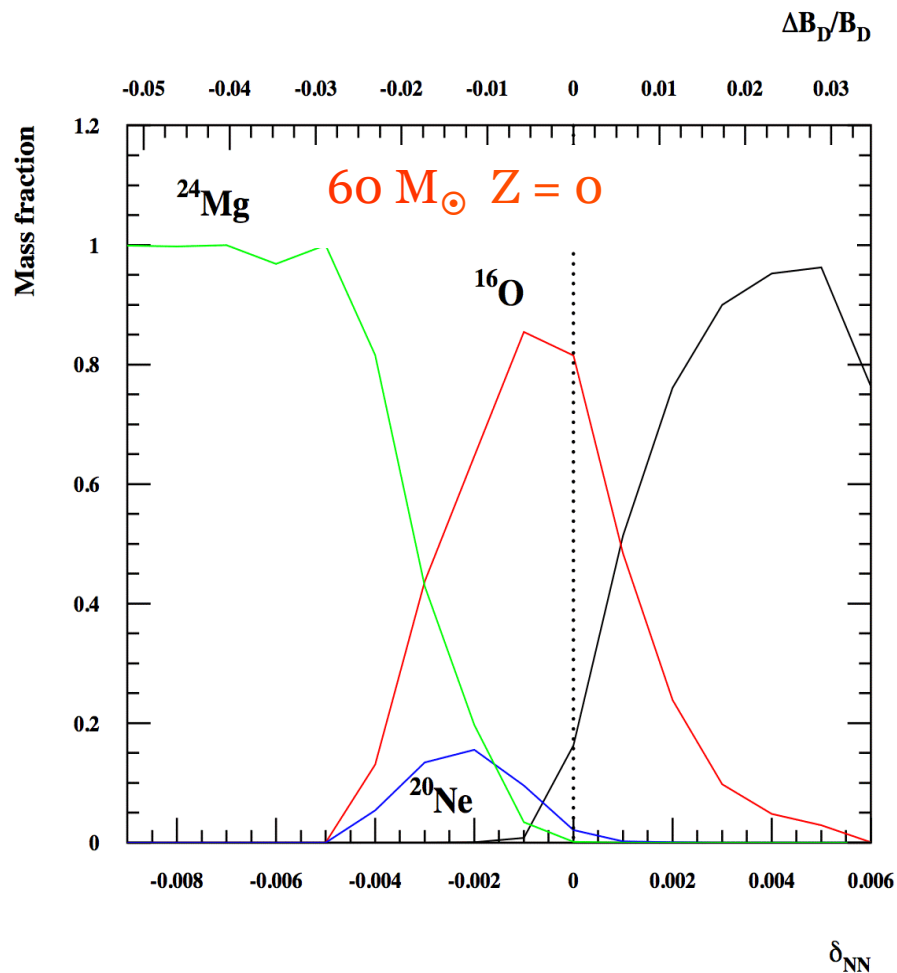


$60 M_{\odot}$ stars / $Z=0$

Composition at the end of core He burning

Stellar evolution of massive Pop. III stars

We choose *typical* masses of 15 and $60 M_{\odot}$ stars/ $Z=0 \Rightarrow$ Very specific stellar evolution



➤ **The standard region:** Both ^{12}C and ^{16}O are produced.

➤ **The ^{16}O region:** The 3α is slower than $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ resulting in a higher T_C and a conversion of most ^{12}C into ^{16}O

➤ **The ^{24}Mg region:** With an even weaker 3α , a higher T_C is achieved and $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}(\alpha,\gamma)^{20}\text{Ne}(\alpha,\gamma)^{24}\text{Mg}$ transforms ^{12}C into ^{24}Mg

➤ **The ^{12}C region:** The 3α is faster than $^{12}\text{C}(\alpha,\gamma)^{16}\text{O}$ and ^{12}C is not transformed into ^{16}O

Constraint

$$^{12}\text{C}/^{16}\text{O} \sim 1 \Rightarrow -0.0005 < \delta_{NN} < 0.0015$$

$$\text{or } -0.003 < \Delta B_D/B_D < 0.009$$

Conclusions

The effect of the variation of fundamental constants on the nuclear physics processes needed to infer BBN predictions & describe the evolution of Pop . III stars have been modelled.

Constraints on the variation of the nuclear interaction

It can be related to fundamental constants (via Deuterium)

Stable $A=5$ & $A=8$ does not affect primordial CNO predictions

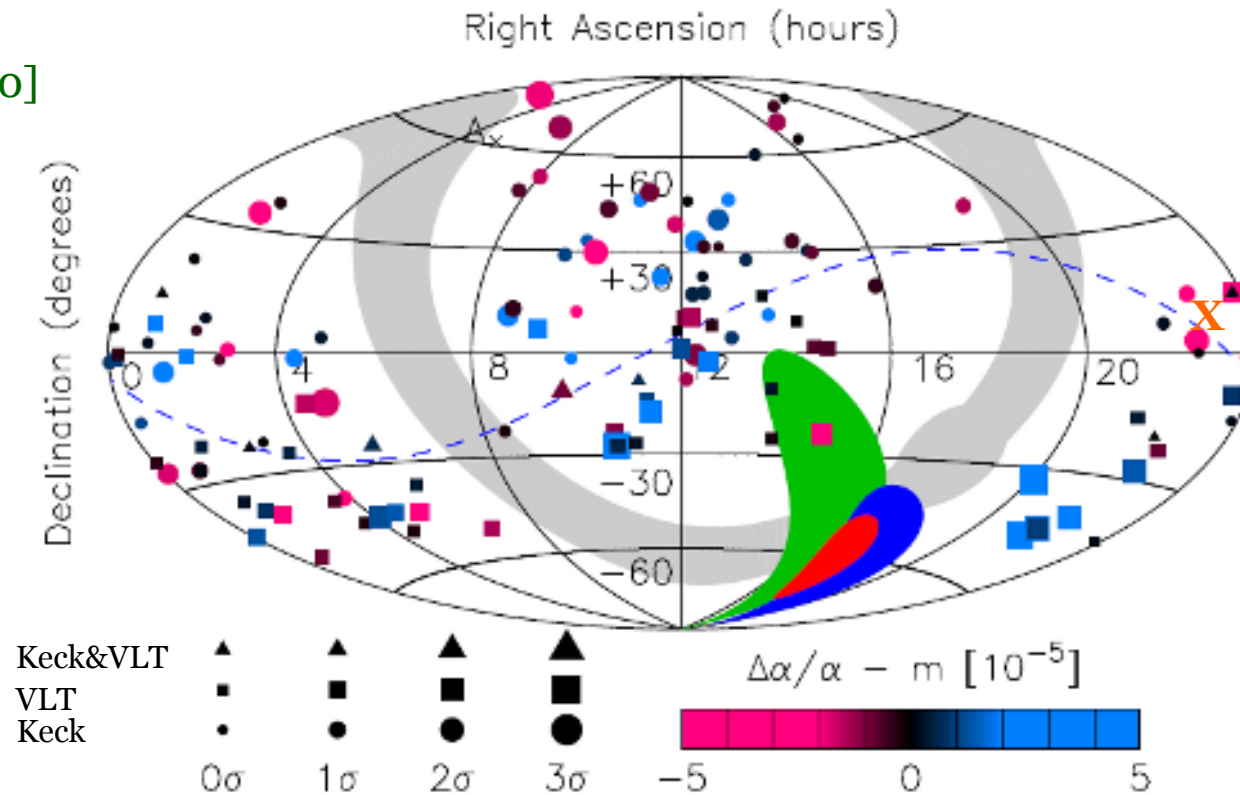
Evolution of Pop. III stars can be significantly affected

The tuning required to get C/O or order 1 is $1/1000$ (Hoyle fine tuning)

Spatial variation

Spatial variation?

[Webb et al., 2010]



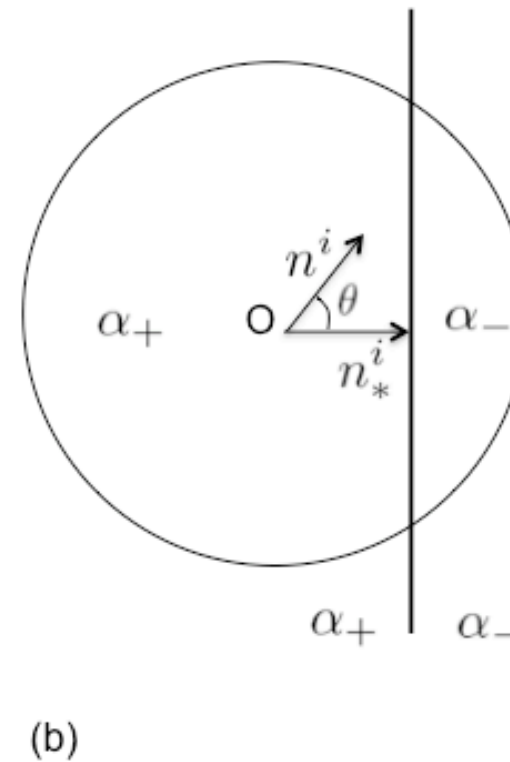
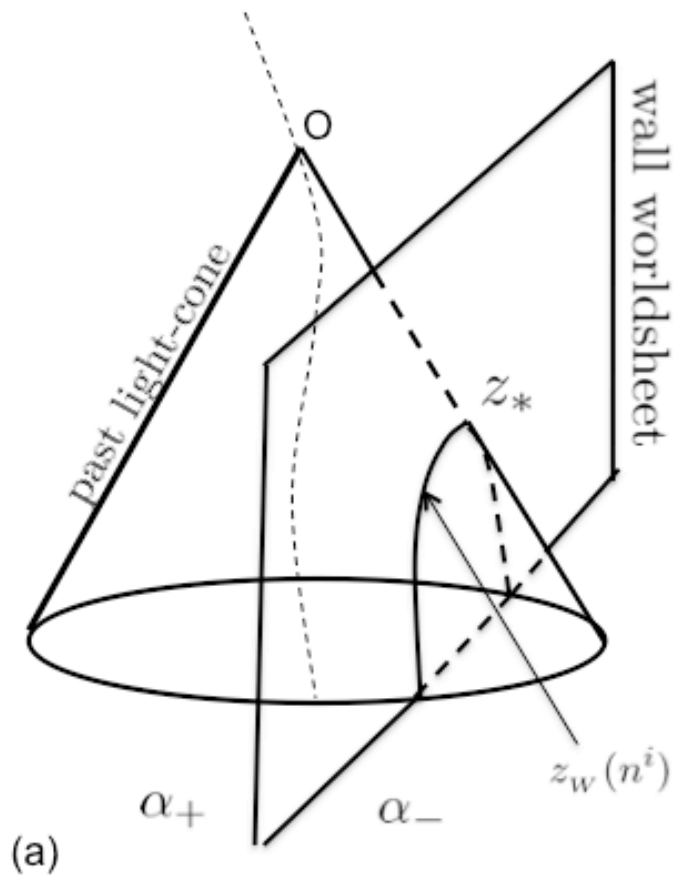
Claim: Dipole in the fine structure constant [« Australian dipole »]

Indeed, this is a logical possibility to reconcile VLT constraints and Keck claims of a variation.

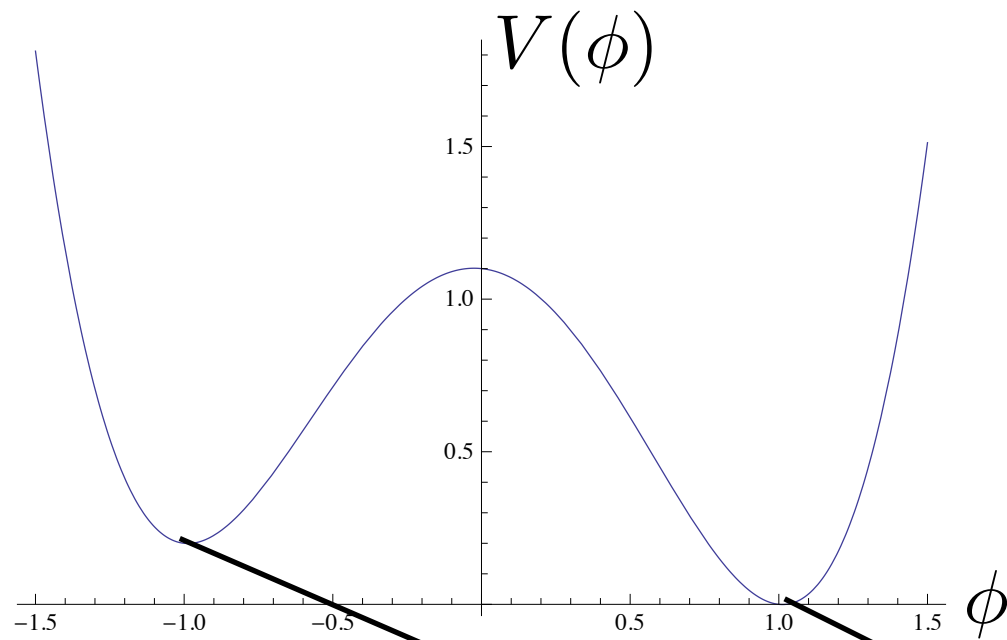
A possible theoretical model

[Olive, Peloso, JPU, 2010]

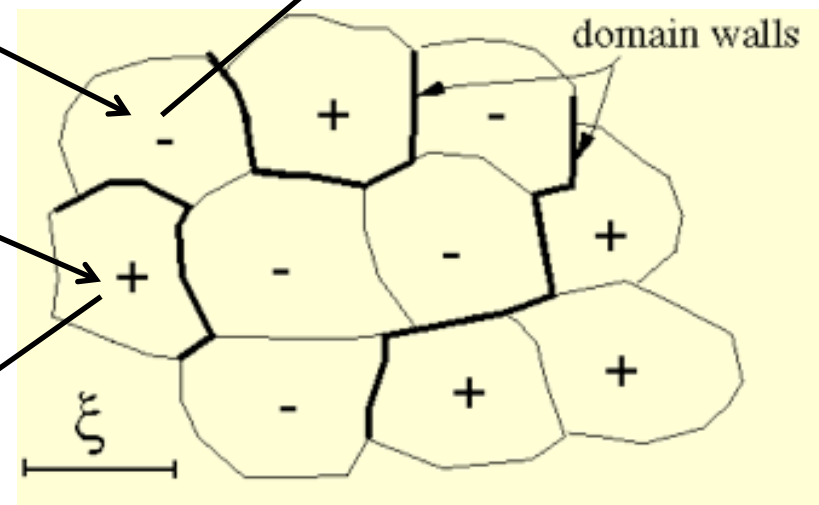
Idea: Spatial discontinuity in the fundamental constant due to a domain wall crossing our Hubble volume.



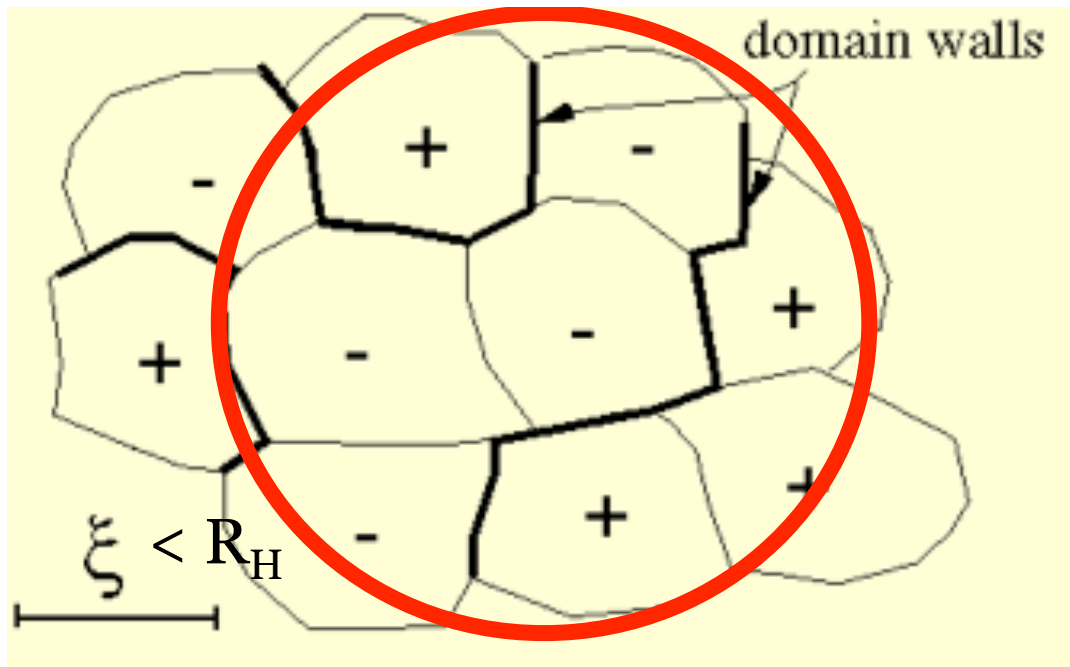
Spatial distribution of the constants



$\alpha(\phi_-), \mu(\phi_-), \dots$

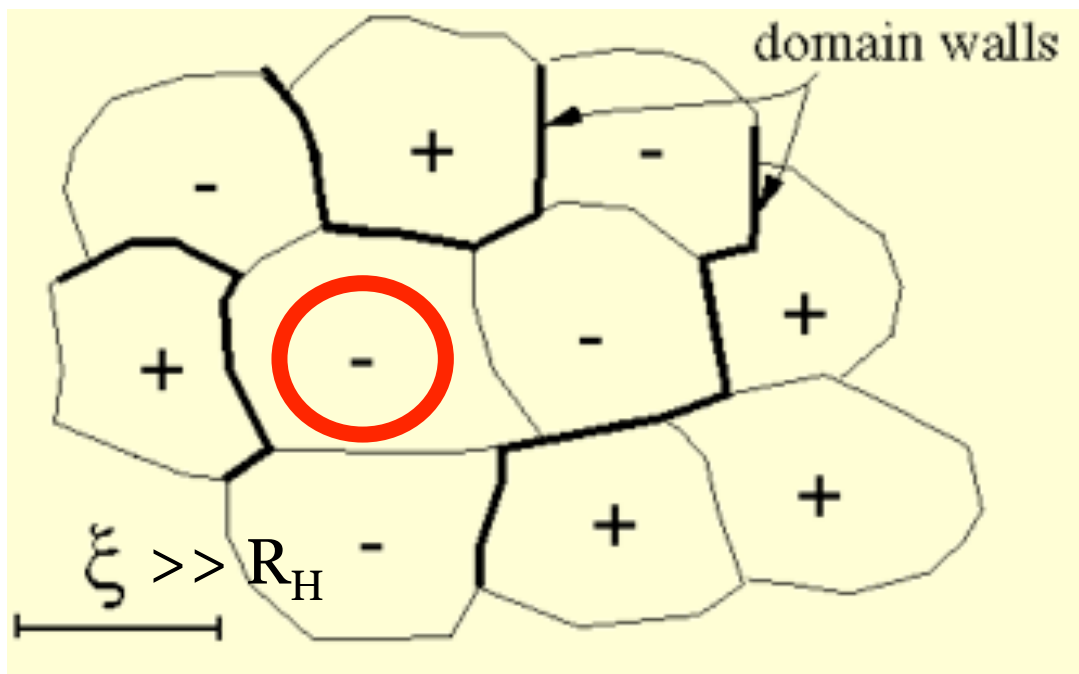


$\alpha(\phi_+), \mu(\phi_+), \dots$



Constants vary on sub-Hubble scales.

- may be detected
- microphysics in principle accessible



Constants vary on super-Hubble scales.

- landscape ?
- exact model of a theory which dynamically gives a distribution of fundamental constants
- no variation on the size of the observable universe

Spatial variation on CMB

If one assumes that some constants have a dipolar variation

$$c_a(\mathbf{n}, z) = c_{0a}(z) + \sum_{i=-1}^1 \delta c_a^{(i)}(z) Y_{1i}(\mathbf{n}).$$

then the CMB temperature can be expanded as

$$\begin{aligned} \Theta(\mathbf{n}) &= \bar{\Theta}[\mathbf{n}, c_a(\mathbf{n})] \\ &= \bar{\Theta} \left[\mathbf{n}, c_{0a} + \sum_{i=-1}^1 \delta c_a^{(i)}(z) Y_{1i}(\mathbf{n}) \right] \\ &\simeq \bar{\Theta}[\mathbf{n}] + \sum_a \sum_{i=-1}^{+1} \frac{\partial \bar{\Theta}[\mathbf{n}]}{\partial c_a} \delta c_a^{(i)}(z) Y_{1i}(\mathbf{n}) \end{aligned}$$

The coefficients of the multipolar expansion are thus

$$a_{\ell m} = \bar{a}_{\ell m} + \sqrt{\frac{3}{4\pi}} \sum_a \sum_i \delta c_a^{(i)} (-1)^m \sum_{LM} \frac{\partial \bar{a}_{LM}}{\partial c_a} \times \sqrt{(2\ell+1)(2L+1)} \begin{pmatrix} \ell & L & 1 \\ -m & M & i \end{pmatrix} \begin{pmatrix} \ell & L & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

Spatial variation on CMB

[Prunet, JPU, Brunier, Bernardeau, 2005]

This implies multipole correlations

$$D_{\ell m}^{(i)} \equiv \langle a_{\ell m} a_{\ell+1 m+i}^* \rangle = f_i(\ell, m) \sum_a \delta c_a^{(i)} \Gamma_\ell^{(a)}$$

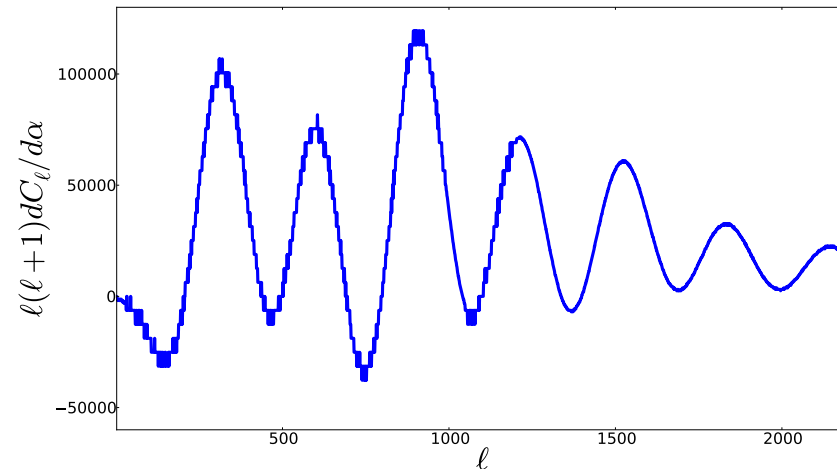
Known functions of l and m

Amplitude of the modulation

$$\Gamma_\ell^{(a)} \equiv \frac{1}{2} \left(\frac{\partial \bar{C}_\ell}{\partial c_a} + \frac{\partial \bar{C}_{\ell+1}}{\partial c_a} \right)$$

$$f_0(\ell, m) = \sqrt{\frac{3}{4\pi}} \frac{\sqrt{(\ell+1)^2 - m^2}}{\sqrt{(2\ell+1)(2\ell+3)}}$$

$$f_1(\ell, m) = \sqrt{\frac{3}{8\pi}} \sqrt{\frac{(\ell+2+m)(\ell+1+m)}{(2\ell+1)(2\ell+3)}}$$



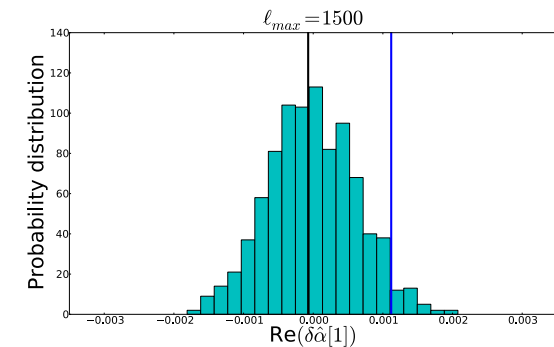
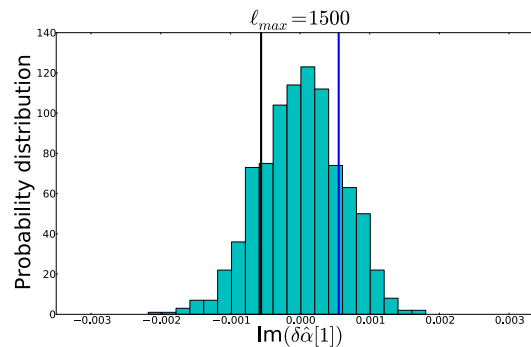
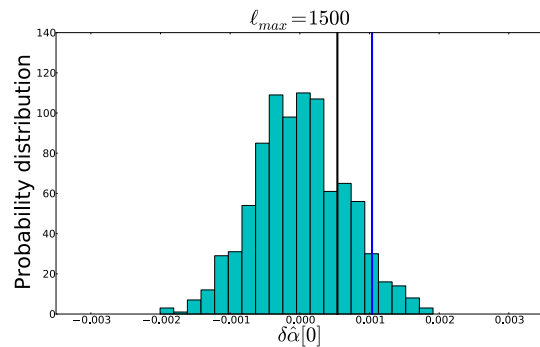
Analysis of *Planck* data

This allows to design an estimator of the D_{lm} [prunet et al (2005); Hansen-Lewis (2009)]

Masking effect also induces l-correlations

Simulations of 10^3 maps with no modulation + Planck masking

Simulation of a CMB with α modulation



Simulated map with $\delta\alpha = 10^{-3}$ / Planck data

The amplitude of a modulation of α is constrained to $\delta\alpha < 6 \times 10^{-4}$ (1σ) at $z = 1000$

First constraint from the CMB

To be compared with $\delta\alpha/\alpha = (0.97 \pm 0.22) \times 10^{-4}$ (4σ) at $z=2$ [webb et al. (2011)]

Conclusions and perspective

Conclusions

In the past years, we have obtained a series of results concerning the variation of fundamental constants:

- Theoretical modelling of g_p ; useful for clock & quasars
- Study of coupled variations in GUT
- First model of pure spatial variations

- CMB
 - improved constraint by a factor 5 compared to WMAP
 - lift the degeneracy between α , m_e and H_0
 - First constraint on spatial variation

- Nuclear physics:
 - BBN: improved constraints; detailed study of $A=5$ & $A=8$
 - Pop III stars: fine tuning at 10^{-3} (anthropic)

Physical systems: new and future

