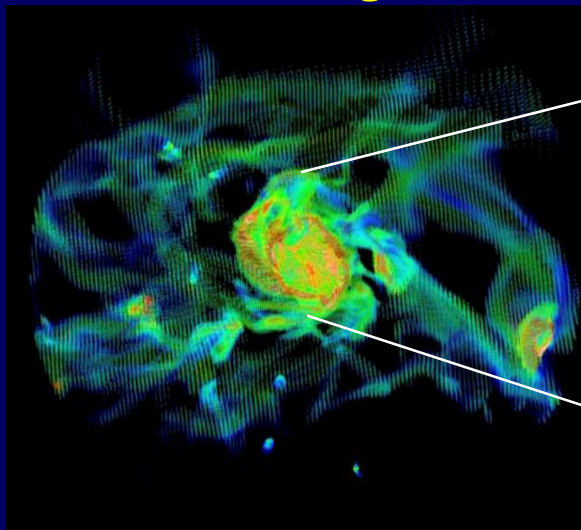


Key Issues at the Peak of Galaxy Formation

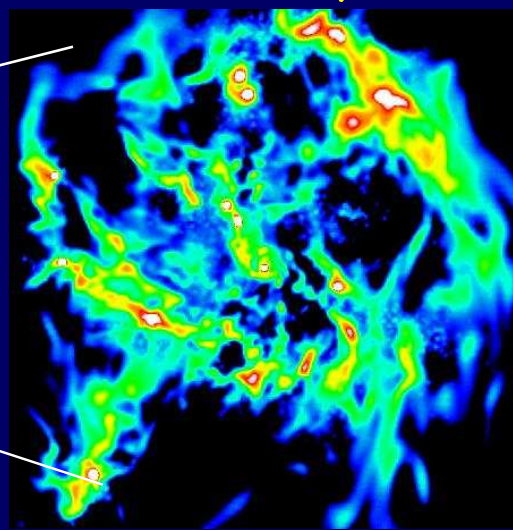
Avishai Dekel
The Hebrew University of Jerusalem

IAP, October 2014

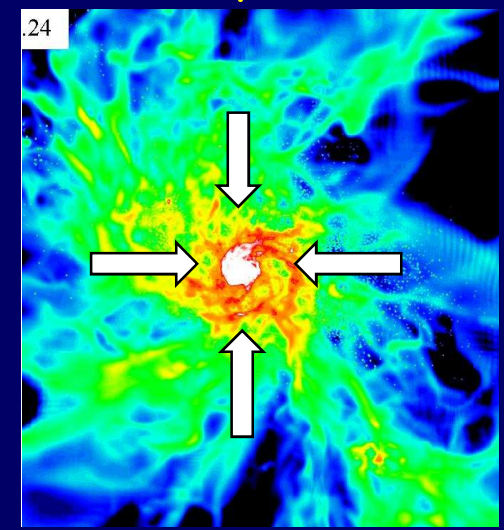
streams - ring - disk



instability



compaction



The most active phase of galaxy formation
is during the first few Gyr, $z=1-4$

Rapid mass assembly: $dM/dt \sim M (1+z)^{2.5}$
→ at $z \sim 2$ significant mass growth per orbital time

High gas fraction: $\sim 50\%$

Fast star formation: $SFR \sim 100 M_{\odot} \text{ yr}^{-1}$

The processes are intensified compared to $z=0$
→ best for studying galaxy formation

Galaxies form in the Cosmic Web

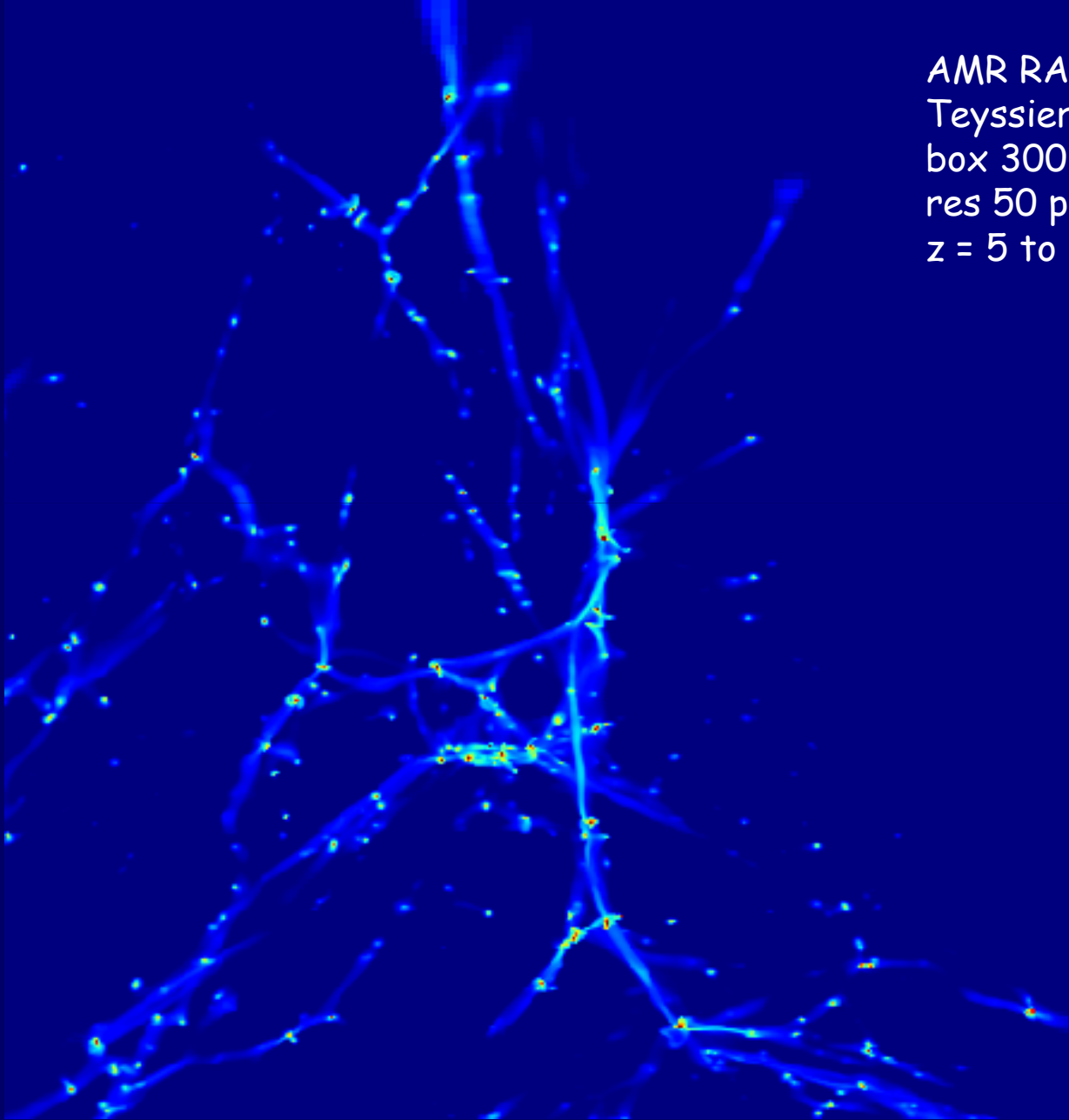
Massive halos (at high-sigma nodes) are fed by relatively thin dense filaments → cold streams

Typical halos reside in relatively thick filaments, fed from all directions

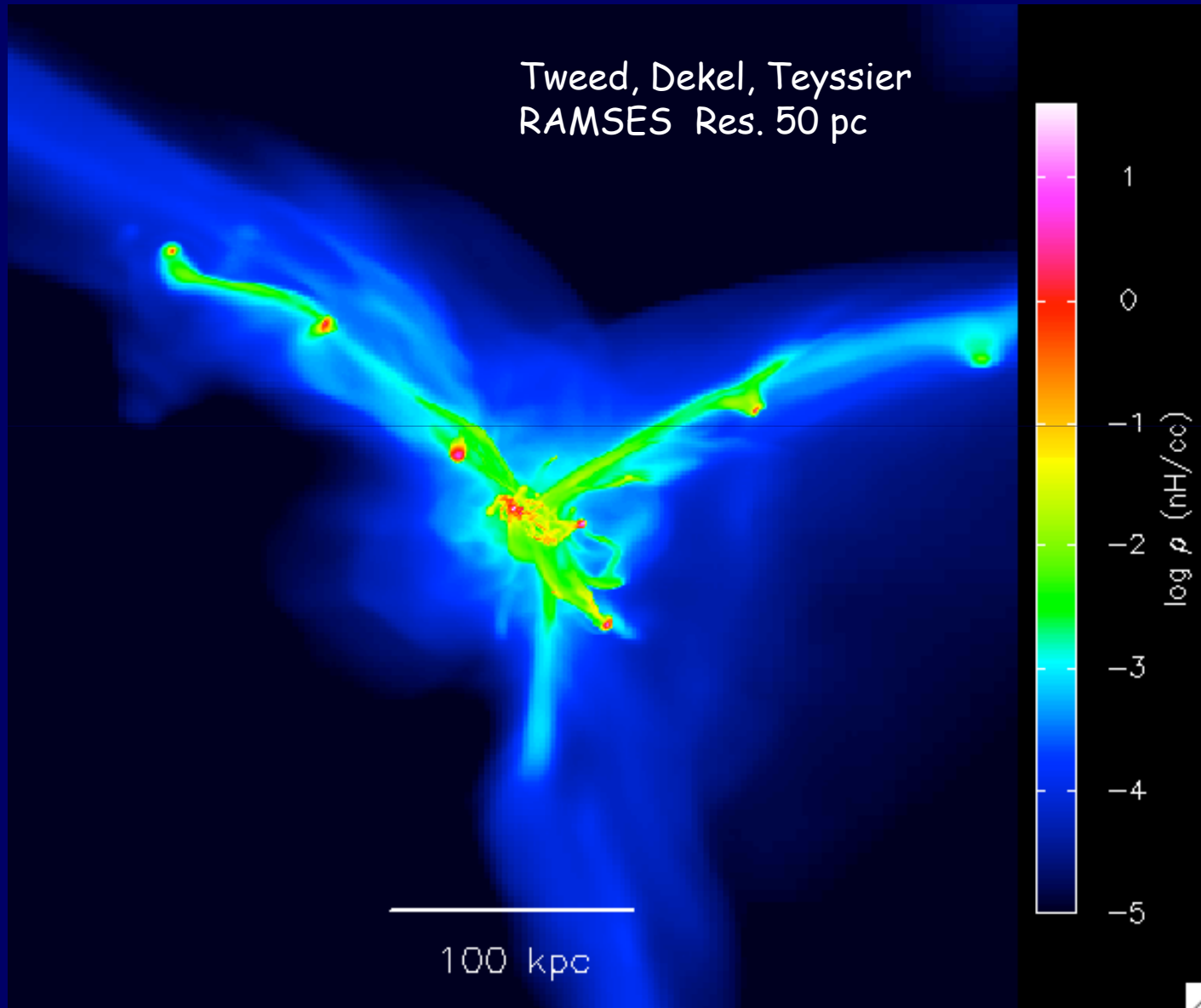
the millenium cosmological simulation

Gas streams along the cosmic web

AMR RAMSES
Teyssier, AD
box 300 kpc
res 50 pc
z = 5 to 2.5

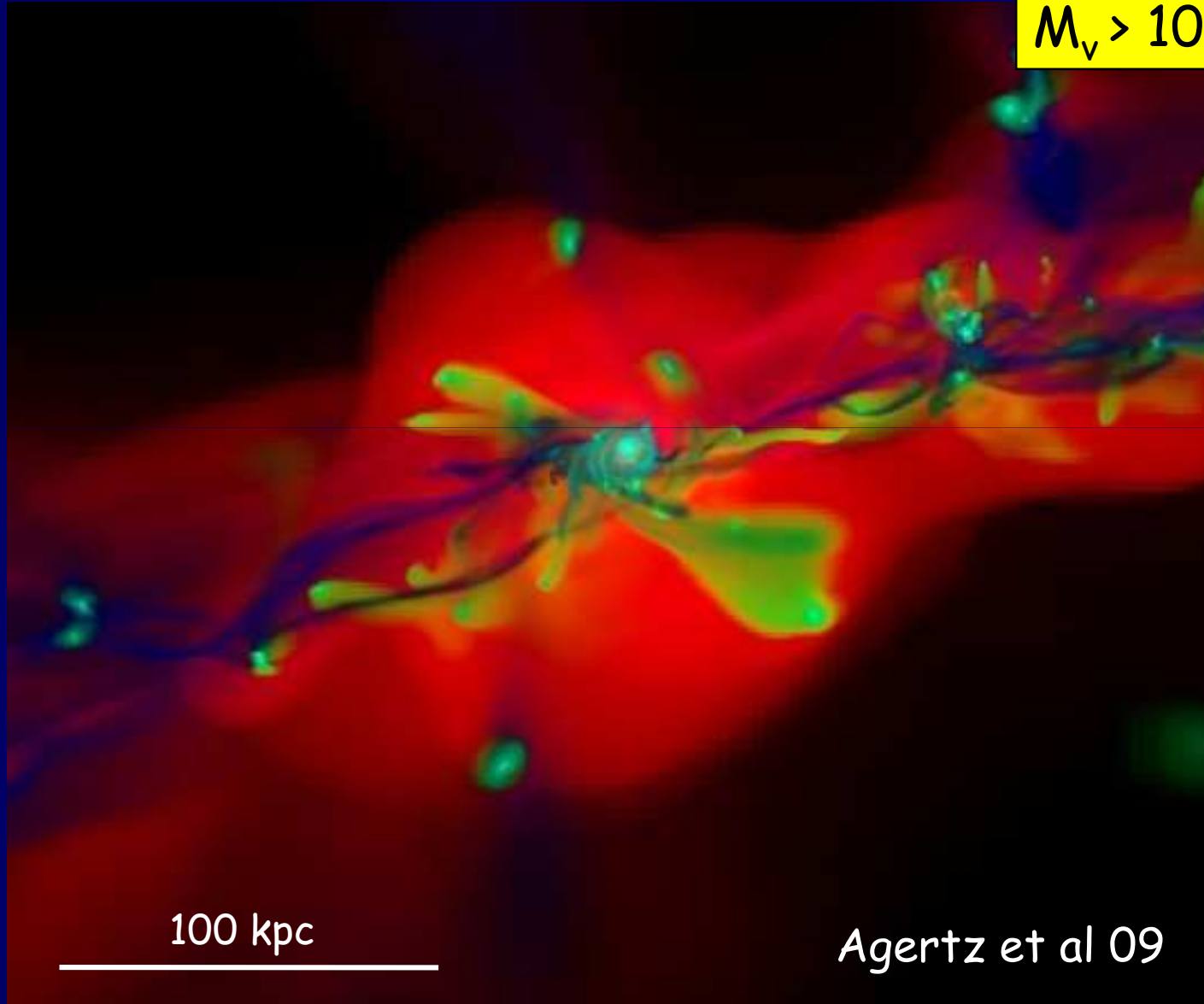


Streams Feeding a Hi-z Galaxy

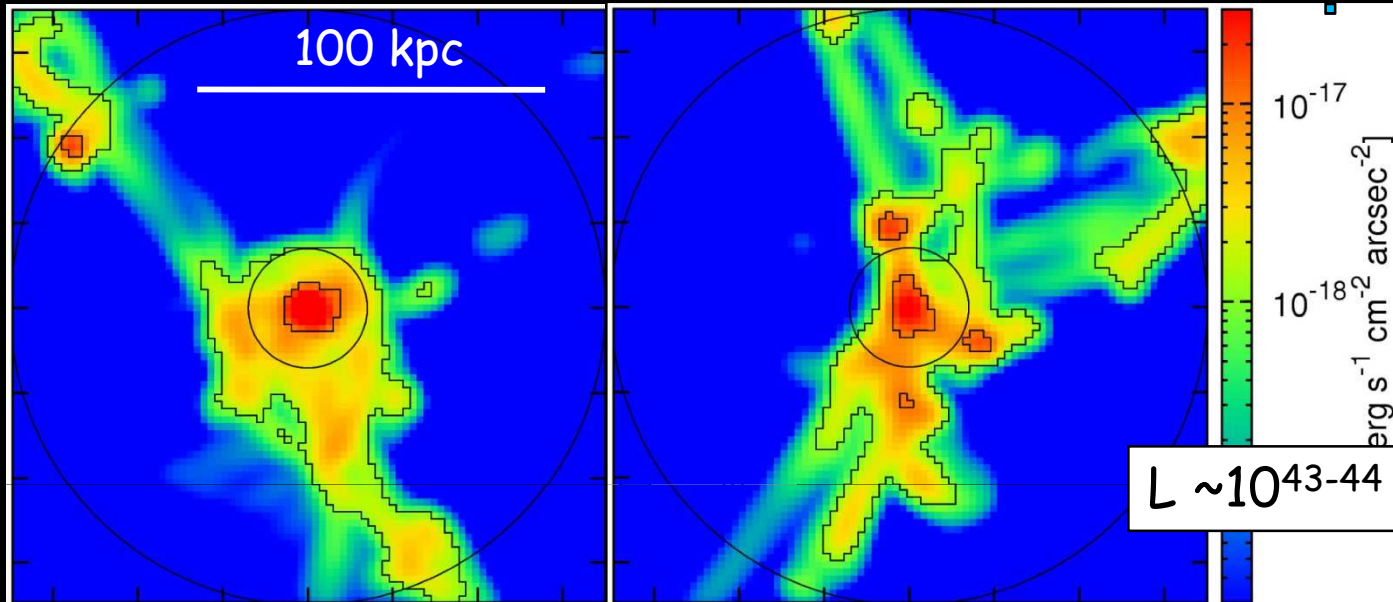


Cold Streams Penetrate through Hot Halos

$M_v > 10^{12} M_\odot$

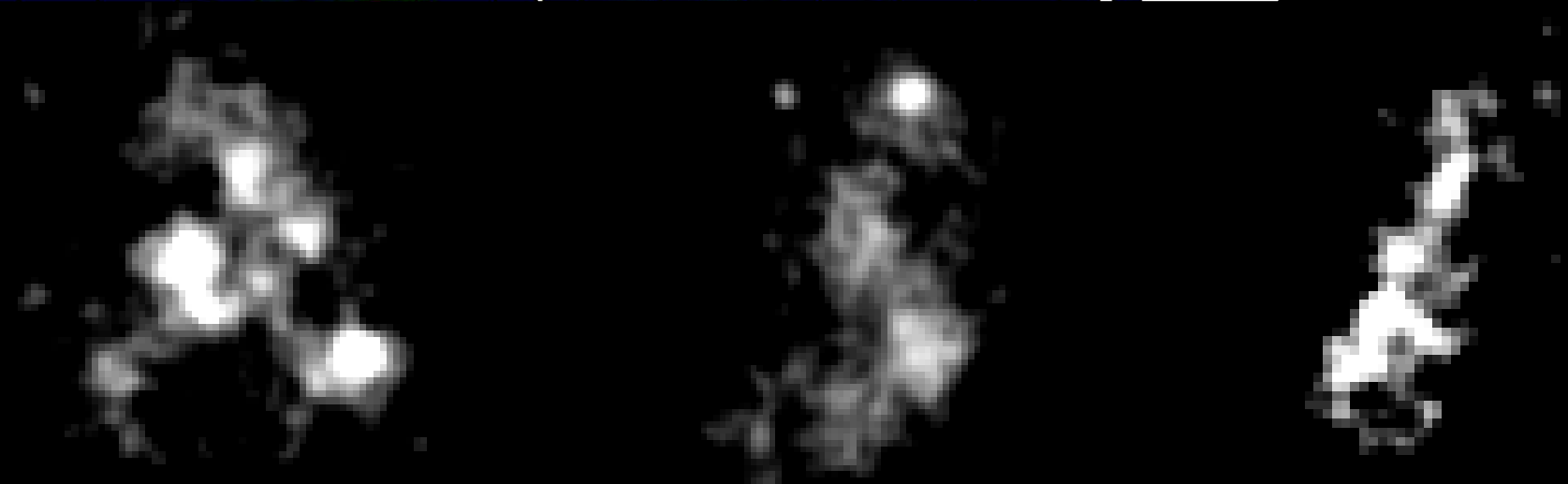


Cold streams Observable: Lyman-alpha Blobs



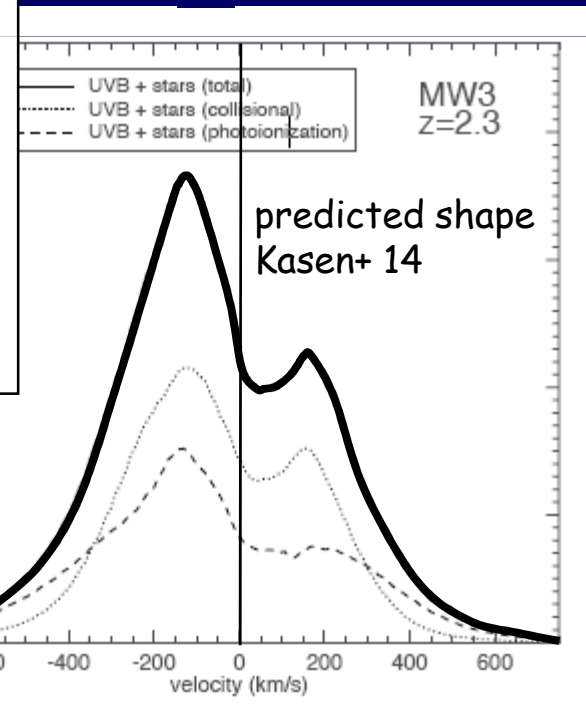
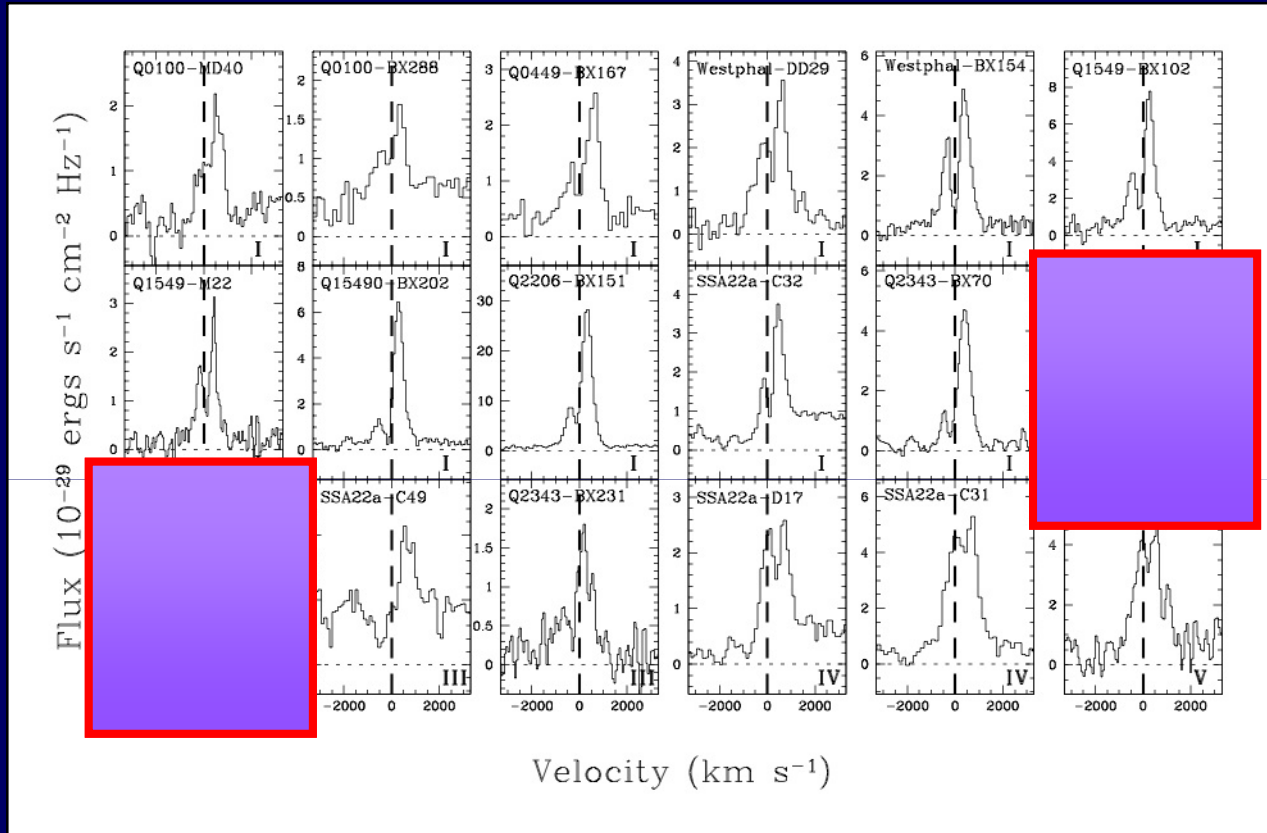
Goerdt, Dekel,
Sternberg, Ceverino,
Teyssier, Primack 10

MUSE?



Matsuda et al 06-09

Detection of Inflow in Ly α Emission

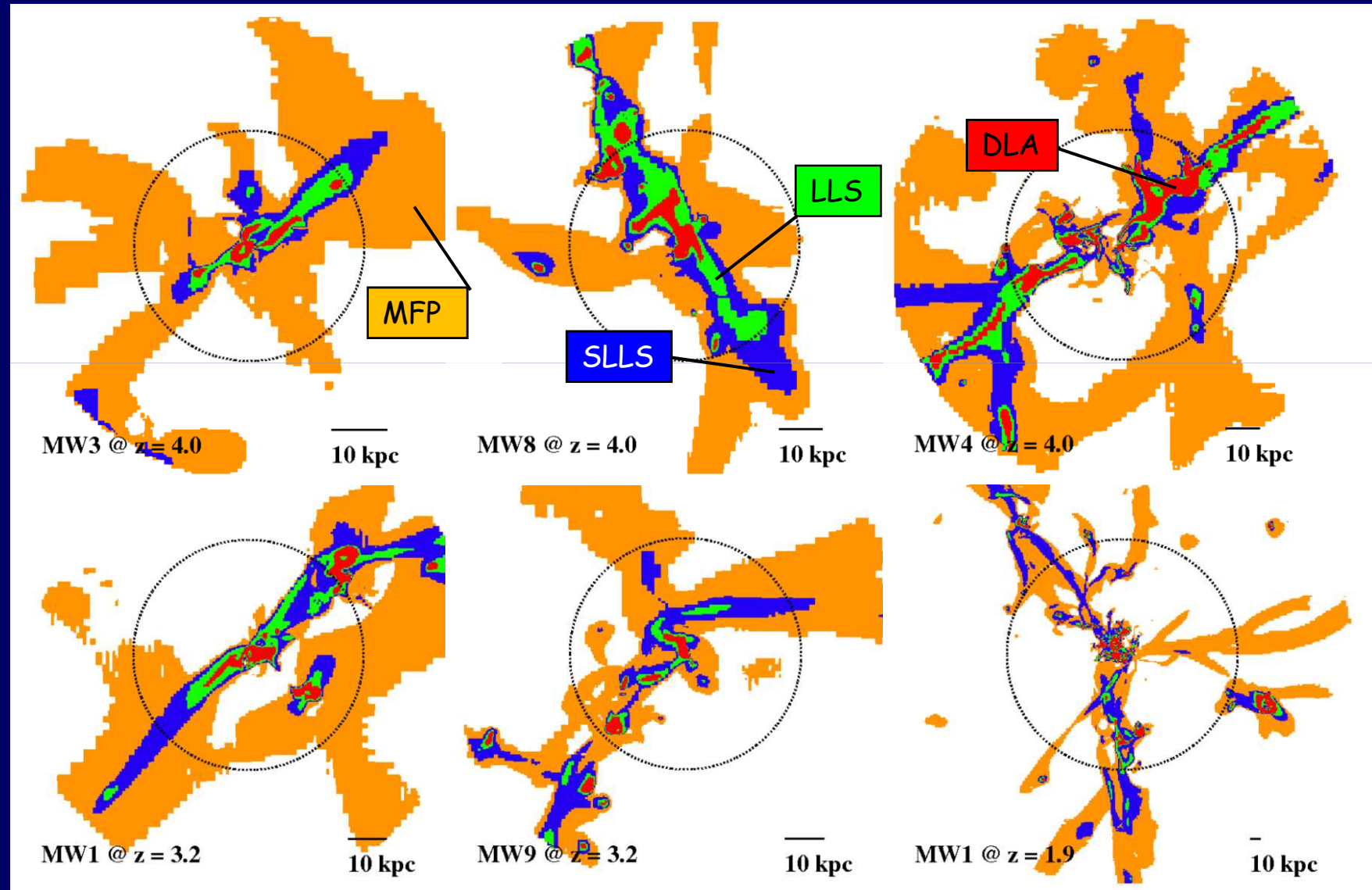


Kulas, Shapley+ 2012: Systemic z from H α

Most are red-shifted \rightarrow outflows
 Some are blue-shifted \rightarrow inflow

Cold Streams & Pancakes in Ly- α Absorption

Fumagalli, Prochaska, Kasen, Dekel, Ceverino, Primack 11



High- z massive galaxies are fed by intense thin streams of smooth gas and frequent mergers (minor, major)

New challenges for galaxy formation

1. Galaxy angular momentum from the cosmic web
2. Violent disk instability: nonlinear, stimulated
3. Quenching by compaction + hot halo

AMR Cosmological Simulations

Cosmological box, RAMSES (Teyssier), resolution 1 kpc

Zoom-in galaxies, ART (Kravtsov, Klypin), RAMSES (Teyssier)

Ceverino, Dekel, Primack:

- 50 pc res. (30 galaxies)
- 25 pc res., lower SFR, w/o rad. fdbk (2x30 galaxies)
- same with stronger RP feedback



DeGraf, Dekel, Gabor, Bournaud:

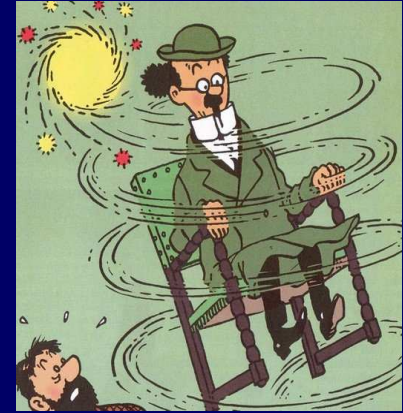
- with BHs and AGN feedback (isolated and cosmological)

Isolated galaxies, resolution 1-10 pc, RAMSES, Bournaud et al.

HUJI: Ceverino, Mandelker, Danovich, Tweed, Zolotov, DeGraf, Inoue
Groups of Krumholz+, Burkert+, Bournaud+, Teyssier+, Primack+



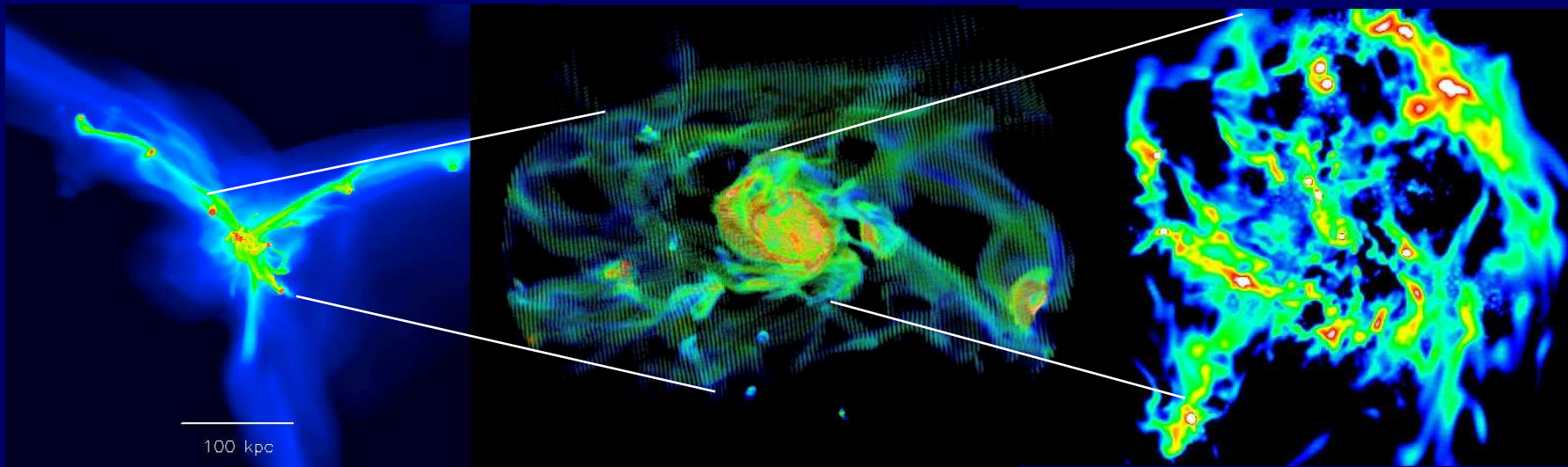
Angular Momentum Buildup in High- z Galaxies



Prof. tournesol



Pichon, Pogosyan, Devriendt, Dubois, Colombi+ 2011-2014
Stewart, Bullock+ 2011, 2013
Danovich, Dekel, Hahn, Teyssier, Ceverino, Primack 12, 14



Standard Picture: Spherical Collapse

Rees & Ostriker 77, Silk 77, White & Rees 78, Fall & Efstathiou 80 ...

Proto-halo expansion, turnaround,
collapse to a virialized DM halo

AM by tidal torques (TTT) prior to maximum expansion: $\lambda_{\text{gas}} \sim \lambda_{\text{DM}} \sim 0.04$

Spherical gas infall into the halo with the DM
Virial shock heating to $T_v \sim 10^6 \text{K}$

$$\lambda_{\text{gas}} \sim \lambda_{\text{DM}}$$

Radiative cooling, cylindrical accretion to disk

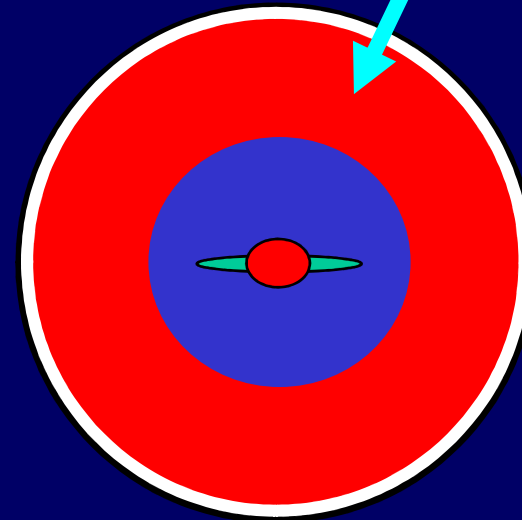
AM is conserved

Halo AM determines disk size and structure

$$\text{const.} = J_{\text{gas}} / M_{\text{gas}} \sim \lambda R_{\text{vir}} V_{\text{vir}} \sim R_{\text{disk}} V_{\text{disk}}$$

→

$$\frac{R_{\text{disk}}}{R_{\text{vir}}} \approx \lambda \approx 0.04$$



Bulge by mergers, disk AM is conserved

Is the naïve model of smooth cylindrical
infall with disk spin \sim halo spin (SAM)
valid at high redshift?

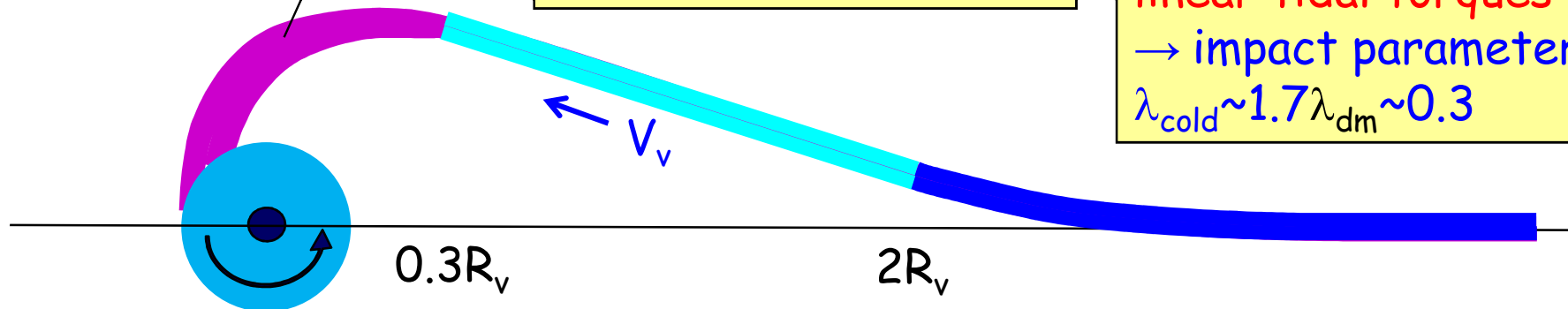
AM Buildup by Cold Streams in 4 Phases

Danovich, Dekel, Hahn+ 14

III. inner halo - extended tilted ring
 non-linear torques, dissipation
 AM loss $\lambda_{\text{cold}} \rightarrow 0.04$ & alignment

II. outer halo
 AM transport, $j \sim \text{const.}$
 $\lambda_{\text{cold}} \sim 3\lambda_{\text{dm}} \sim 0.1$ DM mix

I. cosmic web
 linear tidal torques
 \rightarrow impact parameter
 $\lambda_{\text{cold}} \sim 1.7\lambda_{\text{dm}} \sim 0.3$

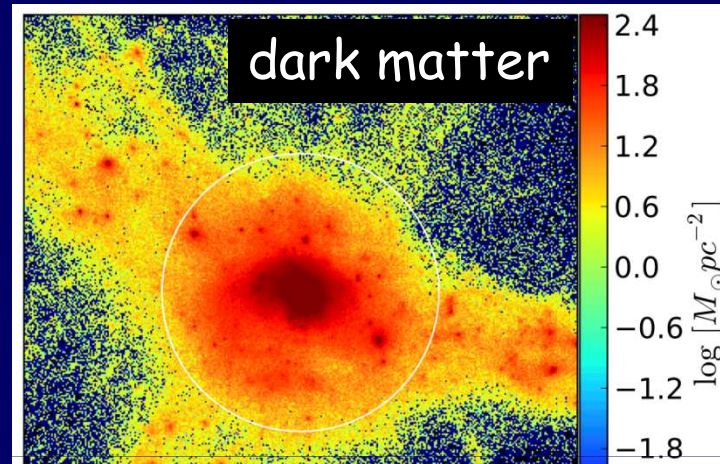
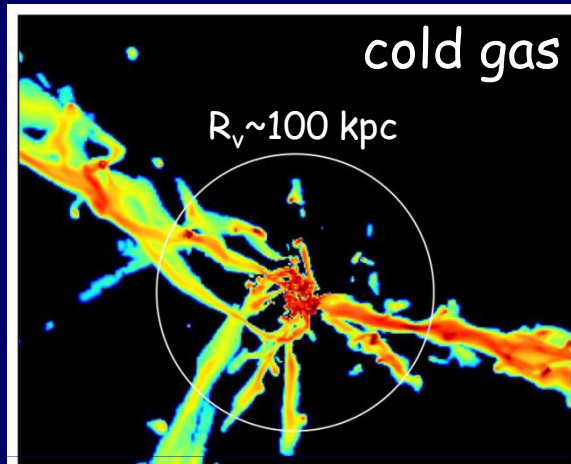


IV. inner disc (+ bulge)
 disk instability, outflows
 $\lambda_{\text{baryons}} \sim 0.03$

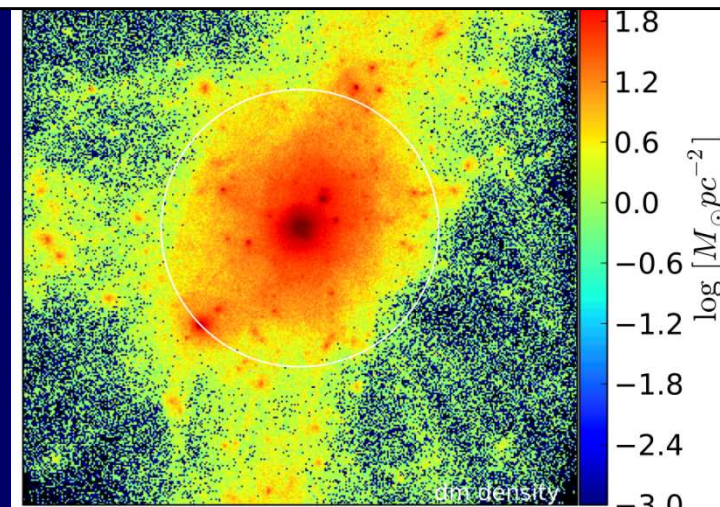
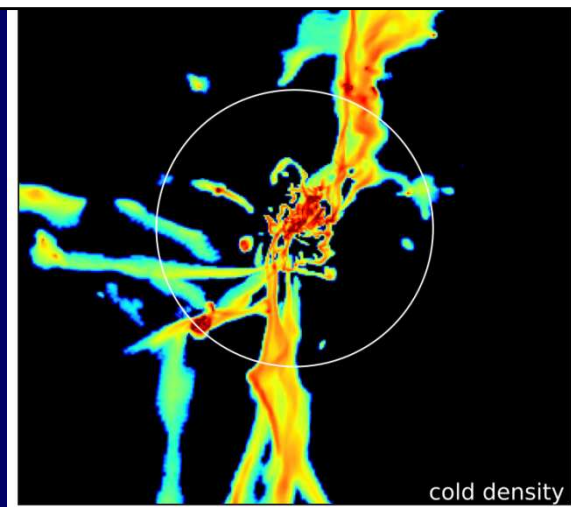
spin parameter $\lambda \sim \frac{J/M}{\sqrt{2}R_v V_v}$

TTT outside the halo: $\lambda_{\text{cold}} \sim 1.7 \lambda_{\text{DM}}$ due to quadrupole moment of inertia

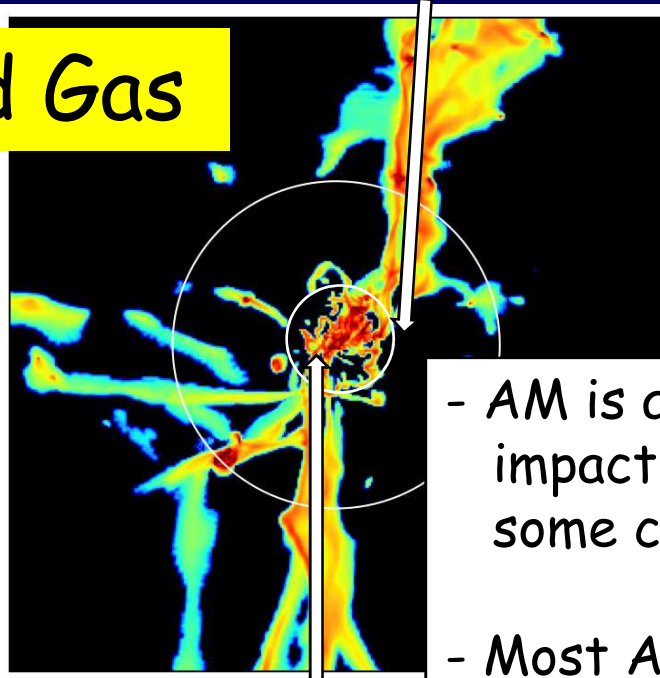
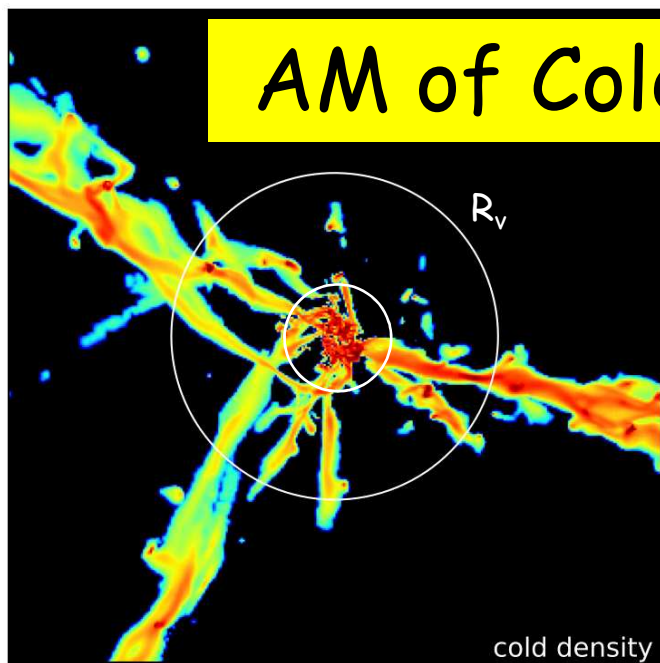
$$J_i \propto t \varepsilon_{ijk} T_j I_{lk}$$



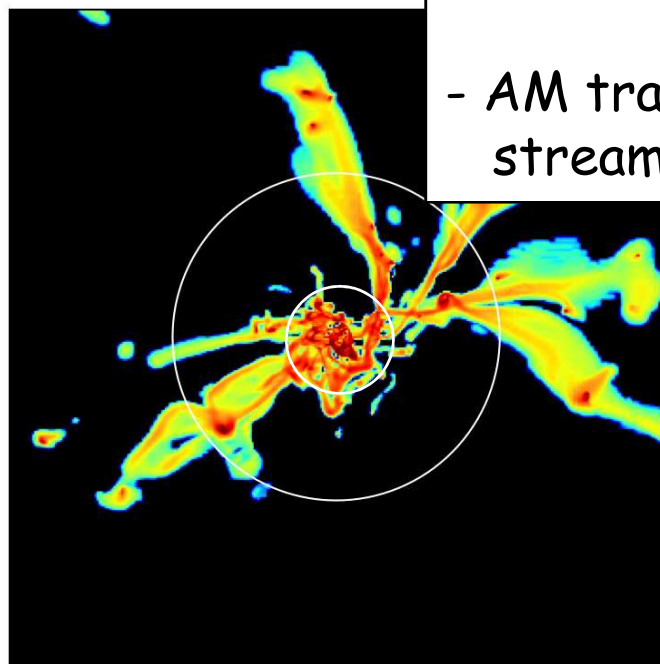
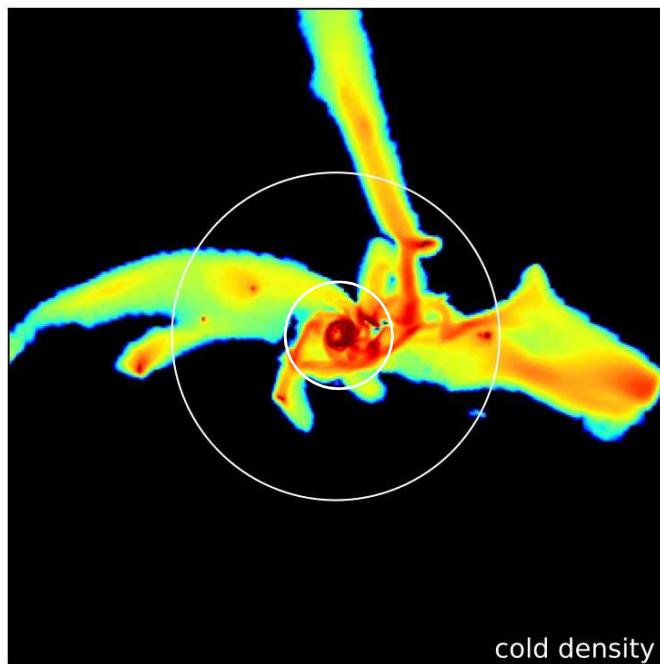
TTT is applied at max-expansion along the streams, after pre-collapse of gas to the filaments' cords



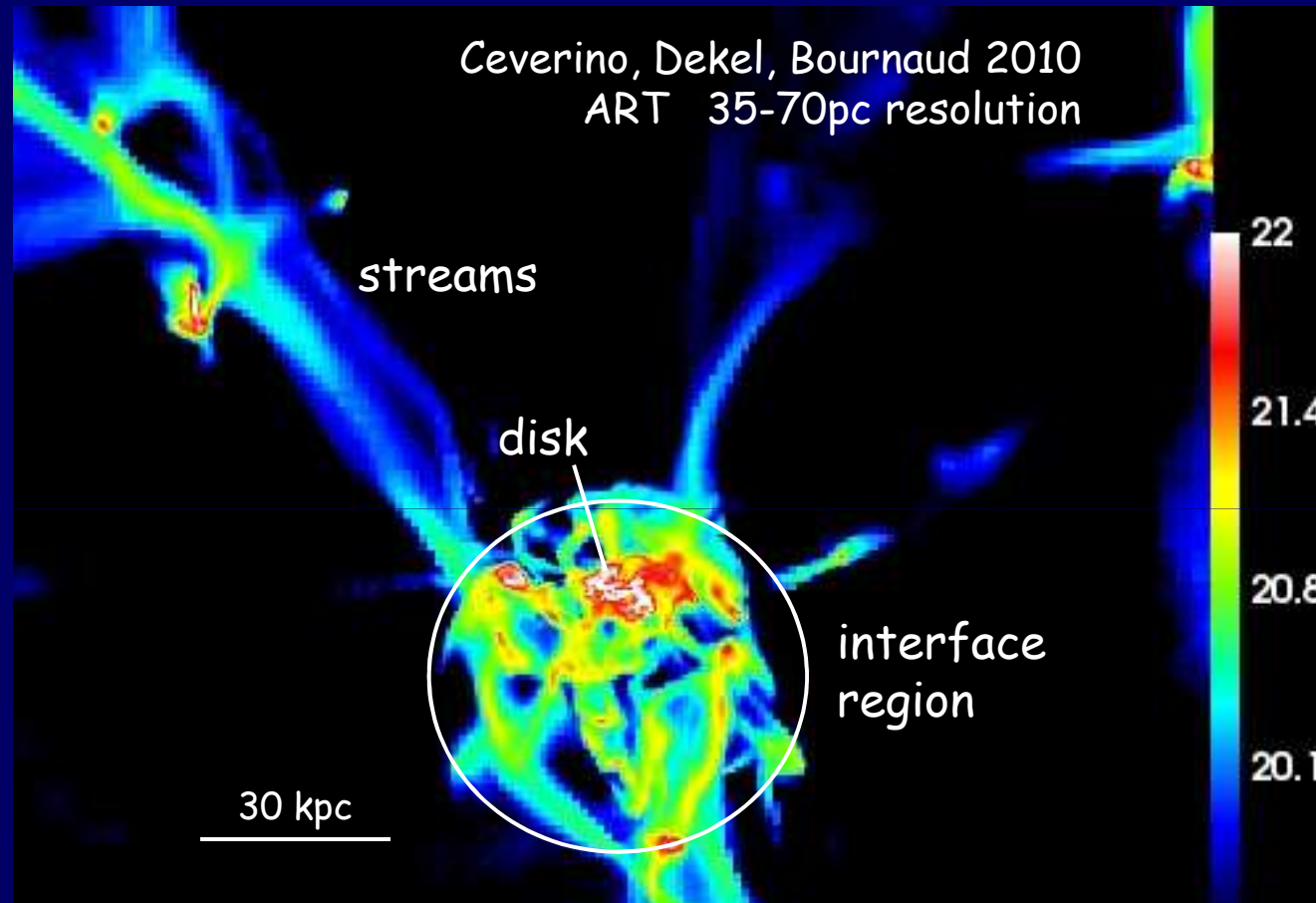
AM of Cold Gas



- AM is associated with impact parameters $< 0.3R_v$, some counter-rotating
- Most AM in one stream
- AM transport by straight streams to inner halo



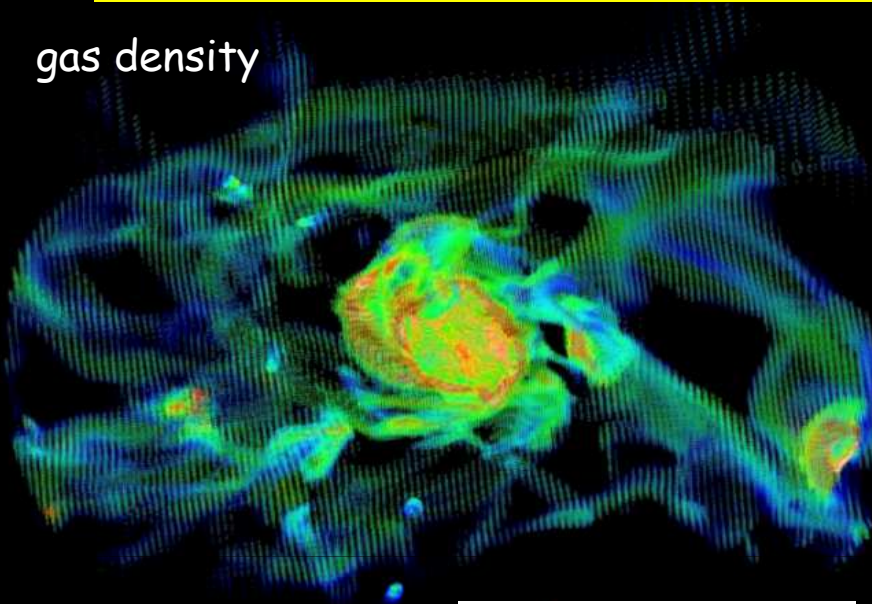
How do the streams join the disk?



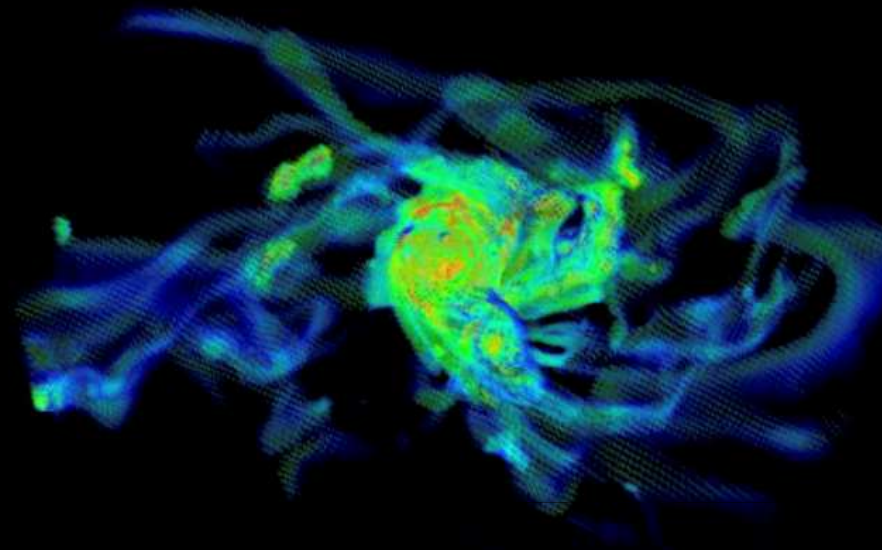
A messy interface region:
breakup due to shocks, hydro and thermal instabilities,
collisions between streams and clumps, heating

An Extended Tilted Ring about the Disk

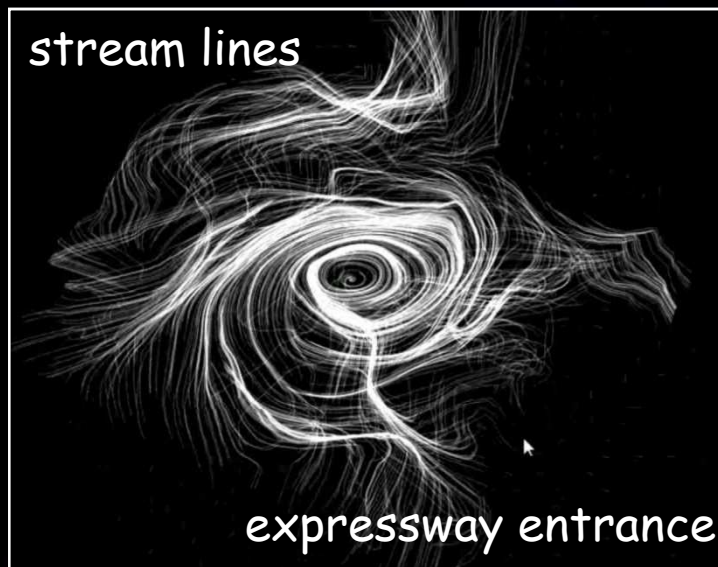
gas density



30 kpc

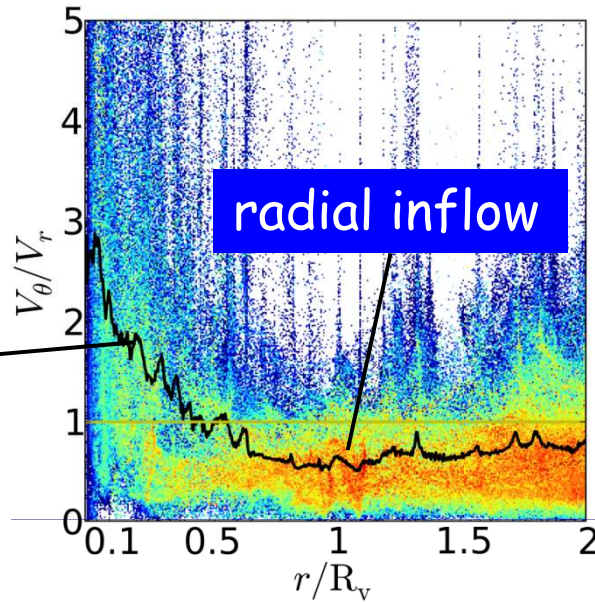


stream lines



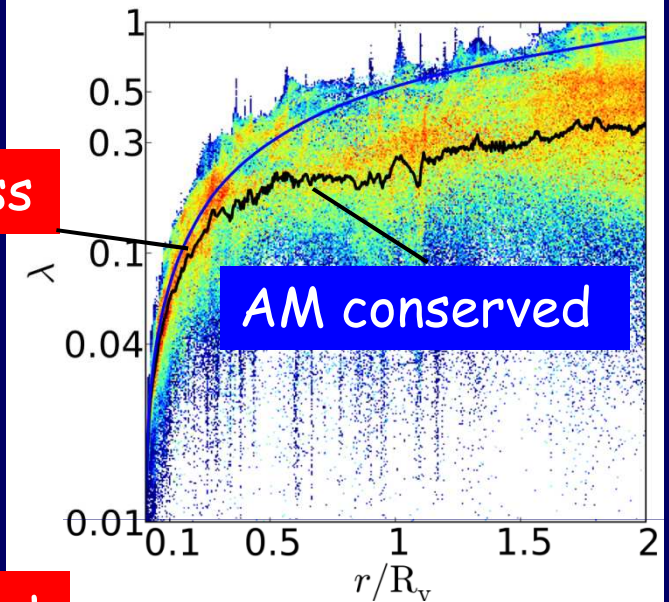
expressway entrance

A tilted rotating ring in the inner halo



circular orbits

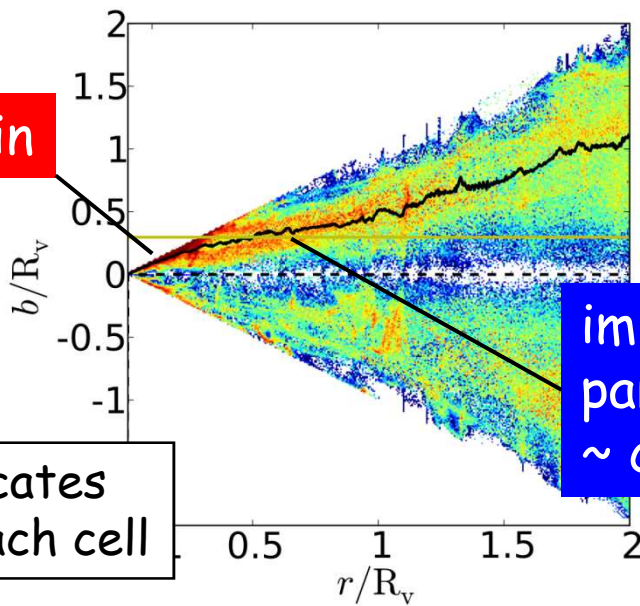
AM loss



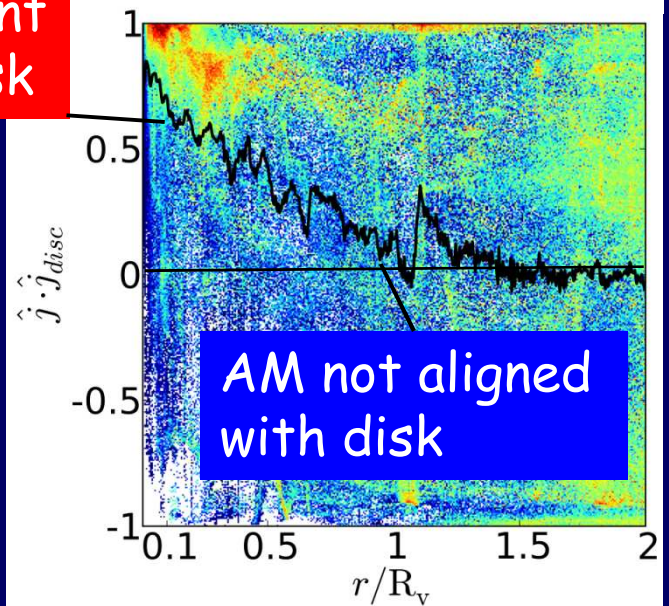
AM conserved

alignment with disk

spiral in



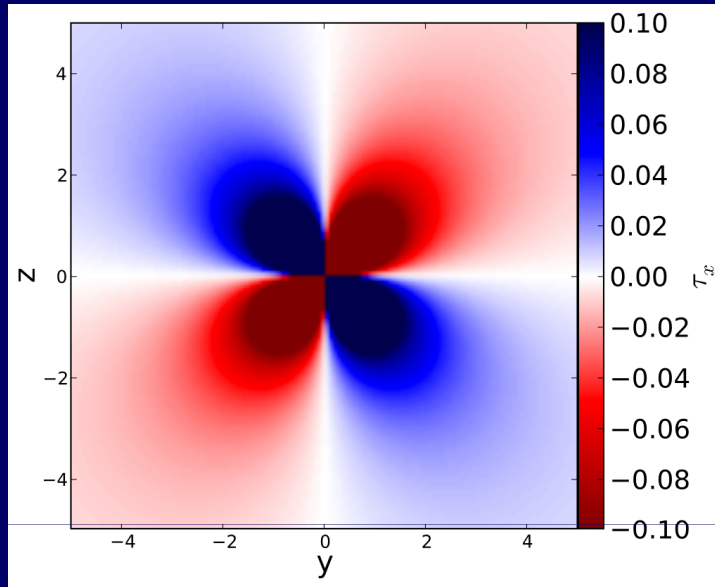
impact parameter ~ const.



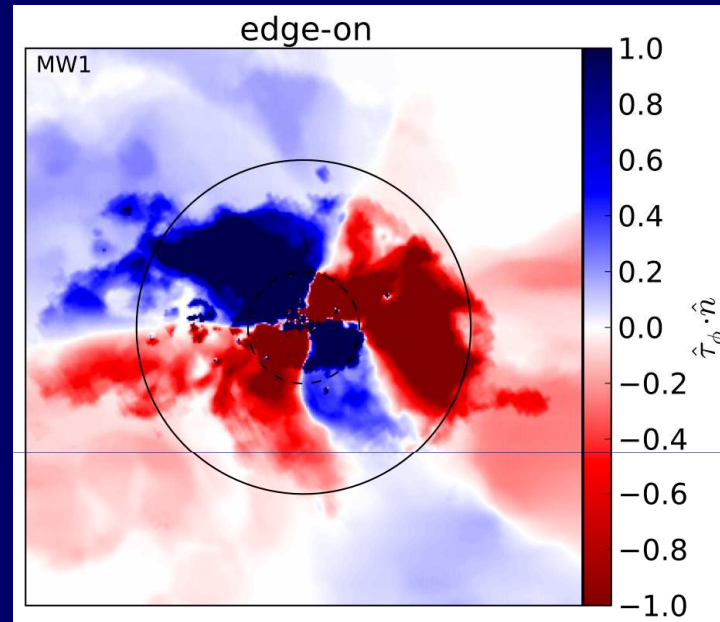
AM not aligned with disk

color indicates mass in each cell

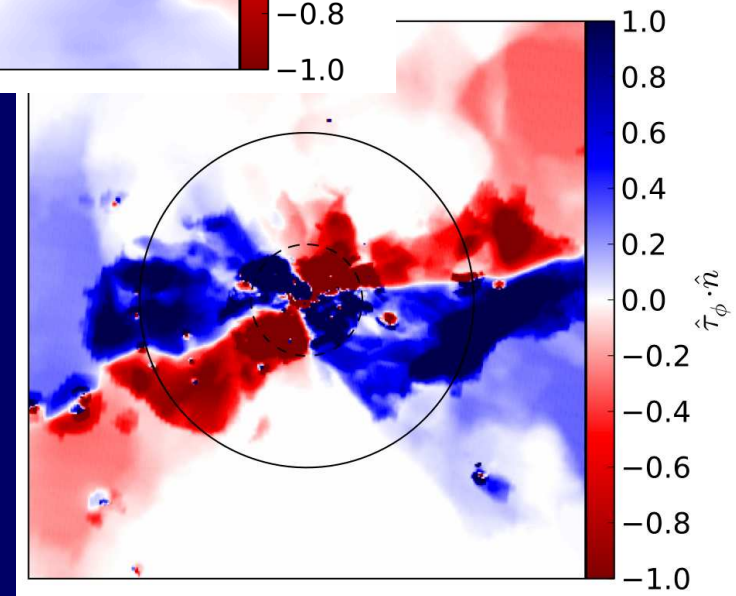
AM Exchange in the Ring: Torques by Disk



torques by an idealized disk

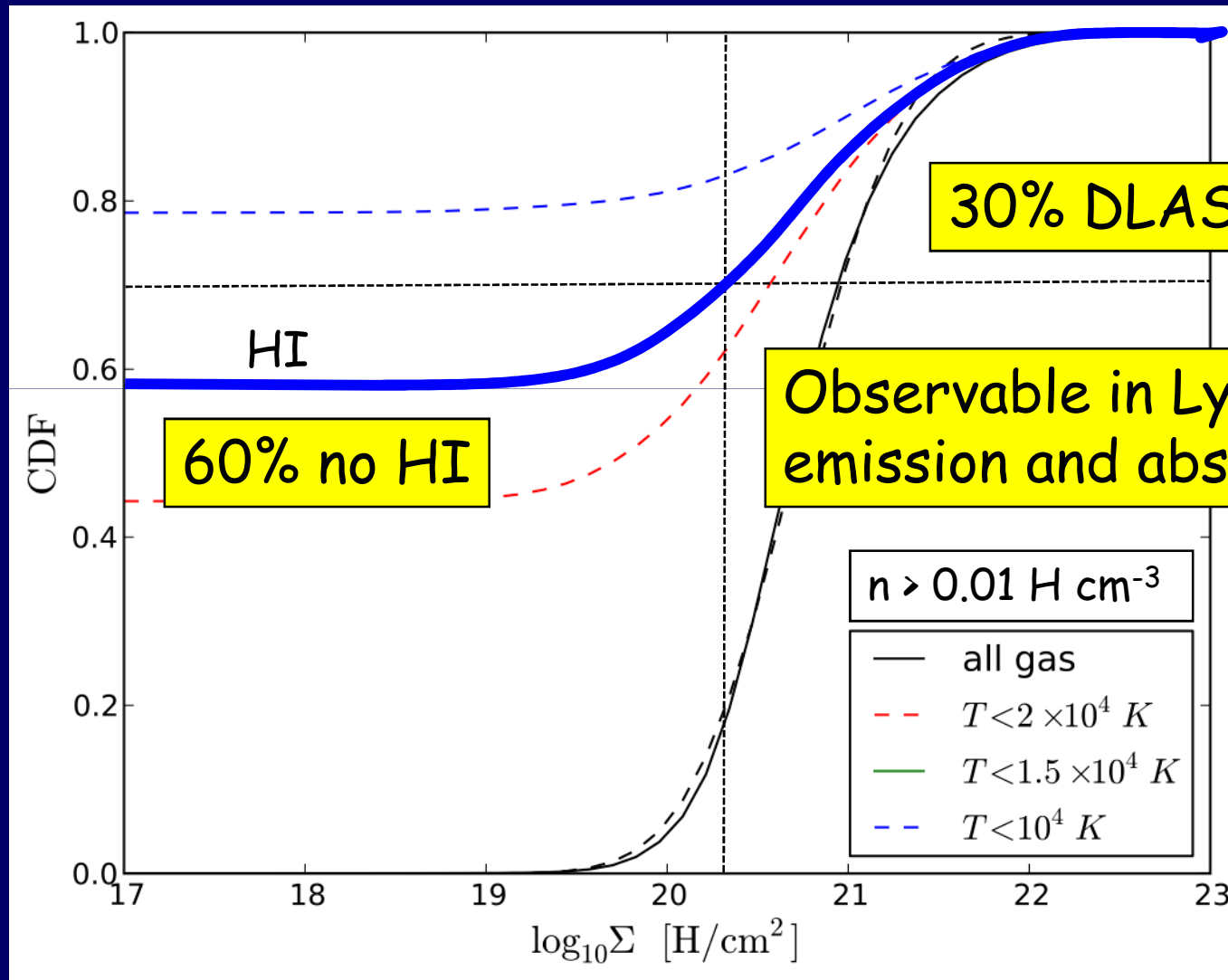


Torques in the simulated galaxies



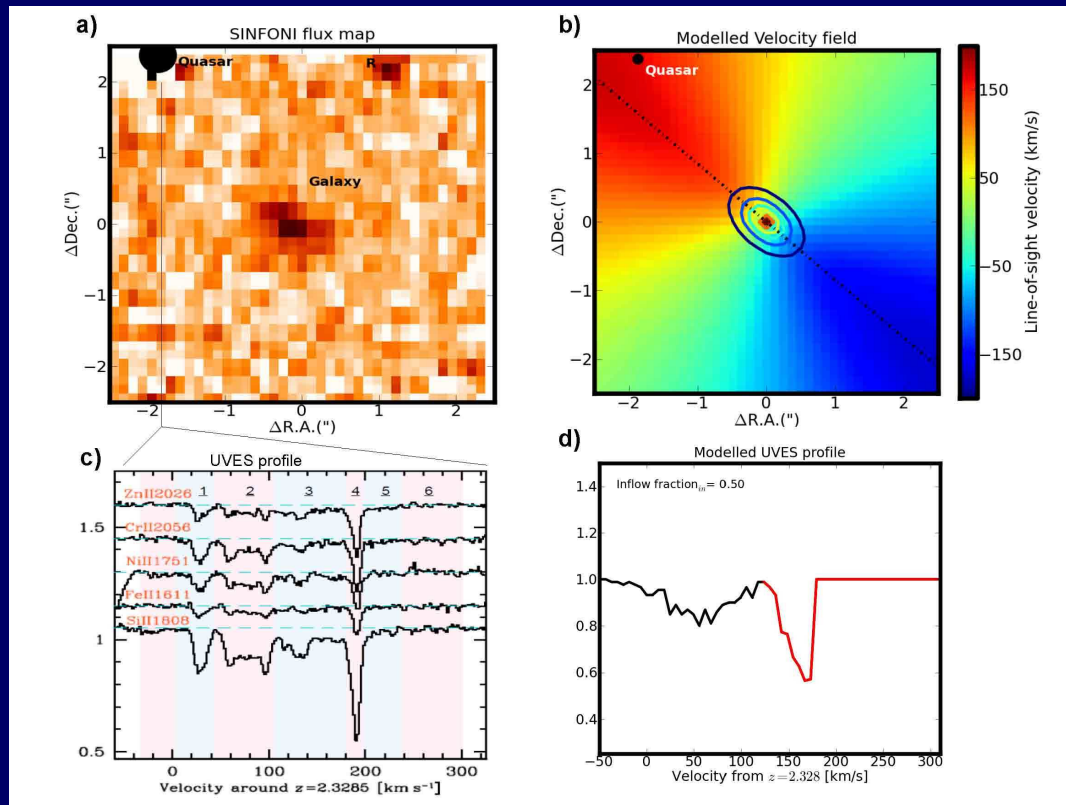
Extended Ring: HI Column Density

Random lines of sight through $(0.1-0.3)R_v$

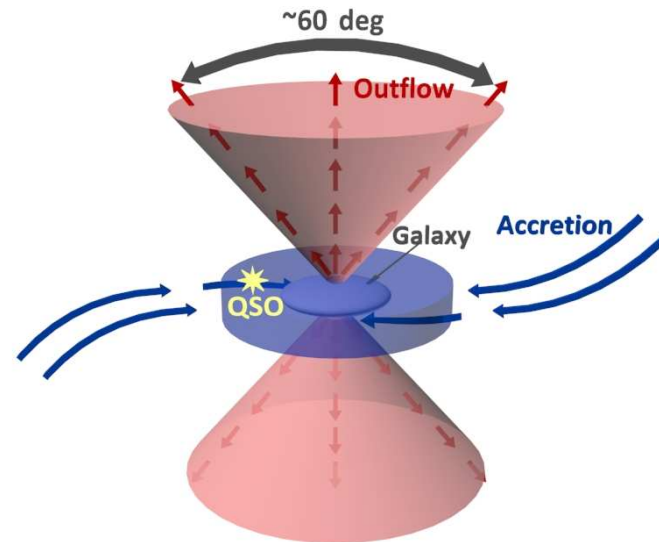


Detection of an Extended Ring?

Bouche+ 2013



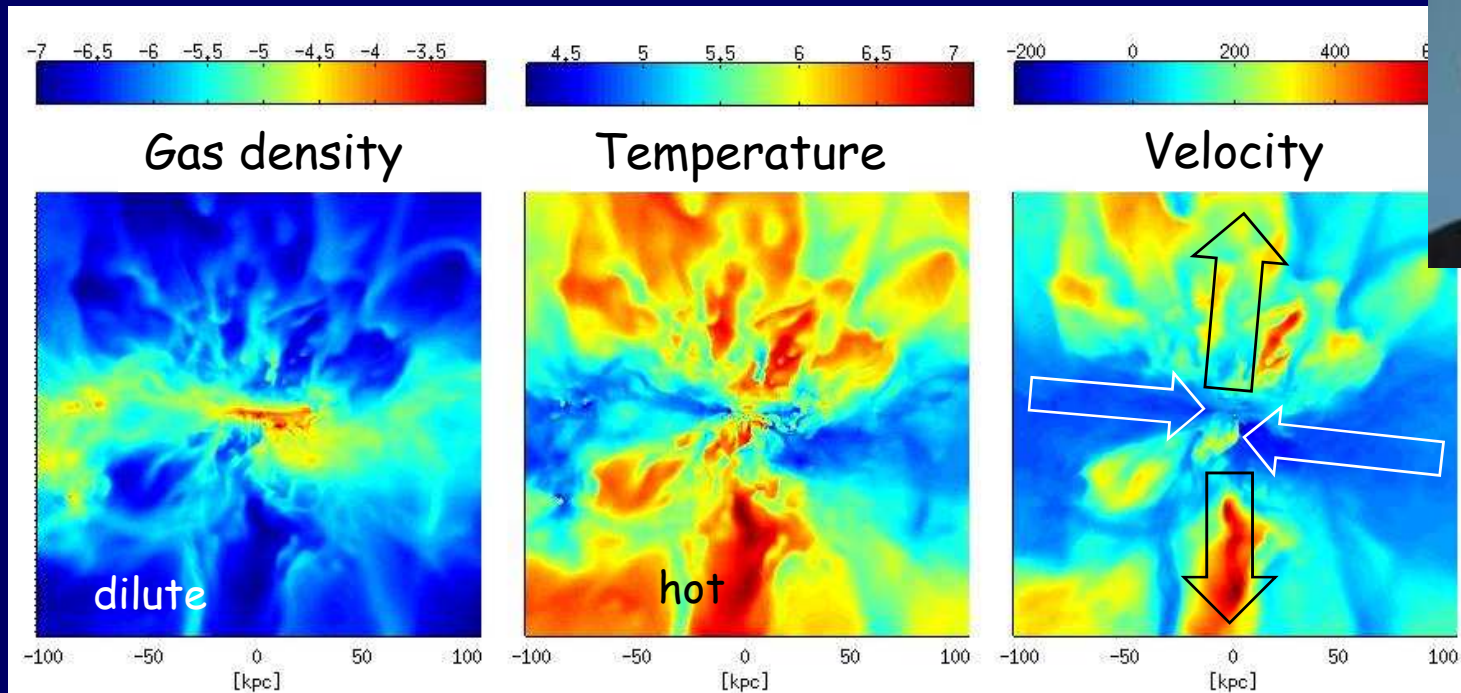
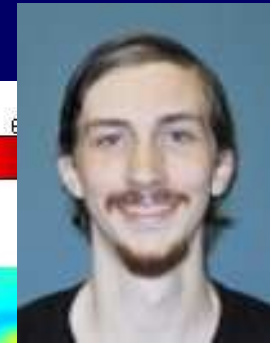
$z=2.3$
 Low- Z gas
 26 kpc from center
 $V=180$ km/s



Crighton+ 2013 $z=2.4$, 54 kpc
 Steidel+ 2002, Kacprzak+ 2010

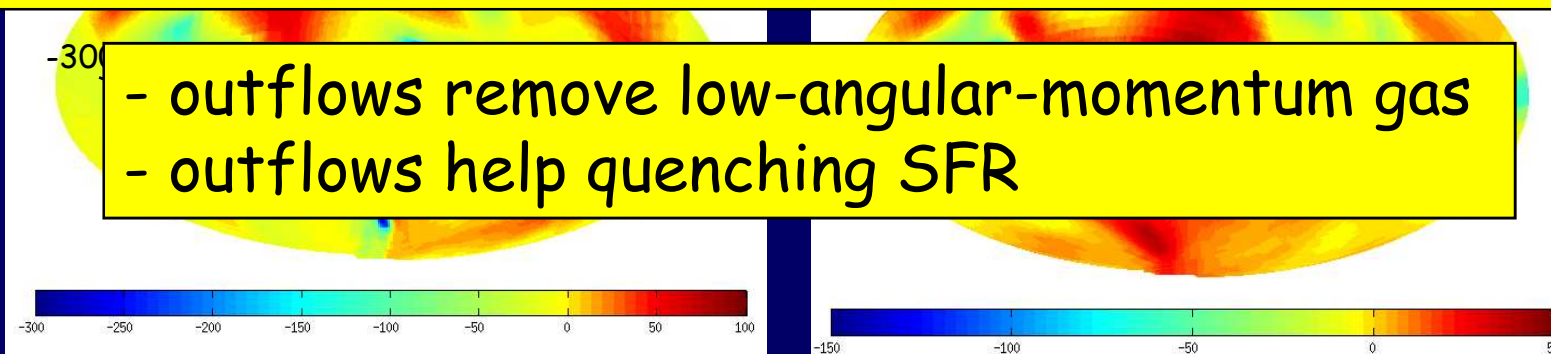
Outflows do not halt the Inflows

DeGraf+ 14



- Dense, cold, metal-poor inflows penetrate into the galaxy
- Hot, metal-rich, fast outflows fly through the dilute CGM

- outflows remove low-angular-momentum gas
- outflows help quenching SFR



AM Evolution in Disks

- Gas-rich \rightarrow violent disk instability (VDI) (Noguchi 99; Dekel+ 09)
- \rightarrow torques \rightarrow AM outflow and mass inflow (Gammie 01)
- \rightarrow massive spheroids (+BHs) with low AM (Genzel+ 08; Bournaud+11; Dekel+ 13)

Stellar and AGN feedback \rightarrow outflows remove low-AM gas from galaxy centers (Maller & Dekel 02; Governato+ 10; Guedes+ 11)

$$\lambda_{\text{gal}} < \lambda_{\text{disk}} \sim 0.03$$

λ_{disk} is only slightly smaller than λ_{DM} \rightarrow the naive model is a crude approximation despite the different AM evolution

Conclusions: Angular-Momentum Buildup

High- z massive galaxies form at cosmic-web nodes
Fed by ~ 3 co-planar **streams** penetrating hot CGM

4 Phases of AM buildup:

- effective tidal **torques** on pre-collapsed gas streams,
- AM **transport** through outer halo to inner halo
- spiral-in through an **extended tilted rotating ring (DLAS?)**
- redistribution within the disk by VDI and feedback

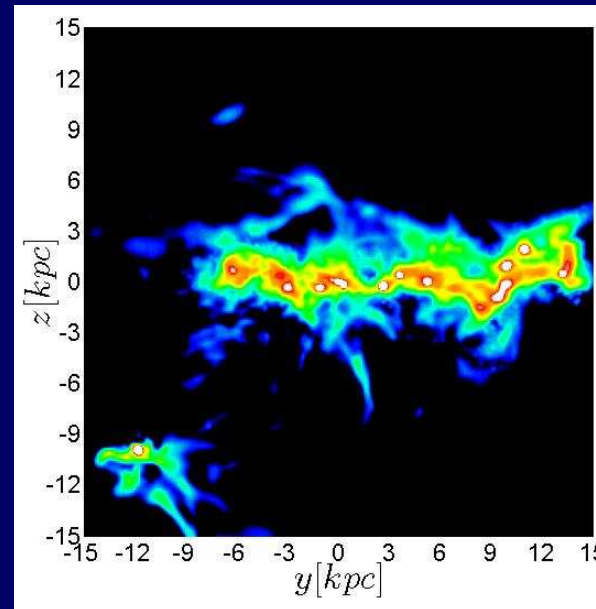
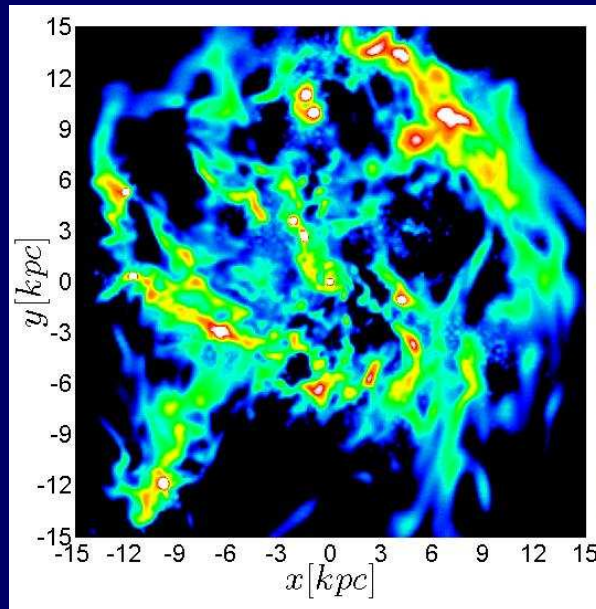
disk spin \sim halo spin (within $\times 2$) despite different evolution
→ moderate changes to semi-analytic models



Violent Disk Instability: Nonlinear and Stimulated



Dekel, Sari, Ceverino 2009; Ceverino+ 2010, 2012
Mandelker+ 2014; Moody+ 2014; Forbes+ 2014a,b
Dekel, Bournaud, Mandelker+ 2014;
Inoue+ 2014



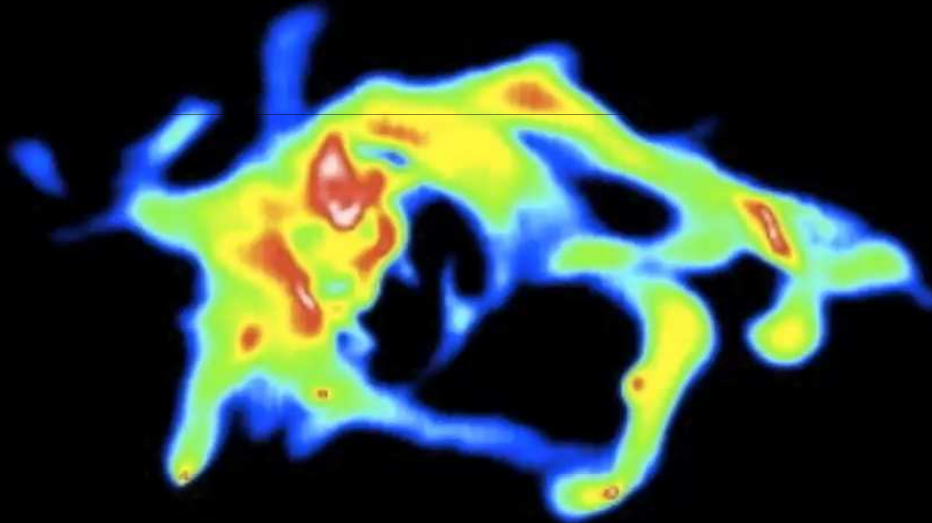
Clumpy Disk

Ceverino, Dekel et al.

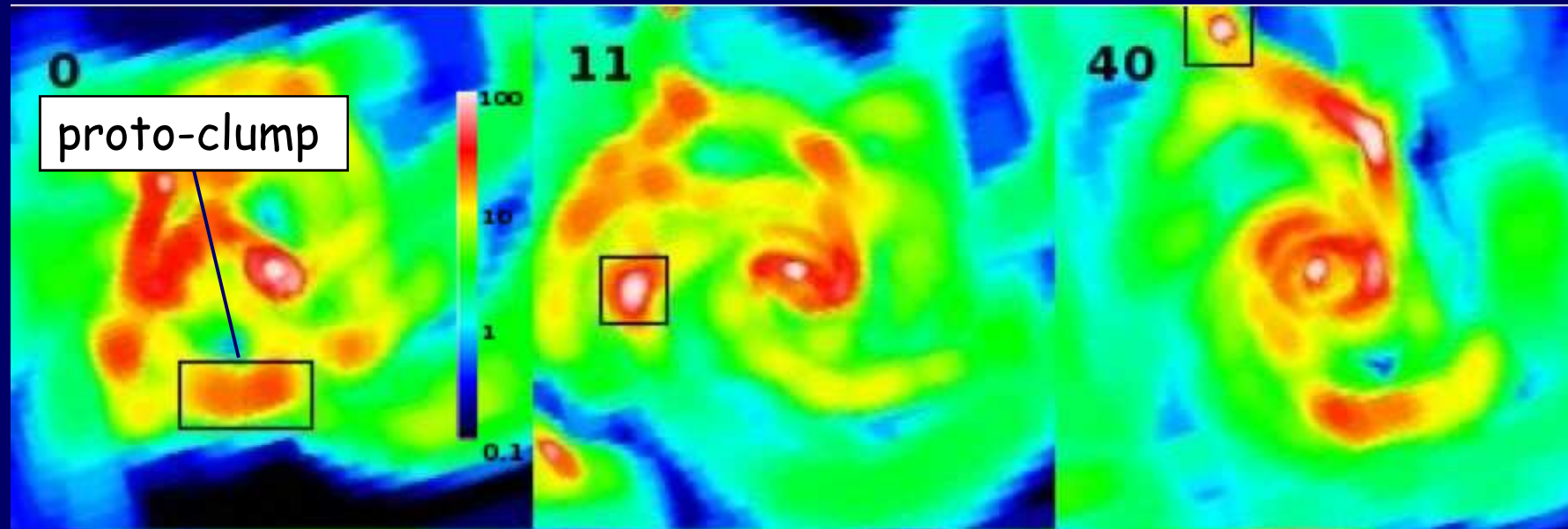
10 kpc

$z=4-2.1$

Record=284.00



Local Instability: Forces on Protoclump



pressure
prevents small clumps

rotation
prevents big clumps

$$Q \propto \frac{\sigma \Omega}{\Sigma} \approx 1$$

self-gravity attraction

Gravity wins when $Q < 1$

Violent Disk Instability (VDI) at High z

High gas density \rightarrow disk unstable

Giant clumps and transient features
 \rightarrow rapid evolution on dynamical time

$$Q \propto \frac{\sigma \Omega}{G \Sigma} \leq 1$$

$$R_{\text{clump}} \propto \frac{G \Sigma}{\Omega^2}$$

Toomre 64

Isolated galaxies:

Noguchi 99

Immeli+ 04a,b

Bournaud, Elmegreen+ 06, 08

Hopkins+ 12

Bournaud+ 13

In cosmology:

Dekel, Sari, Ceverino 09

Agertz et al. 09

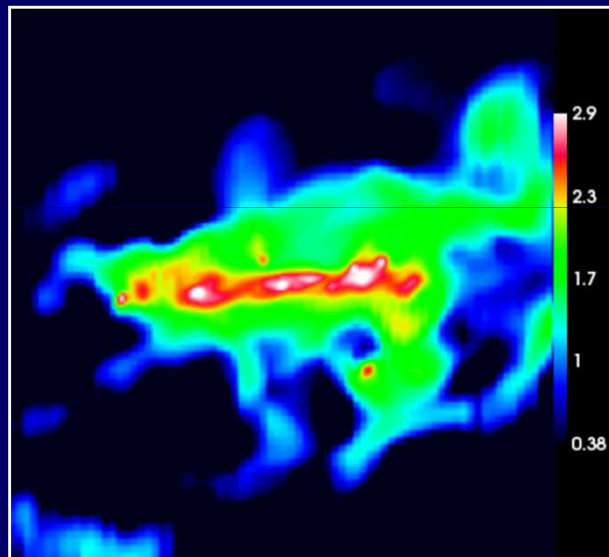
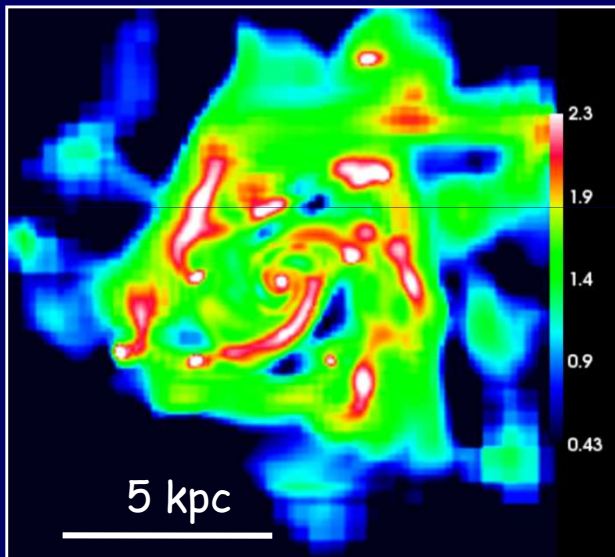
Ceverino, AD, Bournaud 10

Ceverino+ 11

Cacciato, AD, Genel 12a,b

Genel+ 12

Forbes et al. 12, 13, 14



Self-regulated at $Q \sim 1$ by torques and inflow \rightarrow turbulent with high $\sigma/V \sim 1/5$

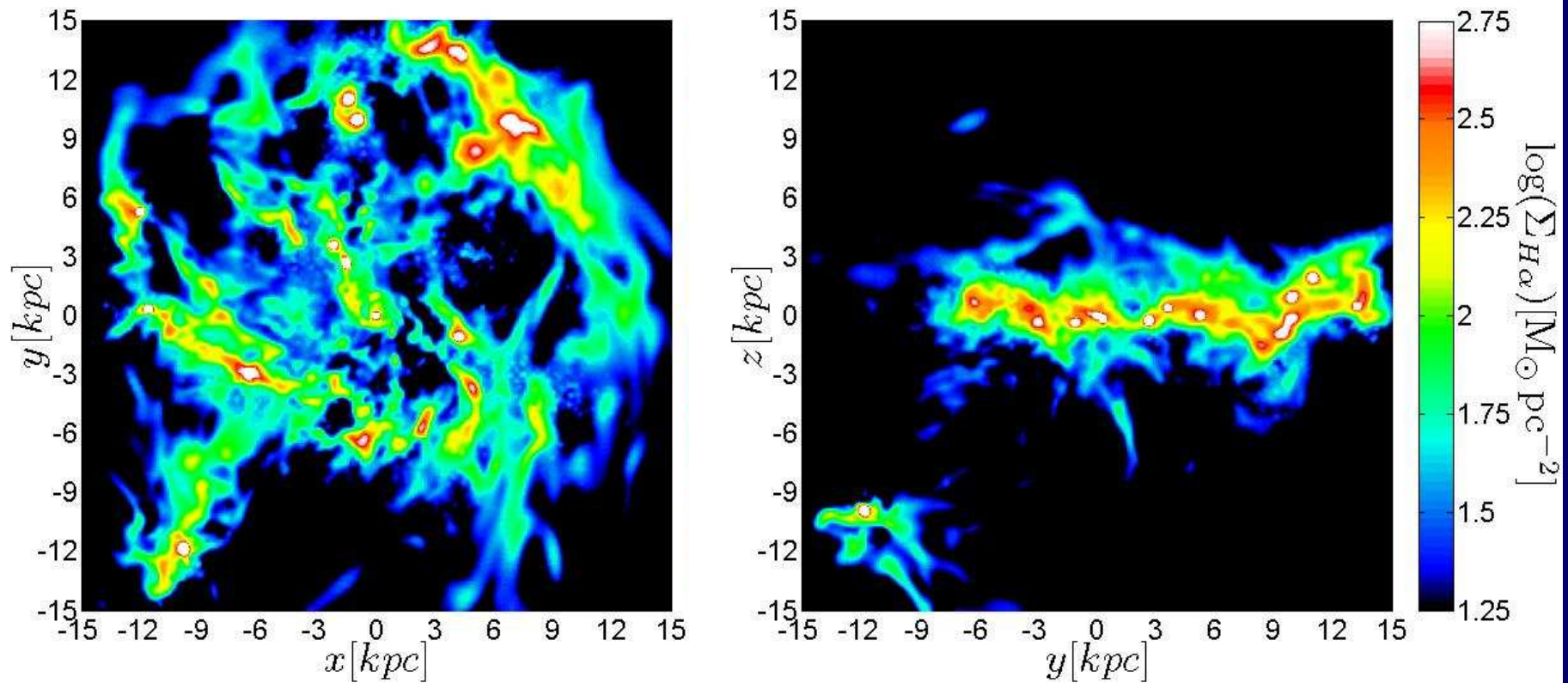
Inflow in disk \rightarrow compact bulge and BH

Steady state: disk draining and replenishment, **bulge \sim disk**

Violent Disk Instability (VDI) at High z

Ceverino+ ART-AMR cosmological simulations at 25pc resolution

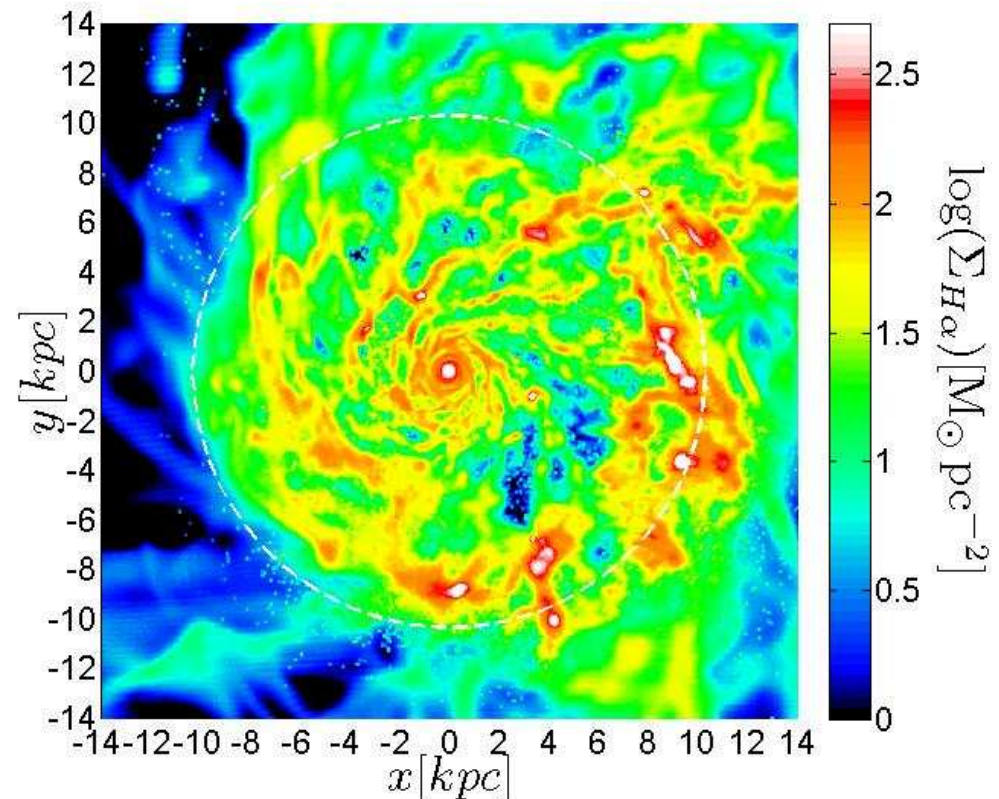
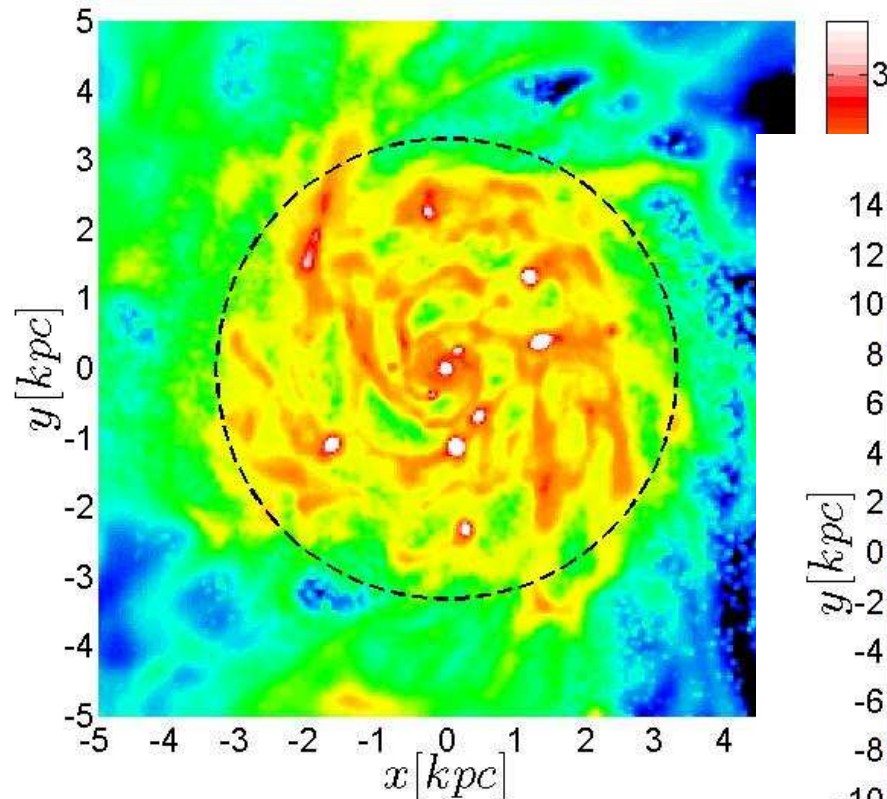
highly perturbed, clumpy rotating disk: $H/R \sim \sigma/V \sim f_{\text{cold}} \sim 0.2$



Violent Disk Instability (VDI) at High z

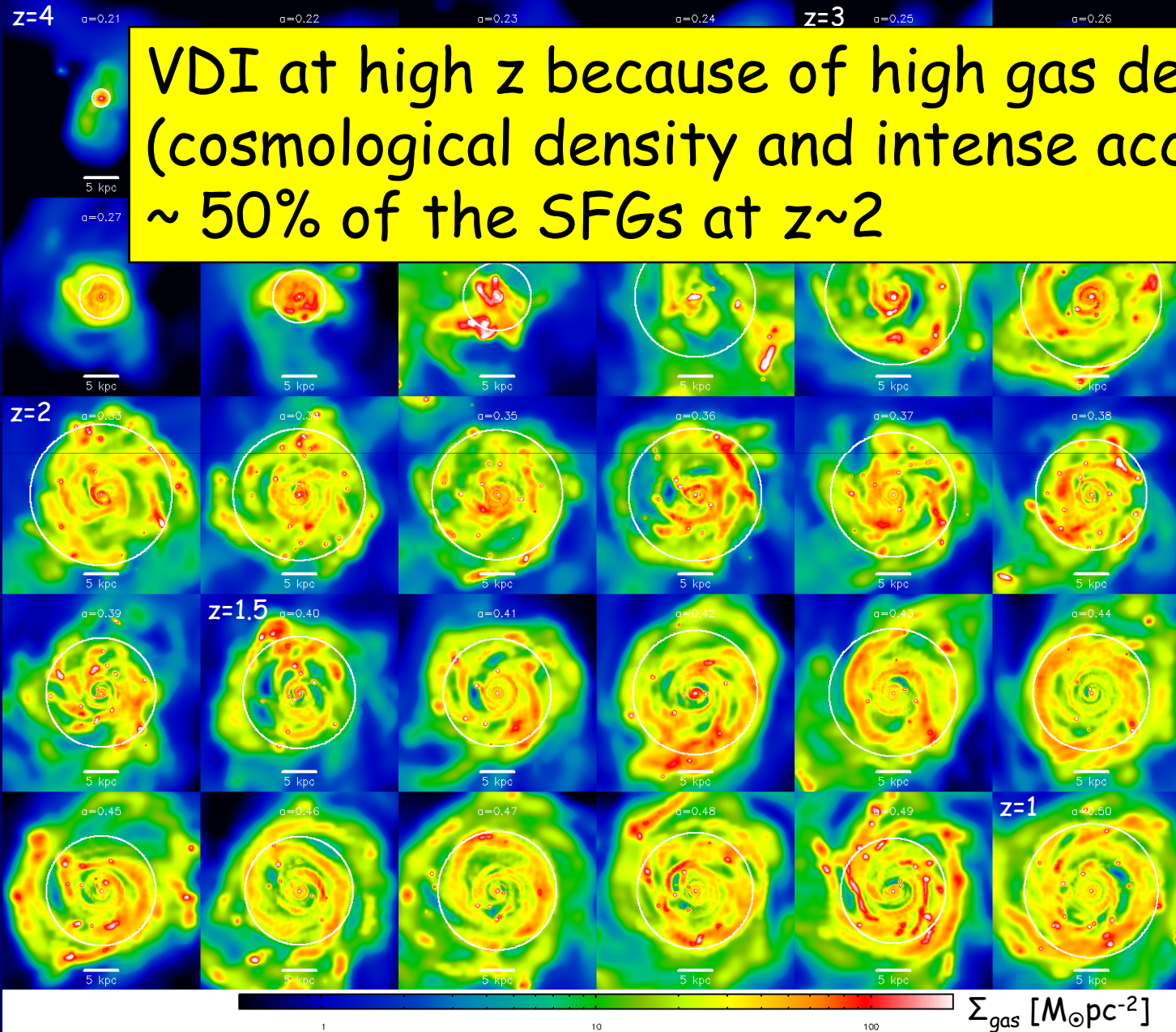
Ceverino+ ART-AMR cosmological simulations at 25pc resolution

$$H/R \sim \sigma/V \sim f_{\text{cold}} \sim 0.2$$



Clumpy Disk in a cosmological steady state

VDI at high z because of high gas density
(cosmological density and intense accretion)
 $\sim 50\%$ of the SFGs at $z \sim 2$

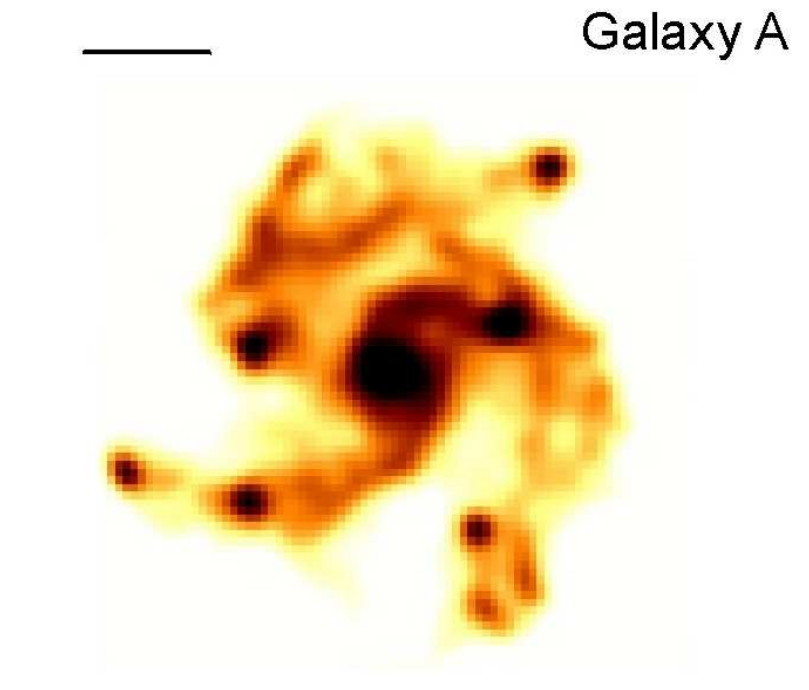
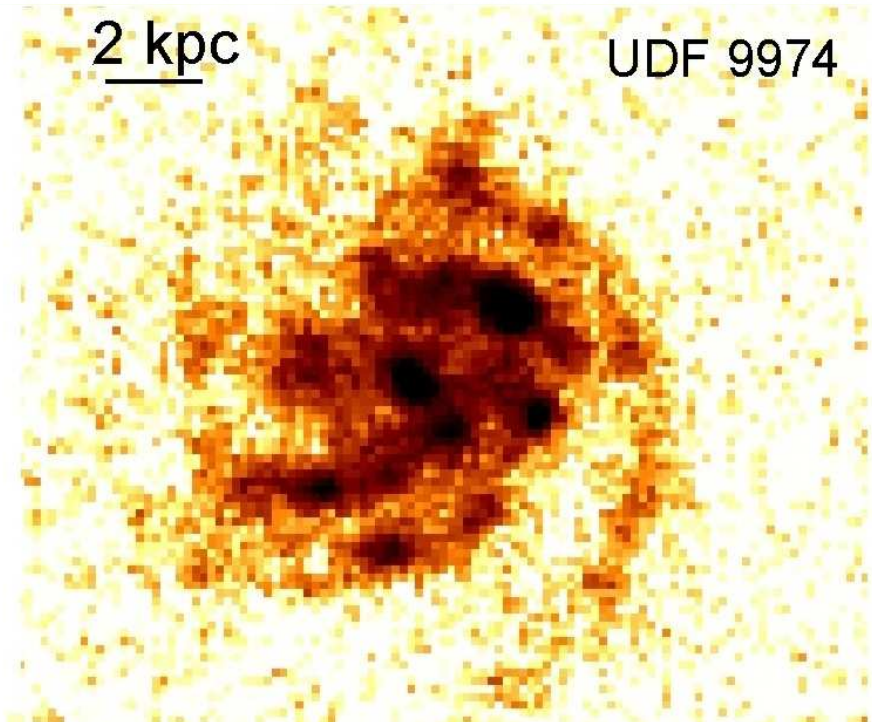
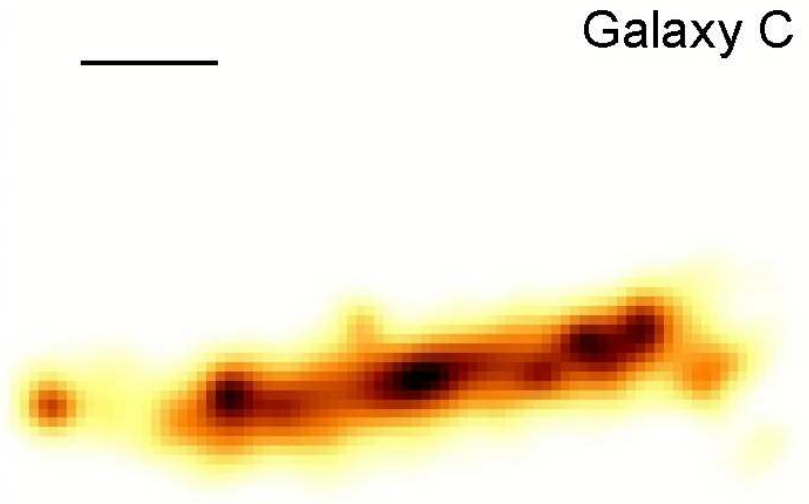
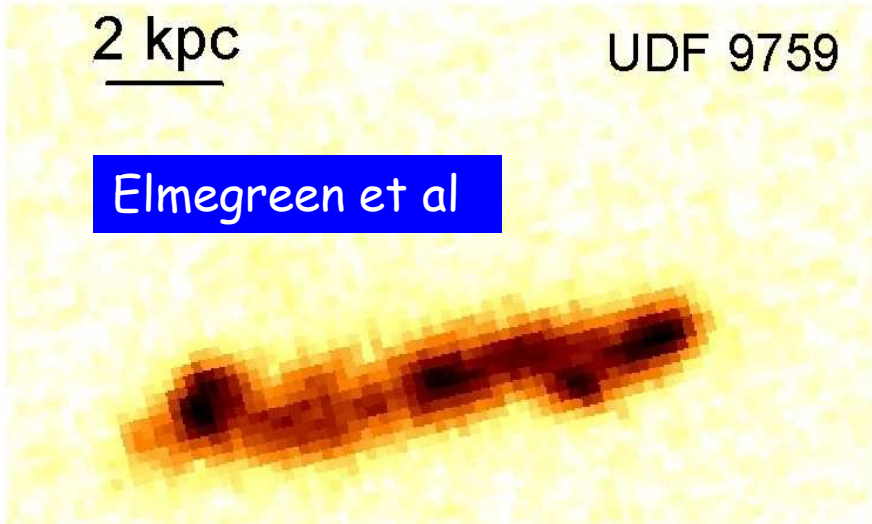


Dekel, Sari,
Ceverino 09;

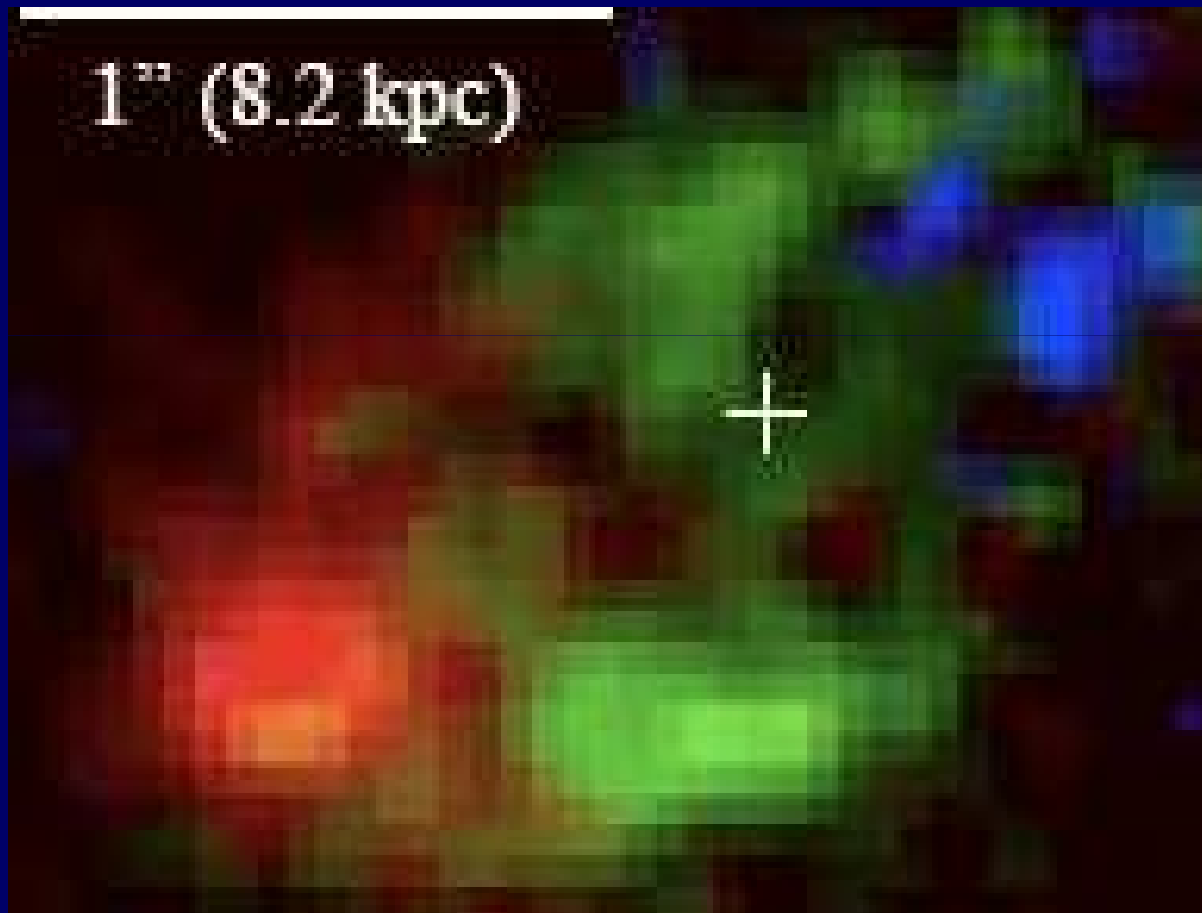
Ceverino, Dekel,
Bournaud 10

Mandelker et al. 14

Observations vs. Simulations



A typical star-forming galaxy at $z=2$:
clumpy, rotating, extended disk & a bulge



H α star-forming
regions

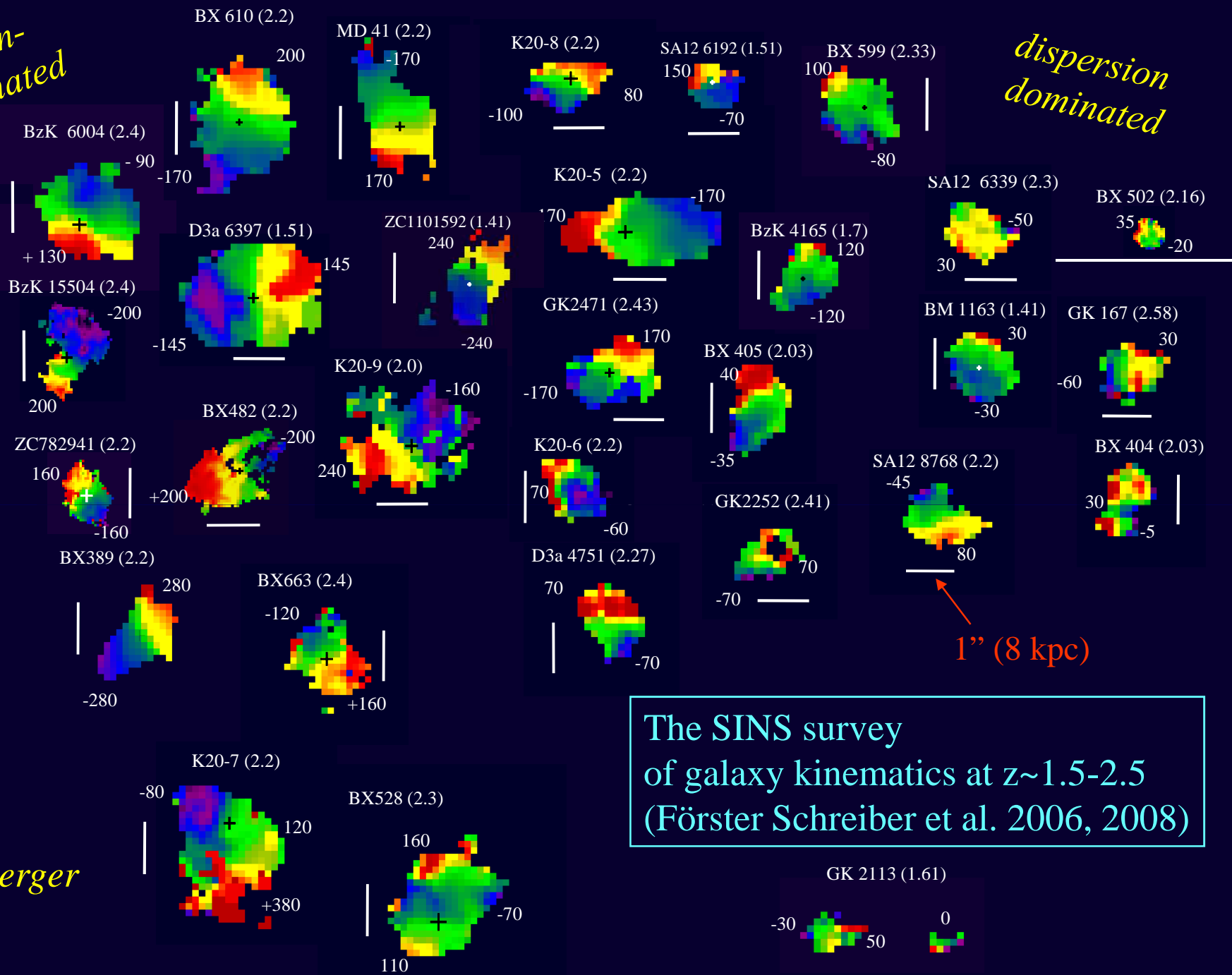
color-code
velocity field

Genzel et al 08

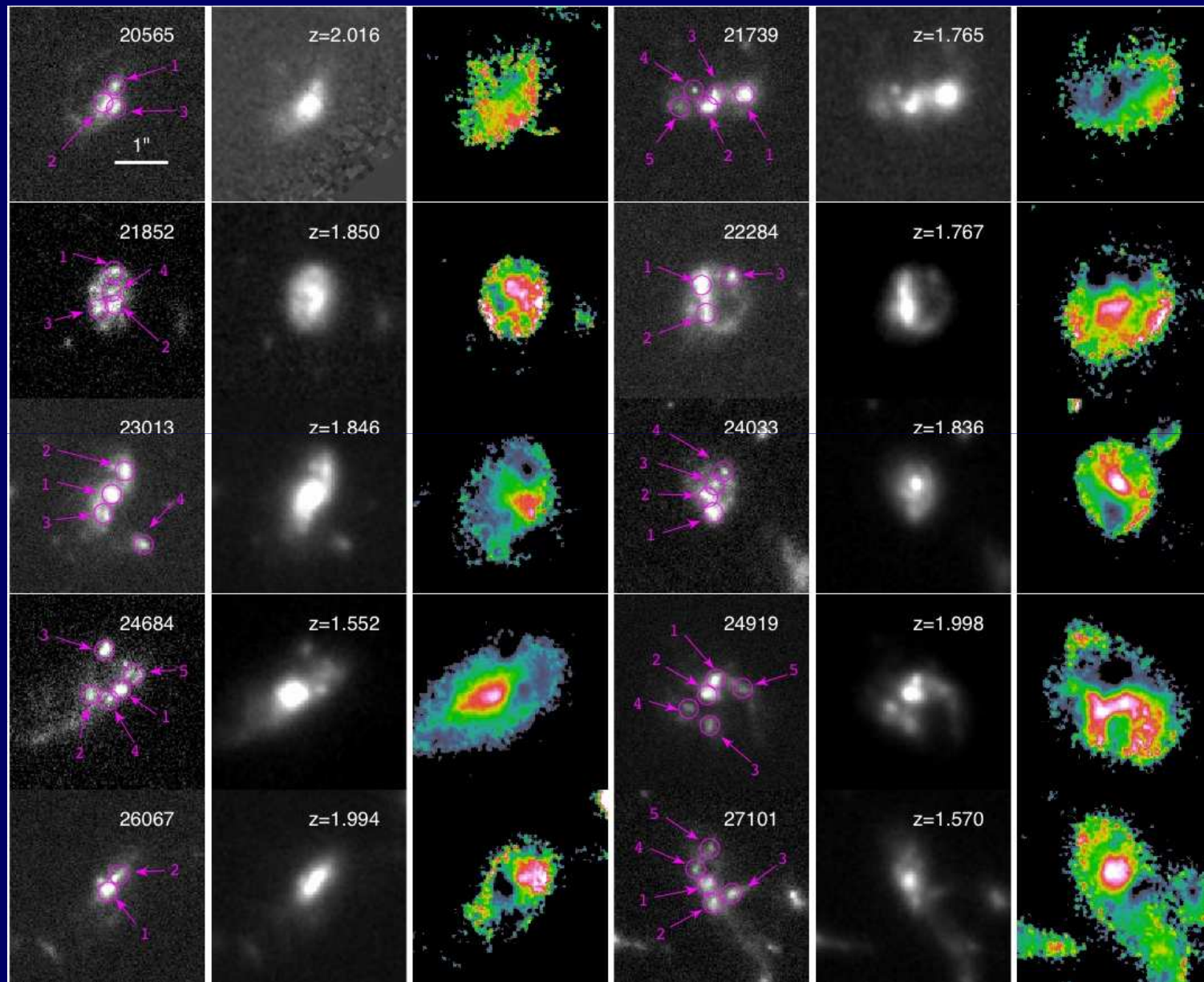
rotation-dominated

dispersion dominated

merger



High-z Disks with Giant Clumps



Guo et al. 12
CANDELS

Clumps in VDI Disks

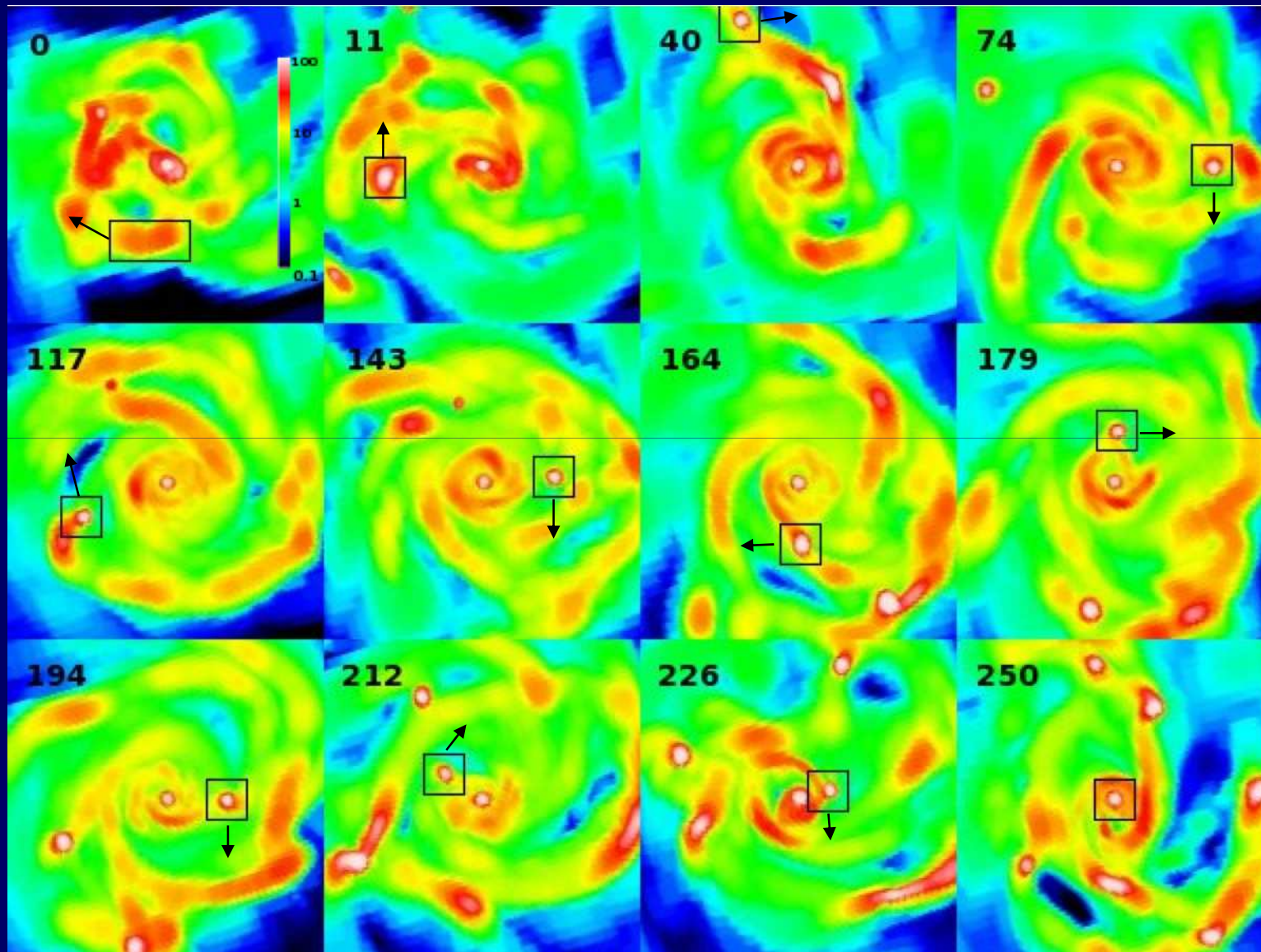
Mandelker+14
Bournaud+14
Dekel, Krumholz 14
Moody+14, Snyder+14



- **bulge** ~ **disk** in cosmological steady state
- giant clumps $M \sim 10^8 - 9 M_{\odot}$ $R < 0.5 \text{ kpc}$
- **in-situ** (gaseous, SFR) and **ex-situ** (stellar, mergers)
- half the SFR in clumps
- **migration** to center in $\sim 300 \text{ Myr}$ \rightarrow gas+ young stars
- **clumps** $> 10^8 M_{\odot}$ **survive** outflows with mass \sim constant
 $\eta \sim 1-2$ winds, gas accretion, tidal stripping
- less massive clumps disrupt
- VDI feed gas & stars to the **bulge and BH**

Expect a gradient of clump mass and age/color

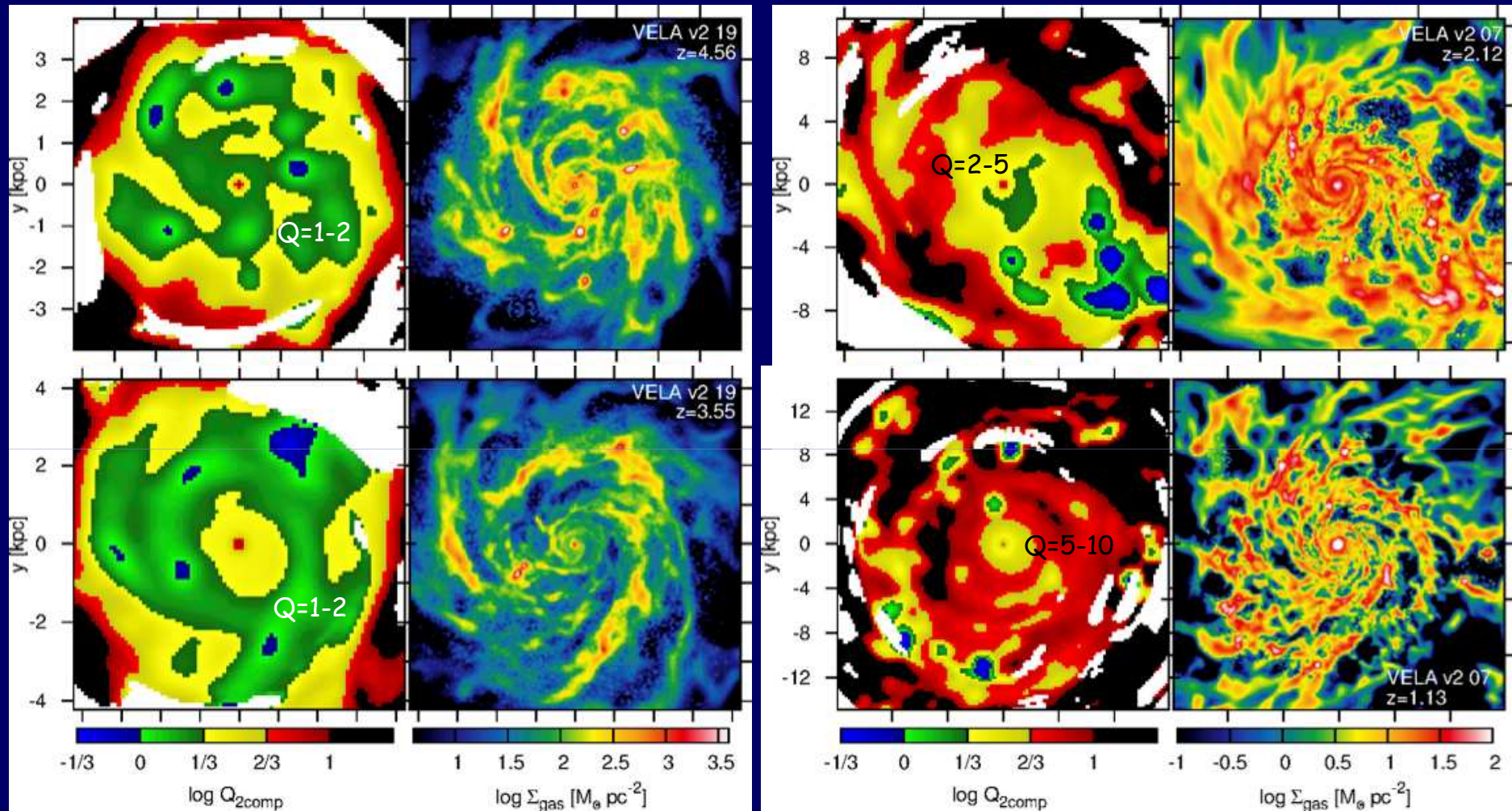
Clump Formation & Migration



Ceverino, Dekel, Bournaud 10

Violent Instability with $Q \sim 2-3$

Toomre $Q \propto \Omega \sigma / \Sigma \approx 1$



Nonlinear instability - stimulated by intense inflows with minor mergers, or by the non-linear clumps themselves

Stimulated Non-linear Instability

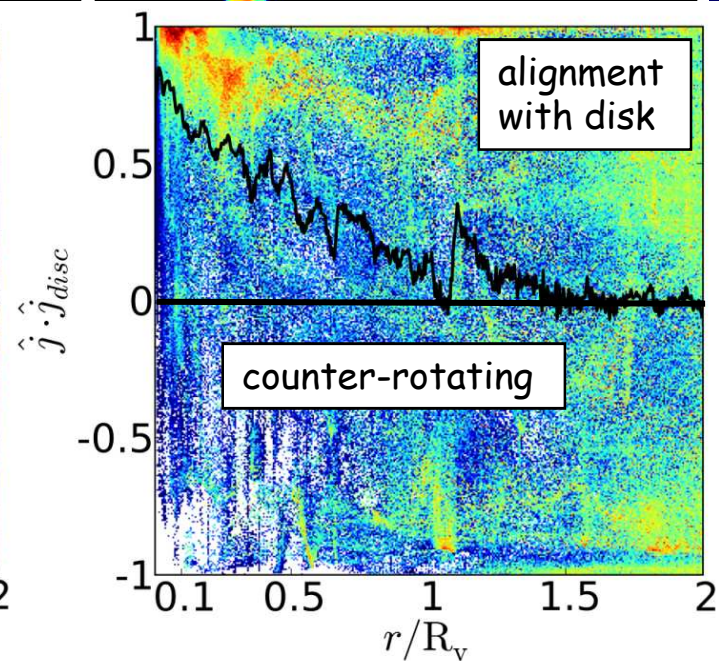
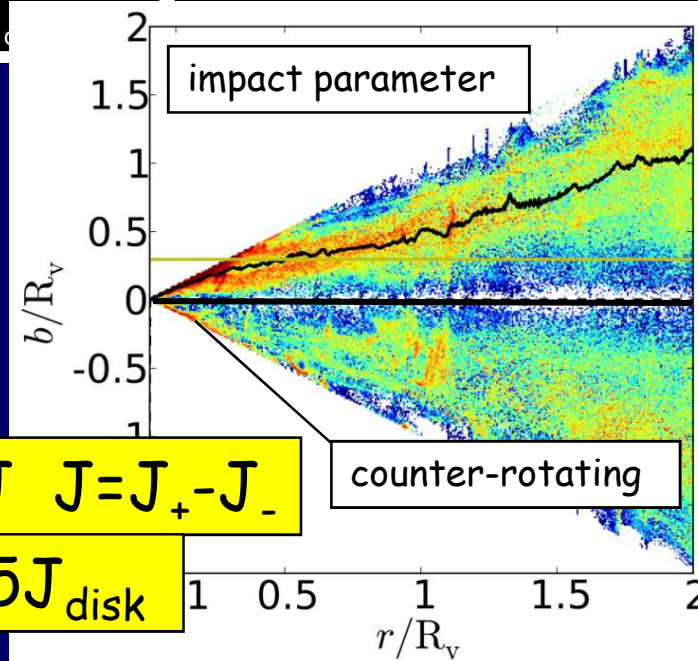
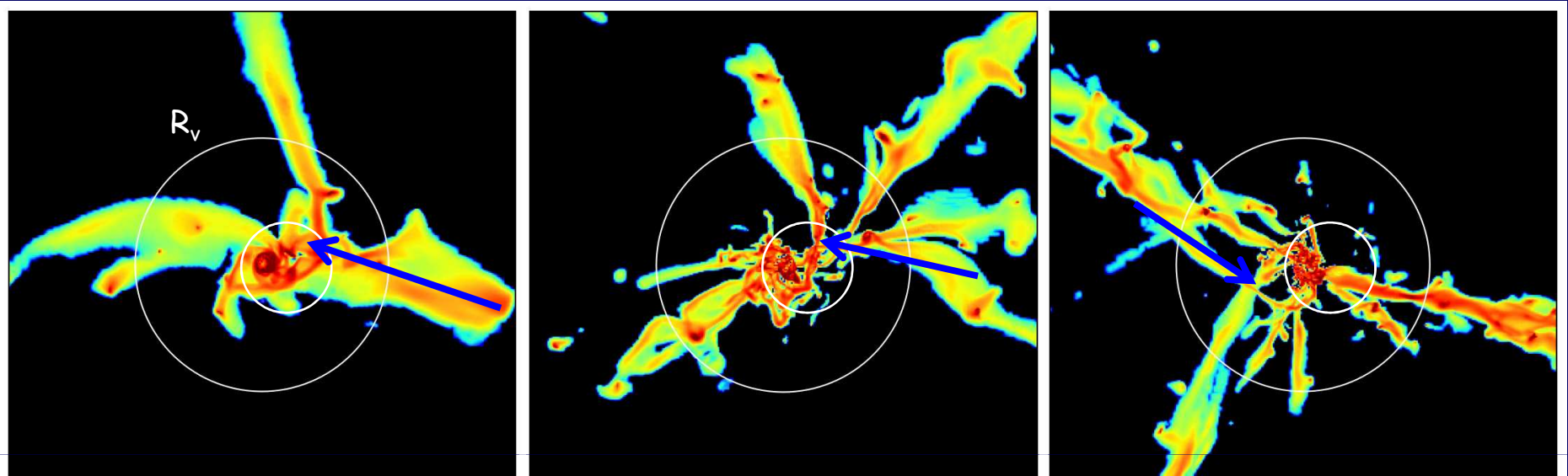
Toomre

$$Q \propto \frac{\sigma \Omega}{\Sigma} \approx 1$$

Tentative ideas for $Q > 1$ instability:

- Rapid decay of turbulence (Elmegreen)
no time for pressure buildup against clump self-gravity
- Irregular rotation - counter-rotating streams (Lin)
no centrifugal force against clump self-gravity
- Compression modes of turbulence (Bournaud, Renaud)
by tidal compression? trigger local collapse
- Clumps generate new clumps

Counter-rotating Streams



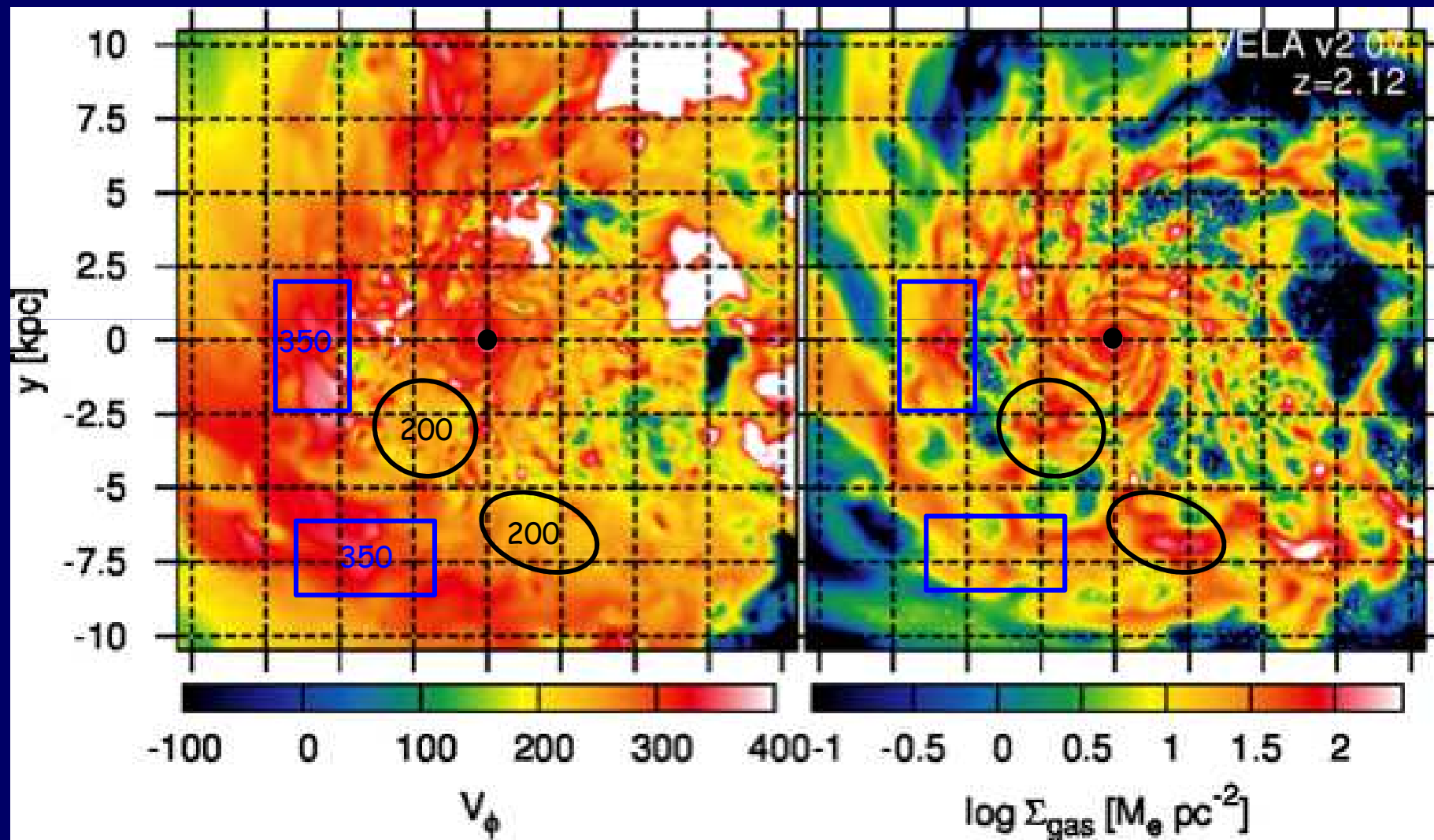
inflow: $J_- \sim 0.7J$ $J = J_+ - J_-$

in t_{orbit} : $J_- \sim 0.15J_{disk}$

Irregular Rotation

local rotation velocity

density



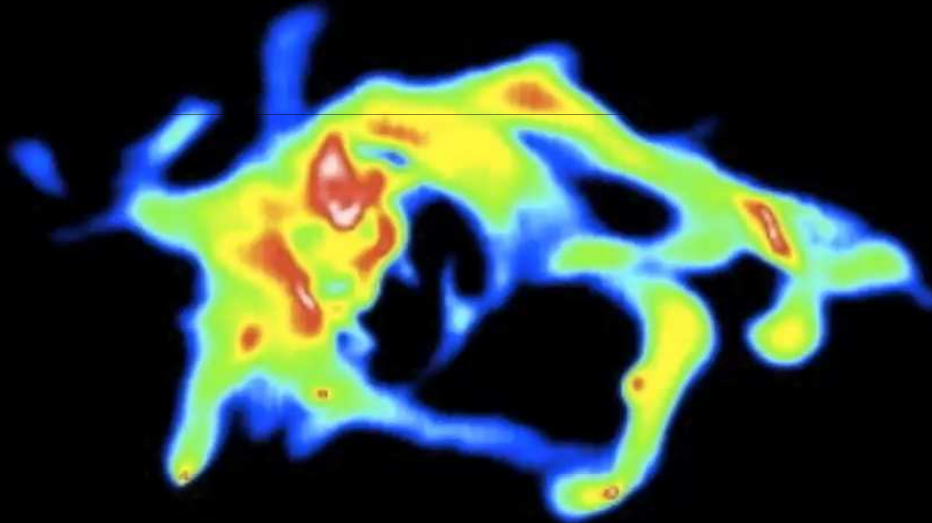
Clumpy Disk

Ceverino, Dekel et al.

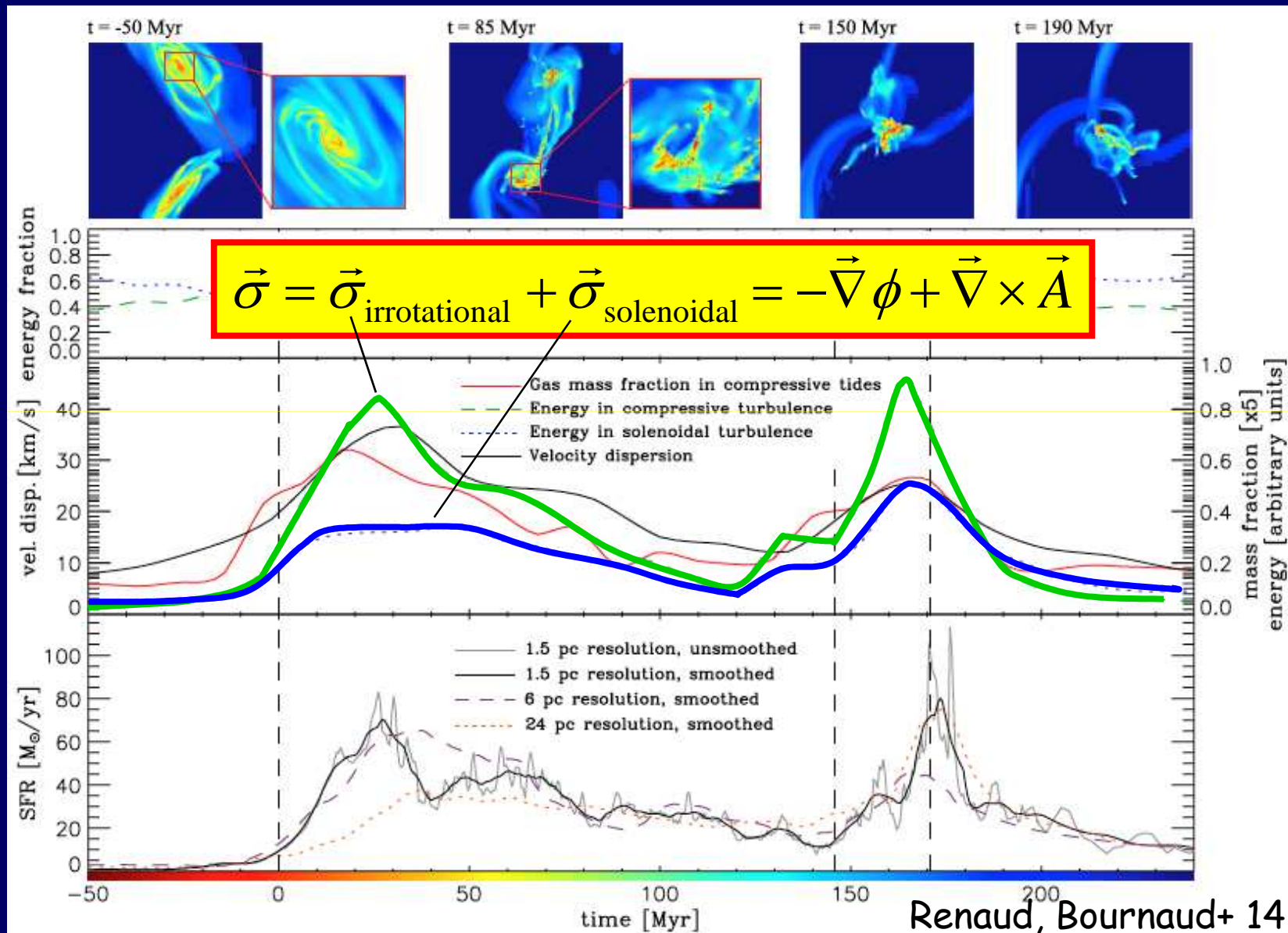
10 kpc

$z=4-2.1$

Record=284.00



Compression Modes of Turbulence: Merger



Conclusions: Stimulated Non-linear VDI

Typical SFGs have perturbed rotating disks undergoing violent disk instability (VDI)

- Massive clumps ($>10^8 M_{\odot}$) **survive feedback**
- off-center in-situ **young clumps** <300 Myr, showing **age/gas gradient**
- older **ex-situ clumps**

Nonlinear instability

Stimulated by inflow+mergers? Compressive turbulence?
Irregular rotation due to counter-rotating streams?

VDI and (minor) mergers actually work in concert



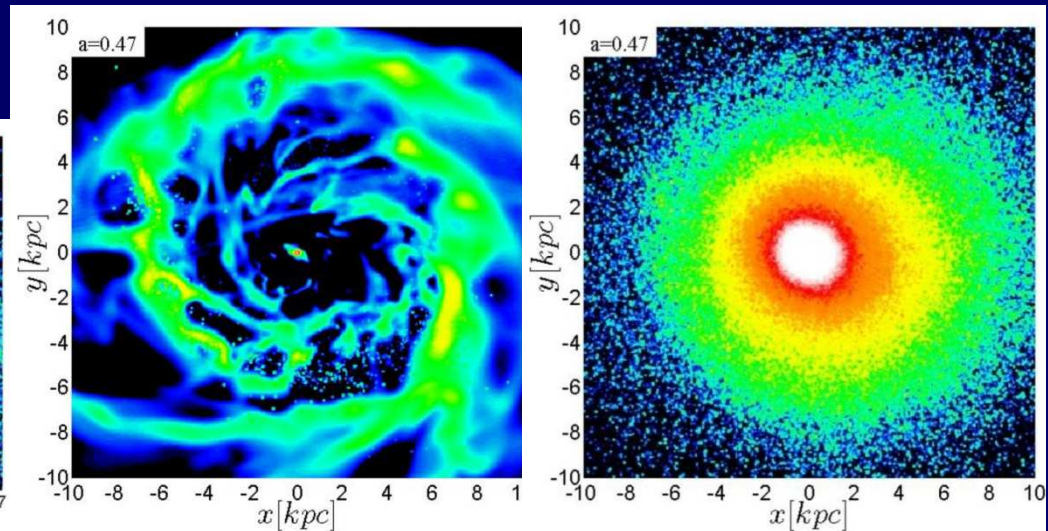
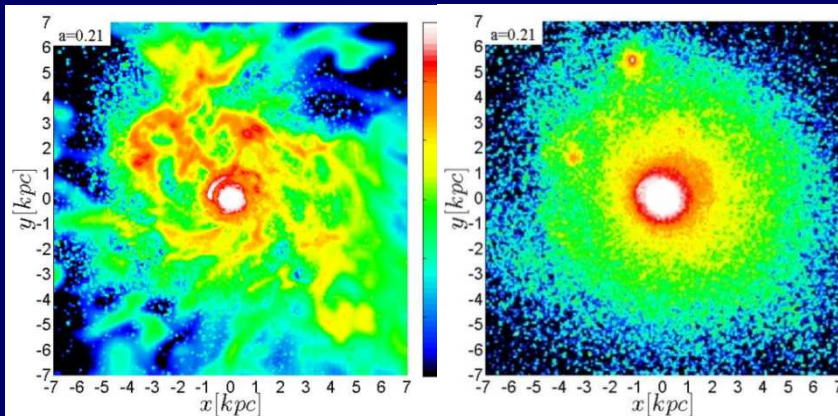
Quenching by Compaction and by a Hot Halo



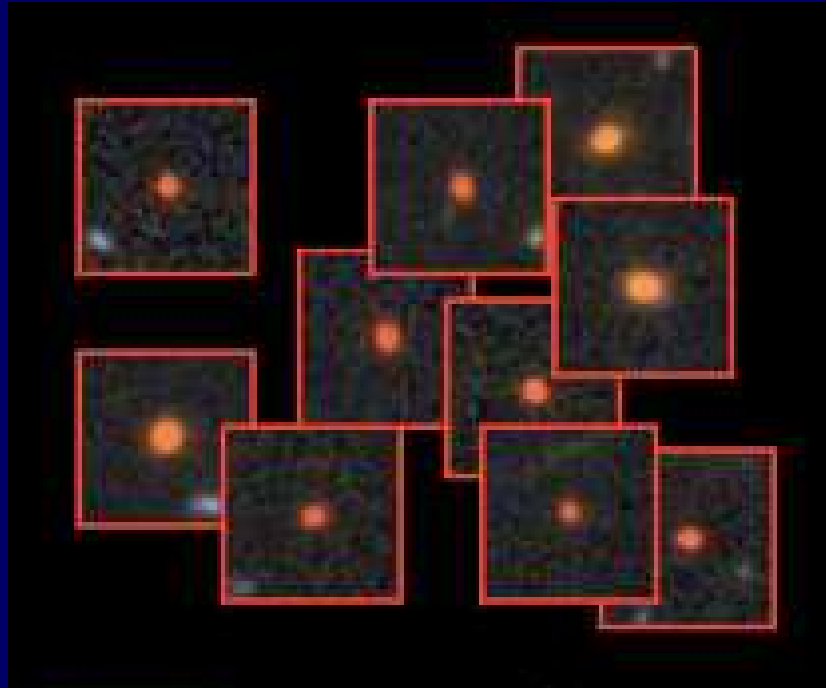
Dekel & Burkert 2014; Zolotov et al. 2014
Dekel & Birnboim 2003, 2006

Red Nugget

"Blue" Nugget



Red Nuggets



$z \sim 2$ $M \sim 10^{11} M_{\odot}$ $R_e \sim 1$ kpc low-SFR

Van Dokkum+08,10,14, Damjanov+09, Newman+10,
Damjanov+11, Whitaker+12, Bruce+12, ...

Wet Compaction

Dekel & Burkert 2013; Zolotov et al. 2014

Compact stellar spheroid → **dissipative** “wet” inflow to a “blue nugget”
by **mergers and/or VDI**

Inflow is “wet” if $t_{\text{inflow}} \ll t_{\text{sfr}}$

Inflow in self-regulated VDI disk $Q \sim 1$, evaluated by torques,
dynamical friction, clump encounters, energy conservation, ...

$$t_{\text{inflow}} \approx f_{\text{cold}}^{-2} t_{\text{dyn}} \approx (V/\sigma)^2 t_{\text{dyn}} \approx 10 t_{\text{dyn}}$$

Gammie 01; Dekel, Sari, Ceverino 09

$$\frac{M_{\text{cold}}}{M_{\text{tot}}} \equiv f_{\text{cold}} \approx \frac{\sigma}{V}$$

**Wetness
parameter**

$$W \equiv \frac{t_{\text{sfr}}}{t_{\text{inflow}}} \approx \varepsilon_{\text{sfr}}^{-1} f_{\text{cold}}^2 > 1$$

$$\varepsilon_{\text{sfr}} \leq 0.02 \quad \delta \geq 0.2$$

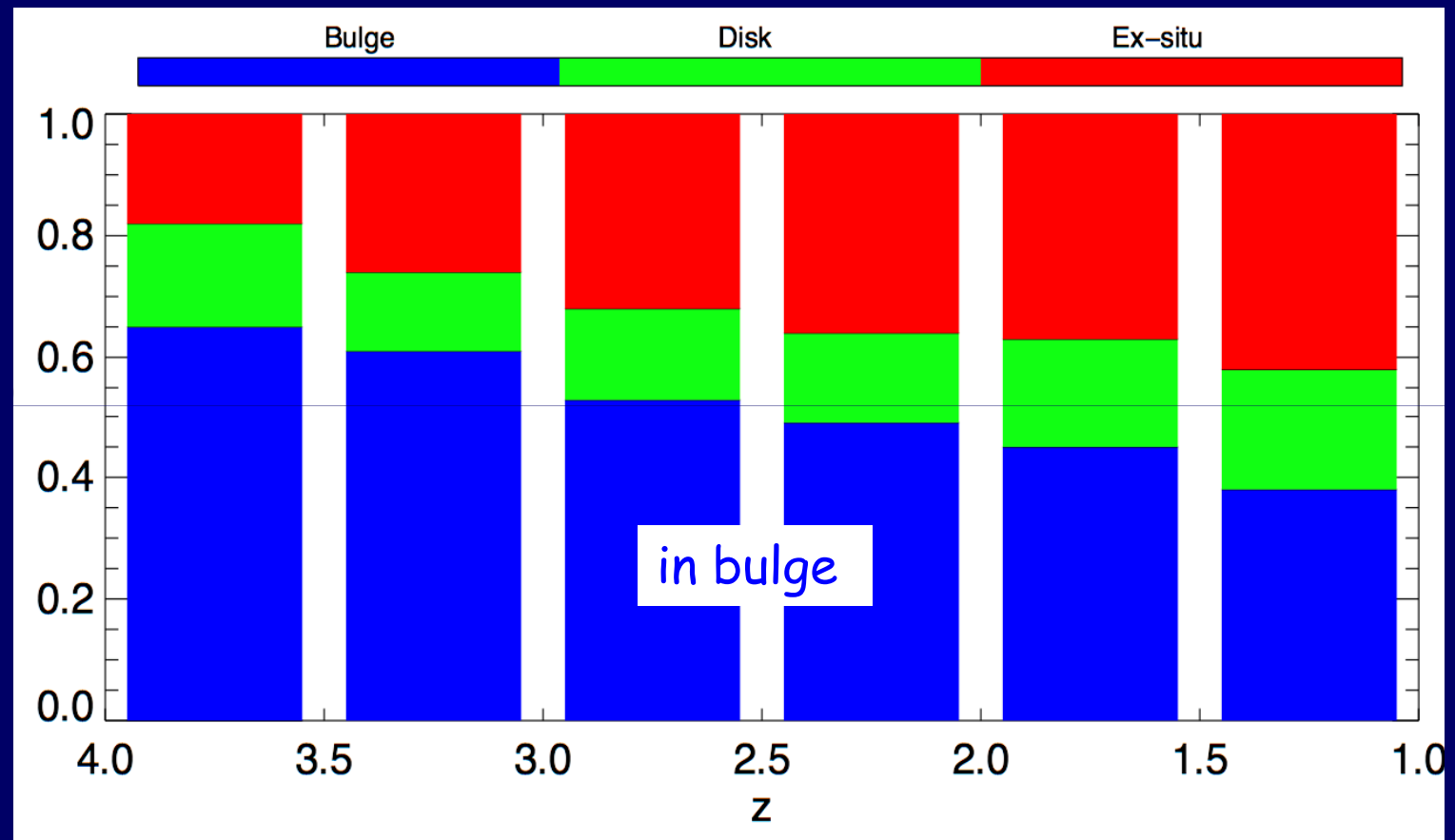
Expect compact nuggets:

- at **high z**, where f_{gas} is high
- for **low spin** λ , where initial R_{gas} is low

Wet Origin of Bulge in Simulations

Zolotov, Dekel, Mandelker, Tweed, DeGraf, Ceverino, Primack 2014

Fraction of bulge stars born in different components

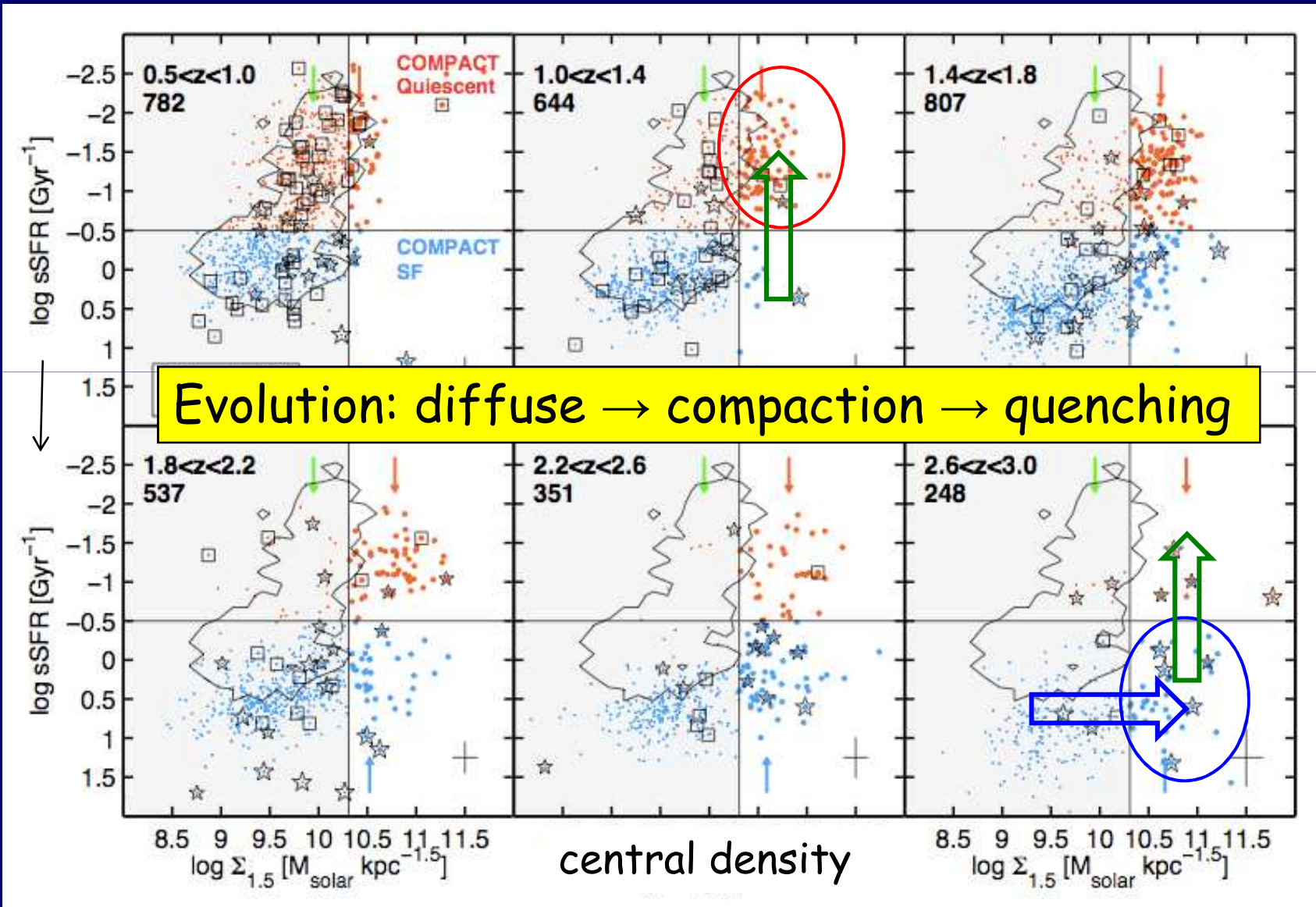


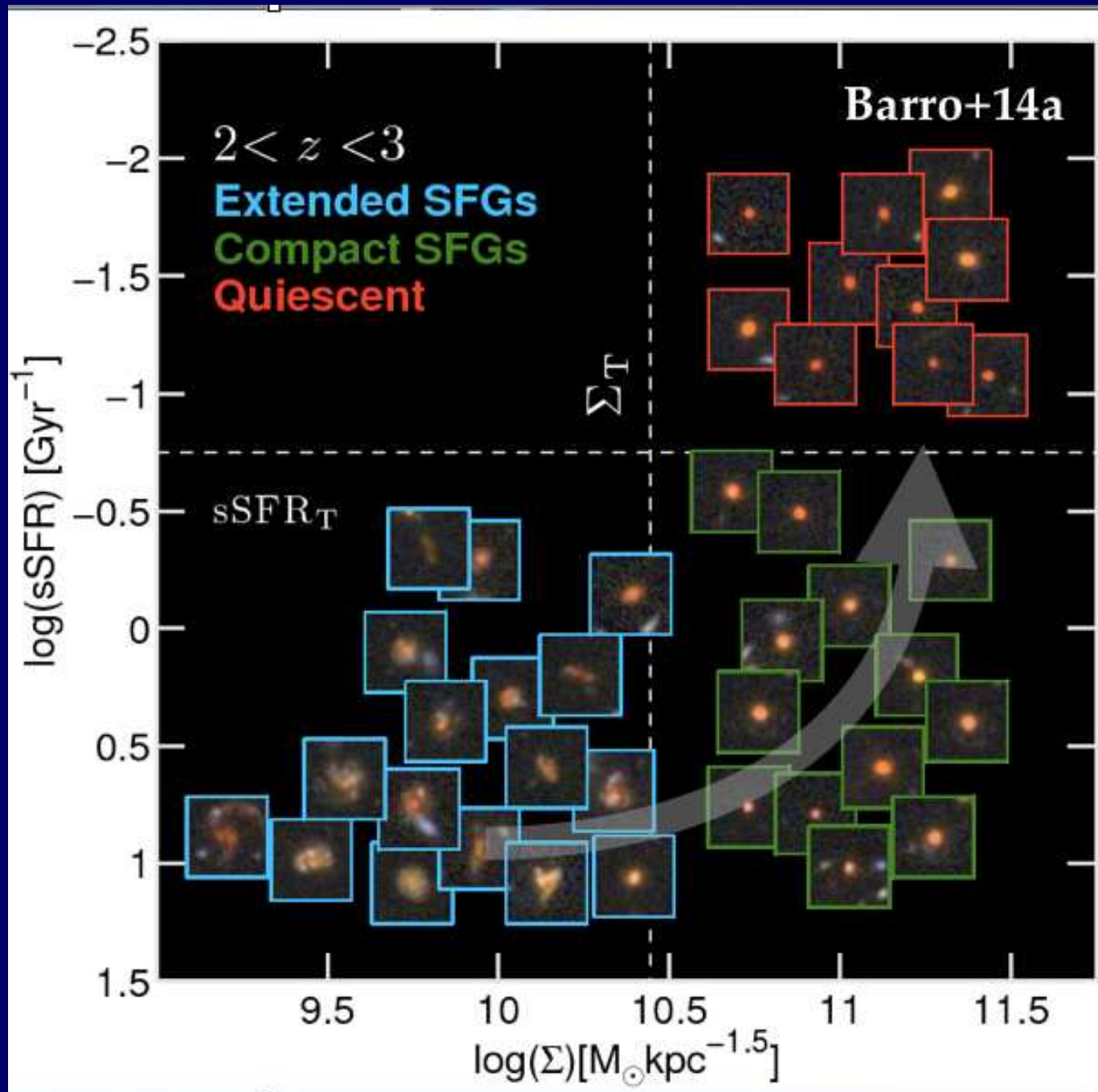
65-38% of the bulge stars formed in the bulge → wet inflow

Driven by wet VDI or wet mergers

Observations: Blue Nuggets -> Red Nuggets

Barro+ 13 CANDELS z=1-3





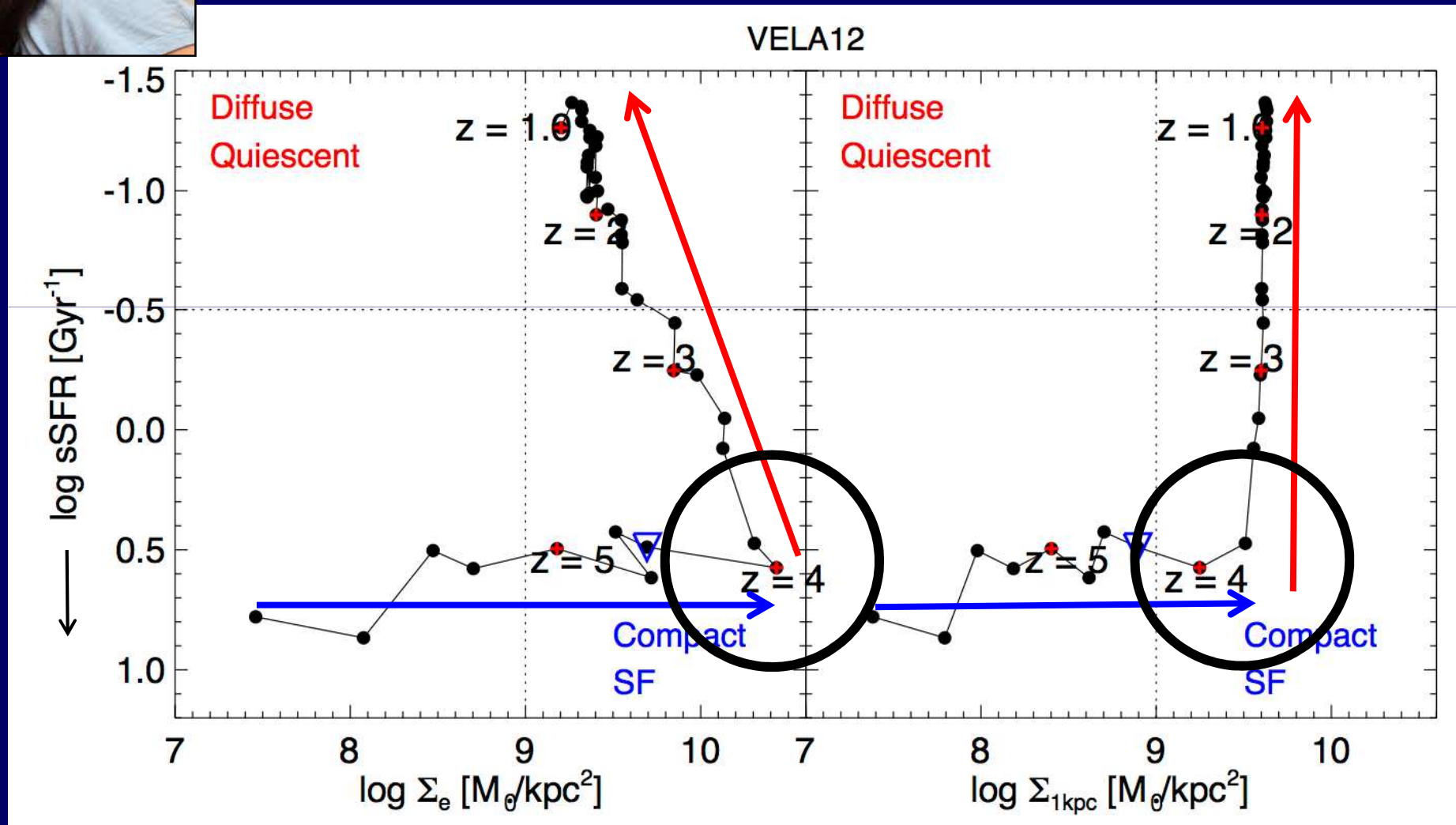
RN

BN

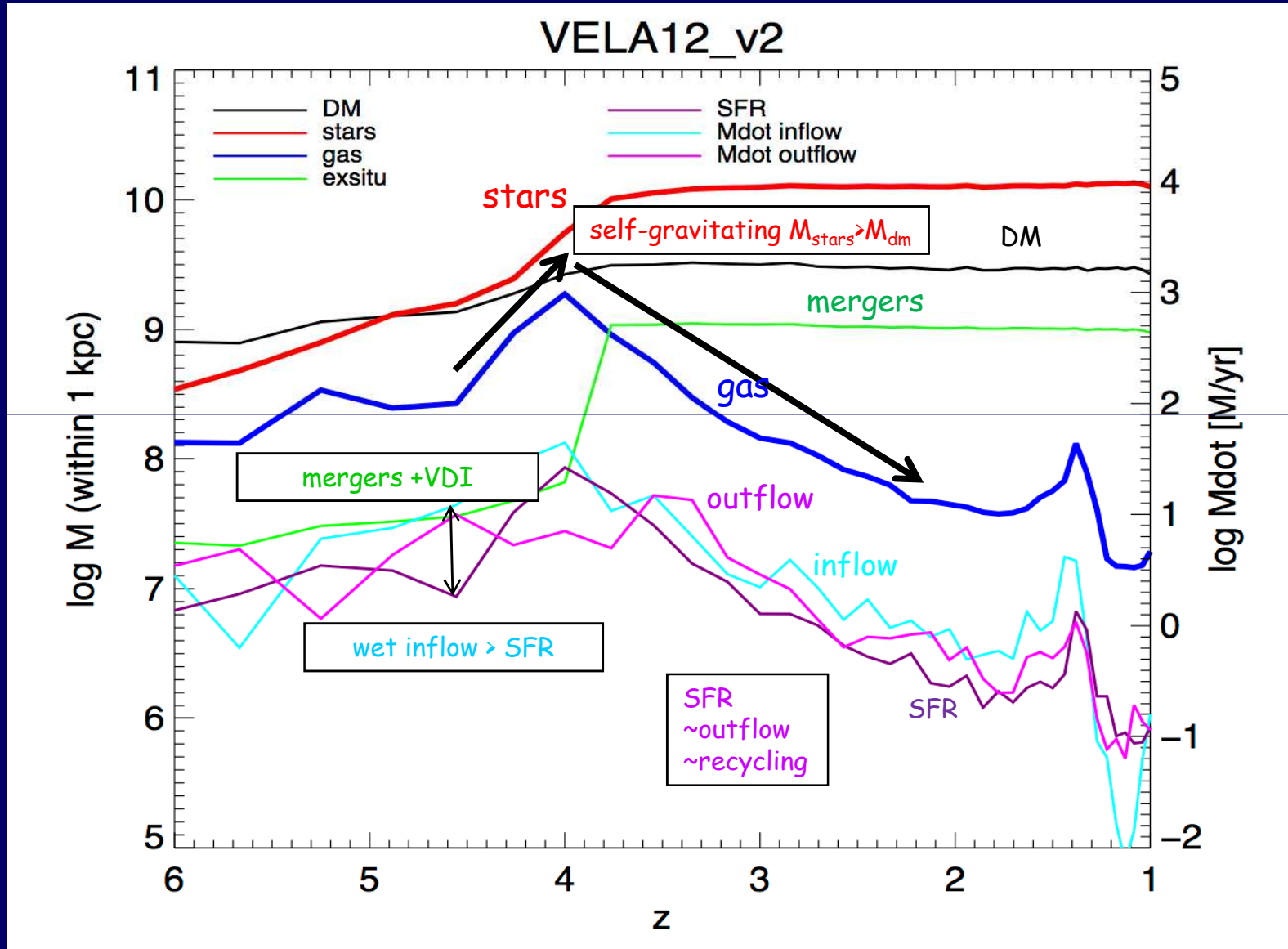


Compaction and quenching

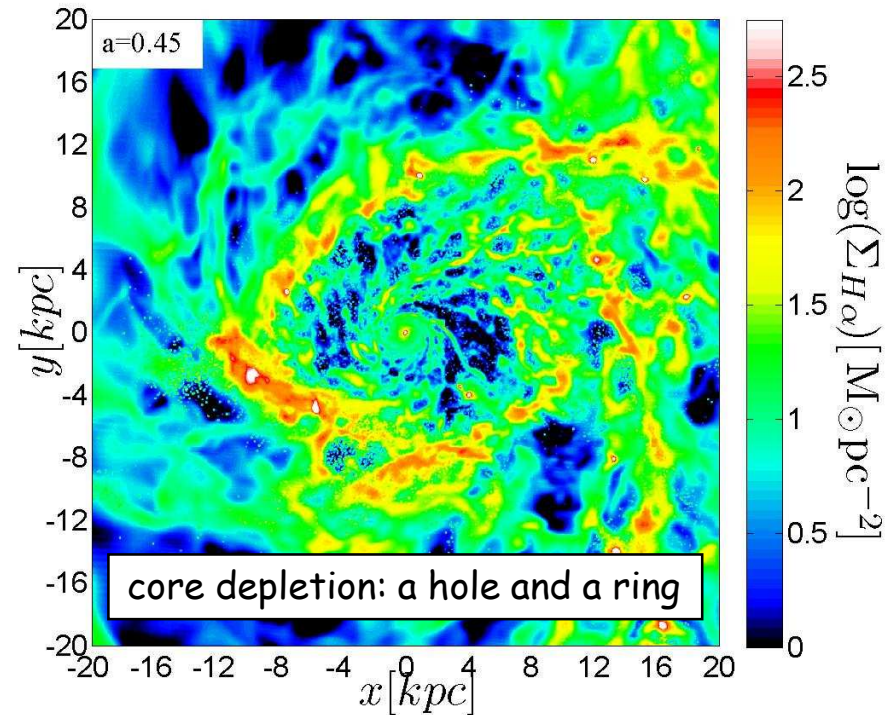
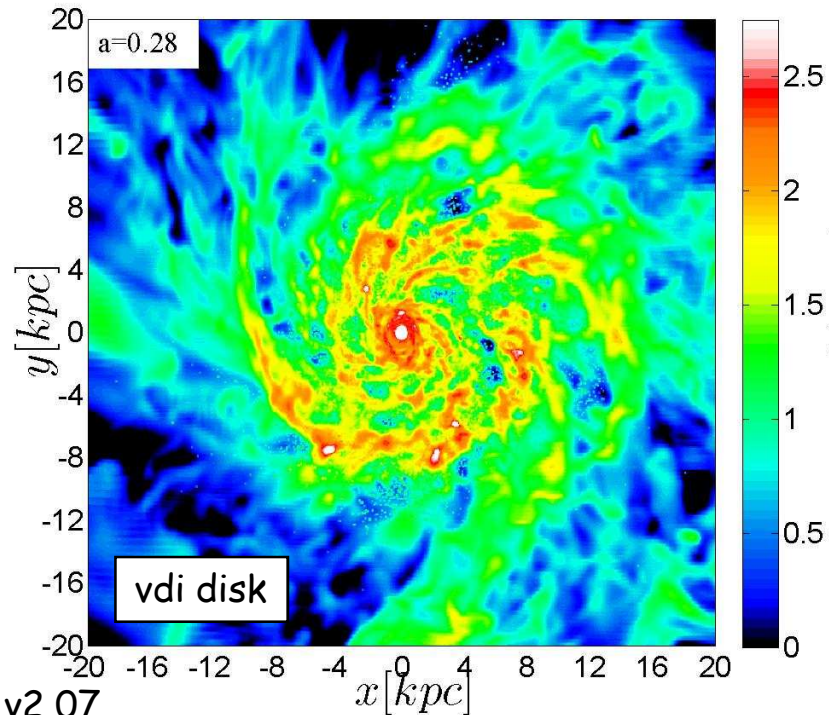
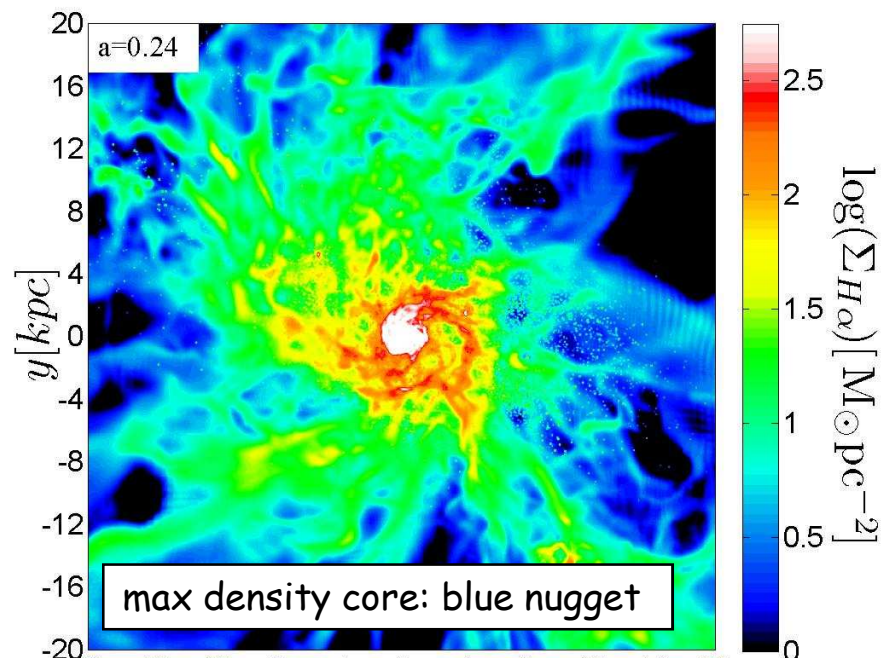
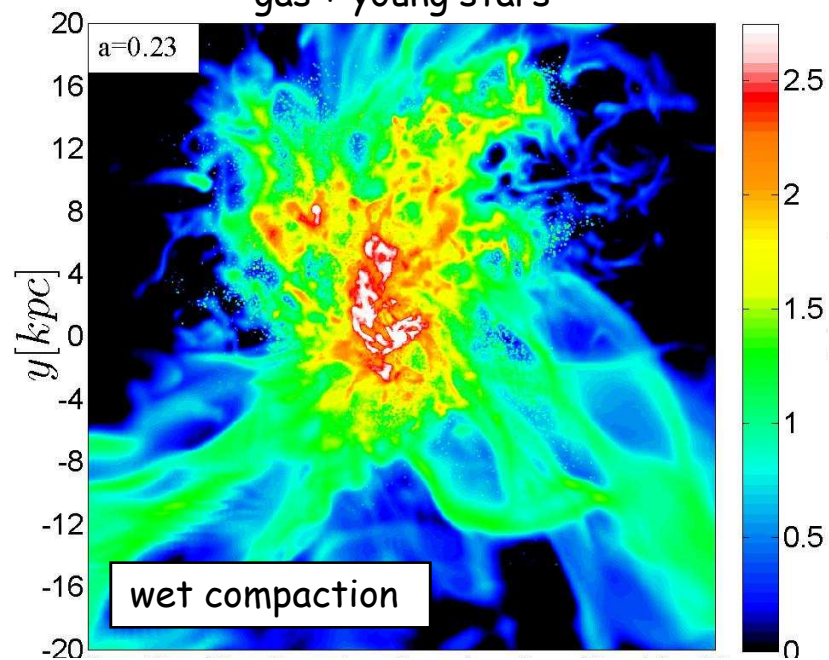
Zolotov+ 14 ART cosmological simulations, res. 25pc, rad fdbk, no AGN



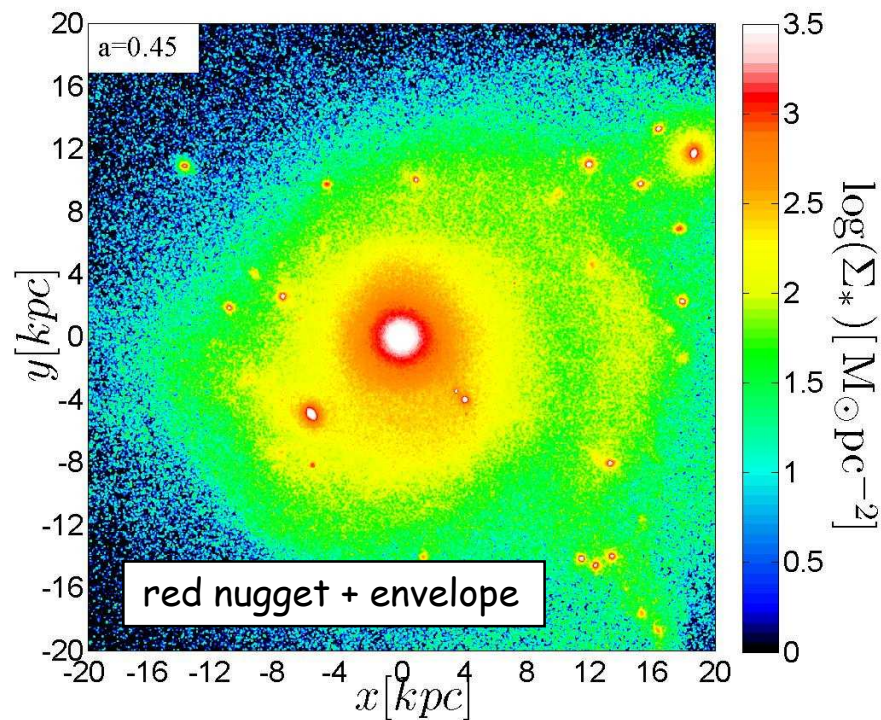
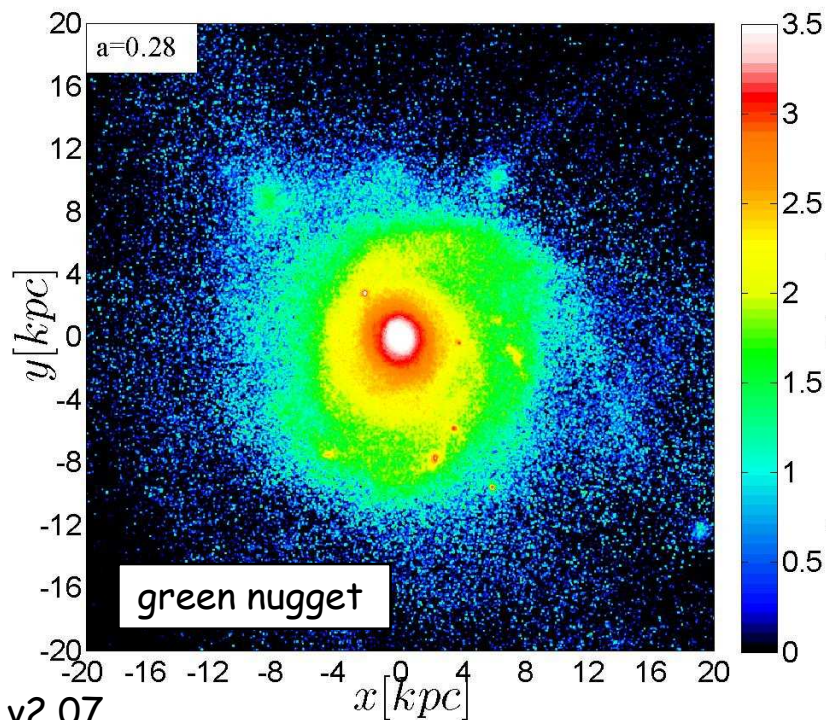
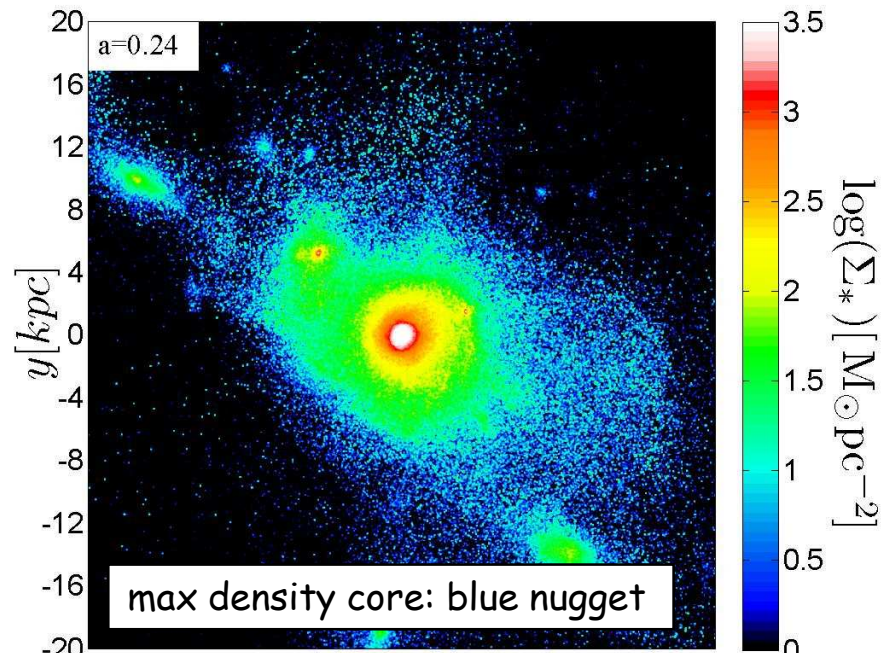
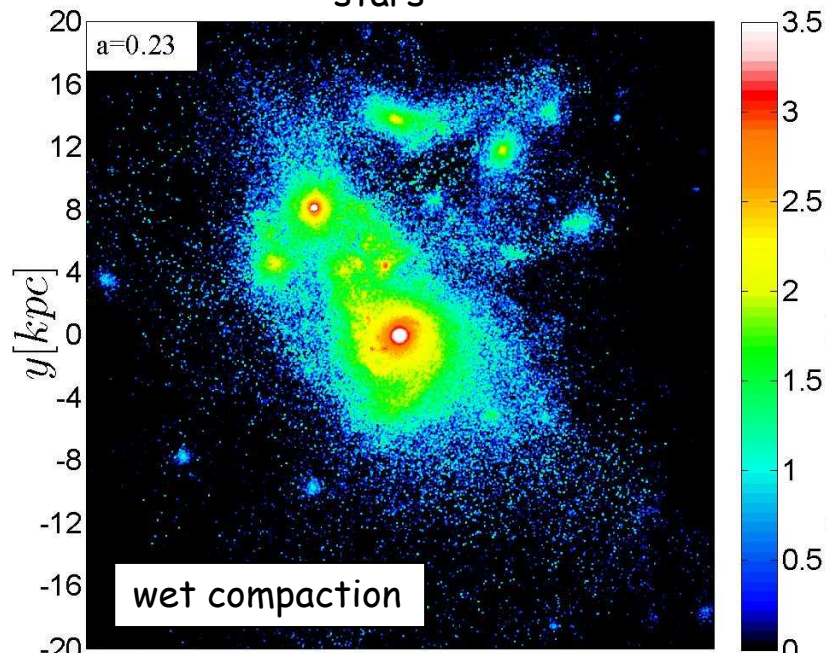
Compaction and quenching



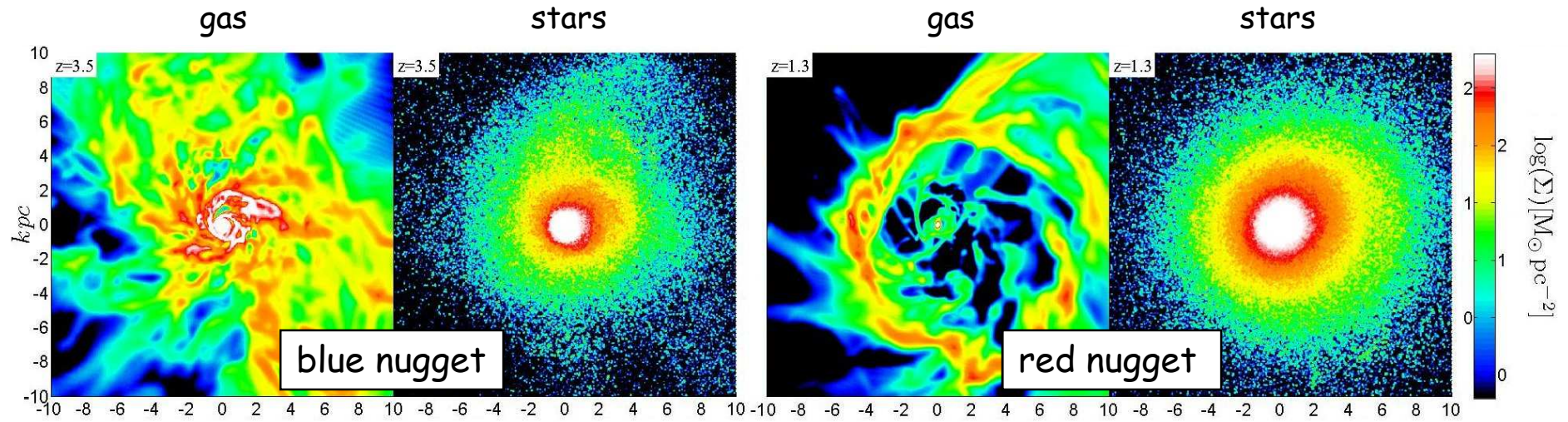
gas + young stars



stars



Blue Nugget - Red Nugget



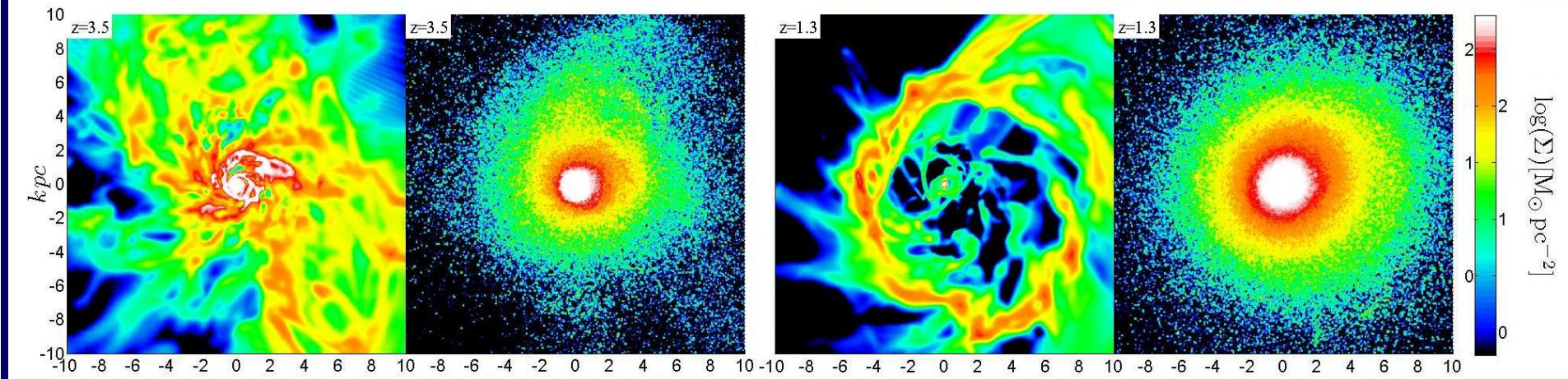
dense gas core \rightarrow dense stellar core

gas depletion from core,
gas ring may form,
 \rightarrow inside-out quenching

stellar core remains dense
from BN to RN

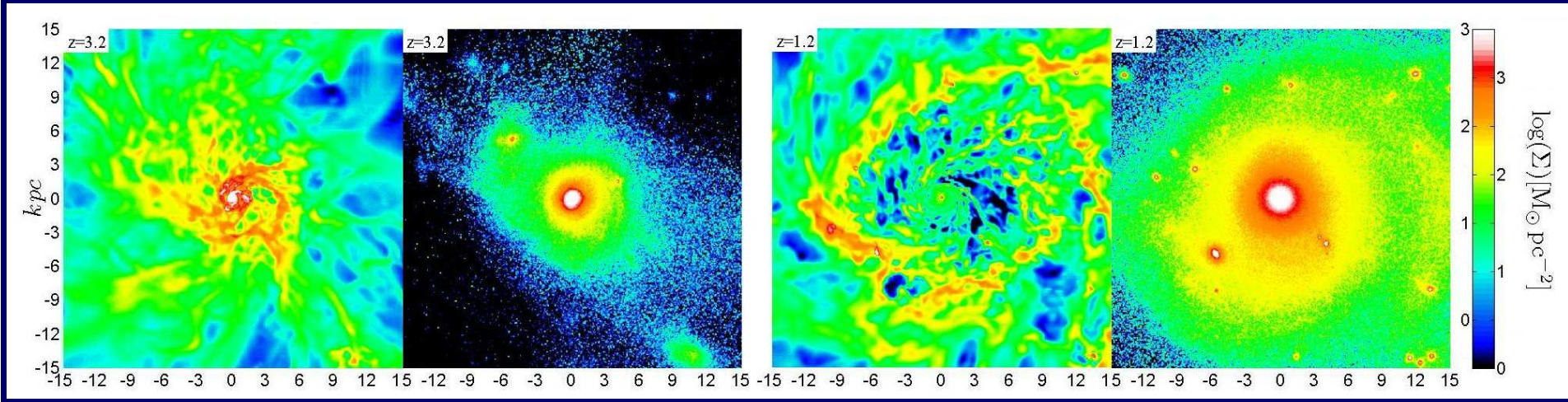
Blue Nugget - Red Nugget

naked red nugget

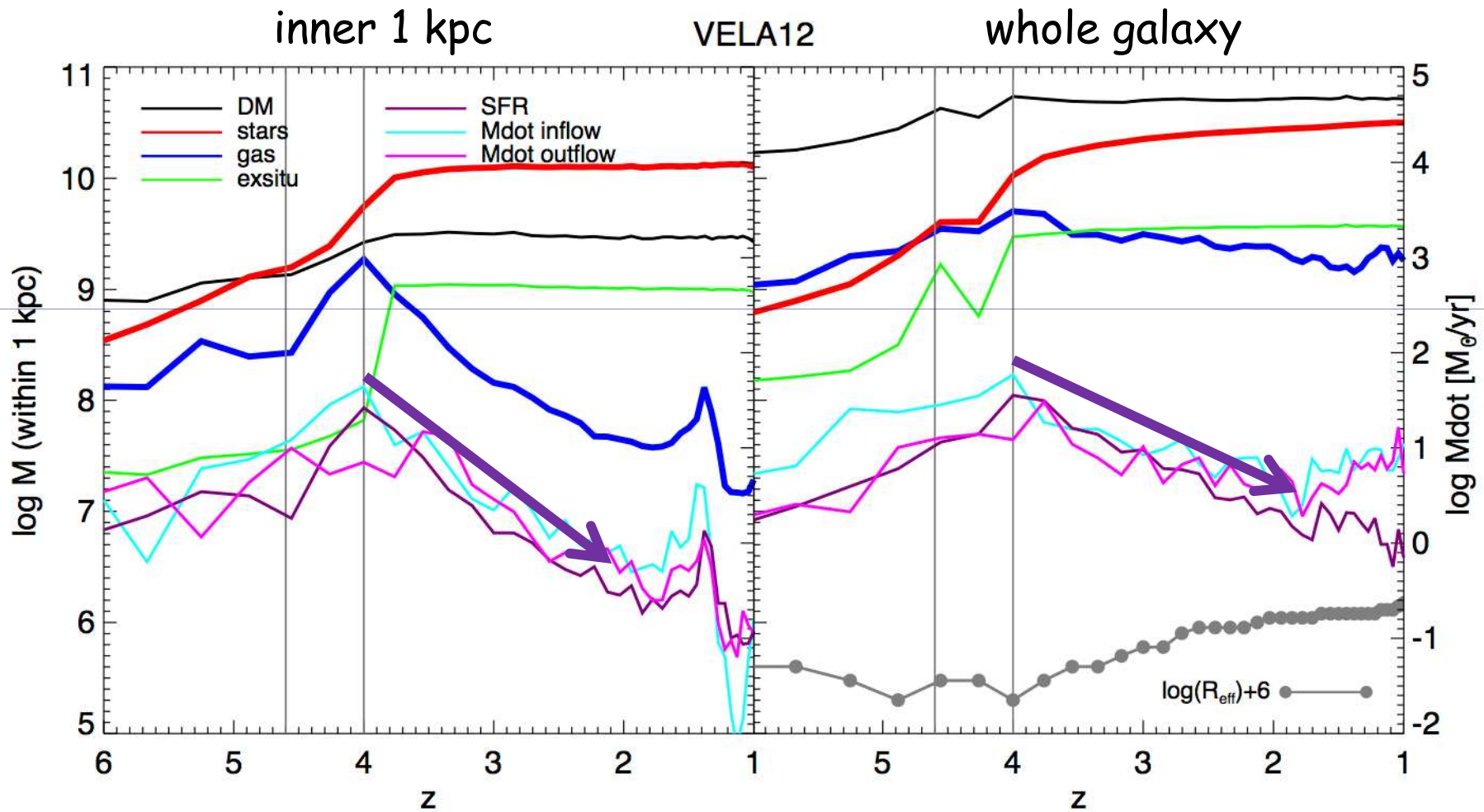


a stellar envelope may gradually grow by dry mergers

red nugget + envelope = elliptical



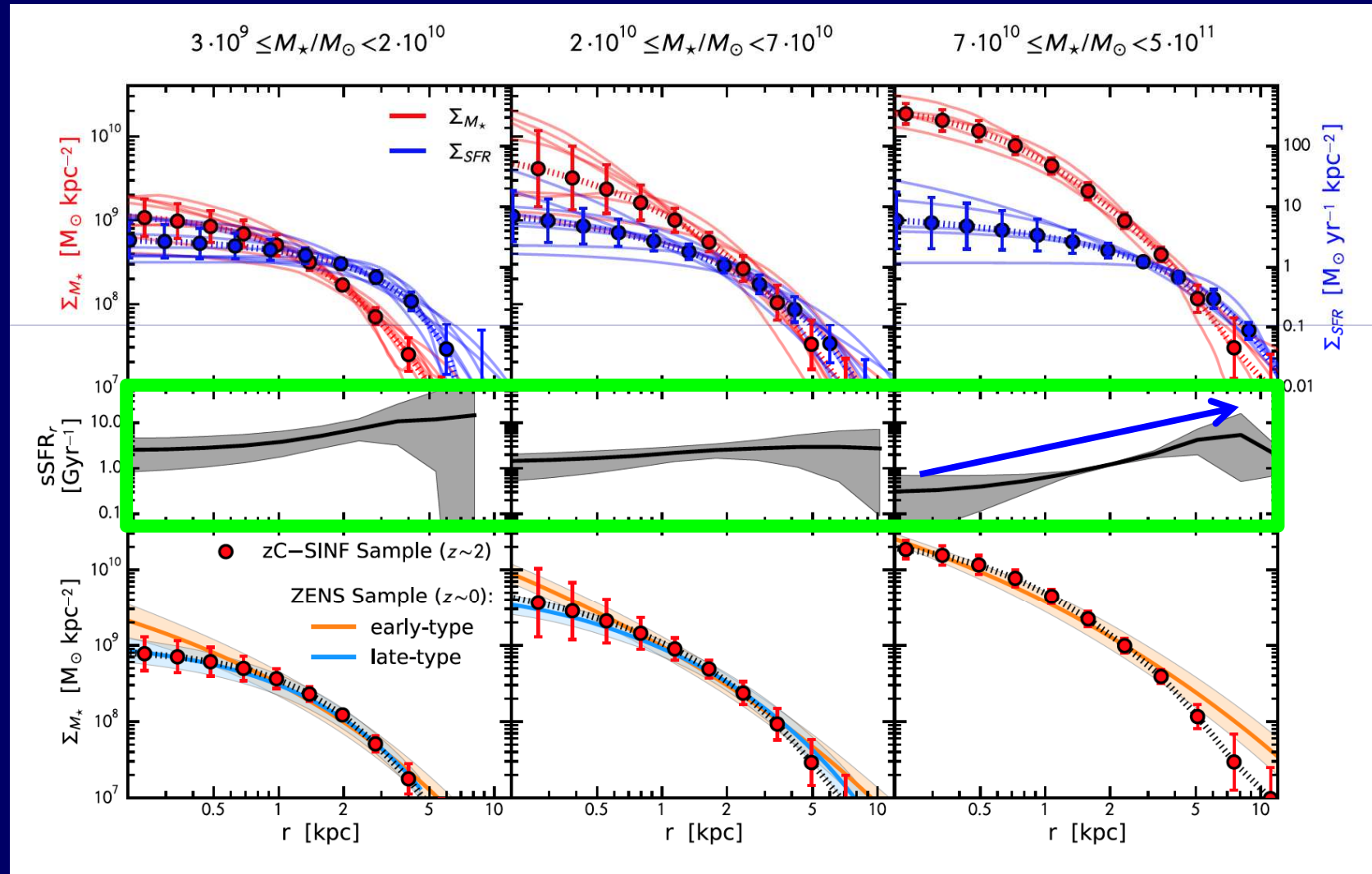
Inside-Out Quenching: Slower Quenching in the Outer Disk



Inside-Out Quenching

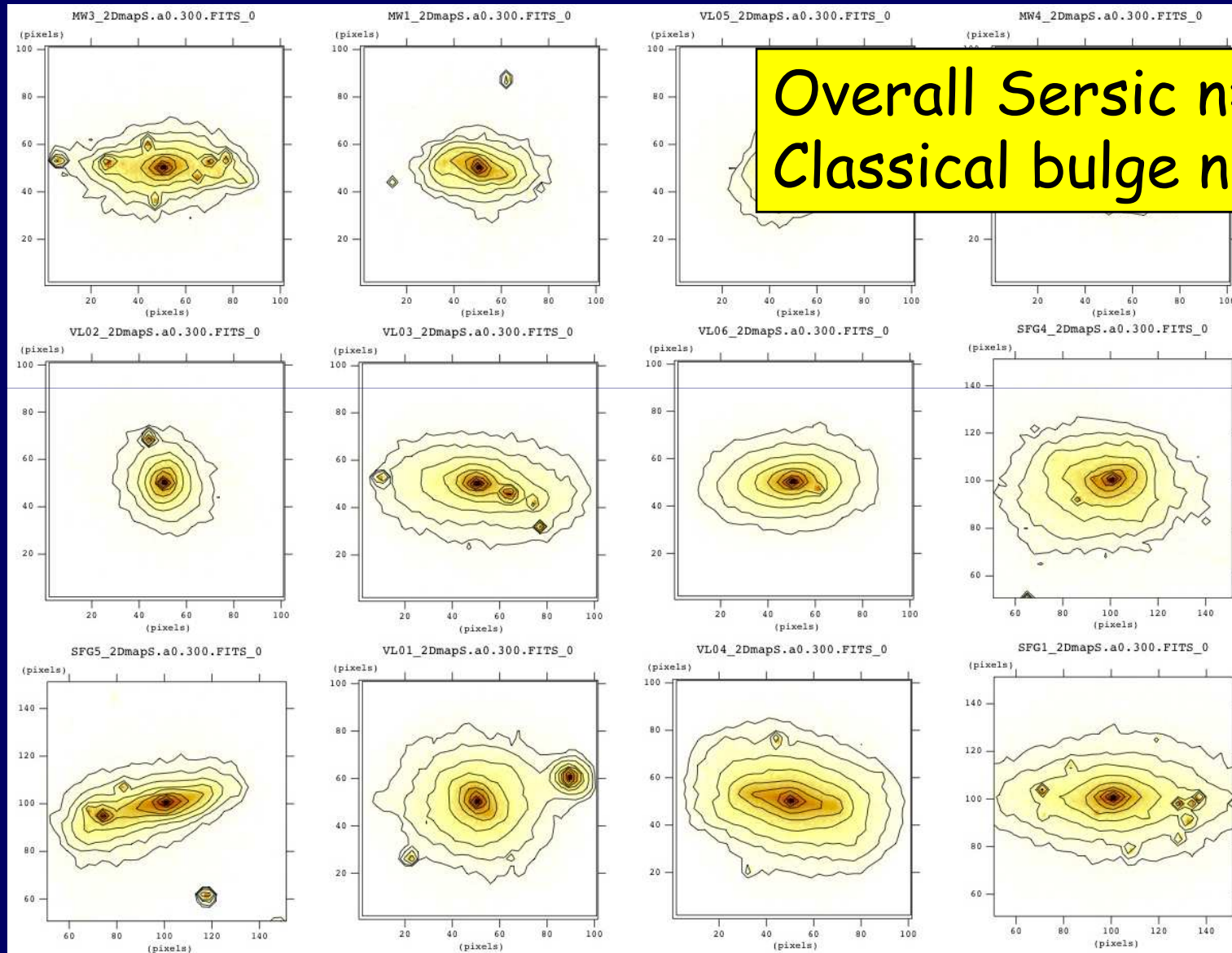
Tacchella+ 2014

profiles of sSFR ($=\text{SFR}/M_{\text{star}}$) at $z\sim 2$ galaxies

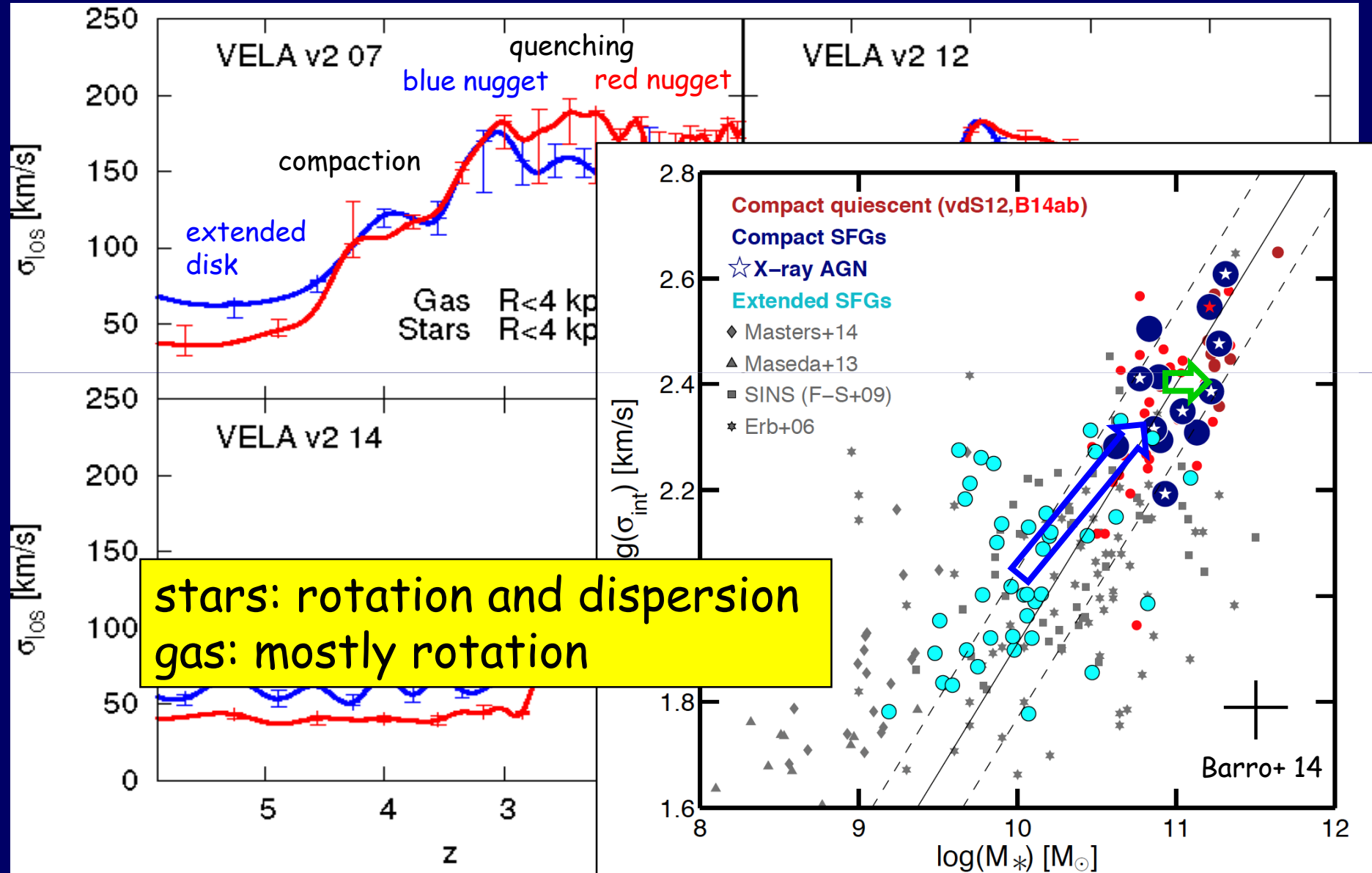


Stellar Component at $z=2.3$, edge-on

Ceverino+ 2014



"line width" evolution in simulated galaxies



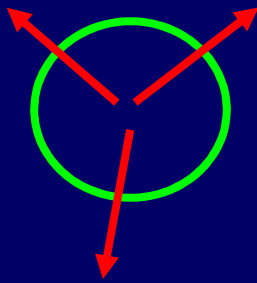
The Trigger of wet Compaction?

- VDI-driven inflow
- Mergers
- tidal compression
- Counter-rotating streams

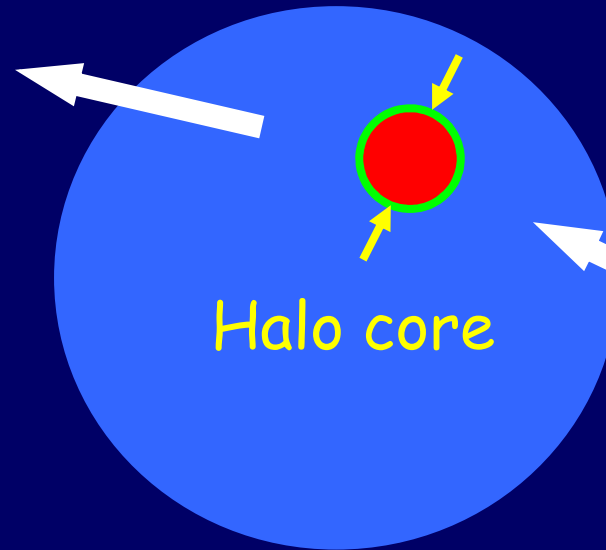
Compaction by Tidal Compression

Dekel, Devor, Hetzroni 2003

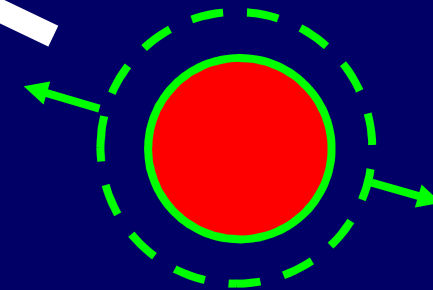
gas depletion
- quenching



compression - compaction
- star formation - outflow

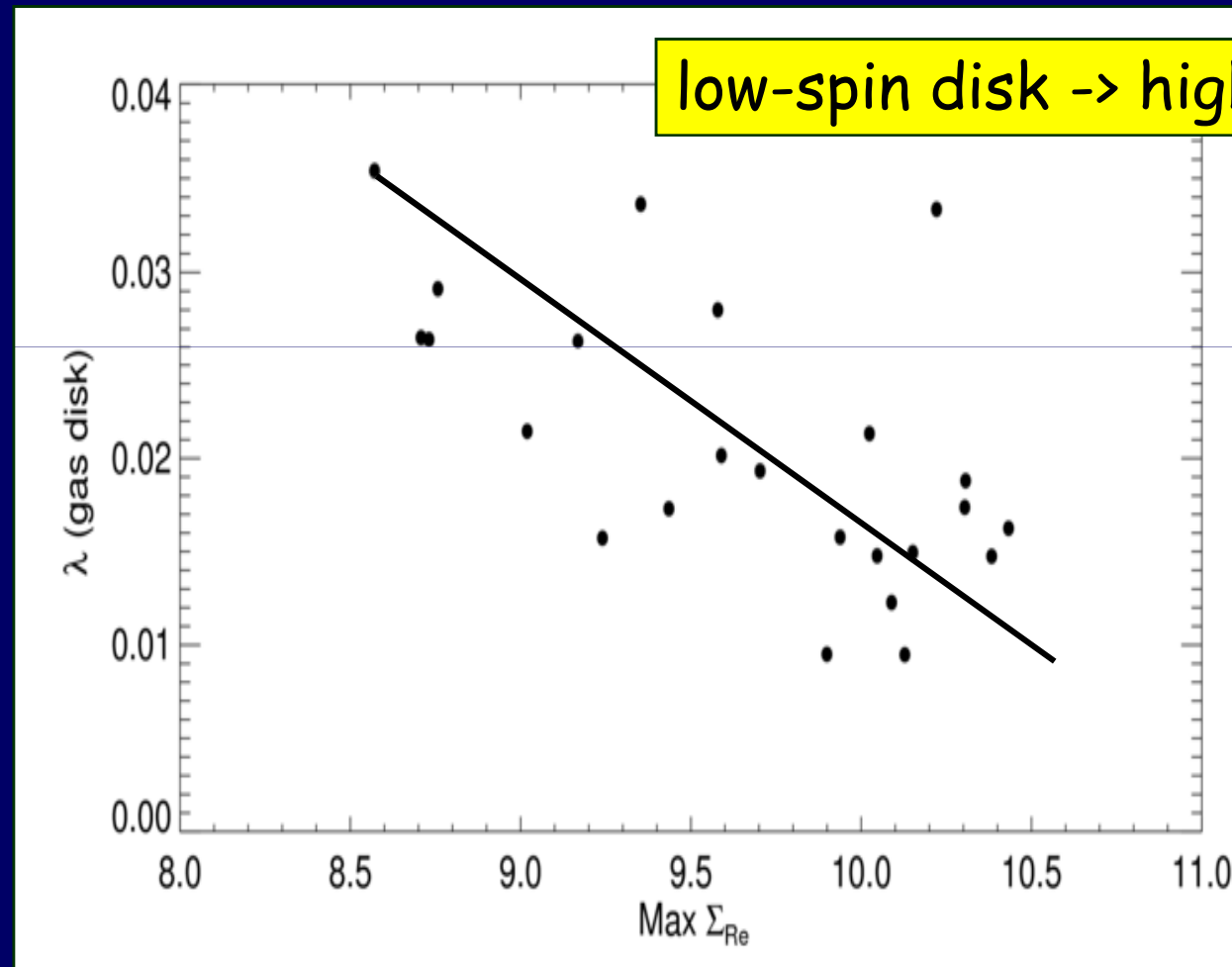


stripping

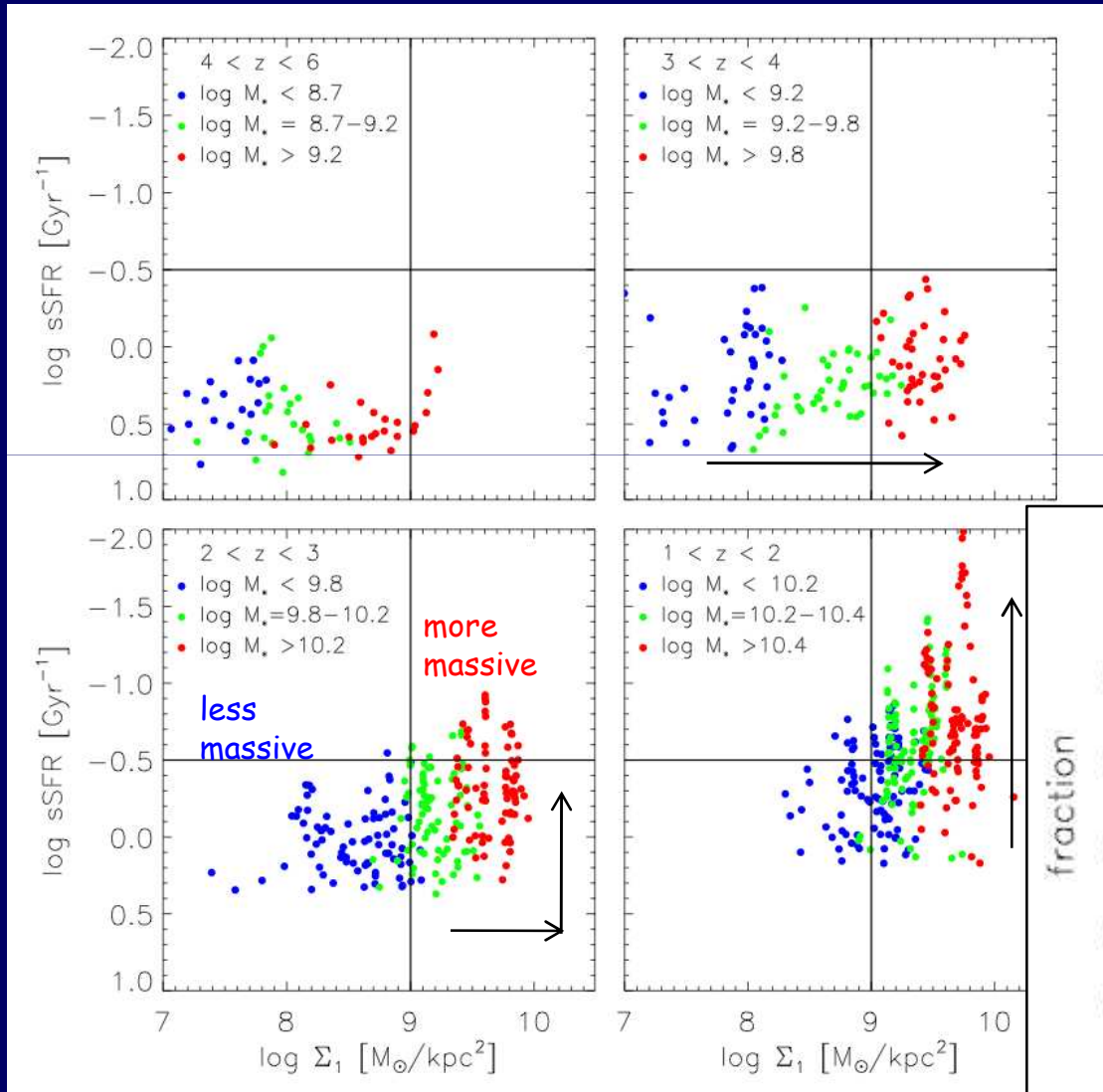


Blue Nuggets by Wet Inflow: Low Spin

Simulations confirm model predictions Dekel, Burkert 14; Zolotov+ 14



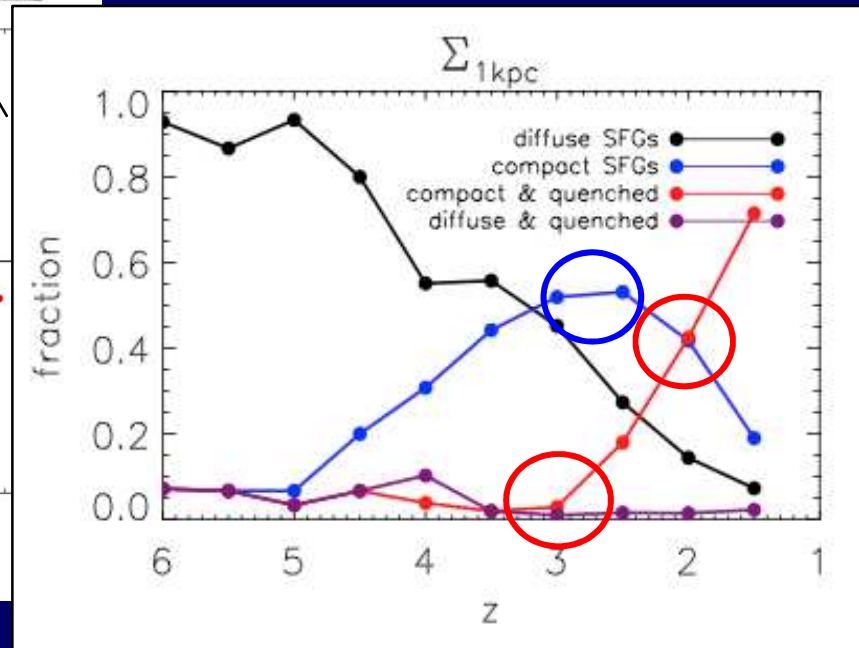
Diffuse SFG -> "Blue" Nuggets -> Red Nuggets



Downsizing:

More massive galaxies

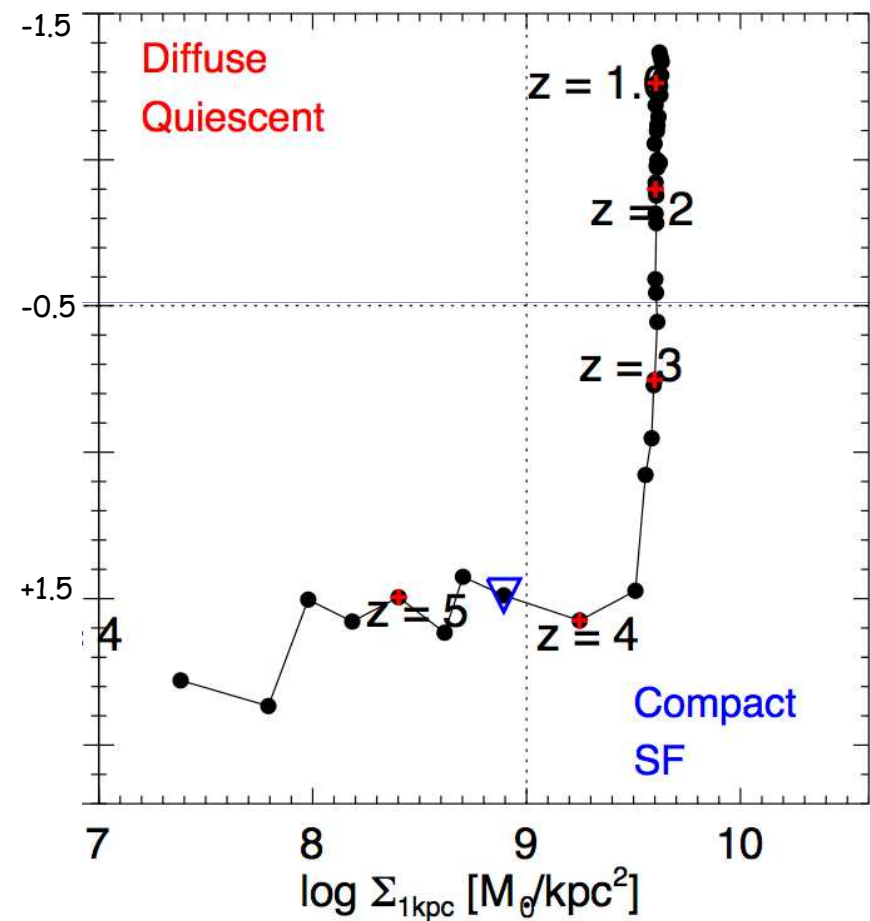
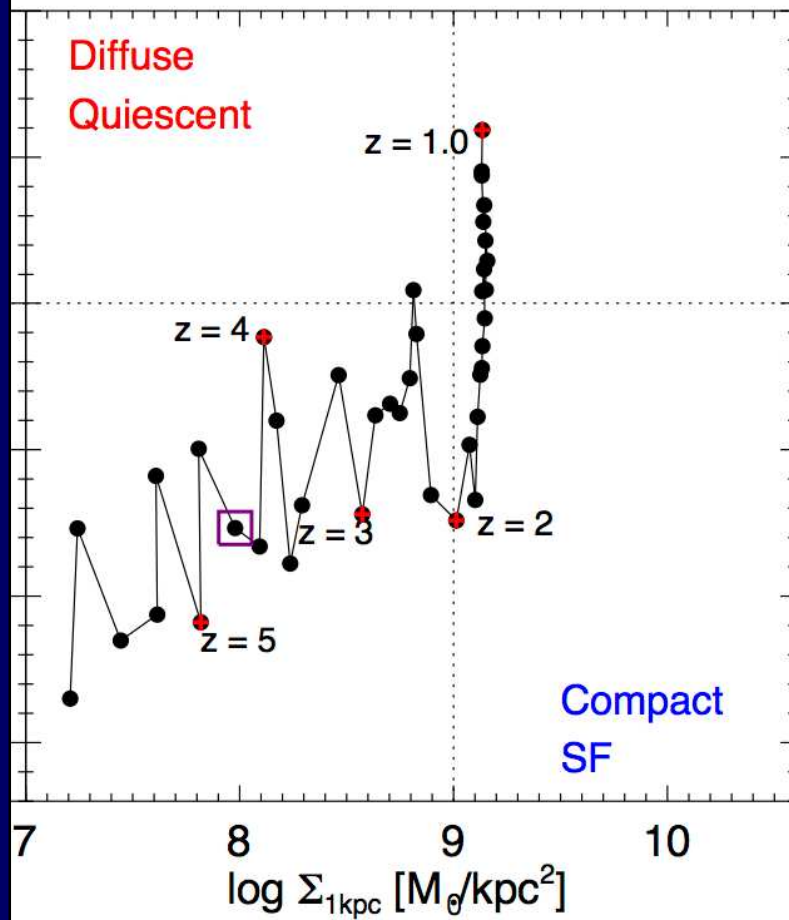
- compactify earlier
- to higher density Σ_1, Σ_e
- quench more efficiently



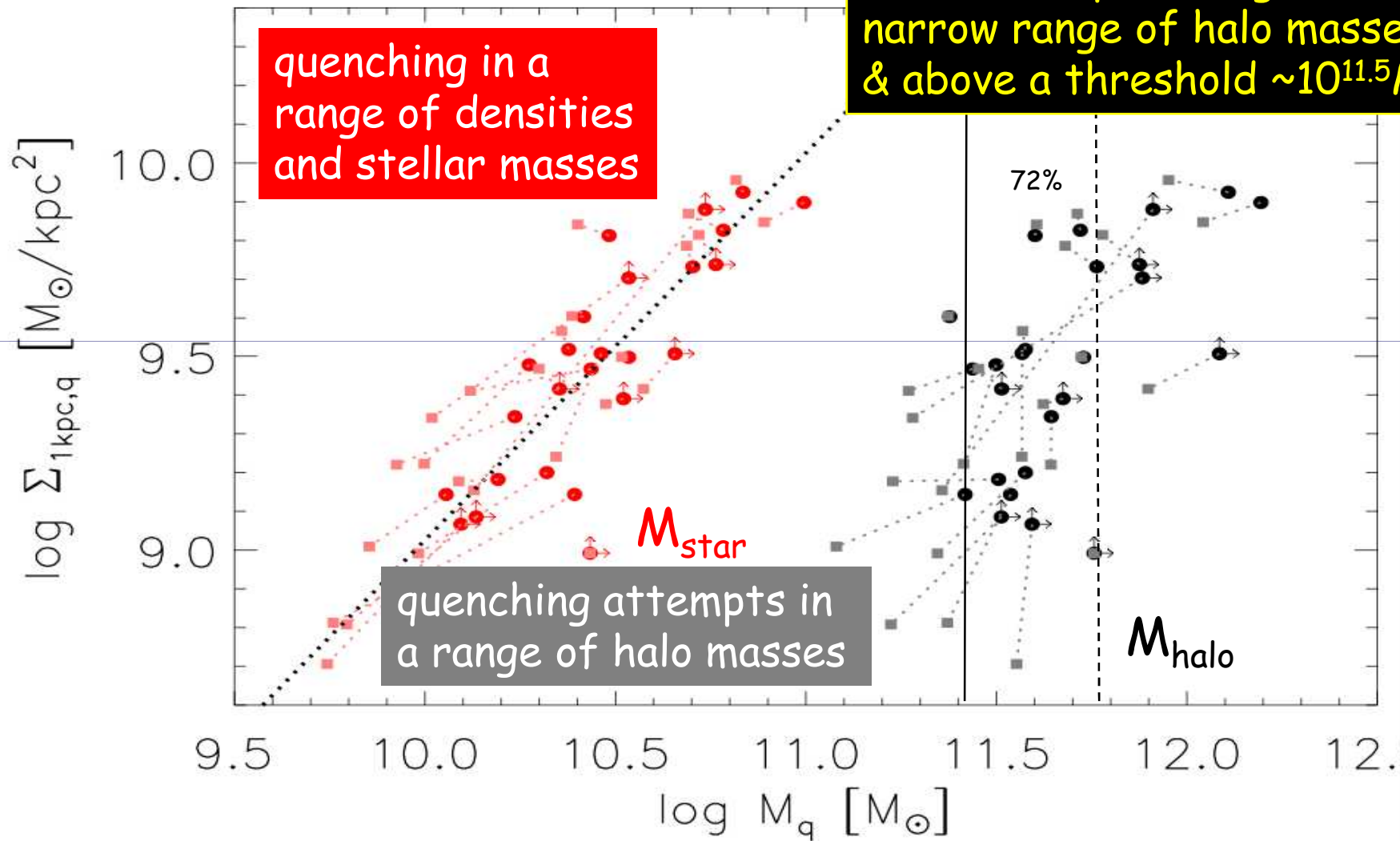
Hesitant vs. Decisive Quenching

low mass, low z

high mass, high z



Mass and Central Density at Quenching



Virial Shock Heating

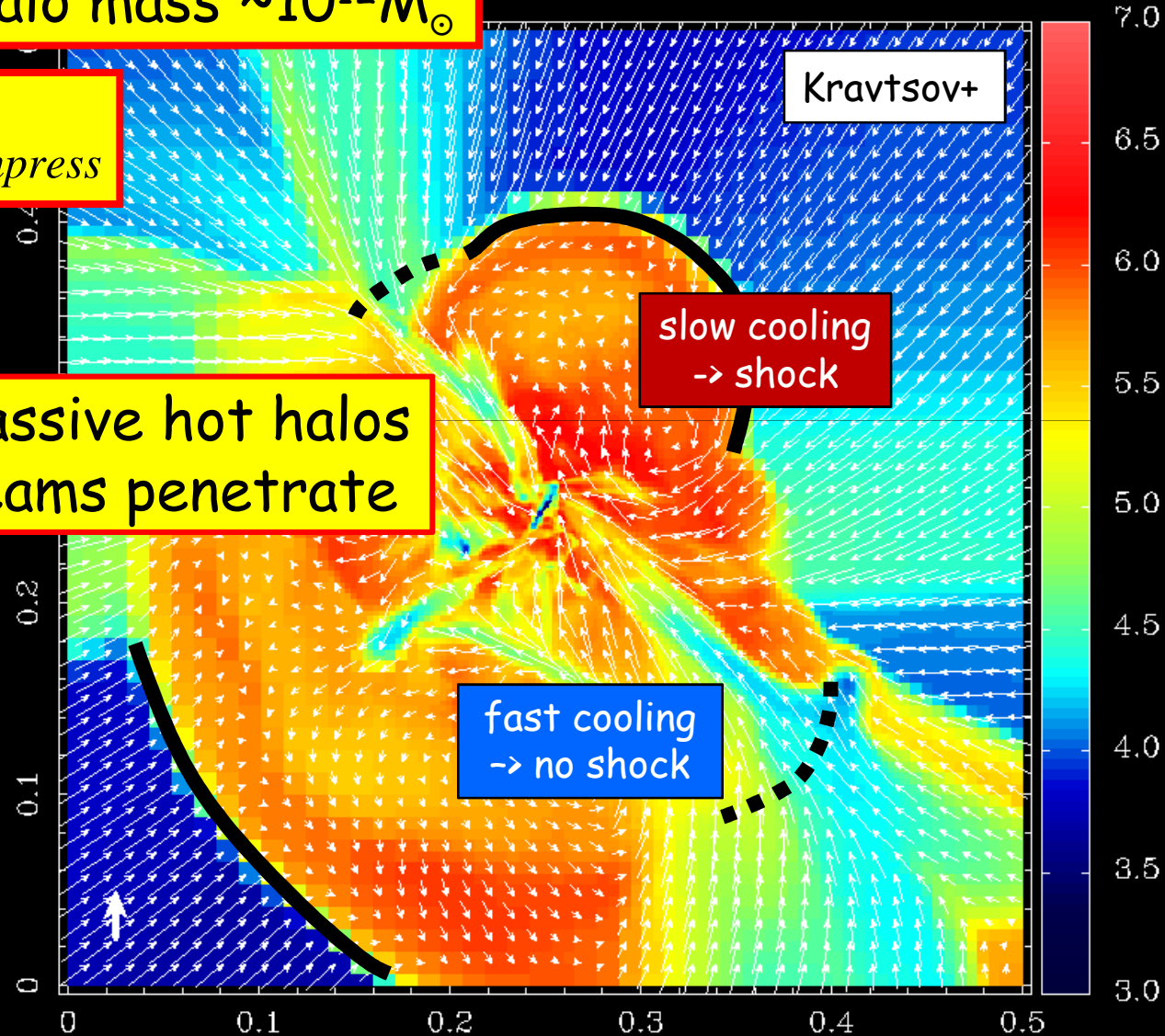
Dekel & Birnboim 2006

$\log(T[\text{K}])$

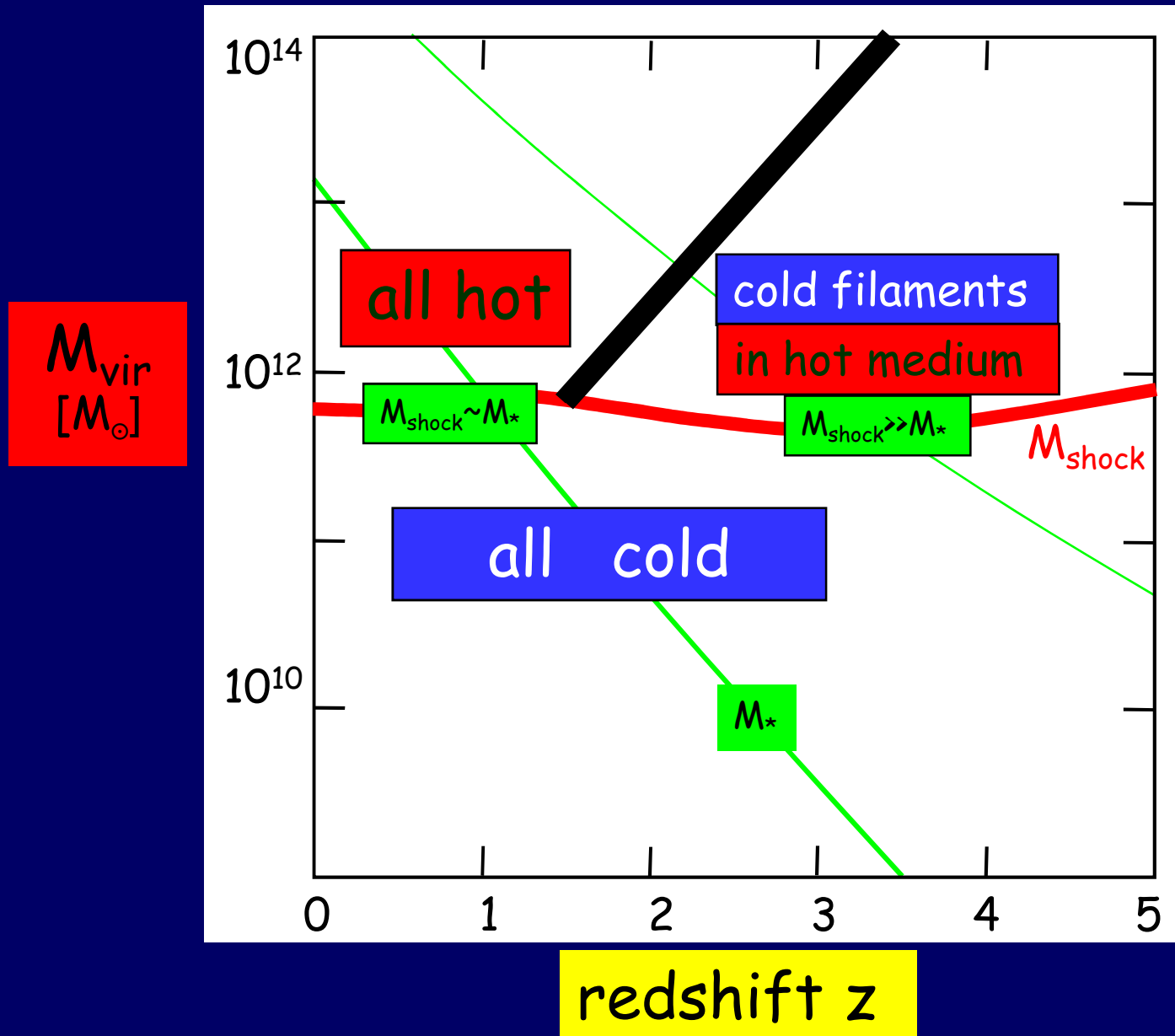
critical halo mass $\sim 10^{12} M_{\odot}$

$$t_{cool}^{-1} < t_{compress}^{-1}$$

in hi-z massive hot halos
cold streams penetrate



Cold Streams in Big Galaxies at High z



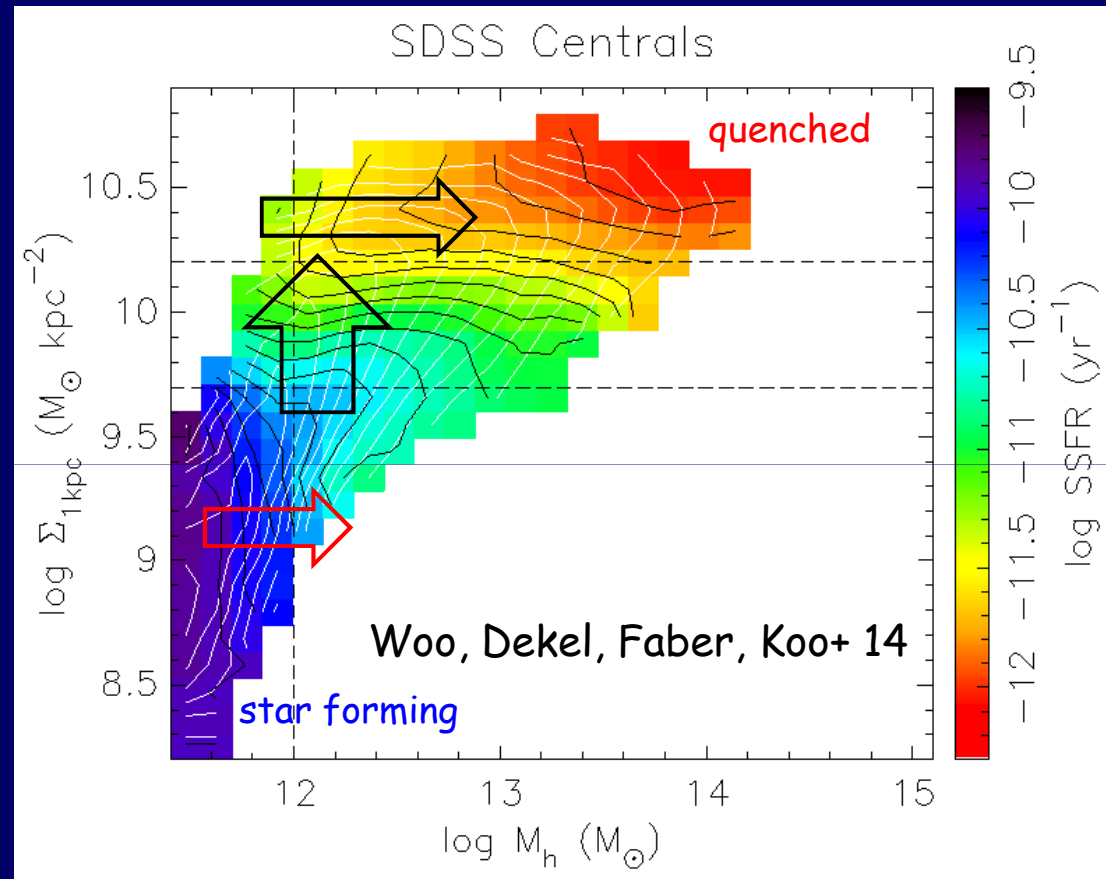
Dekel &
Birnboim 06

Two Quenching Mechanisms: Bulge & Halo



Compact gaseous bulge
→ gas removal by high SFR, outflow, AGN, Q-quenching

In halos $> 10^{12} M_{\odot}$
→ long-term shutdown of gas supply by virial shock heating



Compact bulge and halo quenching

But each can quench by itself

The Quenching Mechanism

Wet compaction triggered by streams/mergers/VDI in low-spin galaxies: $\text{inflow} > \text{SFR} + \text{outflow}$

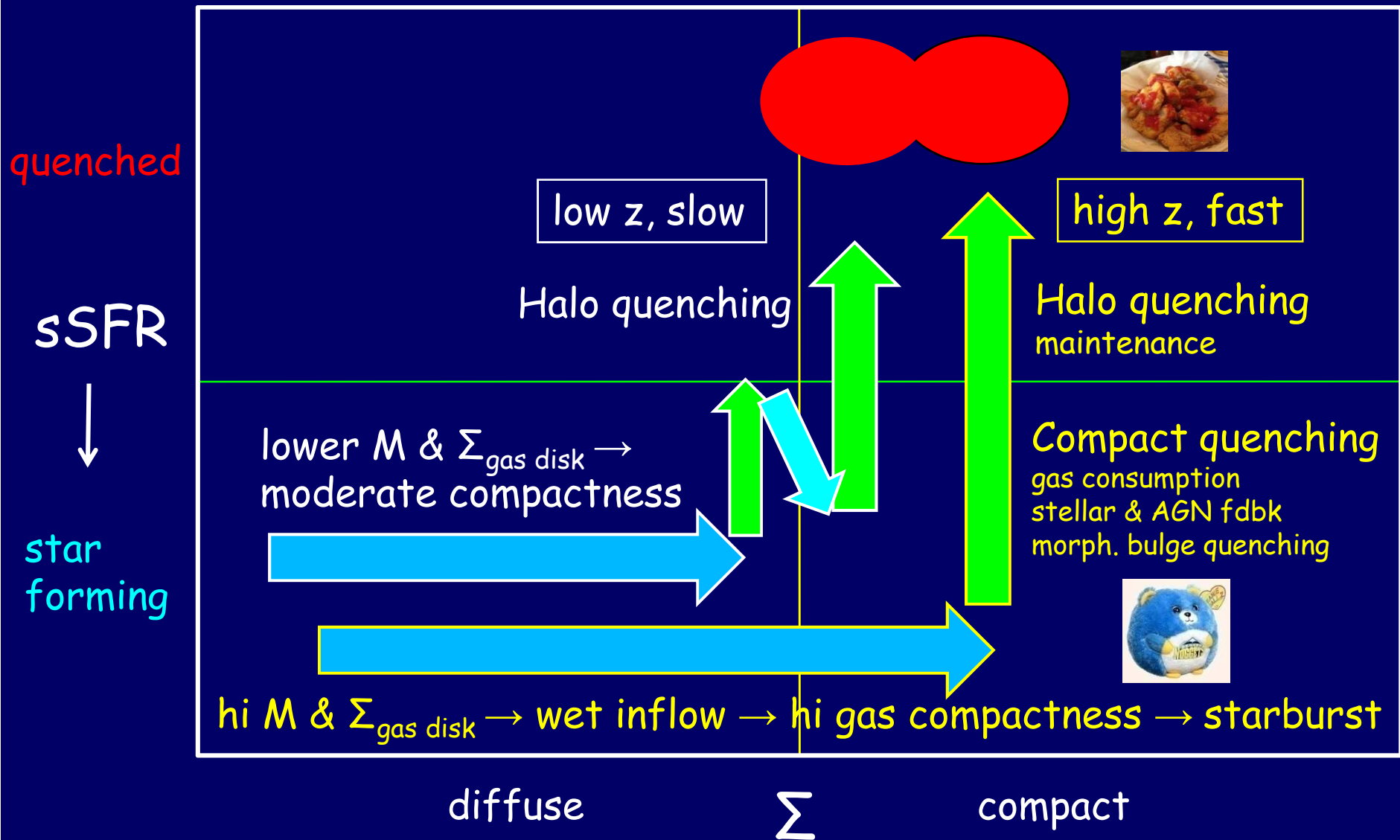
High SFR and no gas supply to the center:
 $\text{inflow} < \text{SFR} + \text{outflow} \rightarrow \text{quenching attempt}$

- disk has shrunk \rightarrow no immediate gas supply
- massive bulge suppresses VDI-driven inflow
- AGN outflow

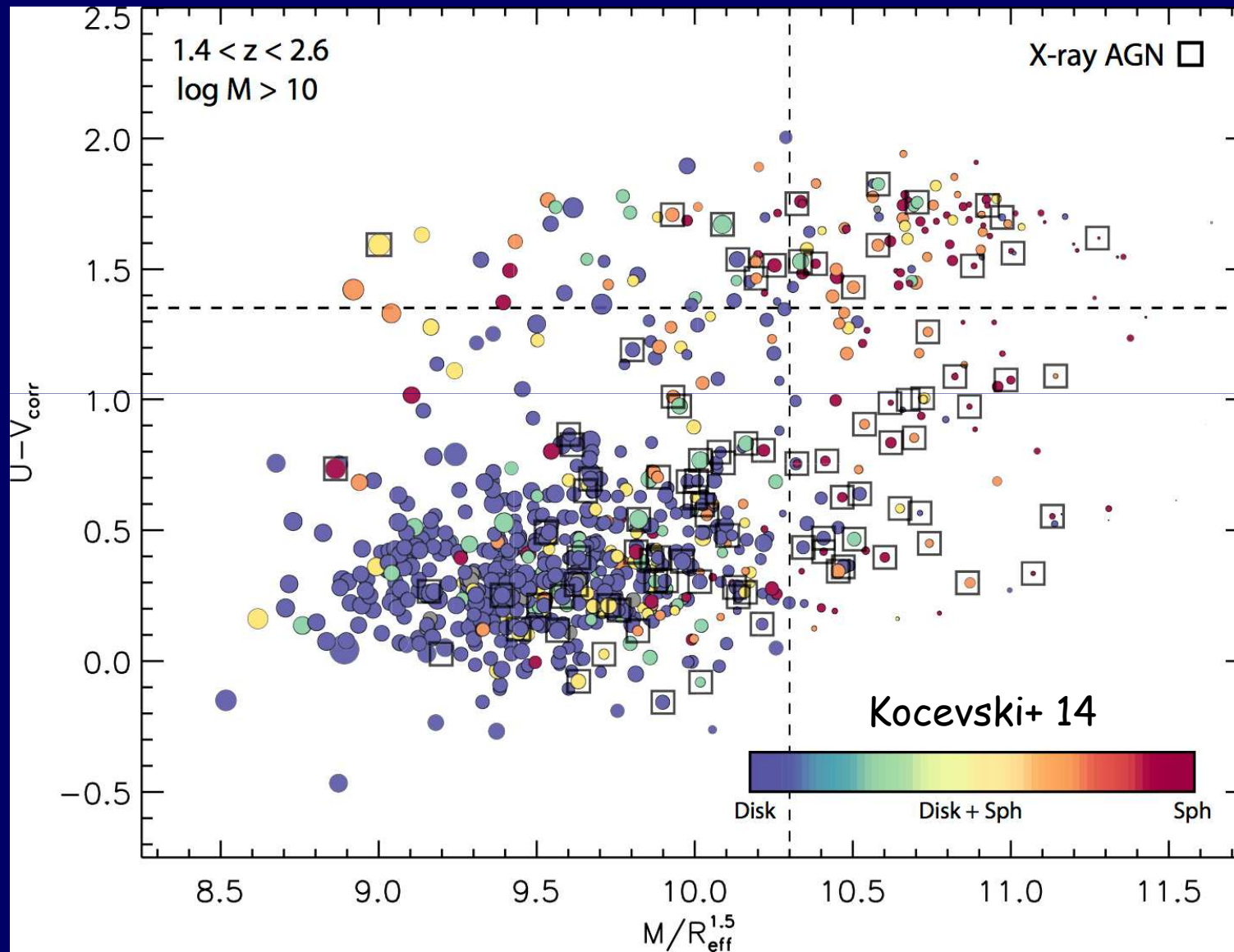
If halo is massive (hot) \rightarrow no further gas supply
 \rightarrow long-term quenching

If halo is less massive \rightarrow gas supply to a new disk
 \rightarrow new compaction and SFR ... until the halo is massive (hot)

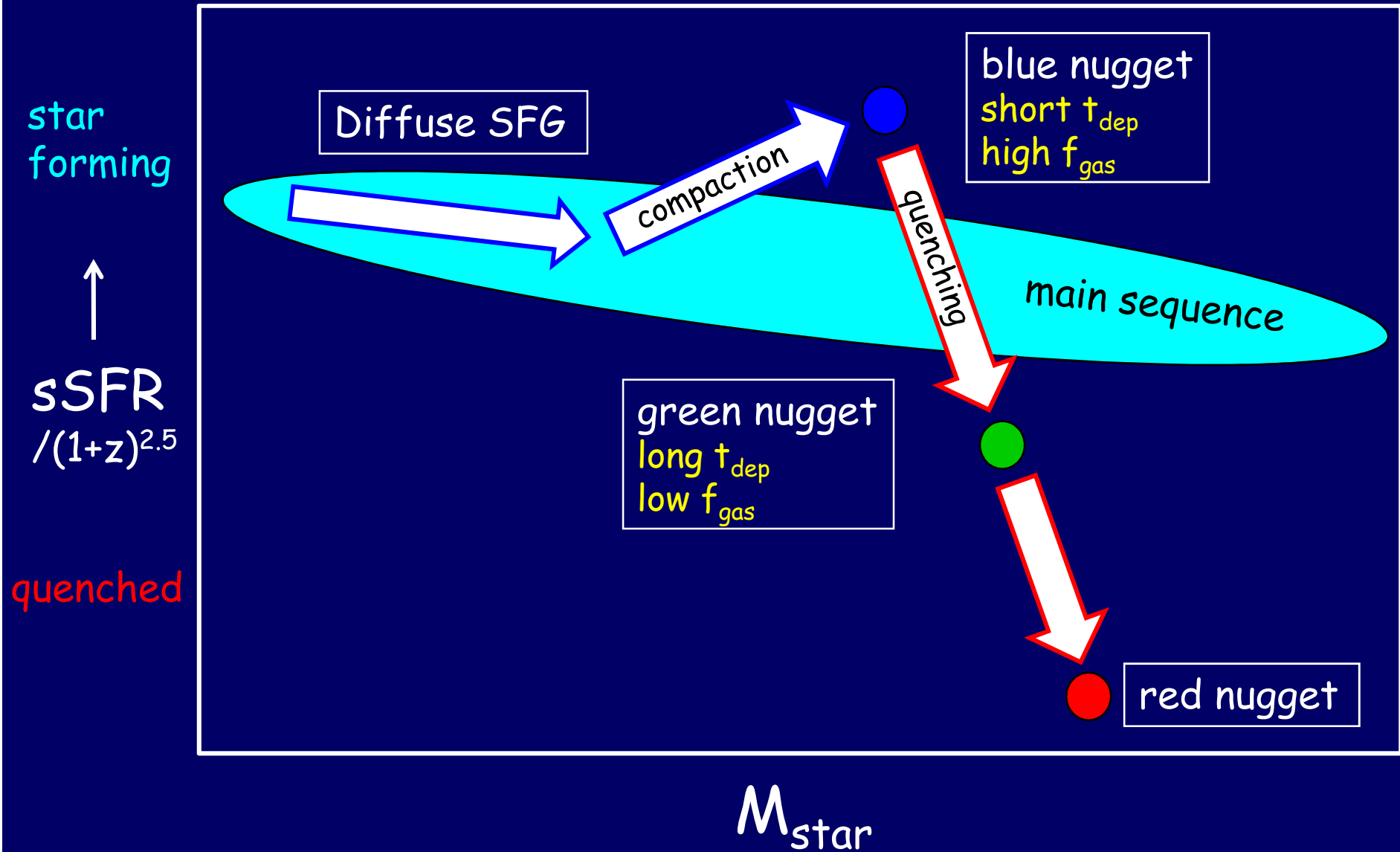
Modes of Evolution



Role of AGN Feedback in Quenching?



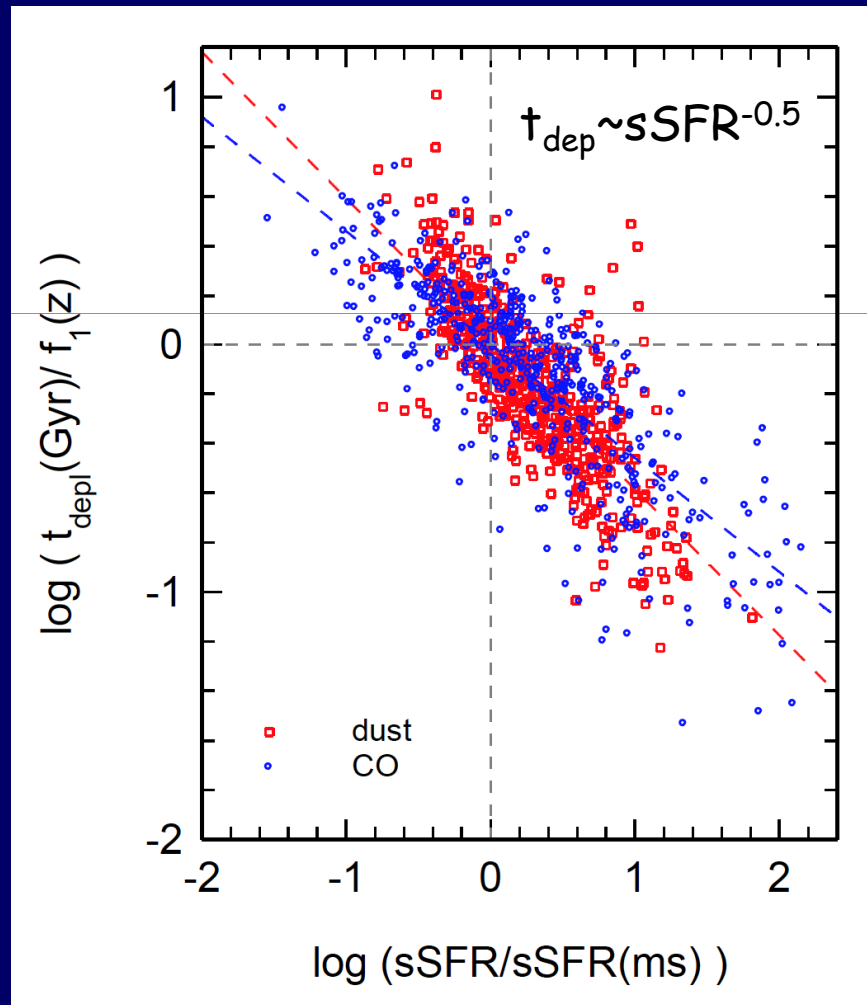
Gradients Across the Main Sequence



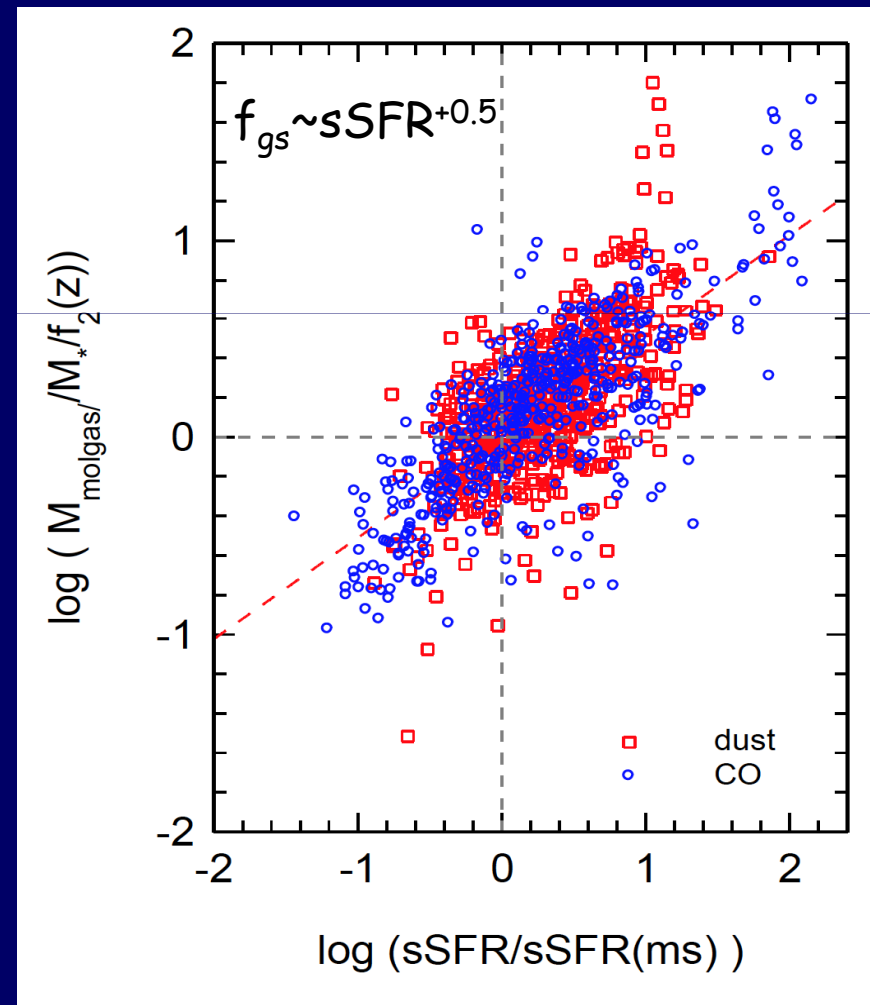
Gradients Across the Main Sequence

Genzel+ 2014 $z=0-2.5$

$$\tau_{\text{depl}} = M_{\text{gas}} / \text{SFR}$$



$$f_{\text{gs}} = M_{\text{gas}} / M_{\text{stars}}$$



Conclusions: Quenching

A characteristic sequence of events at high z in most galaxies:

- **wet compaction** by mergers and VDI to **compact SFGs** ("blue" nuggets)
rotating flattened spheroids with high dispersion,
above the main-sequence, short depletion time, high gas fraction
- high SFR+AGN, outflows, massive self-gravitating bulge →
fast **quenching inside-out to compact spheroids** (red nuggets)
+gas rings
- long-term quenching by **hot massive halo**
- **Quenching downsizing:**
massive galaxies quench earlier, efficiently, at higher central densities
less massive galaxies : hesitant quenching till halo shutdown

Conclusions

High- z massive galaxies at cosmic-web nodes
Fed by ~ 3 co-planar **streams** penetrating hot CGM

Angular-momentum:

- effective tidal **torques** on gas streams, AM transport to inner halo
- spiral-in through an extended rotating tilted **ring (DLAS?)**
- **disk spin** \leq halo spin

Typical SFGs have perturbed rotating disks at violent instability (VDI)

- Massive clumps ($>10^8 M_\odot$) **survive feedback**
- off-center in-situ **young clumps** <300 Myr, showing **age/gas gradient**
- older ex-situ clumps

Nonlinear instability driven by inflow+mergers. Compressive turbulence?

A characteristic sequence of events:

- **wet compaction** by mergers and VDI into **compact SFGs** (blue nuggets)
- high SFR+AGN, outflows, massive self-gravitating bulge
 - > fast **quenching to compact ellipticals** (red nuggets) +gas rings (?)
- long-term quenching by **hot massive halo** -> downsizing of quenching

Conclusions

High- z massive galaxies at cosmic-web nodes
Fed by ~ 3 co-planar **streams** penetrating hot CGM

Angular-momentum:

- effective **tidal torques** on gas streams, AM transport to inner halo
- spiral-in through an extended rotating tilted **ring (DLAS?)**
- **disk spin** \sim **halo spin**, despite different history

Typical SFGs have perturbed rotating disks at violent instability (VDI)

- Massive clumps ($>10^8 M_\odot$) **survive feedback**
- in-situ **young clumps** showing **age gradient**. Older ex-situ clumps

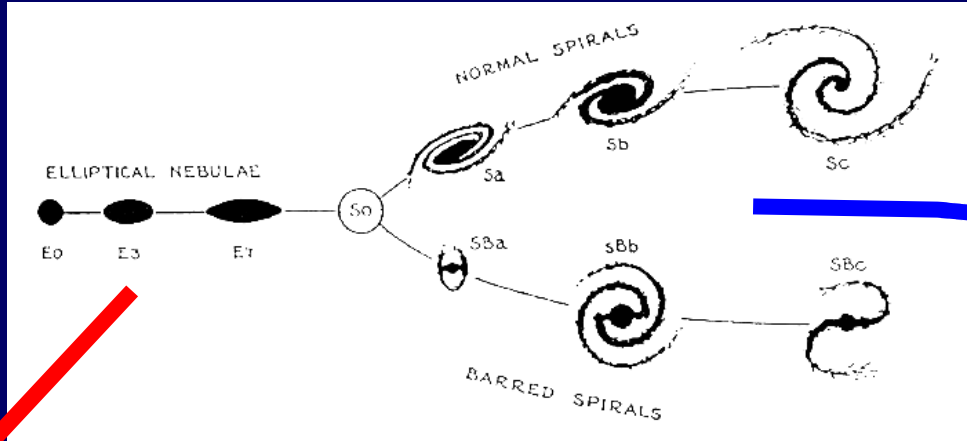
Stimulated nonlinear instability, by inflow+mergers.

compressive turbulence? Compressive tides? counter-rotating streams?

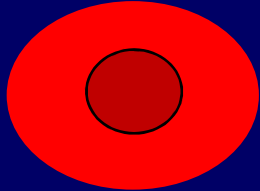
A characteristic sequence of events:

- **wet compaction** by mergers and VDI into **compact SFGs** (blue nuggets)
- high SFR+AGN, outflows, massive self-gravitating bulge
 - > fast **quenching to compact ellipticals** (red nuggets) +gas rings (?)
- long-term quenching by **hot massive halo** -> downsizing of quenching

The High-z "Hubble" Sequence

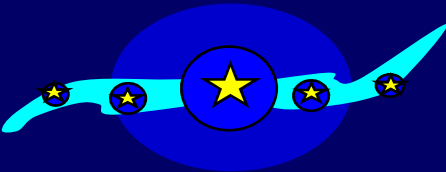


Red nuggets



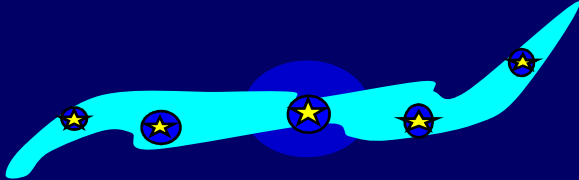
Compact ellipticals

Blue nuggets



Starburst in a compact bulge

Clumpy disks



No bars
Perturbed spiral patterns

$z=0$



$z=2$



www.shutterstock.com · 57051511

$z=8$

