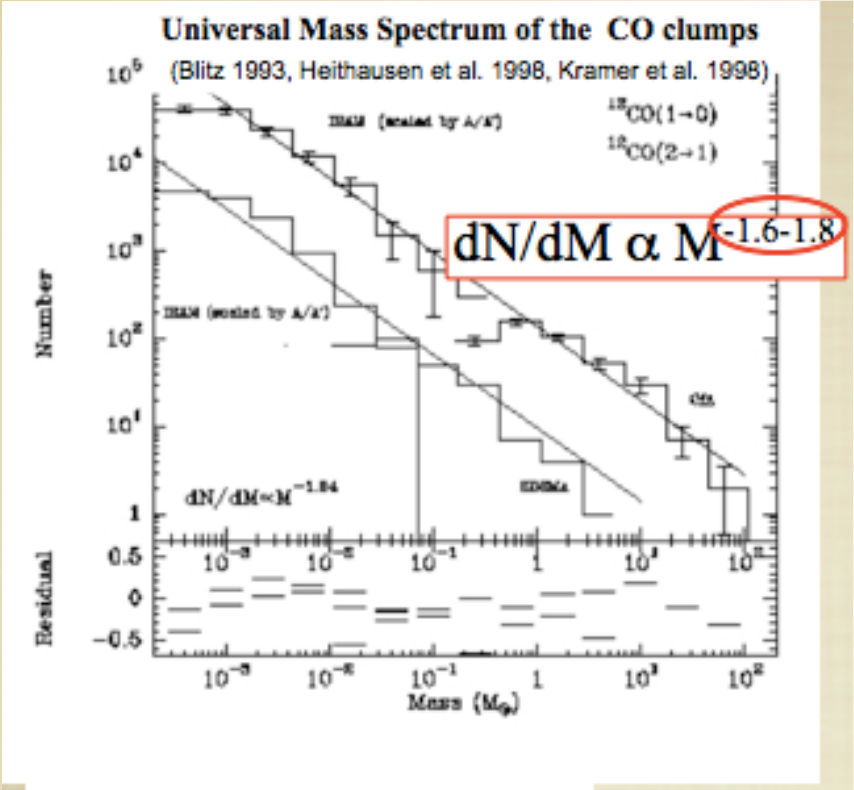
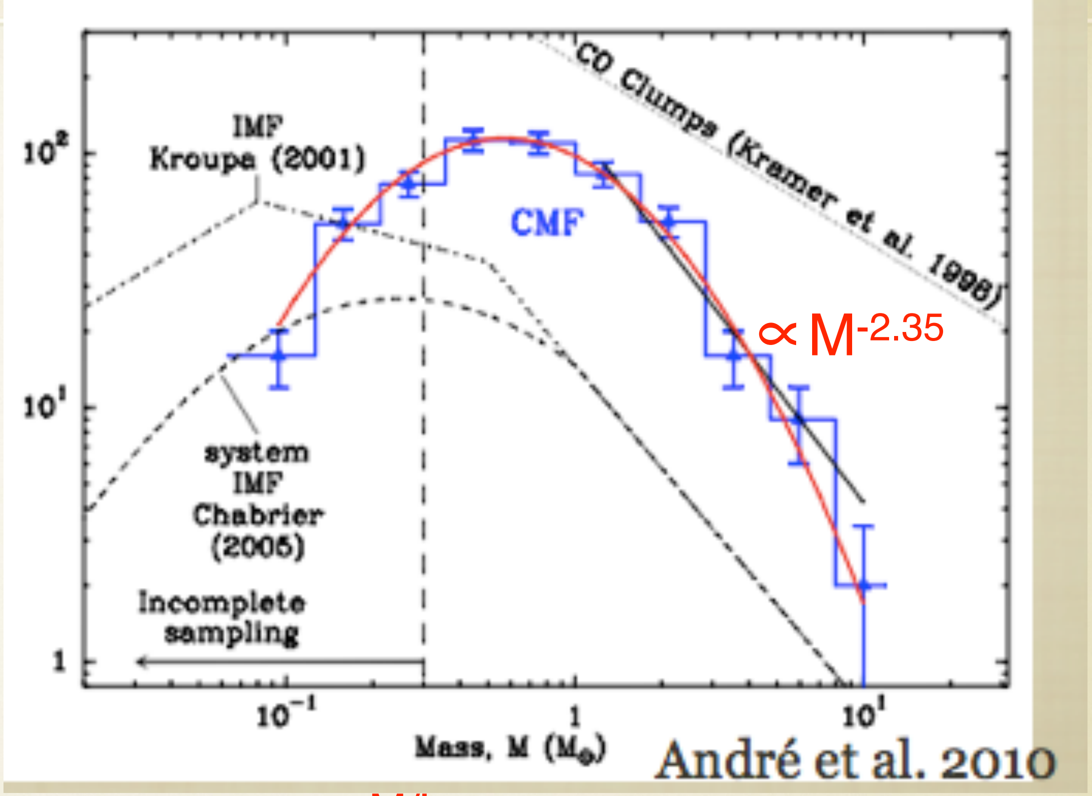
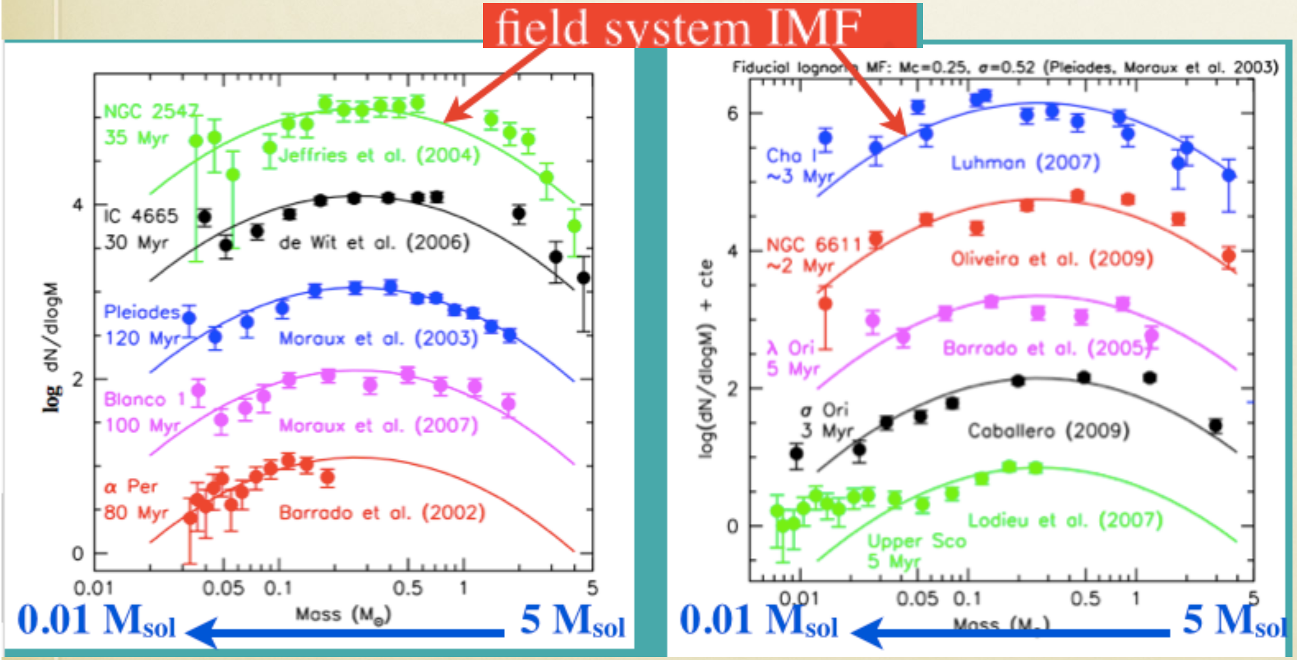
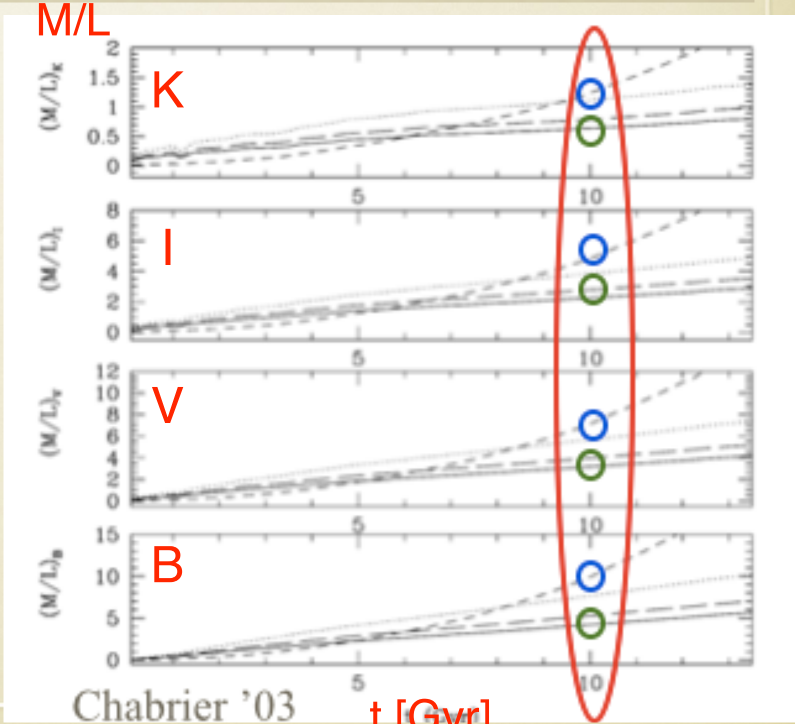
# Is the stellar IMF universal ?

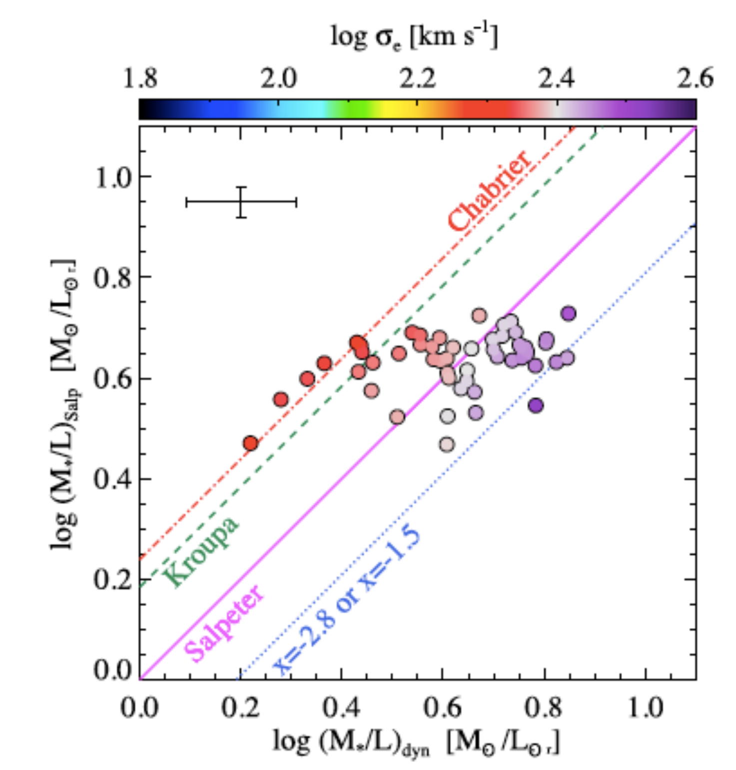
Gilles Chabrier
CRAL, ENS-Lyon


Universal Mass Spectrum of the CO clumps
(Blitz 1993, Heithausen et al. 1998, Kramer et al. 1998)
$^{12}$CO(1→0)
$^{12}$CO(2→1)
dN/dM α M$^{-1.6-1.8}$
dN/dM∝M$^{-1.84}$
Number
Residual
Mass (M$_{\odot}$)

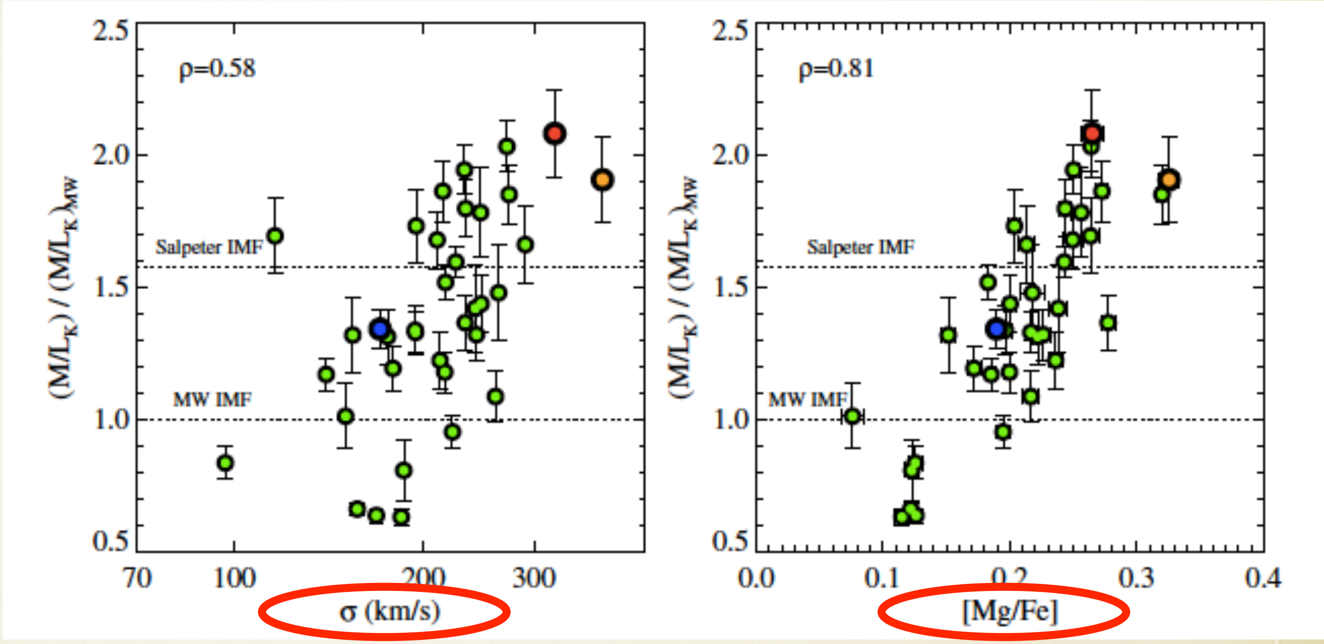

IMF
Kroupa (2001)
CO Clumps (Kramer et al. 1998)
CMF
∝ M$^{-2.35}$
system
IMF
Chabrier
(2005)
Incomplete
sampling
Mass, M (M$_{\odot}$)
André et al. 2010


field system IMF
NGC 2547
35 Myr
Jeffries et al. (2004)
IC 4665
30 Myr
de Wit et al. (2006)
Pleiades
120 Myr
Moraux et al. (2003)
Blanco 1
100 Myr
Moraux et al. (2007)
α Per
80 Myr
Barrado et al. (2002)
log dN/dlogM
Mass (M$_{\odot}$)
0.01 M$_{sol}$
5 M$_{sol}$
Fiducial lognormal MF: Mc=0.25, σ=0.52 (Pleiades, Moraux et al. 2003)
Cha I
~3 Myr
Luhman (2007)
NGC 6611
~2 Myr
Oliveira et al. (2009)
λ Ori
5 Myr
Barrado et al. (2005)
σ Ori
3 Myr
Caballero (2009)
Upper Sco
5 Myr
Lodieu et al. (2007)
log(dN/dlogM) + cte
0.01 M$_{sol}$
5 M$_{sol}$

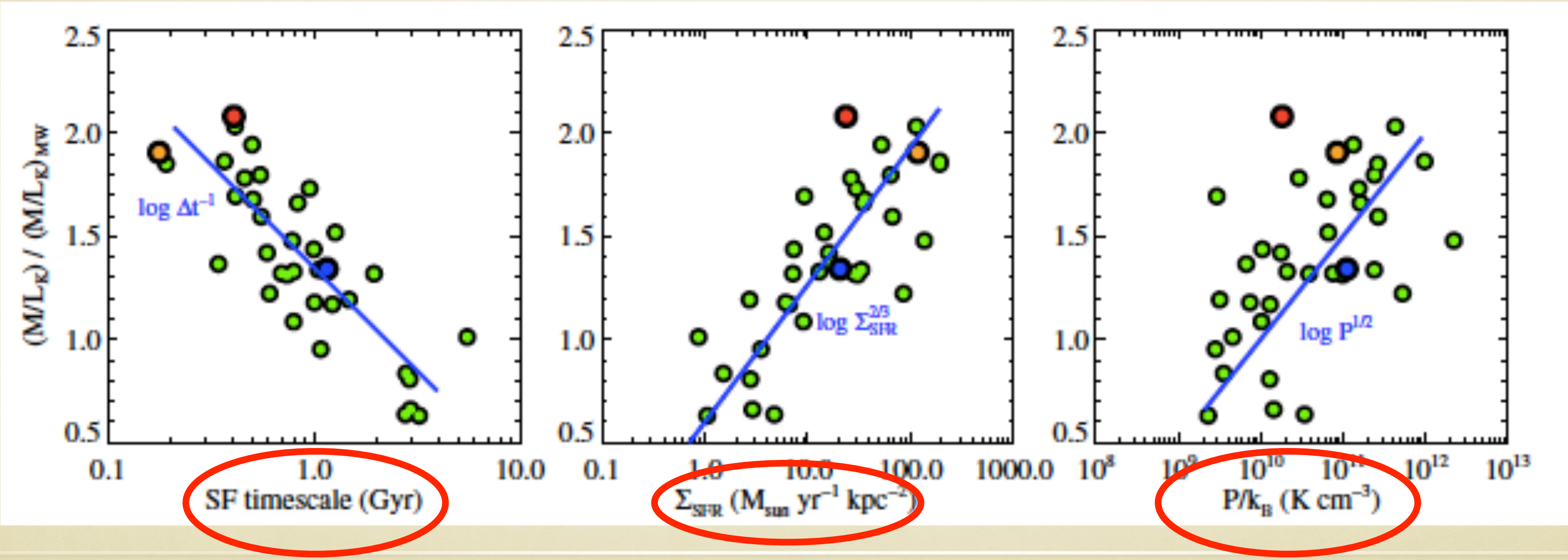

M/L
K
I
V
B
Chabrier '03
t [Gyr]




Cappellari et al. 2012

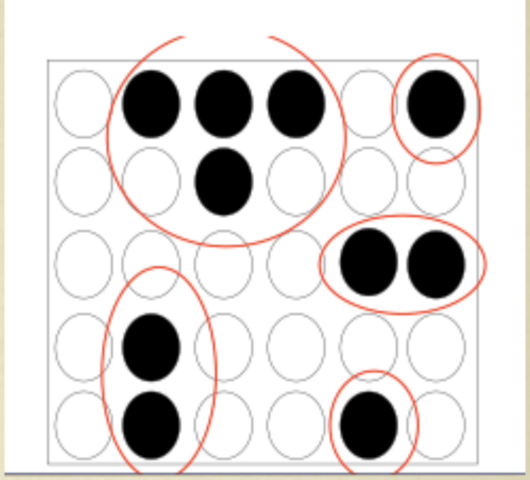



Conroy & van Dokkum 2012




Conroy & van Dokkum 2012

*Principles of Press-Schechter formalism* (used in cosmology to predict the mass spectrum of primordial collapsing structures (galaxies)). Very successful !



- consider a density field, $\delta(\vec{x})$, of density fluctuations, $\delta = \dfrac{\rho - \bar{\rho}}{\bar{\rho}}$,

$$\mathcal{P}(\delta_M)\,\mathrm{d}\delta_M = \frac{1}{\sqrt{2\pi}\,\sigma_M}\exp\left[-\frac{\delta_M^2}{2\sigma_M^2}\right]\mathrm{d}\delta_M$$

- setup a density threshold, $\delta_c$, to determine which perturbations should be considered (collapse time < Hubble time in cosmology)

$$\mathcal{P}(\delta_M > \delta_c) = \frac{1}{\sqrt{2\pi}\,\sigma_M}\int_{\delta_c}^{\infty}\exp\left[-\frac{\delta_M^2}{2\sigma_M^2}\right]\mathrm{d}\delta_M = \frac{1}{2}\mathrm{erfc}\left[\frac{\delta_c}{2\sigma_M}\right]$$

- sum over all the corresponding fluctuations

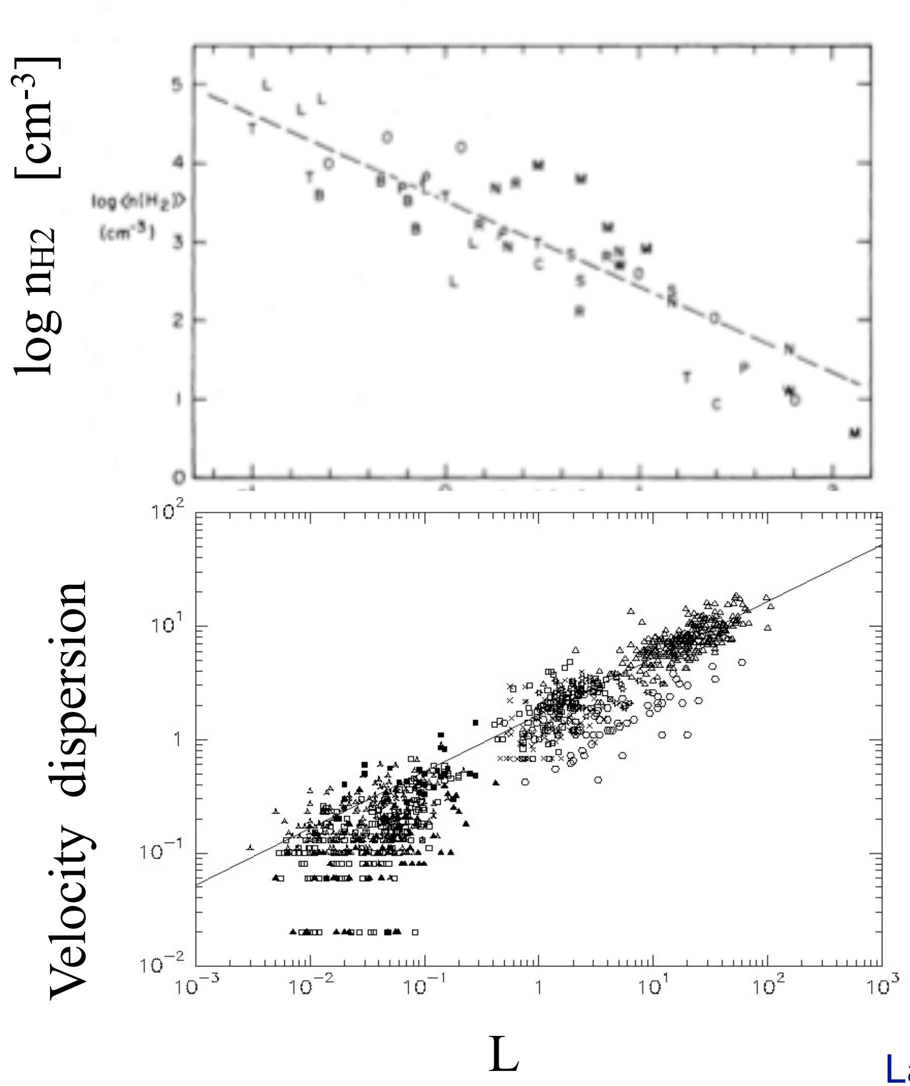
$$n(M,t)\,\mathrm{d}M = 2\frac{\bar{\rho}}{M}\frac{\partial\mathcal{P}(>\delta_c)}{\partial M}\mathrm{d}M = \sqrt{\frac{2}{\pi}}\frac{\bar{\rho}}{M^2}\frac{\delta_c}{\sigma_M}\exp\left(-\frac{\delta_c^2}{2\sigma_M^2}\right)\left|\frac{\mathrm{d}\ln\sigma_M}{\mathrm{d}\ln M}\right|\mathrm{d}M$$

# Which density fluctuation statistics for molecular clouds ?

*At large scale, molecular clouds dominated by supersonic turbulence*

Density versus size of CO clumps

Velocity dispersion versus size of CO clumps



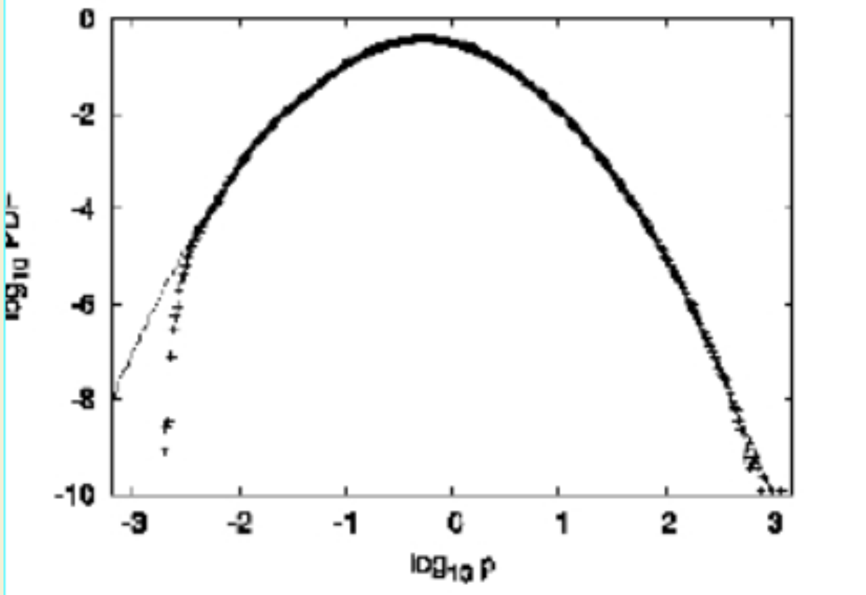

$\langle n \rangle \propto L^{-0.7\ \ -1}$

$V_{rms} \propto L^{0.4\ -\ 0.5}$

Larson ’81;
Hennebelle & Falgarone ’12

$$\bar{n} = (d_0 \times 10^3\ \mathrm{cm}^{-3}) \left(\frac{L}{1\mathrm{pc}}\right)^{-0.7}, \quad V_{\mathrm{rms}} = (u_0 \times 0.8\ \mathrm{km\ s}^{-1}) \left(\frac{L}{1\mathrm{pc}}\right)^{\eta}.$$

# PDF of compressible turbulence




$$\delta = \ln(\rho / \overline{\rho})$$

A lognormal distribution:

$$P(\delta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left( -\frac{(\delta + \sigma^2/2)^2}{2\sigma^2} \right)$$

$$\mathcal{P}_{\log\rho} \propto k^{n'}$$
$$\mathcal{P}_V \propto k^n$$

Kolmogorov (n=3.66) < **n'~ n ~ 3.8** < Burgers (n=4)

Beresnyak et al. '04, Kritsuk et al. '07, Federrath et al. '08

**Step 1: we ignore gravity**

***structures due to turbulence-induced fluct'ns (scale free)***

Clumps are defined as unbound structures having a density above some **constant** density threshold $\delta_c = \ln(\varrho_c / \varrho)$

$$\boxed{\frac{dN}{dM} \propto M^{-(2-\frac{n'-3}{3})} \times \exp\left(-(\frac{\delta_c - \frac{\sigma^2}{2}}{2\sigma^2})^2\right)}$$

***power law***

***gaussian truncation at large scale***
$(R \to L_i \Rightarrow \sigma \to 0)$

$$\boxed{n = 3.8 \Rightarrow \frac{dN}{dM} \propto M^{-1.7}}$$

***Compatible with the observed CO clump distribution***

$$\boxed{\frac{dN}{dM} \propto \frac{1}{R^6}\frac{M}{R^3}^{-1-\frac{1}{2\sigma^2}\ln(M/R^3)} \times \exp(-\frac{\sigma^2}{8})}$$

Combination of *power law* and *lognormal* (dominant at small and large scales)

X $\boxed{M = R(1+\underline{\mathcal{M}_\star^2}R^{2\eta})}$

$$\mathcal{M}_\star = \frac{1}{\sqrt{3}}\frac{V_0}{C_s}\left(\frac{\lambda_J}{1\,\mathrm{pc}}\right)^\eta \quad (\mathcal{M}_\star \sim 1-2)$$

*(see Schmidt et al. 2010)*

$\mathcal{M}_\star$ = Mach number at the *Jeans scale* due to turbulent support
(transition thermal to turbulent support ($\mathcal{M}_\star R^\eta > 1$) around 1 $M_J$)

$$\mathcal{M}_\star \approx 0, \frac{dN}{d\log M} \propto M^{-3}$$

$$\mathcal{M}_\star \geq 1, \frac{dN}{d\log M} \propto M^{-\frac{n+1}{2n-4}}$$

For n=3.8   dN/dM ~ $M^{-2.33}$

$$\eta = \frac{n-3}{2}$$

$n \simeq 3.8 \Rightarrow \eta = 0.4$

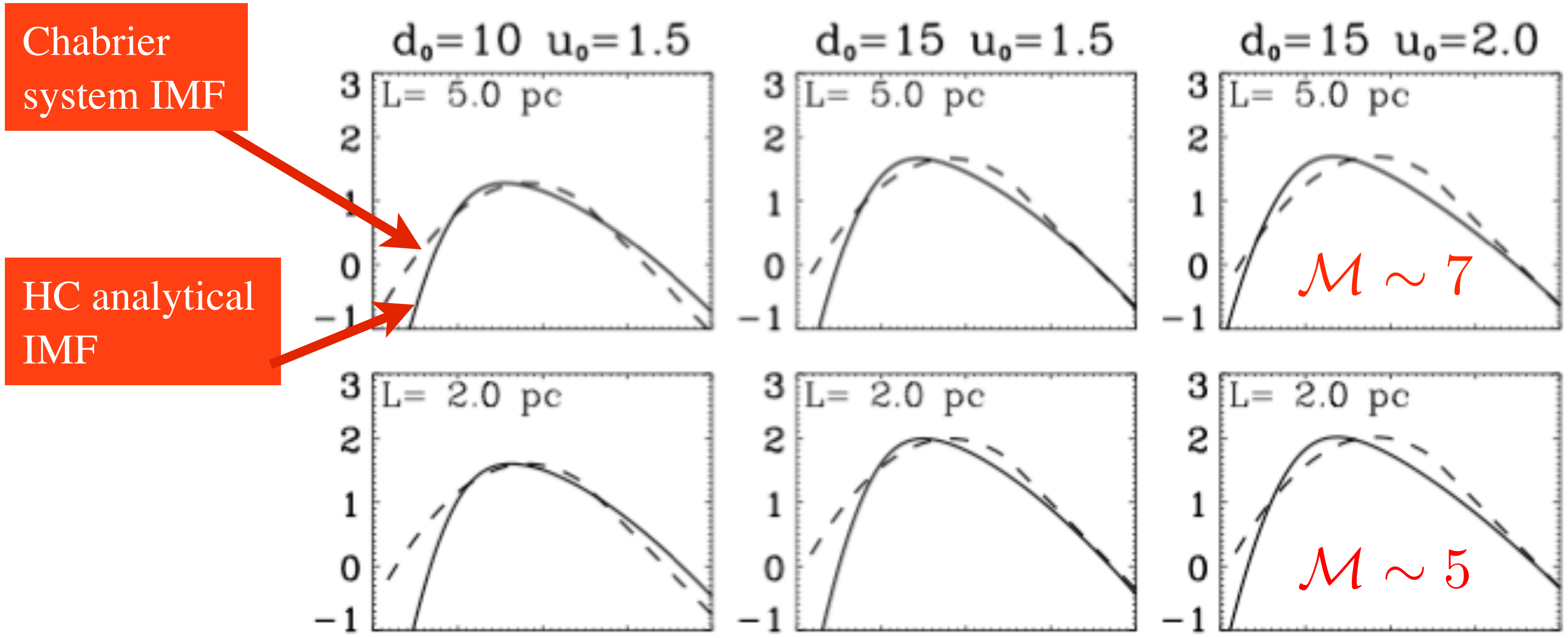
*Hennebelle & Chabrier '08; Chabrier & Hennebelle '11*

$$M \propto R^{2\eta+1} \sim R^{1.8}$$

Obs Herschel: (André et ak. '10, Konyves et al. '10):

$$M \propto R^{\beta} \quad \beta \sim 1 - 2$$

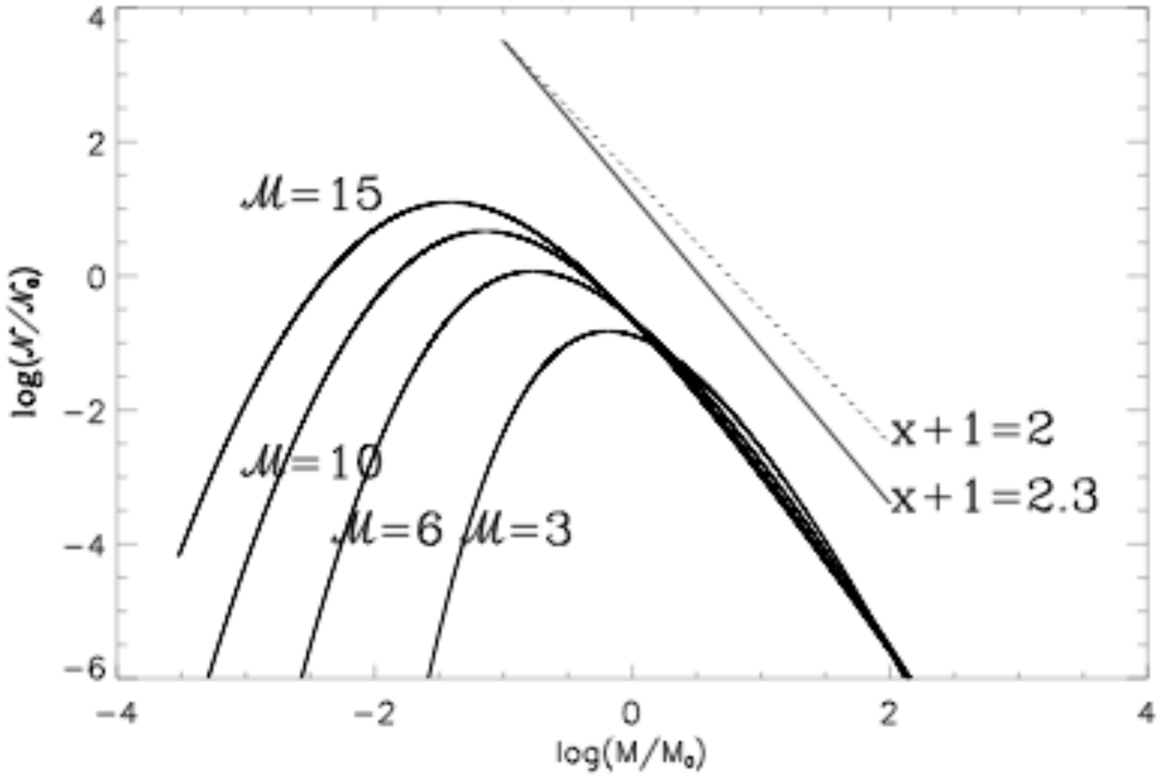
$$\bar{n} = (d_0 \times 10^3\ \mathrm{cm}^{-3}) \left(\frac{L}{1\mathrm{pc}}\right)^{-0.7}, \quad V_{\rm rms} = (u_0 \times 0.8\ \mathrm{km\ s}^{-1}) \left(\frac{L}{1\mathrm{pc}}\right)^{\eta}.$$




Hennebelle & Chabrier '09

I

## Influence of the global Mach number on the CMF




The larger the Mach number, the larger the number of small cores (brown dwarfs)

$$\sigma^2 = \mathrm{Ln}[1 + (b\mathcal{M})^2]$$

II

$$\frac{dn}{dM} \propto \left(\frac{M_R}{R^3}\right)^{-1-\frac{1}{2\sigma^2}\ln(M_R/R^3)}$$

$$\mathcal{M}_\star^2 \widetilde{R}^{2\eta} \gg 1 \ : \ \frac{dn}{dM} \propto M^{-\alpha}$$

$$\alpha = \alpha_1 + \alpha_2$$

$$\begin{cases} \alpha_1 = \frac{n+1}{n-2} \simeq \underline{2.66} \\ \alpha_2 = 3\frac{n-5}{(n-2)^2}\frac{\ln(\mathcal{M}_*)}{\sigma^2} \simeq \underline{-1.11\,(\ln \mathcal{M}_*)/\sigma^2} \end{cases}$$

MW : $\mathcal{M} \sim 3-8,\ \mathcal{M}_*^2 \sim 2 \Rightarrow \alpha_2 \simeq -0.15\ -\ -0.3$

$$\boxed{\bar{n} \gtrsim (4.0 \times 10^3)\left(\frac{T}{10\,\mathrm{K}}\right)\left(\frac{L_c}{10\,\mathrm{pc}}\right)^{-2}\left(\frac{\mathcal{M}}{10}\right)^{4.8}\ \mathrm{cm}^{-3}}$$

$$\boxed{\alpha_2 \ll \alpha_1 \Rightarrow \alpha \simeq \alpha_1 \simeq 2.7}$$

$$T = 60K, L_c = 100\mathrm{pc}, \mathcal{M} = 60, \rightarrow \bar{n} \gtrsim 10^6\,\mathrm{cm}^{-3}$$

Chabrier, Hennebelle & Charlot, ApJ 2014

III

# Characteristic mass (=peak) of the IMF

$$\boxed{M_{peak} = \frac{M_J}{1+(b\mathcal{M})^2} \propto \bar{\rho}^{-1/2}\mathcal{M}^{-2}}$$

$$\bar{\rho} \propto d_0 \times L^{-\eta_d} \qquad V_{rms} \propto V_0 \times L^{+\eta}$$

Jeans Mach

$$\eta_d \approx 0.7 - 1.0 \qquad \eta \approx 0.4$$

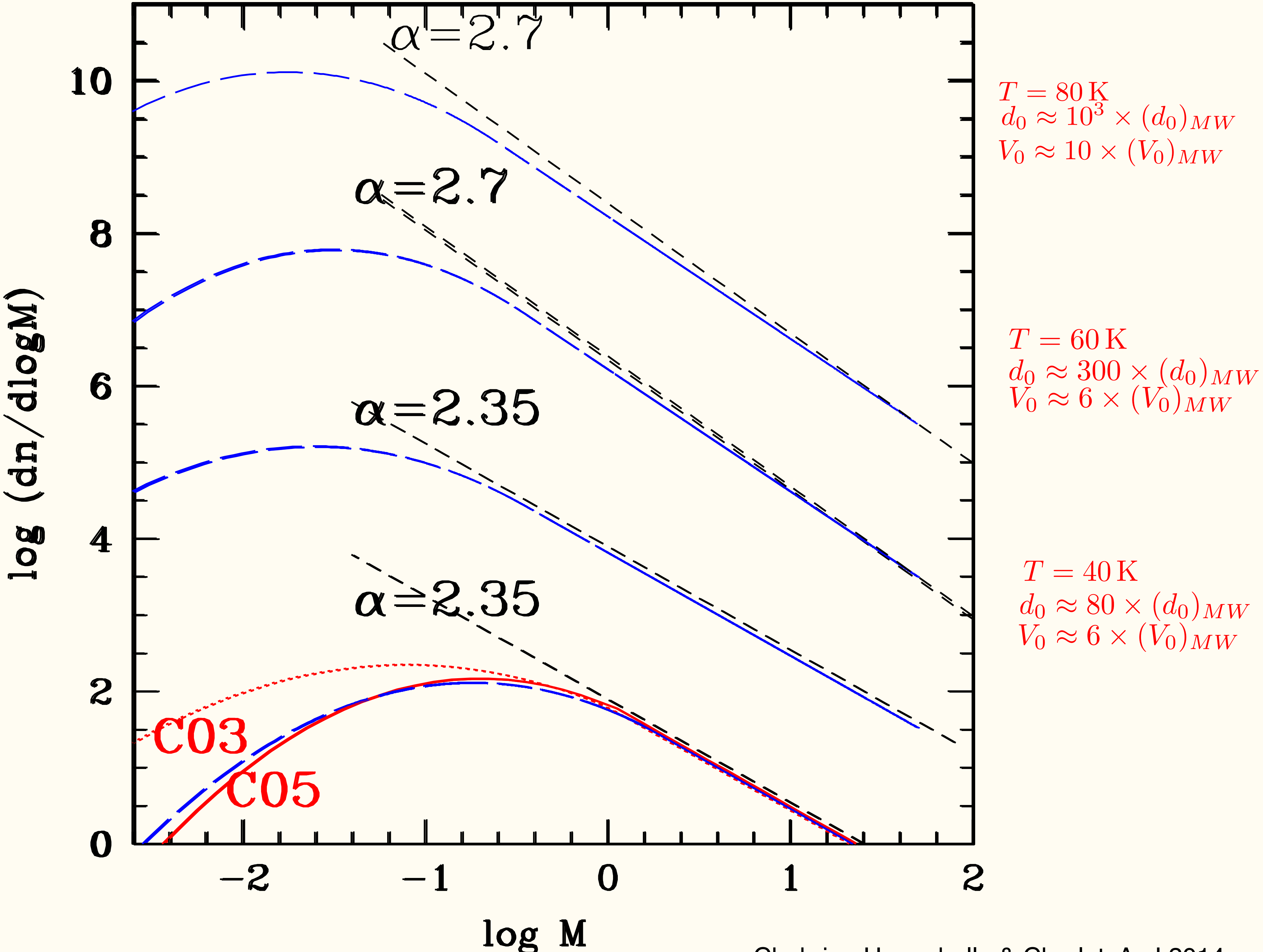
$$\boxed{\Rightarrow M_{peak} \propto [d_0^{-0.7} V_0^{-2}] \times \quad M_c^{-0.15} - M_c^{-0.20}}$$

**Weak variation of the IMF for given cloud characteristic conditions (gas mean density, large scale velocity dispersion) !**

$$d_0 \approx 10^3 \left(\frac{\Sigma_0}{10\, M_\odot \mathrm{pc}^{-2}}\right) \approx 1.8 \times 10^3 \left(\frac{P/k_B}{10^4\, \mathrm{K\, cm}^{-3}}\right)^{1/2} \mathrm{km\, s}^{-1}$$

$$V_0 \approx 0.82 \left(\frac{\Sigma_0}{10\, M_\odot \mathrm{pc}^{-2}}\right)^{1/2} \approx 1.1 \left(\frac{P/k_B}{10^4\, \mathrm{K\, cm}^{-3}}\right)^{1/4} \mathrm{km\, s}^{-1}$$

HC08, HC13, CHC14

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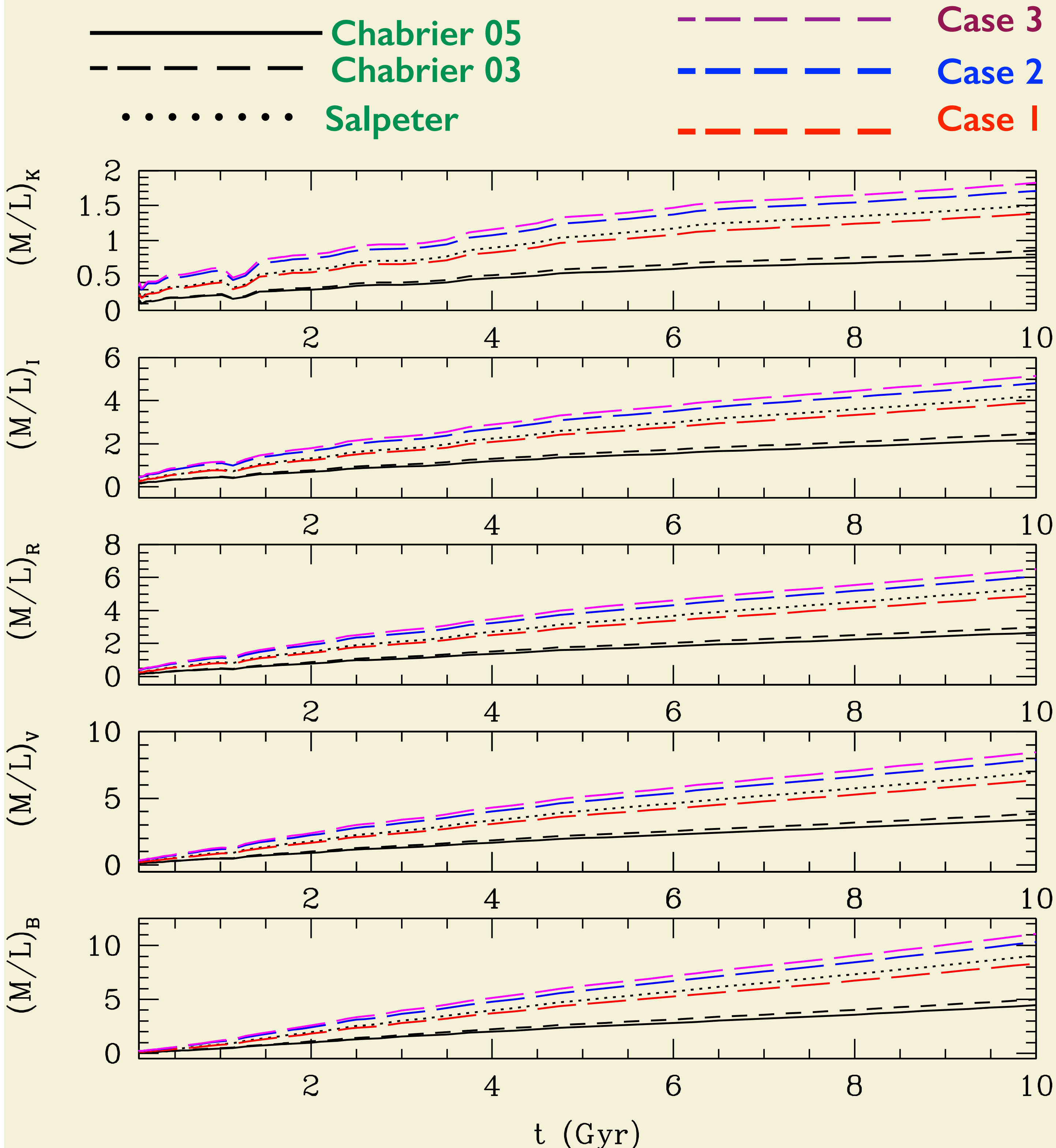
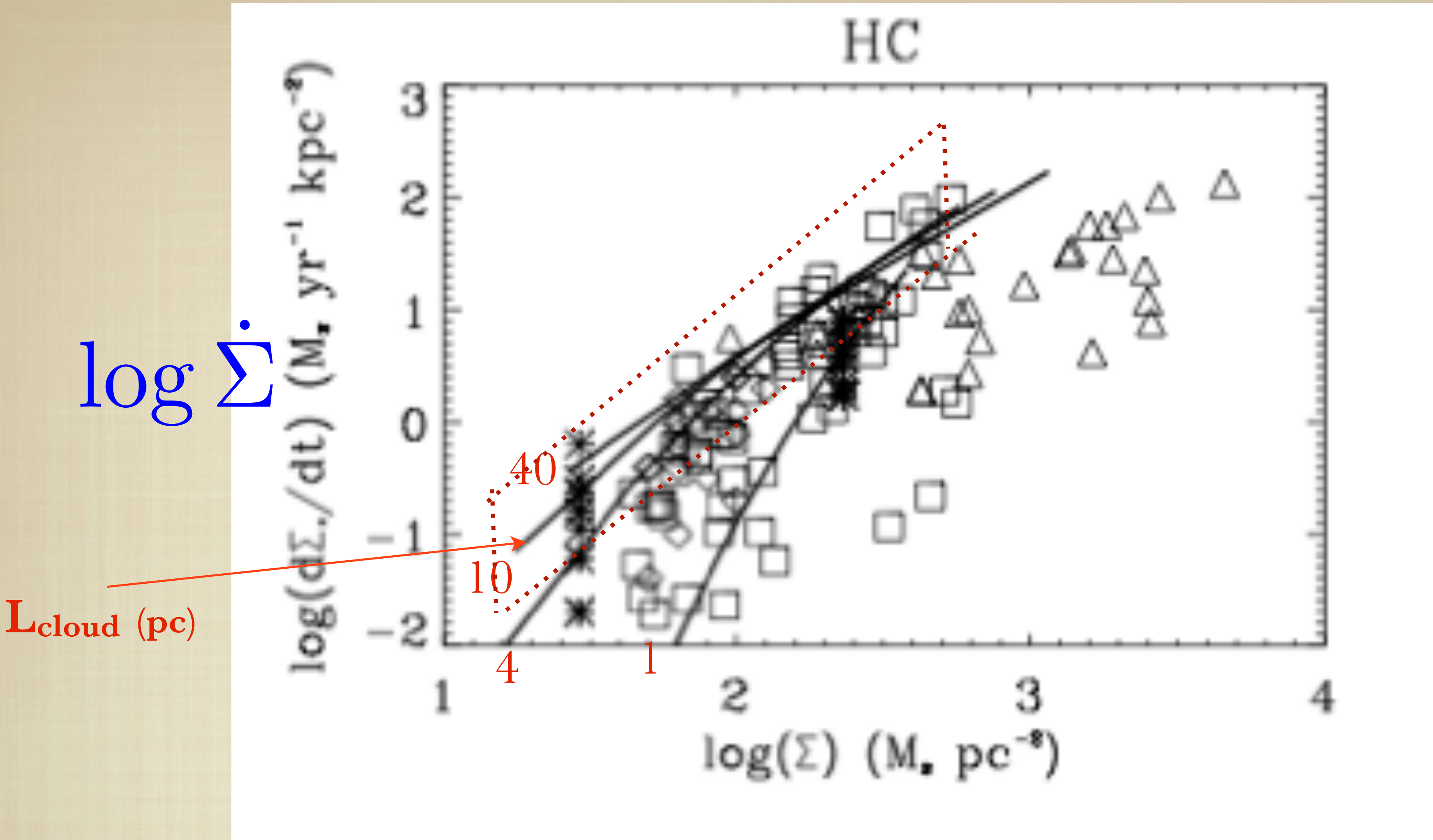



| | B | V | R | I | K | $\Upsilon/\Upsilon_{*,MW}$ |
|---|---|---|---|---|---|---|
| MW | 4.7 | 3.6 | 2.8 | 2.3 | 0.8 | |
| Salpeter | 9.1 | 6.9 | 5.4 | 4.2 | 1.5 | $\sim$ 1.9 |
| Case 1 | 8.3 | 6.4 | 4.9 | 3.9 | 1.4 | $\sim$ 1.7 |
| Case 2 | 10.3 | 7.9 | 6.1 | 4.8 | 1.7 | $\sim$ 2.1-2.2 |
| Case 3 | 11.1 | 8.5 | 6.5 | 5.1 | 1.8 | $\sim$ 2.3 |

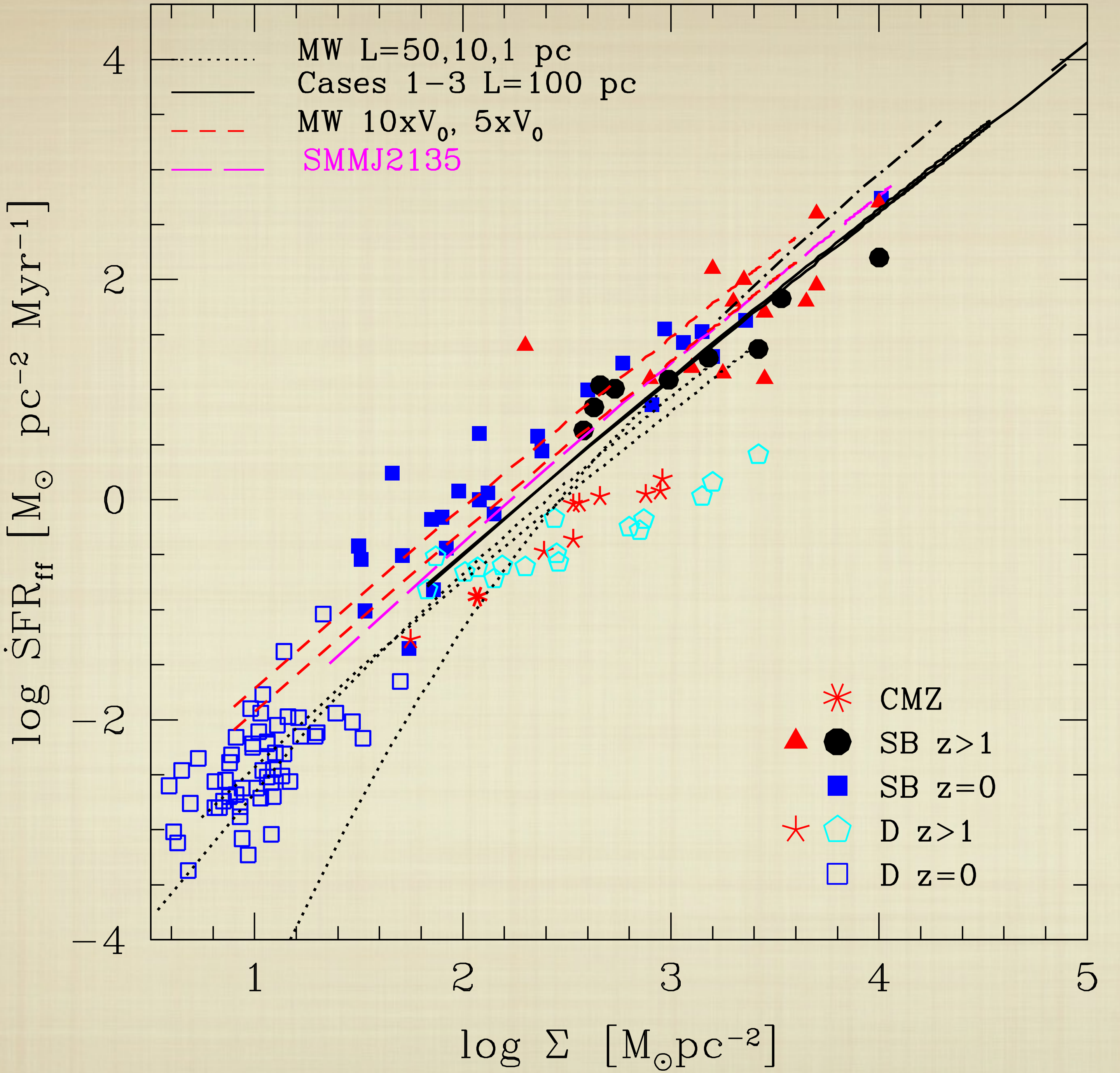



* Lada et al. ’10

□ Evans et al. ’09
+ Heiderman et al. ’10

Gutermuth et al. ’11

Hennebelle & Chabrier 2011, 2012




Chabrier, Hennebelle & Charlot, ApJ 2014

# Conclusion

- IMF well described by a gravo-turbulent picture of star formation (HC, Hopkins):
  - turbulence sets up the **initial field of density fluctuations**
  - gravity selects those **dense enough to collapse** in a **turbulent medium** (virial cond'n)

  « **universal process** » (only depends on turbulence $P_{\log\rho}$ and $P_V$)

- Entails a **power law** + a **lognormal** contributions

- Characteristics of the IMF (peak mass, width) DO **depend on the environment** (cloud, galaxy):

$$\mathbf{T,\ d_0, V_0 <\!\!-\!\!> \Sigma_0} \quad (\equiv \mathbf{P_{ext}}, \dot{M}_{acc})$$

- In **very dense** (compact$\Rightarrow$ **burst-like** star f'n) AND **turbulent** environments, the high-mass tail can reach

$$\boldsymbol{\alpha \sim 2.7}$$

- Adequately reproduces M/L + SFR