A physicist's approach to city modelling

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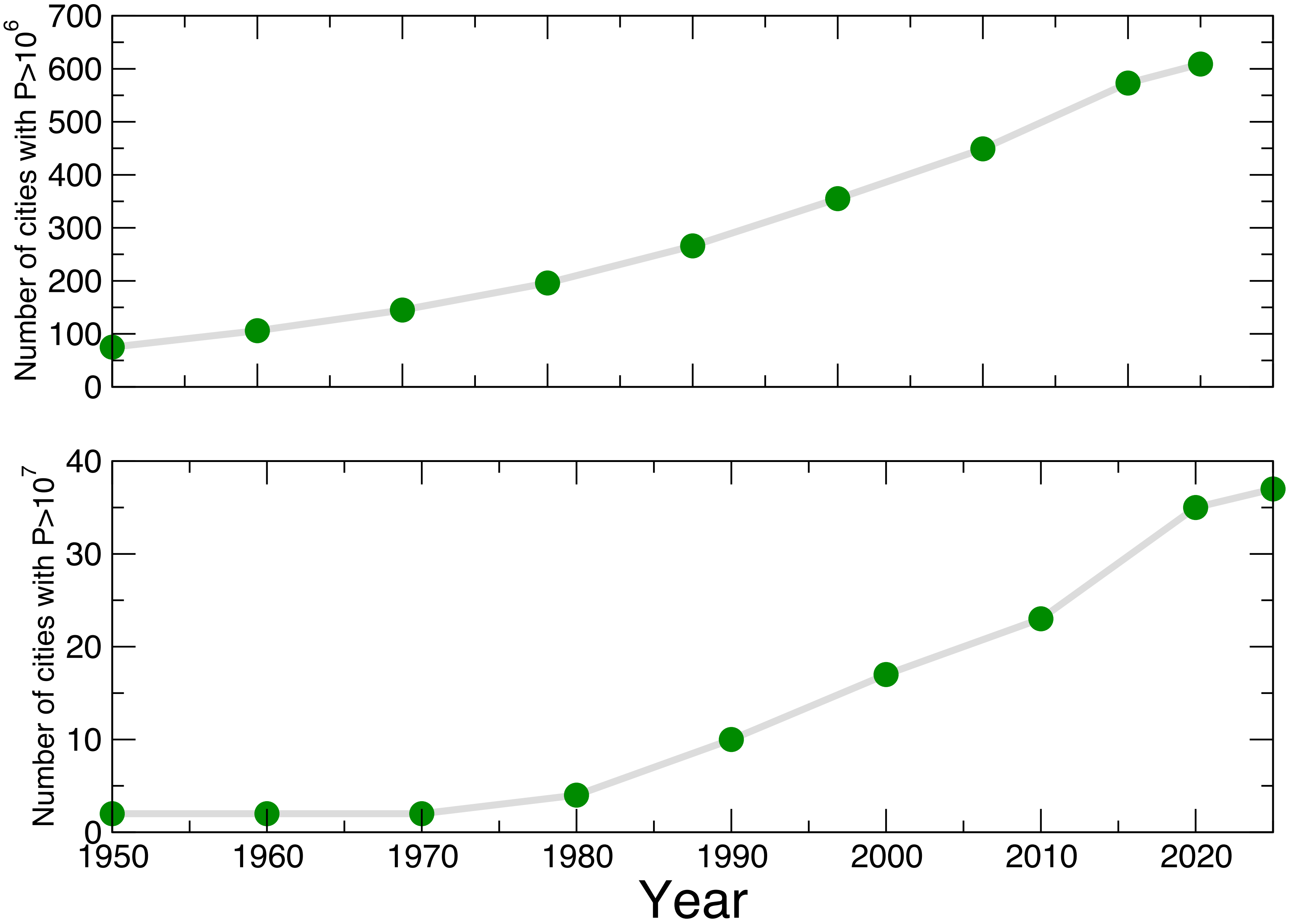
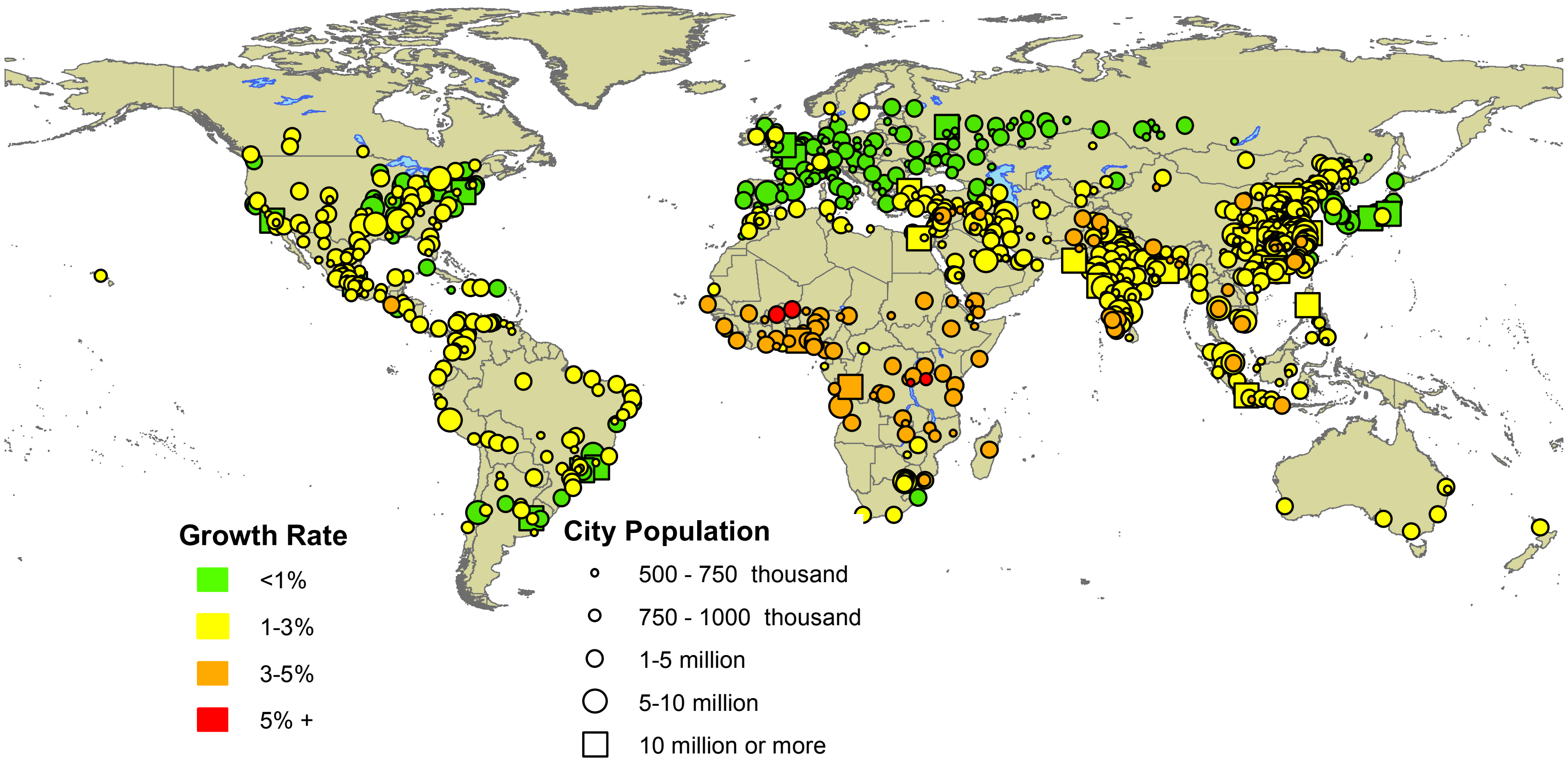
Importance of cities: megacities



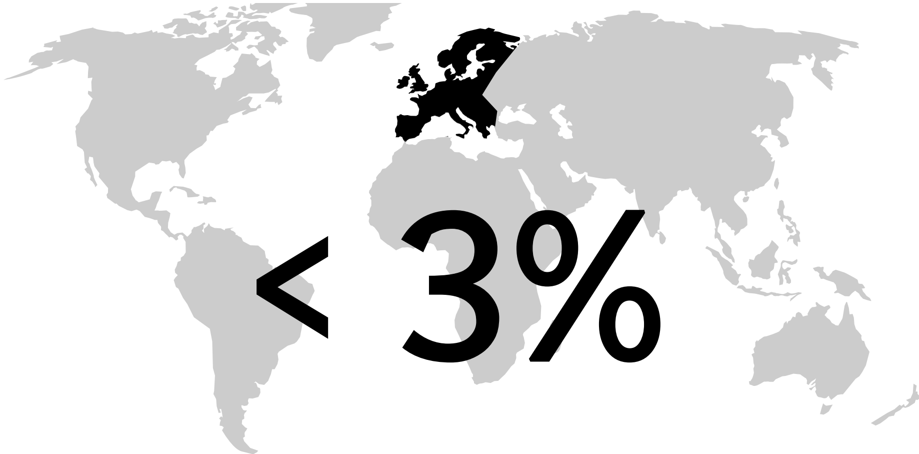
Data from www.geohive.com (UN data)

Heterogeneous distribution of growth rates



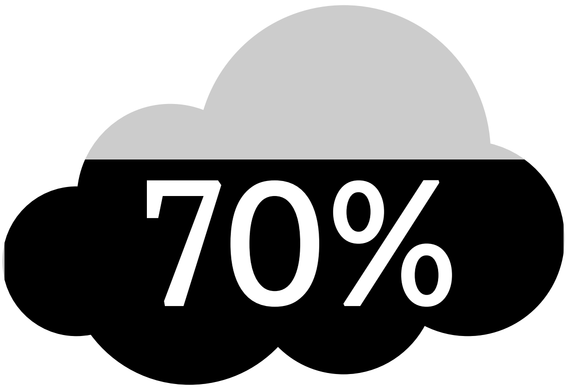
Cities are about concentration

Urbanized area



NASA 2001

CO2 emissions



ONU-HABITAT 2011

GDP

80%

ONU 2011

Urbanism: a lot of "theory" .. But many practical problems !

- Social and economical problems (spatial segregation, crime, accessibility, etc.)

- Mobility: congestion, pollution, …

- Sustainability of urban structures ?

- We need robust models, and a better understanding of the "physics" of cities
- Build a "science of cities" validated by data

Science and cities: state of the art

Number of parameters

Urban economics (and physics):
Very abstract models, empirical tests ?

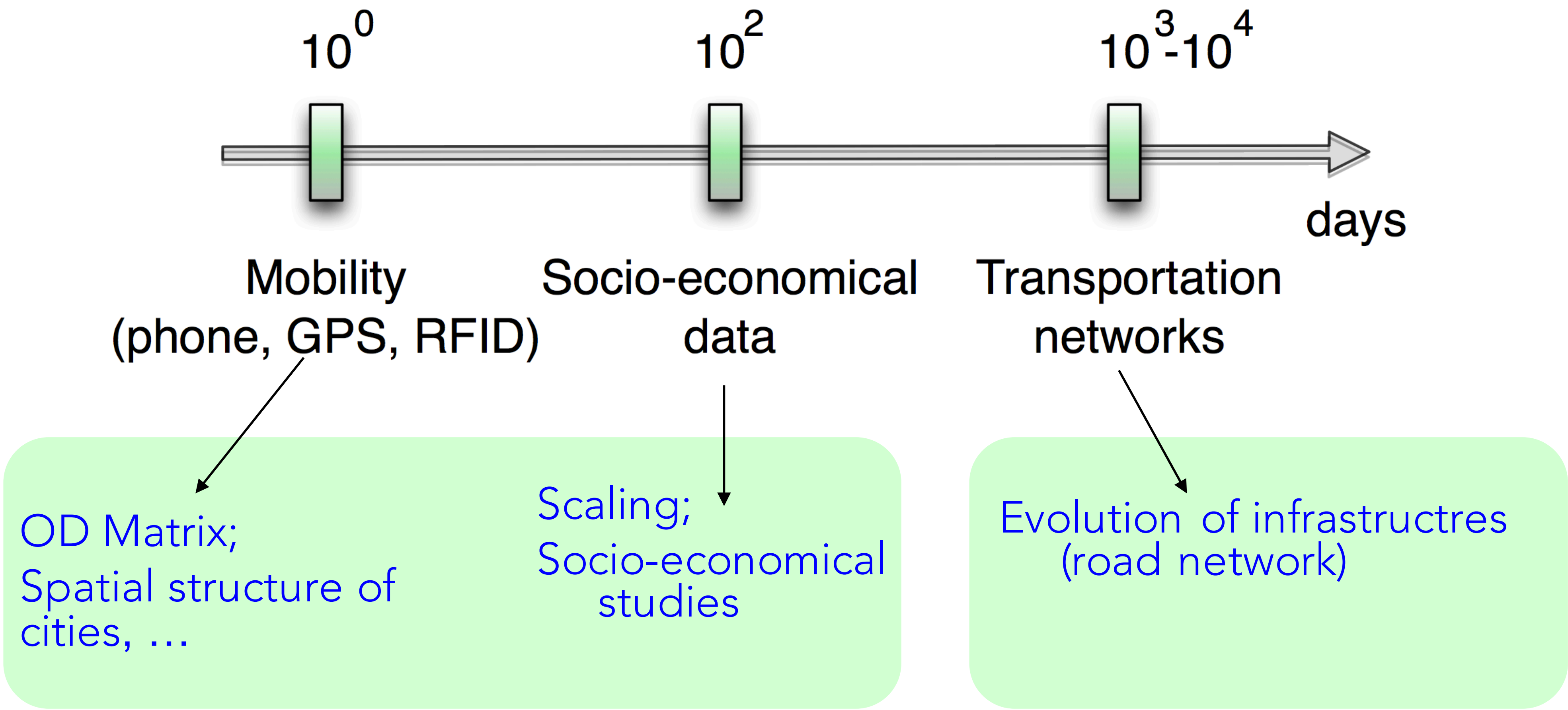
Simulations, agent-based models (LUTI models): Validity ? Strong perturbation ?
Machine learning ?

Minimal model: the smallest number of parameters and the largerst number of verified predictions

- Loop theory-observation necessary
- "Machine learning": black box, output difficult to interpret…

A new science of cities

- Game changer ? Urban data !
- Different scales (and different processes)

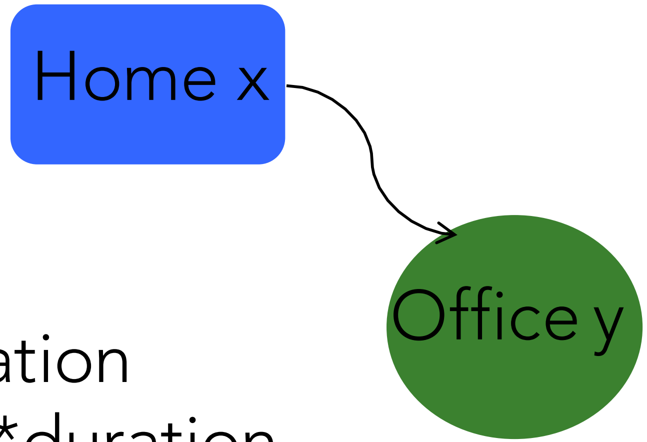


Revisiting Spatial economics: the Fujita-Ogawa model (1982)

- A model for the spatial structure of cities: an agent will choose to live in *x* and work in *y* such that

$$Z_0(x, y) = W(y) - C_R(x) - G_T(x, y)$$

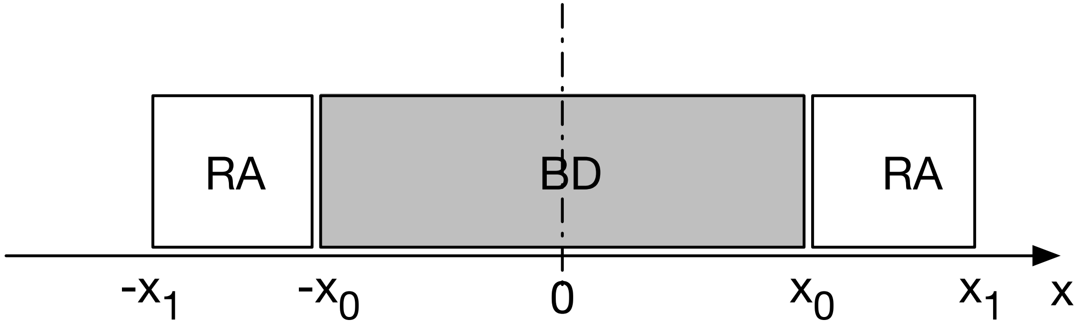
is maximum



- W(y) is the wage at y
- C_R(x) is the rent at x
- G_T(x,y) is the generalized transportation cost from x to y =monetary cost+V*duration

- …and a similar equation for companies (maximum profit)

Spatial economics: Fujita-Ogawa (1982)



- Main result: monocentric configuration stable if

$$\frac{t}{k} \leq \alpha$$

- t: transport cost
- 1/α interaction distance between firms

- Effect of congestion: larger cost t

Spatial economics: Fujita-Ogawa (1982)

- There are many problems with this model:
  - Not dynamical: optimization. We want an out-of-equilibrium model
  - No congestion (!) We want to include congestion (for car traffic). Only one transport mode – we want to include mode choice
  - No empirical test. Extract testable predictions (see the book: Spatial Economics, by Fujita, Krugman, Venables)

Spatial economics: Fujita-Ogawa (1982)

- This model is unable to predict the spatial structure of cities in general
- We will "simplify" the problem and discuss two phenomena:

  - (1) the evolution of car traffic with population
  - (2) the number of activity centers….(if time allows)

I. Modeling car traffic in cities

Car traffic: Newman-Kenworthy 1989

- Many problems:
  - Data availability ?
  - Reproductibility ?
  - Interpretation and use ?
  - Theoretical foundation ?

$$Q_{gas} \approx \frac{1}{\rho^{\sigma}} \quad ?$$

Newman & Kenworthy.
Gasoline consumption and cities: a comparison of US cities with a global survey.
J. Am. Plann. Assoc 1989

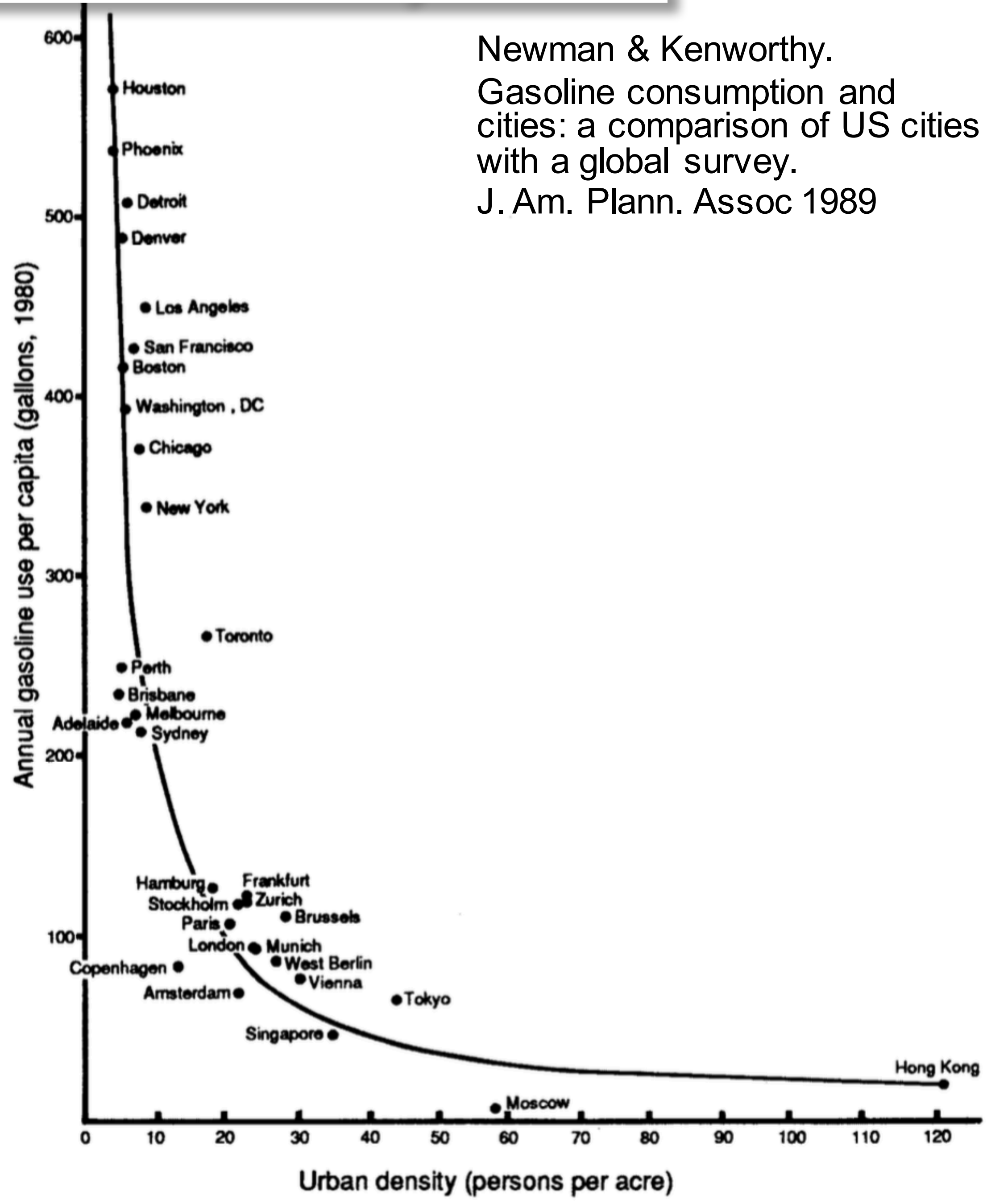


Figure 1. Gasoline use per capita versus population density (1980)

Modeling car traffic in cities

- Theoretical approach with testable predictions ?

- Ingredients:
  - Budget optimization: maximize (Fujita & Ogawa)

$$\max(Z_0 = \cancel{W(y)} - \cancel{C_R(x)} - G_T(x,y))$$

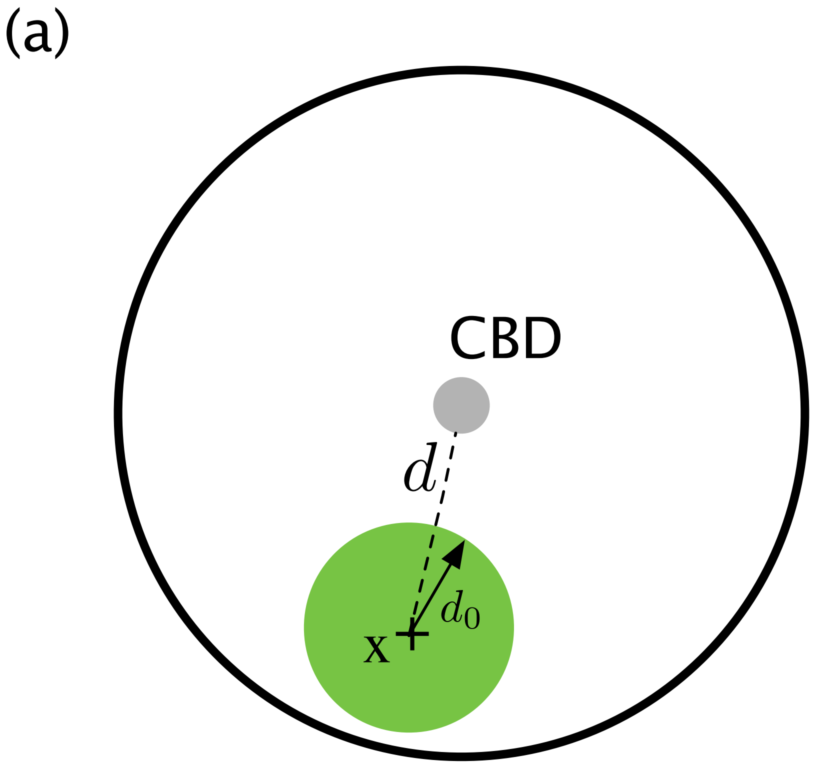
  - Individuals randomly located across the city
  - Monocentric case: same wage at the CBD

$$\max(Z_0) \Rightarrow \min G_T$$

  - Minimum computed over the different modes: mass rapid transit (subway) and private car

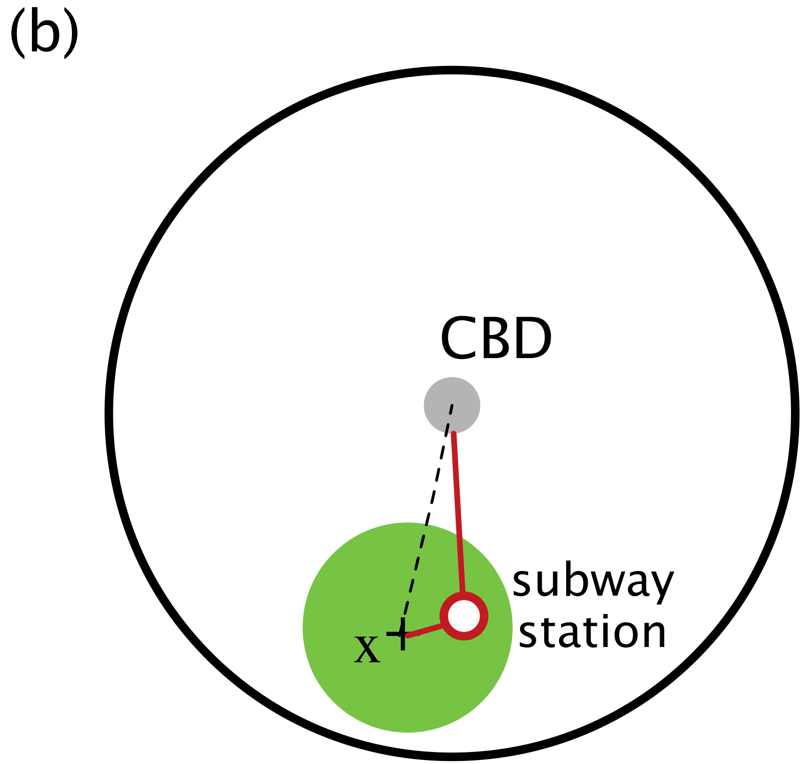
Modeling car traffic in cities

- Probability p to have access (<1km) to a MRT station



Probability 1–p:
no subway station

$$\text{Cost} = G_{car}(x)$$



Probability p:
subway station

$$\text{Cost} = \min(G_{car}(x), G_{MRT}(x))$$

Generalized cost

- Generalized costs: **monetary cost+V*time**

- C_c Monetary cost: price of the car, insurance, etc…

- V value of time=amount of money willing to pay in order to save one hour of time; increasing with income (typically a fraction of the income).

- Large V => time is the most important
- Small V => money is the most important

Generalized cost: car

- The time to go from x to y separated by d(x,y) is (Bureau of Public Roads function)

$$\tau(x,y) = \frac{d(x,y)}{\overline{v}}\left[1+\left(\frac{T(x,y)}{C}\right)^{\mu}\right]$$

where:

$\overline{v}$ is the average free flow velocity

T(x,y) is the traffic

C is the capacity of the road system

μ is an exponent >1 characterizing the sensitivity to congestion

- With congestion time=d/v is not valid anymore !

Generalized cost: car

- The time for a trip of length d, on road of capacity C and free flow velocity v_c, and with traffic T is then

$$\tau(d) = \frac{d}{v_c}\left[1 + \left(\frac{T}{C}\right)^{\mu}\right]$$

- The generalized cost for the car is then

$$G_{car} = C_c + V\frac{d}{v_c}\left(1 + T^{\mu}\right)$$

Generalized cost: MRT (subway)

- We neglect monetary costs (compared to C_c)
- V value of time, for a distance d, trip duration

$$\tau(d) = f + \frac{d}{v_m}$$

- Driving is faster: $v_c > v_m$ (typically 40 vs 30km/h) but more expensive
- f: walking+waiting time

- Generalized cost for the MRT

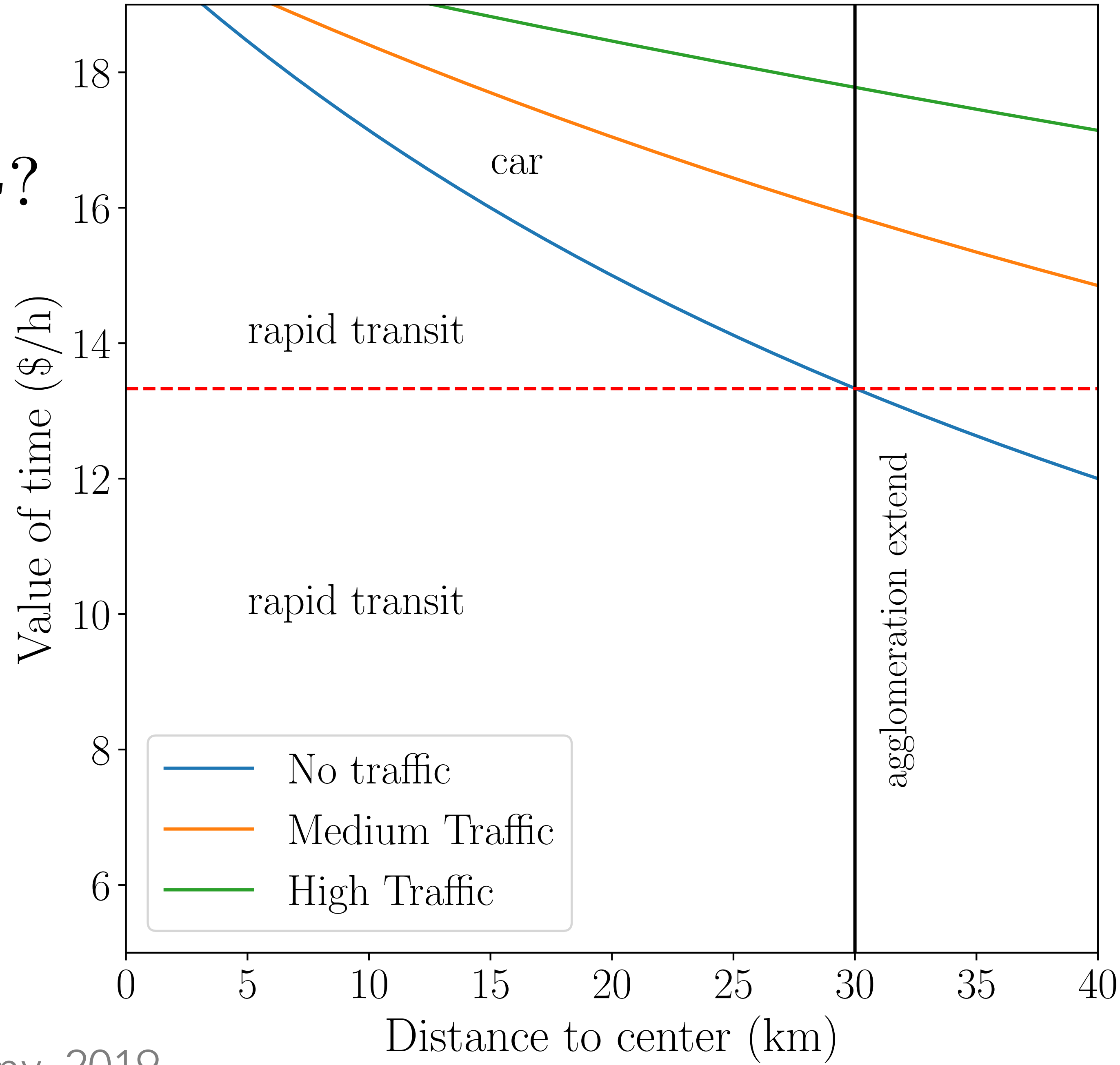
$$G_{MRT} = V\left(f + \frac{d}{v_m}\right)$$

Modeling car traffic in cities

- Mode choice:

$$G_{car} > G_{MRT}?$$

- Choice of mode depends on V



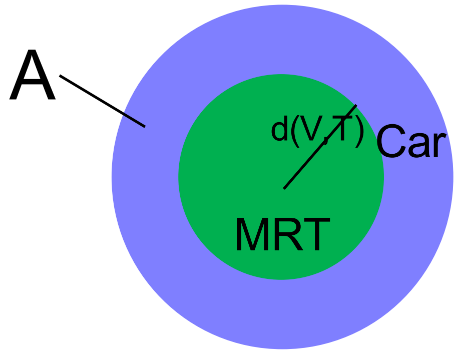
Verbavatz & Barthelemy, 2019

Critical distance and traffic

- Writing $G_{MRT}=G_{car}$ gives a critical distance ($L \sim A^{1/2}$, A area of the city)

$$d(V,T) = \min\left(L, \frac{\frac{C_c}{V} - f}{\frac{1}{v_m} - \frac{1}{v_c}}\left(1 + \left(\frac{T}{C}\right)^{\mu}\right)\right)$$

- If d<d(V,T) => MRT
- If d>d(V,T) => car



- Writing d(V,T*)=L gives the critical maximum traffic T* above which MRT is beneficial in the whole city

$$T^* = CF\left(v_m, v_c, \frac{1}{L}\left(\frac{C_c}{V} - f\right)\right)$$

Evolution equation for the car traffic T

- Population P increases
- Probability p to have access (<1km) to a MRT station

- For T<T*:

$$\frac{\mathrm{d}T}{\mathrm{d}P} = 1 - p + p\left(1 - \frac{\pi d(V,T)^2}{A}\right)$$

No access to MRT

Access to MRT and in the « car regime »

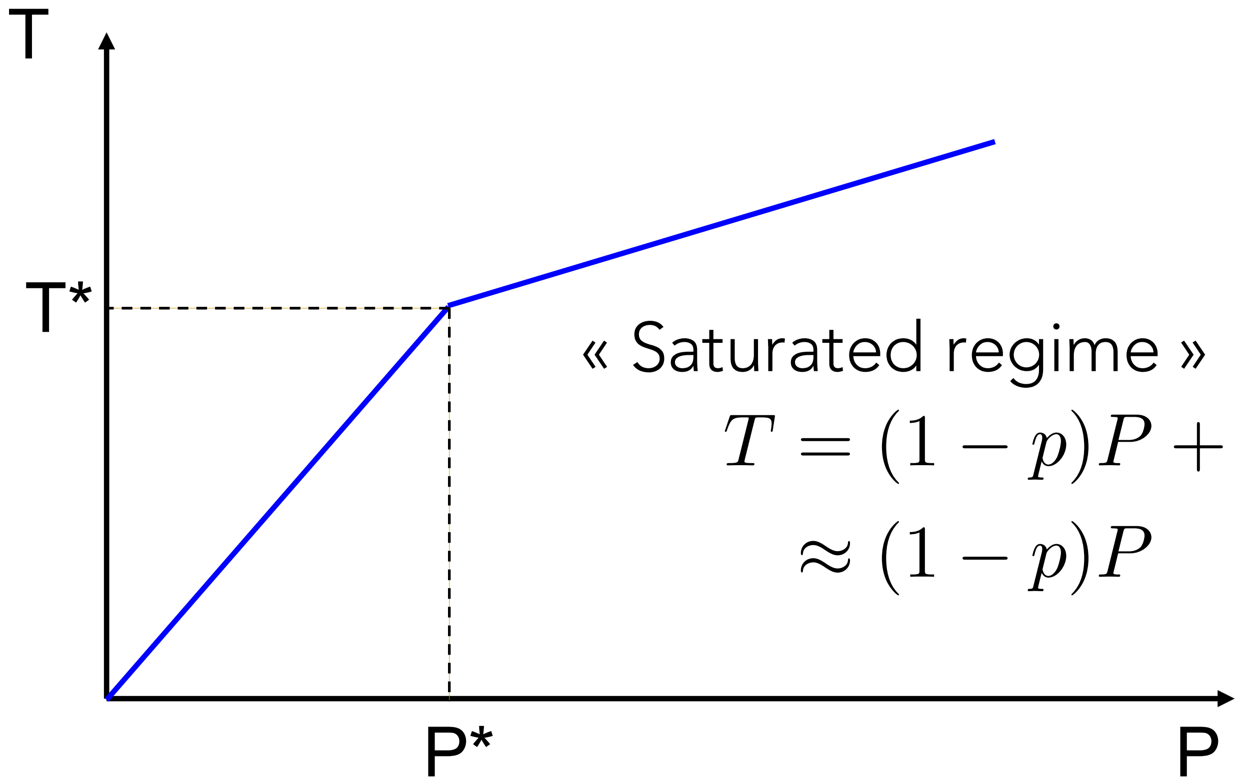
- For T>T* and P>P*:

$$\frac{\mathrm{d}T}{\mathrm{d}P} = 1 - p$$

Verbavatz & Barthelemy, 2019

Modeling car traffic in cities

- Evolution equation for the traffic



« Saturated regime »

$$T = (1-p)P + \text{const}$$
$$\approx (1-p)P$$

Results of the model

- For P>P* the only source of car traffic comes from individuals who do not have access to MRT

$$T \approx (1-p)P$$

where p is the proba to have access to MRT

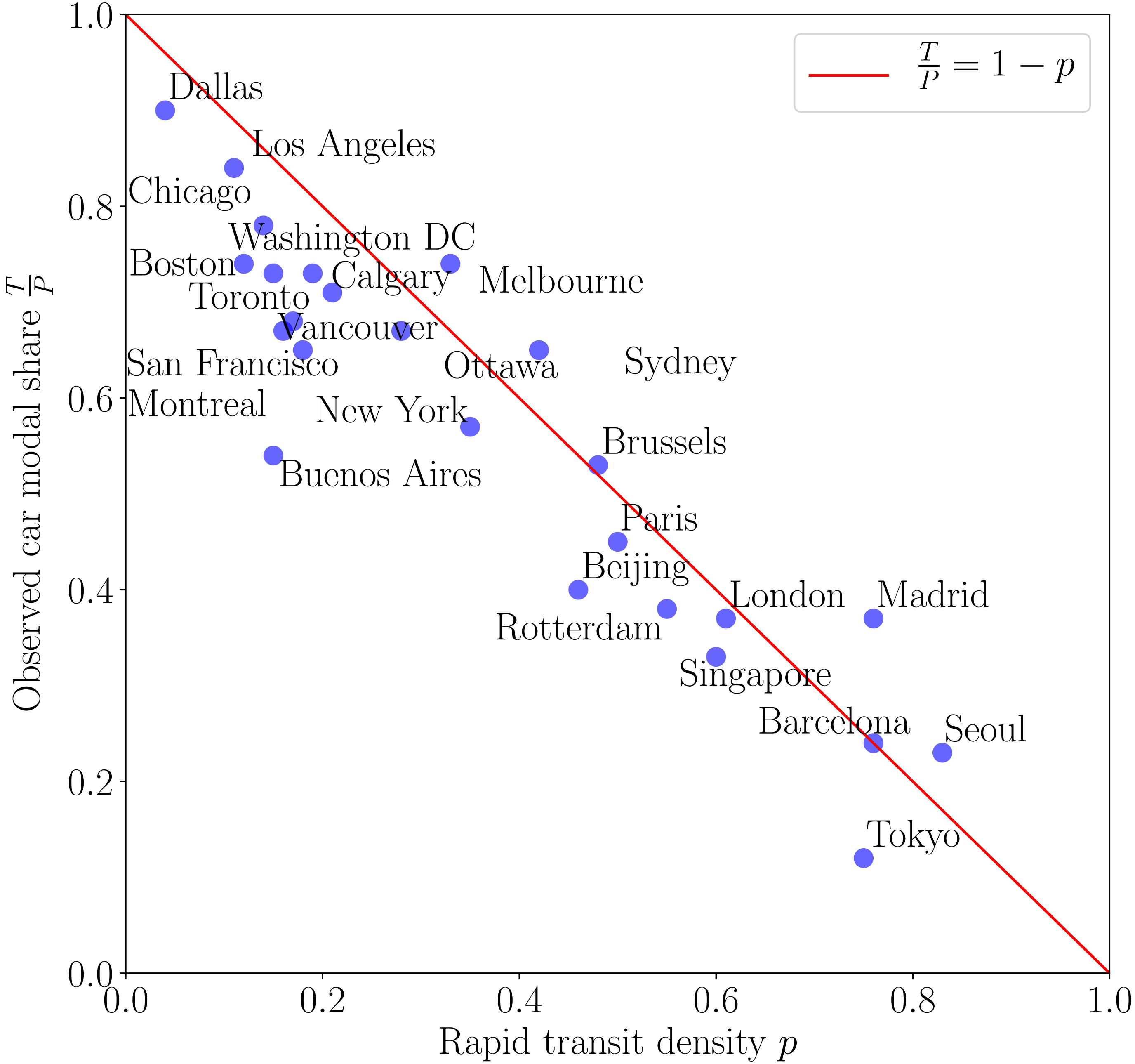
- P* depends on the details of the city and individuals, usually small).

- For most large cities, the traffic is « saturated »: the only car drivers do not have an alternative

- We got the data for 25 cities in the world (bottleneck: p)

Verbavatz & Barthelemy, 2019

Modeling car traffic in cities



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CO_2 emissions

- CO_2 emissions proportional to the time spent driving

$$Q_{CO_2} \propto \sum_{drivers\ i} d(x_i) \left[1 + \left(\frac{T}{C}\right)^{\mu}\right]$$
$$\propto g\sqrt{A}(1-p)P[1+\tau]$$

where τ the average delay due to traffic jams (data available:TomTom)

Marseille 40%

- We then obtain

$$\boxed{Q_{CO_2}/P \approx (1-p)\sqrt{A}(1+\tau)}$$

Public transport density

Size of city

Effect of congestion

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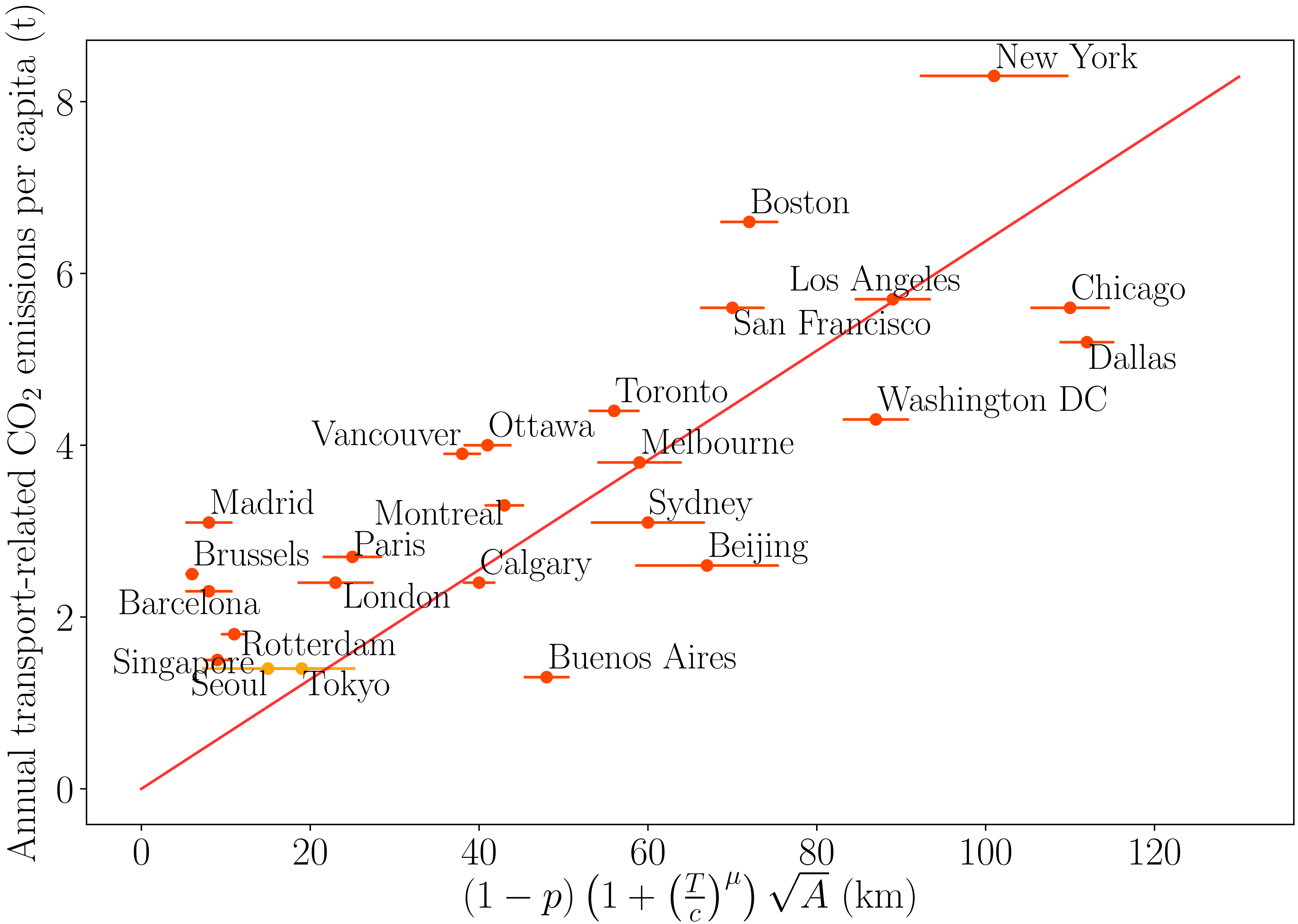
CO_2 emissions in cities



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Modelling car traffic in cities

- We assume $Q_{gas} \propto Q_{CO_2}$
- From $Q_{gas}/P \approx (1-p)\sqrt{A}(1+\tau)$ we obtain

$$Q_{gas}/P \approx \frac{\sqrt{P}}{\sqrt{\rho}} \approx \frac{1}{\sqrt{\rho}}$$

where ρ=P/A is the average urban density

- We « understand » here the result of Newman and Kenworthy

Verbavatz & Barthelemy, 2019

Discussion

- The model predicts that it is not the density that controls gasoline consumption (and CO_2 emissions due to transport) but:
  - the density of public transport
  - car congestion
  - the area size of the city

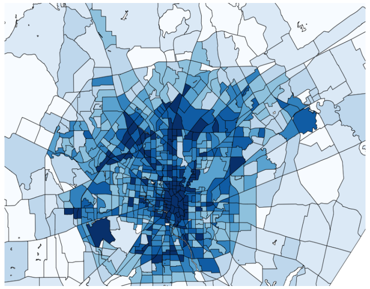
- In general increasing the density in order to decrease CO_2 emissions is.. wrong !
  - If P increases (at A fixed)=>Q_{CO2} increases (congestion)
  - We have to decrease A or more realistically increase p or density at MRT stations

This simple model helped us to point to the relevant parameters…

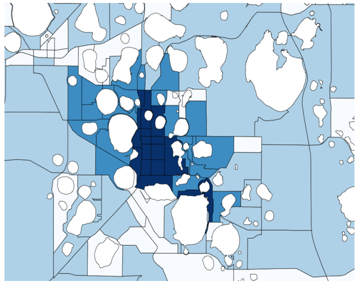
Verbavatz & Barthelemy, 2019

II. Polycentric structure of cities

Polycentric structure: empirical result

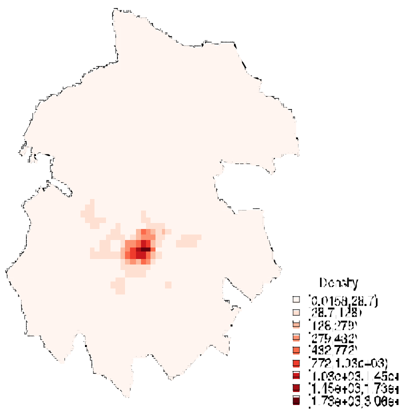


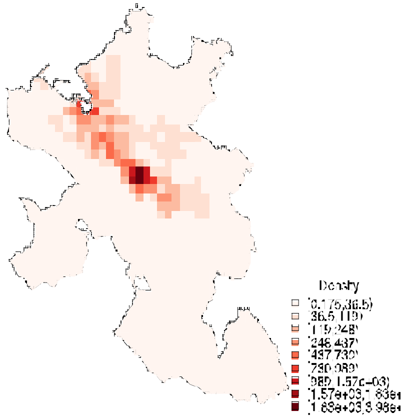
San Antonio (TX), USA



Winter Haven (FL), USA

# of employees per zip code, USA (9,000 cities)





Mobile phone data: density of users (urban areas in spain)

Number H of activity centers

$$H \sim P^{\beta}$$

- Exponent **β**~0.5-0.6

The spatial structure of activity in cities

- We have a polycentric structure, evolving with P
- We can count the number H of centers

$$H \sim P^{\beta} \quad \beta \approx 0.5 - 0.6$$

Computing β ?

- Mobility is the key: we need to model how individuals choose their home and work place
- Problem largely studied in geography, and in spatial economics: Edge City model (Krugman 1996), Fujita-Ogawa model (1982)
- Revisiting Fujita-Ogawa: predicting the value of β

Spatial economics: the edge city model (Krugman 1996)

- The important ingredient is the 'market potential'

$$\Pi(x) = \int K(x-z)\rho(z)\mathrm{d}z$$

- Describes the spillovers due to the density in z
- Specifically

$$K(u) = A(u) - B(u)$$

- The average market potential is

$$\overline{\Pi} = \frac{1}{\Omega}\int \Pi(x)\rho(x)\mathrm{d}x$$

Spatial economics: the edge city model (Krugman 1996)

- The equation for the evolution of business density is

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = \gamma \left(\Pi(x) - \overline{\Pi}\right)$$

- Linearize around flat situation $\rho(x) = \rho_0 + \delta\rho(x)$

$$\delta\rho(k) \sim \mathrm{e}^{\gamma K(k)t}$$

- At least one maximum at k=k*; the number of hotspots is then:

$$H \sim \Omega k_*^2$$

- Scaling with the population ?
- Link micromotives-macrobehavior ?

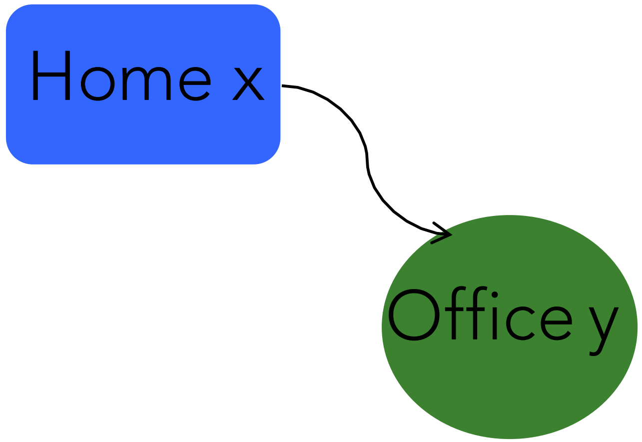
Spatial economics: Fujita-Ogawa (1982)

- A model for the spatial structure of cities: an agent will choose to live in *x* and work in *y* such that

$$Z_0(x,y) = W(y) - \cancel{C_R}(x) - C_T(x,y)$$

is maximum

- $W(y)$ is the wage at y
- $C_R(x)$ is the rent at x
- $C_T(x,y)$ is the transportation cost



- Assumptions and simplifications:
  - Assume that home is uniformly distributed (x): find a job !
  - We have now to discuss W(y) and C_T

Estimating the wage

- The wage results from a large number of interactions: complex quantity !

- In physics (heavy ions) replacing a complex quantity by a random variable is useful and sometimes accurate (Wigner 55) ! (=> theory of random matrices)

- We then choose:

$$W(y) = s\eta(y)$$

where s sets the salary scale and η is a random variable

- Note: the disorder is quenched here

Louf, MB, PRL 2013

Generalized cost: car

- The time to go from x to y separated by d(x,y) is (Bureau of Public Roads function)

$$\tau(x,y) = \frac{d(x,y)}{\overline{v}}\left[1 + \left(\frac{T(x,y)}{C}\right)^{\mu}\right]$$

where:

$\overline{v}$ is the average free flow velocity
T(x,y) is the traffic
C is the capacity of the road system
μ is an exponent >1 characterizing the sensitivity to congestion

- We write

$$C_T(x,y) \propto \tau(x,y)$$

Summary: the model

- Every time step, add a new individual at a random x

- The individual will choose to work in *y* (among N_c possible centers) such that

$$Z(x,y) = \eta(y) - \frac{d(x,y)}{\ell}\left[1 + \left(\frac{T(x,y)}{C}\right)^{\mu}\right]$$

is maximum

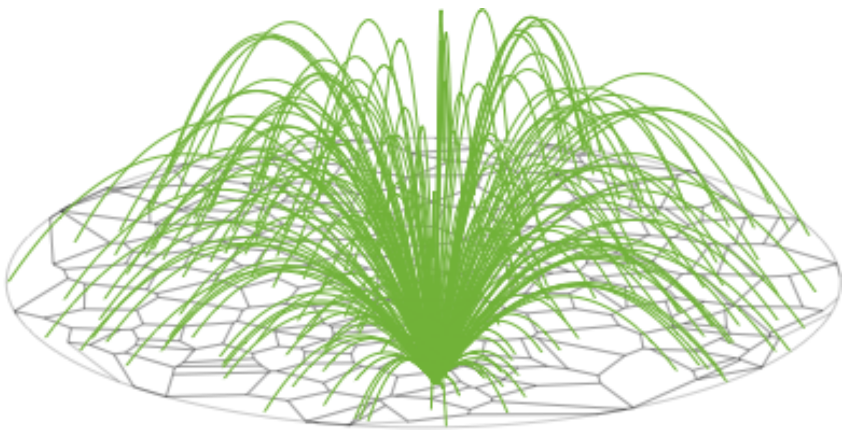
- $\eta(y)$ is the wage at y --> random

- $C_T(x,y)$ is the transportation cost from x to y: depends on the traffic from x to y --> congestion effects

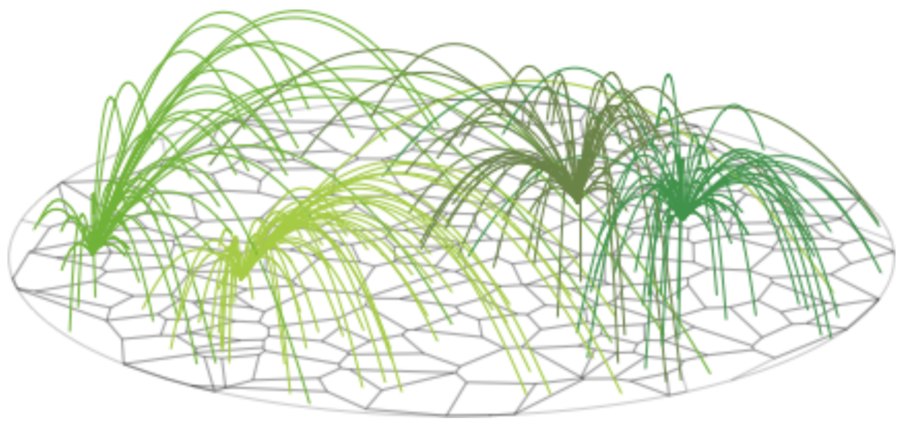
Louf, MB, PRL 2013

Results

- Depending on the values of parameters, we see two types of mobility patterns: Monocentric vs. polycentric



Monocentric



Polycentric

Monocentric-polycentric transition

- Start with one center $\eta_1 > \eta_j$
- All other subcenters have a zero traffic T(j)=0
- The number of individuals P increases, T(1) increases and at a certain point there is another j such that:

$$Z(i,j) > Z(i,1)$$

Or:

$$\eta_j - \frac{d_{ij}}{\ell} > \eta_1 - \frac{d_{i1}}{\ell}\left[1 + \left(\frac{P}{c}\right)^{\mu}\right]$$

Monocentric-polycentric transition

- Critical value for the population: effect of congestion !

$$P > P^* = C \left(\frac{\ell}{\sqrt{A} N_c} \right)^{1/\mu}$$

- C: capacity of the road system sets the scale

- If ℓ is too small, $P^* < 1$ and the monocentric regime is never stable

Results: scaling for the number of centers

- We obtain the average population for which a kth subcenter appears is:

$$\overline{P}_k = P^*(k-1)^{\frac{\mu+1}{\mu}}$$

which implies:

$$\boxed{H \sim \left(\frac{P}{P*}\right)^{\frac{\mu}{\mu+1}}}$$

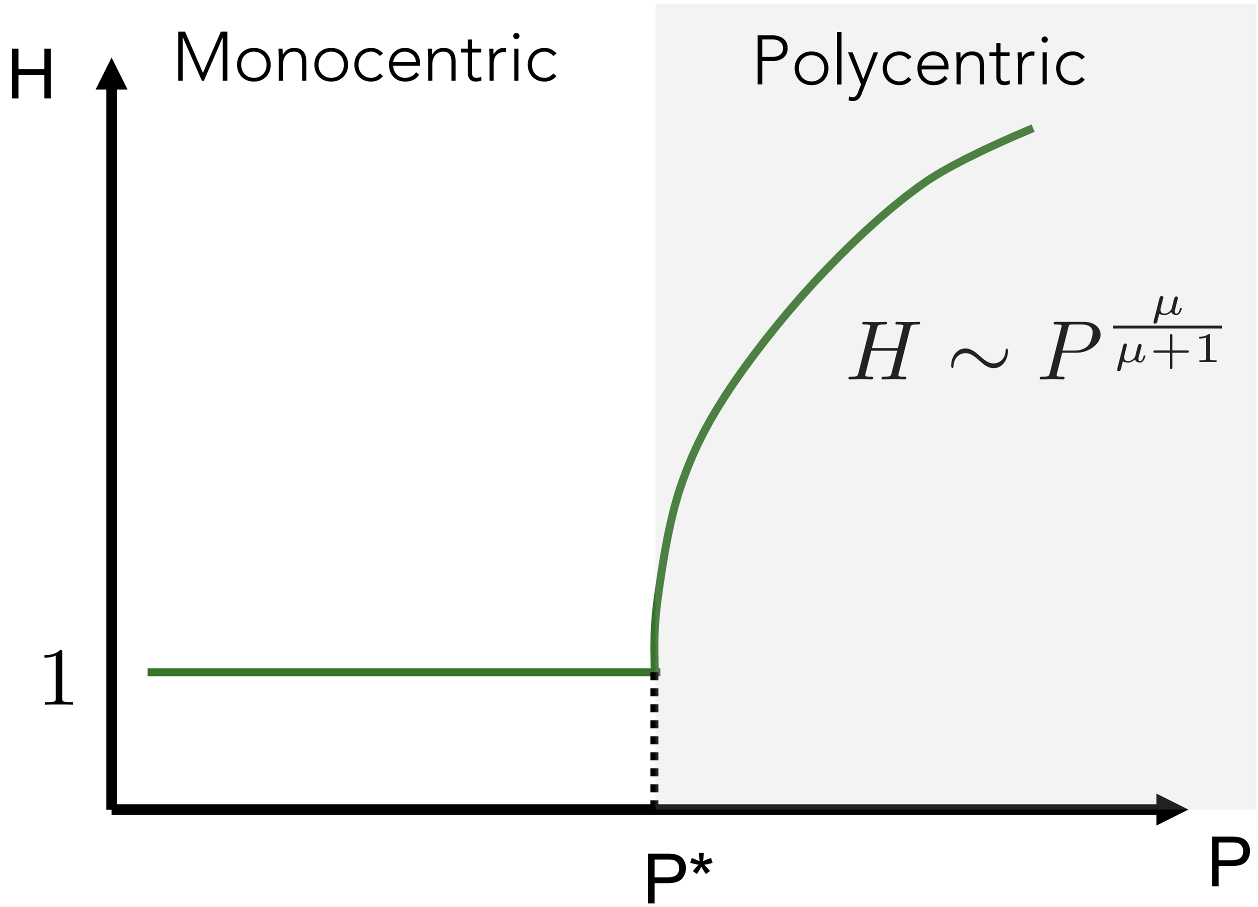
Sublinear relation !

- From US employment data (9000 cities)

$$H \sim P^{0.64} \quad (\Rightarrow \mu \simeq 2)$$

'Urban transition: Phase diagram'

Number of hotspots H versus population P

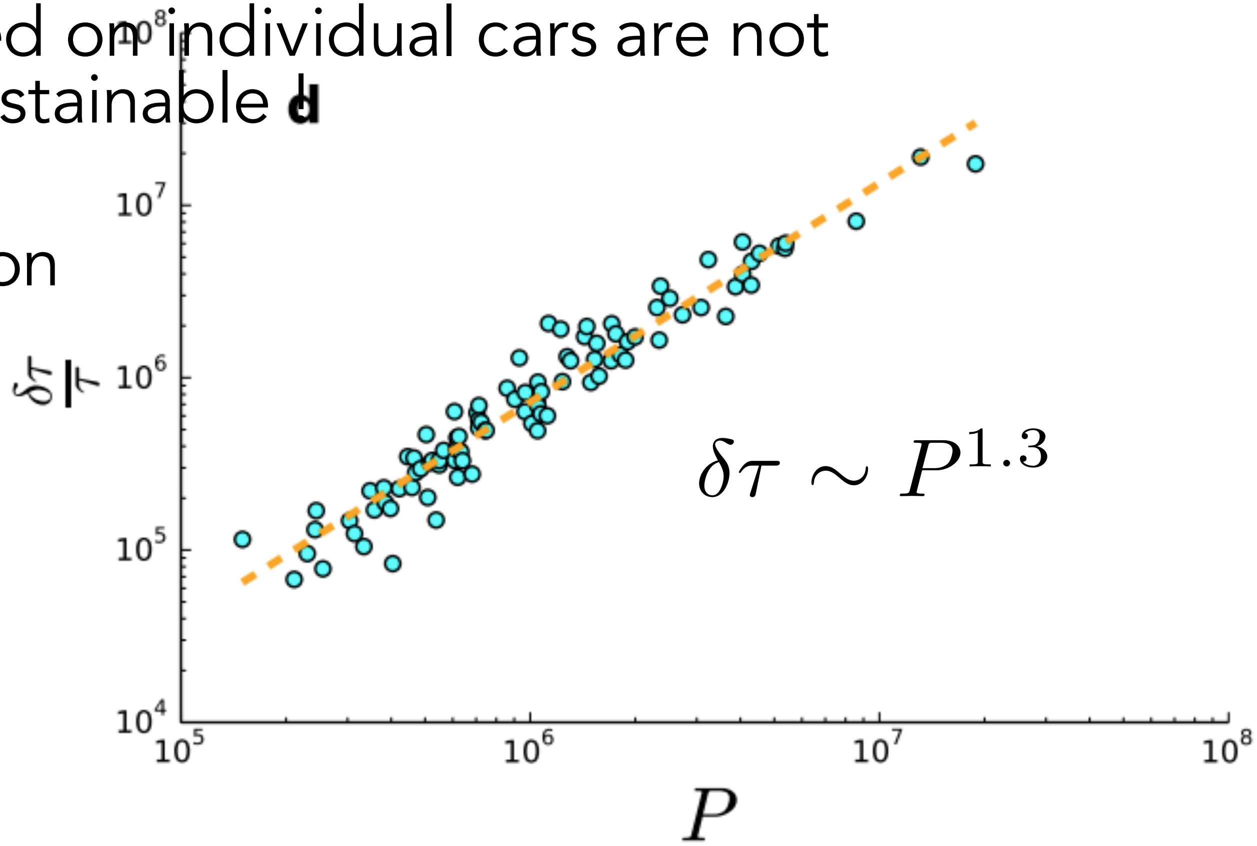


Important cause of polycentricity: traffic congestion

Discussion

- We observe a scaling of H with P which can be explained by a simple model integrating congestion
- Polycentrism is the natural response of cities to congestion, but not enough !
- For large P: Effect of congestion becomes very large
  => large cities based on individual cars are not
  economically sustainable !

Delay due to congestion
(US cities)



Louf, MB (2013, 2014)

Perspectives

- Science needs data ! Availability of data is critical for Science and also for improving our societies

- Parsimonious models translate for us the information hidden in large datasets, and provide guides to explore data, phenomena and to identify critical factors

- Future: modeling of complex systems…
  - Machine learning is useful for practical applications but do not improve so far (!) our knowledge
  - Mathematical modeling assisted by artificial intelligence ?
  - Or is this the end of mathematical modeling ?

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Mathematicians, computer scientists (27%)
Geographers, urbanists, GIS experts, historian (27%)
Economists (13%)
Physicists (33%)

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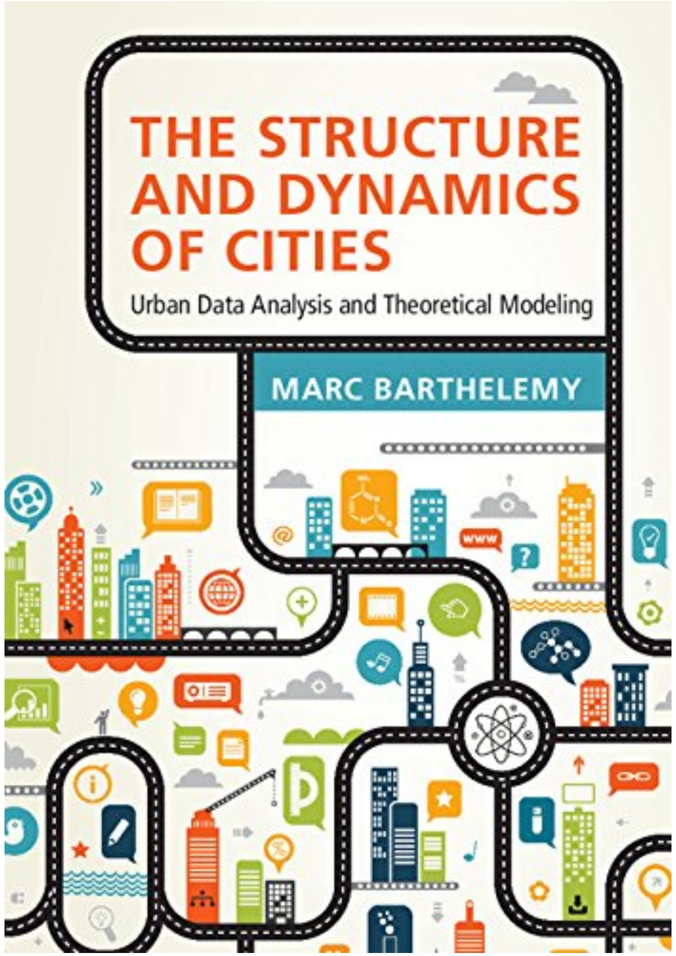

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