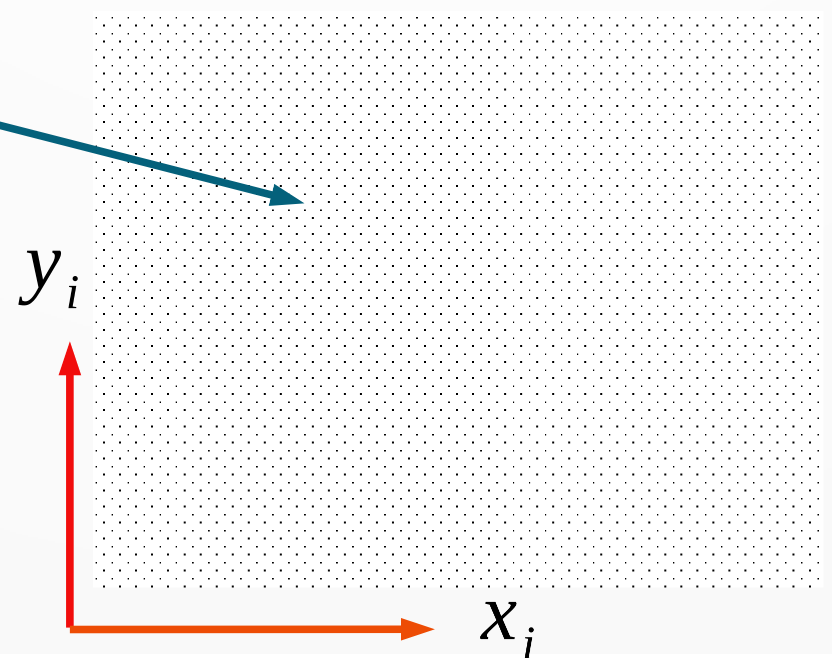
Numerical experiments

Starting from the lens equation $\vec{r}_S = \vec{r} - \vec{\nabla}\,\phi$

We define a mapping of the lens plane in Cartesian coordinates:

$\vec{r}_i = (x_i, y_i)$

The lens plane is covered
With a regular grid



For each $\vec{r}_i$ we use
The lens equation
To esimate the
corresponding $\vec{r}_{S,i}$

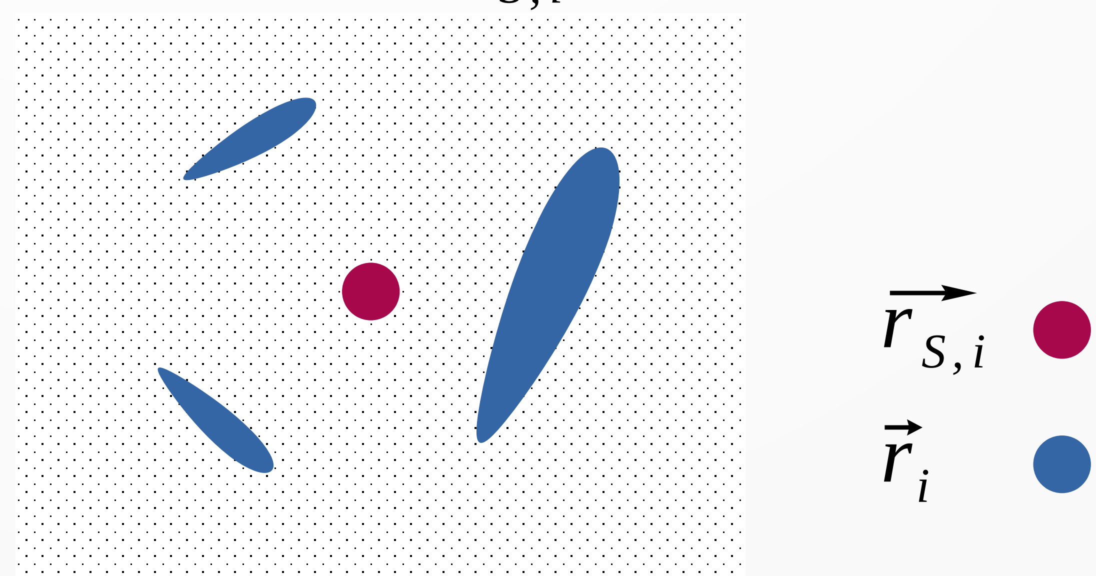
Simple ray tracing

The image of the source corresponds to the points where $\vec{r}_{S,i}$ belonging to the area Occupied by the source. The points $\vec{r}_i$ associated to $\vec{r}_{S,i}$ is the image of the source

For a circular source with center, $\vec{r}_0$
And radius, R_0

$$\left(\vec{r}_{S,i} - \vec{r}_0\right)^2 = R_0^2$$

The brightness of the pixels at $\vec{r}_i$ in the image is precisely the source brightness At the corresponding point in the source plane $\vec{r}_{S,i}$



Make a simple code

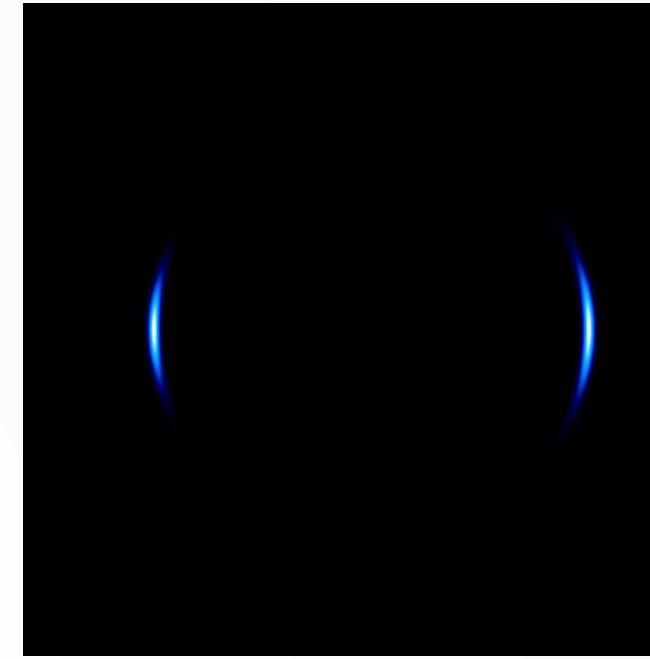
Define 2 loops in the lens plane along x and y coordinates
For each point (x,y) evaluate $\left(x_S, y_S\right)$ by using the lens equation

If $\sqrt{\left(x_S^2 + y_s^2\right)} \leq R_0$ then the associated points (x,y) belong to the image

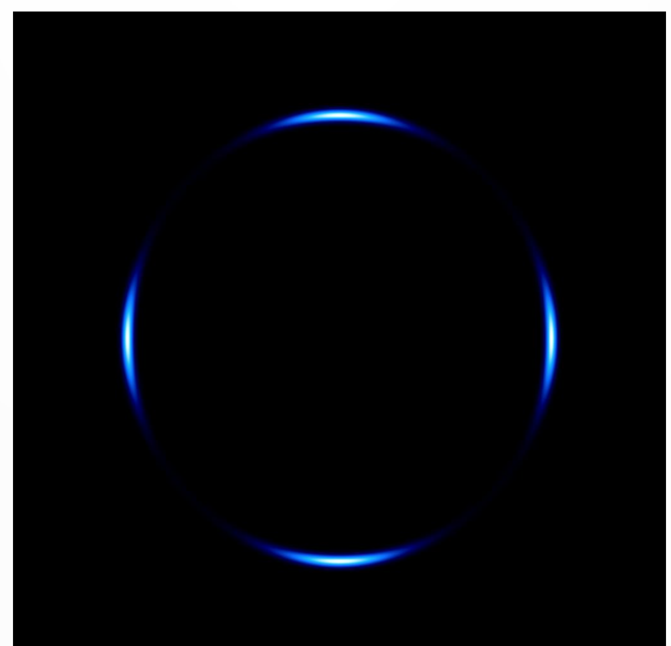
Of the circular source of constant brightness with radius R_0

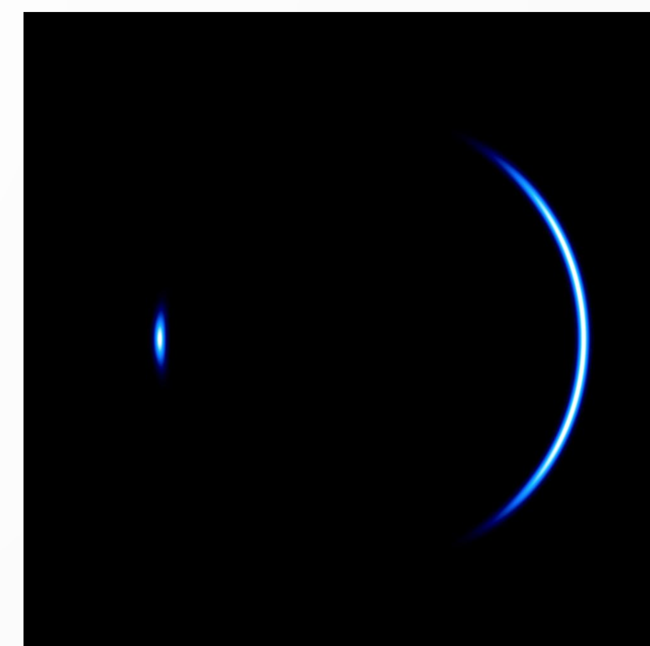
For a general source assign the value of the source at $\left(x_S, y_S\right)$ to the image point (x,y).

Circular potential
Left: source centered
Right: 0.1 Re offset



Isothermal elliptical potential
Left: source centered
Right: 0.1 Re offset= 2η





Use the ray tracing method with various potential models

$$\text{Point mass}: \phi = \frac{1}{2}\log\left(x^2 + y^2\right)$$

$$\text{Isothermal}: \phi = \sqrt{\left(x^2 + y^2\right)}$$

$$\begin{cases} x_s = x - \dfrac{\partial \phi}{\partial x} \\ y_s = y - \dfrac{\partial \phi}{\partial y} \end{cases}$$

$$\text{Elliptical isothermal}: \phi = \sqrt{\left(\left(1-\eta\right)x^2 + \left(1+\eta\right)y^2\right)}$$

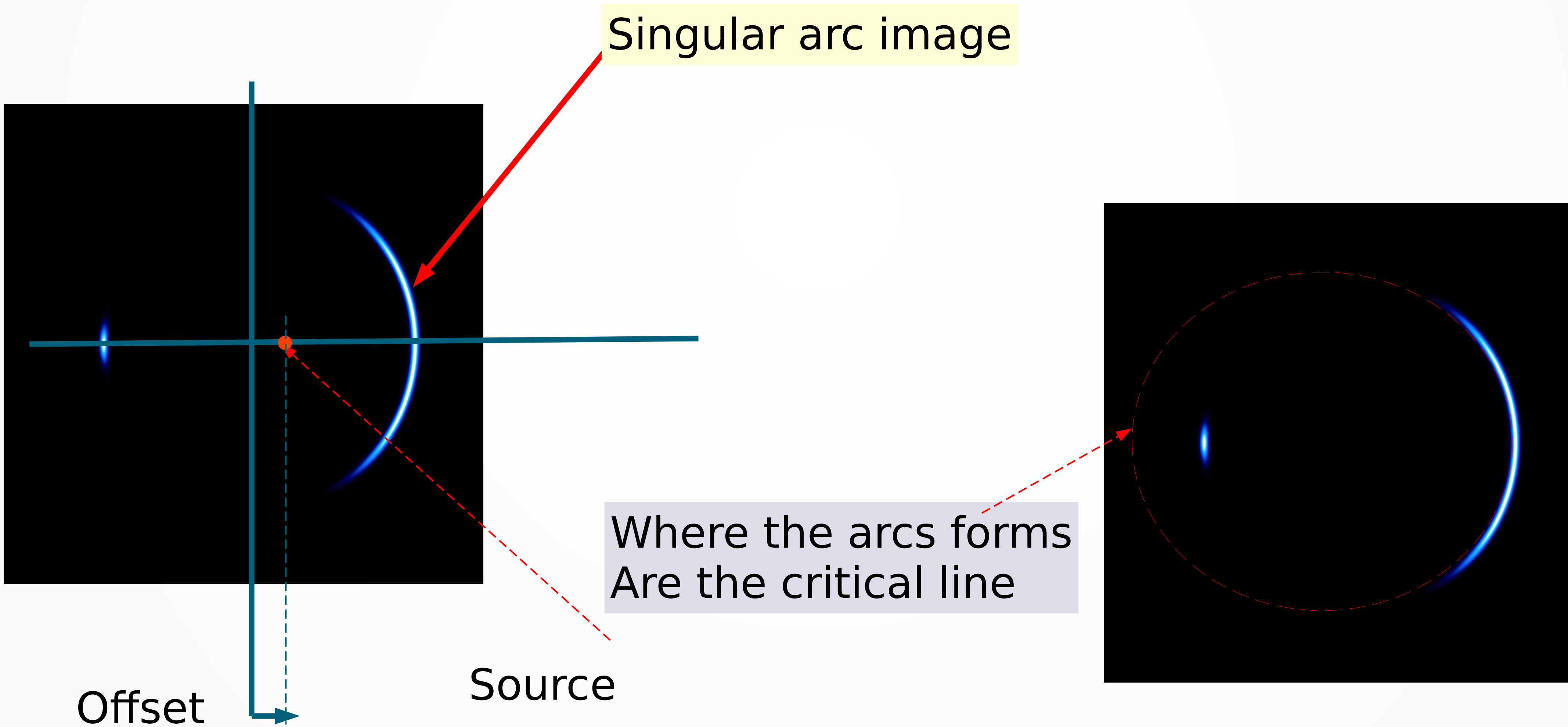
These potential models have Einstein radius normalized to unity
Thus explore the lens plane on a grid with a typical range of the order of unity.

The image of a source at the center of a circular potential will be
A full circle (the Einstein circle)

An interesting case: the isothermal potential with the
source position: $\left(2\eta, 0\right)$

Reconstructing the amplification map and the caustics for the isothermal potential

The caustics are the source positions where the amplification is infinite
For an infinitely small source, the ratio between the source size and image size are infinite



Reconstructing the amplification map and the caustics

We will now divide the lens plane using a regular grid
Each pixel in this mapping is a small source

By using the former ray-tracing technique we sample the lens plane in the (x,y) coordinates on a grid at the scale of the Einstein circle.
For each (x,y) we estimate the source plane coordinates $\left(x_S, y_S\right)$.

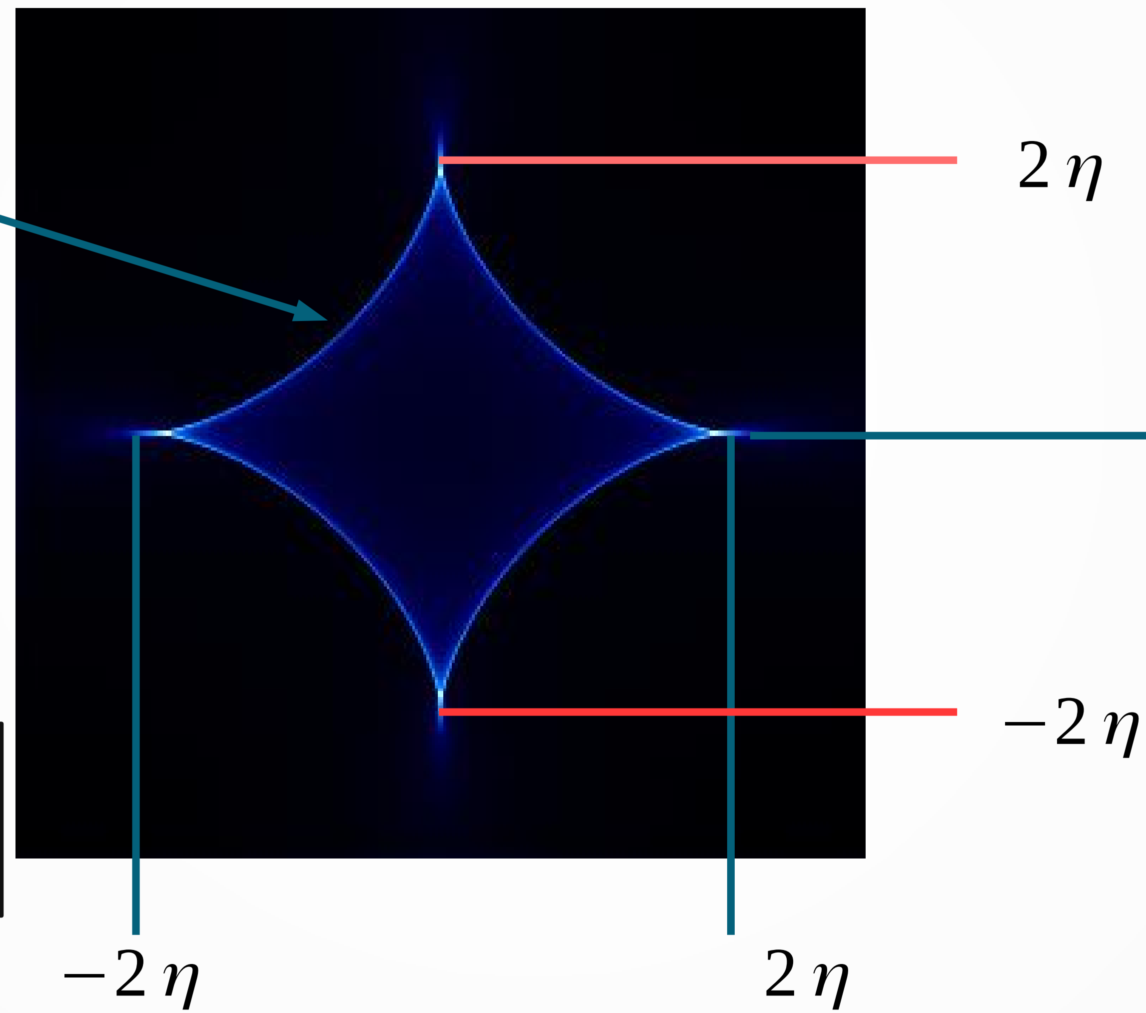
For each $\left(x_S, y_S\right)$ we calculate the pixel position in the source plane grid
The count for this pixel is updated and increased.

The result is an image of the total amplification as a function of the source position. A particularly interesting case is the elliptical isothermal potential
The source position area to explore is of the order of a fraction of the Einstein radius

η typical value: 0.05 to 0.1

The amplification map for the isothermal elliptical potential

3 images here
2 smaller image
+1 large arc
FOLD CAUSTIC



2 images here
1 small image
+1 large arc
CUSP CAUSTIC

Experiment: reconstruct
The image for a fold caustic