Numerical reconstruction of double lens using ray-tracing

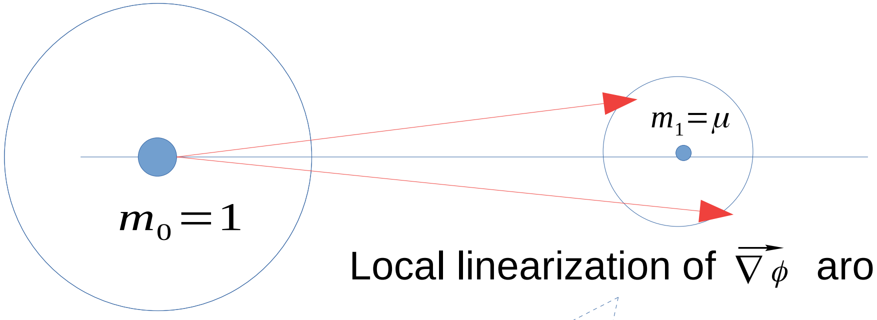
1) The potential and basic scale of the problem

2) practical implementation of the ray-tracing method to reconstruct the caustics around the planet

3) Estimating the different lensing regimes as a function of the components separation

4) Additional problem: evaluating the caustics near the main component due to the perturbation by the planetary companion

(1) A short reminder of the basics of the problem



Local linearization of $\vec{\nabla}\phi$ around m_1

$$\phi=\mu\log(r)+\frac{1}{2}\log\left((x+x_1)^2+y^2\right)\simeq\log(x_1)+\mu\log(r)+\frac{x}{x_1}+\frac{y^2-x^2}{2x_1^2}$$

Displacement term

Distortion (shear)

Expansion at order 2 in $\frac{1}{x_1}$

In this approximation the solution for the critical lines and caustics
Is analytical

$$J = \frac{\partial x_s}{\partial x}\frac{\partial y_s}{\partial y} - \frac{\partial x_s}{\partial y}\frac{\partial y_s}{\partial x}$$

with

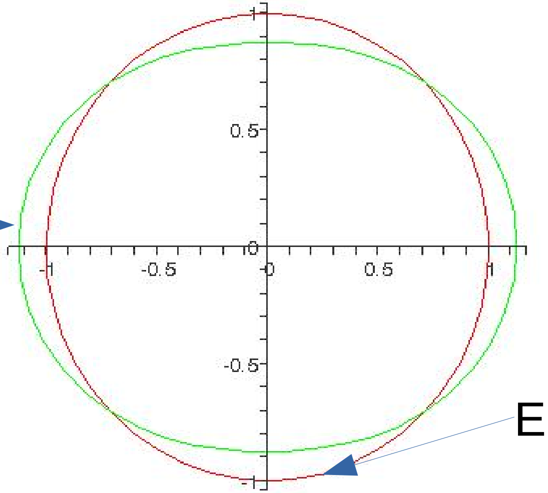
$$\vec{r}_S = \vec{r} - \vec{\nabla}\phi$$

Expansion at order 2 in $\frac{1}{x_1}$

$$J = \frac{r^4 - \mu^2}{r^4} - 2\mu\frac{\cos(2\theta)}{r^2 x_1^2}$$

Critical lines: J=0

$$r = \sqrt{\mu}\left(1 + \cos\frac{(2\theta)}{2x_1^2}\right)$$



Einstein ring

Caustics: transformation of the critical line in source plane coordinates

Take critical line equation $r=\sqrt{\mu}\left(1+\cos\frac{(2\,\theta)}{2\,x_1^2}\right)$

Insert in lens equation $\vec{r}_S=\vec{r}-\vec{\nabla}\,\phi$

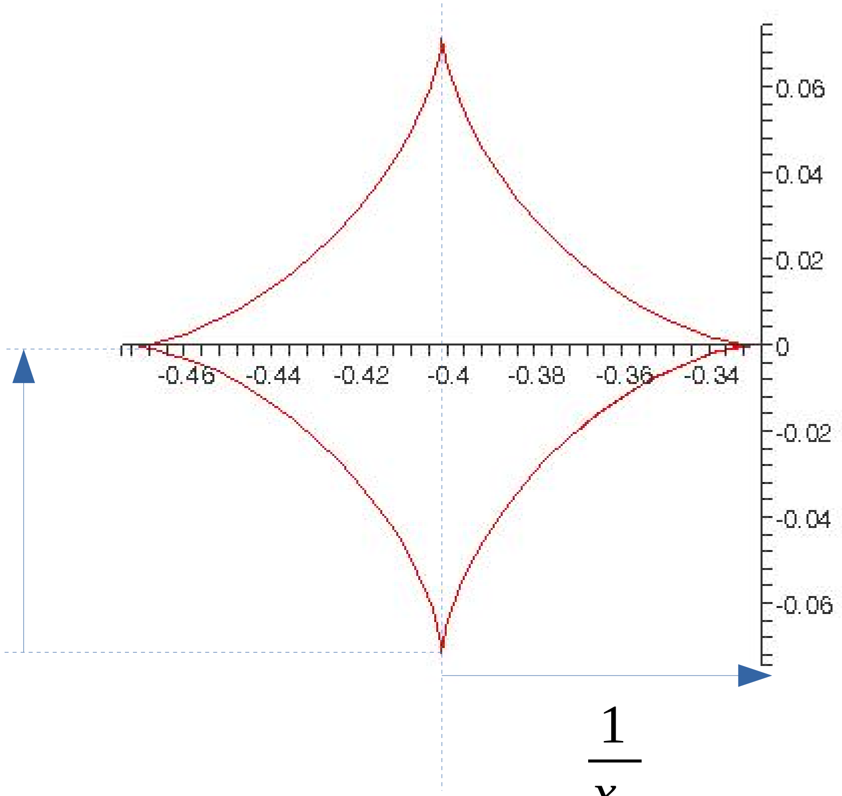
Expansion at order 2 in $\frac{1}{x_1}$

$$\begin{cases} x_S=-1/x_1+\sqrt{\mu}\,\dfrac{3\cos(\theta)+\cos(3\,\theta)}{2\,x_1^2} \\ y_S=\sqrt{\mu}\,\dfrac{-3\sin(\theta)+\sin(3\,\theta)}{2\,x_1^2} \end{cases}$$

Numerical application: shape of the caustics

$x_1 = 2.5 \quad ; \quad \mu = 0.05$

Amplitude $2\dfrac{\sqrt{\mu}}{x_1^2}$



$\dfrac{1}{x_1}$

(2) Caustics reconstruction

The typical size of the caustic grid: a few times: $\dfrac{2\sqrt{\mu}}{x_1^2}$

The small of-centering of the caustic grid : $-\dfrac{1}{x_1}$

The size of the grid in the lens plane: a few times: $\sqrt{\mu}$

(The size may have to larger when the components are closer)

(3) Estimate various lensing regime

For simplicity take a Jupiter sized planet, $\mu \simeq 0.01$

Explore the progressive assymetric deformation of the caustics

Starting from, $x_1 \simeq 3$, and going to x_1 of the order of unity

Take care to increase the size of the grid in the lens plane to capture all possible rays

(4) The small caustics near the main component And the global reconstruction of the two caustics

This time reconstruct the caustic not only near the small component but
For the whole system
Increase the caustic grid size in the X direction

Increase the grid size in the lens plane to cover properly the whole area
Make an image of the global caustic system

Observe the merging of the cautics for separations close to unity

The computing cost will increase

What you should get

Separation 1.5
No merging

Separation 1.05
Full merging

Separation 1.25
Caustics starts to merge