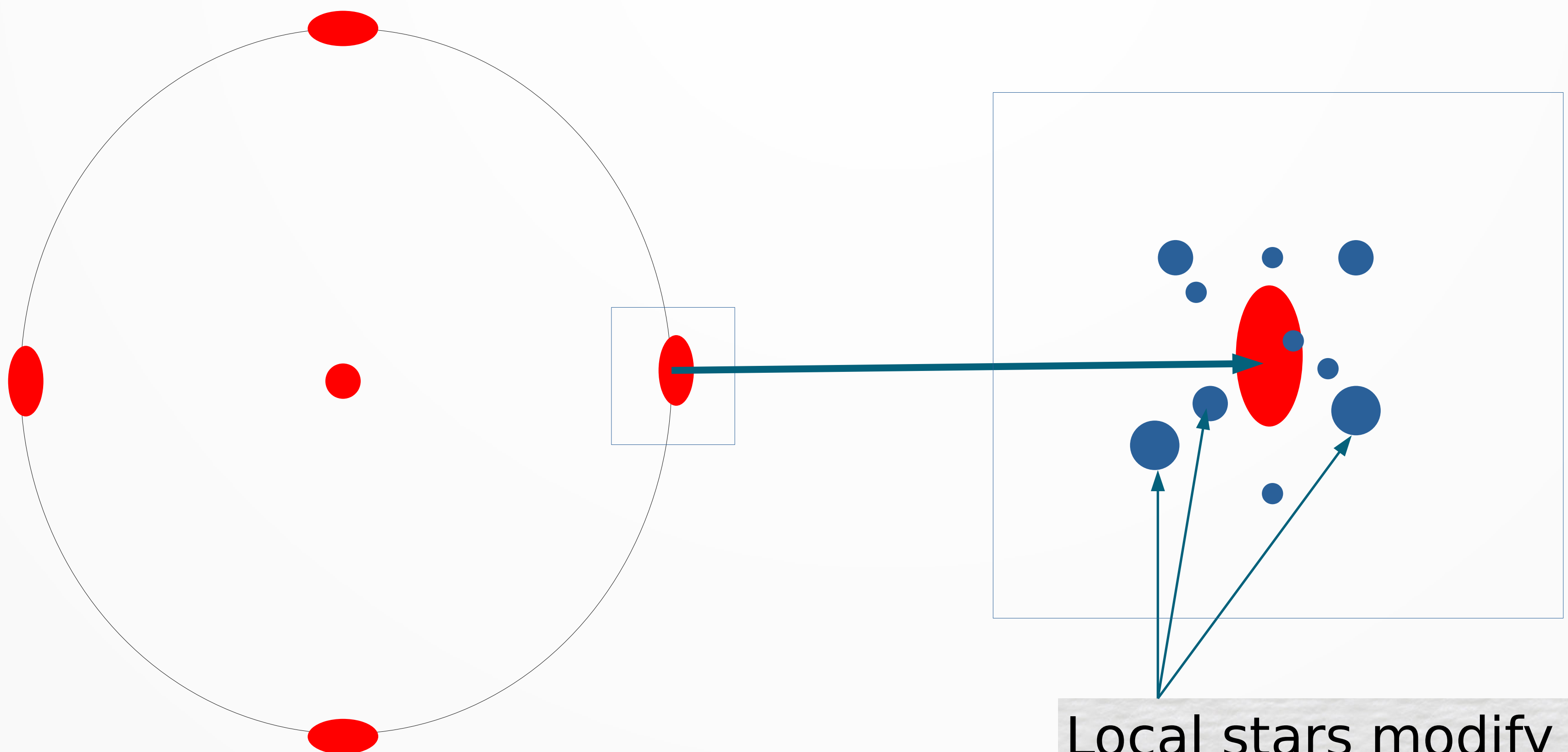
The network of micro-caustics due to stars in systems like the Einstein-Cross

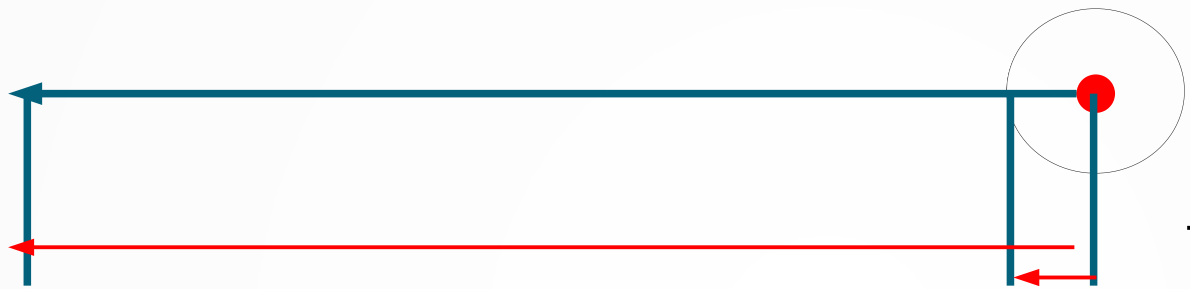


The 4 images of a quasar at the center of an elliptical potential

Local stars modify the potential at small scale. Graininess of the Local potential.

The basics of the problem

Let consider a single star in the field smooth average field of the galaxy



$$\vec{r}_S = \vec{r} - \vec{\nabla}\phi$$

The lens equation in vicinity of the star Einstein radius

Center of the galaxy

In this problem $\vec{r}_0$ is close to the Einstein radius of the galaxy which for convenience has been set to unity by using an appropriate renormalization of the coordinate system

$\phi = \phi_{star} + \phi_{galaxy}$ Near the star Einstein circle: $\phi_{galaxy} \simeq \phi_{galaxy}(\vec{r}_0) + \vec{\nabla}\phi_{galaxy}(\vec{r}_0).\vec{r} + \vec{r}.J(\vec{r}_0)\vec{r}$

$$\vec{r}_S = \vec{r} - \vec{\nabla}\phi \simeq \vec{r}_S = \vec{r} - \vec{\nabla}\phi_{star} - \vec{\nabla}\phi_{galaxy}(\vec{r}_0) - J(\vec{r}_0)\vec{r}$$

At the star Einstein circle: $\vec{\nabla}\phi_{star} \simeq \sqrt{(M_{star})}$

$\vec{\nabla}\phi_{galaxy}(\vec{r}_0)$ Is a source offset term

At the star Einstein circle: $J(\vec{r}_0)\vec{r} \simeq 2\eta\cos(2\theta) r_E \simeq 2\eta\cos(2\theta)\sqrt{(M_{star})}$

As a consequence we have a situation where the galactic mean field acts as a small perturbation of the local star field, leading to a typical caustic with size of the order of a few times $\eta r_E = \eta\sqrt{M_{star}}$

Thus for a large number of stellar perturbators a network of small caustics Associated with each star should appear. In practice the problem is highly Linear and complex and require a numerical resolution

Numerical implementation

Basic scale

The scale of the micro-caustics are close to the Einstein radius of stars

The smooth average potential of the galaxy reads: $\phi_{gal} = \sqrt{(1-\eta)x^2 + (1+\eta)y^2}$

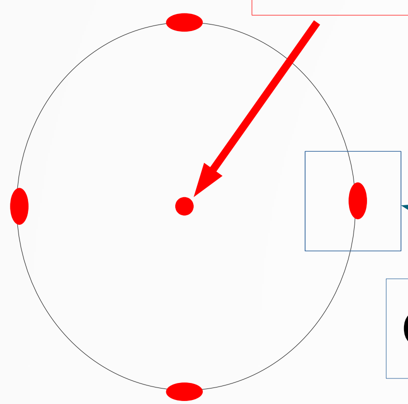
The Einstein radius of this potential is normalized to unity.

Considering a typical stellar mass of: $M_{star} = 10^{-10} M_{gal}$

The stellar Einstein radius is: $R_E^{star} = R_E^{gal} \sqrt{\frac{M_{star}}{M_{gal}}} = 10^{-5}$

Practical implementation

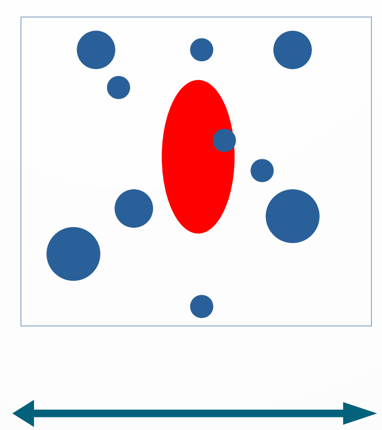
The source is at the center of the galaxy



Choose one image with coordinates (x,y)

Modify the lens potential near (x,y)
By adding point mass with typical
mass $M_{star} \simeq 10^{-10} M_{gal}$

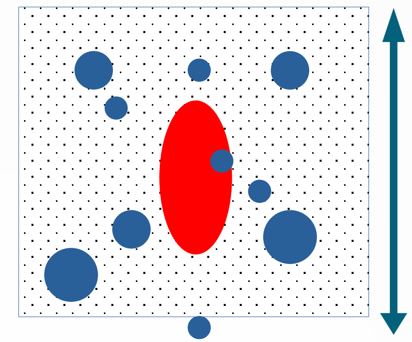
and typical distances $R_E^{star} \simeq 10^{-5}$



Box size ~ $10\, R_E^{star}$

Explore the caustic structure using the ray tracing technique

Use a local grid in the lens plane



Box size ~ $10\,R_E^{star}$

Transport to the source plane using the lens equation

Count the entries on a local grid
in the source plane.
Typical size of the grid in the
source plane ~ $10\,R_E^{star}$