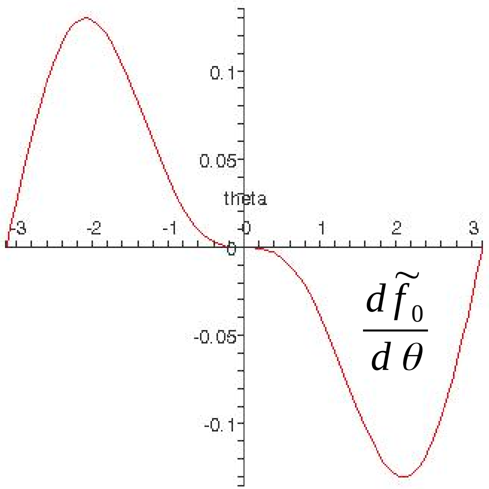
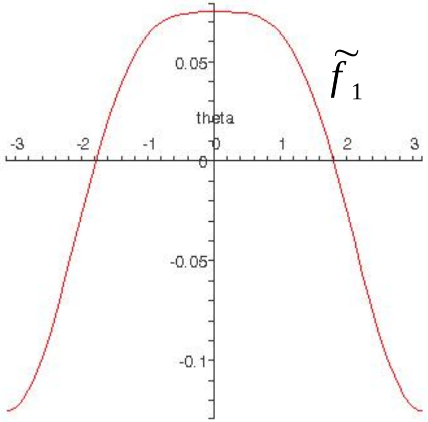
Pseudo isothermal potential $\phi=\sqrt{(1-\eta)x^2+(1+\eta)y^2}\simeq r\left(1-\frac{\eta}{2}\cos(2\theta)\right)$

$$\left\{\begin{array}{l} f_1= -\frac{\eta}{2}\cos(2\theta) \\ \frac{df_0}{d\theta}=\left[\frac{\partial\phi}{\partial\theta}\right]_{[r=1]} = \eta\sin(2\theta) \end{array}\right. \longrightarrow \tilde{f}_i=f_i+x_0\cos(\theta)+y_0\sin(\theta) \longrightarrow \left\{\begin{array}{l} \tilde{f}_1= -\frac{\eta}{2}\cos(2\theta)+x_0\cos(\theta)+y_0\sin(\theta) \\ \frac{\partial\tilde{f}_0}{\partial\theta}= \eta\sin(2\theta)-x_0\sin(\theta)+y_0\cos(\theta) \end{array}\right.$$





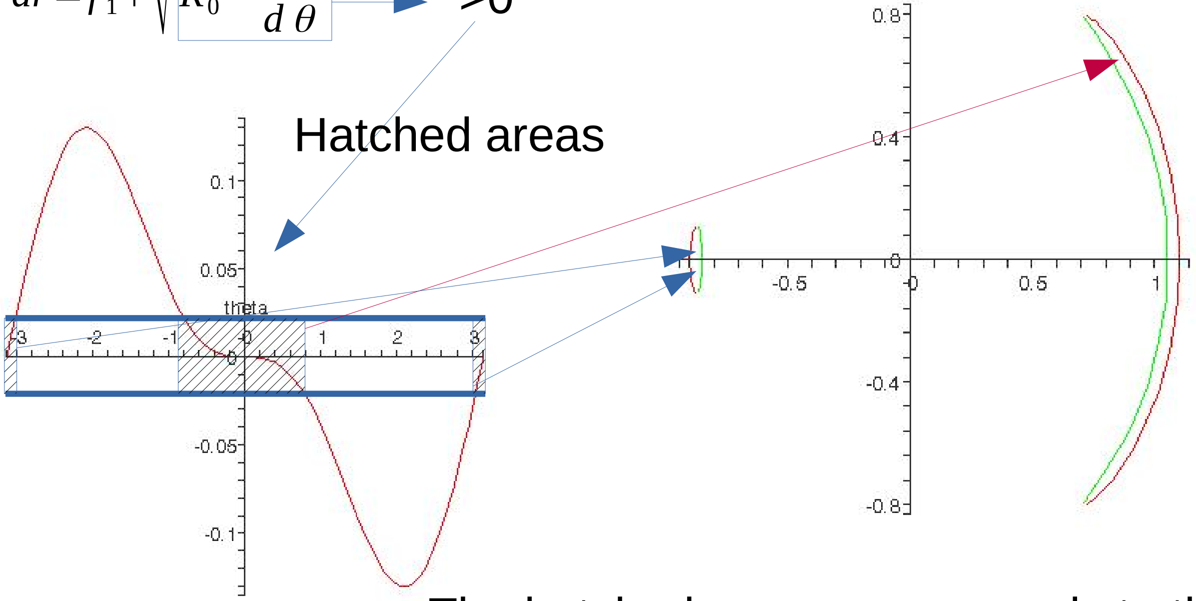
Cusp caustic

$\eta=0.05,\quad x_0=2\eta,\quad y_0=0,\quad \theta=0$

$$\eta = 0.05, \quad x_0 = 2\eta, \quad y_0 = 0, \quad \theta = 0, \quad R_0 = 0.02$$

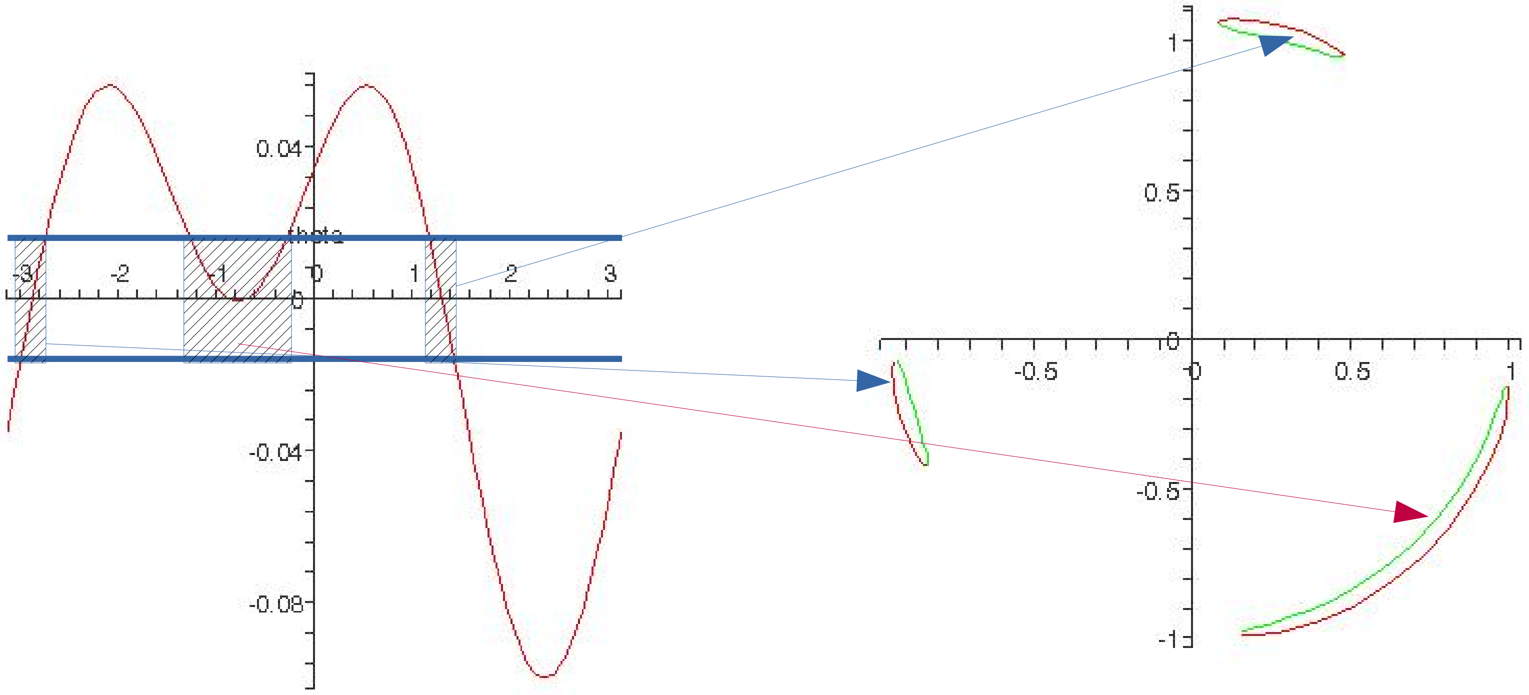
$$dr = \tilde{f}_1 \mp \sqrt{R_0^2 - \frac{d\tilde{f}_0}{d\theta}}$$

→ >0



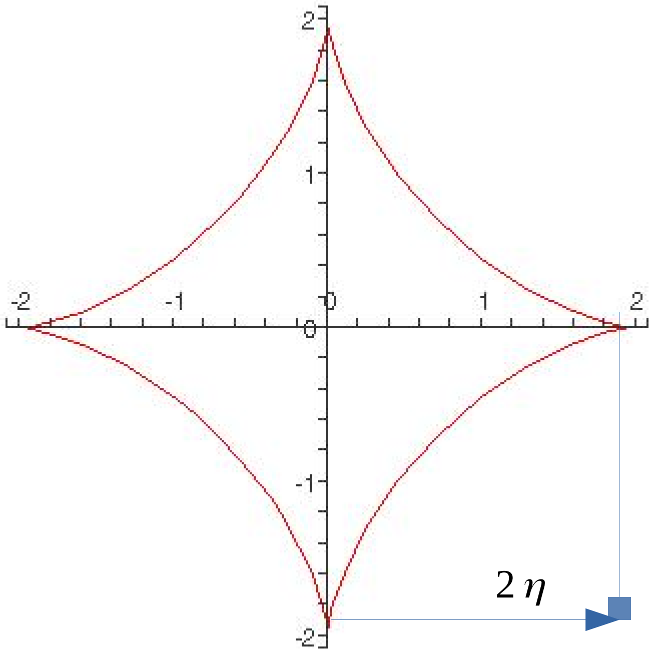
The hatched areas corresponds to the 2 images

Fold caustic $\eta = 0.05, \quad x_0 \simeq 0.035, \quad y_0 \simeq 0.035, \quad \theta = 0, \quad R_0 = 0.02$

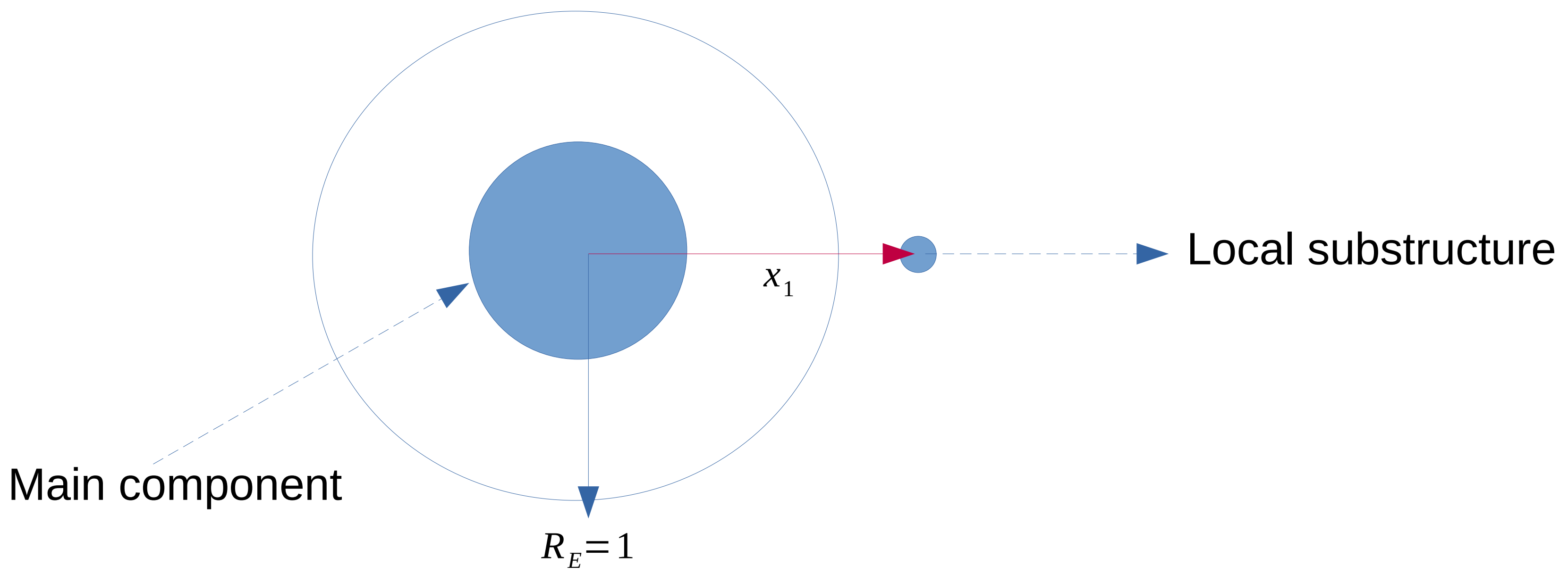


Caustics for the pseudo isothermal potential

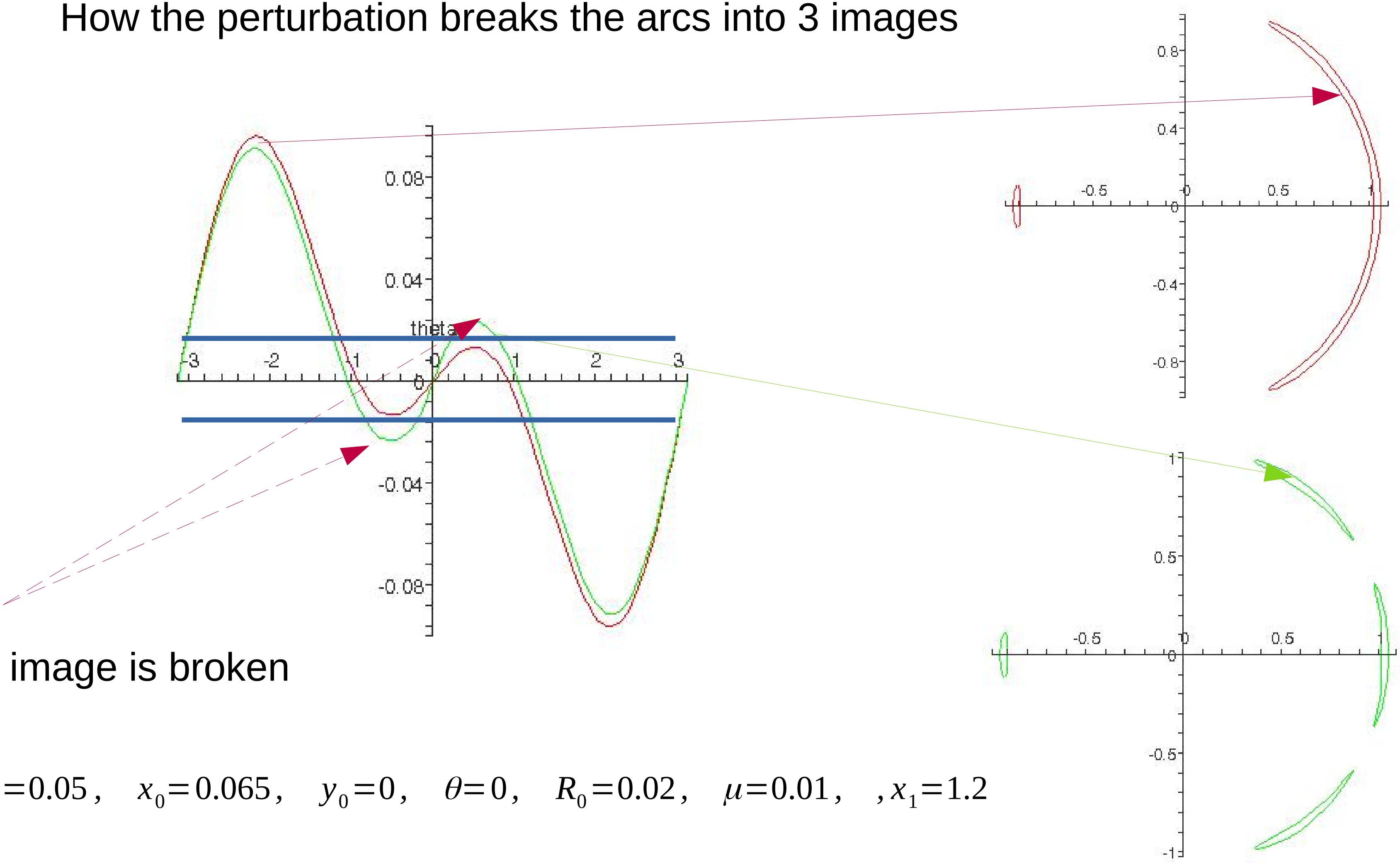
$$\begin{cases} x_S = \dfrac{d^2 f_0}{d\theta^2}\cos\theta + \dfrac{d f_0}{d\theta}\sin\theta = \dfrac{\eta}{2}\left(\cos 3\theta + 3\cos\theta\right) \\ y_S = \dfrac{d^2 f_0}{d\theta^2}\sin\theta - \dfrac{d f_0}{d\theta}\cos\theta = \dfrac{\eta}{2}\left(\sin 3\theta - 3\sin\theta\right) \end{cases}$$



Potential perturbed by local substructure $\phi = \phi_{iso} + \phi_{pert} \simeq r\left(1 - \frac{\eta}{2}\cos(2\theta)\right) + \mu\sqrt{(x - x_1)^2 + y^2}$



How the perturbation breaks the arcs into 3 images



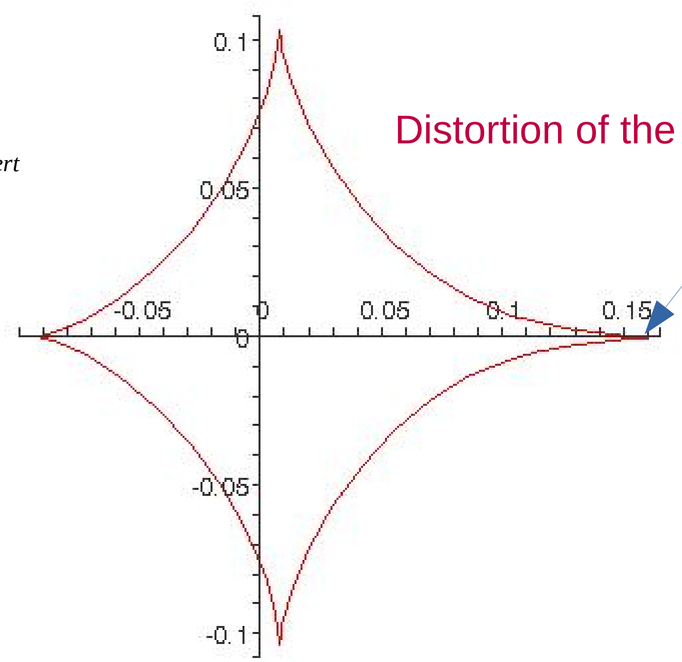
arcs image is broken

$$\eta=0.05, \quad x_0=0.065, \quad y_0=0, \quad \theta=0, \quad R_0=0.02, \quad \mu=0.01, \quad ,x_1=1.2$$

The effect of the perturbation on caustics

$$\phi = \phi_{iso} + \phi_{pert} \Rightarrow \frac{d f_0}{d\theta} = \left[\frac{d f_0}{d\theta}\right]_{iso} + \left[\frac{d f_0}{d\theta}\right]_{pert}$$

$$\begin{cases} x_S = \dfrac{d^2 f_0}{d\theta^2}\cos\theta + \dfrac{d f_0}{d\theta}\sin\theta \\ y_S = \dfrac{d^2 f_0}{d\theta^2}\sin\theta - \dfrac{d f_0}{d\theta}\cos\theta \end{cases}$$

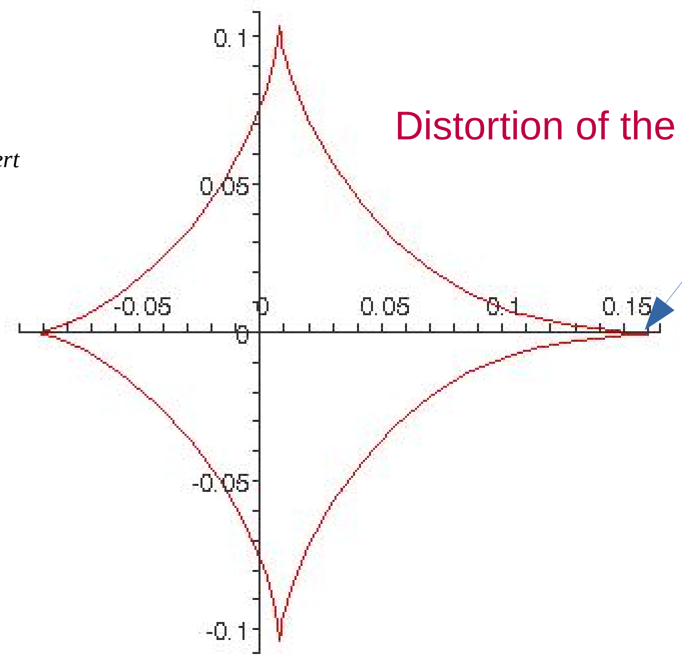


Distortion of the caustic due to the perturbator

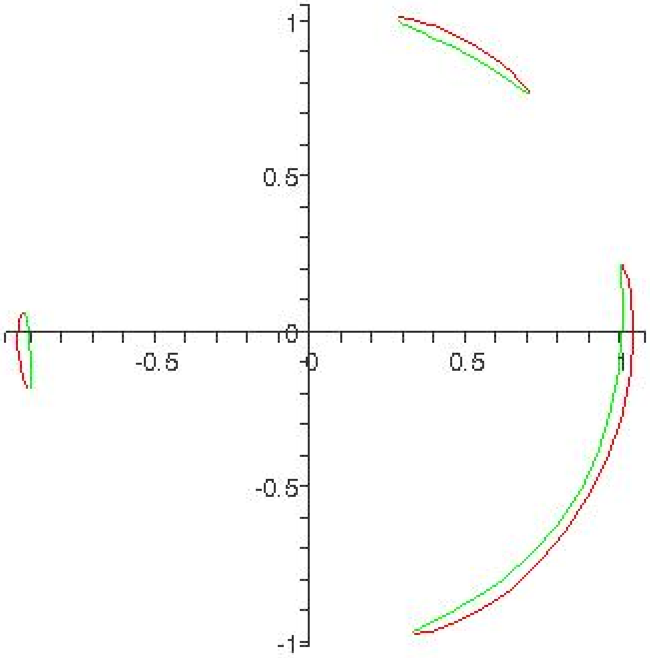
The effect of the perturbation on caustics

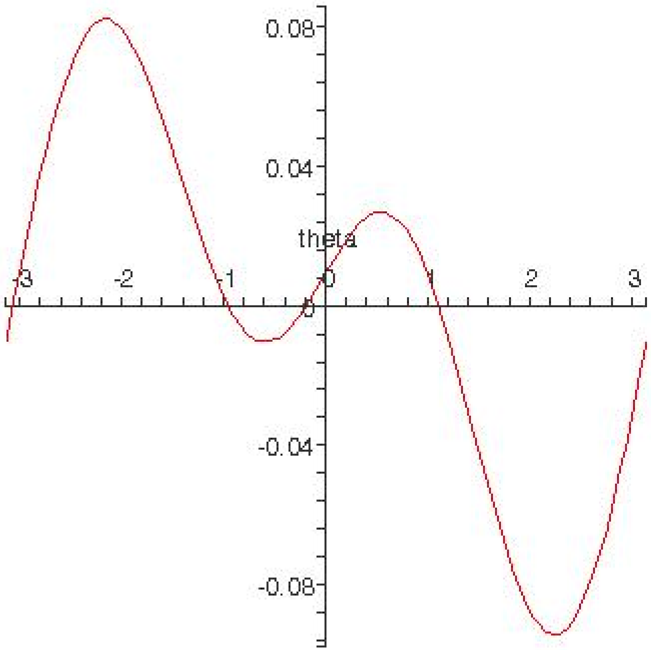
$$\phi = \phi_{iso} + \phi_{pert} \Rightarrow \frac{d f_0}{d\theta} = \left[\frac{d f_0}{d\theta}\right]_{iso} + \left[\frac{d f_0}{d\theta}\right]_{pert}$$

$$\begin{cases} x_S = \dfrac{d^2 f_0}{d\theta^2}\cos\theta + \dfrac{d f_0}{d\theta}\sin\theta \\ y_S = \dfrac{d^2 f_0}{d\theta^2}\sin\theta - \dfrac{d f_0}{d\theta}\cos\theta \end{cases}$$



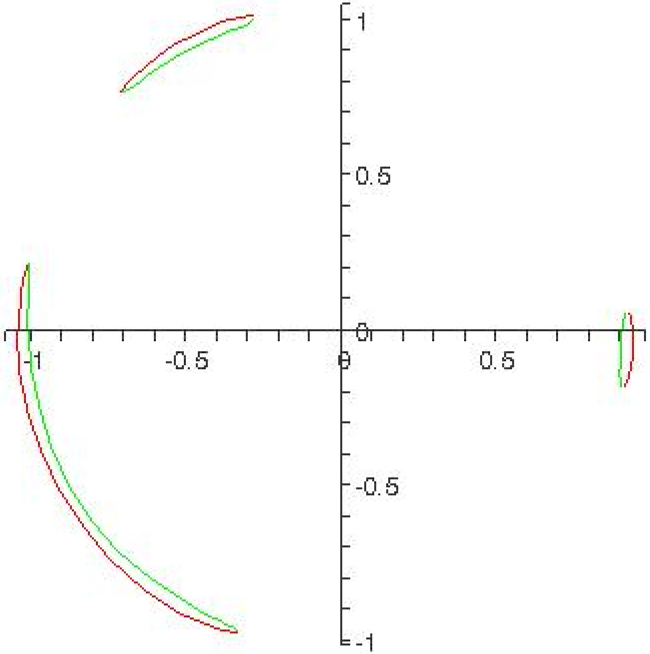
Find the corresponding solution for the image field

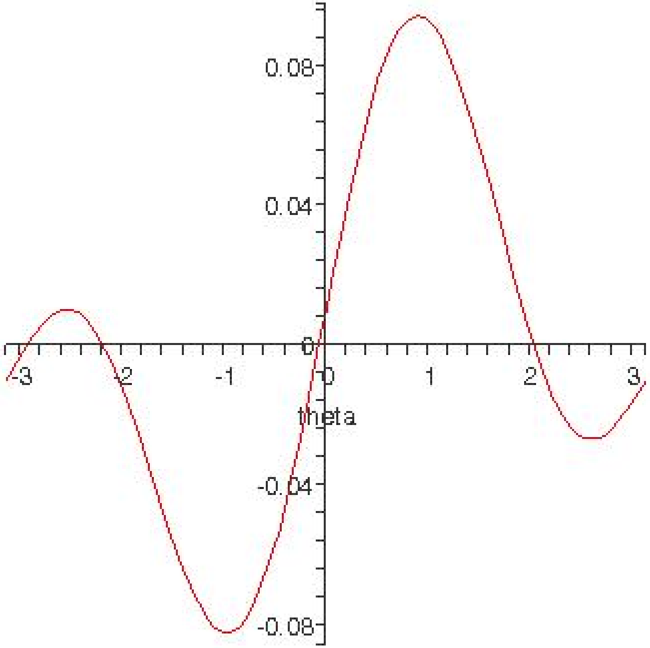




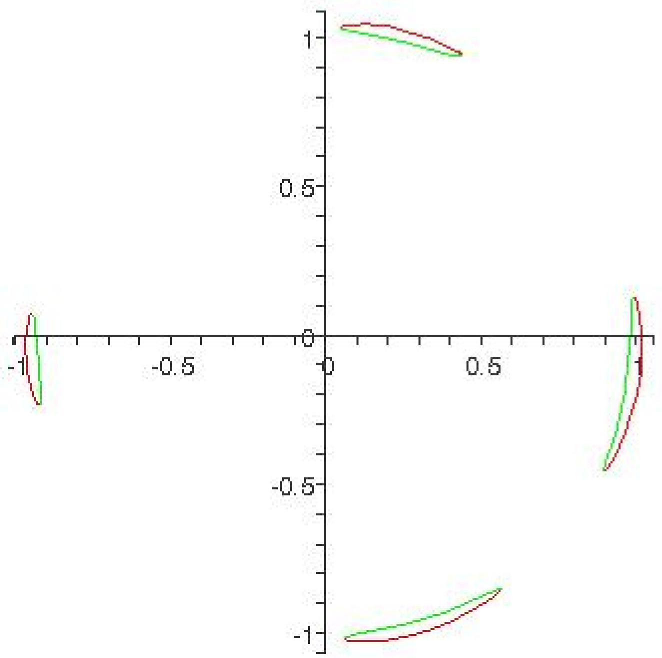
Note the field is periodic thus: $\int_{\theta=0}^{\theta=2\pi} \frac{d f_0}{d\theta} d\theta = 0$ the mean value of f_0 is zero

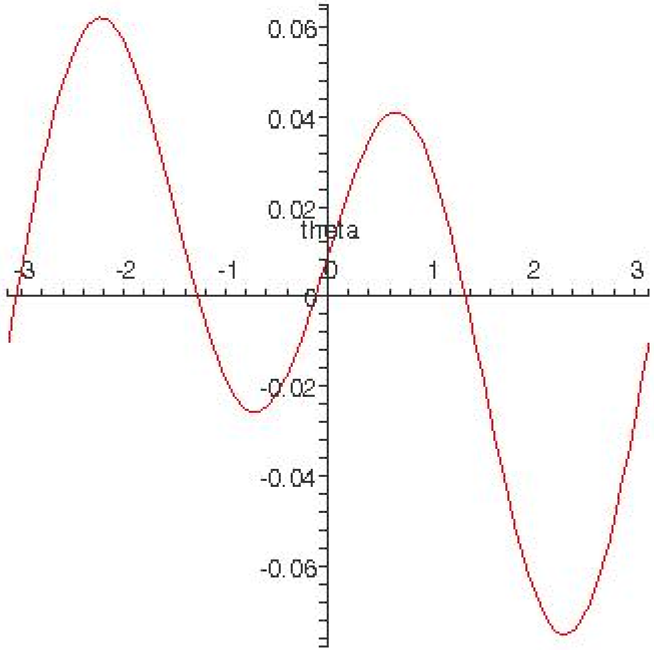
Find the corresponding solution for the image field



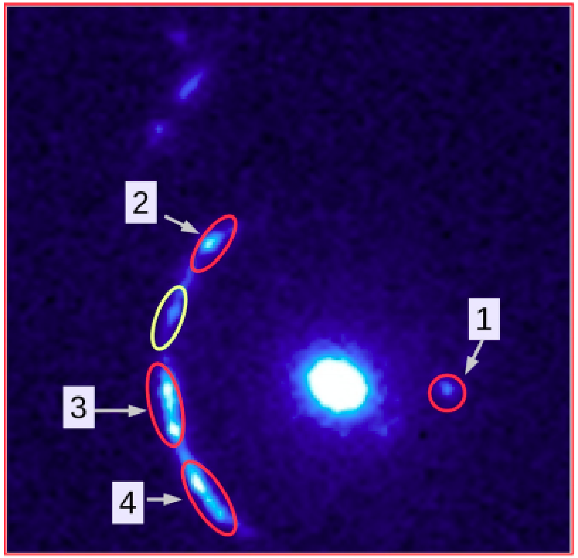


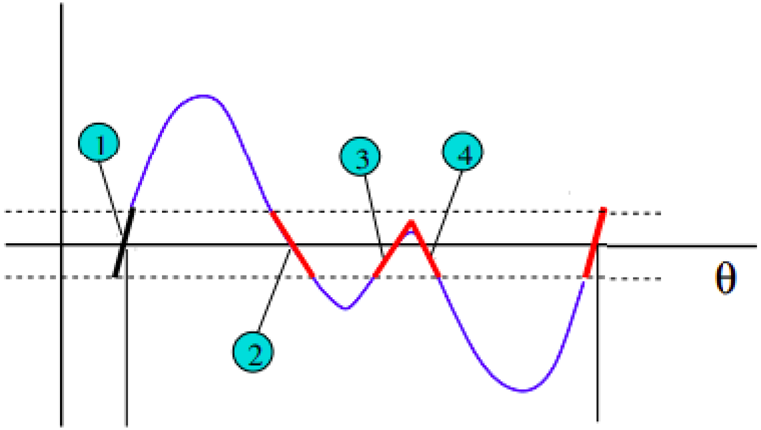
Find the corresponding solution for the image field

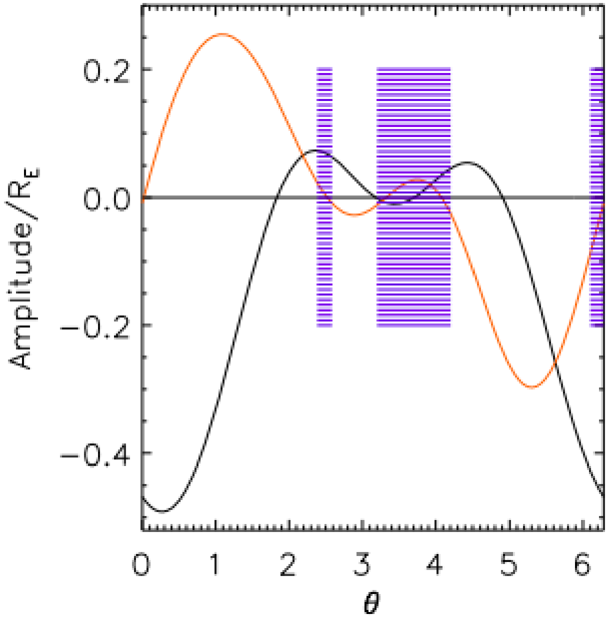




Alard (2010)

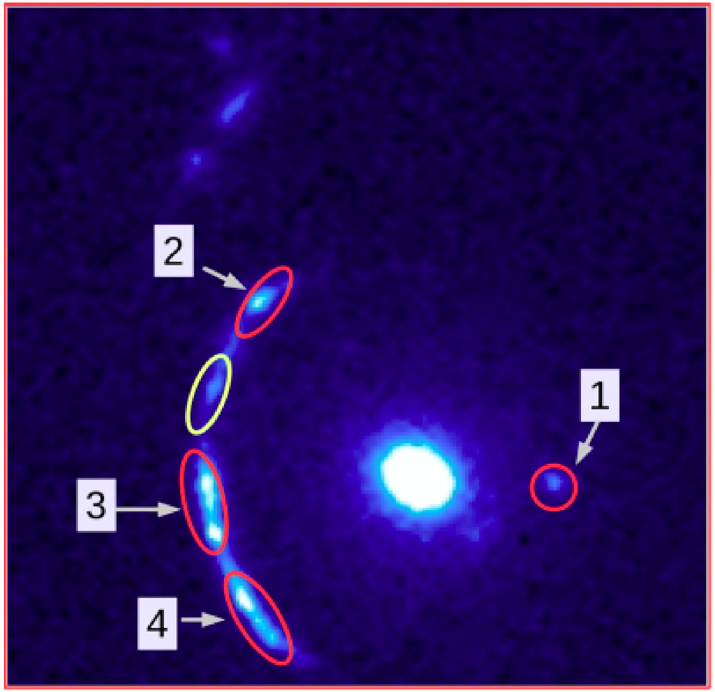


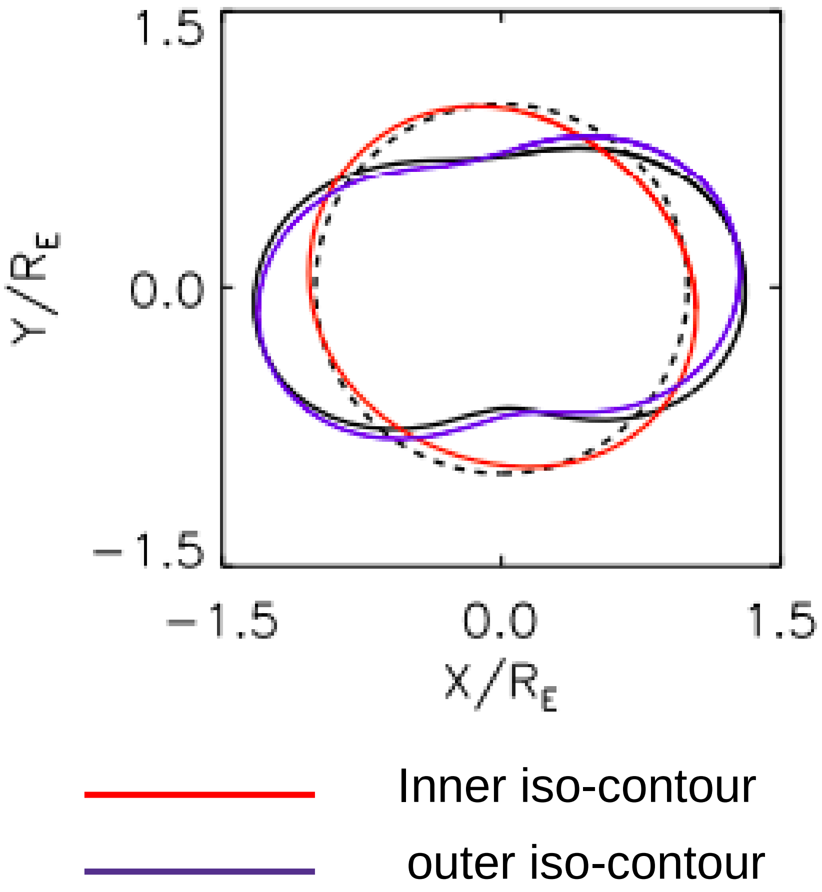




The lens system and the reconstruction
Of the 2 fields

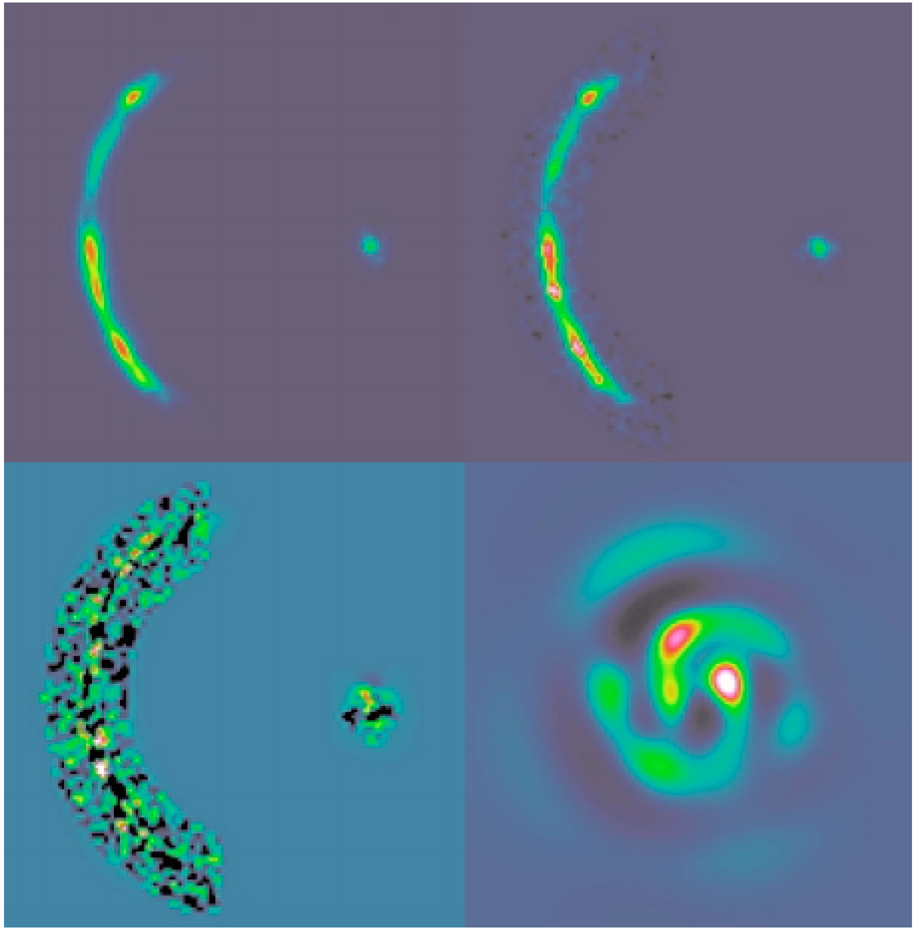
The reconstruction of the potential iso-contours

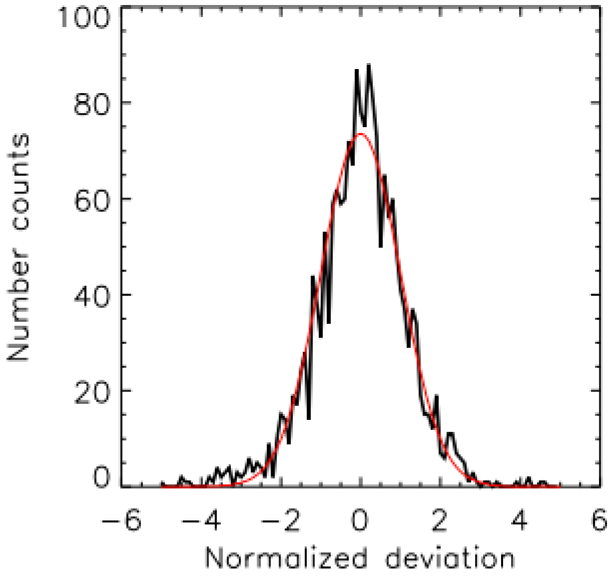




Alard (2010)

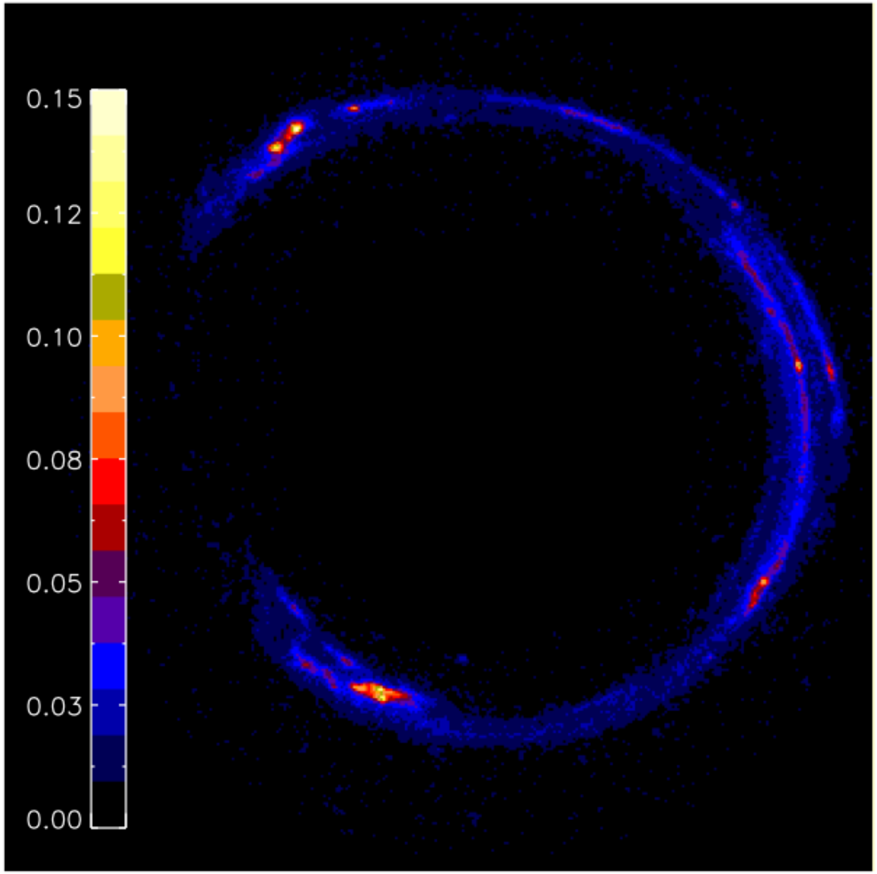
Image and source reconstruction



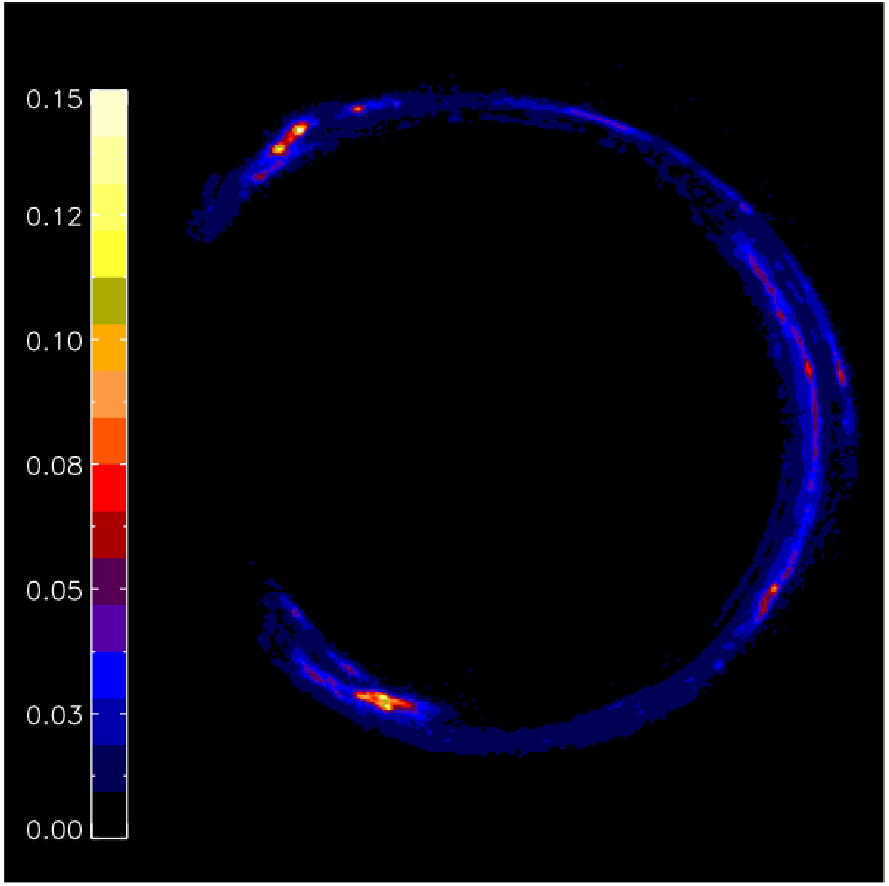


Alard (2010)

Reconstruction of the cosmic horseshoe

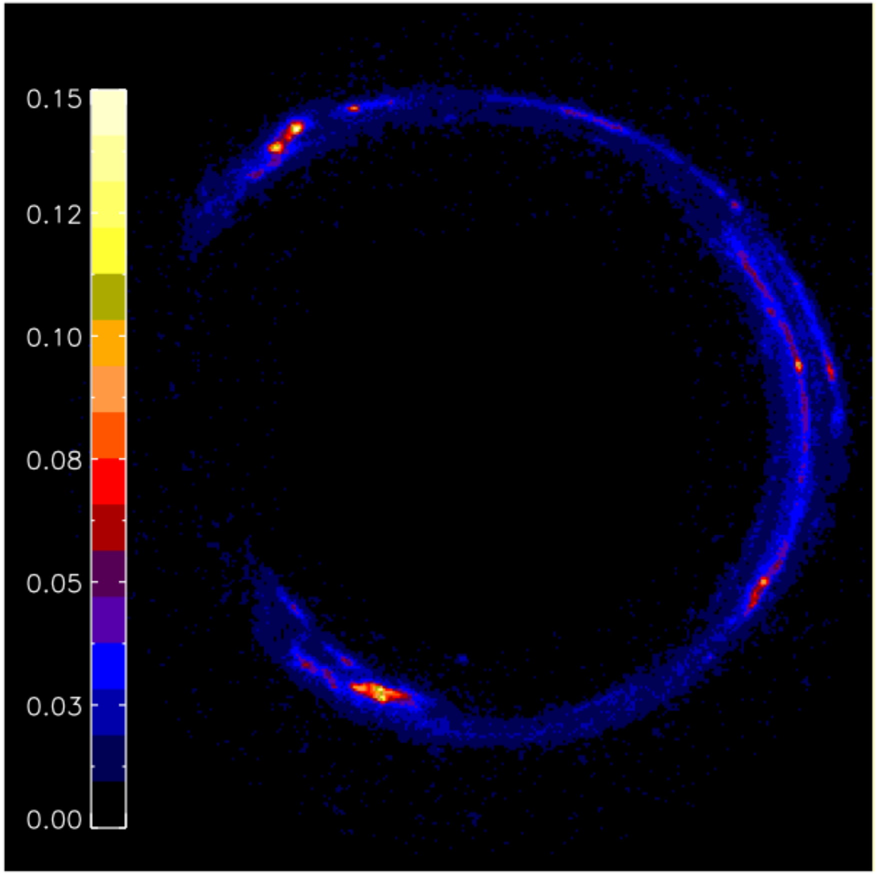


Original (HST data)

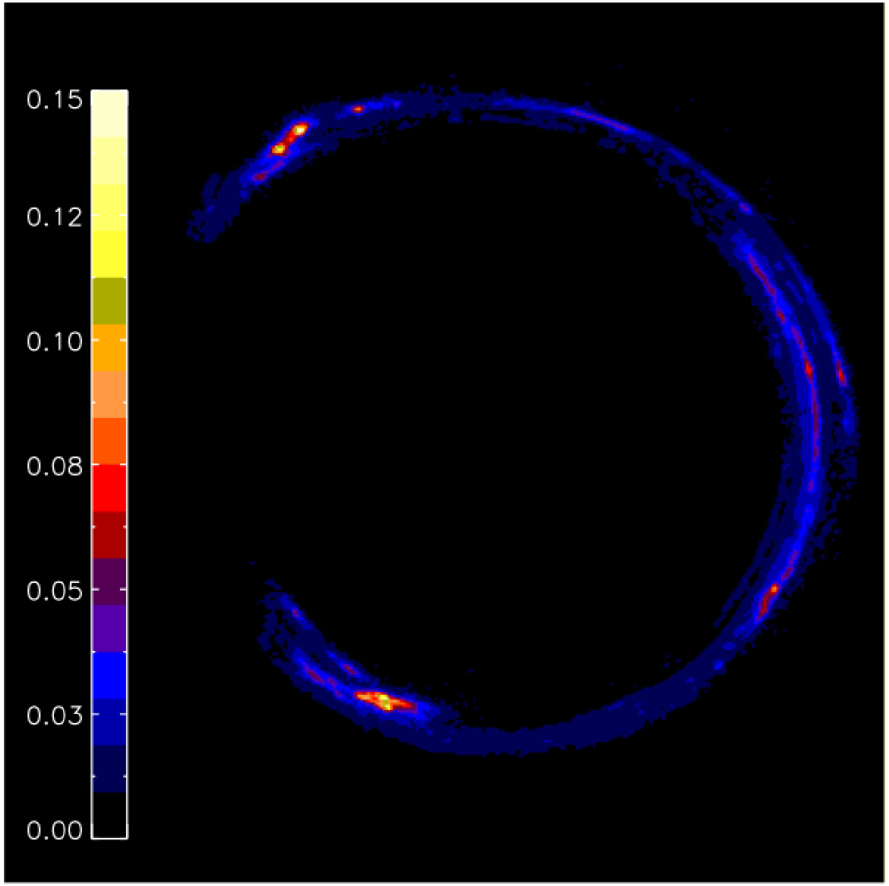


Reconstructed

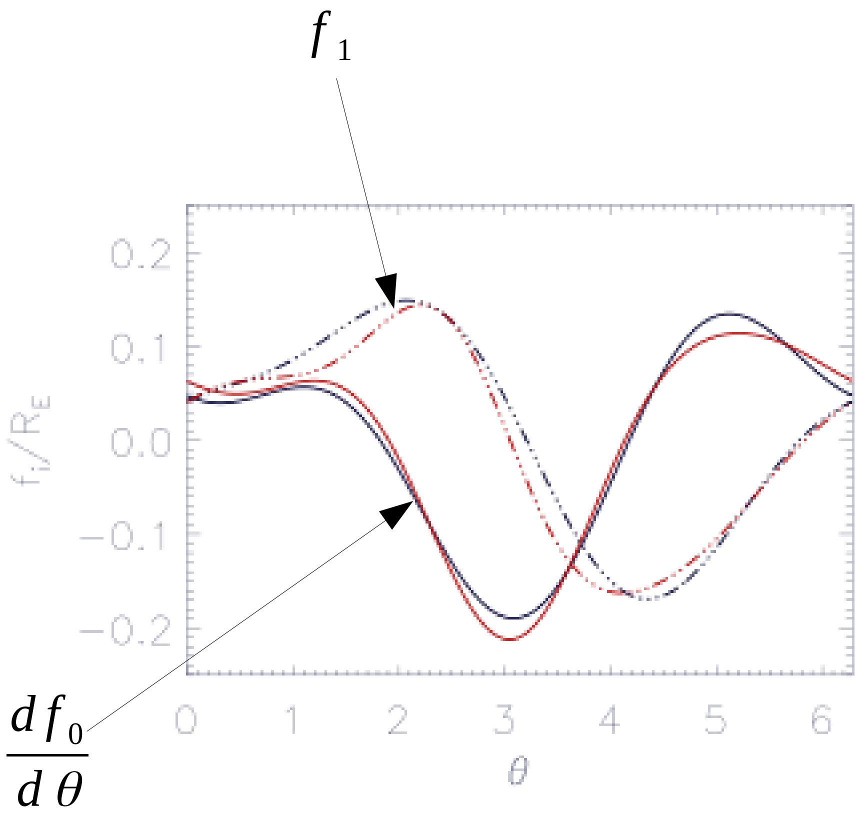
Reconstruction of the cosmic horseshoe



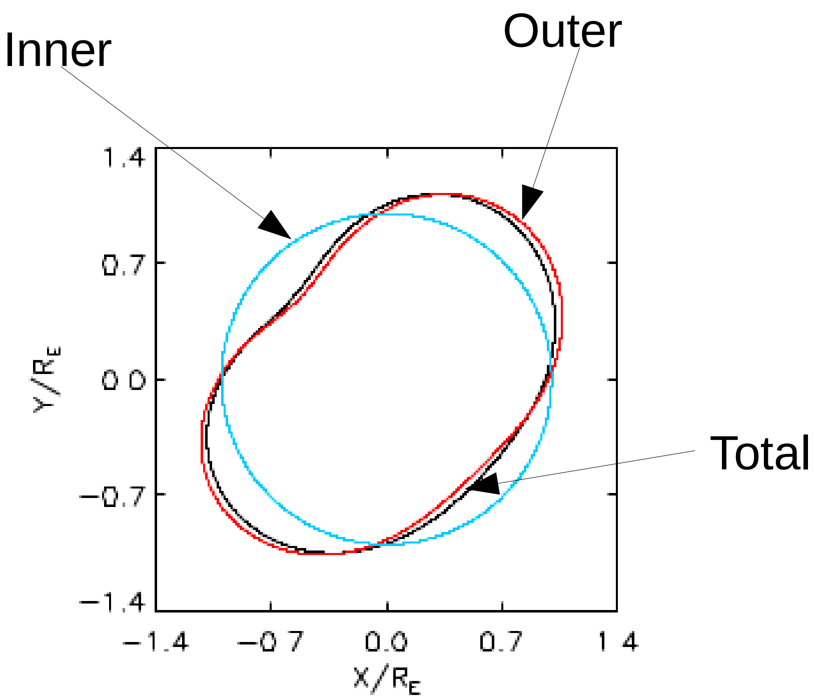
Original (HST data)



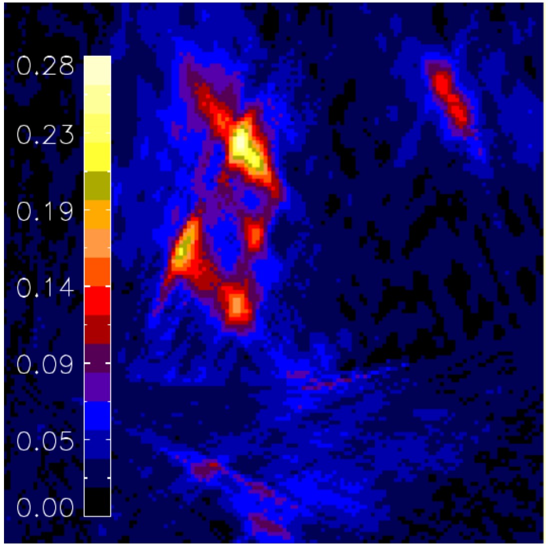
Reconstructed



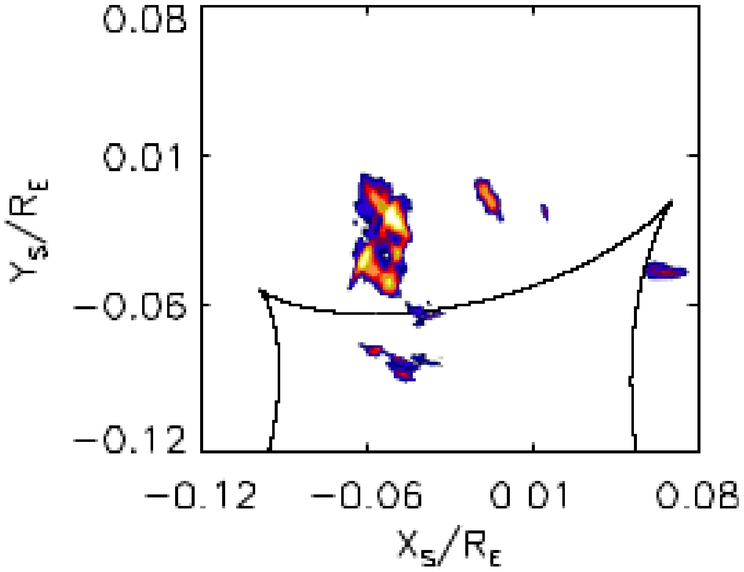
Solution for the fields



Potential iso-contours

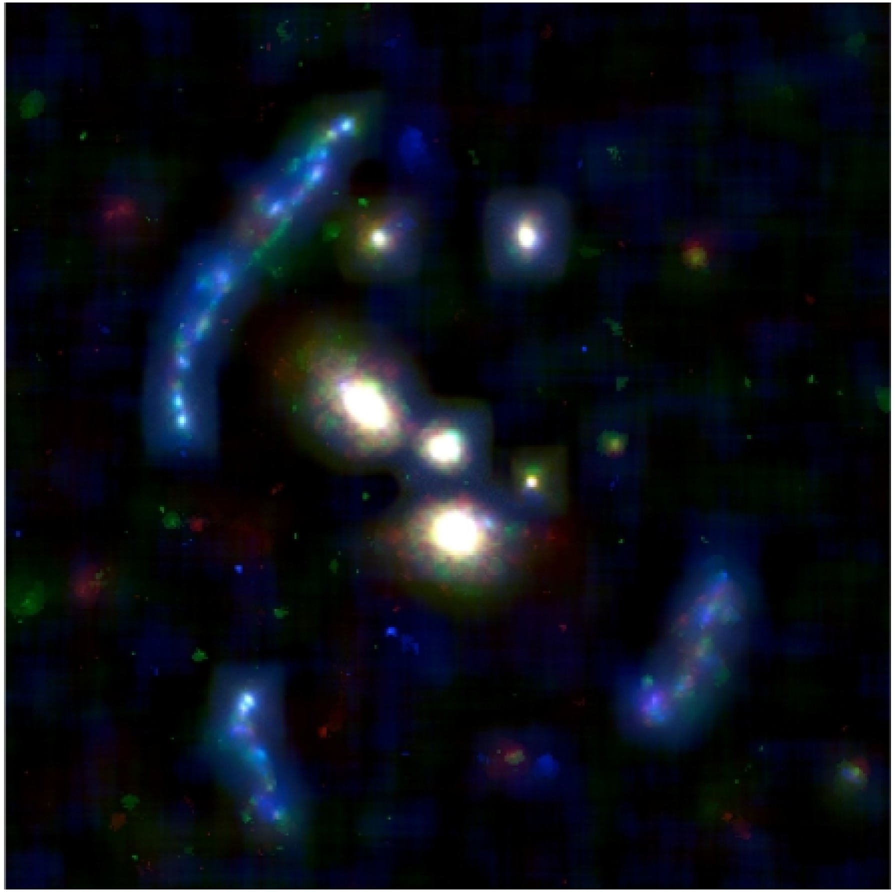


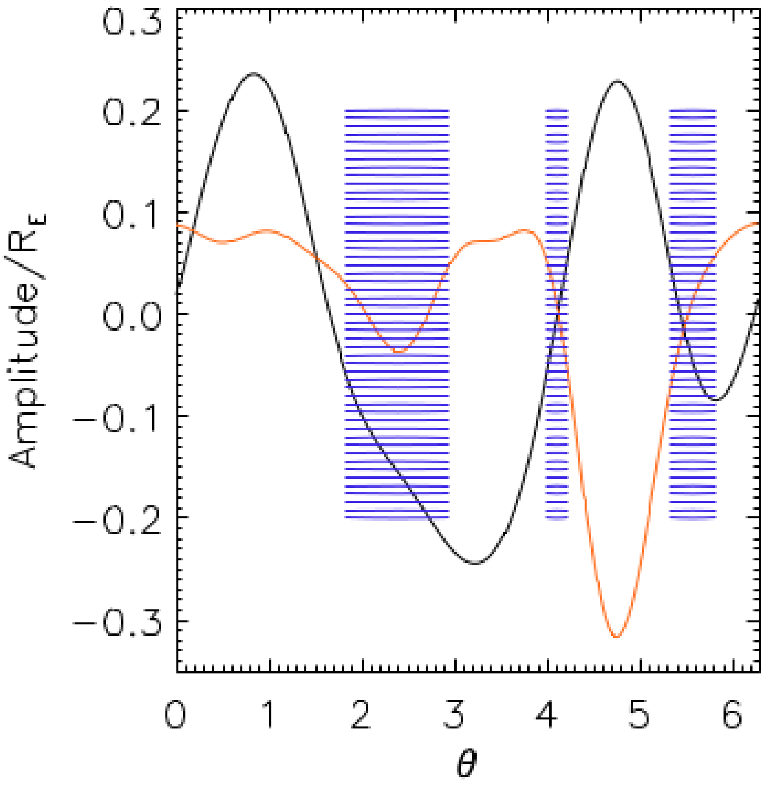
Source reconstruction



Source/caustic configuration

Alard (2009)

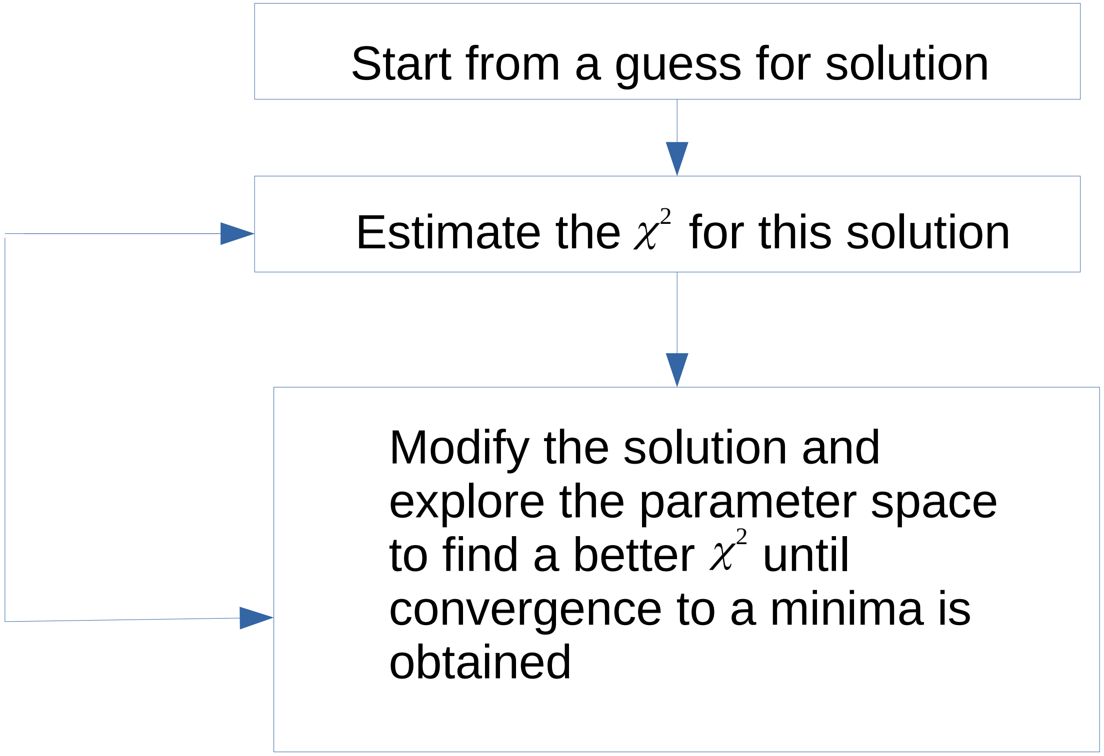




Fields reconstruction for the lens

In practice how do these reconstructions work ?

They work by a χ^2 descent method (the simplex for instance)



How do we estimate the χ^2 for a given potential ?

The lens potential is known but the source model is unkown

In this case the source reconstruction is a linear least-square problem

Represent the source as a linear sum of local basis functions - $Source = \sum_i a_i B_i(x_S, y_S)$

Estimate the image of each basis function using ray-tracing $C_i(x, y)$ *image of* $B_i(x, y)$

Convolve each image of the basis function with the PSF model $D_i(x, y) = C_i(x, y) * PSF(x, y)$

Make a linear least-square fit of the arc image $I_{arc}(x, y) = \sum_i a_i D_i(x, y)$

Estimate $\chi^2 = \int \left[I_{arc}(x, y) - \sum_i a_i D_i(x, y) \right]^2 dx\, dy$