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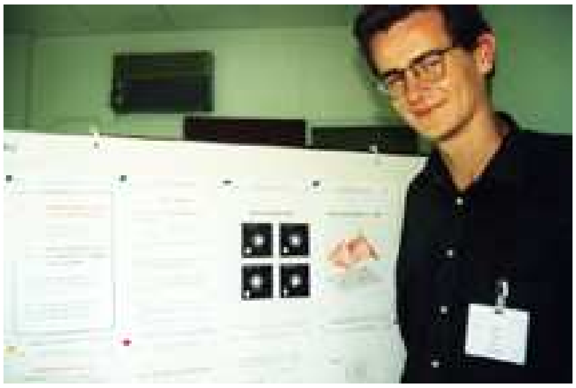
A CO-ANALYSIS METHOD OF WEAK GRAVITATIONAL LENSING AND SUNYAEV-ZEL'DOVICH OBSERVATIONS



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We describe briefly a new method, allowing to perform a joint and self-consistent perturbative analysis of Sunyaev-Zel'dovich and gravitational weak lensing observations, and to deduce from it an X-ray brightness prediction.

1 Introduction

Clusters of galaxies are the largest gravitationally bound structures in the Universe and a natural probe of cosmological scenarii of structure formation. Cluster surveys constitute now a wide observational database covering a broad wavelength range, which permit to investigate each component (dark matter (DM), intra-cluster medium (ICM), stellar content (galaxies)) in details. However, a joint analysis of various types of data is still a subtle task. In particular, although recent spectacular progresses in weak lensing[6,1] (WL) and Sunyaev-Zel'dovich[8,9,2] (SZ) observational techniques entail a new look on DM and ICM in clusters, the co-analysis methods of this data set are still in their infancy.
Since it is promising for both the understanding of the physical interplay of the ICM and DM and its cosmological implications, several groups have tackled this task [10,7] (Castander, Grego and Holder in this proceedings). In this poster, we present a new method which uses simultaneously WL and SZ data to perform a true co-analysis. Our approach provides X-ray predictions and permits to test very general physical hypothesis about the gas (hydrostatic equilibrium, global thermodynamic equilibrium).
Section 2 briefly describes the principle. Details and discussion are in sections 3 and 4 respectively.

2 General principles

Observations only provide $2-D$ projected quantities (e.g. gravitational potential, gas pression,...). They must be related to physical processes via $3-D$ equations (e.g. hydrostatic equilibrium, equation of state) which have no convenient equivalents for projected $2-D$ quantities. In particular, projection along the line of sight does not provide an equation of state

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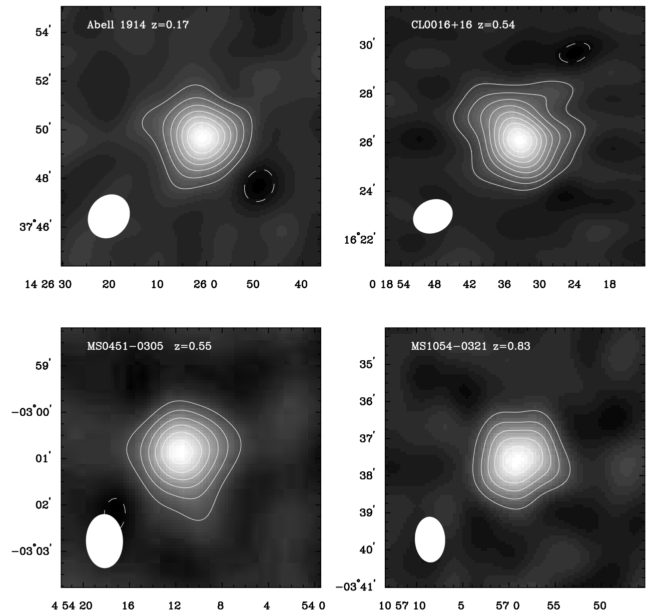

Figure 1: Images of the SZ effect observed towards four galaxy clusters with various redshifts. The contours correspond to 1.5 to 5 times the noise level. Data taken with the low-noise cm-wave receiver installed on the OVRO and BIMA mm-wave interferometric arrays [4].

or an hydrostatic equilibrium equation relating projected quantities. Therefore, if we want to compare WL, SZ and X-ray data, we have to deproject, explicitly or not, the relevant physical quantities ($P_g, T_g, \rho_g \ldots$ where $_g$ stands henceforth for the gas). This can be done only by using strong assumptions : either by using parametric models (e.g. a β model) or by assuming a mere geometrical hypothesis, a parametric model usually encompassing a geometrical assumption. Since our motivation is to use WL and SZ images (see fig. 1), we choose the geometric approach in order to use as general physical grounds as possible and to avoid theoretical biases as much as possible [5].

Our basic idea is the following. The quasi-circular (ellipsoidal) shapes of y-maps in SZ images and projected gravitational potentials in WL data obtained for some clusters result of an intrinsic quasi-sphericity (spheroidality) of the $3-D$ cluster structure (see fig. 1 for an illustration). The deviation to perfect symmetry is then considered as a perturbation which can be expressed by perturbing linearly the original symmetric model (which we know how to deproject). Then, a self-consistent inversion procedure allows us to infer, from observations, a set of complementary projected thermodynamical quantities, namely gas density and gas temperature. For both of them, we deduce zeroth order perfectly symmetrical (spherical or spheroidal) terms and first order (non symmetrical) corrections. By relating the properties of the gas and the gravitational potential we can then predict the X-ray luminosity map to first order.

3 Method

3.1 Hypothesis

Let us first assume that SZ and WL observations are described respectively by a y- and ϕ-function, which denote the amplitude of the measured SZ effect and the reconstructed gravitational potential. (R, φ) are polar coordinates in the image plane, and $_0$ and $_1$ suffixes mark respectively zeroth order terms (perfectly symmetrical) and first order corrections (non sym-

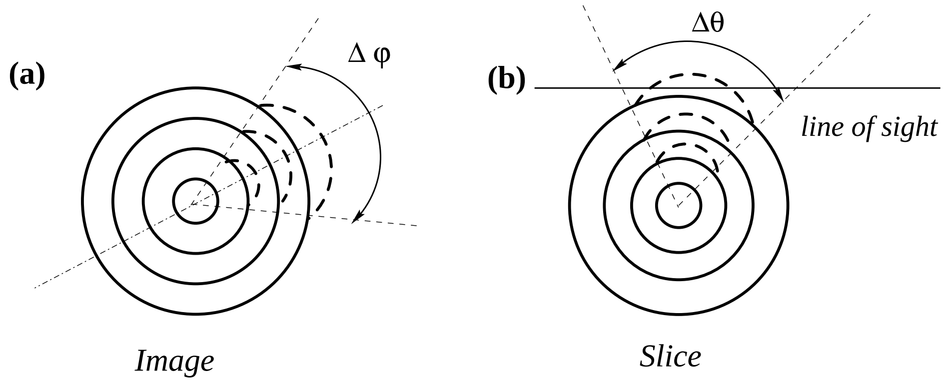

Figure 2: We represent schematically in (a) an image corresponding to our hypothesis. The full line corresponds to the perfectly circular $2-D$ term, e.g. $\phi_{DM,0}$, and the dashed line to the first perturbative correction to it, e.g. $\phi_{DM,1}m(\varphi)$, $\Delta\varphi$ represents the observed angular extent. In (b) we represent a schematic slice in the $3-D$ potential responsible for this image. This slice has been performed along the dash-two-dotted plane indicated on figure (a). Here again, the full line corresponds to the perfectly circular $3-D$ term, e.g. $\Phi_{DM,0}$, and the dashed line to the first perturbative correction to it, e.g. $\Phi_{DM,1}f(\theta,\varphi)$. The line of sight direction is indicated by the full thin line.

metrical):

$$
\begin{aligned}
y(R,\varphi) &= y_0(R) + \varepsilon y_1(R)\ m(\varphi) \\
\phi(R,\varphi) &= \phi_0(R) + \varepsilon \phi_1(R)\ n(\varphi)\ .
\end{aligned}
$$

In this illustrative development we will deal only with the spherical model, but the spheroidal case can be treated in a similar way, provided it is properly parametrised. We then assume that this is representative from an intrinsic $3-D$ *linearly perturbed spherical symmetry*, whose perturbations can be separated in an angular and a radial part. Consequently the gas pressure P_g, the gravitational potential Φ_{DM} (which we assume to be only due to the dark matter), the gas density ρ_g and the gas temperature T_g might be written as:

$$
\begin{aligned}
P_g(r,\theta,\varphi) = P_{g,0}(r) &+ \varepsilon P_{g,1}(r) f(\theta,\varphi) \\
\Phi_{DM}(r,\theta,\varphi) = \Phi_{DM,0}(r) &+ \varepsilon \Phi_{DM,1}(r) h(\theta,\varphi) \\
\rho_g(r,\theta,\varphi) = \rho_{g,0}(r) &+ \varepsilon \rho_{g,1}(r) g(\theta,\varphi) \\
T_g(r,\theta,\varphi) = T_{g,0}(r) &+ \varepsilon T_{g,1}(r) k(\theta,\varphi)\ .
\end{aligned}
$$

Assuming furthermore the *hydrostatic equilibrium*, the *equality of the ionic and electronic temperature* and the validity of the usual gas *equation of state* one can easily show that without restrictions:

$$
f(\theta,\varphi) = h(\theta,\varphi) = g(\theta,\varphi) = k(\theta,\varphi)\ .
$$

At this point, we can express SZ and WL observations, which provide the projected gas pressure and a projected gravitational potential. If $\alpha \equiv \frac{\sigma_T}{m_e c^2}$ then:

$$
\begin{aligned}
y(R,\varphi) &= \alpha \int P_{g,0}(r) dl +\ \varepsilon\alpha \int P_{g,1}(r) f(\theta,\varphi) dl \\
&\equiv y_0(R) +\ \varepsilon y_1(R) m(\varphi) \\
\phi_{DM}(R,\varphi) &= \int \Phi_{DM,0}(r) dl + \varepsilon \int \Phi_{DM,1}(r) f(\theta,\varphi) dl \\
&\equiv \phi_0(R) +\ \varepsilon \Phi_1(R) m(\varphi)\ .
\end{aligned}
$$

The crucial point now is to find how one can deduce from these observations zeroth order and first order maps. Let us analyze, for instance, the projected gas density that we note:

$$
\begin{aligned}
D_g(R,\varphi) &= \int \rho_{g,0}(r)dl + \varepsilon \int \rho_{g,1}(r) f(\theta,\varphi) dl \\
&\equiv D_{g,0}(R) + D_{g,1}(R,\varphi) \; .
\end{aligned}
$$

3.2 Zeroth order term

Had we the knowledge of $y_0(R)$ and $\phi_0(R)$, we could deproject them easily by use of the standard Abel's transform to get $P_{g,0}(r)$ and $\Phi_{DM,0}(r)$. Then, expressing to first order in ε the hydrostatic equilibrium equation:

$$
P'_{g,0}(r) = -\rho_{g,0}(r)\Phi'_{DM,0}(r) \; ,
$$

we would get easily $\rho_{g,0}(r)$ and thus $D_{g,0}(R)$ in terms of $P'_{g,0}(r)$ and $\Phi'_{DM,0}(r)$.
To gain access to these terms $y_0(R)$ and $\phi_0(R)$, we can just average over a succession of annulus the observed y and ϕ maps. We circularize this way the observed image (the amplitude of the averaged first order terms is negligible).

3.3 First order term

In order to compute a first order correction term to the previous one, namely $D_{g,1}(R,\varphi) = \int \rho_{g,1}(r) f(\theta,\varphi) dl$, we express the hydrostatic equilibrium equation to second order in ε :

$$
\rho'_{g,0}(r)\Phi_{DM,1}(r) = \rho_{g,1}(r)\Phi'_{DM,0}(r) \; ,
$$

which leads to $\int \rho_{g,1}(r) f(\theta,\varphi) dl = \int \frac{\rho'_{g,0}(r)}{\Phi'_{DM,0}(r)} \; \Phi_{DM,1}(r) f(\theta,\varphi) dl$, which can be approximated the following :

$$
\begin{aligned}
\int \rho_{g,1}(r) f(\theta,\varphi) dl &\simeq \left(\frac{\rho'_{g,0}(r)}{\Phi'_{DM,0}(r)}\right)_R \int \Phi_{DM,1}(r) f(\theta,\varphi) dl \\
&\simeq \left(\frac{\rho'_{g,0}(r)}{\Phi'_{DM,0}(r)}\right)_R \phi_1(R) m(\varphi) \\
&\simeq \left(\frac{\rho'_{g,0}(r)}{\Phi'_{DM,0}(r)}\right)_R (\phi_{DM,1}(R,\varphi) - \phi_0(R)) \; ,
\end{aligned}
$$

where $(f(r))_R$ denotes the values of f at the maximal radius r observed at R. The validity of this approximation is discussed in more details and numerically demonstrated somewhere else[3]. It has the obvious advantage to express the correction term to the zeroth order projected gas density in terms of the observed $\phi_1(R)$ first order terms and zeroth order order profiles derived above. Note that we choose to express this correction in terms of WL data but, using the equation of state we could have expressed it identically in terms of the SZ data. This last step is crucial to our method since we relate in this way the non symmetric part of various quantities.

Similarly, we can compute a second order map of projected gas temperature:

$$
\begin{aligned}
\zeta_g(R,\varphi) &= \int T_{g,0}(r) \; dl + \varepsilon \int T_{g,1}(r) f(\theta,\varphi) dl \\
&\equiv \zeta_{g,0}(R) + \zeta_{g,1}(R,\varphi)
\end{aligned}
$$

3.4 Predicted X-ray surface brightness map

The joint use of SZ and WL data produces all the physical quantities regarding the dynamical and thermodynamical stage of the cluster. So, we can in principle recover its X-ray properties as well. Assuming a given X emission model, the surface brightness is

$$b_X(E) = \frac{1}{4\pi(1+z)^3} \int n_e^2 \Lambda(E, T_e)\ dl\ .$$

Hence we have

$$\begin{aligned} b_X(E) &\propto \int n_e^2 T_e\ dl \\ &\propto \int \rho_g^2 T_g\ dl \\ &\propto \int \rho_{g,0}^2 T_{g,0} dl + \varepsilon \int (\frac{2}{\beta} P_{g,0}\rho_{g,1} + \rho_{g,0}^2 T_{g,1}) f(\theta, \varphi) dl \end{aligned}$$

If we thus use the maps derived previously we can therefore predict this quantity and compare it with X-ray surface brightness maps.

4 Discussion

Throughout this work we relied on very general physical hypothesis (hydrostatic equilibrium, equation of state) and a more geometrical hypothesis, namely a linear perturbation description. Since the second one constitutes a mere descriptive assumption, whose validity is demonstrated in view of observations, and since we worked in a fully self-consistent manner, our method brings a test of the first general physical hypothesis.
The numerical qualification of our hypothesis is under progress and will be soon presented[3]. More than a discussion of the method, we will as well properly discuss noise issues and systematics. This work should constitute a first step towards a full deprojection which will be presented elsewhere.

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