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A SCALING RELATION BETWEEN POTENTIAL ENERGY AND MASS FOR CLUSTER ELLIPTICAL GALAXIES



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We show from theoretical calculations that elliptical galaxies obey a scaling relation between potential energy and mass. We have also shown that their specific entropy is quasi-constant. These two laws define two 2-manifolds in the space defined by the three Sérsic law parameters (intensity Σ_0, scale a and shape ν). Elliptical galaxies are distributed on a thin line, which is the intersection of these two 2-manifolds. The scaling relation between potential energy and mass allows to naturally explain the origin of the observed correlation between absolute magnitude and mean surface brightness. From the theoretical location of elliptical galaxies in the thin line traced by the two laws, the photometrical plane can be naturally derived. These two relations are indeed observed in 142 galaxies belonging to three nearby clusters (with the assumption that light traces mass) and in simulations of dark halos. They have many consequences on galaxy formation, evolution and cosmology, and should allow to derive a new distance indicator.

1 Introduction

Elliptical galaxies present a striking regularity in their global luminosity distributions: their light profiles can be described by simple functions, such as the Sérsic law [13]:

$$\Sigma = \Sigma_0 \ \exp[\ -(R/a)^{\nu}\ ] \tag{1}$$

They are supposed to be formed under non collisional processes, where dissipation is expected to be negligible. Under these circumstances, both the initial conditions and the gravitational forces are expected to have a crucial influence on their properties.

We have shown in previous papers [9,10]:

- that the specific entropy of a sample of cluster ellipticals is constant;

- that another physical law must be operating to explain why ellipticals reside in a very thin, almost linear region of the so-called entropic plane.

In this contribution we show that this second law, which arises from a scaling relation between potential energy and mass, allows us to naturally derive the observed absolute magnitude/surface brightness correlation. We also show that, by taking into account the uniqueness of the specific entropy, it is also possible to obtain a theoretical explanation for the observed photometric plane.

2 A scaling relation between potential energy and mass

We consider a region presently supposed to be the location of an elliptical galaxy. Quantities are indexed with i – for initial and with p – for present.

The total mass and total energy are conserved during the formation and subsequent evolution:

$$M_i = M_p = M \tag{2}$$

$$E_{tot,i} = E_{tot,p} = E_{tot} \tag{3}$$

If T and U are the kinetical and potential energies, the total energy at the initial time is:

$$E_{tot} = T_i + U_i \simeq U_i \tag{4}$$

since the kinetical energy is negligible compared to the potential energy.

At the present stage, we have:

$$T_p + U_p = E_{tot} \tag{5}$$

$$2T_p + U_p = 0 \tag{6}$$

Eq. 7 is the virial relation (not applicable in the i-case).

Some algebra leads to the usual relation for a collapsing gravitational system:

$$U_p \simeq 2U_i \tag{7}$$

In the initial linear stage all quantities such as the typical perturbation length λ_i, the mass M, the potential energy U are related to one another:

$$M \propto \lambda_i^3 \quad \text{and} \quad U_i \propto M^2/\lambda_i \,,$$

so we have:

$$U_i \propto \frac{M^2}{M^{1/3}} \propto M^{5/3} \tag{8}$$

Therefore:

$$U_p \propto M^{5/3} \tag{9}$$

We then define e as:

$$e = \ln(U_p) - 5/3 \ln(M) \tag{10}$$

so that Eq. (9) becomes:

$$e = e_0 \tag{11}$$

where e_0 is a constant to be determined by observations.

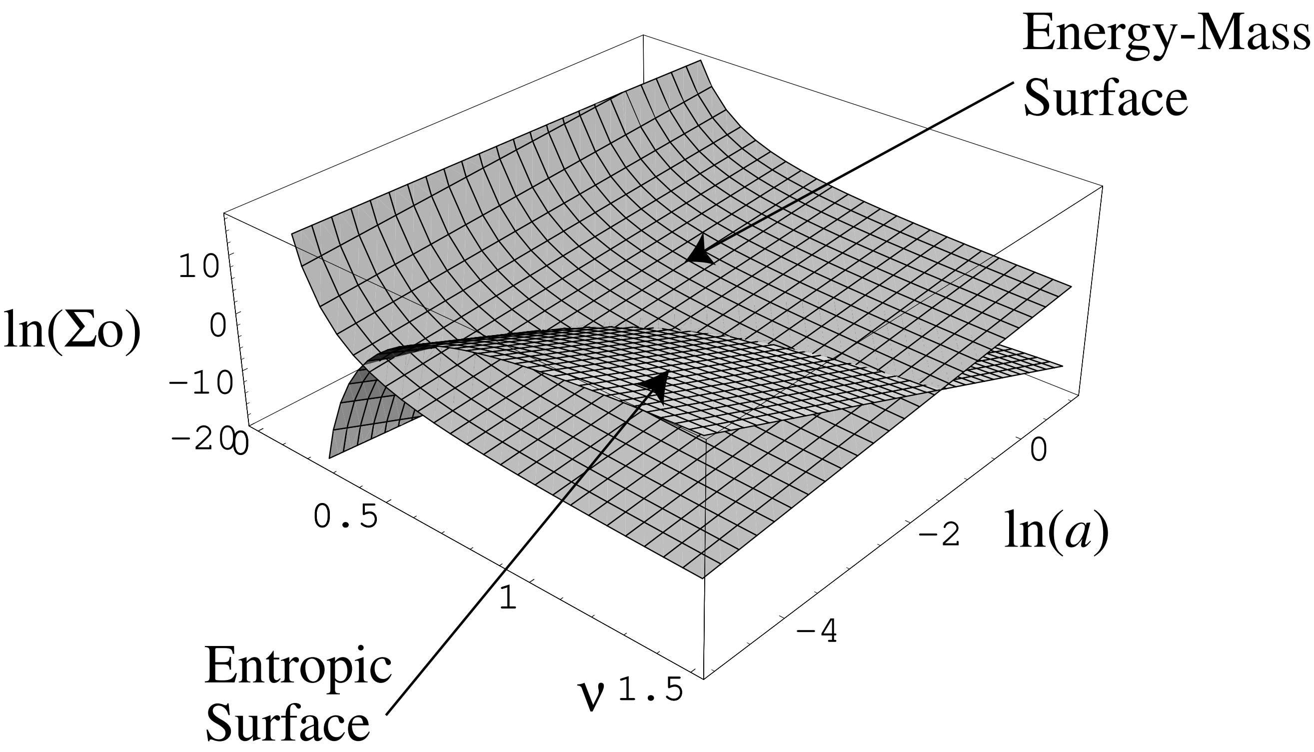

Figure 1: 3-D representation of the specific entropy and energy-mass surfaces, using the coordinates:[$\log \Sigma_0, \log a, \nu$].

3 Relations between the Sérsic parameters

To describe the brightness surface of elliptical galaxies, we choose a Sérsic law, a generalization of the de Vaucouleurs profile, given in Eq. (1). Using the parameters $[a, \nu, \Sigma_0]$, all quantities of astronomical interest can be calculated [3,9].

In a previous paper [9], we admitted the existence of a state of quasi-constant entropy and calculated the specific entropy $s = S/M$ (the entropy normalized by the mass) by deriving it from the observed light (mass) distribution. The specific entropy s is calculated by assuming that the stars obey the equations of state of an ideal gas and using the standard thermodynamical definition of the entropy. The assumption of an ideal gas can be eliminated by adopting the microscopic Boltzmann-Gibbs definition [10]. In both cases, the specific entropy s is constant:

$$s = s_0 \tag{12}$$

where s_0 is to be determined by observations.

The relations given by Eqs. (11) and (12) define two 2-manifolds in the Sérsic parameter space $[\nu, a, \Sigma_0]$. The intersection of the specific entropy surface and of the energy-mass surface is a line: the entropy-energy line. We show in Fig. 1 a 3-D sketch of these two surfaces.

The relations obtained imply two-by-two relations between the Sérsic parameters, which we now compare to observations.

4 Predictions faced to observations

4.1 Data set

The set of elliptical galaxies includes 74 galaxies in Coma, 34 in Abell 85 and 34 in Abell 496 (CCD imaging in the V band, and cluster membership confirmed by redshift). We assume

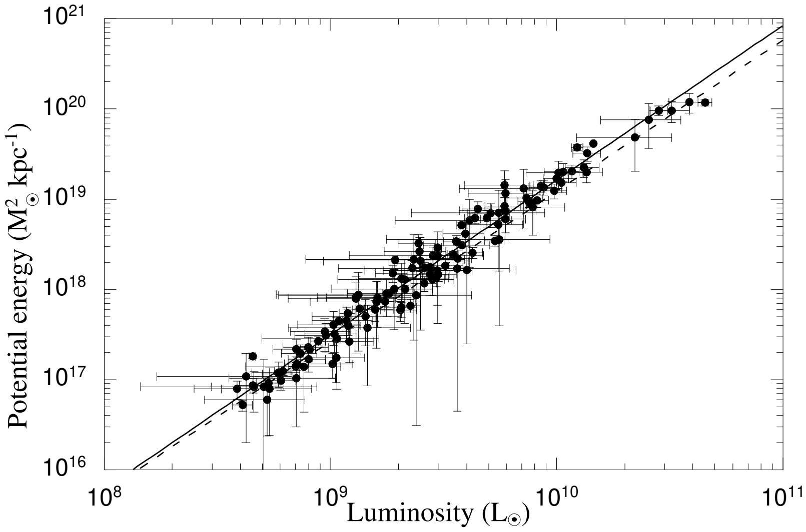

Figure 2: Potential energy versus luminosity. Power-law fits with indexes of 5/3 (full line) and the theoretical prediction with index of 1.69 (dashed line) are superimposed.

$H_0 = 100$ km s^{-1}Mpc^{-1} and $q_0 = 0$ throughout.

4.2 Energy-mass relation

The potential energy is displayed in Fig. 2 as a function of total luminosity. A free power law fit gives an index of 1.69 $\pm$0.01. If we impose the index to be equal to 5/3 (see Eq. 10), the fit is good enough to confirm that Eq. (10) is a good first order approximation.

4.3 Correlations

The relations predicted above can be translated observationally as correlations of the three Sérsic parameters taken two by two. In Fig. 3 we plot the correlation between $[a, \nu]$. The corresponding theoretical relation is superimposed on these figures and is in very good agreement with the data. This is also the case for the relations $[\Sigma_0, \nu]$ and [a, Σ_0].

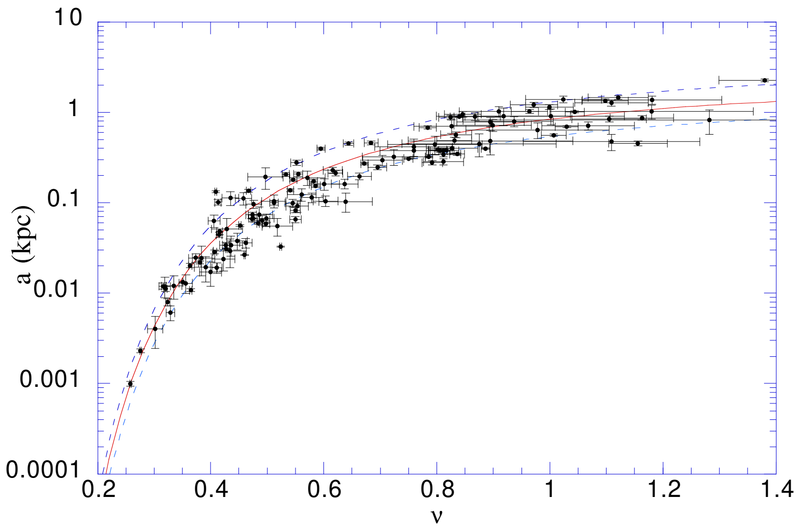

Figure 3: Correlation between the Sérsic parameters [a, ν].

5 From theoretical to observed (previous) correlations

The energy-mass scaling law allows to give a physical motivation to some previous observed correlations.

5.1 The Absolute Magnitude/mean surface brightness correlation

Relation (10) can be rewritten as:

$$e_0 = \frac{1}{3}[\ln M - 3 \ln r_g] , \tag{13}$$

where r_g is the gravitational radius. Introducing the definition of $\langle\mu\rangle_{eff}$:

$$\langle\mu\rangle_{eff} = -2.5 \log L_{\rm tot} + 5 \log R_{eff} + 2.5 \log(2\pi) \tag{14}$$

We introduce the absolute 'magnitude' $\mathcal{M} = -2.5 \log L_{\rm tot}$ in Eq. (14) and we obtain:

$$-\langle\mu\rangle_{eff} = -0.32\mathcal{M} + 13.426 \tag{15}$$

We give in Fig. 4 the observed correlations coming from our data together with the theoretical relation established above, showing that we can recover the observed correlations from the theory.

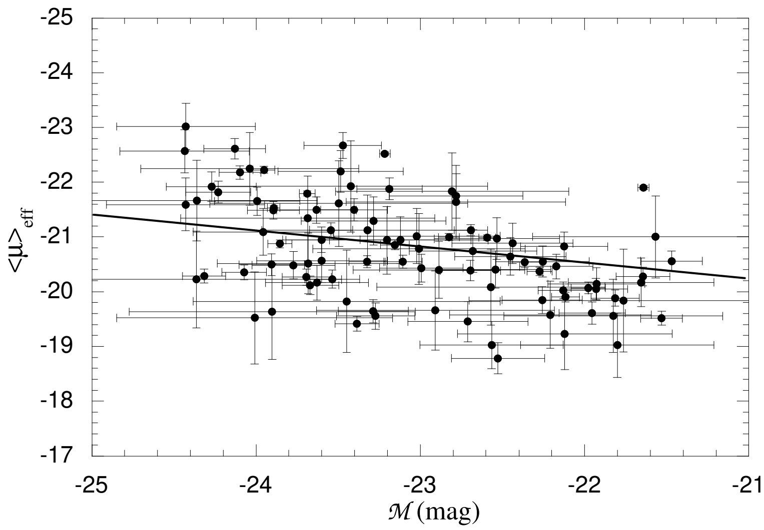

Figure 4: Distribution of the mean effective magnitude versus absolute magnitude. The theoretical relation given by eq. (15) is superimposed.

5.2 The Photometric plane

Whereas the previous relations have been obtained only by using the scaling law between potential energy and mass, in the derivation of the photometric plane we also have to make use of the uniqueness of the specific entropy.

Khosroshahi et al.[6] have fit a set of E and of spiral bulges with the Sérsic profile and shown that they lie in a "plane":

$$\begin{aligned}(0.173 \pm 0.025) \log R_{eff} - (0.069 \pm 0.007)\mu_0 = \\ - \log \nu - (1.18 \pm 0.05)\end{aligned} \tag{16}$$

(with $\mu_0 = -2.5 \log \Sigma_0$ and $n = 1/\nu$).

We use the definition of $\log R_{eff}$ and μ_0, introduce them in the Eqs. defining the two surfaces and obtain:

$$0.173 \log R_{eff} - 0.069\mu_0 = K_{\mu,5} \tag{17}$$

Eq. (17) is obtained from theoretical relations and Eq. (16) is a fit to the observed data. The only differences between these two relations are (by construction) the two right-hand sides, i.e. $(-\log \nu)$ can be compared to $K_{\mu,5}(\nu)$. The forms of the two functions are compared in Figure (5) in the interval used by Khosroshahi et al. , [0.25, 0.7]. The good agreement observed between the observed points our theoretical relation suggests that the observed properties of ellipticals (and with a larger dispersion spiral bulges) do obey the two derived laws.

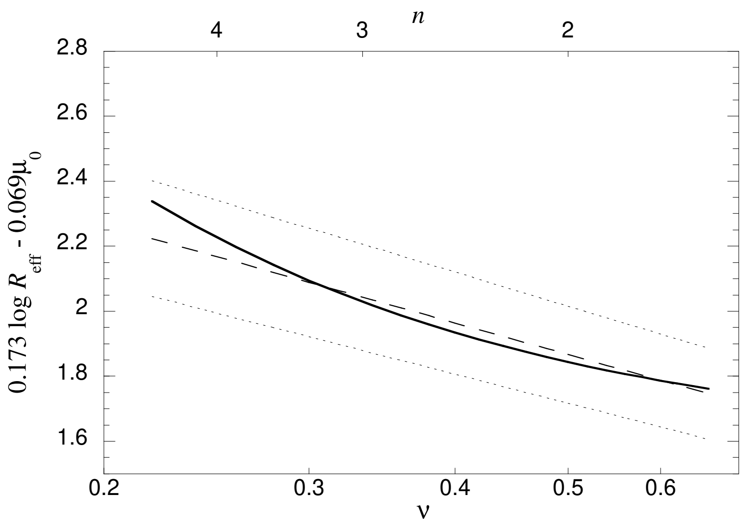

Figure 5: Theoretical relation (right-hand side of eq. 17) superimposed on the fit obtained by khosroshahi et al. 2000 (right-hand side of eq. 16). The two lines correspond to 8% errors on the fit. n=1/ν is given at the top of the figure to allow direct comparison with auther authors.

5.3 The Faber-Jackson relation

By considering the Virial Theorem together with the scaling law between potential energy and mass we can obtain:

$$\langle V \rangle^2 \propto M^{2/3} \quad \text{or} \quad M \propto \langle V \rangle^3 \,, \tag{18}$$

with $\langle V \rangle = \sqrt{2T/M}$. The similar relation linking comparable observables is the well-known Faber-Jackson relation [2]:

$$L \propto \sigma^{\alpha} \,, \tag{19}$$

with α varying from 2 to 4 (see, e.g., de Zeeuw & Franx [14]). It is nevertheless difficult to translate the theoretical quantity $\langle V \rangle$ into the observable quantity σ; this has been attempted for instance by Graham & Colless [4] and Graham [5]. This is in fact part of the broader problem of translating the virial theorem, based on theoretical quantities, into the fundamental plane, which is linked to observable quantities [1,4,12].

6 Dark matter halos

As a check of Eq. (10) we have computed the potential energy of DM halos found in N–body simulations, and we have plotted it against their mass (see Fig. (6)). In the selected sphere,

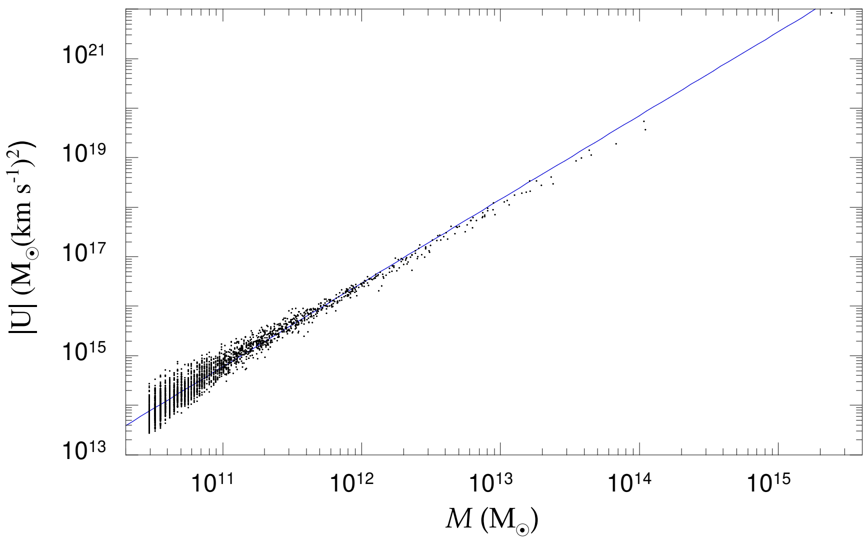

Figure 6: Potential energy–mass relation for about 2600 DM halos selected in a high–resolution N–body simulation. We used a sphere of radius $r = 7\,\mathrm{Mpc}/h$, with $h = 0.7$) around a very massive DM halo ($M \simeq 2.3\,10^{15}\,M_\odot/h$, virial radius $R_{vir} \simeq 2.7\,\mathrm{Mpc}/h$) has been used. The mass particle of the simulation is of about $2\,10^9\,M_\odot/h$, thus allowing to resolve halos down to $2\,10^{10}\,M_\odot/h$ (assuming a minimum mass of 10 particles per halo). The simulation is for a ΛCMD cosmological model, with density parameters $\Omega_0 = 0.3$ and $\Omega_\Lambda = 0.7$, and the Hubble constant H= $100\,h^{-1}$ km/s Mpc^{-1}. Potential energy is in units of $M_\odot$ (km/s)$^{-2}$, masses are in $M_\odot$. The red line is the best fit to the data, with a slope of 1.69 ± 0.01, in very good agreement with the expected value of $5/3 = 1.7$.

about 2600 halos of masses ranging from $2.0\,10^{10}$ to $2.3\,10^{15}\,M_\odot/h$ are found. The slope of the best fitting line is 1.69 ± 0.01, in very good agreement with the expected value ($5/3 = 1.67$).

7 Conclusions

We have shown both from theoretical reasons and from observations, that elliptical galaxies obey the same scaling relation between the potential energy and mass (luminosity). This physical law allows us to naturally derive:

- the correlation between the absolute magnitude and the central brigntness;
- the Faber-Jackson relation.
- the Photometric Plane (also considering that elliptical galaxies share the same specific entropy).

We have verified the validity os these resuls for

- photometric data on cluster ellipticals;
- simulations of dark matter halos;
- the hot gas distribution in clusters as traced by the X-ray emission (see Demarco, these proceedings).

These two laws constitute a theoretical background to a number of physical properties of elliptical galaxies, and have clear implications for cosmology and models of galaxy formation and evolution. We also note that the energy-entropy line can be used as a distance indicator method. These results will be described elsewhere [11].

Acknowledgments

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References

1. Capelato H. V., de Carvalho R.R., Carlberg R.G., 1995, ApJ 451, 525
2. Faber S.M., Jackson R.E., 1976, ApJ 204, 668
3. Gerbal D., Lima Neto G.B., Márquez I., Verhagen H., 1997, MNRAS 285, L41
4. Graham A., Colless M. 1997, MNRAS 287, 221
5. Graham A., 1998, MNRAS 295, 933
6. Khosroshahi H.G., Wadadekar Y., Kembhavi A., Mobasher B. 2000, ApJ, 531, L103
7. Kormendy J., 1977, ApJ 218, 333
8. Kormendy J., Djorgovski, S., 1989, ARAA 27,235
9. Lima Neto G.B., Gerbal D., Márquez I., 1999, MNRAS 309, 481 (Paper I)
10. Márquez I., Lima Neto G.B., Capelato H., Durret F., Gerbal D. 2000, A&A 353, 873
11. Márquez I., Lima Neto G.B., Capelato H., Durret F., Gerbal D., Lanzoni, B. 2000b, in preparation
12. Prugniel Ph., Simien F., 1997, A&A 321, 111
13. Sérsic J.L., 1968, "Atlas de galaxias australes, Observatorio Astronómico de Córdoba", Argentina
14. de Zeeuw T., Franx M., 1991, ARAA 29, 275