BIGGER RIP WITH NO DARK ENERGY

There are two ways to accommodate the accelerating expansion in the Friedmann equation (we will assume $k=0$, a flat universe):

(1) Keep GR and add a DE term to the RHS

(2) Modify GR and omit DE.

The more conservative approach is (1) based on the spectacular success of GR at Solar System scales.

However, there are two reasons to entertain seriously possibility (2):

a) The Universe has size 10^{28} cm compared to the Solar System 10^{14} cm. There is no independent evidence that GR survives this huge extrapolation.

b) DE has bizarre properties so its avoidance might be considered.

DGP gravity suggested by

G. Dvali, G. Gabadadze and M. Porrati
PL B485, 208 (2000). hep-th/0005016.

is one modification of GR.

A 3-brane is embedded in Minkowski M_5

with an intrinsic curvature term :

$$\boxed{S = M(t)^3 \int d^5X \sqrt{G}\, R^{(5)} + M_{Pl}^2 \int d^4x \sqrt{g}\, R}$$

$R^{(5)}$ = 5-dimensional scalar curvature
$M(t)$ = time-dependent 5-dim. Planck mass.
G = determinant of 5-dim metric

At small distances, Newton's law is recovered on the brane.

At large distances $F \sim \frac{1}{r^3}$ in 5-dim.

The length scale where the two regimes cross is

$$\boxed{L(t) = \frac{M_{Pl}^2}{M(t)^3}}$$

The t-dependence represents a generalization of the original DGP approach.

4-dimensional coordinates are labeled by
$i, k = 0, 1, 2, 3$

Einstein's equations in empty space are modified to

$$\boxed{\left(R^{ik} - \frac{1}{2} R g^{ik}\right) + \frac{2\sqrt{G}}{4(\varepsilon)\sqrt{g}} \left[\!\left[\left(R^{(s)ik} - \frac{1}{2} G^{ik} R^{(s)}\right)\right]\!\right] = 0}$$

where the notation is

$$\boxed{\int dx \, [[(F(x))]] \equiv f(x)\, S(x)}$$

Generalising the Schwarzschild solution leads to a modified potential $V(r)$:

$$\boxed{V(r) = -\frac{GM}{r} - \frac{2\sqrt{2}\,\sqrt{r_g r}}{L}}$$

$L \gg r \gg r_g$
weak gravity regime

where

$$r_g = 2GM$$

is the Schwarzschild radius

Fractional change in V is

$$\boxed{\left|\frac{\Delta V}{V}\right| = \sqrt{\frac{8r^3}{L^2 r_g}}}$$

To explore this we assume a power-law time dependence

$$\boxed{L(t) = L(t_0)\, T(t)^p}$$

where $T(t) = \left(\dfrac{t_{rip} - t}{t_{rip} - t_0} \right)$

We assume $p > 0$ so that $L(t)$ decreases with increasing t. (for $p < 0$ it increases)

A gravitationally-bound system will become unbound at $t = t_u$ estimated by $\left|\frac{\Delta V}{V}\right| = 1$

$$\boxed{\frac{8 r^3}{L(r_u)^2 r_g} = 1}$$

$$\boxed{t_{rip} - t_u = \frac{1}{8} \left(\frac{8 \ell_0^3}{L_0^2 r_g} \right)^{\frac{1}{2\beta}}}$$

where $\gamma = (t_{rip} - t_0)^{-1}$ and $\ell_0 =$ characteristic scale of bound system.

At a later time $t = t_{caus}$ the previously bound system became causally disconnected until t_{rip}:

$$\boxed{t_{rip} - t_{caus} = \frac{\ell_0}{c} \left(\frac{a(t_{caus})}{a(t_u)} \right)}$$

Let us consider an example for $\beta = 1$ (Big Rip)

With $p = 1$ $\quad$ $L(t_0) = H_0^{-1} = 1.3 \times 10^{28}$ cm

$\gamma = (20\,\mathrm{Gy})^{-1}$: $\quad$ ($t$ime $t_{rip} - t_0 = 20$ Gy)

	l_0(cm)	r_g(cm)	$t_{rip} - t_u$	$t_{rip} - t_{caus}$
GALAXY	5×10^{22}	3×10^{16}	100My	4My
SUN-EARTH	1.5×10^{13}	3×10^{5}	2mos	31 hr
EARTH-MOON	3.5×10^{10}	0.86	2wks	1hr

As another example:

$p = 11$ $\quad$ $L(t_0) = 1.3 \times 10^{28}$ cm

$\gamma = (50\,\mathrm{Gy})^{-1}$: $\quad$ $t_{rip} - t_0 = 50$ Gy

	l_0(cm)	r_g(cm)	$t_{rip} - t_u$	$t_{rip} - t_{caus}$
GALAXY	5×10^{22}	3×10^{16}	250My	7My
SUN-EARTH	1.5×10^{13}	3×10^{5}	5mos	2 days
EARTH-MOON	3.5×10^{10}	0.86	1mo	2hrs

With a longer wait until rip disintegration of structure and causal disconnection occur correspondingly earlier before eventual Rip.

$p = 1$ resembles the Big Rip, so now we investigate general $p > 1$ which gives a Bigger Rip: a more singular $a(t)$ at $t \to t_{rip}$

As a specific example we will look at $p = 2$ but develop the formalism for general p.

The modified Friedmann equation for DGP gravity is

$$\boxed{H^2 - \frac{H}{L(t)} = 0}$$

leading to

$$\boxed{\frac{\dot{a}}{a} = H = H(t_0)\,\frac{1}{T^p}}$$

Define as before $\gamma = -dT/dt$ then

$$\ln a(t) = -\int_1^T \frac{dT}{\gamma L_0 T^p}$$

For $\underline{p=1}$ $\quad a(t) = T^{-\frac{1}{\gamma L_0}}$ $\quad$ like $\omega < -1$ DE

but for $p > 1$ there is the Bigger Rip

$$\boxed{a(t) = a(t_0) \exp\left[\left(\frac{1}{T^{p-1}} - 1\right)\frac{1}{(p-1)\gamma L_0}\right]}$$

which diverges more essentially as $T \to 1$

Inversion gives:

$$\boxed{T = \left[1 + (p-1)\gamma L_0 \ln a(t)\right]^{-\frac{1}{(p-1)}}}$$

We shall regard this as <u>no dark energy</u> although any function can be re-expressed as some $W_{eff}(t)$

$$\boxed{W_{eff}(t) = -1 - \frac{2}{3}\left(\frac{p\gamma L_0}{1+(p-1)\gamma L_0 \ln a(t)}\right)}$$

So that

$$W_{eff}(t_0) = -1 - \frac{2}{3} p \gamma L_0$$

and

$$W_{eff}(t \to t_{rip}) \to -1$$

e.g. for $p=1$ $\gamma = (20\,Gy)^{-1}$

$$W_{eff}(t_0) = \underline{\underline{-1.47}}$$

<u>is</u> allowed with this parametrization (see below)

Here is an example for $p=2$

$$\gamma = (20\ \mathrm{Gy})^{-1} \qquad L_0 = 1.3 \times 10^{28}\ \mathrm{cm}$$

	l_0 (cm)	r_g (cm)	$t_{rip} - t_{U}$	$t_{caus} - t_U$
GALAXY	5×10^{22}	3×10^{16}	2.37 Gy	1.1 Gy
SUN-EARTH	1.5×10^{13}	2.95×10^{5}	9.6×10^{4} y	7 y
EARTH-MOON	3.5×10^{10}	0.86	2.5×10^{4} y	6 MOS

Compared to the $p=1$ Big Rip the $p=2$ Bigger Rip with other parameters same leads to more rapid expansion,

earlier t_U

and much earlier t_{caus} (not long after t_U : why we tabulate $t_{caus} - t_U$)

Observational Constraints.

We must include (especially for the past) the DM. Including all components gives the generalised Friedmann equation

$$\boxed{\left(H^2 + \frac{k}{a^2}\right) = \left(\sqrt{\frac{\rho_m}{3M_{Pl}^2} + \frac{1}{4L^2}} + \frac{1}{2L}\right)^2}$$

Defining $\Omega_M = \frac{\rho_m}{\rho_c} = \rho_{m0}(1+z)^3$:

$$H^2 = H_0^2 \left[\Omega_k (1+z)^2 + \left(\sqrt{\Omega_L} + \sqrt{\Omega_L + \Omega_M (1+z)^3} \right)^2 \right]$$

At the present time:

$$\boxed{\Omega_k + \left(\sqrt{\Omega_L} + \sqrt{\Omega_L + \Omega_M}\right)^2 = 1}$$

This allows us to compare to the SNe1a data:

We show the cases $\underline{\gamma = 1/(15 G_y)}$

and $\gamma = \underline{1/(30 G_y)}$ with $p = 2$

Shown (next transparency) are in Ω_L–Ω_m plot

95% CL (<u>dotted</u>)

99% CL (<u>dashed</u>)

<u>Solid line is flat $k=0$</u> (using modified constraint!)

(The lowest plot is for L = constant.)

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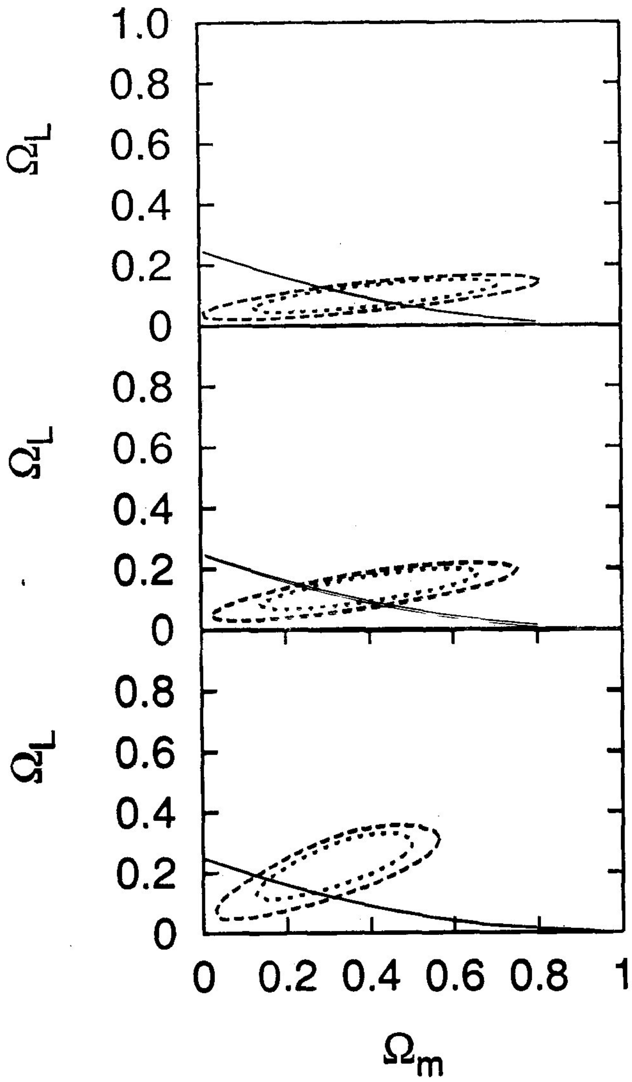

Figure 1: Constraint from the SNeIa observation [8] in the Ω_L-Ω_m plane for the case with the constant L (bottom), $\gamma = 1/15$(Gyr) (middle) and $\gamma = 1/30$(Gyr) (top). Here we take $p = 2$. Contours are for 95 % (dotted line) and 99 % (dashed line) C.L. constraints respectively. The solid line indicates parameters which give a flat universe.

The final figure shows the $\gamma - \Omega_M$ plane for a $k=0$ flat universe.

Putting $\Omega_M = 0.3$ leads to $\boxed{\frac{1}{\gamma} > 14\,Gy}$

Note that the effective $w_{eff}(t_0)$ can be more negative than for $w =$ constant

e.g. $\Omega_M = 0.3$, $\frac{1}{\gamma} = 14\,Gy$ permits for $p = 2$

$$w(t_0) = -1 - \frac{2}{3} p \gamma t_0 = -2.9$$

(c.f. $w > -1.2$ for constant case)

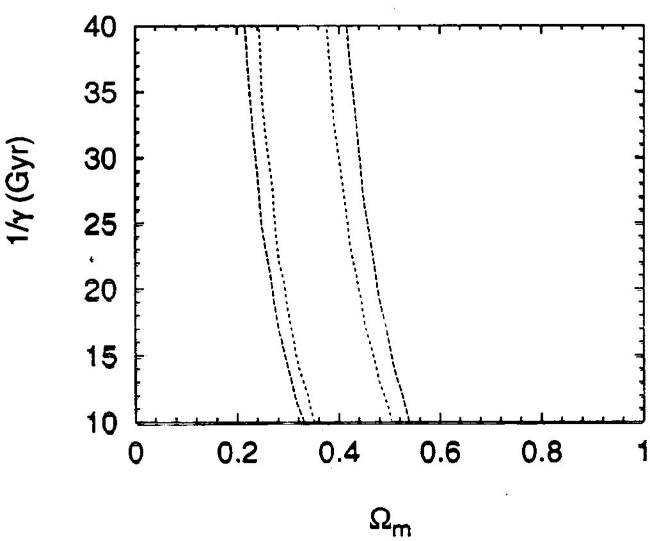

Figure 2: Constraint from SNeIa observation in the γ-$\Omega_{\rm m}$ plane. Contours are for 95 % (dotted line) and 99 % (dashed line) C.L. constraints respectively. In this figure, we assume a flat universe and $p = 2$.

DISCUSSION

The Big and Bigger Rip can be philosophically more attractive than the standard model. Intricately-formed structure from the past is systematically disintegrated and causally disconnected as we approach the $t \to t_{rip}$.

Here we have modified the LHS (geometry) of the Friedmann equation. Although the term can be reinterpreted on the RHS there is no simple $w = \frac{p}{\rho}$ as in a conventional dark energy.