Dynamical Formation of Black Hole Binaries in Star Clusters

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- Based on the following papers
  - Lee (1995) MNRAS, 272, 605
  - Lee (2001) CQG, 18, 3977
  - Bae, Lee & Kim et al. 2014, MNRAS, 440, 2714
  - Hong & Lee 2015, MNRAS, 448, 754
  - Park et al. 2017, MNRAS, in press

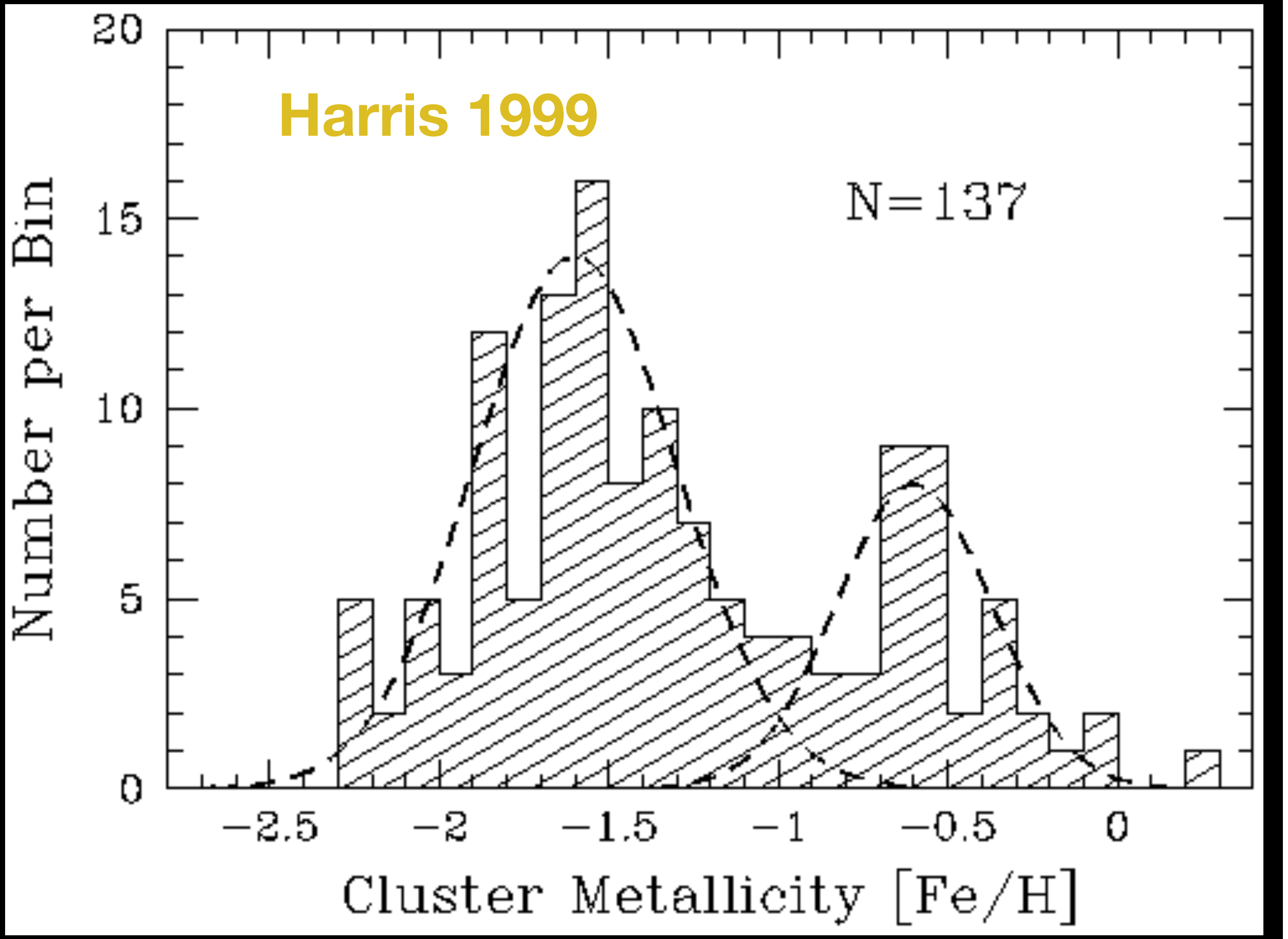
Outline

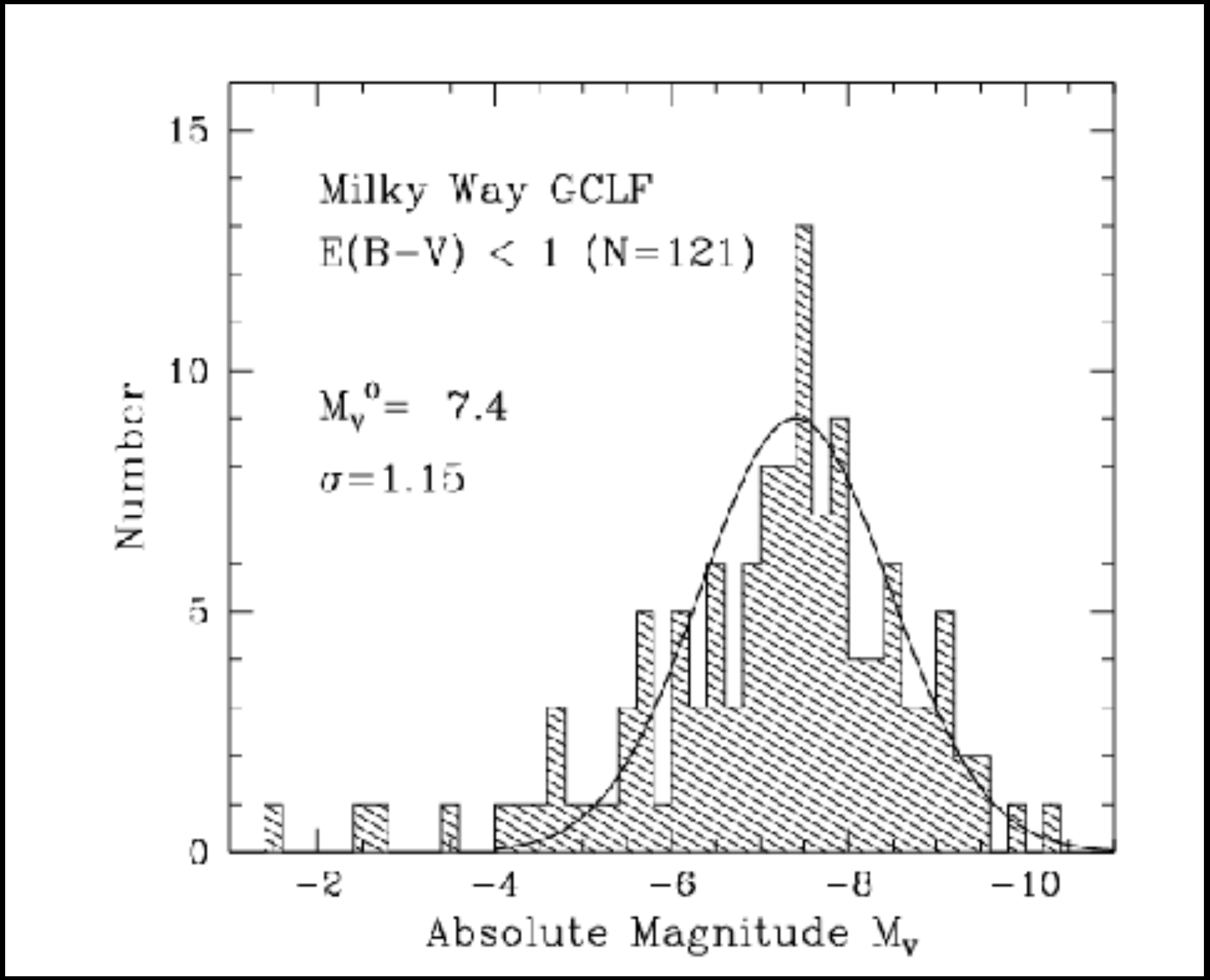
- Globular Clusters
- Dynamical Formation of BH Binaries in Globular Clusters
- Properties of the Dynamical Binaries
- Summary

Short Description of Globular Clusters

- Nearly spherical objects
- Extremely dense systems
- Considered to be the oldest objects in the Galaxy
- Our Galaxy has ~150 clusters, and about 50 more could be hidden by Galactic extinction.
- Stars are formed at the same time
    - GCs have been thought to be ideal objects to study N-body dynamics and stellar evolution

Physical Parameters





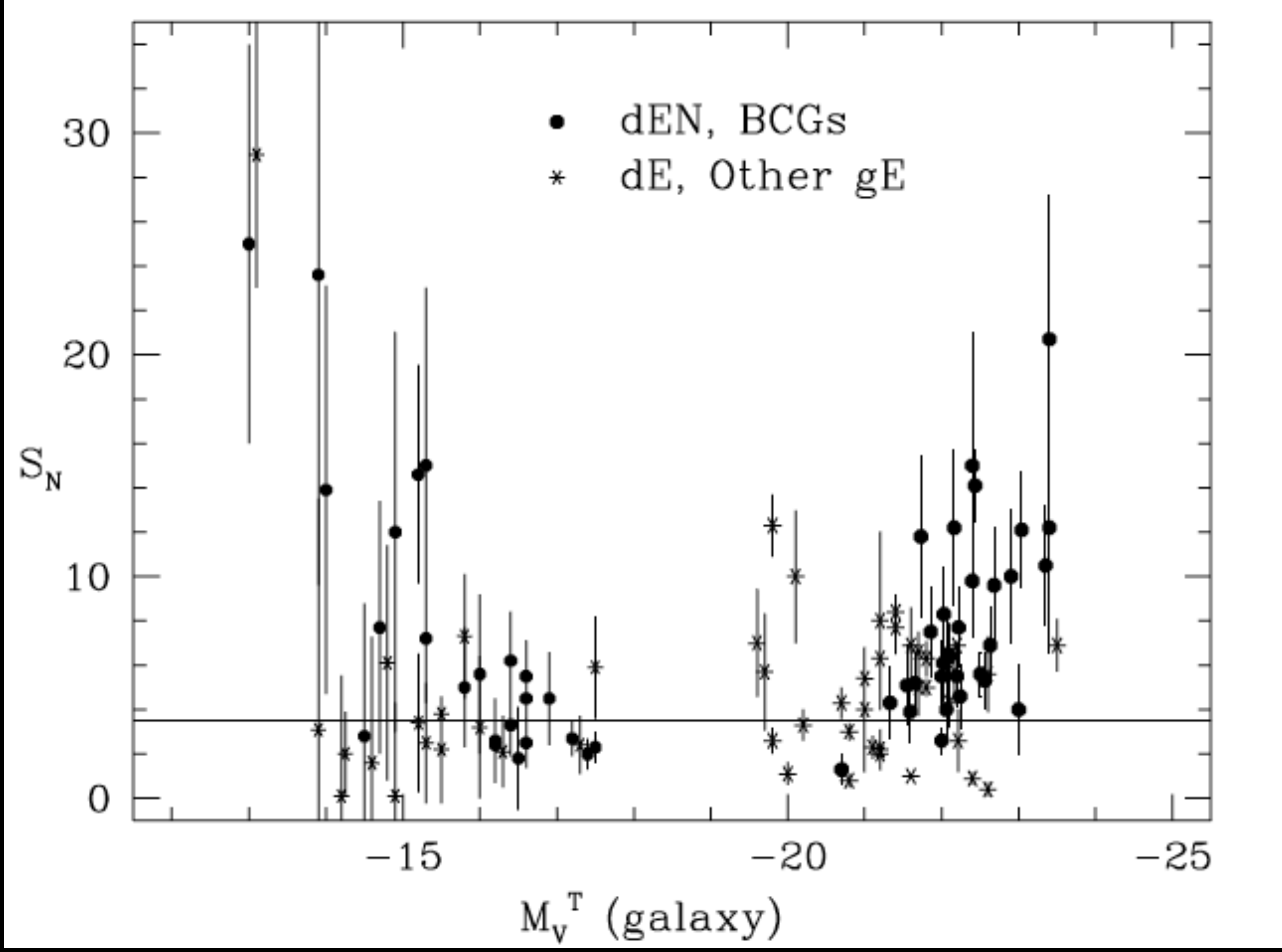
- Typical masses: 10^4 - 10^6 M_{sun}.
- Metallicity is lower than the Sun
- Current luminosity function is nearly log-normal: maybe a consequence of long dynamical evolution

Globular Clusters Host Many Compact Binaries

- Millisecond pulsars
  - ~80 times more frequent per stellar mass than the Galaxy at large
- X-ray binaries
  - Also over abundant in globular clusters by a large factor

Most Galaxies also Host Globular Clusters:

- Specific frequency S_N: number of GCs per luminosity
- $S_N \sim 1$ for spirals, and much higher for ellipticals
- Are extragalactic GCs similar to Galactic GC?
- What is the mean S_N?



Harris 1999

Dynamics of Globular Clusters

- Self-gravitating
- Composed of mostly point masses
- Weak field (i.e., Newtonian dynamics)
- General evolution is well understood, but we do not know the details mostly due to
  - Long range nature of the gravity
  - Poor understanding of the initial conditions
  - Complicated microphysics such as stellar evolution, effects of external fields, etc.

Time scales

- Dynamical time: time for a star to make a single trip across the entire system

$$t_{dyn} \approx \frac{v}{R}$$

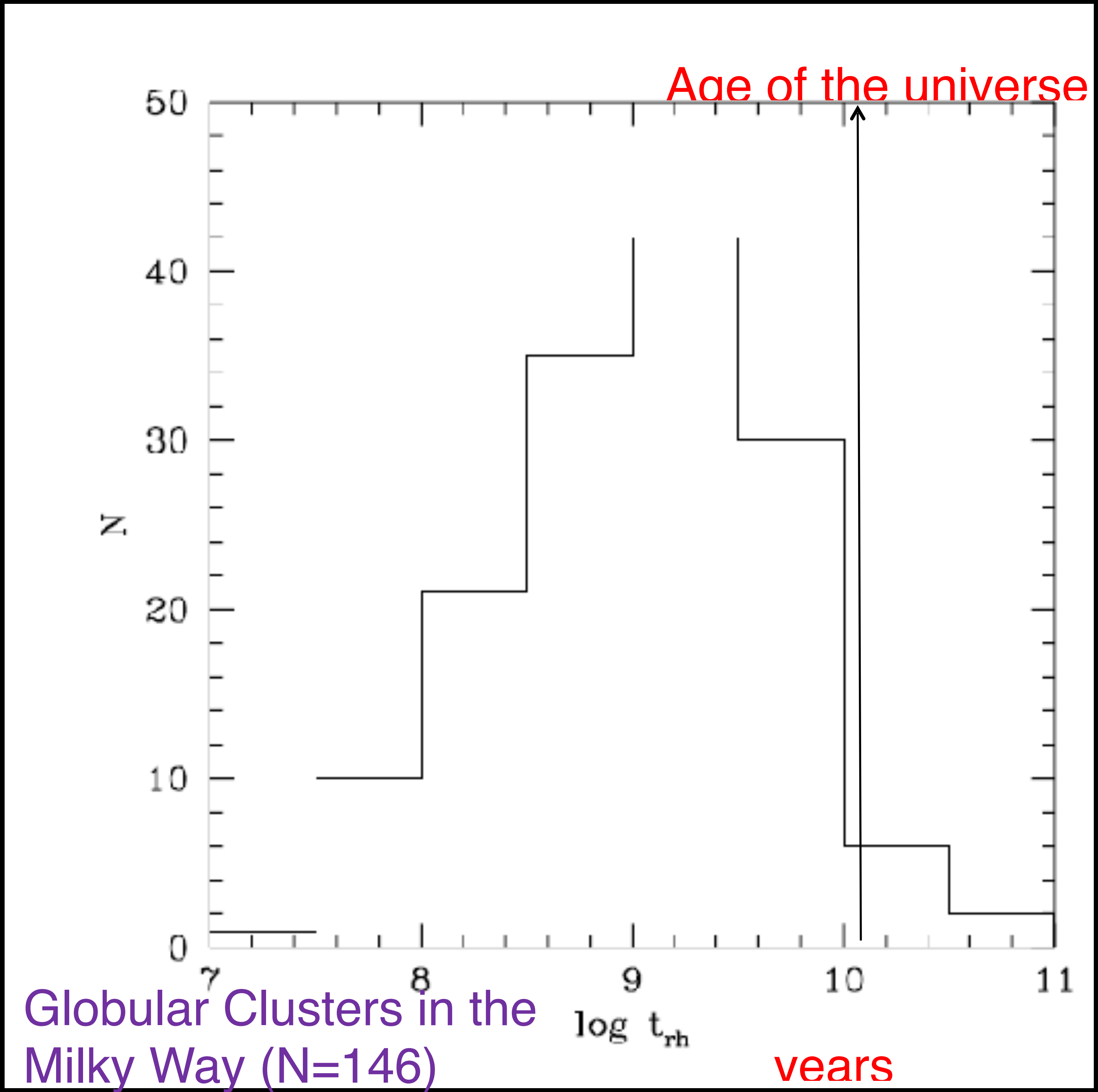
- Relaxation time: time for a star to lose its initial memory of orbit (or energy)
- If the system is self-gravitating (i.e., in virial equilibrium)

$$t_{\rm rel} \approx \frac{0.1N}{\ln N} t_{dyn}$$

—> $t_{rel} >> t_{dyn}$ if N becomes large (> 1000)

Relaxation Times of Globular Clusters

- During $t << t_{rel}$, stellar orbit is determined by the smooth potential of the system: collisionless
- In the long run, the stellar orbit will deviate from the original one: collisional
- Dynamical evolution takes place in relaxation time scales
- Most of the globular clusters have t_{rel} < age of the universe
  - Dyamical evolution has to be taken into account

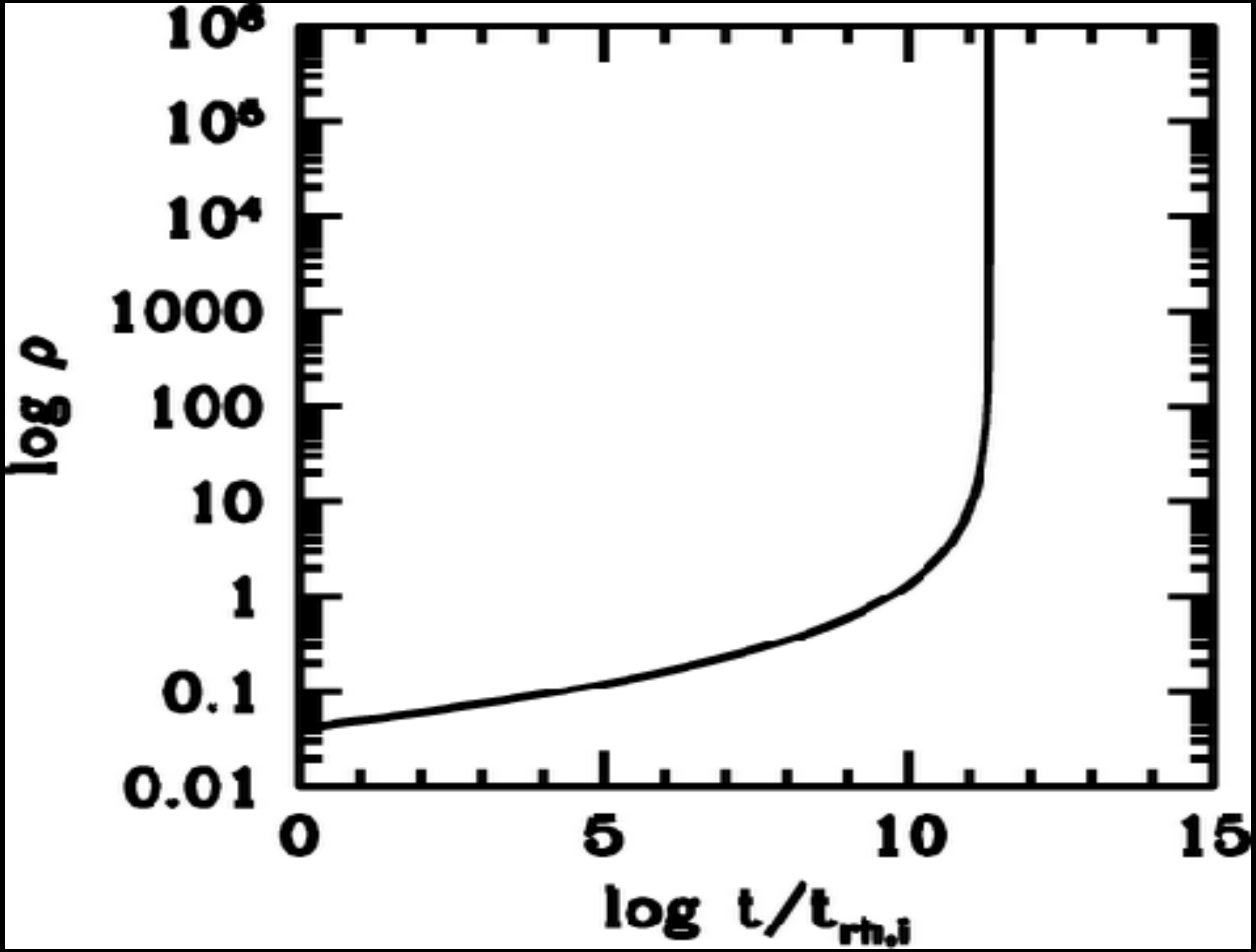


$$t_{rh} = 0.138 \frac{N^{1/2} r_h^{3/2}}{m^{1/2} G^{1/2} \ln(\gamma N)}$$

Course of Evolution

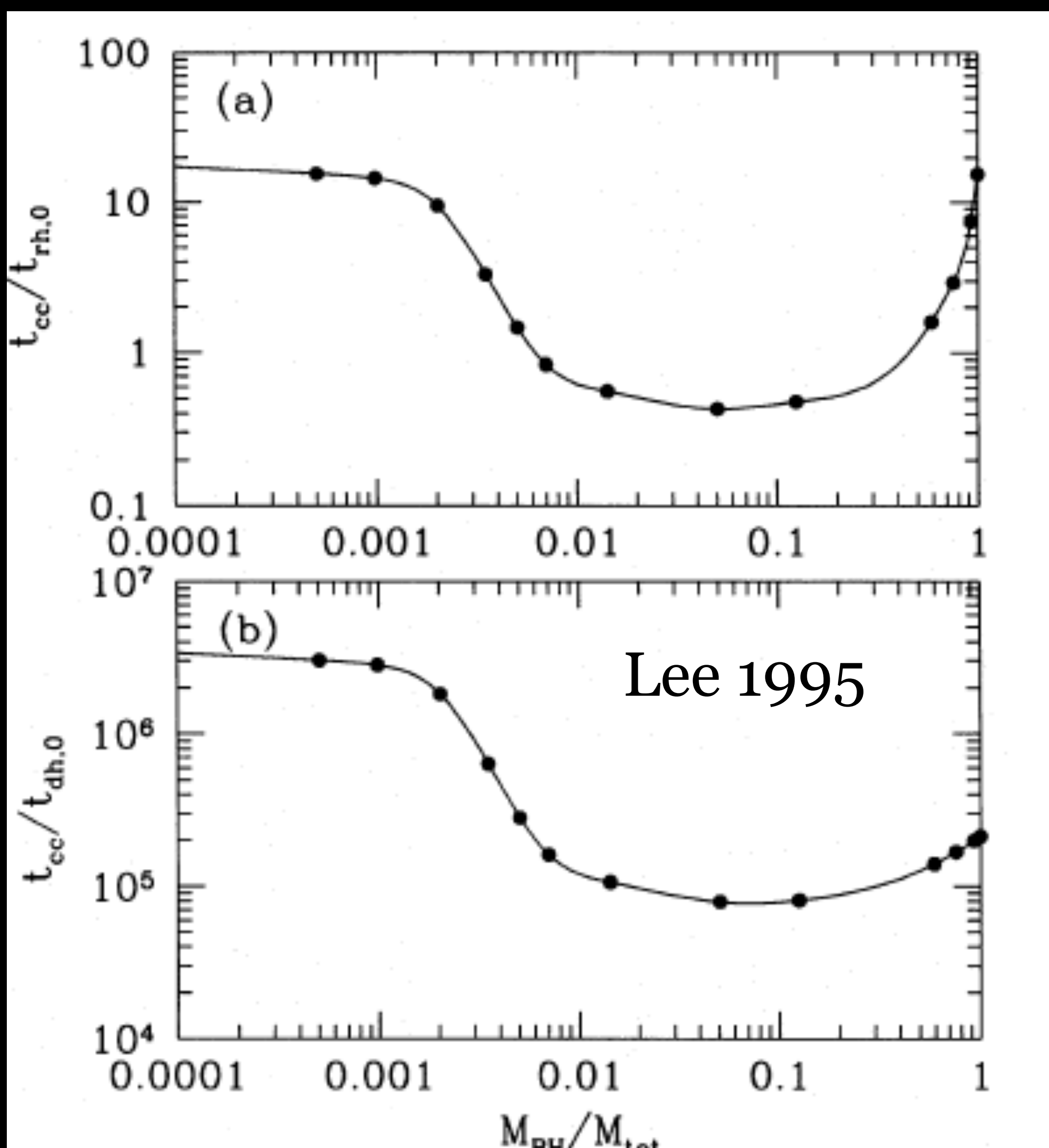
- Self-gravitating systems undergo core-collapse via two-body relaxation

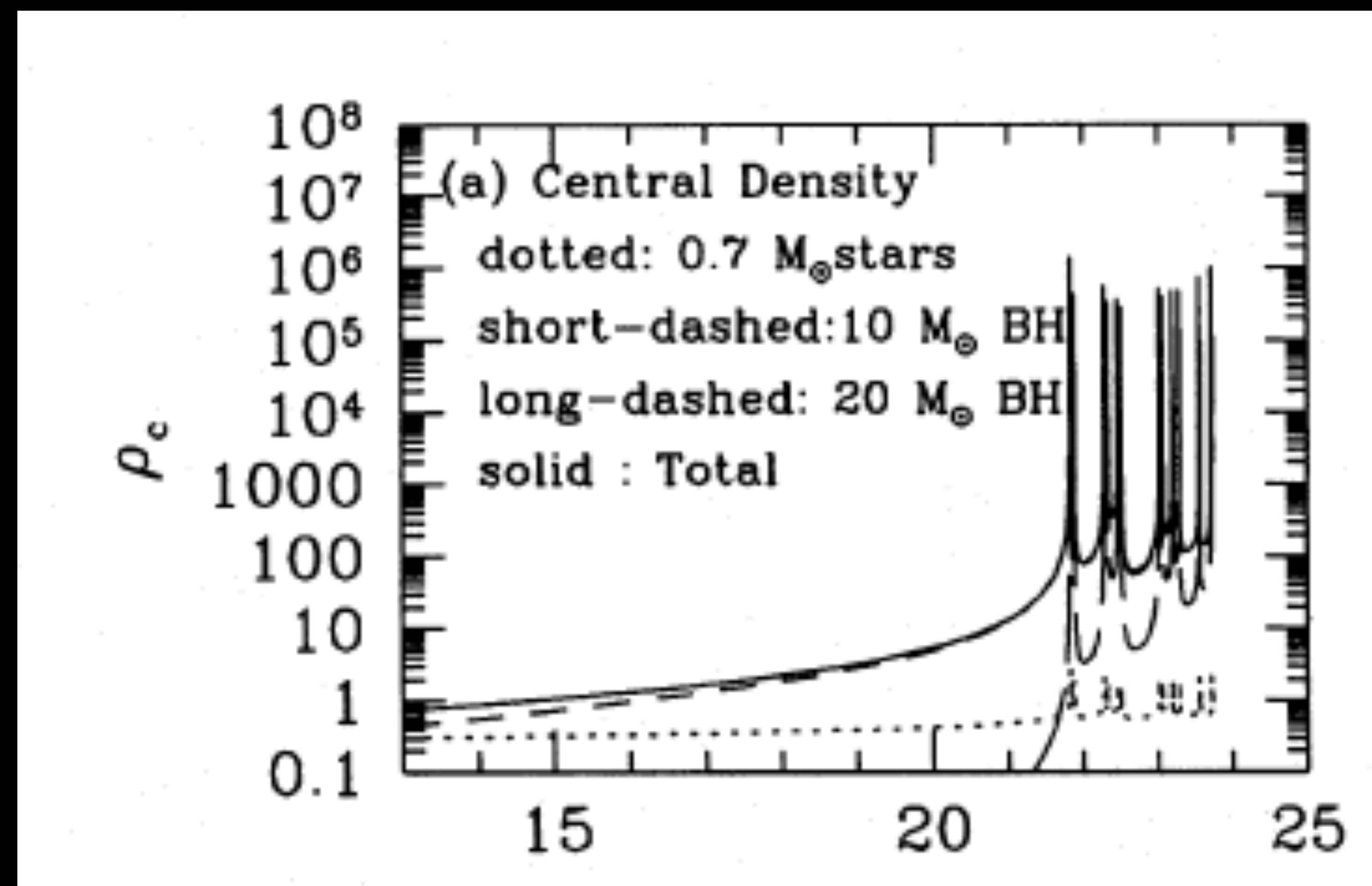
- The central part becomes very dense
- The time scale for this is ~10 t_{rh}



Based on the integration of Fokker-Planck equation

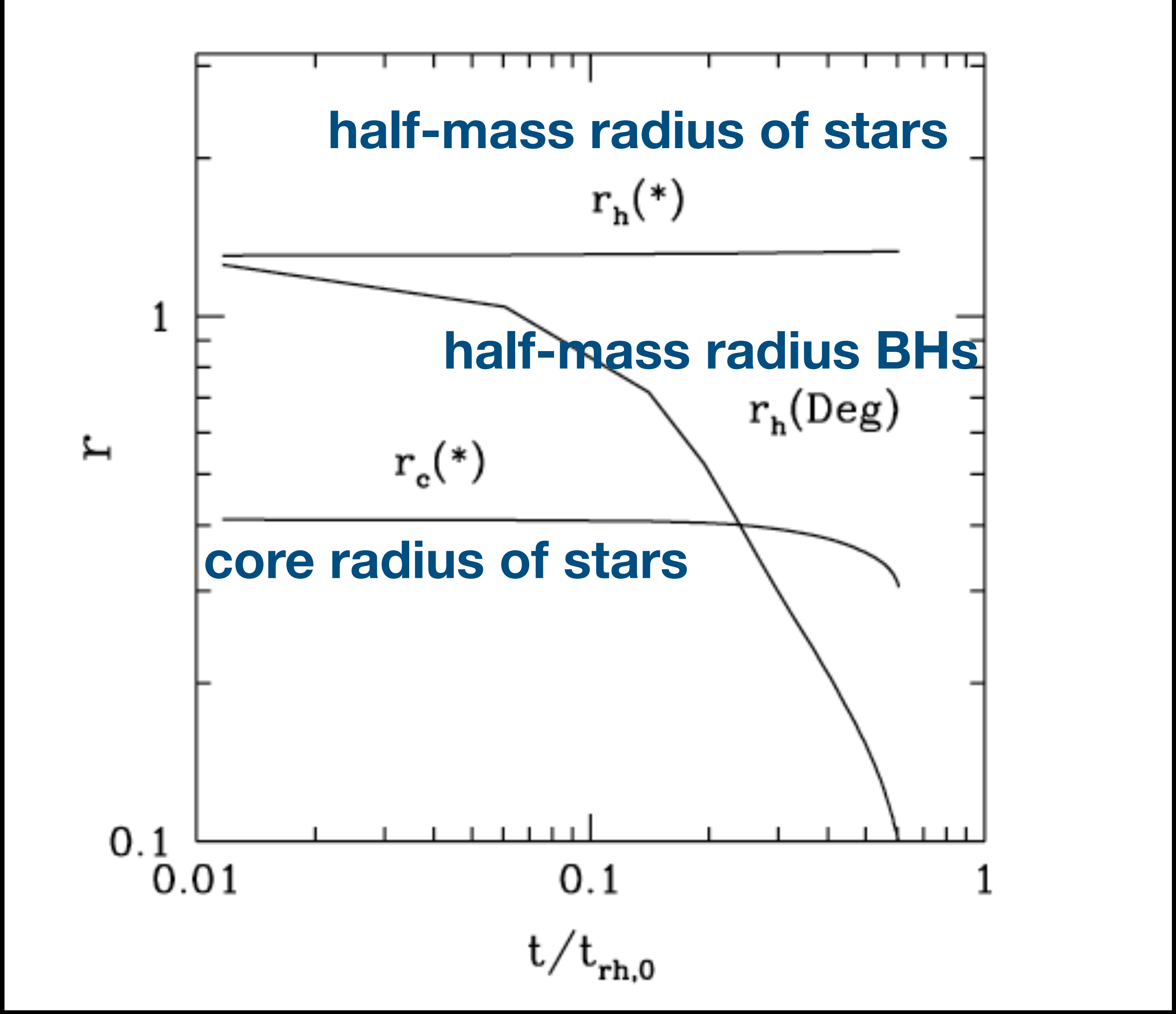
Example of Cluster Evolution with Black Holes





Dynamical Friction

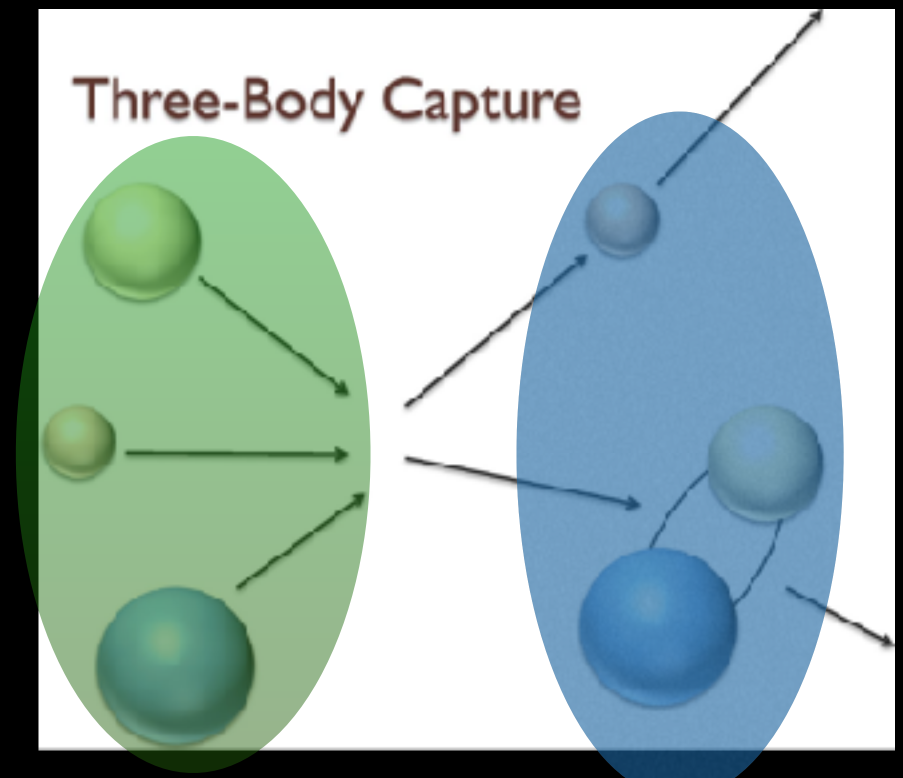
- Black holes become the most massive component within ~10^7 years in globular clusters
- In order to reach equipartition, massive component lose energy and settle into the central region (dynamical friction)
- Friction time scale is ~$m_*/m_{\rm BH}$ < 1/10 times relaxation time
  - Central parts are completely dominated by BHs.

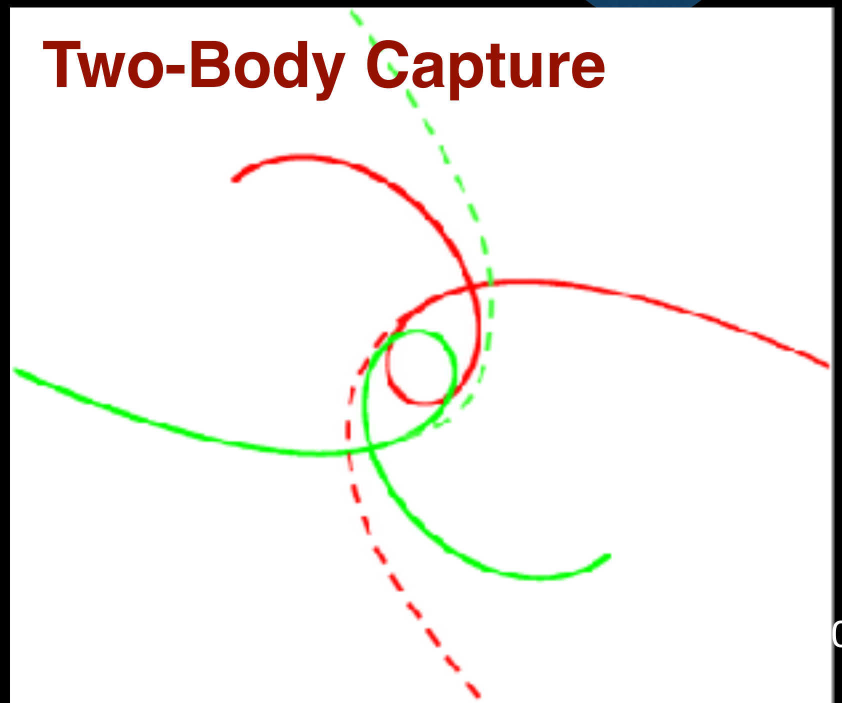


Lee 2001 $m_* = 0.7\mathrm{M}_\odot, \quad m_{deg} = 10\mathrm{M}_\odot$

Formation Processes of Black Hole Binaries

- In extremely dense systems purely dynamical processes can lead to the formation of compact binaries
  - Three-body processes
  - Two-body capture





Formation Rates of BH Binaries

- Gravitational Wave Capture in parabolic approximation

$$\Sigma_{cap} \approx 17 \frac{G^2 m^2}{c^{10/7} v_\infty^{18/7}} \qquad \left(\frac{dn}{dt}\right)_{cap} = \frac{1}{2} < n^2 \Sigma_{cap} v_{rel} >$$

- Three-body processes (Goodman & Hut 1983)

$$\left(\frac{dn}{dt}\right)_{3B} \approx C \times n^3 \frac{(Gm)^5}{\sigma^9}, \quad C \sim 0.2$$

Which is more efficient?

- Capture versus 3-body processes

$$\frac{(dn/dt)_{Cap}}{(dn/dt)_{3B}} \approx 10^7 \left(\frac{10^5 \mathrm{pc}^{-3}}{n_B}\right) \left(\frac{\sigma}{100 \mathrm{km/s}}\right)^{52/7}$$

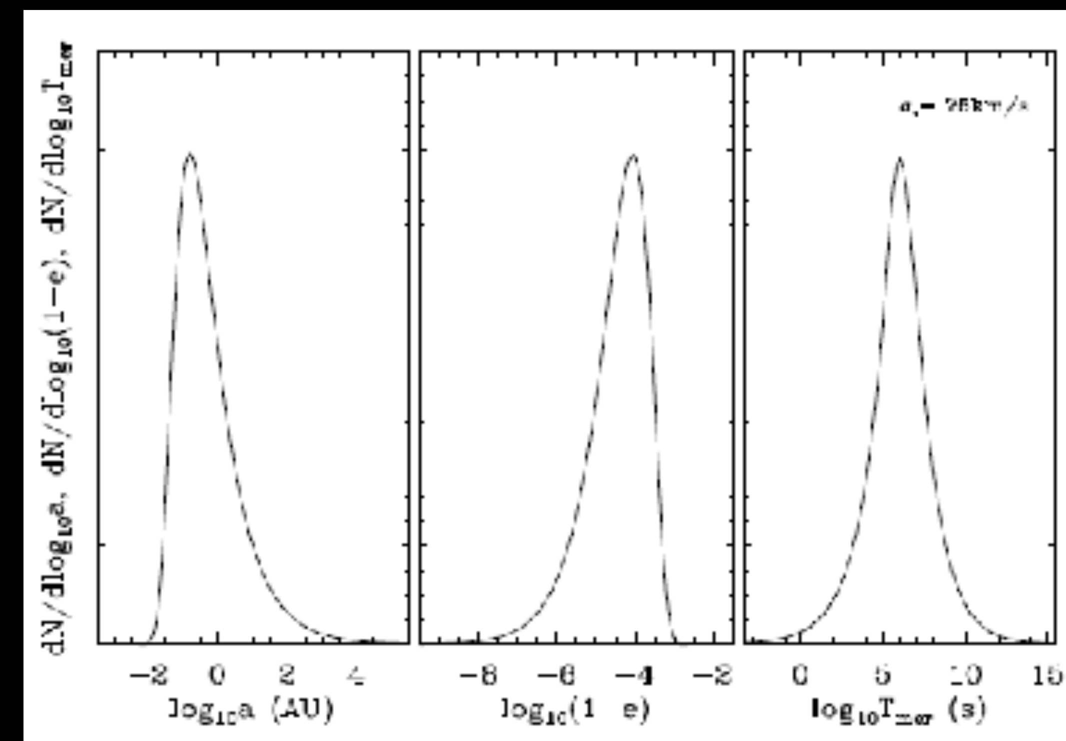
- Globular clusters : $\sigma \sim 10$ km/s, and black holes would have even smaller σ
  - Three-body processes are more efficient
- Galactic Nuclei: $\sigma \sim 100$ km/s
  - Direct capture is more efficient

Dynamical Evolution and Binary Formation

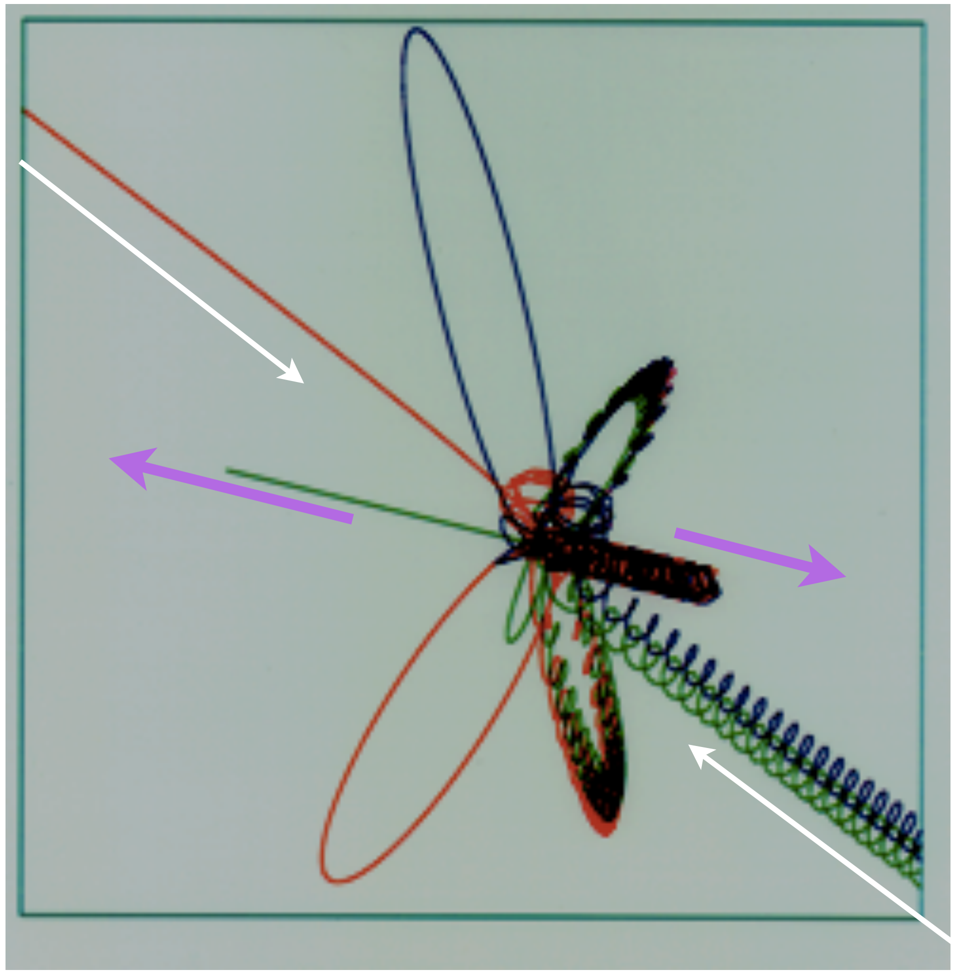
- Globular clusters are not static objects, but evolve in time
  - Core collapse followed by dynamical friction.
  - Black holes become the most massive component in short time (<few times 10^7 years).
- Large fraction of the black holes form binaries through three-body processes

Orbits of Dynamical Binaries

- Captured Binaries:
  - Very eccentric: $1 - e \sim 10 \left(\frac{\sigma}{c}\right)^{10/7}$
  - Semi-major axis: $a \sim 0.05 \frac{Gm}{\sigma^2}$
  - Merge very quickly
- Three-body binaries
  - Moderately eccentric *f(e) de ~ e de*
  - Semi-major axis: $a \sim 0.2 \frac{Gm}{\sigma^2}$
  - Merging times are very long



Hong & Lee 2015
Captured binaries



Fate of BH Binaries

- As the orbits become tighter, the "collision time" with surrounding stars increases while merger time scale decreases
- The recoil energy due to binary-single encounters also increases

$$K_B \sim \frac{2}{3} \times 0.4 E_B = \frac{2}{15} \frac{Gm^2}{a}$$

- If $K_B > 1/2\, m v_{\rm esc}^2$, the binaries get ejected.
- At some point recoil energy exceeds the escape energy from the cluster

$$a \lesssim a_{crit} = \frac{Gm}{15 v_{esc}^2}$$

- Depending on $v_{\rm esc}$, ejection or merger take place first.

Hardness of Binaries

- It is useful to define hardness parameter *x*, as a binding energy of binaries normalized by the average kinetic energy of the background.

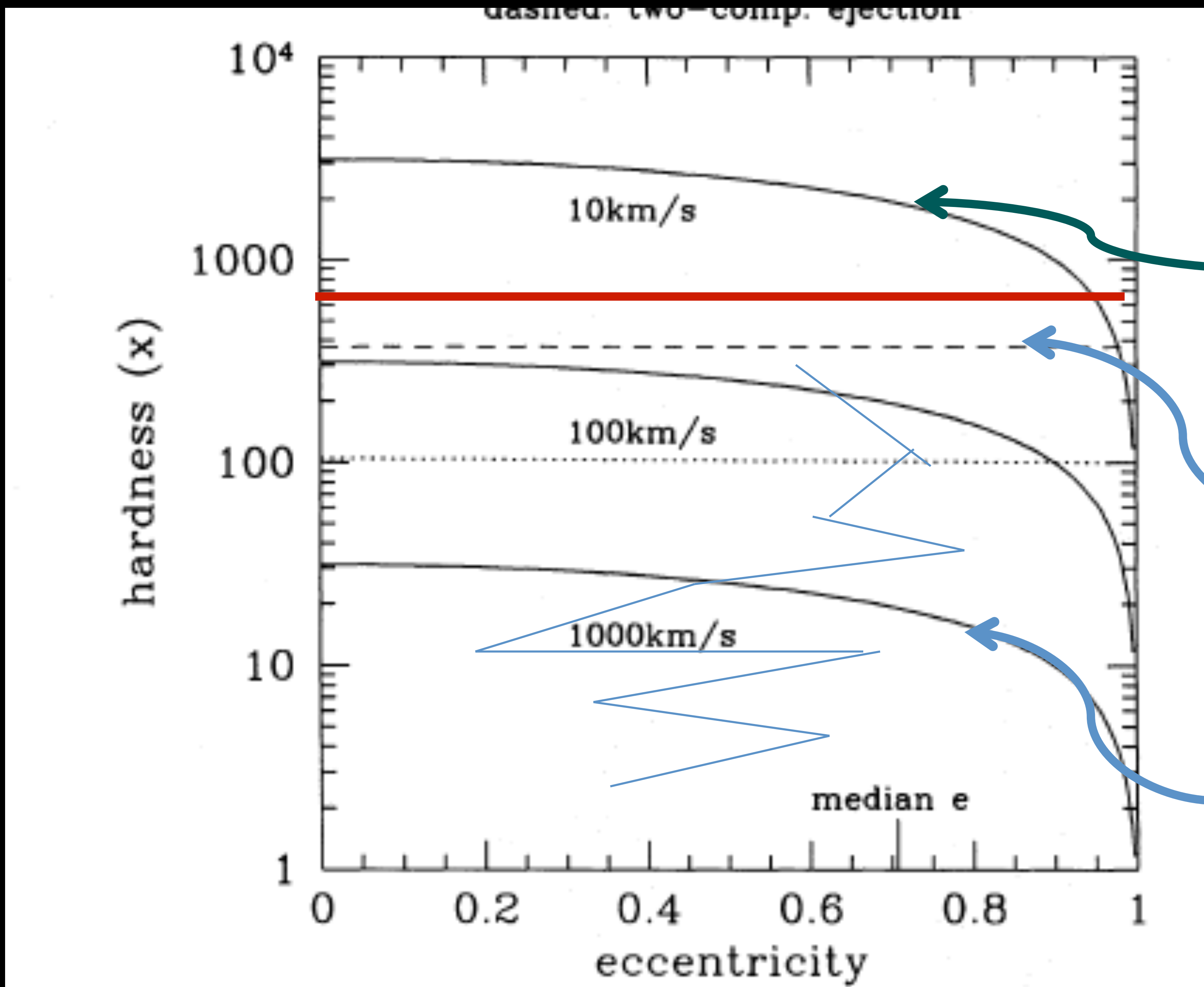
$$x \equiv \frac{Gm_1 m_2/2a}{3m_* \sigma^2}, \quad m_* : \text{stellar mass}$$

- The condition for ejection corresponds to

$$x_{crit} \sim 5 \left(\frac{m}{m_*}\right) \left(\frac{v_{esc}^2}{\sigma^2}\right) \qquad \left(\frac{v_{esc}^2}{\sigma^2}\right) \sim \ 12$$

- Therefore $x_{crit} \sim 60 \left(\frac{m}{m_*}\right)$

Schematic Evolution of Binary Parameters in *e*-*x* space



Critical hardness for 'merger' before next interaction (depends on velocity dispersion)

Critical hardness for 'ejection'

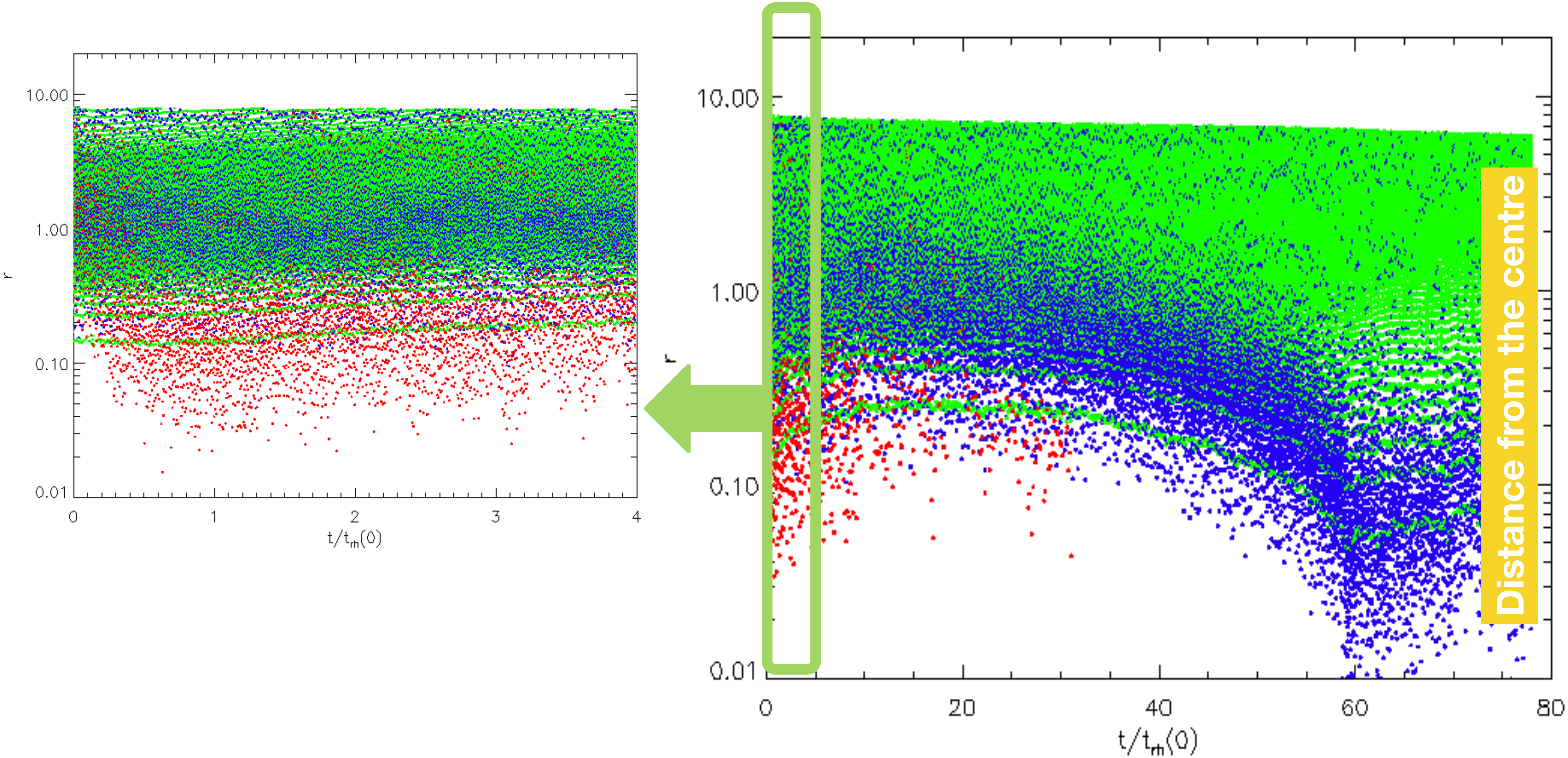
Evolution of binary in (*e*,*x*) space

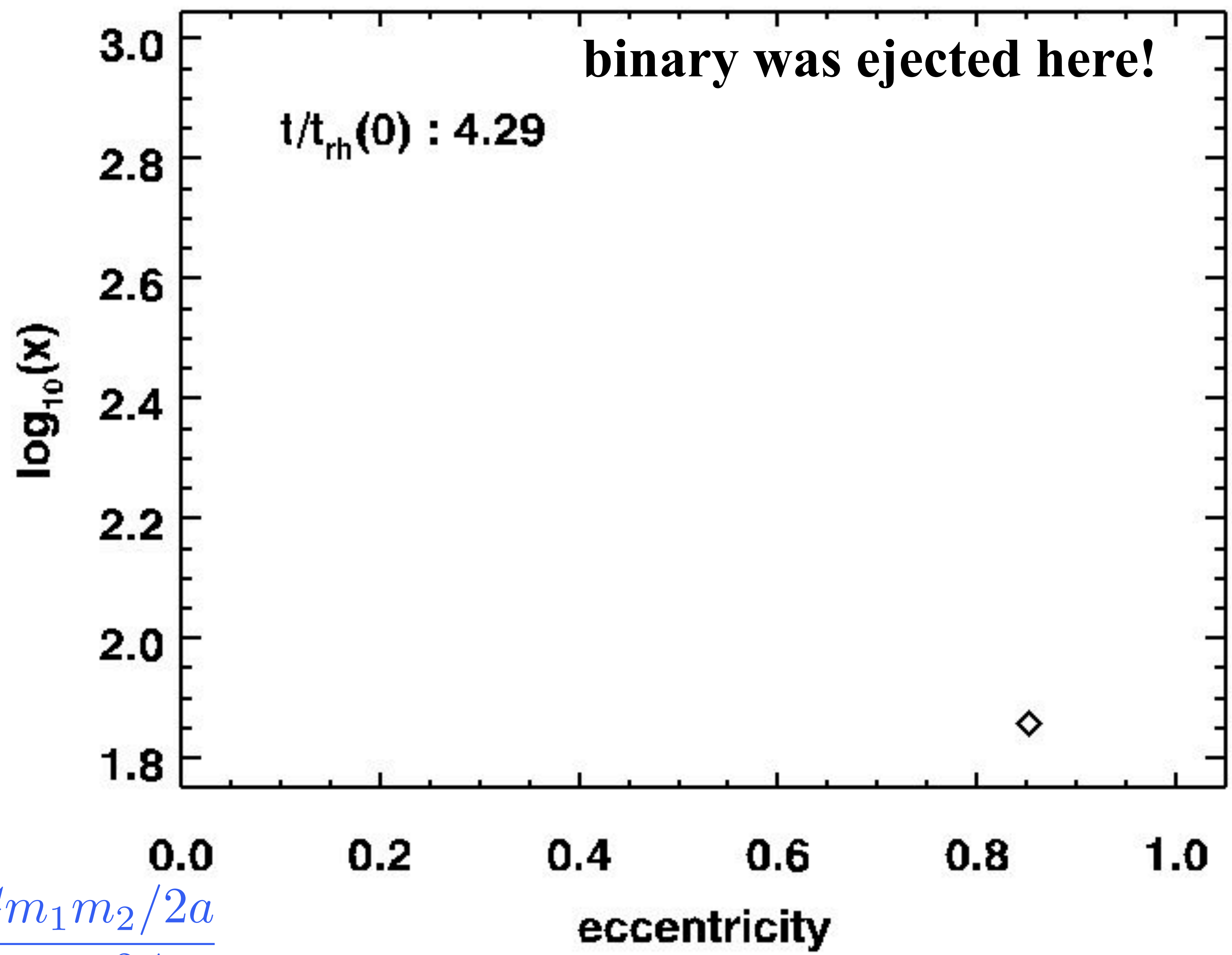
Numerical Simulation of BH Binary Formation and Evolution

- Direct N-body simulations
- Spherical, non-rotating models in static tidal field
- Composed of ordinary and degenerate stars
    - Ordinary stars: 0.7 $M_\odot$ (actual average mass could be lower)
    - 1.4 $M_\odot$, representing NS
    - 10 ~ 40 $M_\odot$, representing BH
        - Single component
        - Two component
        - Continuous mass spectrum
- N=5,000 ~ 50,000
- No Primordial binaries

Some Concerns of the Simulations

- N<< actual number of stars in GC
  - We try to look for properties that are independent of N
- No stellar evolution
  - Stellar evolution is important only in the very early phase of evolution (~10^7 - 10^8 years)
- No initial BH binaries
  - Soft binaries easily disrupted
  - Hard binaries behave same way as dynamical binaries, and therefore our results provide conservative lower limits
- No mass function for normal stars
- No direct capture is included
  - Ratio between three-body processes and direct capture depends strongly on N
  - Direct capture will be investigated separately with Fokker-Planck code

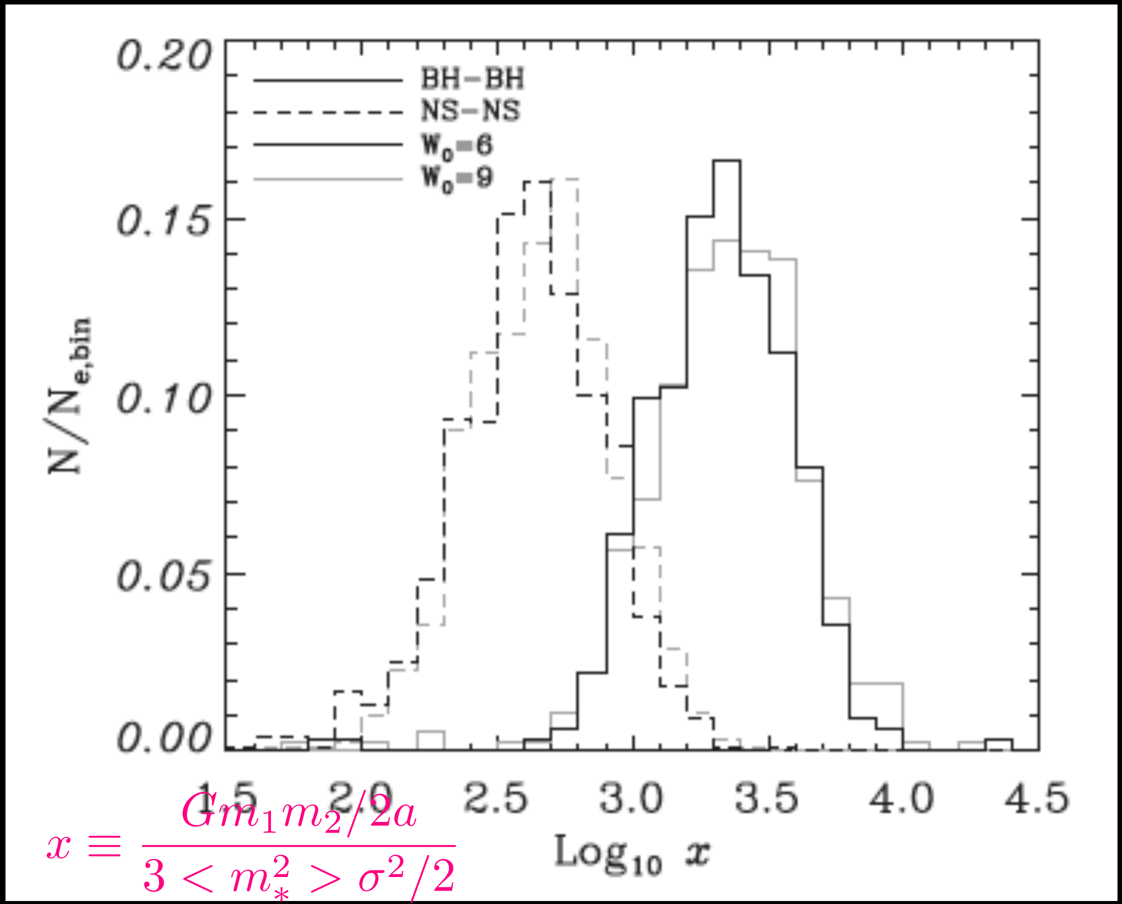

10.00
1.00
0.10
0.01
r
0
1
2
3
4
t/t_rh(0)
10.00
1.00
0.10
0.01
r
0
20
40
60
80
t/t_rh(0)
Distance from the centre

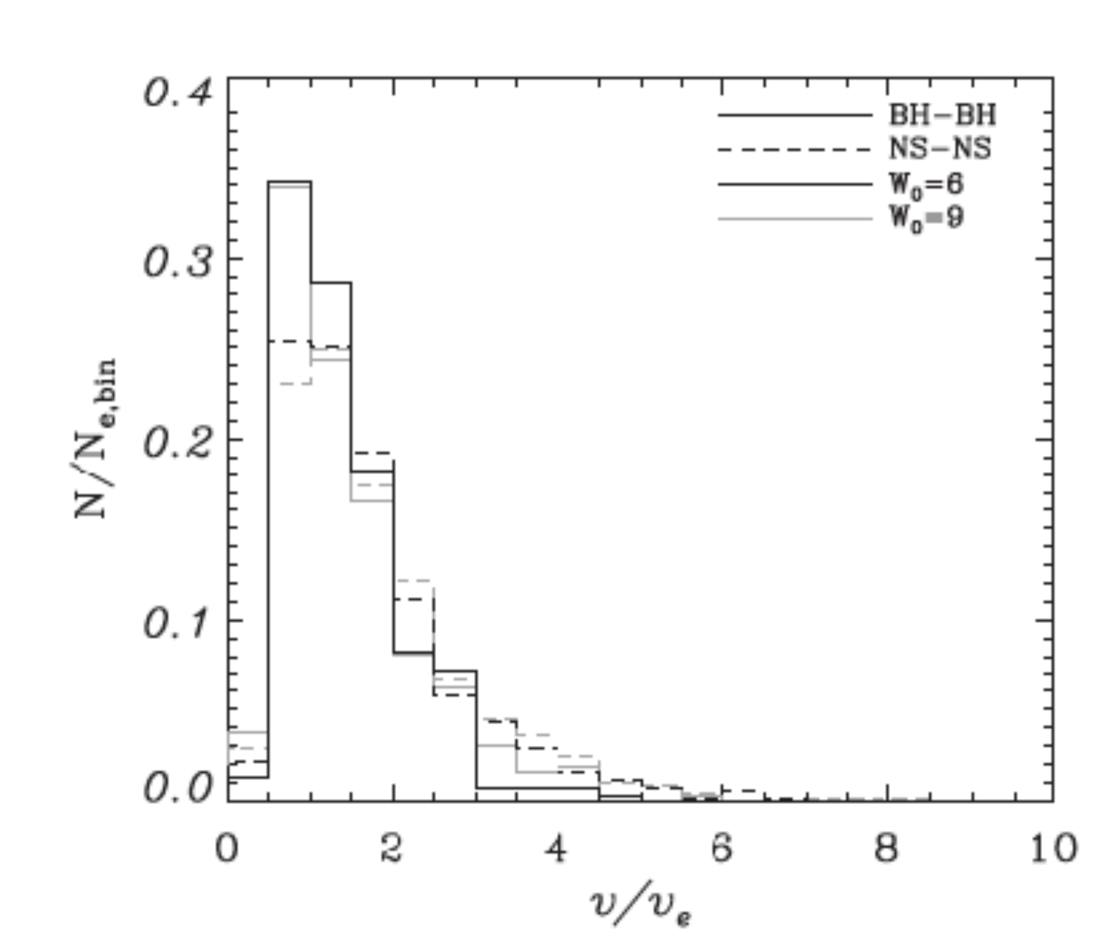


$$x \equiv \frac{Gm_1 m_2/2a}{3m_* \sigma^2/2}$$

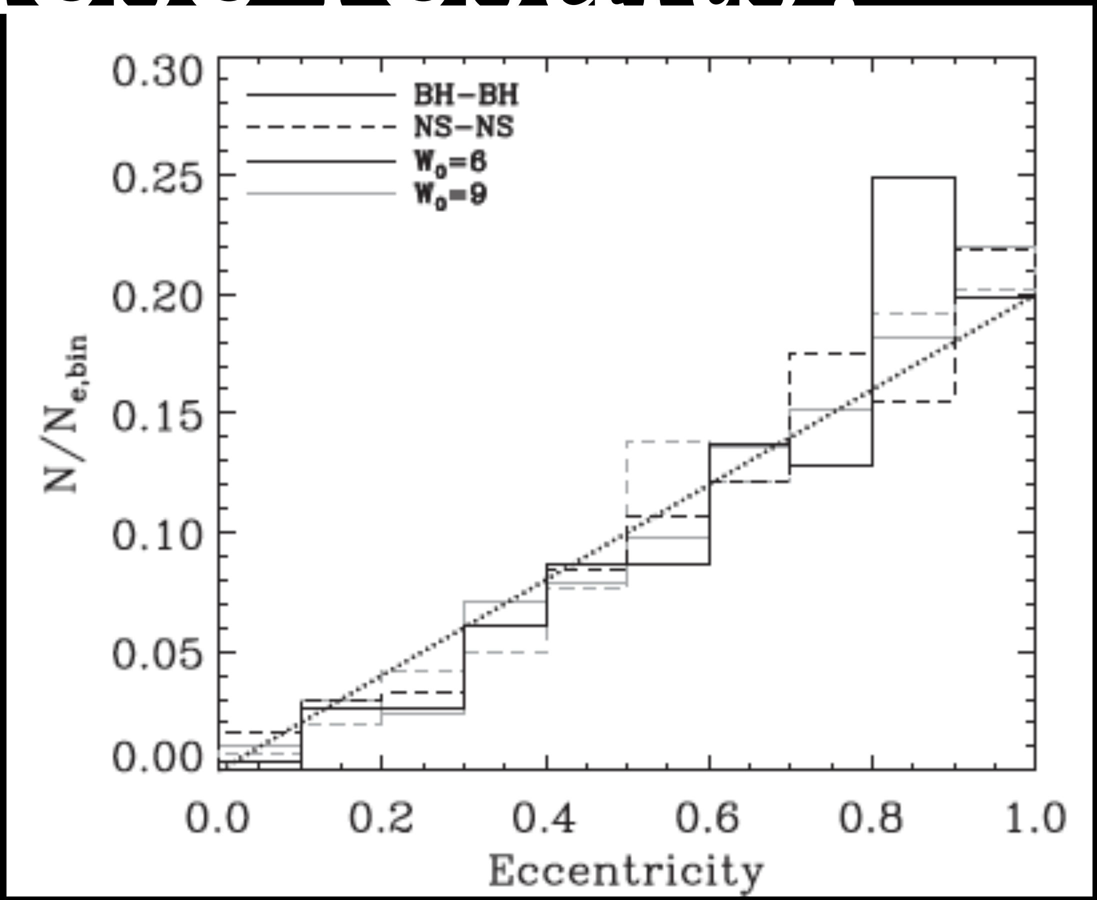
Properties of ejected binaries

- Ejection occurs as the binaries become very hard, consistent with the predictions
  - But actual distribution of x is broad
- The relative velocity to the cluster is $> v_e$.

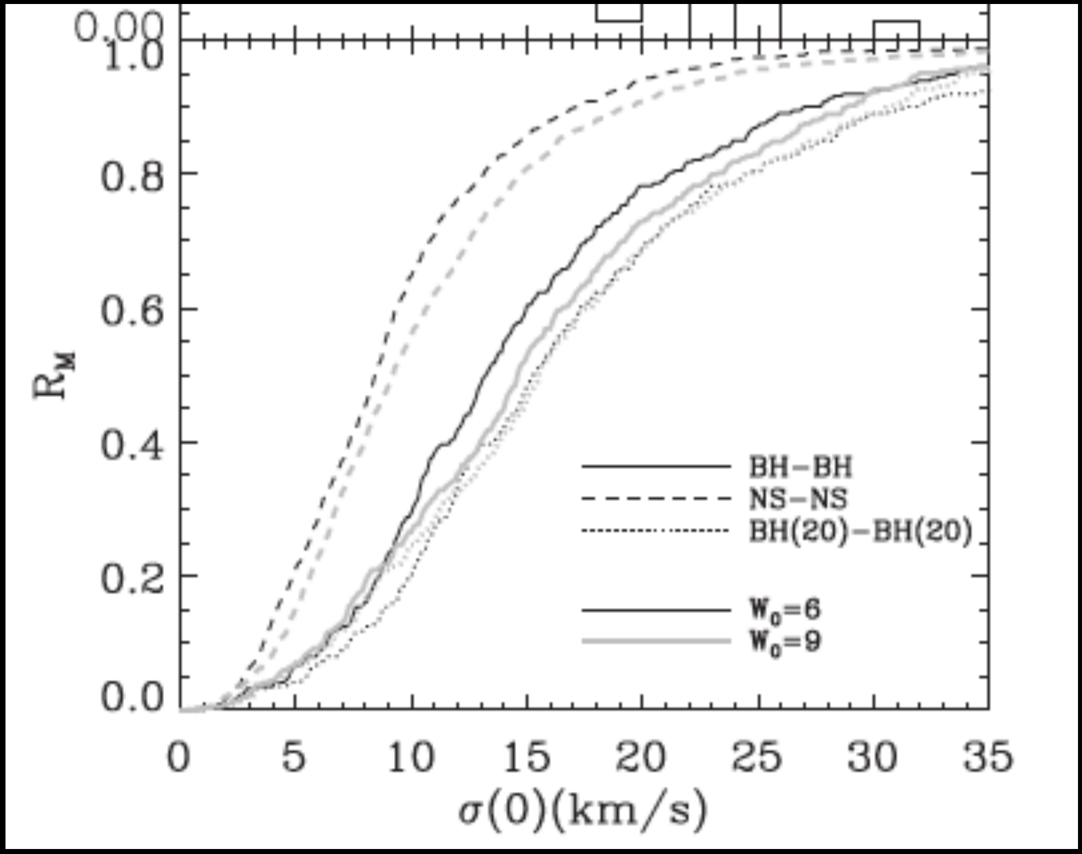




How can we use these results?

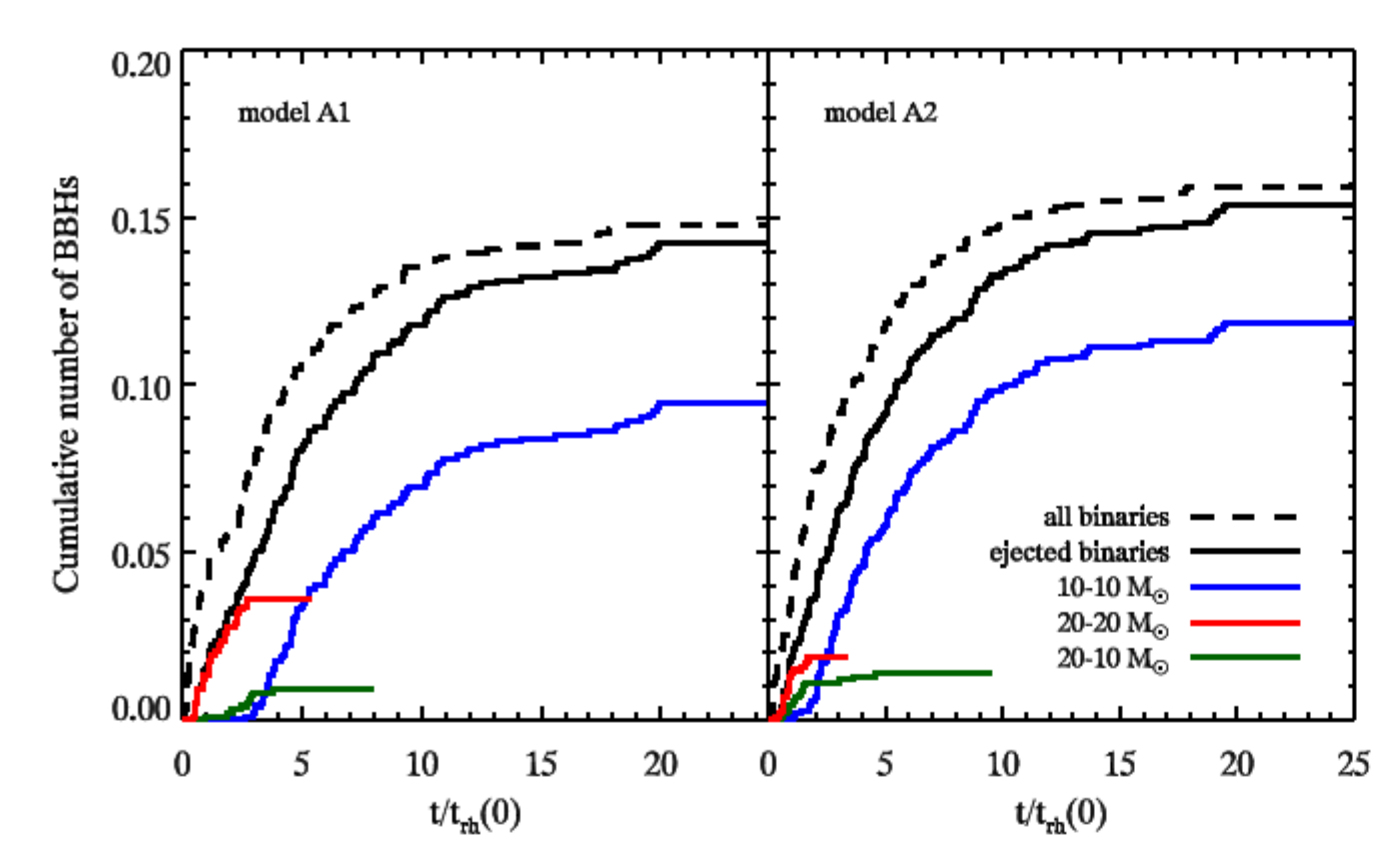


- The distribution of orbital separation depends only on the velocity dispersion
- The eccentricity distribution is almost invariant of any parameters (i.e., thermal distribution)
- Consequently 'merging fraction' depends on the central velocity dispersion



Bae, Kim & Lee 2014

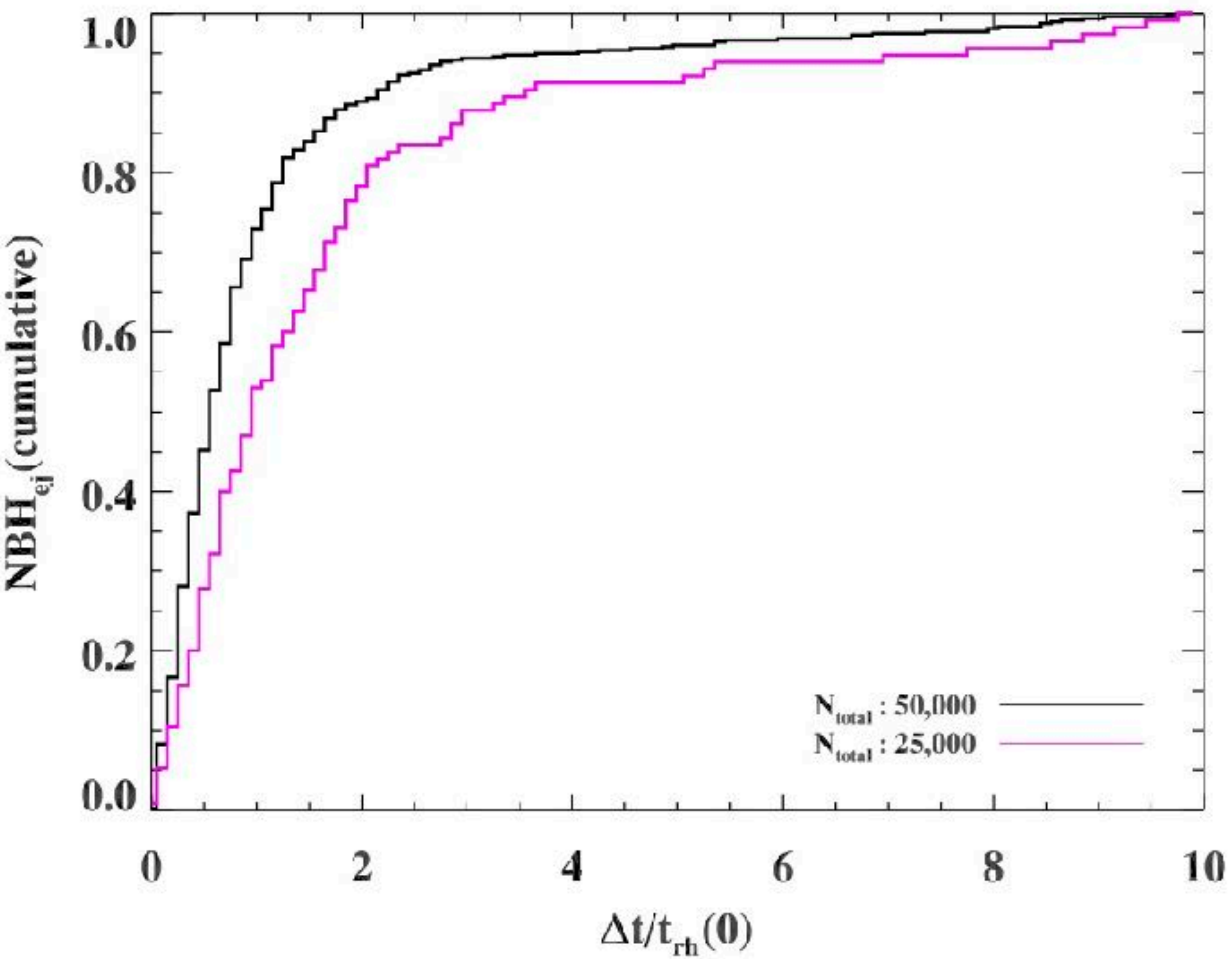
Models with two-mass BH: 10 & 20 M☉



- Higher mass BHs form binaries first and the lower mass ones follow
- Time gap between formation and ejection is a few t_{rh}, but it could be smaller for realistic systems

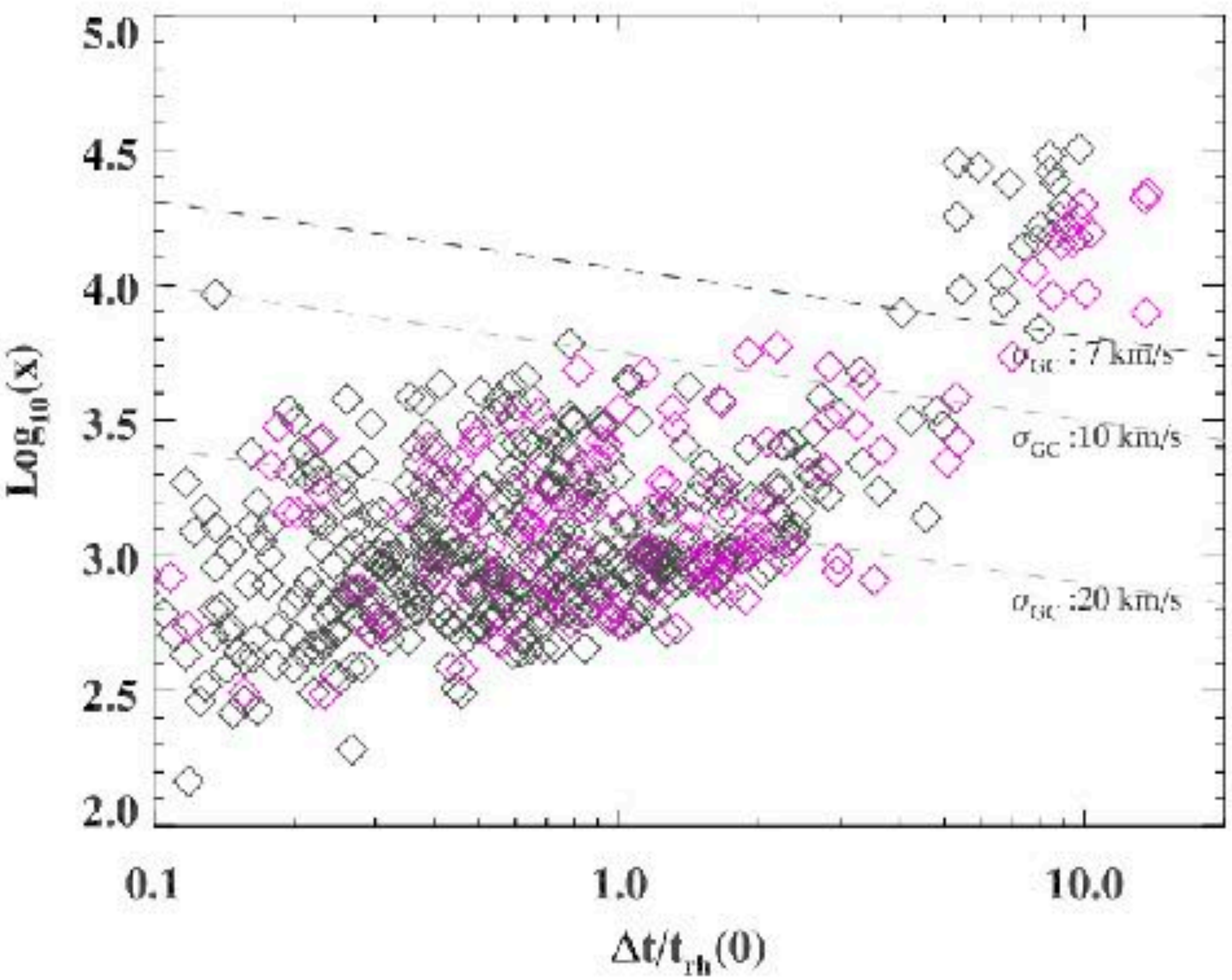
Park et al., (2017)

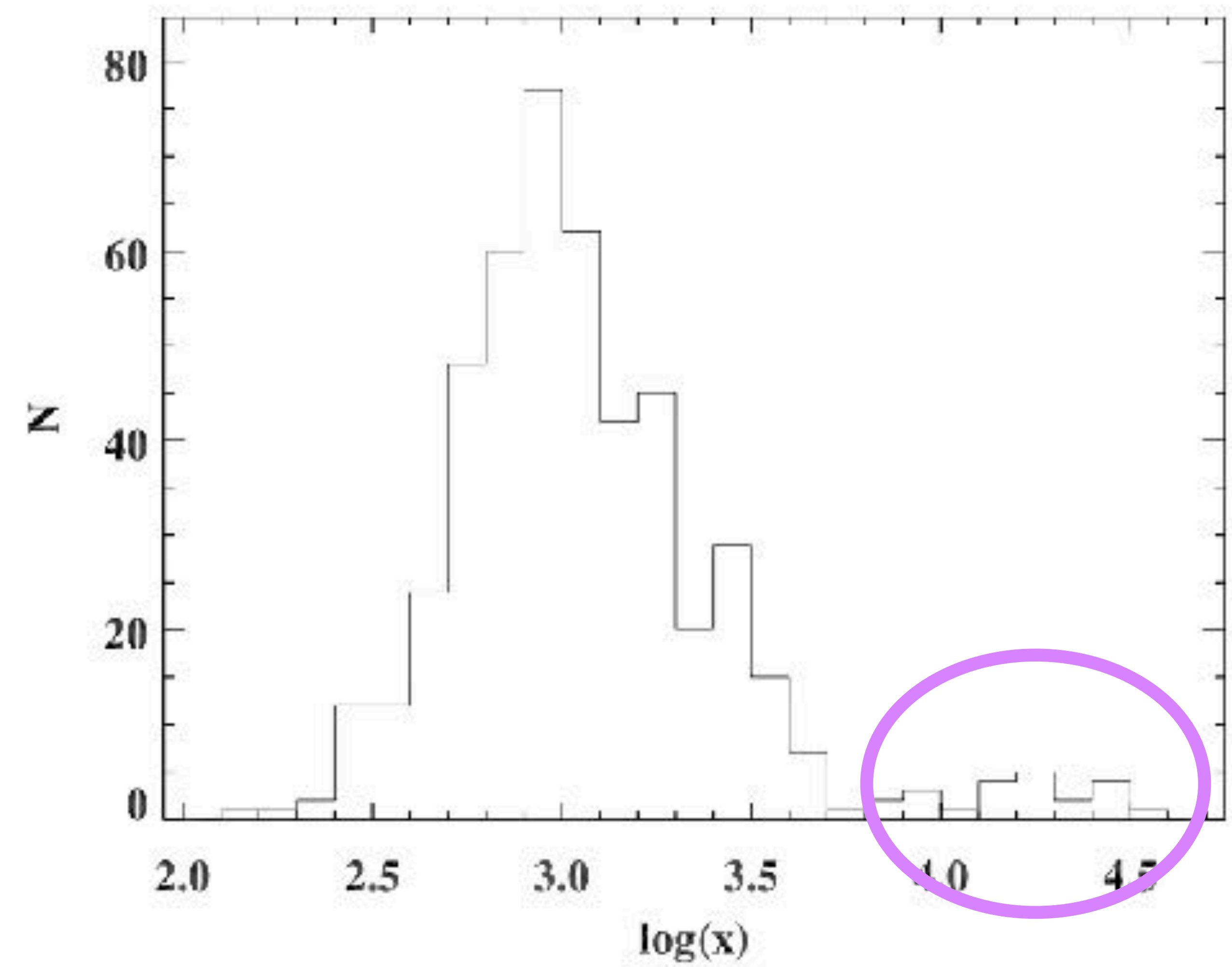
Time gap between formation and ejection



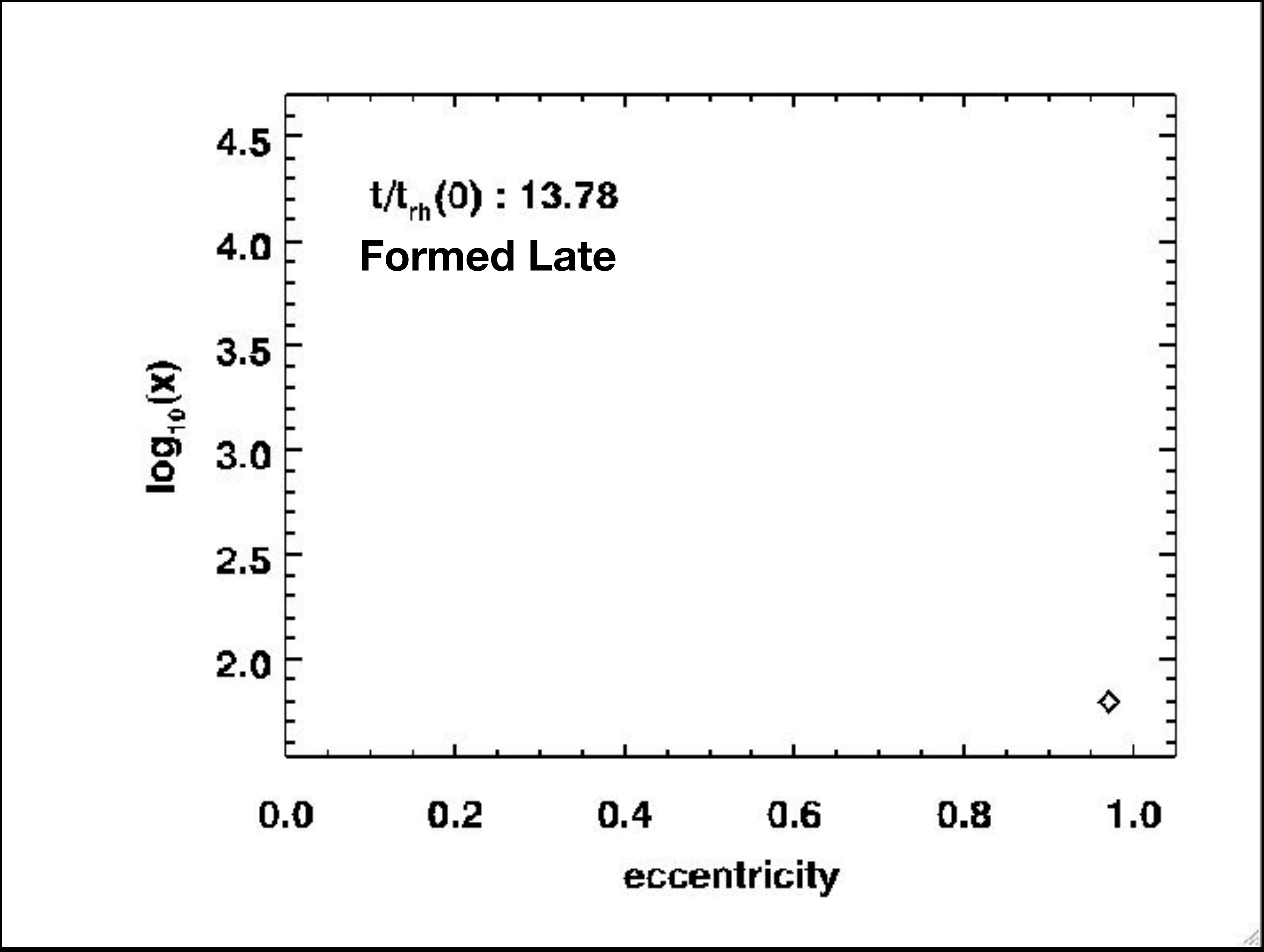
- As N grows, Δt becomes smaller.
- There is a long tail of large Δt
- Those with large Δt would merge in the cluster, rather than get ejected.

Long lived binaries are very hard ones



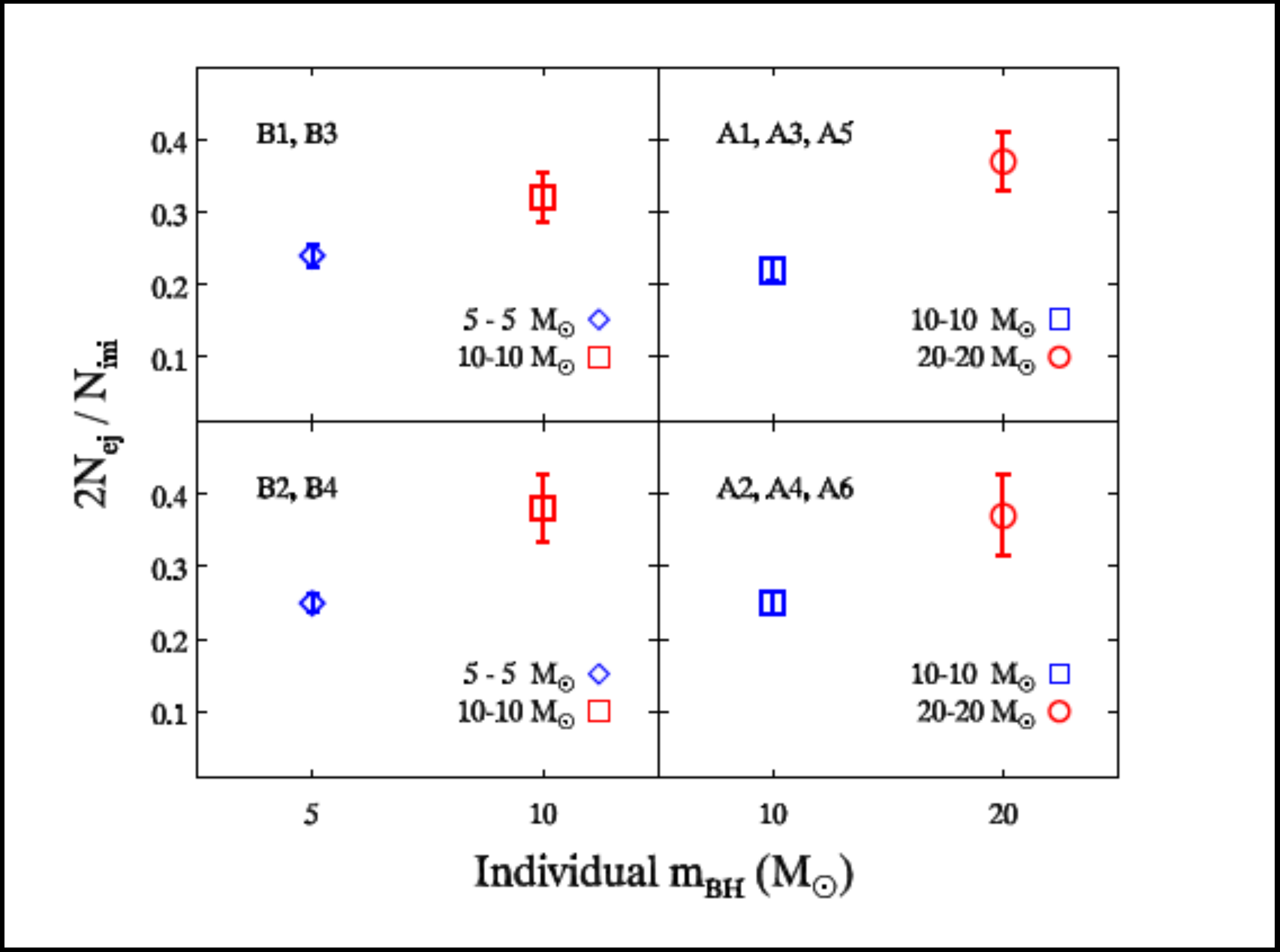


Orbital Evolution of the Long-Lived Binary



- Experipences many weak encounters
- Composed of lower mass BHs

Efficiency of Binary Formation with Mass

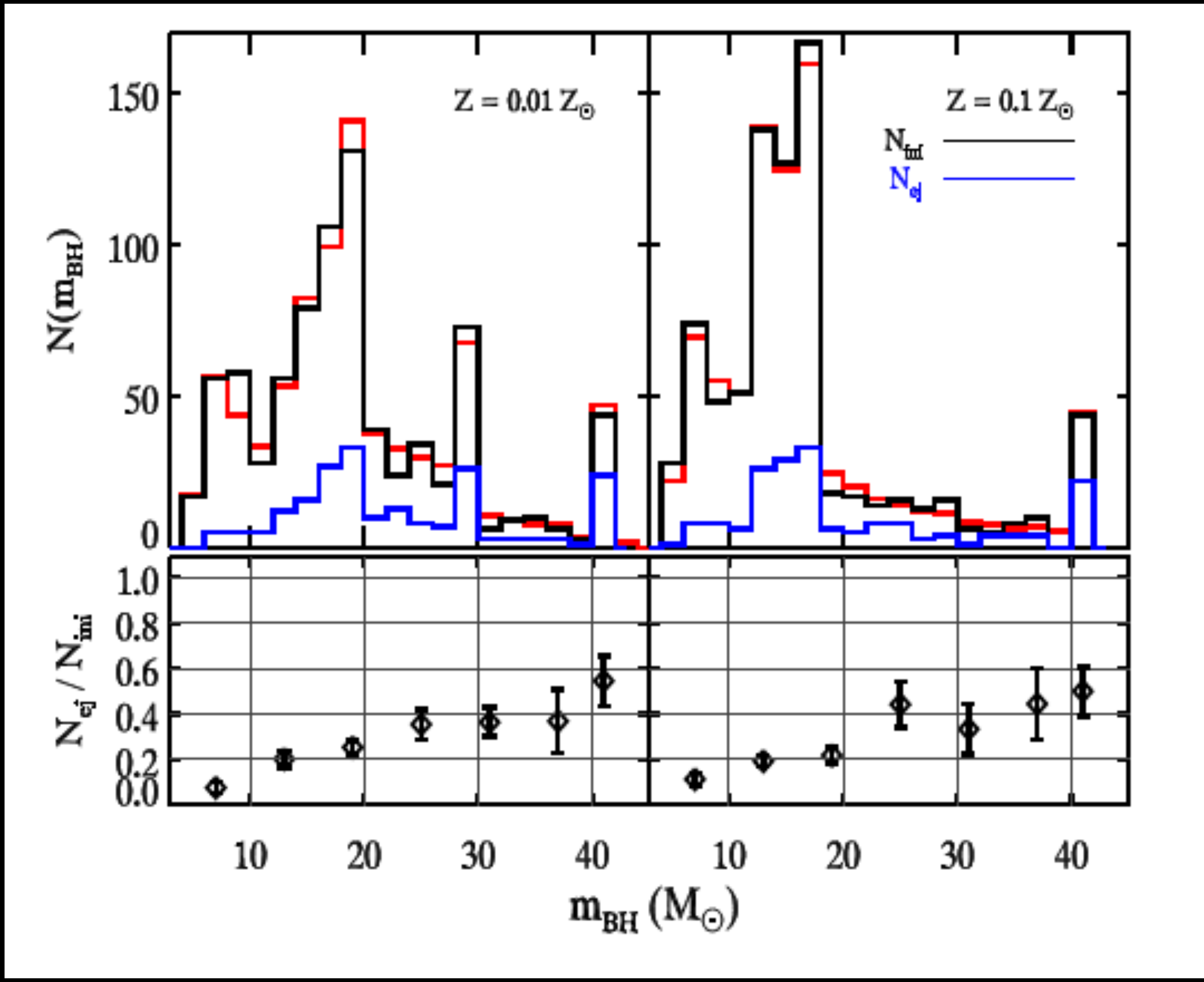


Park et al, (2017)

- Higher mass BHs form binaries more efficiently
- ‘Mass function’ obtained by GW observation would be skewed toward higher mass.

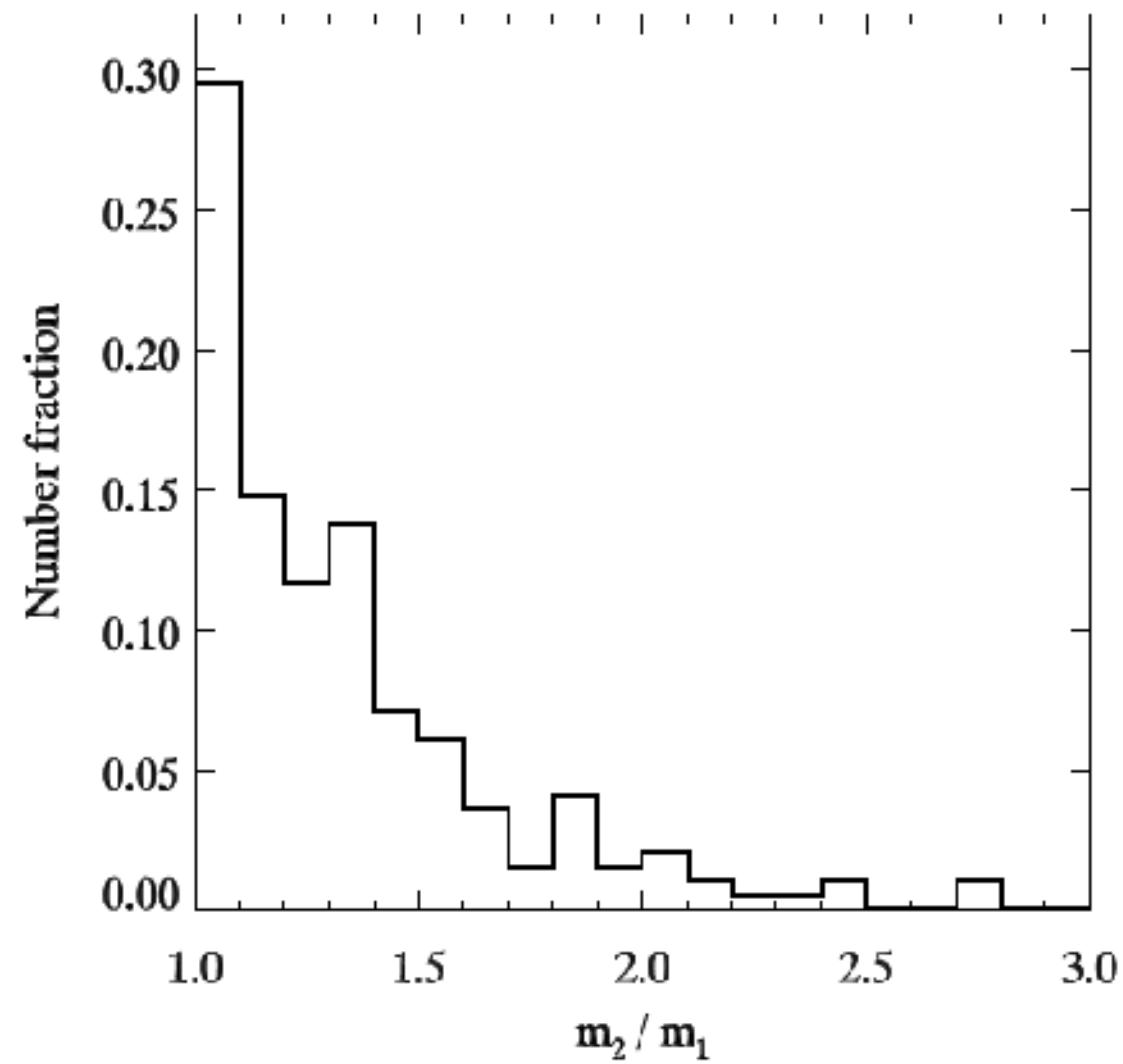
Continuous BH mass function

- BH mass function depends on metallicity
- Biased formation of higher mass BH binaries also seen here



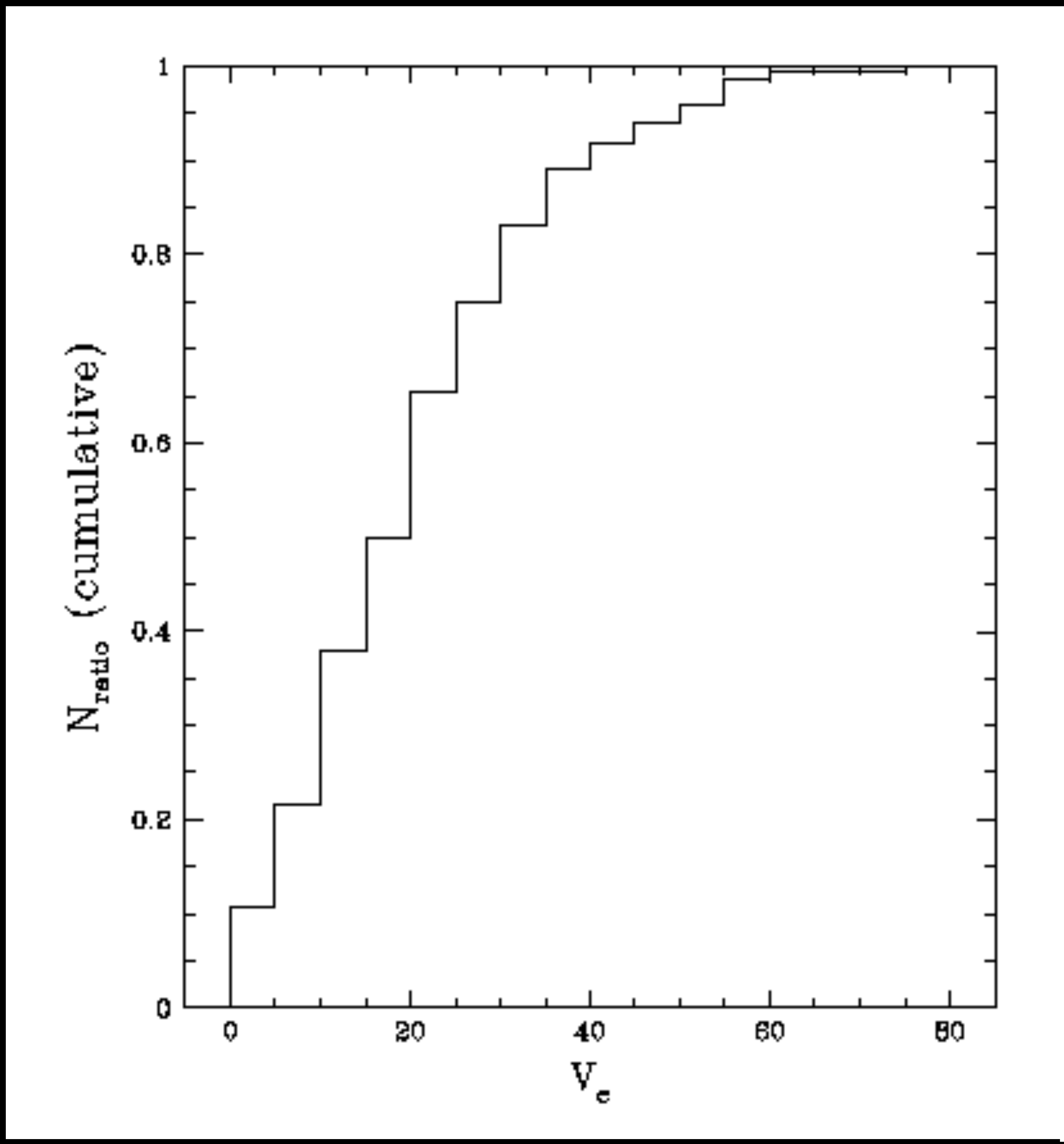
Park et al., (2017)

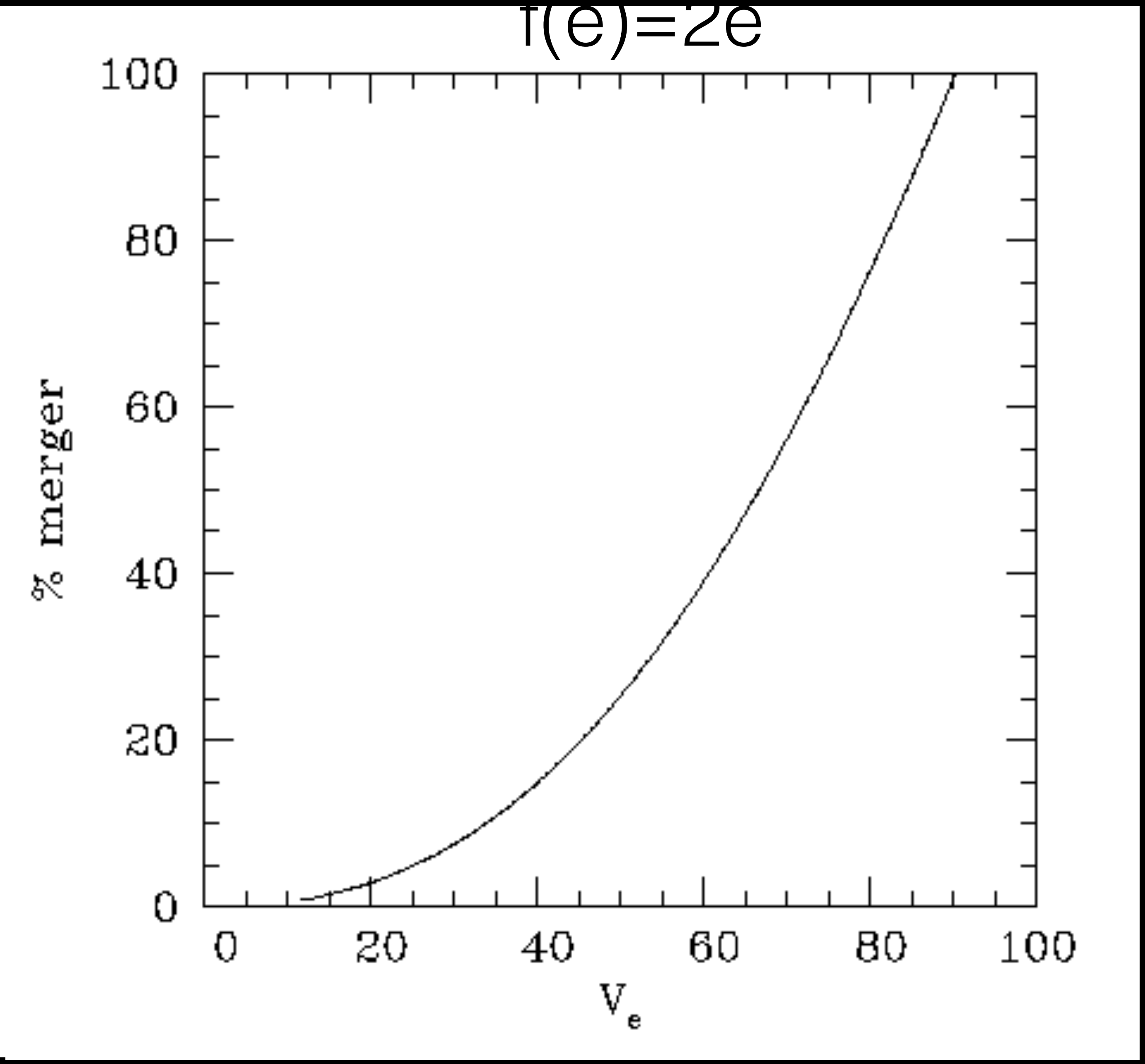
BH mass function by Belczynski et al. (2016)



Number fraction
0.30
0.25
0.20
0.15
0.10
0.05
0.00
1.0
1.5
2.0
2.5
3.0
m2 / m1

Galactic Globular Clusters





Median V_{esc} = 20 km/sec

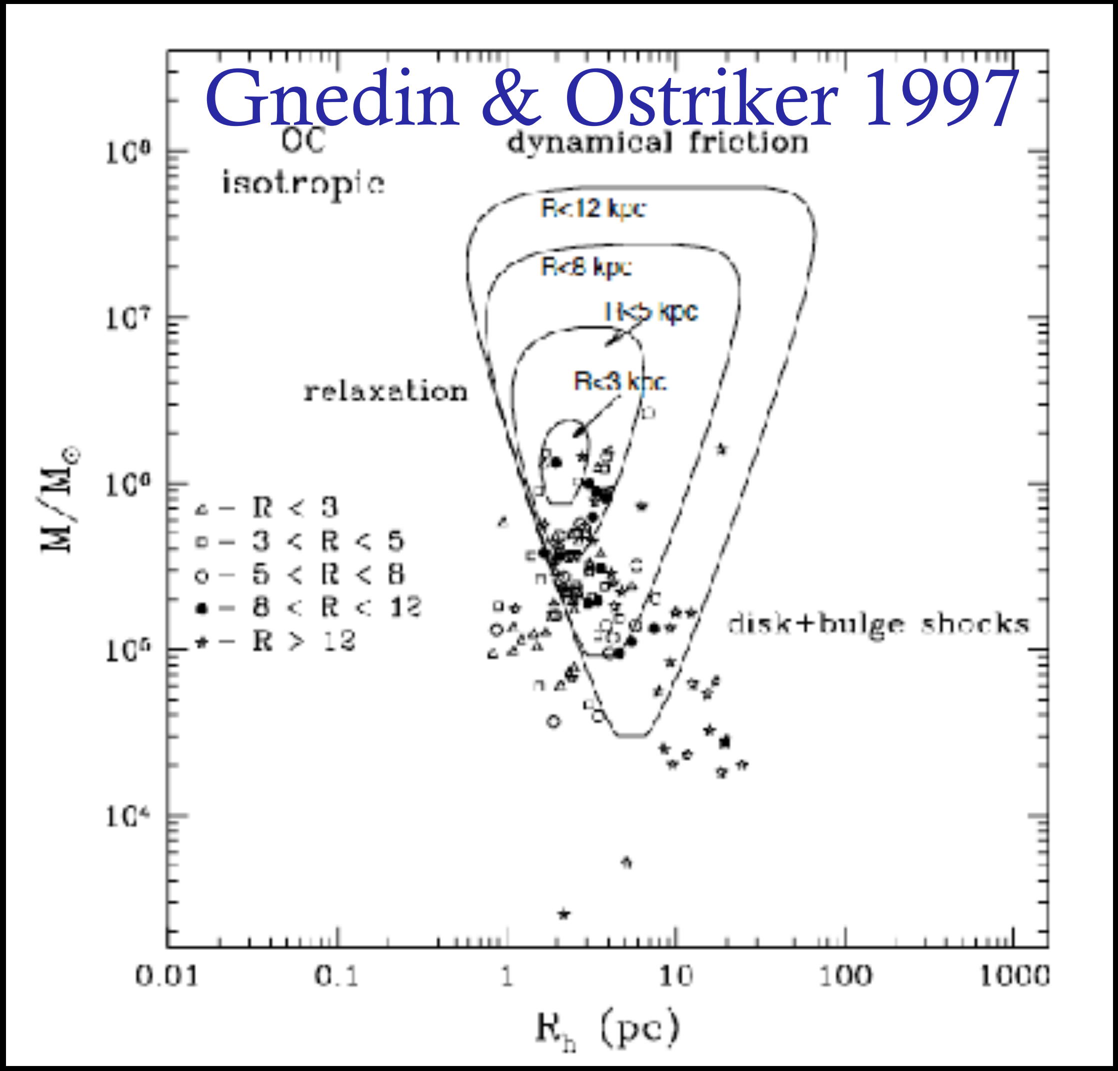
Fraction of merger binaries within Hubble time

Estimation of merger rate

- Assume 0.45 % NS (but 10% of this remains) and 0.18% BH by number in GC
- 15% of these objects escape in the form of compact binaries
- We used 142 clusters in the catalogue by Harris (2010) with mass and velocity dispersion http://physwww.mcmaster.ca/~harris/Databases.html
  - We computed the number of binaries whose merging time is shorter than Hubble time for each cluster and add them up
- Number density of globular clusters $n_{GC} = 8.7\ h^3\ Mpc^{-3}$ and h=0.72,
- Results
  - $0.1 - 1\ yr^{-1}$ within advanced detector range
  - $\sim 10\ yr^{-1}\ Gpc^{-3}$ for BH-BH merger
  - Very small number of NS-BH binaries

Uncertainties

- Initial Mass function could have been top heavy for in low metallicity environments
- Clusters with relatively large σ could have produced direct capture binaries
- Many GCs have been already disrupted due to
  - Galactic tidal field
  - Tidal Shocks
  - Dynamical friction



Planned Improvements

- More careful assessment of dependence on N
- Effects of the mass function of the ordinary stars
  - We assumed 0.7 Msun, but more realistic mean mass is much lower
- The rotation of the clusters could accelerate the dynamical
- Consideration of captured binaries.

Summary

- Black hole binaries can be formed by dynamical processes in globular clusters efficiently
- Most of them are formed by three-body processes.
- Some of them will merge in Hubble time after getting ejected
  - Mass ratios are likely to be less than 2
  - Massive BHs have higher chances in forming binaries
  - Merger rate is estimated to be ~ 10 yr^{-1} Gpc^{-3}
  - Some binaries could have been merged within the cluster (<10%). They are mostly composed of lower mass BHs.
- Substantial uncertainties in the estimated rate exist, but actual rate could be higher