The inflationary origin of the seeds of cosmic structure: quantum theory and the need for novel physics.

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Plan: (Very condensed report of various years of work !)

1) Inflation, the great success.... and the problem.
2) The usual answers. (Please let us leave the the discussion on this part for the end). Some people get annoyed because they see this as "just philosophy". Bear with me to see it is not.
3) Our approach. A word about Dynamical Collapse Theories.
4) The formal implementation. (Very brief)
5) The practical implementation. (Brief)
6) Collapse schemes and detailed predictions. Comparing with observations.
7) Other results, The QG connection (speculations motivated by Penrose's ideas).
8) More on the usual answers (As time allows it). A situation were we can see analogous conceptual problems: Mini-Mott.

1) Cosmic Inflation:

Contemporary cosmology includes inflation as one of its most attractive components: The inclusion of an inflationary stage leads to a natural explanation for the seeds of cosmic structure in terms of quantum fluctuations .

Basics Inflation: A period of accelerated expansion, that takes the universe from relative generic post Plank era initial data to a stage where it is well described (with exponential accuracy in the number of e-folds) by a flat Robertson Walker space-time.

$$dS^2 = a(\eta)^2\{-d\eta^2 + d\vec{x}^2\}$$

Advantages: Resolves various naturalness problems: Flatness, Horizons, and GUT relics.

The biggest one is the natural generation of the seeds of cosmic structure from “quantum fluctuations”.

However, how exactly does this happen? How do the inhomogeneities arise from the quantum uncertainties?

I will describe our approach (briefly contrasting with the usual one, which we feel that it is not truly satisfactory.)

The Data

The CMB photons emitted by the LSS. They are essentially at a local a $T \approx 3000K^0$. but are subjected to the redshift by the cosmological expansion down to $T \approx 2.7K^0$. However, besides that, there is an extra red shift associated with their emergence from the local well in the Newtonian potential.

Then:
$\frac{\delta T}{T_0}(\theta, \varphi) = \frac{1}{3}\psi(\eta_D, \vec{x}_D)$, gives us a picture of Newtonian Potential on the LSS.

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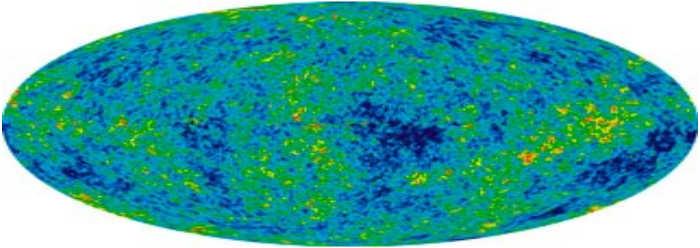

We characterize this map in terms of the spherical harmonic functions, and write: $\frac{\delta T}{T_0}(\theta, \varphi) = \sum_{lm} \alpha_{lm} Y_{lm}(\theta, \varphi)$.
The coefficients are thus :

$$\alpha_{lm} = \frac{1}{3} \int d\Omega^2 \psi(\eta_D, \vec{x}_D) Y^*_{lm}(\theta, \varphi) \tag{1}$$

This is what is measured. (Please do not confuse the fact that the satellite rotates, with the notion that one only extracts an average: The measurements allow us to extract the detailed map we saw and from which we extract the individual quantities α_{lm}).
On the other hand, the quantity that is often the focus of the analysis is:

$$C_l = \frac{1}{2l+1} \sum_m |\alpha_{lm}|^2. \tag{2}$$

The analysis leads to a remarkable agreement with observations:

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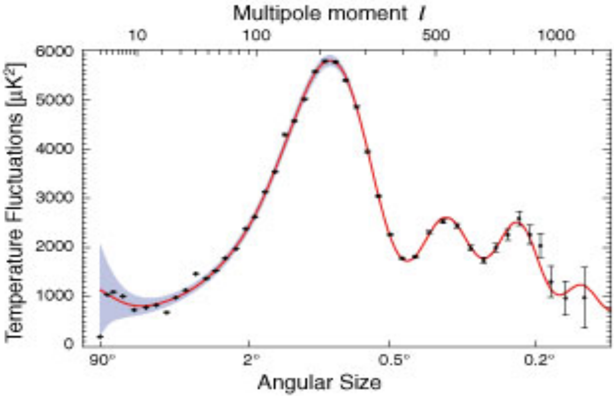


These are supposed to represent the primordial inhomogeneities which evolved into all the structure in our Universe: galaxies, stars planets, etc... AND THE THEORY FITS VERY WELL WITH THE OBSERVATIONS. One is then very tempted to say "well that is it. What else do we want?". An attitude that is hard to blame.

However, let us consider the following: The Universe was H&I, (both in the part that could be described at the "classical level", and the quantum level) as a result of inflation. But we end with a situation which is not: Contains the primordial inhomogeneities which will result in our Universe structure and the conditions that permit our own existence.

How does this happen if the dynamics of the closed system does not break those symmetries.? I say we might want to understand this, and answer the above question in a fully satisfactory way. If our theory really does it, then fine. If not, we can take this as a starting point to further inquiry.

After all, Inflation resulted from demanding more from cosmology that what the Old Big Bang Theory was providing (like naturalness).

2) THE USUAL ANSWERS: See also Penrose's "Shadows of the Mind" Ch 6 Let's discuss this part at the end!

a) As in any situation involving QM: We measure.
The problem with this view, is that the conditions that made possible our own existence would be said to result of our actions.

b)Environment-induced decoherence + many worlds Interpretations (MWI). i) Requires identification of D.O.F as an " environment" (and traced over). That would entail using our own limitations to measure things, as part of the argument. ii) The situation is not described now by one element of the diagonal density matrix, but by all of them, and as such, the situation is still symmetric. We need something like MWI. iii) But MWI relies on a mind whose state of consciousness determines the alternatives into which the word splits.
c) Consistent (or de-cohering) Histories. Answer depends on the questions we ask.

d) This is just Philosophy. One would have thought so. However as we will see it is not, as it leads in principle to predictions that depend on the answers.

How do we explain the breakdown of the symmetry? Decoherence?

Consider a particle in a state with sharp localization at $\vec{X}_0 = (D, 0, 0)$: $|\Psi_{\vec{X}_0}\rangle$. Now let's say that it has a spin in the y direction $|+y\rangle$ so the state is $|\Psi_{\vec{X}_0}\rangle \otimes |+y\rangle$. The result of rotating this state by 180^o about the z axis, is the state $|\Psi_{-\vec{X}_0}\rangle \otimes |-y\rangle$. Now consider that the system is prepared in the supposition given by:

$$\frac{1}{\sqrt{2}}(|\Psi_{\vec{X}_0}\rangle \otimes |+y\rangle + |\Psi_{-\vec{X}_0}\rangle \otimes |-y\rangle)$$

The state is invariant under rotations of 180^o about the z axis.
The reduced density matrix that results from ignoring (tracing over) the spin degree of freedom is:

$$\frac{1}{2}(|\Psi_{\vec{X}_0}\rangle\langle\Psi_{\vec{X}_0}| + |\Psi_{-\vec{X}_0}\rangle\langle\Psi_{-\vec{X}_0}|)$$

I.e. perfect decoherence. Now does this mean the particle is either at $\vec{X}_0$ or at $-\vec{X}_0$? No!.
Does this mean the invariance of the system's state has somehow disappeared? No!

In the case of our cosmological problem, the environment would correspond to the DOF of other fields, or some particular modes of the inflaton field deemed to be ‘non observable” (by us). The point is, however, that the whole state involving the full set of modes is symmetric as inflation is supposed to drive all fields to their vacuum state (the geometric accelerated expansion affects the inflaton and other fields in the same way).

Most people working on this topic compute the so called decoherence functionals, apparently without focussing too much on these issues.

However, even W Zurek tells us: *“The interpretation based on the ideas of decoherence and ein-selection has not really been spelled out to date in any detail. I have made a few half-hearted attempts in this direction, but, frankly, I was hoping to postpone this task, since the ultimate questions tend to involve such “anthropic” attributes of the “observership” as “perception,” “awareness,” or “consciousness,” which, at present, cannot be modeled with a desirable degree of rigor.”*

We can not use *our own* observational limitations as part of the argument (i.e. the appeal to an unobservable set of DOF we declare to constitute the environment) which through decoherence explains the very conditions that lead to *us*.

We need to understand the breakdown of the initial homogeneity and isotropy if we really want to understand the source of the seeds of the cosmic structure (which eventually lead to galaxies stars and planets, where we can find the conditions for the emergence of life and eventually intelligent :-) .., beings like ourselves.) .

3) OUR APPROACH: The situation we face here is unique (Quantum + Gravity + Observations) (Thanks J.M. !!)
We want to be able to point to a physical process that occurs in time as explaining the emergence of the seeds of structure. After all emergence means : Something that was not there at a time, is there at a later time.

Collapse Theories: GRW, Pearle, Diosi, Penrose and now Weinberg.
We propose to add to the standard inflationary paradigm, a quantum collapse of the wave function as a self induced process.

NOTE, HOWEVER, THAT EVEN IGNORING THE PROBLEMS AND ACCEPTING ONE OF THE ALTERNATIVES a) b) or c) , what they indicate is that the relevant state is not the H& I vacuum state $|0\rangle$ but some other state $|\xi\rangle$ Thus, one should characterize such state and extract the spectrum from it. Not from $|0\rangle$!!. I think even the advocates of such postures would have to agree with us on this point.

Most people working in the field believe that the results are the same no matter how one approaches the issue (the statistics are the "same"). It turns out that this is not a correct assumption.

4) The Proposal:

The idea is that at the quantum level gravity is VERY different, and at large scales leaves something that looks like a collapse of the quantum wave function matter fields. (Inspired by Penrose and Diosi's ideas).

Thus the inflationary regime is one where gravity already has a good classical description but matter fields might still require a full quantum treatment.
The setting will thus naturally be semiclassical Einstein's gravity (with the extra element: THE COLLAPSE): i.e., besides U we have sometimes, spontaneous jumps:
$....|0\rangle_{k_1} \otimes |0\rangle_{k_2} \otimes |0\rangle_{k_3} \otimes .... \rightarrow ....|\Xi\rangle_{k_1} \otimes |0\rangle_{k_2} \otimes |0\rangle_{k_3} \otimes .....$

ASSUME: There is an underlying Quantum Theory of Gravity, (probably with no notion of time as in LQG), however, by the "time" we recover space-time concepts, the semiclassical treatment is a very good one. Its regime of validity includes the inflationary regime as long as $R << 1/l^2_{Plank}$.

More precisely we will rely on the notion of *Semiclassical Self-consistent Configuration* (SSC).

DEFINITION: The set $g_{\mu\nu}(x), \hat{\varphi}(x), \hat{\pi}(x), \mathcal{H}, |\xi\rangle \in H$ represents a SSC if and only if $\hat{\varphi}(x)$, $\hat{\pi}(x)$ and $\mathcal{H}$ correspond a to quantum field theory constructed over a space-time with metric $g_{\mu\nu}(x)$ and the state $|\xi\rangle$ in $\mathcal{H}$ is such that:

$$G_{\mu\nu}[g(x)] = 8\pi G\langle\xi|\hat{T}_{\mu\nu}[g(x), \hat{\varphi}(x), \hat{\pi}(x)]|\xi\rangle.$$

It is a GR version of Shrödinger-Newton equation.

This however can not describe the transition from a H&I SSC to one that is not. For that we need to add a collapse: A collapse will be a transition from one SSC to another.
So instead of just "state jumps" we need: $....SSC1.... \rightarrow ....SSC2....$

That involves changing the state, and thus the space-time, and thus the Hilbert space where the state "lives" and is a bit complex.

Space-time is thus treated as classical and in our case (working in a specific gauge and ignoring the tensor perturbations):

$$ds^2 = a^2(\eta)\left[-(1+2\psi)d\eta^2 + (1-2\psi)\delta_{ij}dx^i dx^j\right], \psi(\eta,\vec{x}) \ll 1$$

The scalar field is treated at the level of quantum field theory on a curved space-time, so we write:

$$\hat{\phi}(x) = \sum_{\alpha}\left(\hat{a}_\alpha u_\alpha(x) + \hat{a}_\alpha^\dagger u_\alpha^*(x)\right), \tag{3}$$

with the functions $u_\alpha(x)$ a complete set of normal modes orthonormal with respect to the symplectic product.

Note that this construction depends on ψ. Is a self referential situation.

Working up to the first order in the Newtonian potential the equations for the normal modes simplify to

$$(1-2\psi)(\ddot{u}_{\vec{k}}+2\mathcal{H}\dot{u}_{\vec{k}})-(1+2\psi)\Delta u_{\vec{k}}-4\dot{\psi}\dot{u}_{\vec{k}}+a^2m^2u_{\vec{k}}=0, \quad (4)$$

$$\int_{\eta=\text{const.}}\left[u_{\vec{k}}(\partial_\eta u^*_{\vec{k}'})-(\partial_\eta u_{\vec{k}})u^*_{\vec{k}'}\right](1-4\psi)d^3x=i\hbar a^{-2}\delta_{\vec{k}\vec{k}'}. \quad (5)$$

Construct the modes for a “generic” ψ and then look for a state in the Hilbert space leading to a self consistent solution for the GR equations controlling $a(\eta)$ and ψ. This is nontrivial, but is a well defined problem. We have constructed explicitly the SSC for the H& I case where $\psi=0$ and, for the case involving the excitation of just one nontrivial mode $\psi=F(\eta)Cos(\vec{k_0}.\vec{x})$ and studied the transition from one SSC to the other (to appear in JCAP arXiv:1106.1176 [gr-qc]). In practice, and while working just to first order perturbation, we can work with a single QFT construction.

5) PRACTICAL TREATMENT:

We have checked that this is equivalent at the lowest order in perturbation theory.
We again split the treatment into that of a classical homogeneous ('background') part and an in-homogeneous part ('fluctuation'), i.e. $g = g_0 + \delta g$, $\phi = \phi_0 + \delta\phi$.
The background is taken again to be Friedmann-Robertson universe, and the homogeneous scalar field $\phi_0(\eta)$. In the previous more precise treatment this corresponds to the zero mode of the quantum field.
The big difference will be in the spatially dependent perturbations. Here the theory indicates we should quantize the scalar field but not the metric perturbation.
We will set $a = 1$ at the'present cosmological time", and assume that inflationary regime ends at a value of $\eta = \eta_0$, negative and very small in absolute terms.

Semiclassical Einstein's equations, at lowest order lead to

$$\nabla^2\Psi = 4\pi G\dot{\phi}_0\langle\delta\dot{\phi}\rangle = s\langle\delta\dot{\phi}\rangle, \quad (6)$$

where $s \equiv 4\pi G\dot{\phi}_0$.

Consider the quantum theory of the field $\delta\phi$. Work with the rescaled field variable $y = a\delta\phi$ and its conjugate momentum $\pi = \delta\dot{\phi}/a$. (Set the problem in a box of side L, and $L \to \infty$ at the end).

We decompose the field and momentum operators as:

$$y(\eta,\vec{x}) = \frac{1}{L^3}\sum_{\vec{k}} e^{i\vec{k}\cdot\vec{x}}\hat{y}_k(\eta), \qquad \pi_y(\eta,\vec{x}) = \frac{1}{L^3}\sum_{\vec{k}} e^{i\vec{k}\cdot\vec{x}}\hat{\pi}_k(\eta),$$

where

$$\hat{y}_k(\eta) \equiv y_k(\eta)\hat{a}_k + \bar{y}_k(\eta)\hat{a}^{+}_{-k}; \qquad \hat{\pi}_k(\eta) \equiv g_k(\eta)\hat{a}_k + \bar{g}_k(\eta)\hat{a}^{\dagger}_{-k}$$

The usual choice of modes: $y_k(\eta) = \frac{1}{\sqrt{2k}}\left(1 - \frac{i}{\eta k}\right)\exp(-ik\eta)$ $g_k(\eta) = -i\sqrt{\frac{k}{2}}\exp(-ik\eta)$, leads to the Bunch Davies vacuum: the state defined by $\hat{a}_k|0\rangle = 0$.

Note that $\langle 0|\hat{y}_k(\eta)|0\rangle = 0$ and $\langle 0|\hat{\pi}_k(\eta)|0\rangle = 0$.
The collapse will modify the state and thus expectation values of the operators $\hat{y}_k(\eta)$ and $\hat{\pi}_k(\eta)$.

Now we need to specify the rules according to which collapse happens. That is: the state $|\Theta\rangle$ after the collapse. This is thought to be controlled by novel physics so we must try to make and "educated guess", and hopefully then contrast with data.

We will assume that after the collapse, the expectation values of the field and momentum operators in each mode, will be related to the uncertainties of the pre-collapse state (these quantities for the vacuum are NOT zero).

In the vacuum state, $\hat{y}_k$ and $\hat{\pi}_k$ characterized by Gaussian wave functions centered at 0 with spread Δy_k and $\Delta \pi_{y_k}$, respectively.

6) We will want to consider various possibilities for the detailed form of this collapse. Thus, for their generic form, associated with the ideas above, we assume that at time η_k^c the part of the state corresponding to the mode $\vec{k}$ undergoes a sudden jump so immediately afterwards:

$$\langle \hat{y}_k(\eta_k^c) \rangle_\Theta = A \ \ x_{k,1} \sqrt{\Delta \hat{y}_k}$$
$$\langle \hat{\pi}_k(\eta_k^c) \rangle_\Theta = B \ \ x_{k,2} \sqrt{\Delta \hat{\pi}_k^y}$$

where $x_{k,1}, x_{k,2}$ are (single specific values) selected randomly from within a Gaussian distribution centered at zero with spread one.

Model 1): *the symmetric model* $A = B = 1$.

Model 2): *the Newtonian model.* $A = 0$, $B = 1$.

Finally using the evolution equations for the expectation values (i.e. using Ehrenfest's Theorem) we obtain $\langle \hat{y}_k(\eta) \rangle$ and $\langle \hat{\pi}_k(\eta) \rangle$ for the state that resulted from the collapse for all later times.

Analysis of the Phenomenology

The semi-classical version of the perturbed Einstein's equation that, in our case, leads to $\nabla^2\Psi = 4\pi G\dot{\phi}_0\langle\delta\dot{\phi}\rangle$ indicates that the Fourier components at the conformal time η are given by:

$$\Psi_k(\eta) = -(s/ak^2)\langle\hat{\pi}_k(\eta)\rangle$$

Prior to the collapse, the state is the vacuum and $\langle 0|\hat{\pi}_k(\eta)|0\rangle = 0$ so we have:

$$\Psi_k(\eta) = 0$$

But after the collapse we have:

$$\Psi_k(\eta) = -(s/ak^2)\langle\Theta|\hat{\pi}_k(\eta)|\Theta\rangle \neq 0$$

And thus we can reconstruct the Newtonian potential (for times after the collapse)

$$\Psi(\eta,\vec{x}) = \frac{1}{L^3}\sum_{\vec{k}} e^{i\vec{k}\cdot\vec{x}}\Psi_k(\eta)$$

The quantity we want to focus on is the "Newtonian potential" on the surface of last scattering: $\Psi(\eta_D, \vec{x}_D)$, where η_D is the conformal time at decoupling and $\vec{x}_D$ are co-moving coordinates of points on the last scattering surface corresponding to us as observers.
This quantity is identified with the temperature fluctuations on the surface of last scattering.
Thus :

$$\alpha_{lm} = \int \Psi(\eta_D, \vec{x}_D) Y^*_{lm} d^2\Omega.$$

Now, we have:

$$\Psi(\eta, \vec{x}) = \sum_{\vec{k}} \frac{sU(k)}{k^2} \sqrt{\frac{\hbar k}{L^3}} \frac{1}{2a} F(\vec{k}) e^{i\vec{k}\cdot\vec{x}}$$

where $F(\vec{k})$ contains the information about the type of collapse scheme one is considering, as well as the time at which the collapse of the wave function for the mode $\vec{k}$ occurs.

The factor $U(k)$ represents known physics like the acoustic oscillations of the plasma (i.e. corresponds to the transfer functions).

Now, putting all this together we find,

$$\alpha_{lm} = s\sqrt{\frac{\hbar}{L^3}}\frac{1}{2a}\sum_{\vec{k}}\frac{U(k)\sqrt{k}}{k^2}F(\vec{k})4\pi i^l j_l(|\vec{k}|R_D)Y_{lm}(\hat{k}),$$

where $j_l(x)$ is the spherical Bessel function of the first kind, $R_D \equiv ||\vec{x_D}||$, and $\hat{k}$ indicates the direction of the vector $\vec{k}$.

Thus α_{lm} is the sum of complex contributions from all the modes, i.e. the equivalent to a two dimensional random walk, whose total displacement corresponds to the observational quantity.
We then evaluate the most likely value of such quantity:

$$|\alpha_{lm}|^2_{M.L.} = \frac{s^2\hbar}{2\pi a^2}\int\frac{U(k)^2C(k)}{k^4}j_l^2((|\vec{k}|R_D)k^3dk$$

The function $C(k)$ encodes information contained in $F(k)$. For each model of collapse it has a slightly different functional form.
It turns out that in order to get a reasonable spectrum, we have one single simple option: z_k must be almost independent of k, That is: $\eta_k^c = z/k$.

This result shows that the details of the collapse have observational consequences!!

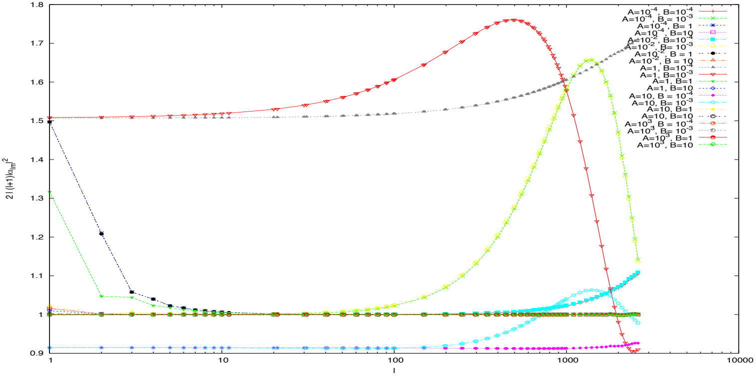
For Model 1) we have

$$C(k)^{(1)} = 1 + \frac{2}{z_k^2}\sin^2\Delta_k + \frac{1}{z_k}\sin(2\Delta_k),$$ where $\Delta_k = k\eta - z_k$, $z_k = \eta_k^c k$ with η representing the conformal time of observation, and η_k^c the conformal time of collapse of the mode k.

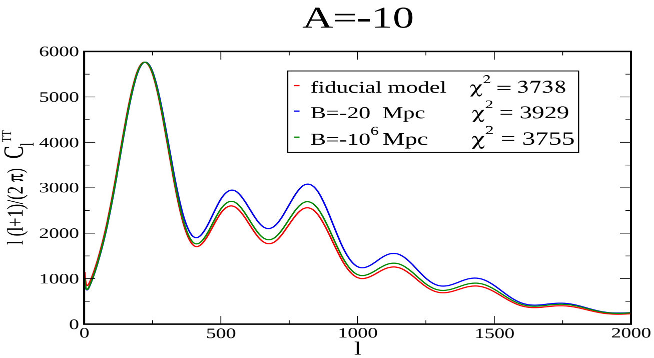
For Model 2) we find:

$$C(k)^{(2)} = 1 + \sin^2\Delta_k\left(1 - \frac{1}{z_k^2}\right) - \frac{1}{z_k}\sin(2\Delta_k),$$

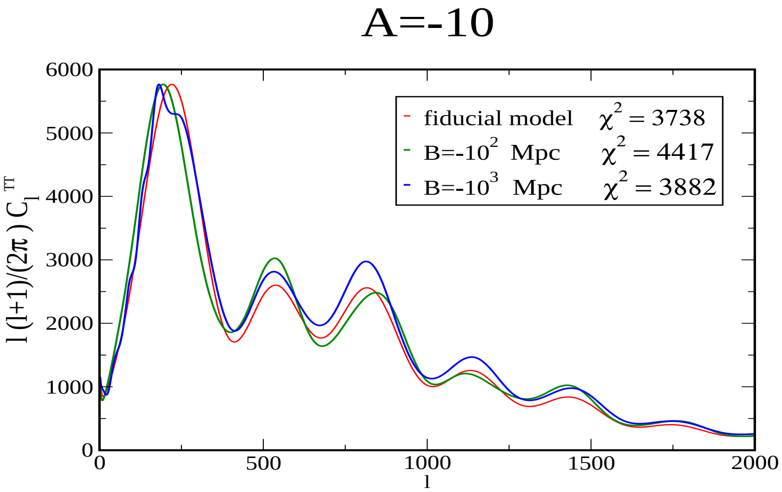
We have carried out preliminary exploration (with A de Unanue, PRD 2008) of the effects of departures from the pattern $\eta_k^c = z/k$, but assuming $\eta_k^c = A/k + B$. This first analysis simply ignores the effects of late physics and compares the predicted spectrum with a flat one.



We are now finishing (with S, Landau & C. Sccocola) a much more detailed analysis incorporating the well understood late time physics (acoustic oscillations, etc) and comparing directly with the observational data. For Model 1:



For Model 2


A=-10
fiducial model $\chi^2 = 3738$
B=-10^2 Mpc $\chi^2 = 4417$
B=-10^3 Mpc $\chi^2 = 3882$
l (l+1)/(2π) C_l^{TT}
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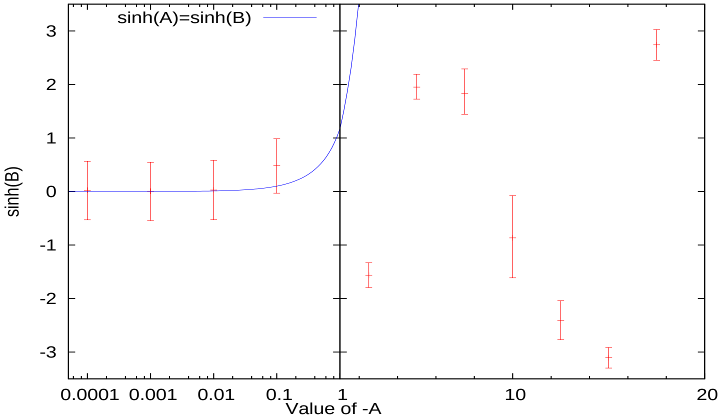
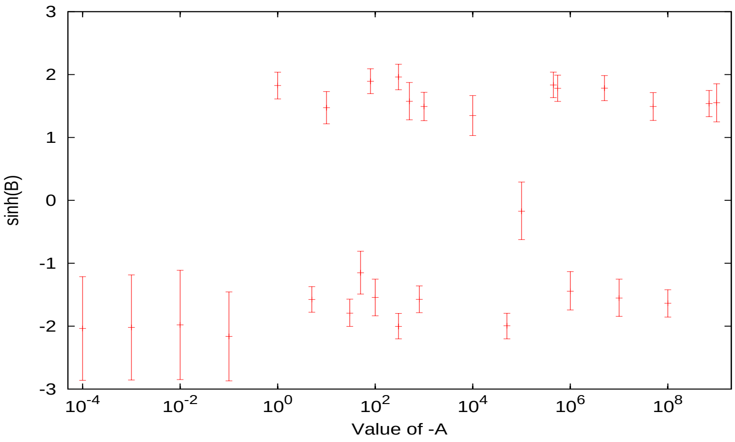
For Model 1



For Model 2



A version of ‘Penrose’s mechanism’ for collapse in the cosmological setting

Penrose has advocated the idea of a collapse of the wave functions as a dynamical process related to gravitational interaction. The suggestion: collapse into one of two quantum mechanical alternatives would take place when the gravitational interaction energy between them exceeds a certain threshold.

A very naive realization of Penrose’s ideas in the present setting could be obtained as follows: each mode would collapse by the action of the gravitational interaction between its own possible realizations. In our case, one could estimate the interaction energy $E_I(k, \eta)$ by considering two representatives of the possible collapsed states on opposite sides of the Gaussian associated with the vacuum. We will denote the two alternatives by the indices (1) and (2).

We interpret Ψ as the Newtonian potential and, the matter density $\rho = a^{-2}\dot{\phi}_0\dot{\delta}\phi = a^{-3}\dot{\phi}_0\pi^y$.

Then the relevant energy is given by :

$$E_I(\eta) = \int \Psi^{(1)} \rho^{(2)} dV = \int \Psi^{(1)}(x,\eta)\rho^{(2)}(x,\eta) a^3 d^3x = \int \Psi^{(1)}(x,\eta)\dot{\phi}_0(\pi^y(x,\eta))^{(2)} d^3x$$

where $\Psi^{(1)}$ is Newtonian potential that would have arisen if the system had collapsed into the alternative (1), and $\rho^{(2)}$ represents the density perturbation associated with a collapse into the alternative (2).

Viewing each mode's collapse as occurring independently, the trigger for the collapse of mode k would be the condition that this energy $E_I(k,\eta) = (\pi\hbar G/ak)(\dot{\phi}_0)^2$ reaches the value of the Planck Mass M_p.

This leads to:

$$z_k = \eta_k^c k = \frac{\pi}{9}(\hbar V')^2 (H_I M_p)^{-3} = \frac{\epsilon}{8\sqrt{6\pi}}(\tilde{V})^{1/2} \equiv z^c$$

which is independent of k, and, thus, as we have seen, this leads to a roughly scale invariant spectrum of fluctuations in accordance with observations. TEST OF CONCEPT.

7) MORE ON THE COLLAPSE MODELS AND IDEAS.

i) No tensor modes. (In the semiclassical approach we favor. This can also be tested.)

ii) Might offer a solution to the Fine Tuning problem for the inflaton Potential. (CQG, 27, 225017 (2010).

iii) Multiple collapses. More information about the post-collapse states .(CQG, 28, 155010 (2011))

iv) New views on the study of Non-Gaussianities. Novel possibilities, and approaches. (arXiv:1107.3054 [astro-ph.CO].)

v) Very Speculative Ideas connecting with QG and the problem of time: Wheeler de Witt or LQG are timeless theories. To recover time, we must resort to identifying an observable that acts as a physical clock. When the evolution of the state for other variables is cast in terms of a physical clock an approx. Schrëdinger eq. is recovered. But it is not 100% Unitary. Can this be the place where a collapse fits with the rest of our theories?

8) MORE ON THE INTERPRETATIONAL PROBLEM:

(if we have time)

In fact, we could have decided to compute directly the quantities most directly observed: our specific CMB map (characterized by its coefficients):

$$\alpha_{lm} = \frac{1}{3} \int d\Omega^2 \psi(\eta_D, \vec{x}_D) Y^*_{lm}(\theta, \varphi)$$

Identifying $\psi(\eta_D, \vec{x}_D)$ with $\langle 0|\hat{\psi}_{\vec{k}}(\eta_R)|0\rangle$, we find that $\alpha_{lm} = 0$. THIS SEEMS LIKE A PROBLEM, OR DOES IT NOT?.

One could dismiss this by saying: Well that is only the average value over universes. That is, one would take the view that the vacuum state (i.e its unitary evolution) does not represent the state of our universe. That it is just " like when we measure anything"... perhaps, but then, we must acknowledge that there must be some measurement involved. What measurement? By whom?

Perhaps, the view is that the vacuum state does not represent our Universe, but some ensemble. If so, what is the state that represents our universe? And why should we not use that state in analyzing the spectrum?

Any way, we should not trust the analysis that leads to $\alpha_{lm} = 0$. Is not this quantity a weighted average over spatial directions? should this not be equal to the weighted ensemble average? isn't that zero?. If so...

Why should we trust some predictions of the formalism and not others ?

In fact, to be able to trust the analysis, we need to find the physical reason behind the breakdown of the initial symmetry. (even if the symmetry was broken in one part in e^{80} that is not relevant). Often this issue is hidden from view by the fact that one is dealing with complex situations involving large numbers of D.O.F.

But this does not mean that the conceptual problem simply goes away.

What helps us focus here on the issue, despite the large number of D.O.F., is the symmetry.

People often refer to so called analogous situations:
Example 1 Radioactive α Decay of an spherically symmetric atomic nucleus in a bubble chamber.
How is it that the outgoing spherical wave function characterizing such decay, could be reconciled with the observational fact that the emitted α particles lead to straight tracks in the bubble chambers ?
Problem considered by Sir. N. F. Mott in 1929 in the following manner:

Initially, we have the unstable nucleus located at ($\vec{X} = \vec{0}$) in the state $|\Psi^{+}\rangle$ (spherically symmetric). Decays to the nuclear ground state $|\Psi^{0}\rangle$, plus an α particle in the state $|\Xi_{\alpha}\rangle$, which is also spherically symmetric . One considers then two hydrogen atoms with nuclei fixed at $\vec{a}_1$ and $\vec{a}_2$, while the electrons are in the corresponding ground states . The analysis focuses on the degree of alignment of the origin and the points ($\vec{a}_2 \approx c\vec{a}_1$) if both atoms become excited by the interaction with the α particle.

The result is that the probability of both atoms to be excited is $\neq 0$ only if there is a large degree of alignment, which then explains the experimental finding of straight α tracks in the bubble chamber.

At first sight, this seems like a clear example of an initial state with a given symmetry ($|\Psi^+\rangle$) evolving to a final state lacking it, despite the fact that the Hamiltonian (governing the decay $|\Psi^+\rangle \to |\Psi^0\rangle|\Xi_\alpha\rangle$ and the dynamics of the α particle) preserves that symmetry.

A second look reveals, to start, that the localization of the hydrogen nuclei breaks the symmetry. The discussion, in fact, is based not just on what we said before, but also on the Hamiltonian for the joint evolution of the α particle and the 2 electrons (of the localized hydrogen atoms).
In fact, the analysis by Mott relies, implicitly, on the projection postulate in connection with measurement: This is employed while computing probabilities, by projecting on the sub-space corresponding to both atoms being excited.
If we were to replace such atoms by some hypothetical detectors having spherical wave functions (say spherical shells with radius r_i), a similar calculation would not yield straight lines but spherical patterns of excitation. We would find that, with a certain probability, the shells $i^{th} \& j^{th}$ would be excited, but symmetry would remain intact. In our problem with the inflationary cosmology, the situation is closer to the latter than to the former.

Simplified Model: Mini-Mott:

Consider two, double level detectors $|-\rangle$ (ground) y $|+\rangle$ (excited) located in $x = x_1$ y $x = -x_1$: Initially, they are in their ground states, and there is a particle with an initial state corresponding to a wave packet $\psi(x, 0)$ centered at the origin and symmetric under $x \to -x$. The Hamiltonian for the free particle:

$$\hat{H}_P = \hat{p}^2/2M \tag{7}$$

while that for each detector is

$$\hat{H}_i = \epsilon \hat{I}_p \otimes \{|+\rangle^{(i)}\langle +|^{(i)} - |-\rangle^{(i)}\langle -|^{(i)}\}. \tag{8}$$

where $i = 1, 2$. The hamiltonian for particle -detector 1 interaction is:

$$\hat{H}_{P1} = \frac{g}{\sqrt{2}}\delta(x - x_1\hat{I}_p) \otimes (|+\rangle^{(1)}\langle -|^{(1)} + |-\rangle^{(1)}\langle +|^{(1)}) \otimes I_2 \tag{9}$$

analogously for detector 2.

Then, we consider Schrödinger's equation for the initial condition:

$\Psi(0) = \sum_x \psi(x,0)|x\rangle \otimes |-\rangle^{(1)} \otimes |-\rangle^{(2)}$

Thus, after some time t we have:

$$\Psi(t) = \sum_x \psi_1(x,t)|x\rangle \otimes |+\rangle^{(1)} \otimes |-\rangle^{(2)} + \sum_x \psi_2(x,t)|x\rangle \otimes |-\rangle^{(1)} \otimes |+\rangle^{(2)}$$

$$+ \sum_x \psi_0(x,t)|x\rangle \otimes |-\rangle^{(1)} \otimes |-\rangle^{(2)} + \sum_x \psi_D(x,t)|x\rangle \otimes |+\rangle^{(1)} \otimes |+\rangle^{(2)}$$

The first 2 terms seem to be easily interpreted, while the last two represent the failure of detection and double detection (or bounce) usually very small amplitude g^2.

Thus, we could think that the first 2 terms indicate the high probability of breakdown of the symmetry: Either detector 1 or 2 became excited. Just using a Bohr-like interpretation, we are done. However, besides indicating that these are detectors, we must specify how they are used. In other words, one must determine which basis (or observable) is the appropriate one to describe their behavior.

Let us focus on the ambiguities by considering an
Alternative description
Simply work with the basis:

$$|U\rangle \equiv |+\rangle^{(1)} \otimes |+\rangle^{(2)} \tag{10}$$

$$|D\rangle \equiv |-\rangle^{(1)} \otimes |-\rangle^{(2)} \tag{11}$$

$$|S\rangle \equiv \frac{1}{\sqrt{2}}[|+\rangle^{(1)} \otimes |-\rangle^{(2)} + |-\rangle^{(1)} \otimes |+\rangle^{(2)}] \tag{12}$$

$$|A\rangle \equiv |\frac{1}{\sqrt{2}}[|+\rangle^{(1)} \otimes |-\rangle^{(2)} - |-\rangle^{(1)} \otimes |+\rangle^{(2)}] \tag{13}$$

These are more convenient to discuss the symmetry issues .

In terms of the new states:

The Hamiltonian for the detectors:

$$\hat{H}_1 + \hat{H}_2 = 2\epsilon \hat{I}_p \otimes \{|U\rangle\langle U| - |D\rangle\langle D|\}. \tag{14}$$

(the other eigen-states correspond to he eigenvalue 0.)

The interaction Hamiltonian

$$\hat{H}_{P1} + \hat{H}_{P2} = \frac{g}{\sqrt{2}}[\{\delta(x - x_1\hat{I}_p) + \delta(x - x_2\hat{I}_p)\} \otimes (|U\rangle + |D\rangle)\langle S| \tag{15}$$

$$+\{\delta(x - x_1\hat{I}_p) - \delta(x - x_2\hat{I}_p)\} \otimes (|U\rangle + |D\rangle)\langle A|] + h.c. \tag{16}$$

This structure revels that a wave function that is symmetric $x \to -x$ and $1 \to 2$ can not excite the antisymmetric state of the detectors. In particular, the second term would give no contribution.
In fact, we can write the solution to the problem as :

$$\Psi(t) = \sum_x \psi_s(x,t)|x\rangle \otimes |S\rangle + \sum_x \psi_0(x,t)|x\rangle \otimes |D\rangle + \sum_x \psi_D(x,t)|x\rangle \otimes |U\rangle$$

Here the question is: why would it be wrong to consider this picture involving the full Hilbert space and the particle- detectors interactions, in terms of this basis where we view the two detectors as simply a more complex single one.

In this way we see that the initial symmetry is not broken when the detectors are considered at the quantum level and are initially also in a symmetric state. At least not, without the introduction of some additional postulate.