Probing Inflation with Primordial Messengers

Matteo R. Fasiello
IFT Madrid

October 25th 2020, IAP
Inflation

- Inflation, the idea
- Single-field slow-roll scenario: successes and signatures
- The importance of upcoming observations
- Axion inflation as a case study
- The “cosmological collider”
- Conclusions & Future work
Inflation

\[ 3M_P^2 H^2 \sim \rho_X \]

must satisfy: \[ w_X \equiv \frac{p_X}{\rho_X} < -\frac{1}{3} \]

What we learn

\[
\begin{align*}
\frac{\ddot{a}}{a} &= -\frac{1}{2M_P^2} (\rho + 3p) \\
\dot{\rho} + 3H(\rho + p) &= 0
\end{align*}
\]

acceleration!

special case \[ p \approx -\rho \]
Inflation

Simplest realization: single-scalar field in slow-roll

Scalar field:

\[ p_{\phi} = \frac{\dot{\phi}^2}{2} - V(\phi) \approx -V(\phi) \]
\[ \rho_{\phi} = \frac{\dot{\phi}^2}{2} + V(\phi) \approx V(\phi) \]

\[ \dot{\phi}^2 \ll V \]
\[ p_{\phi} \approx -\rho_{\phi} \]

“slow-roll” phase: potential is nearly flat
Slow-roll

\( \epsilon \equiv -\frac{\dot{H}}{H^2} \simeq \frac{M_P^2}{2} \left( \frac{V'}{V} \right)^2 \simeq \frac{3}{2} \frac{\phi^2}{V} \ll 1 \)

\( |\eta| \equiv \frac{|\dot{\epsilon}|}{H\epsilon} \simeq -\frac{2}{3} \left( \frac{V''}{H^2} \right) + 4\epsilon \ll 1 \)
Background + Fluctuations

\[ \phi(\vec{x}, t) = \bar{\phi}(t) + \delta \phi(\vec{x}, t) \]

- Classical homogeneous background
- Quantum fluctuations
- Perturbation modes are stretched by the expansion, become super horizon and freeze out to their value at horizon exit

\[ \Delta T, \delta \rho \]

cosmological perturbations
Metric Fluctuations

\[ ds^2 = (-dt^2 + a(t)^2 [e^{2\zeta} \delta_{ij} + \gamma_{ij} dx^i dx^j]) \]

- scalar fluctuations
- tensor perturbations
Primordial power spectra
(minimal scenario)

scalar fluctuations

\[ P_\zeta(k) = \frac{1}{8\pi^2} \frac{1}{\epsilon} \frac{H^2}{M_{\text{pl}}^2} \left( \frac{k}{k_*} \right)^{n_s-1} \]

\[ 0.9649 \pm 0.0042 \]
\[ 2.2 \times 10^{-9} \]
\[ [k_* = 0.05 \text{ Mpc}^{-1}, 68\% \text{C.L.}] \]

from Planck measurements of CMB anisotropies

\[ n_s - 1 \simeq -2\epsilon - \eta \]
Primordial power spectra
(vacuum fluctuations)

tensor fluctuations

energy scale of inflation

\[ \mathcal{P}^{\text{vacuum}}(k) = \frac{2}{\pi^2} \frac{H^2}{M_{\text{pl}}^2} \left( \frac{k}{k_*} \right) \]

\[ r \equiv \frac{\mathcal{P}_\gamma}{\mathcal{P}_\zeta} \] tensor-to-scalar ratio

\[ n_T \simeq -2\epsilon \simeq -r/8 \]

red tilt

\[ n_T \]

bounds

\[ r < 0.056 \text{ (95\%CL, Planck)} \]

current

\[ r < 0.01 \text{ (CMB–S3)} \]

future

\[ r < 0.001 \text{ (CMB–S4)} \]
Crossing Qualitative Thresholds

\[ n_s - 1 \simeq -2\epsilon - \eta \]

agnostic/naive

\[ \epsilon \sim \eta \quad \implies \quad r \gtrsim 10^{-2} \]

compelling models
e.g. Starobinsky

\[ 1 - n_s \simeq \frac{2}{N}, \quad r \simeq \frac{12}{N^2} \]

\[ r \gtrsim 10^{-3} \]
Single-field Inflation is doing well
Why go beyond the single-field scenario?

- Interpreting observations
  - What to infer from GW detection?
    - E.g. $r \leftrightarrow H$ relation
- Likely
  - String theory
  - Flux compactifications
  - 4D EFT with many moduli fields
- Interesting
  - Signatures of new content on GW spectrum:
    - PS: scale-dependence, chirality,
    - n-G: (amplitude, shape, angular..)
Focus

1 (class of) model(s): axion inflation

1 probe: primordial gravitational waves
Natural Inflation

\[ \mathcal{L} = \sqrt{-g} \left[ R[g] - (\partial \phi)^2 - \mu^4 (1 + \cos[\phi/f]) \right] \]

- axion-like potential
- simple
- (technically) natural: shift symmetry
- viable for \( f \gtrsim M_P \)

[Freese, Frieman, Olinto]
Chromo Natural Inflation

\[ \mathcal{L} \supset -\frac{1}{4} F^2 + \frac{\lambda \phi}{4f} F \tilde{F} - (\partial \phi)^2 - U_{\text{axion}}(\phi) \]

[Freese, Frieman, Olinto]
[...]

\[ f \ll M_P \] realization

very interesting GW signatures!
Extension of **Chromo Natural Inflation**

\[ \mathcal{L} \supset L_{\text{inflaton}} - \frac{1}{4} F^2 + \frac{\lambda \chi}{4f} F \tilde{F} - (\partial \chi)^2 - U_{\text{axion}}(\chi) \]

- \( f \ll M_P \) realization
- same interesting GW spectrum
- observationally viable
(Primordial) Gravitational Waves

\[ G_{\mu\nu} = 8\pi G T_{\mu\nu} \]

\[ ds^2 = -dt^2 + a^2(t) (\delta_{ij} + \gamma_{ij}) \, dx^i \, dx^j \]

\[ \gamma^i_j = \partial_i \gamma_{ij} = 0 \quad \text{two polarization states} \]

\[ \ddot{\gamma}_{ij} + 3H \dot{\gamma}_{ij} + k^2 \gamma_{ij} = 16\pi G \Pi^T_{ij} \]

anisotropic stress-energy tensor
Primordial GW in our Model

\[ \mathcal{L} \supset \mathcal{L}_{\text{inflaton}} - \frac{1}{4} F^2 + \frac{\lambda \chi}{4f} F \tilde{F} - (\partial \chi)^2 - U_{\text{axion}}(\chi) \]

\[ \begin{aligned} A_0^a &= 0 \\ A_i^a &= aQ \delta_i^a \\ \delta A_i^a &= t_{ai} + \cdots \end{aligned} \]

\[ \ddot{\gamma}_{ij}^\lambda + 3H \dot{\gamma}_{ij}^\lambda + k^2 \gamma_{ij}^\lambda \propto t_{ij}^\lambda + \cdots + \cdots \]

[Dimastrogiovanni, MF, Fujita]

\[ P_{\lambda}^{\text{sourced}} \gtrsim P_{\lambda}^{\text{vacuum}} \]

now possible!
\[
\begin{align*}
\text{metric} & \quad \Psi''_{R,L} + \left(1 - \frac{2}{x^2}\right) \Psi_{R,L} = \mathcal{O}(1)(t_{R,L}) \\
\text{gauge} & \quad t''_{R,L} + \left[1 + \frac{2m_Q \xi}{x^2} \mp \frac{2}{x}(m_Q + \xi)\right] t_{R,L} = \tilde{\mathcal{O}}(1)(\Psi_{R,L})
\end{align*}
\]

\[\xi = \frac{\lambda \dot{\chi}}{2fH}\]

\[x \sim -k\tau\]
Laser Interferometers: new frontier to test primordial physics (GW) at small scales

LISA: $10^{-4}\text{Hz} \lesssim f \lesssim 10^{-1}\text{Hz}$

LIGO+: $1\text{Hz} \lesssim f \lesssim 10^3\text{Hz}$
Testing Amplitude & Scale Dependence

```
  h^2 \Omega_{GW}
```

```
f (Hz)
```

```
10^{-19} 10^{-14} 10^{-9} 10^{-4} 10^0 10^4 10^6
```

```
```

“\hspace{1cm}” freedom in parameter space
Chirality

(background +) Chern-Simons coupling \( \lambda \chi \frac{F \tilde{F}}{4f} \)

\[
\dot{t}_{ij}^{L/R} \pm \lambda(\ldots)t_{ij}^{L/R} + \cdots = 0
\]

\[
\gamma_{ij}^{L} \neq \gamma_{ij}^{R}
\]

chiral spectrum

\[
\mathcal{P}_{\gamma}^{L} \neq \mathcal{P}_{\gamma}^{R}
\]
Chirality

CMB tests

no chirality

\[ \langle BT \rangle = 0 = \langle EB \rangle \]

chirality

\[ \langle BT \rangle \neq 0 \neq \langle EB \rangle \]
FIG. 2: Left panel: C_{BB}' for the same three sets of parameters used in Figure 1: (blue: $r^\ast = 2, r = 0.07, k_p = 0.005 \text{ Mpc}^{-1}$), (orange: $r^\ast = 2, r = 0.07, k_p = 0.0005 \text{ Mpc}^{-1}$), (green: $r^\ast = 2, r = 0.07, k_p = 7 \times 10^{-5} \text{ Mpc}^{-1}$) compared to the LiteBIRD noise spectrum with 2% foregrounds (solid black) and without foregrounds (dotted black), the lensing BB spectrum (dashed black), and the standard vacuum fluctuation $C_{BB}'(r = 0.07)$ consistent with the BKP $r < 0.07$ (95% C.L.) (solid grey). The axion-SU(2) spectra contain a contribution from vacuum fluctuations with $r^\ast = 10^{-5}$. Right panel: $|C_{TB}'|$ (solid colour) and $|C_{EB}'|$ (dashed colour) spectra for the same three sets of parameters. Shown in black is an example of the spurious TB signal induced by polarimeter miscalibration for an angle of one arcminute, as discussed in § III C.

FIG. 3: Signal-to-noise of TB + EB spectra assuming perfect delensing and no foreground contamination or instrumental noise. The black dashed line indicates the bounds placed by $r^\ast < 0.07$. Left panel: $k_p = 5 \times 10^{-3} \text{ Mpc}^{-1}$. Right panel: $k_p = 7 \times 10^{-5} \text{ Mpc}^{-1}$.

\[ P_{h\text{sourced}} = r_* P_{\zeta} \text{Exp} \left[ - \frac{1}{2\sigma^2} \ln^2 \left( \frac{k}{k_p} \right) \right] \]
Extended Chromo-Natural

gravitational waves forecast: LiteBIRD

Komatsu et al 2017
FIG. 10: Signal-to-noise contours obtained using Equation 23 for a BBO-like experiment described in § IV C. The primordial spectrum has $k_p \sim 10^{13} \text{ Mpc}^{-1}$.

The TB and EB correlations that result from the chiral tensor spectrum. We found that LiteBIRD would be able to detect the chirality for $r^* > 0.03$, whilst $r^* < 0.07$ is required by current observations. The addition of Stage 4 observations has little effect as such a survey would be limited to $` > 30$, but the primordial chiral signal is contained almost entirely within $2 < ` < 30$. Further, we found that for cosmic-variance limited observations the maximum achievable signal-to-noise for $r^* < 0.07$ would be $\sim 3$. From these studies we conclude that the ability of CMB two-point statistics to determine the presence of a chiral GWB is fairly limited.

However, in this study we have not fully leveraged the scale-dependence of the axion-SU(2) model. Single-field slow-roll expects the tensor spectrum to have a tilt given by the self-consistency relation $n_T = r/8$. If future measurements of the BB spectrum were able to form a long enough lever arm to constrain $n_T$, a deviation from the self-consistency relation would be strong evidence for an alternative GW production mechanism during inflation. For LiteBIRD we find that for a peak wavenumber in the range $k_p \sim 7 \times 10^5$ to $7 \times 10^3 \text{ Mpc}^{-1}$ the primordial BB spectrum is detectable with $(S/N)_{BB} \sim 1$ for $r^* \sim 10^3$. However, the projected sensitivity on $n_T$ for LiteBIRD is $\sim 0.04$, which is not sufficient to test deviations from the self-consistency relation.

Another characteristic of the axion-SU(2) model of Ref. [13] is its intrinsic non-Gaussianity. Some studies have recently shown that higher order statistics of B-modes, such as the BBB bispectrum, may yield a significant improvement for the axion-U(1) model [12, 19]. An analysis of the CMB non-Gaussianity for the axion-SU(2) model is therefore in order [61].

In § IV we showed that interferometers may provide a complementary probe to the CMB at much smaller scales $\sim 10^{12} \text{ Mpc}$, even for the relatively flat spectra required by the attractor behaviour of the background axion field coupled to the SU(2) gauge field. This takes advantage of the scale-dependence of the axion-SU(2) model, which allows the spectrum to have a large excursion at some scale $k_p$, e.g. as shown in Figure 7, making the cosmological GWB of the axion-SU(2) model a viable target for interferometers with current sensitivities. We went on to consider two designs of an advanced stage LISA-like mission proposed by Ref. [33] which are sensitive to both the intensity and circular polarization of the GWB. We found that such experiments would be able to detect a chiral GWB to high significance for a large region of the model's parameter space inaccessible to the CMB.

Acknowledgments
BT would like to acknowledge the support of the University of Oxford-Kavli IPMU Fellowship and an STFC studentship. MS was supported in part by a Grant-in-Aid for JSPS Research under Grant No. 27-10917. The work of TF is partially supported by the JSPS Overseas Research Fellowships, Grant No. 27-154. Numerical computations

Komatsu et al 2017
Chirality

Interferometers tests

- cross-correlation between interferometers at different locations
  
  [Smith, Caldwell 2017]

- recent work on LISA: use kinematically induced dipole
  
  [Seto 2006]
  [Domcke et al 2019]
n-G in Axion-Gauge Field Model

\[ \langle h_R(\vec{k}_1) h_R(\vec{k}_2) h_R(\vec{k}_3) \rangle = (2\pi)^3 \delta^{(3)} \left( \sum_{i=1}^{3} \vec{k}_i \right) B_h(k_1, k_2, k_3) \]

\[ \Psi = \text{GW} \]
\[ t = \text{tensor SU(2)} \]

\[ \frac{B_h}{P_\zeta^2} \lesssim r^2 10^6 \]

sourced nG tensors
much larger than in SFSR

- \( m_Q = 3.45 \)
- \( \epsilon_B \simeq 10^{-5} \)
- \( H \simeq 10^{13} \text{ GeV} \)
- \( r_{\text{vac}} \simeq 0.002 \)
- \( r_{\text{sourced}} \simeq 0.04 \)
Primordial GW to test inflationary particle content

scale-dependence

chirality

...non-Gaussianity

more later

Axion-gauge field models
general approach: inflationary particle content
Organizing Principles for extra particle content: the mass

(effective) mass range

\[ m \gg H \]

fields integrated out, some remnants

Achucarro et al 2012
Burgess et al 2013
MF et al 2013
Silverstein 2017

\[ m \lesssim H \]

immediate and detectable effects
Organizing Principles for particle content: the spin

consequences on the mass range

Particles as unitary irr. rep of spacetime isometry group, dS

principal series

\[ \frac{m^2}{H^2} \geq \left( s - \frac{1}{2} \right)^2 \]

complementary

\[ s(s - 1) < \frac{m^2}{H^2} < \left( s - \frac{1}{2} \right)^2 \]

discrete series

\[ \frac{m^2}{H^2} = s(s - 1) - t(t - 1) \]

\[ s \geq t ; \ s, t, = 0, 1, 2, \ldots \]
Mass & Spin

spin-2 example can source tensors linearly!

\[ m^2 = 0 \quad \checkmark \quad m^2 \geq 2H^2 \]

+ 

interactive spin-2 fields ==> at most 1 is massless

[Boulanger, Damour, Gualtieri, Hennaux (2000)]
Extra spin-2 field is a massive graviton

know how to write it non-linearly

\[ S_{\text{tot}} = S_\phi + \int d^4x \left[ \sqrt{-g} M_P^2 R[g] + \sqrt{-f} M_f^2 R[f] - m^2 M^2 \sqrt{-g} \beta_n E_n \left( \sqrt{g^{-1} f} \right) \right] \]

[de Rham, Gabadadze, Tolley (2011)]
[Hassan, Rosen (2011)]

ghost-free

+ well-known use for late-time acceleration, \( m \sim H_0 \)

check the unitarity bound

&

use in inflationary context
Unitarity bound

\[
\tilde{m}^2 \left[ 1 + \left( \frac{H_f/M_f}{H/M_P} \right)^2 \right] \geq 2H^2
\]

somewhat weakened constraint but

\[ m \sim H \]

extra spin-2 fields tend to decay quickly

[MF, Tolley (2012&2013)]

[MF, Tolley (2012&2013)]

[MF, Tolley (2012&2013)]

extra spin-2 fields tend to decay quickly

[m \sim H]

not the end of the story!

[m \sim H]

not the end of the story!

extra spin-2 fields tend to decay quickly

[m \sim H]

not the end of the story!

extra spin-2 fields tend to decay quickly

[m \sim H]

not the end of the story!

extra spin-2 fields tend to decay quickly

[m \sim H]

not the end of the story!

extra spin-2 fields tend to decay quickly

[m \sim H]

not the end of the story!

extra spin-2 fields tend to decay quickly

[m \sim H]

not the end of the story!

extra spin-2 fields tend to decay quickly

[m \sim H]

not the end of the story!

extra spin-2 fields tend to decay quickly

[m \sim H]

not the end of the story!
How can we probe info on Mass & Spin?
so far

\[ \langle \zeta_{k_1} \zeta_{k_2} \rangle \equiv \frac{2\pi}{k^3} \mathcal{P}(k) \delta^{(3)}(k_1 + k_2) \]

n>2-point functions probe interactions

\[ \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle \equiv (2\pi)^3 \delta^{(3)}(k_1 + k_2 + k_3) B(k_1, k_2, k_3) \]

Amplitude

\[ f_{NL} \sim B/P^2 \]
Squeezed Bispectrum: single-field inflation

$$
\lim_{k_1 \to 0} \frac{1}{P_\zeta(k_1)} \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = -k_2 \cdot \frac{\partial}{\partial k_2} \langle \zeta_{k_2} \zeta_{k_3} \rangle
$$

Standard consistency relation for single-field inflation [Maldacena, 2003]

$$
\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle \Big|_{k_1 \ll k_3} \propto \frac{1}{k_1^3 k_3^3} \sum_{n=0} \ b_n \left( \frac{k_1}{k_3} \right)^n \propto f_{\text{NL}}
$$

Physical information from n=2

Qualitative threshold for LSS surveys $f_{\text{NL}} \sim 1$
Squeezed Bispectrum: new physics

extra particle content $\Rightarrow$ non-analytical scaling $\Rightarrow$ directly probe new physics

$\langle \zeta k_1 \zeta k_2 \zeta k_3 \rangle \bigg|_{k_1 \ll k_3} \propto \frac{1}{k_1^3 k_3^3} \left( \frac{k_1}{k_3} \right)^{3/2-\nu_s} P_s(\hat{k}_1 \cdot \hat{k}_3)$

standard

non-analytical scaling

$\nu_s = \mu_s = \sqrt{\frac{m^2}{H^2} - \left( s - \frac{1}{2} \right)^2}$

extra angular dependence

info on mass & spin!

[Noumi et al 2012]
[Arkani-Hamed, Maldacena 2015]
[Kehagias, Riotto 2015]
Squeezed Bispectrum: new physics

\[ \langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle_{k_1 \ll k_3} \propto \frac{1}{k_1^3 k_3^3} \frac{1}{\mu_s} e^{-\pi \mu_s} \left( \frac{k_1}{k_3} \right)^{3/2} P_s(\hat{k}_1 \cdot \hat{k}_3) \cos \left[ \mu_s \ln \left( \frac{k_1}{k_3} \right) \right] \]

direct mass suppression

non-analytical scaling

\[ m \geq \frac{3}{2} H \]

crucial fact for \( s \geq 2 \) spinning fields

\[ m \gtrsim H \]
Tensor-scalar-scalar Bispectrum

\[
\langle \gamma_{k_L} \zeta_{k_S} \zeta_{k_S} \rangle \bigg|_{k_L \ll k_S} \propto \frac{1}{k_L^3 k_S^3} \left( \frac{k_L}{k_S} \right)^{3/2 - \nu_s} E_2^\lambda (\hat{k}_L \cdot \hat{k}_S) P_s^\lambda(\hat{k}_L \cdot \hat{k}_S)
\]

non-analytical scaling, CRs breaking

\[
\nu_s = \sqrt{\left( s - \frac{1}{2} \right)^2 - \frac{m^2}{H^2}}
\]

extra angular dependence

Connections with “tensor fossils” as diagnostic of new physics

\[
P_\zeta(k, x_c) |_{\gamma_L} = P_\zeta(k) \left( 1 + Q_{\ell m}(x_c, k) \hat{k}_\ell \hat{k}_m \right)
\]

[Dimastrogiovanni, MF, Jeong, Kamionkowski 2014]
[Dimastrogiovanni, MF, Kamionkowski 2016]
Crucial for non-Gaussianity at small scales (e.g. LISA)

\[ P_{\gamma}^{\text{tot}}(k, x_c) \bigg|_{\gamma_L} = P_{\gamma}(k) \left( 1 + Q_{lm}(x_c, k) \hat{k}_l \hat{k}_m \right) \]

[Dimastrogiovanni, MF, Tasinato, PRL 2020]

\[ Q_{lm}(x_c, k) \equiv \int \frac{d^3 q}{(2\pi)^3} e^{i \mathbf{q} \cdot \mathbf{x}_c} \sum_{\lambda} \left[ \frac{\tilde{B}^{\text{sq}}(k, q)}{2 P_{\gamma}(k) P_{\gamma}^\lambda(q)} \right] \epsilon_{\lambda m} (-\hat{q}) \gamma_{-\lambda} \]
Caveat: only squeezed non-G

- propagation effects typically de-correlate primordial non-Gaussianities
  
  [Bartolo, De Luca, Franciolini, Lewis, Peloso, Riotto, 2019]

- an important exception is the ultra-squeezed regime (e.g. long mode horizon-size)
  
  [Dimastrogiovanni, MF, Tasinato]

- application: correlate STT-sourced GW anisotropies with CMB anis. to test primordial origin
  
  [Adshead, Afshordi, Dimastrogiovanni, MF, Lim, Tasinato]

\[ \delta F_{NL}^{sq} \sim \frac{2.8 \times 10^3}{\text{SNR}_{SGWB}} \]
Recap

extra fields can be probed via squeezed bispectrum because they break consistency relations

spinning ==> richer set of signatures
but, typically
spinning ==> mass bounds ==> suppression

[Biagetti, Dimastrogiovanni, MF 2017]
One crucial ingredient

the mass, the spin... the coupling

\[ \exists \text{1 field that doesn’t decay: the inflaton} \]

non-minimal coupling to the inflaton!

Effective Field Theory Approach

[Iacconi, MF et al, 2019]

[Dimastrogiovanni, MF, Tasinato, Wands 2018]

[Bordin, Creminelli, Khmelnitsky, Senatore 2018]
Examples

quasi-single-field

\[ S_m = \int d^4 x \sqrt{-g} \left[ -\frac{1}{2} (R + \sigma)^2 (\partial_\mu \theta)^2 - \frac{1}{2} (\partial_\mu \sigma)^2 - V_{sr}(\theta) - V(\sigma) \right] \]

[Chen, Wang 2009] + ...

scalar sector

inflaton

extra

(gauge) vector field

\[ U(1), SU(2) \ldots \]

\[ I(\phi)F^2 \quad \text{or} \quad I(\phi)F \tilde{F} \]

strongly affects tensor sector ==> chiral GW at LISA scales
The EFT approach

philosophy and cooking instructions

- unitarity bounds on spinning particles masses are dictated by dS isometries
- inflation needs to end $\iff$ dS iso are broken by inflaton

[Cheung et al 2007]

- couple directly to the inflaton any otherwise massive field that you want to make effectively lighter
- non-linearly realized symmetries prescribe inflaton $\iff$ extra field(s) coupling(s)
The EFT approach can be implemented for generic extra spin

it is an EFT of fluctuations around FLRW

\begin{align*}
S[\sigma] &= \frac{1}{4} \int d^4 x a^3 \left[ (\dot{\sigma}^{ij})^2 - c_2^2 (\partial_i \sigma^{jk})^2 / a^2 - \frac{3}{2} (c_0^2 - c_2^2) (\partial_i \sigma^{ij})^2 / a^2 - m^2 (\sigma^{ij})^2 \right] \\
S_{\text{int}} &= \int d^4 x \sqrt{-g} \left[ -\frac{\rho}{2\epsilon H a^2} \partial_i \partial_j \pi_c \sigma^{ij} + \frac{1}{2} \rho \gamma_{c ij} \sigma^{ij} \\
&\hspace{1cm} - \frac{\rho}{2\epsilon H^2 M_P a^2} (\partial_i \pi_c \partial_j \pi_c \dot{\sigma}^{ij} + 2H \partial_i \pi_c \partial_j \pi_c \sigma^{ij}) + \frac{\tilde{\rho}}{\epsilon H^2 M_P a^2} \hat{\pi}_c \partial_i \partial_j \pi_c \sigma^{ij} - \mu (\sigma^{ij})^3 \right] \\
\text{[Bordin et al 2018]}
\end{align*}
\[ S_{\text{int}} = \int d^4x \sqrt{-g} \left[ -\frac{\rho}{2\epsilon H a^2} \partial_i \partial_j \pi_c \sigma^{ij} + \frac{1}{2} \rho \gamma_{cij} \sigma^{ij} \right. \\
- \frac{\rho}{2\epsilon H^2 M_P a^2} (\partial_i \pi_c \partial_j \pi_c \dot{\sigma}^{ij} + 2H \partial_i \pi_c \partial_j \pi_c \sigma^{ij}) + \frac{\tilde{\rho}}{\epsilon H^2 M_P a^2} \mathring{\pi}_c \partial_i \partial_j \pi_c \sigma^{ij} - \mu (\sigma^{ij})^3 \left. \right] \]

**Extra spin-2 case**

\[ P_\gamma(k) = \frac{4H^2}{M_P^2 k^3} \left[ 1 + \frac{C_\gamma(\nu)}{c_\sigma^{2\nu}} \left( \frac{\rho}{H} \right)^2 \right] \]

[Bordin et al 2018]
\[ \frac{\rho}{H} \ll 1 \quad \text{perturbative treatment of quadratic mixing} \]
\[ \frac{\mu}{H} \ll 1 \quad \text{L}_3 < \text{L}_2 \]
\[ \frac{\rho}{\sqrt{\epsilon}H} \ll 1 \quad \text{small radiative corrections to sigma mass} \]
\[ c_\sigma \gtrsim 10^{-2} \quad \text{tensor nG limits as well} \]

\[ f_{\text{nl}}^{\text{eq}} \approx \begin{cases} 
\frac{77782}{\sqrt{r}} r^2 \approx 1143 & \text{for } c_\sigma = 0.1 \\
\frac{155563}{\sqrt{r}} r^2 \approx 2286 & \text{for } c_\sigma = 0.05 \\
\frac{777817}{\sqrt{r}} r^2 \approx 11431 & \text{for } c_\sigma = 0.01 
\end{cases} \]

[Dimastrogiovanni, MF, Tasinato, Wands 2018]
Small scales signatures?
time-dependent sound speeds \( \{c_0, c_1, c_2\} \), \( s_i = \frac{\dot{c}_i}{Hc_i} \)

Why? Integrating out heavy fields may result into \( c_s < 1 \) for the remaining light field(s)

\[
\begin{align*}
\bullet \quad c_1^2 &= \frac{1}{4} c_2^2 + \frac{3}{4} c_0^2 \\
\bullet \quad s_0 &= \frac{4}{3} \frac{c_1^2}{c_0^2} s_1 - \frac{1}{3} \frac{c_2^2}{c_0^2} s_2 \\
\bullet \quad \text{perturbativity bound: } c_2 > 10^{-4}, \text{ sets a bound on } s_2 = \frac{\dot{c}_2}{Hc_2}
\end{align*}
\]
Consequence: scale dependent $P_\zeta$ &/or $P_\gamma$

$$P_\gamma(k) \propto \frac{1}{c_2^{2\nu}} \left( \frac{k}{k_*} \right)^{-2\nu}$$

if $s_2 < 0$ the sourced contribution is blue-tilted:

can this signal be detected?

Example: $\{s_2 < 0, s_1 = 0, s_0 > 0\}$

$$\begin{cases} 
  c_2(k) &= c_2 \big|_{in} \left( \frac{k}{a_0H_0} \right)^{s_2} \\
  c_0(k) &= \sqrt{\frac{4}{3}c_1^2 - \frac{1}{3}c_2(k)^2}
\end{cases}$$

parameters: $\{H, c_2 |_{in}, c_1, \frac{m}{H}\}$
Example of parameter space analysis

\[ H = 10^{13} \text{ GeV}, \; c_1 = 0.85, \; \frac{m}{H} = 0.54, \; c_2|_{in} = 10^{-1} \]

\[
\rho \quad \begin{array}{c}
10^{-1} \\
10^{-2} \\
10^{-3} \\
10^{-4}
\end{array} \\
\frac{H}{H} \\
\begin{array}{c}
0.00 \\
0.02 \\
0.04 \\
0.06 \\
0.08 \\
0.10 \\
0.12 \\
0.14
\end{array}
\]

- \[ r < 0.056 \]
- Gradient instabilities
- \( P_\zeta(k_*)_{\text{vac}} > P_\zeta(k_*)_{\text{source}} \)
- LIGO
- LISA

Region above red line: excluded by bounds
Region below blue line: can be excluded by LISA

Example of parameter space analysis.
The same analysis can be performed also considering SKA
Conclusions

Cosmological probes will soon cross qualitative thresholds e.g. on $r, f_{NL}$

Lots we can learn on inflationary field content, strong connection with particle physics

Prepare theory to meet experiments

What is Compelling & Testable?
Thank You!
Back-up Slides
Observational bounds/sensitivities for SGWB

- CMB: Henro-Versillé et al 1408.5299
- BBN:
- aLIGO (O1): Abbott et al 1612.02029
- PTA: Lentati et al 1504.03692
- aLIGO (Design):
- LISA: Amaro-Seoane et al 1702.00786

Graph showing frequency (f) vs. h^2 Ω_{GW} with data points and lines indicating sensitivities and bounds.
Backreaction Under Control

$r > 0.07$

$R_{GW} < 1$

$g = 10^{-2}$

$H = 3 \times 10^{13} \text{ GeV}$

$H = 10^{13} \text{ GeV}$

$H = 3 \times 10^{12} \text{ GeV}$

$H = 10^{12} \text{ GeV}$

$\varepsilon_B$
Scalar bispectrum: current bounds

$$f_{NL}^{\text{local}} = -0.9 \pm 5.1 \quad f_{NL}^{\text{equil}} = -26 \pm 47 \quad f_{NL}^{\text{ortho}} = -38 \pm 24$$

[68 % CL]

Scalar bispectrum: future bounds

- **LSST**
- **SKA**
- **SPHEREx**

$$\sigma (f_{NL}^{\text{local}}) \approx 1$$

**21-cm**

$$\sigma (f_{NL}^{\text{local}}) \lesssim 10^{-1}$$

[Munoz, Ali-Haïmoud, Kamionkowski]

Tensor bispectrum

- **Planck**

$$f_{NL}^{\text{tens}} = (8 \pm 11) \times 10^2$$

[68 % CL]

$$f_{NL}^{\text{tens}} \equiv \frac{B_{+++}(k, k, k)}{(18/5)P_{\zeta}^2(k)}$$

(parity violating models / roughly equilateral)

- **LiteBIRD**

$$\sigma (f_{NL}^{\text{tens}}) = \text{a few}$$

(possibly also with **PICO**)
Tensor-Scalar-Scalar bispectrum

\[ f_{\gamma \zeta \zeta} = \frac{B_{\gamma \zeta \zeta}}{P_{\zeta}^2} \]

Local shape — temperature data [Shiraishi, Liguori, Fergusson]

\[ f_{\text{NL}} = -48 \pm 28 \quad [68\% \text{ CL}] \]

\[ f_{\gamma \zeta \zeta} \longleftrightarrow \sqrt{r} f_{\text{NL}} \]

Improvement expected from Planck to CMB-S4 (from BTT):

<table>
<thead>
<tr>
<th></th>
<th>CMB-S4</th>
<th>Relative improvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>\text{Local} \ (r = 0.01)</td>
<td>\sigma(\sqrt{r} f_{\text{NL}}) = 0.7</td>
<td>25.3</td>
</tr>
<tr>
<td>\text{Equilateral} \ (r = 0.01)</td>
<td>\sigma(\sqrt{r} f_{\text{NL}}) = 14.7</td>
<td>13.7</td>
</tr>
</tbody>
</table>

[CMB-S4 Science Book]