Probing cosmic string networks with gravitational waves

Pierre Auclair
auclair@apc.in2p3.fr
Under the supervision of Danièle Steer and Chiara Caprini

Laboratoire Astroparticule et Cosmologie, Université de Paris

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Outline

Introduction to cosmic strings

Gravitational wave emission from cosmic strings

The loop distribution: beyond the Nambu-Goto approximation

Conclusion

References
Introduction to cosmic strings

References:
(Nielsen & Olesen, 1973)
(Kibble, 1976)
(Vilenkin & Shellard, 2001)
(Ringeval, Sakellariadou, & Bouchet, 2007)
(Ringeval, 2010)
(Vachaspati, Pogosian, & Steer, 2015)
Cosmic strings (Kibble, 1976)

1D topological defects

- Cosmic strings are 1D topological defects that may appear after a symmetry breaking phase transition.
- After the phase transition the field *falls* into the new vacuum manifold $\mathcal{M}$.
- Strings arise if $\mathcal{M}$ is not simply connected, i.e. $\mathcal{M}$ contains holes around which loops can be trapped.
- We expect strings to be formed in most models of spontaneous symmetry breaking.

*Figure:* String formation in the "Mexican hat" potential $V(|\phi|)$. Figure taken from (Ringeval, 2010)
Cosmic strings (Kibble, 1976)

1D topological defects

- Cosmic strings are 1D topological defects that may appear after a symmetry breaking phase transition.
- After the phase transition the field *falls* into the new vacuum manifold $M$.
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As an example, the Lagrangian for the Nielsen-Olesen string (Nielsen & Olesen, 1973)

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_{\mu} \phi)^* D^{\mu} \phi - \frac{\lambda}{4} (|\phi|^2 - \eta^2)$$
Nambu-Goto strings: the one-dimensional limit

- The width of the string is very small compared to the other length scales in the problem, and the thin string limit is commonly adopted.
- Then the string is simply modeled as a line with mass per unit length $\mu \propto T^2$ using the Nambu-Goto action which minimizes the area swept by the string

$$S = -\mu \int \sqrt{-\det(\gamma)} d^2\zeta$$

$\zeta^a = (t, \zeta)$ and $\gamma_{ab}$ the induced metric on the string

<table>
<thead>
<tr>
<th>Energy scale</th>
<th>Width</th>
<th>Linear density</th>
</tr>
</thead>
<tbody>
<tr>
<td>GUT: $10^{16}$ GeV</td>
<td>$2 \times 10^{-32}$ m</td>
<td>$G\mu \approx 10^{-6}$</td>
</tr>
<tr>
<td>$3 \times 10^{10}$ GeV</td>
<td>$5 \times 10^{-27}$ m</td>
<td>$G\mu \approx 10^{-17}$</td>
</tr>
<tr>
<td>$10^8$ GeV</td>
<td>$2 \times 10^{-24}$ m</td>
<td>$G\mu \approx 10^{-22}$</td>
</tr>
<tr>
<td>EW: 100 GeV</td>
<td>$2 \times 10^{-18}$ m</td>
<td>$G\mu \approx 10^{-34}$</td>
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</tbody>
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Closed loops of cosmic strings
Oscillation and gravitational wave emissions

The general solution for a Nambu-Goto string in a Minkowski background is

\[ \vec{X}(t, \zeta) = \frac{1}{2} \left[ \vec{a}(\zeta - t) + \vec{b}(t + \zeta) \right] \]

\[ \vec{a}'^2 = \vec{b}'^2 = 1 \]

For a closed loop \( X^\mu(t, \zeta + \ell) = X^\mu(t, \zeta) \). One can show that the loop oscillates with a period \( T = \frac{\ell}{2} \).

These oscillations lead to a gravitational radiation. The quadrupole formula can give a rough estimate of the power emitted (Vilenkin & Shellard, 2001)

\[ \dot{E} \approx G \left( \frac{d^3D}{dt^3} \right)^2 \approx GM^2 L^4 \omega^6 \approx \Gamma G \mu^2 \]

in which \( D \approx ML^2 \) is the quadrupole moment, \( M = \mu L \) is the mass and \( \omega \approx \frac{1}{L} \) the characteristic frequency.

**NOTE**: it does not depend on the loop length!
Typical properties of cosmic strings

Loop formation and scaling

- When strings intersect, they change partner
- Analytical arguments and numerical simulations show the existence of an attractor solution independent of initial conditions called **scaling**
- During scaling, all length-scales are proportional to $t$ cosmic time.
- In particular, it means loop can survive until today

$$\rho_\infty \propto t^{-2} \propto \begin{cases} a^{-4} & \text{during radiation era} \\ a^{-3} & \text{during matter era} \end{cases}$$

Figure: (Ringeval et al., 2007)
Observational signatures of cosmic strings
Selection of observational signatures

- CMB: line discontinuities in the temperature or polarization patterns, and statistical methods based on calculations of various correlation functions. $G\mu < \text{few} \times 10^{-7}$
- 21-cm: brightness fluctuations or spatial correlations between the 21 cm and CMB anisotropies. Future experiments can in principle constrain $G\mu \approx 10^{-10} - 10^{-12}$
- The metric around a cosmic string can result in characteristic lensing patterns of distant light sources.

Figure: CLS-1, discovered in 2003, raised a lot of interest from the cosmic strings community but turned out to be two similar galaxies close to each other.
Gravitational wave emission from cosmic strings

References in this section:
(Vachaspati & Vilenkin, 1985)
(Damour & Vilenkin, 2001)
(Siemens et al., 2006)
(Blanco-Pillado & Olum, 2017)
(Abbott et al., 2018)
(Collaboration & the Virgo Collaboration, 2019)
(Auclair, Blanco-Pillado, et al., 2019)
A typical loop will have a number of kinks and cusps, and the spectrum of high frequency gravitational radiation emitted from a string depends on these features:

- **Kinks** are discontinuities in the tangent vector of the string. Kinks are formed when strings intercommute and travel along the string at the speed of light, \( q = \frac{5}{3} \).
- **Cusps** travel instantly at the speed of light, \( q = \frac{4}{3} \).

The waveform of the gravitational wave arriving at the detector is known (Damour & Vilenkin, 2001)

\[
h_q(\ell, z, f) = A_q(\ell, z, f) f^{-q} \quad , \quad A_q = g_{1,q} \frac{G\mu \ell^{2-q}}{(1 + z)^{q-1} r(z)}
\]
Rate of bursts

For a given loop distribution, you can estimate the GW burst rate (Siemens et al., 2006)

\[
\frac{d^2 R_q}{dV d\ell} = \frac{1}{1+z} \times \frac{d^3 \nu_q}{dt d\ell dV} \times \Delta_q
\]

as a function of

- \(\Delta_q\) geometrical factor for the fraction of GWs you can access (linked to a beaming angle)
- \(\frac{d^3 \nu_q}{dt d\ell dV} = \frac{2}{\ell} N_q \frac{d^2 N}{d\ell dV}\) number of events per space time volume per unit length
- \(N_q\) mean number of events per oscillation, which is supposed to be a fixed number.
- \(z\) redshift at emission

The effective burst rate in the detector depends on its sensitivity.

\[
R_q = \int dA_q \ e_q(A_q) \frac{dR_q}{dA_q} (G\mu, N_q)
\]
LIGO/Virgo burst search during O1

The parameter space \((G\mu, N_q)\), is scanned and excluded at a 95% level when \(R_q\) exceeds \(2.996/T_{\text{obs}}\) which is the rate expected from a random Poisson process over an observation time \(T_{\text{obs}}\).

- No cosmic string burst detected during O1 and O2 runs
- Allows to put upper bounds on the string tension which are not very competitive with respect to the Stochastic Background of GW
- We are currently involved in the LIGO/Virgo collaboration to produce constraints for the O3 run

**Figure**: (Abbott et al., 2018)
Emission of gravitational waves by a cosmic string loop

\[ \dot{E} = \Gamma G \mu^2, \quad \Gamma = \sum_m P_m = \mathcal{O}(50) \]

- All the energy radiated by loops is converted to gravitational waves
- An effective average power \( P_m \) emitted in mode \( m \) determined by simulations and/or analytical arguments

The high frequency regime is dominated by contributions from burst-like events

\[ P_m \propto \begin{cases} m^{-4/3} & \text{for cusps} \\ m^{-5/3} & \text{for kinks} \end{cases} \]

Low-frequency modes are dominated by the oscillations of the loops

Figure: Averaged power spectrum determined numerically in (Blanco-Pillado & Olum, 2017)
The stochastic background of gravitational waves

The uncorrelated sum of all the GW signals produced by cosmic string loops during the History of the Universe constitutes a Stochastic Background of GW.

We can estimate this background using energetic arguments:

\[ \Omega_{GW}(\ln f) = \frac{8\pi G}{3H_0^2} f \rho_{GW} \]

\[ \rho_{GW}(f) = \int_0^{t_0} \frac{dt}{[1 + z(t)]^4} P_{gw}(t, f') \frac{\partial f'}{\partial f} \]

\[ P_{gw}[t, f'] = G\mu^2 \sum_m \frac{2m}{fr^2} P_m \frac{d^2N}{d\ell dV}[\frac{2m}{f'}, t] \]

The loop distribution \( \frac{d^2N}{d\ell dV} \) remains to be specified, more in the next section.
• The constraint from burst is less stringent than the one from stochastic
• The intercommutation probability $p$ is set to 1 in the present seminar
• There is a huge disparity between different models especially on these relatively high-frequency experiments. More on that later

**Figure:** 95% confidence exclusion regions (Abbott et al., 2018)
Projected constraints for LISA (Auclair, Blanco-Pillado, et al., 2019)

Analysis done within the LISA cosmology working group

$$\Omega_{gw}(f \to \infty) \propto \sqrt{\frac{G\mu}{\Gamma}}$$

Figure: A comparison of the LISA sensitivity curve to the predicted SBGW. LISA will probe strings with tensions higher than $G\mu = 10^{-17}$ with little dependence on the cosmic string model.
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The loop distribution: beyond the Nambu-Goto approximation

References
(Hindmarsh, Stuckey, & Bevis, 2009)
(Vachaspati, 2010)
(Blanco-Pillado, Olum, & Shlaer, 2011)
(Mota & Hindmarsh, 2015)
(Matsunami, Pogosian, Saurabh, & Vachaspati, 2019)
(Auclair, Steer, & Vachaspati, 2019)
• So far we have studied Nambu-Goto strings, ie. infinitely thin strings
• Large-scale field-theory simulations find that cosmic strings decay rapidly into particles (Hindmarsh et al., 2009)
• High resolution field theory simulation of single loops tend to show that their lifetime is actually longer that previously expected (Matsunami et al., 2019)
• The rate at which strings emit particles has been measured in high-resolution numerical simulations
• We propose a first step to bridge the gap between Nambu-Goto strings and field-theory strings

Figure: Energy of a loop with the initial size of 390 lattice spacings plotted vs time. (Matsunami et al., 2019)
Energy budget for a cosmic string loop

We parametrize the energy lost by an average loop with $J$, remember that for cosmic string loops, $E = \mu \ell$

$$\frac{D\ell}{Dt} = -\Gamma \mu J(\ell)$$

Where

- $J(\ell) = 1$ if GW emission is the only channel for losing energy
- $J(\ell) = 1 + \frac{\ell_k}{\ell}$ if kinks are present on the loop
- $J(\ell) = 1 + \sqrt{\frac{\ell_c}{\ell}}$ if cusps are present on the string
- $J(\ell) = \Theta(\ell - \ell_V)$ in the case of superconducting strings

$$\ell_k \sim \beta_k \frac{w}{\Gamma G\mu} \propto (G\mu)^{-3/2}, \quad \ell_c \sim \beta_c \frac{w}{(\Gamma G\mu)^2} \propto (G\mu)^{-5/2}, \quad \ell_V = \frac{N}{\sqrt{\mu}}$$
Modeling the loop distribution with a continuity equation (Auclair, Steer, & Vachaspati, 2019)

Non self-intersecting loops are produced from the network of infinite strings and then lose energy

\[
\frac{\partial}{\partial t} \left( a^3 \frac{d^2 N}{d\ell dV} \right) + \frac{\partial}{\partial \ell} \left( a^3 \frac{D\ell}{Dt} \frac{d^2 N}{d\ell dV} \right) = a^3 P(\ell, t)
\]

which, in terms of our length-dependent energy-loss channel becomes

\[
\frac{\partial}{\partial t} \left( a^3 \frac{d^2 N}{d\ell dV} \right) - \Gamma G_\mu \frac{\partial}{\partial \ell} \left( a^3 J(\ell) \frac{d^2 N}{d\ell dV} \right) = a^3 P(\ell, t)
\]

Introducing the new variables

\[
\tau \equiv \Gamma G_\mu t, \quad \xi \equiv \int \frac{d\ell}{J(\ell)}.
\]

the continuity equation becomes

\[
\left( \frac{\partial}{\partial \tau} \bigg|_\xi - \frac{\partial}{\partial \xi} \bigg|_\tau \right) \left( \Gamma G_\mu J a^3 \frac{d^2 N}{d\ell dV} \right) = a^3 J P,
\]
Modeling the loop distribution

Solution for a \(\delta\)-function loop production function

The shape of the loop production function (LPF) has been studied in numerical simulations but it is still a matter of debate. Simplest choice coming from the standard one-scale model is to assume

\[
\mathcal{P}(\ell, t) = Ct^{-5} \delta\left(\frac{\ell}{t} - \alpha\right)
\]

which seems to reproduce well (Blanco-Pillado et al., 2011) and can be used as a Green’s function for more elaborate LPF. The loop formation time \(t^*_\star\) satisfies the following equation

\[
\Gamma G\mu t^*_\star + \xi(\alpha t^*_\star) = \Gamma G\mu t + \xi(\ell),
\]

and the loop distribution is given by

\[
t^4 \frac{d^2 \mathcal{N}}{d\ell dV} = C \frac{1}{\mathcal{J}(\ell)} \frac{\mathcal{J}(\alpha t^*_\star)}{\alpha + \Gamma G\mu \mathcal{J}(\alpha t^*_\star)} \left(\frac{t^*_\star}{t}\right)^{-4} \left(\frac{a(t^*_\star)}{a(t)}\right)^3.
\]

If \(J(\ell) = 1\) then \(\xi(\ell) = \ell\) it reduces to the standard scaling Nambu-Goto loop distribution for a delta-function loop production function

\[
t^4 \frac{d^2 \mathcal{N}}{d\ell dV} = C \frac{(\alpha + \Gamma G\mu)^{3-3\nu}}{(\gamma + \Gamma G\mu)^{4-3\nu}}
\]
Consequences on the number of loops
Modeling the loop number density with both GW and particle emission

(a) Influence of kinks, $G\mu = 10^{-17}$

(b) Influence of cusps, $G\mu = 10^{-17}$

Figure: From bottom to top, the curves show snapshots of the loop distribution at redshifts $z = 10^{13}, 10^{11}, 10^9, 10^7, 10^5$, and the black curve is the scaling NG loop distribution
Impact on the SBGW

Breaking of the high frequency plateau

A consequence of the introduction of $\ell_k, \ell_c$ is that the high frequency plateau is cutoff at

$$f = \sqrt{\frac{2H_0\sqrt{\Omega_{\text{rad}}c}}{\ell_{c,k}\Gamma G\mu}}$$
Particle emission bounds

Injected energy by cosmic strings (Mota & Hindmarsh, 2015; Vachaspati, 2010)

- The emitted particles are heavy and in the dark particle physics sector corresponding to the fields that make up the string
- We assume that there is some interaction of the dark sector with the standard model sector

The energy density injected by cosmic strings per unit of time

\[
\Phi_H(t) = \int_0^{\alpha t} P_{c,k} \frac{d^2 N}{d\ell' dV} d\ell'
\]

in which

\[
P_k = \Gamma G \mu \frac{\ell_k}{\ell} \quad P_c = \Gamma G \mu \sqrt{\frac{\ell_c}{\ell}}
\]

Then the emitted particle radiation will eventually decay, and a significant fraction of the energy \( f_{\text{eff}} \sim 1 \) will cascade down into \( \gamma \)-rays.

\[
\omega_{\text{DGRB}} = f_{\text{eff}} \int_{t_c}^{t_0} \frac{\Phi_H(t)}{(1 + z)^4} dt
\]
Contribution of cosmic strings to the Diffuse Gamma-Ray Background

Constraints from Fermi-LAT

\[ \omega_{\text{DGRB}}^{\text{obs}} \leq 5.8 \times 10^{-7} \text{ eV.cm}^{-3} \]

(a) Contribution from kinks, for small \( G\mu \), \( \omega_{\text{DGRB}} \propto \mu^{9/8} \) and \( \mu^{-2} \log \mu \) for large \( G\mu \)

(b) Contribution from cusps, for small \( G\mu \), \( \omega_{\text{DGRB}} \propto \mu^{13/12} \) and \( \mu^{-5/3} \) for large \( G\mu \)
Conclusion

Summary

- Cosmic strings are a general prediction of most symmetry-breaking models.
- **Scaling** means that the network of cosmic strings survives for a very long time.
- Gravitational wave astronomy is one of the most promising techniques to probe for cosmic strings, especially with the space-based detector LISA which will be able to probe cosmic strings with tension $G\mu \geq 10^{-17}$.
- We have tried to go beyond the Nambu-Goto approximation by taking into account the emission of particles which seems to dominate in Field-Theory simulations on small scales.
- Our analysis show that this phenomenon has little effect on the Stochastic Background.
- We have also checked that this emission of particles does not violate bounds for the diffuse Gamma-Ray Background.
Conclusion

Future developments

- It is important to evaluate more carefully the prevalence of kinks versus cusps on cosmological string loops.
- It would also be interesting to study other loop production functions, particularly power-law LPF which predict a larger number of small loops; hence one might expect a larger gamma ray background from strings.
- We are also applying these tools to the study of vortons together with Danièle Steer, Patrick Peter and Christophe Ringeval.
Thank you
References I


References II


References III

