# Primordial black holes in an early matter era and stochastic inflation

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This talk is based on [1912.01638], [2001.08220] and [2006.14597]

# Introduction

- Primordial Black Holes are relevant dark matter candidates. They are interesting because they do not require physics beyond inflation.

- A large window of masses remains viable[1]

$$10^{-16} M_\odot \lesssim M_{\rm PBH} \lesssim 10^{-11} M_\odot \, .$$

- Their astrophysical signatures (gravitational waves, lensing, etc.) could be probed within the next decade. [2]

- We wish to determine the effects on the PBH abundance of
  1. The equation of state of the Universe at the time of their formation.
  2. The stochastic inflation formalism.

- We explore these aspects in the context of three different inflationary models.

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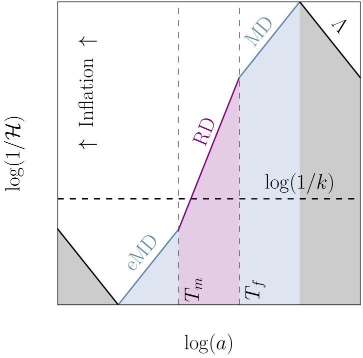
[1] B. Carr, et al. [0912.5297], A. Arbey et al. [1906.04750], H. Niikura et al. [1701.02151], A. Katz et al. [1807.11495]

[2] M. Sasaki et al. [1801.05235]

# Primordial Black Holes

## Primordial Black Holes are...

Black holes formed in the early universe by mechanisms different to the usual stellar collapse. There are many possibilities, from the collision of vacuum bubbles and collapse of topological defects to inflation.




For PBHs to form, we need large density fluctuations = = . These are produced during inflation. We assume transitions are instantaneous.

Fluctuations leave the horizon and induce collapse upon re-entry.

# Collapse in the Radiation Era

The mass of the PBHs that form is proportional to the total energy in a Hubble patch, and thus depends on the scale of the fluctuation, [3]

$$M_{\rm PBH} = \frac{4}{3} \quad H^{\;\;3};$$

with . 1 because of causality. After a short calculation,

$$M_{\rm PBH} = 10^{18}\text{g} \quad \frac{}{0{:}2} \quad \frac{g(T_f)}{106{:}75}^{1=6} \quad \frac{k}{7 \quad 10^{13}\text{Mpc}^{\;\;1}}^{2} :$$

The fraction of DM in the form of PBHs is ($M_{\rm PBH} = M_{\rm PBH}(k)$)

$$f_{\rm PBH} = \frac{(M_{\rm PBH})}{8 \quad 10^{\;\;16}} \quad \frac{}{0{:}2}^{3=2} \quad \frac{g(T_f)}{106{:}75}^{1=4} \quad \frac{M_{\rm PBH}}{10^{18}\text{g}}^{1=2} ;$$

with $(M_{\rm PBH}) \quad \exp[\quad {}^2_c{=}2P_R(k)]$ for Gaussian fluctuations.

[3] B. Carr [10.1086/153853]

The power spectrum $P_R(k)$ tells us how these fluctuations are distributed, can be computed from inflation, and is what CMB experiments measure.

## How do we get PBHs from inflation?

Fluctuations at CMB scales do not produce enough PBHs to explain all DM. We need to enhance the power spectrum at small scales.

Roughly speaking $P\,(k) \quad P_R(k) \quad H^4 = \_^2$,

# The Simplest Model

Consider a scalar eld coupled to gravity in the Jordan frame[4]

$$S = \int d^4x \sqrt{\quad g} \quad \frac{1}{2}(M_p^2 + \quad ^2)R + \frac{1}{2}g \quad \partial \quad \partial \quad \quad V(\quad) \quad :$$

We can get rid of the coupling to $R$ by rede ning the elds,

$$^2 \quad 1 + \quad ^2 = M_p^2 ;$$

$$g \quad \to \quad ^2[\quad] g \quad ;$$

$$^2 \frac{dh}{d} = \quad \quad ^2 + \frac{3}{2} M_p^2 \quad \frac{d \quad ^2}{d} \quad ^2 \Big]^{1=2} ;$$

where $h$ is such that the kinetic term is canonically normalized. The resulting potential is (denominator helps t CMB)

$$U(h) \quad \frac{V}{\quad 4} = \frac{a_2 \quad ^2 + a_3 \quad ^3 + a_4 \quad ^4}{(1 + \quad ^2 = M_p^2)^2} \quad = \quad (h)$$

[4] G. Ballesteros and M. Taoso [1709.05565], G. Ballesteros et al. [2001.08220]

By adjusting the tilt of $P_R$ at CMB scales we run into problems with evaporation bounds, $n_s^{poly} \simeq 0.949$ but $n_s^{CDM} = 0.9649 \quad 0.0042$.

1. Extend CDM, since $n_s^{CDM + N_{eff} + dn_s/d\log(k)} = 0.950 \quad 0.011$ [5]

2. Add higher-dimensional operators $_n \; ^n = \; ^{n \quad 4}$ (expected anyway)

[5] Y. Akrami et al. [1807.06211]

# Subtleties

## Do non-Gaussianities matter?

In this case, no. There are no constraints on the potential parameters, so a large change in the abundance

$$(M_{PBH}) \quad \exp[\quad {}^{2}_{c} = 2 P_R(k)]$$

can be compensated with a small change in the power spectrum[7] (beware of stochastic e ects, however[8]). The $n_s$ problem occurs at large eld values, whereas the peak in the spectrum is at low eld values.

## The gauge problem for gravitational waves

Induced GWs are gauge-dependent[9]. There are two relevant observables, $_{GW}$ and , neither of which is properly understood beyond leading order.

[7] V. De Luca et al. [1904.00970], M. Taoso and A. Urbano [2102.03610]
[8] J. M. Ezquiaga et al. [1912.05399]
[9] V. De Luca et al. [1911.09689], Y. Lu et al. [2006.03450]

# The Stochastic Formalism

## What is Stochastic In ation?

In stochastic in ation, quantum uctuations backreact on the classical trajectory of the in aton, modifying its background evolution[10],

$$\frac{d}{dN} = \frac{@V}{3H^2} + \frac{H}{2} \quad ! \quad P_R \quad 1 \quad \text{(slow roll)}$$

The eld is split into a coarse-grained part and a perturbation,

$$(t;x) = (t) + \underbrace{\int \frac{d^3k}{(2\ )^{3=2}} W[k \quad k\ (t)] \Big[ a_k \quad _k(t) e^{ik\ x} + hc: \Big]}_{^{\wedge}_Q(t;x)} ;$$

where $k \ = \ aH$ is a cuto that separates classical, superhorizon modes from quantum, subhorizon modes. Fields satisfy the Langevin equations,

$$\frac{d}{dt} = \frac{}{a^3} + \quad ; \qquad \text{and} \qquad \frac{d}{dt} = a^3 \frac{dV}{d} + \quad :$$

[10] G. Ballesteros, and M. Taoso [1709.05565], A. Starobinsky [10.1007/3-540-16452-9-6], M. Biagetti et al. [1804.07124], J. M. Ezquiaga and J. García-Bellido [1805.06731]

# Analytical Model

The enhancement of the power spectrum due to a near-in ection point in the potential can be understood by considering a three-region model,

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H _

The potential, trajectory, etc. can all be reconstructed.

This behaviour is generic for all models of this type.

The evolution of the perturbations can then be obtained by solving the Mukhanov-Sasaki equation in each region and matching the solutions.

The noise and eld perturbations are classical stochastic variables described by their statistical moments. The power spectrum is

$$P_R = \frac{1}{2\ \mathrm{cl}}\ D \quad + 2h\ \ \mathrm{st}\ \ \mathrm{st}\, i \quad 2(\ \mathrm{cl}\ \ \ \mathrm{cl})h\ \ {}^{2}_{\mathrm{st}}\, i\ :$$

The biggest advantage of this approach is that we can nd explicit analytical expressions for the noise matrix $_{ab}\quad h\ \ _a\ \ _b i$

| ! 0 | region I | region II | region III |
|---|---|---|---|
| (N) | $\frac{H^2}{4\ ^2}$ | $\frac{H^2}{4\ ^2} e^{\ 2\ \ \mathrm{II}(N\ \ N_{in})}$ | $\frac{H^2}{4\ ^2} e^{\ 2\ \ \mathrm{III}(N\ \ N_{end})\ \ 2\ \ \mathrm{II}\ \ N}$ |
| (N) | 0 | $\frac{H^2}{4\ ^2}\ \ \mathrm{II}\ e^{\ 2\ \ \mathrm{II}(N\ \ N_{in})}$ | $\frac{H^2}{4\ ^2}\ \ \mathrm{III}\ e^{\ 2\ \ \mathrm{III}(N\ \ N_{end})\ \ 2\ \ \mathrm{II}\ \ N}$ |
| (N) | 0 | $\frac{H^2}{4\ ^2}\ \ {}^{2}_{\mathrm{II}}\, e^{\ 2\ \ \mathrm{II}(N\ \ N_{in})}$ | $\frac{H^2}{4\ ^2}\ \ {}^{2}_{\mathrm{III}}\, e^{\ 2\ \ \mathrm{III}(N\ \ N_{end})\ \ 2\ \ \mathrm{II}\ \ N}$ |

The above can be used to show $2h\ \ \mathrm{st}\ \ \mathrm{st}\, i \quad 2(\ \mathrm{cl}\ \ \ \mathrm{cl})h\ \ {}^{2}_{\mathrm{st}}\, i = 0$.

We have shown that the power spectrum coincides, at the linear level, wit the perturbative result in the ! 0 limit. Numerical results below.

$$P_R = \frac{1}{2\ _{cl}} D + \underbrace{2\langle\ _{st}\ \ _{st}\rangle - 2(\ _{cl}\ \ _{cl})\langle\ ^{2}_{st}\rangle}_{0} :$$

# More subtleties

## This result is valid at leading order

We have identi ed the coarse-grained  eld with the background  eld

$$+ \ \hat{\ }_{Q} \ ' \quad {}_{bg} + \quad {}_{st} :$$

A proper numerical computation involves re-calculating the coarse-grained  eld at each time step[11].

## Stochastic in ation a ects non-Gaussianities

We focused on the power spectrum, but in stochastic in ation it is possible to compute the full PDF. Analyses in perfect USR suggest an exponential tail[12]

1. Can the change in abundance still be reabsorbed in the spectrum?
2. What happens outside of perfect USR? Is there a tail? Is it model-dependent?

[11] D. Figueroa et al. [2012.06551]
[12] J. M. Ezquiaga et al. [1912.05399], C. Pattison et al. [2101.05741], M. Biagetti et al. [2105.07810]

# Collapse in Matter Domination

If collapse occurs during an early matter-dominated era, the abundance is

$$f_{RD} / \quad _{RD} M_{RD}^{1=2} \qquad f_{MD} / \quad _{MD} T_m$$

The function represents the fraction of energy density that collapses. This function has very di erent forms in MD and RD[13],

$$_{RD}(k) / \ \frac{1}{\sqrt{P}} \int_{_c}^{1} \text{exp} \left[ \frac{2}{2P} \right] d \ ;$$

$$_{MD}(k) / I \ ^6 P \ \text{exp} \left[ \frac{I^4}{P}^{1=3} \right] :$$

The latter takes into account the non-sphericity and angular momentum (related to I ) of the collapsing cloud.

## Intuitively...

Collapse is easier during an eMD era because of the lack of radiation pressu (roughly speaking).

[13] T. Harada et al. [1609.01588], T. Harada et al. [1707.03595]

Collapse during matter-domination has two big advantages,

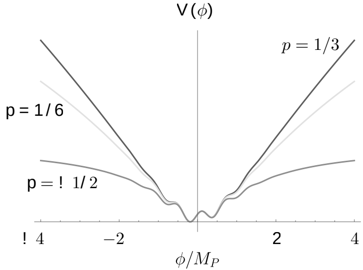
1. The power spectrum required to get a significant PBH abundance is much smaller than in RD ($P_{RD} \sim 10^{-2}$ vs $P_{MD} \sim 10^{-4}$).
2. The abundance is much less sensitive to small changes in $P_R$, since is different.

# Numerical Examples

Consider the following axion monodromy-inspired potential,

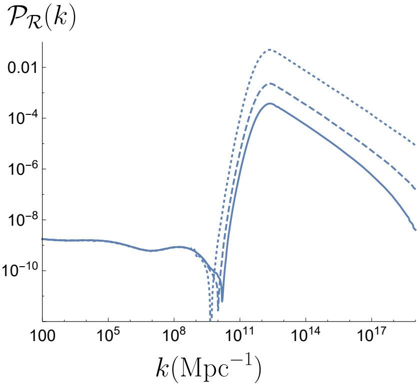
$$V(\ ) = m^2 f^2 \left( \frac{1}{2p}\frac{F^2}{f^2}\left[ 1 + \left(1 + \frac{{}^2}{F^2}\right)^p \right] + \ e^{\left(\frac{}{}\right)^p} \cos\left(\frac{}{f} + \ \right) \right) + V_0 ;$$

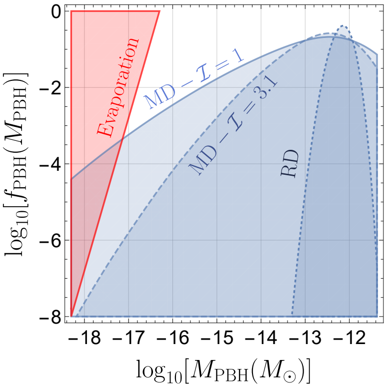
This potential features oscillations and is quadratic near the minimum, ensuring a long epoch of matter domination if reheating is perturbative.




The motivation is to reduce tuning by having several minima.

The mechanism is illustrated in these examples. All parameters are identical except for (controls the depth of minima in the potential), and *I* (from the function).




Thus, the main advantage of MD is also its biggest drawback: lack of suppression makes it more di□cult to evade evaporation bounds.

# Even more subtleties

## There are many uncertainties

1. The efficiency factor .
2. The angular momentum of the collapsing cloud $l$.
3. The effect of non-Gaussianities.
4. Inhomogeneities.
5. ...

These are the best formulas available at the moment (for MD), but there is still a long way to go. The focus is usually on RD.

## No numerical simulations yet

The first step towards determining how good these estimates really are is to perform numerical simulations, which only exist in RD at the moment.[14]

[14] See, however, E. de Jong, et al. [2109.04896]

# Conclusions

- The simplest potential that can produce PBHs is viable, provided CDM is extended, or higher-dimensional operators are considered.
- If dark matter is in the form of PBHs, the corresponding GW signal should be observable by LISA and DECIGO if they form during RD.
- We have shown, both analytically and numerically that, at leading order, stochastic inflation does not affect the power spectrum, even in the presence of a USR phase.
- PBH formation in an early matter-dominated era has significant advantages, namely, that a smaller enhancement of the power spectrum is required, and the potential parameters are less tuned.
- However, the fact that the PBH abundance function is less suppressed in this scenario makes it more difficult to evade evaporation bounds.

# Paths for future work

- We are currently trying to determine how the GW signal is affected if PBHs form in an eMD era.
- The gauge problem of induced gravitational waves remains open. Solving it involves understanding two different observables ( $_{GW}$ and ) beyond leading order.
- The effect of the stochastic formalism on the probability distribution and differences between this and the classical calculations are still to be studied.
- There are many sources of uncertainty in the MD formulas. A more thorough analytical description is necessary, together with numerical simulations.